$$\begin{split} i\hbar\frac{\partial\psi}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi\\ i\hbar\frac{\partial\psi}{\partial t} &= -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi + V\psi\\ x &= L_xx' = Lx' \qquad y = L_yy' = \alpha L_xy' = \alpha Ly'\\ i\hbar\frac{\partial\psi}{\partial t} &= -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{L^2\partial x'^2} + \frac{\partial^2}{\alpha^2L^2\partial y'^2}\right)\psi + V\psi\\ t &= \frac{mL^2}{\hbar}\tau\\ i\hbar\frac{\partial\psi}{\partial\tau} &= -\frac{\hbar^2}{2mL^2}\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\alpha^2\partial y'^2}\right)\psi + V\psi\\ i\hbar\frac{\partial\psi}{\partial\tau} &= \left[-\frac{\hbar^2}{2mL^2}\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\alpha^2\partial y'^2}\right)\psi + V\psi\right]\frac{mL^2}{\hbar}\\ i\hbar\frac{\partial\psi}{\partial\tau} &= -\frac{\hbar}{2}\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\alpha^2\partial y'^2}\right)\psi + \frac{mL^2}{\hbar}V\psi\\ i\frac{\partial\psi}{\partial\tau} &= -\frac{1}{2}\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\alpha^2\partial y'^2}\right)\psi + \frac{mL^2}{\hbar^2}V\psi\\ V' &= \frac{mL^2}{\hbar^2}V\\ i\frac{\partial\psi}{\partial\tau} &= -\frac{1}{2}\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\alpha^2\partial y'^2}\right)\psi + V'\psi\\ \frac{\partial\psi}{\partial\tau} &= -\frac{1}{2}i\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\alpha^2\partial y'^2}\right)\psi + \frac{1}{i}V'\psi\\ \frac{\partial\psi}{\partial\tau} &= \frac{i}{2}\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\alpha^2\partial y'^2}\right)\psi - iV'\psi \end{split}$$