1.

$$a^{[1]} = x \in \mathbb{R}^{n_1}$$

$$a^{[l]} = \sigma(W^{[l]} a^{[l-1]} + b^{[l]}) \in \mathbb{R}^{n_l} \quad \text{for } l = 2, 3, \dots, L$$

find $\nabla a^{[L]}(x)$, where $n_L = 1$

Let $z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]} \Leftrightarrow a^{[l]} = \sigma(z^{[l]})$

$$\Rightarrow \nabla a^{[L]}(x) = \left(\frac{\partial a^{[L]}}{\partial x}\right)^T = \left(\frac{\partial a^{[L]}}{\partial z^{[L]}} \cdot \frac{\partial z^{[L]}}{\partial x}\right)^T$$

$$= \underline{\text{diag}(\sigma'(z^{[L]}))\, \partial(W^{[L]} a^{[L-1]} + b^{[L]})}$$

$$= \sigma'(z^{[L]}) \cdot W^{[L]} \cdot \frac{\partial a^{[L-1]}}{\partial x}$$

$$= \sigma'(z^{[L]}) W^{[L]} \nabla(\sigma'(z^{[L-1]}) W^{[L-1]} \cdots \nabla(\sigma(z^{[2]})) W^{[2]}$$

$W^{[2]}$

2.