Calculus - Chapter 1 Solutions

1. a. Distance = speed · time; The graph can be broken into sections in which the average speeds are approximated and multiplied by the length of time for each section. Smaller sections should give better accuracy and be closer to the exact answer, which is the area under the curve. Answer ≈ 230 miles

   b. The truck's speed will be the absolute value of the slope of its distance function. When the graph is flat, the truck is stopped. When the slope is negative, the truck is driving towards home, and when it is positive, the truck is driving away from home.

   c. The 27 and 53 represent areas under the speed curve, and the 50 represents the slope of the distance function.

2. a. $V = \pi r^2 h = \pi \left(\frac{8}{2}\right)^2 \cdot 6 = \pi \cdot 4^2 \cdot 6 = 96\pi \text{ cm}^3$

   b. $V = \frac{1}{3} Bh = \frac{1}{3}(10 \cdot 10) \cdot 15 = \frac{1}{3} \cdot 1500 = 500 \text{ in.}^3$

   c. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{4.5}{2}\right)^3 = \frac{243}{16}\pi = 15.1875 \text{ m}^3$

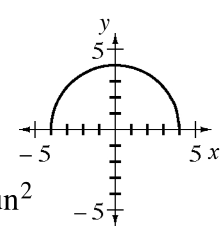
3. a. It is a cylinder.

   b. $V = \pi r^2 h = \pi \cdot 6^2 \cdot 8 = 288\pi \text{ un}^3$

4. a. $A = \frac{bh}{2} = \frac{5 \cdot 5}{2} = \frac{25}{2} = 12.5 \text{ un}^2$

   b. This is the difference between the areas of the two triangles:
   $A(f, 0 \le x \le 7) = \frac{25}{2} - \frac{2 \cdot 2}{2} = \frac{21}{2} = 10.5 \text{ un}^2$

   c. The area of the second triangle must be the area of the first to make their difference, which is $A(f, 0 \le x \le k)$, equal 0. At $k = 10$ the two triangles are congruent and have equal area.



5. See graph at right. This is a semicircle with a radius of 4.

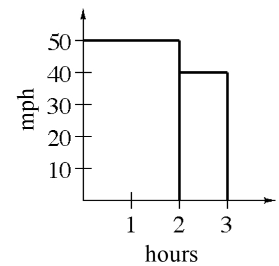
   a. domain: $[-4, 4]$ range: $[0, 4]$

   b. This is a quarter of a circle: $A(g, 0 \le x \le 4) = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \cdot 4^2 = 4\pi \text{ un}^2$

   c. This is double the answer from (b): $A(g, 0 \le x \le 8) = 2(4\pi) = 8\pi \text{ un}^2$

   d. (c) is twice as much as (b).

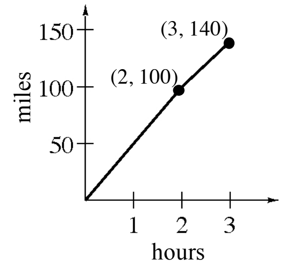
6. a. See graph at far right above.



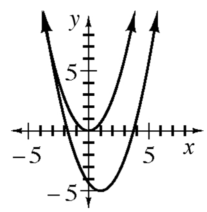
b.

Time	0.5	1	1.5	2	2.5	3
Distance	25	50	75	100	120	140

c. See graph at far right below.



7. a. See graph at right above.
$y = \frac{2}{3}(x-1)^2 - 5$

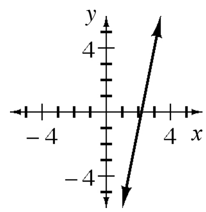


b. $y = \frac{2}{3}(x-1) - 5$

c. Both have a slope of $-\frac{1}{2}$, but one is shifted to the left 2 and up 3.

d. Substitution of $x = -1$ gives $y = -6$.

e. See graph at right below.
Line through (2, –1) with $m = 5$.



8. $y = -3(x+5) - 2$

9. a. point-slope: $y = -\frac{2}{5}\left(x+6\right) + 2$

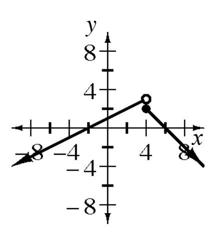
b. slope-intercept: $y = 3x - 6$

c. point-slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-8}{1-2} = \frac{-5}{-1} = 5$, $y = 5(x-1) + 3$ or $y = 5(x-2) + 8$

10. a. $f(-x)$ is the same as $f(x)$ because f is symmetric about the y-axis.

b. $f(-x)$ is the opposite of $f(x)$: f has point symmetry at the origin.

11. See graph at right.



a. The graph jumps at this point or is discontinuous.

b. $f(0) = \frac{1}{2}(0) + 1 = 1$, $f(4) = -4 + 6 = 2$, $f(6) = -6 + 6 = 0$

12. a. Continuous. Intuitively, the graph can be drawn without lifting a pencil.

b. The point (3, 1) is still defined.

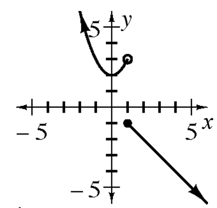
13. The first example gives a large interval with a hole in it. The second example states the same domain as two separate intervals.

14. For $f(x)$, the domain is limited to $x \geq 25$ because of the square root. For $g(x)$, the domain is limited to $x \neq 25$ because of the denominator. For $h(x)$, the domain is limited to $x > 25$ since we can only take logs of positive quantities.

15. A. 2 B. 3 C. 1

16. a. $h(x) = \begin{cases} -(x-2) & \text{for } x \leq 2 \\ x-2 & \text{for } x > 2 \end{cases}$

b. $h(x) = |x+5|$



17. a. See graph at right.

b. The first part must meet the second at $x = 1$.

Two possible answers: $f(x) = \begin{cases} x^2 - 2, & x < 1 \\ -x, & x \geq 1 \end{cases}$ and $f(x) = \begin{cases} x^2 + 1, & x < 1 \\ -x + 4, & x \geq 1 \end{cases}$

18. $\sqrt{1+3} = a(1-1)^2 + b = b,$ so $b = 2$

$a(3-1)^2 + b = -3 + 2 = -1,$

$a \cdot 4 + 2 = -1,$

$4a = -3$

$a = -\frac{3}{4}$

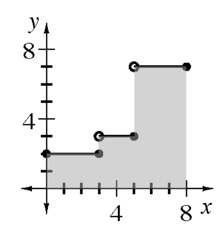
19. a. Possible answer: (3, 0), assuming f is an absolute value function.

b. No; Finite differences are not consistent with a quadratic function.

c. Yes; $f(x) = |x-3|$

d. Yes; Intermediate values are not given.

e. No; It is given that $f(x)$ is a continuous function.



20. a. See graph at right. No. The y-values do not agree at $x = 3$ and $x = 5$.

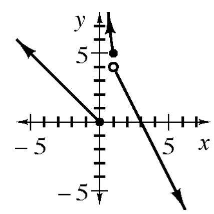
b. $A(g, 0 \leq x \leq 8) = 2 \cdot 3 + 3 \cdot 2 + 7 \cdot 3 = 6 + 6 + 21 = 33 \text{ un}^2$

c. Pieces of the graph resemble "steps."

21. a. $f(0) = 4 - 3(0) = 4$
$f(1) = 4 - 3(1) = 1$
$f(3) = 3^2 = 9$

b. $f(1) = \sqrt{1} = 1$
$f(3) = 3 - 3 = 0$
$f(9.4) = 3 - 9.4 = -6.4$

c. $f(-3) = -(-3) = 3$
$f(0) = -0 = 0$
$f(0.5) = \frac{5}{0.5} = 10$
$f(4) = 6 - 2(4) = -2$

d. 

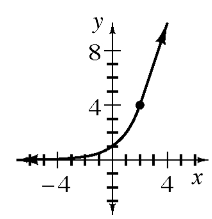
22. a. 78¢, 95¢, 95¢

b. $$y = \begin{cases} 0.44 & \text{for} \quad 0 \le x \le 1 \\ 0.61 & \text{for} \quad 1 < x \le 2 \\ 0.78 & \text{for} \quad 2 < x \le 3 \\ 0.95 & \text{for} \quad 3 < x \le 4 \\ 1.12 & \text{for} \quad 4 < x \le 5 \end{cases}$$

23. a, It is a sphere with radius 3.

b. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{6}{2}\right)^3 = \frac{4}{3}\pi \cdot 27 = 36\pi \text{ un}^2$

24. See graph at right.



a. $D: (-\infty, \infty),\ R: (0, \infty)$

b. Yes, $2^2 = 3(2) - 2$.

c. Yes.

25. a. $A = \frac{0+3}{2} + \frac{3+4}{2} + \frac{4+3}{2} + \frac{3+0}{2} = 1.5 + 3.5 + 3.5 + 1.5 = 10 \text{ un}^2$

b. This is less than the true area.

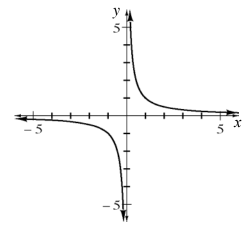
26. a. $\frac{-\sqrt{3}}{2}$ (30º-60º-90º triangle)

b. $\frac{\sqrt{3}}{3}$ (30º-60º-90º triangle)

c. $-\sqrt{2}$ (45º-45º-90º triangle)

d. Undefined (division by 0)

27. See graph at right.



a. As x approaches zero, the function shoots off to negative infinity from the left and positive infinity from the right, because at $x = 0$ we are dividing by zero. $x = 0$ is the asymptote.

b. Division by zero can cause an asymptote. $y = 0$

c. $y = \frac{1}{x-1} + 3$

28. a.

$$\begin{array}{r}
x^2 - x + \frac{4}{x+3} \\
x+3\overline{)\,x^3 + 2x^2 - 3x + 4} \\
\underline{-(x^3 + 3x^2)} \quad\quad\quad \\
-\ x^2 - 3x \quad\ \\
\underline{-(-x^2 - 3x)} \quad\ \\
4
\end{array}$$

b.

$$\begin{array}{r}
x^3 + 2x^2 - x + 1 - \frac{1}{x-2} \\
x-2\overline{)\,x^4 + 0x^3 - 5x^2 + 3x - 3} \\
\underline{-(x^4 - 2x^3)} \quad\quad\quad\quad\quad \\
+2x^3 - 5x^2 \quad\quad\quad \\
\underline{-(2x^3 - 4x^2)} \quad\quad\quad \\
-x^2 + 3x \quad\ \ \\
\underline{-(-x^2 + 2x)} \quad\ \ \\
x - 3 \\
\underline{-(x-2)} \\
-1
\end{array}$$

29. a. The graphs look like $y = x - 2$.

b. If you ignore holes and vertical asymptotes, the graphs would look identical.

c. They all simplify to $x - 2$ but with different remainders.

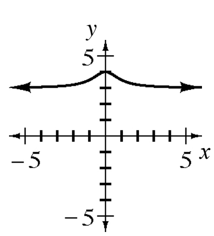
$$\begin{array}{r}
x - 2 \\
x-1\overline{)\,x^2 - 3x + 2} \\
\underline{-(x^2 - 1x)} \quad\ \\
-2x + 2 \\
\underline{-(-2x + 2)} \\
0
\end{array}
\qquad
\begin{array}{r}
x - 2 + \frac{1}{x-1} \\
x-1\overline{)\,x^2 - 3x + 3} \\
\underline{-(x^2 - 1x)} \quad\ \\
-2x + 3 \\
\underline{-(-2x + 2)} \\
1
\end{array}
\qquad
\begin{array}{r}
x - 2\ \frac{5}{x^2+1} \\
x^2+1\overline{)\,x^3 - 2x^2 + x + 3} \\
\underline{-(x^3 \quad\quad\ \ +x)} \quad\ \\
-2x^2 \quad\ + 3 \\
\underline{-(-2x^2 \quad - 2)} \\
5
\end{array}$$

30. a. $y = -x + 2$

b. They are the same except for the fraction.

c. As x gets larger (both in the positive and negative directions), the graph approaches the "slanted" line.

31. See graph at right. $y = 3$



32. We only need to look at large positive and negative values of x.

a. $y = \frac{1}{2}x$

b. $y = 7$

33. a.

$$\begin{array}{r}
2x - 1 + \frac{4}{3x+1} \\
3x+1 \overline{\big)\, 6x^2 - x + 3} \\
\underline{-(6x^2 + 2x)} \\
-3x + 3 \\
\underline{-(-3x - 1)} \\
4
\end{array}$$

b. $y = 2x - 1$

c. The end behavior function is the non-fractional terms of $f(x)$.

34. *i.*

$$\begin{array}{r}
-5x - 10 - \frac{17}{x-2} \\
x-2 \overline{\big)\, -5x^2 + 0x + 3} \\
\underline{-(-5x^2 + 10x)} \\
-10x + 3 \\
\underline{-(-10x + 20)} \\
-17
\end{array}$$

Therefore the end behavior is $y = -5x - 10$.

ii. Since the degree of the numerator and denominator are the same, we only need to look at the first terms to determine the end behavior. $y = \frac{2x}{3x} = \frac{2}{3}$

iii. $y = \frac{x^2 - 4x + 4}{x - 2}$

$y = \frac{(x-2)(x-2)}{(x-2)}$

$y = x - 2$

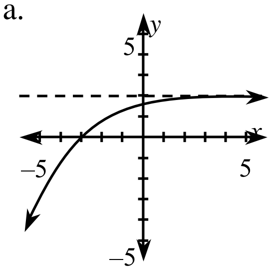
iv. $y = \frac{2x^3 + 2x}{x} = 2x^2 + 2$

a. (*i*) and (*iii*) have slant asymptotes. (*ii*) has a horizontal asymptote. (*iv*) has neither.

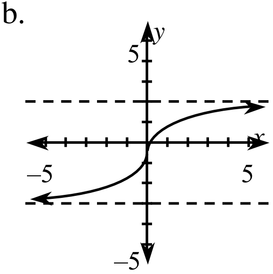
b. Answers vary, but the polynomials in the numerator and denominator must have the same degree. Answers should be of the form: $y = \frac{5x^n + \cdots}{4x^n + \cdots}$.

35. Answers will vary. See graphs below for possible solutions. Part (c) is not possible because a function can have at most two horizontal asymptotes.

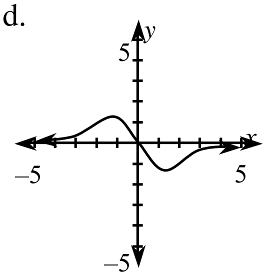
a.



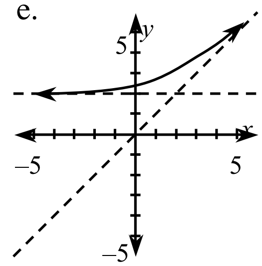
b.



d.



e.



36. a. $A\left(y,-3\le x\le 3\right)\approx\left(\frac{2}{5}(-3)^2+1\right)+\left(\frac{2}{5}(-2)^2+1\right)+\left(\frac{2}{5}(-1)^2+1\right)$

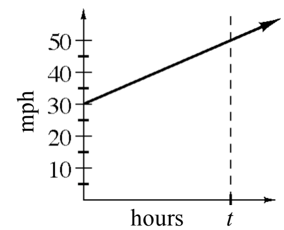
$$+\left(\tfrac{2}{5}\cdot 0^2+1\right)+\left(\tfrac{2}{5}\cdot 1^2+1\right)+\left(\tfrac{2}{5}\cdot 2^2+1\right)=13.6\ \text{un}^2$$

b. The estimate is too high because if we look at the rectangles opposite each other (e.g. from –3 to –2 vs. from 2 to 3), the overestimating rectangle overcompensates for the underestimating rectangle.

c. $A\left(y,-3\le x\le 3\right)\approx\left(\frac{2}{5}(-2)^2+1\right)+\left(\frac{2}{5}(-1)^2+1\right)+\left(\frac{2}{5}\cdot 0^2+1\right)$

$$+\left(\tfrac{2}{5}\cdot 1^2+1\right)+\left(\tfrac{2}{5}\cdot 2^2+1\right)+\left(\tfrac{2}{5}\cdot 3^2+1\right)=13.6\ \text{un}^2$$

d. It was true in this instance because of its symmetry about the middle (about $x = 0$), but would not be true in general. E.g., it does not work for $A(y, 0\le x\le 3)$ for the given y.



37. a. See graph at right.

b. Distance traveled.

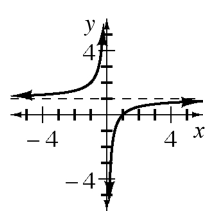
c. The units for the base are miles.
The units for the height are $\frac{\text{miles}}{\text{hour}}$. Therefore $\text{miles}=\frac{\text{miles}}{\text{hr}}\bullet\text{hr}$.

d. It is a trapezoid with bases of 30 and 70, and a height of 2.
$A=\frac{1}{2}(30+70)(2)=100\ \text{miles}$

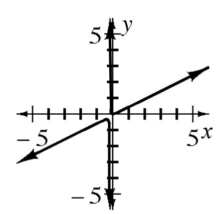
38. a. We cannot divide by zero. Therefore $x^2\ne 0 \Rightarrow x\ne 0$ and $D:\{x : x\ne 0\}$.
Since x^2 is always positive, $\frac{3}{x^2}>0 \Rightarrow \frac{3}{x^2}+1>1$. Therefore $R:\{y : y>1\}$.

b. $f(-x)=\frac{3}{(-x)^2}+1=\frac{3}{x^2}+1$, $f(\sqrt{x})=\frac{3}{(\sqrt{x})^2}+1=\frac{3}{x}+1$ if $x>0$, $f(x+h)=\frac{3}{(x+h)^2}+1$

39. a. $y=1$



b. $y=\frac{x}{2}$



40. a. $x^2+1\ne 0, x^2\ne -1,$ so $D=(-\infty,\infty)$

b. $x\ne 0$ and $x+1\ne 0, x\ne 0, x\ne -1 : D=\left\{x : x\ne 0 \text{ and } x\ne -1\right\}$

c. $x^2-9\ge 0, x^2\ge 9, x\ge 3$ or $x\ge -3 : D=(-\infty,-3]\bigcup[3,\infty)$

d. $x-3>0, x>3,$ and $x+4>0, x>-4,$ so $D=(3,\infty)$

41. a. $\sqrt{k^7} = (k^7)^{1/2} = k^{7/2}$

b. $\sqrt[3]{t^4} = (t^4)^{1/3} = t^{4/3}$

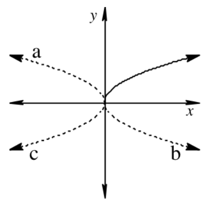
c. $(\sqrt{n})^4 = (n^{1/2})^4 = n^2$

d. $\sqrt[5]{b^{31}} = (b^{31})^{1/5} = b^{31/5}$

42. a. A cone

b. $V = \frac{1}{3}bh = \frac{1}{3}\pi \cdot 5^2 \cdot 4 = \frac{100\pi}{3}$ un^3

43. a-c. See graph at right.
d. a: even, b: neither, c: odd

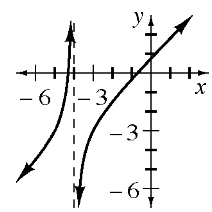


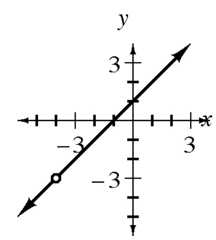
44. a. See graphs at right.

b. f has a V.A. at $x = -4$ and g has a hole at $(-4, -3)$.

c. f: $D:\{x : x \neq 4\}$, $R:\{y : \text{all reals}\}$

g: $D:\{x : x \neq 4\}$, $R:\{y : y \neq 3\}$





45. For any x^* where $q(x^*) = 0$, there will be an asymptote if $p(x^*) \neq 0$. If $p(x^*) = 0$, then for the types of functions shown in the problem, there will be a hole. However the complete answer is that there will be a hole if there are at least as many factors of $(x - x^*)$ in $p(x)$ as there are in $q(x)$, and an asymptote otherwise.

46. a. They are not completely equal because the differ at $x = -3$. Their graphs are identical except at $x = -3$.

b. Two possible answers: If $x \neq 2$ then $\frac{x^2-5x+6}{x-2} = x - 3$.

Or: If $f(x) = \begin{cases} -1, & x = 2 \\ \frac{x^2-5x+6}{x-2} & \text{otherwise} \end{cases}$ then $f(x) = x - 3$.

c. $\frac{(x-2)(x+2)}{x-2} = x + 2$ if $x \neq 2$

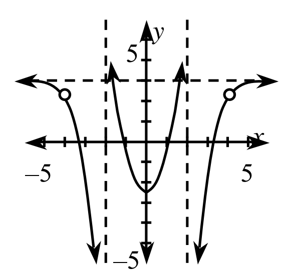
47. a. $y \to -2$

b. $y \to -2$

c. $y \to -\infty$

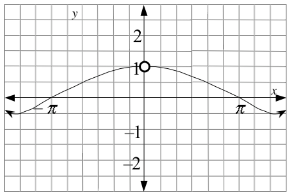
d. $y \to \infty$

e. V.A.: $x = 0$, H.A.: $y = 0$

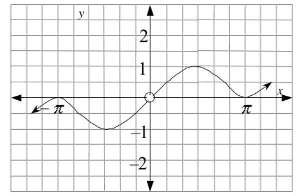


48. See possible graph at right. Answers vary but must have holes at $x = \pm 4$, V.A. at $x = \pm 2$ and H.A. $y = 3$ in both directions.

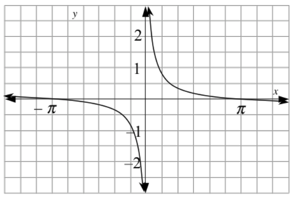
49. a. Not defined: Hole



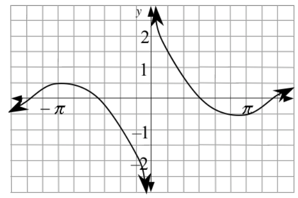
b. Not defined: Hole



c. Not defined: Asymptote

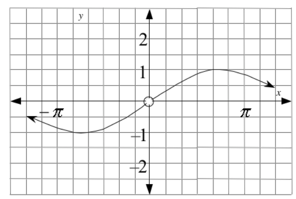


d. Not defined: Asymptote



e. Defined: $\frac{1-\cos 0}{0-1} = \frac{0}{-1} = 0$.

F. Not defined: Hole



50. a. It is a hole because $f(x) = 1$ except at $x = 2$.

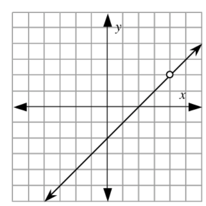
b. It is a hole because $g(x) = x - 2$ except at $x = 2$.

c, It is an asymptote because $h(x) = \frac{1}{x-2}$ and approaches infinity around $x = 2$: as $x \to 2^-, h(x) \to -\infty$ and as $x \to 2^+, h(x) \to \infty$.

d. See graph at right. E.g., $y = \frac{(x-2)(x-4)}{x-4} = \frac{x^2-6x+8}{x-4}$

Answers may differ but must be of the form

$y = \frac{(x-2)r(x)}{r(x)}$ where $r(x) = 0$ only at $x = 4$.



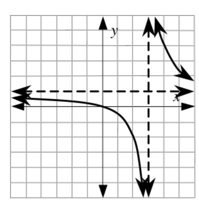
51. a. As $x \to \infty, y \to \infty$.

b. As $x \to -\infty, y \to -\infty$.

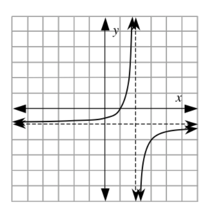
c. As $x \to 1^-, y \to 3$.

d. As $x \to 1^+, y \to 3$.

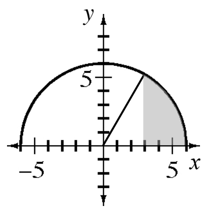
52. Answers can vary: one possible solution: $y = \frac{x}{x-3}$



53. $D = \{x : x \neq 2\}$, $R = \{y : y \neq -1\}$. For example, $y = \frac{1}{x-2} - 1$



54. See graph at right.



a. The triangle is 60º-30º-90º, and the sector is $\frac{1}{6}$ of the circle:

$A = \frac{\pi r^2}{6} - \frac{3 \cdot 3\sqrt{3}}{2} = 6\pi - \frac{9\sqrt{3}}{2} \approx 11.06 \text{ un}^2$

b. This is a quarter circle minus (a):

$A = \frac{\pi r^2}{4} - \left(6\pi - \frac{9\sqrt{3}}{2}\right) = 9\pi - 6\pi + \frac{9\sqrt{3}}{2} = 3\pi + \frac{9\sqrt{3}}{2} \approx 17.22 \text{ un}^2$

c. This is (a) plus twice (b):

$6\pi - \frac{9\sqrt{3}}{2} + 2\left(3\pi + \frac{9\sqrt{3}}{2}\right) = 12\pi + \frac{9\sqrt{3}}{2} \approx 45.49 \text{ un}^2$

55. a. $7t = 26.2,\ t = \frac{26.2}{7} \approx 3.74$ hours

b. ≈ 7 miles per hour; This is her total distance divided by the total time from part (a).

c. 14 miles

d. $7\ \frac{\text{miles}}{\cancel{\text{hour}}} \cdot 2\ \cancel{\text{hours}} = 14 \text{ miles}$

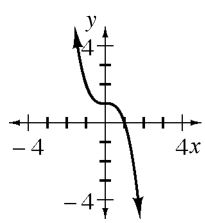
56. a. $(\sqrt{100})^3 = 10^3 = 1000$

b. $27^{4/3} = (\sqrt[3]{27})^4 = 3^4 = 81$

c. $16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$

d. $9^{\frac{4}{2}} = (\sqrt{9})^4 = 3^4 = 81$

57. See graph at right.



a. $y \to -\infty$

b. $y \to \infty$

c. $y \to 1$

58. a. $A(f(x), -3 \le x \le 3 \approx f(-3) + f(-2) + f(-1) + f(0) + f(1) + f(2)$

$= 2(-3)^2 + 1 + 2(-2)^2 + 1 + 2(-1)^2 + 1 + 2(0)^2 + 1 + 2(1)^2 + 1 + 2(2)^2 + 1 = 44 \text{ un}^2$

b. $A(f(x), -3 \le x \le 3) \approx f(-2) + f(-1) + f(0) + f(1) + f(2) + f(3)$

$= 2(-2)^2 + 1 + 2(-1)^2 + 1 + 2(0)^2 + 1 + 2(1)^2 + 1 + 2(2)^2 + 1 + 2(3)^2 + 1 = 44 \text{ un}^2$

c. $A(f(x), -3 \le x \le 3) \approx \frac{f(-3)+f(-2)}{2} + \frac{f(-2)+f(-1)}{2} + \cdots + \frac{f(2)+f(3)}{2}$

$= \frac{28}{2} + \frac{12}{2} + \frac{4}{2} + \frac{4}{2} + \frac{12}{2} + \frac{28}{2} = 44 \text{ un}^2$

The answers were all the same because of the parabola's symmetry, but this does not usually happen.

59. a $\to g(x)$; b $\to h(x)$; c $\to f(x)$

60. a. A sphere.

b. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{243\pi}{2}$

$\frac{\pi d^3}{6} = \frac{243\pi}{2}$

$d^3 = 729$

$d = \sqrt[3]{729} = 9$

c. No

61. a. $D:\{x:-2 \le x \le 4\}$, $R:\{-3 \le y \le -1\}$

b. $D:\{-6 \le x \le 6\}$, $R:\{y:-3 \le y \le 3\}$

c. $f(g(-2)) = f(2) = -1$

d. $g(f(-2)) = g(3) \approx -2.9$

e. $f(f(3)) = f(0) = 1$

f. $f(g(5)) = f(-3)$, which is not defined.

62. a. $h(f(x))$ or $f(h(x))$ b. $g(h(x))$

c. $f(f(x))$ d. $h(g(f(x)))$

63. a. $f: D = (-\infty, \infty), R = (0, \infty)$

$g: 1 - x \ge 0, 1 \ge x, D = (-\infty, 1], R = [0, \infty)$

b. $f(g(x)) = f(\sqrt{1-x}) = 2^{\sqrt{1-x}}$. The domain is the same as for $g: D = (-\infty, 1]$.

c. $g(f(x)) = g(2^x) = \sqrt{1-2^x}$. So $1 - 2^x \ge, 1 \ge 2^x, x \le 0 : D = (-\infty, 0]$.

64. a. $h(j(x)) = x$

b. When x is substituted into the first function and then the value $f(x)$ is substituted into the second function, the result is x. i.e. What the first function does, the second function undoes.

c. To get the inverse, switch the x and the y.

$x = e^y + 2$

$x - 2 = e^y \qquad \Rightarrow g(x) = \ln(x-2)$

$\ln(x-2) = y$

65. a. $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

b. $g^{-1}(7)=5$

66. a.

$$y=2^x$$
$$\log y = x\log 2$$
$$\frac{\log y}{\log 2}=x$$
$$\log_2 y = x$$

b.

$$y=\frac{x+1}{x}$$
$$xy=x+1$$
$$xy-x=1$$
$$x(y-1)=1$$
$$x=\frac{1}{y-1}$$

c. See steps from parts (a) and (b). $f^{-1}(x)=\log_2 x$, $g^{-1}(x)=\frac{1}{x-1}$
The inverse function can be found by solving for x first and then switching the x and y.

67. a. The inverse of $f(x)$ is not a function because $f(-1)=f(1)=8$. Therefore, the inverse would not be a function.

b. *i.* $g^{-1}(2)=1$

ii. $f(g^{-1}(2))=f(1)=8$

iii. $g^{-1}(g(-2))=g^{-1}(-3)=-2$
(Note: The inverse and the original function should cancel each other out.)

c. $j(4)=h^{-1}(4)=3$

68. a. $m=\frac{4-(-2)}{5-(-5)}=\frac{6}{10}=\frac{3}{5}$

$$y=\frac{3}{5}(x-5)+4$$
$$y=\frac{3}{5}x-3+4$$
$$y=\frac{3}{5}x+1$$

b. $y=(x-1)^2-3$

69. a. The function values are 3, 5, 7, 9. They change linearly (add 2 each time).

b. The function values are 10, 7, 4, and 1. They change linearly (subtract 3 each time).

70. a. $f(-1)=f(1)=2,\ f(-2)=f(2)=4,\ f(-3)=f(3)=6$

b. The graph must be even and go through the given values… a possible function is $y=|2x|$.

c. This graph (compared to the previous one) should illustrate that intermediate values are not known.

d. No, the values of $f(x)$ do not increase as a parabolic function increases.

71. a. $x+2\ge 0,\ x\ge -2:\ D:[-2,\infty)$

b. $x\ne 4:\ D:\{x:x\ne 4\}$

c. $x-4>0,\ x>4:\ D:(4,\infty)$

d. $x\ne 0$ and $\frac{2-x}{x}\ge 0:\ 2-x\ge 0,\ 2\ge x:\ D:(0,2]$

72. a. Rectangles:

Left: $A\left(3\sqrt{x+1}, 0\le x\le 6\right)\approx 3\sqrt{0+1}+3\sqrt{1+1}+\ldots+3\sqrt{5+1}\approx 32.495\ \text{un}^2$

Right: $A\left(3\sqrt{x+1}, 0\le x\right)\approx 3\sqrt{1+1}+3\sqrt{2+1}+\ldots+3\sqrt{6+1}\approx 37.43\ \text{un}^2$

Trapezoids:

$A\left(3\sqrt{x+1}, 0\le x\le 6\right)\approx \frac{3\sqrt{0+1}+3\sqrt{1+1}}{2}+\frac{3\sqrt{1+1}+3\sqrt{2+1}}{2}+\ldots+\frac{3\sqrt{5+1}+3\sqrt{6+1}}{2}\approx 34.964\ \text{un}^2$

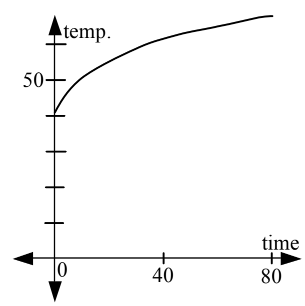
b. Trapezoids and left endpoint rectangles are under estimates. Right endpoint rectangles are overestimates.

c. Trapezoids are more accurate.

73. a. $D=(-\infty,\infty),\ R=[-2,4]$

b. $D=(-\infty,2)\bigcup(2,\infty),\ R=(-\infty,0)\bigcup(0,\infty)$

74. a. See graph at right.



b. At time = 0, because that is when the slope is greatest.

c. We can tell by using the table or looking at the slope: it changes by $\frac{51°-42°}{10\text{ min}-0\text{ min}}=\frac{9°}{10\text{ min}}=\frac{0.9°}{\text{minute}}$.

75. a. Helen is not completely correct (her formula does not work for negative values).

b. $\sqrt{x^2}=|x|$

76. a $\to f(x)$, b $\to h(x)$, c $\to g(x)$

77. $y=\sin x$ is symmetric about the origin.
$y=\cos x$ is symmetric across the y-axis.

78. a. $(-x)^2 = x^2$, even, reflection over y-axis

b. $2(-x)^3 = -2x^3$, odd, rotation about the origin

c. $2+(-x)^4 = 2+x^4$, even, reflection over y-axis

d. $2+(-x)^5 = 2-x^5$, neither

e. $\sin(-2x) = -\sin(2x)$, odd, rotation about the origin

f. $\arctan(-x) = -\arctan(x)$, odd, rotation about the origin

79. Since f is even, $f(-x) = f(x)$. Since g is odd, $g(-x) = -g(x)$.

a. $f(-x)+g(-x) = f(x)-g(x)$, so it is neither even or odd.

b. $f(-x)\cdot g(-x) = f(x)(-g(x)) = -f(x)g(x)$: it is odd.

c. $f(g(-x)) = f(-g(s)) = f(g(x))$: it is even.

d. $g(f(-x)) = g(f(x))$: it is even.

e. $|f(-x)| = |f(x)|$: it is even.

f. $|g(-x)| = |-g(x)| = |g(x)|$: it is even.

80. a.

x	$f(x)$	$g(x)$	$h(x)$
–3	1	1	–1
–2	2	2	–2
–1	1	1	–1
0	0	0	0
1	1	–1	–1
2	2	–2	–2
3	1	–1	–1

b. $h(x)$ is even and also $h(x) = -f(x)$.

c. *i.* impossible; Not given in table.
ii. $g^{-1}(2) = -2$
iii. $f(g(-1)) = f(-3) = 1$

81. The statement does not say anything about vertical asymptotes: *i* only

82. a. $(-x)^{1/3} = \sqrt[3]{-x} = -\sqrt[3]{x}$: odd

b. $-(-x)^2 + 4 = -x^2 + 4$: even

c. $(-x)^3 + (-x)^2 + 1 = -x^2 + x^2 + 1$: neither

83. a. $f(-2)=(-2)^2+5(-2)$
$=4-10=-6$

b. $g(-2)=(-2)+3=1$

c. $f(g(-2))=f(1)=(1)^2+5(1)=6$

d. $g(f(-2))=g(-6)=(-6)+3=-3$

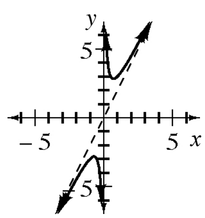
e. $f(f(-2))=f(-6)=(-6)^2+5(-6)$
$=36-30=6$

f. $g(g(-2))=g(1)=1+3=4$

84. a. $A=-\left(\frac{1}{2}\right)(4)(2)+\left(\frac{1}{2}\right)(3)(1.5)=-1.75\ \text{un}^2$

b. $-\left(\frac{1}{2}\right)(4)(2)+\left(\frac{1}{2}\right)(x-4)(0.5x-2)=10$
$-4+\left(\frac{1}{2}\right)(x-4)(0.5x-2)=10$
$\left(\frac{1}{2}\right)(x-4)(0.5x-2)=14$
$0.5x^2-4x+8=28$
$x=\frac{4\pm\sqrt{16-4(0.5)(-20)}}{2(0.5)}=4+\sqrt{56}=4+2\sqrt{14}$

85. See graph at right.
$b(x)=2x$

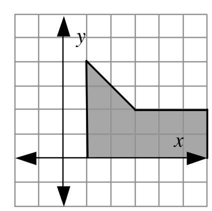


86. See graph at right.
This is a truncated cone next to a cylinder.

$V_{\substack{\text{truncated}\\ \text{cone}}}=\frac{1}{3}\pi(4^2)(4)-\frac{1}{3}\pi(2^2)(2)=\frac{1}{3}\pi(64-8)=\frac{56}{3}\pi$

$V_{\text{cylinder}}=\pi(2^2)(3)=12\pi$

$V_{\text{total}}=\frac{56}{3}\pi+\frac{36}{3}\pi=\frac{92}{3}\pi$



87. As $x\to-\infty$, $y\to\infty$, and as $x\to\infty$, $y\to 1$.

88. $A(y, 0\le x\le 4)=\frac{1}{2}\left(\frac{0}{0+1}+\frac{0.5}{0.5+1}+\frac{1}{1+1}+\cdots\frac{3.5}{3.5+1}\right)\approx 2.171\ \text{un}^2$

89. a. $f(g(x))=f(x+3)=(x+3)^2+5=x^2+6x+9+5=x^2+6x+14$

b. $g(f(x))=g(x^2+5)=(x^2+5)+3=x^2+8$

c. $-6=x^2+5$
$-11=x^2$
No solution

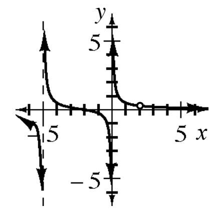
d. $-6=x+3$
$-9=x$

90. a. $y = \frac{x^2-4}{x^3+3x^2-10x} = \frac{(x+2)(x-2)}{x(x+5)(x-2)} = \frac{(x+2)}{x(x+5)}$

hole: $x = 2$

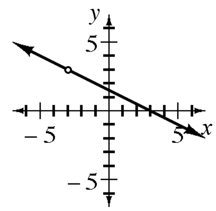
V.A.: $x = 0, -5$

H.A.: $x = 0$ $(-\infty,-5)\cup(-5,0)\cup(0,2)\cup(2,\infty)$



b. $y = \frac{9-x^2}{2x+6} = \frac{(3-x)(3+x)}{2(x+3)} = \frac{3-x}{2} = -\frac{1}{2}x + \frac{3}{2}$

hole: $x = -3$; $(-\infty,-3)\cup(-3,\infty)$

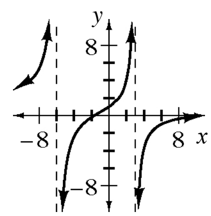


c. $y = \frac{x^2-9x-18}{x^2+3x-18} = \frac{x^2-9x-18}{(x+6)(x-3)}$

V.A.: $x = -6, 3$

H.A: $y = 1$

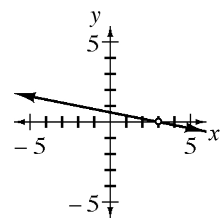
$\{x : x \neq -6, 3\}$



d. $y = \frac{x^2-6x+9}{15-5x} = \frac{(x-3)(x-3)}{5(3-x)} = -\frac{x-3}{5} = -\frac{1}{5}x + \frac{3}{5}$

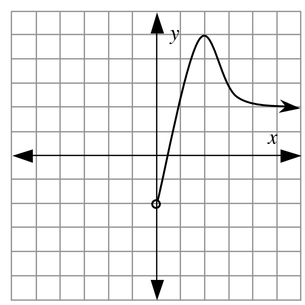
hole: $x = 3$

$\{x : x \neq 3\}$



91. a. See possible graph at right.

$D: 0 < x < \infty,\ R: -2 < y \leq 5$



b. The graph is shifted right 2 and up 1.

$D: 2 < x < \infty,\ R: -1 < y \leq 6$

c. Switch the original domain and range.

$D: -2 < x \leq 5,\ R: y > 0$

94. a. $h(x) = \left((x+2)-3\right)^2 + 4 - 5 = (x-1)^2 - 1 = x^2 - 2x + 1 - 1 = x^2 - 2x$

b. $k(x) = \frac{1}{(3-x)-4} - 2 + 2 = \frac{1}{3-x-4} = -\frac{1}{1+x}$

c. For g, D: $\{x : x \neq 4\}$, R: $\{y : y \neq 2\}$ (because $\frac{1}{x-4}$ can be anything but 0)

For k, D: $\{x : x \neq -1\}$, R: $\{y : y \neq 0\}$ ($\frac{1}{1+x}$ cannot be 0)

95. e. D: all real numbers, R: $y > 3$, For the inverse, just exchange the domain and range.

f. The inverses of functions (a) and (c) are not functions, since they have y-values for which there are multiple x-values.

96. Δy increases by 2 each time x increases by 1.

97. a.

x	–3	–2	–1	0	1	2	3
$f(x)$	28	15	6	1	0	3	10

Δy	–13	–9	–5	–1	3	7

Δy increases by 4 each time x increases by 1.

b.

x	–3	–2	–1	0	1	2	3
$f(x)$	–21	–6	3	6	3	–6	–21

Δy	15	9	3	–3	–9	–15

Δy decreases by 6 each time x increases by 1.

98. For a parabola, Δy increases by a constant amount, namely by $2a$, for each unit increase in x. So parabolas change in a linear way (Δy is linear).

99. $f(x) = a$: no change, $\Delta y = 0$ always
$f(x) = ax + b$: constant change, $\Delta y = a$ always
$f(x) = ax^3 + bx^2 + cx + d$: change varies, Δy is quadratic

100. x^n changes in the pattern of x^{n-1}, i.e. a polynomial of degree $n - 1$.

101. $y = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$ The graph changes from a line with slope –1 to a line with slope 1.

102. a. $x^2 + 4 \neq 0$, D: $(-\infty, \infty)$

b. $x^2 + x \neq 0,\ x(x+1) \neq 0,\ x \neq 0, -1,$ and $x + 2 \geq 0,\ x \geq -2.$
So D: $\{x : x \geq -2 \text{ and } x \neq 0, -1\}$

103. Substitute $y-3=x$:

$$y^2=6(y-3)$$
$$y^2=6y-18$$
$$y^2-6y+18=0$$
$$(y-3)^2-9+18=0$$
$$(y-3)^2+9=0$$

There are no solutions: e is correct.

104. a. $x=\frac{7\pi}{6}$: (30º-60º-90º triangle)

b. $x=0$

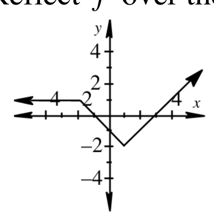
c. $x=\frac{3\pi}{4}$: (45º-45º-90º triangle)

105. a. $f(2)=2(2)^2-3=8-3=5$

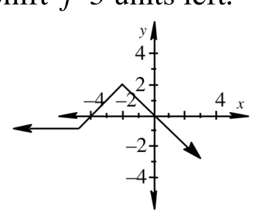
b. 2, switch the x and y from part (a).

c. $$\left[2(x+2)^2-3\right]-\left[2(x-2)^2-3\right]=64$$
$$2x^2+8x+8-3-\left(2x^2-8x+8-3\right)=64$$
$$2x^2+8x+5-2x^2+8x-5=64$$
$$16x=64$$
$$x=4$$

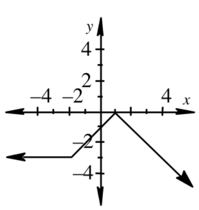
106. a. Reflect f over the x-axis.



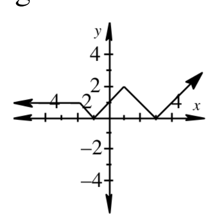
b. Shift f 3 units left.



c. Shift f 2 units down.



D. Reflect negative values over x-axis.



107. For $f(x)=\frac{1}{3}x+1$ and $f^{-1}(x)=3(x-1)$, the areas are: $2\cdot(\frac{1}{3}(3)+1+\frac{1}{3}(5)+1)=2\cdot\frac{14}{3}=\frac{28}{3}$ and, since $f^{-1}(2)=3$ and $f^{-1}(\frac{8}{3})=5$, $2\cdot(2+\frac{8}{3})=2\cdot\frac{14}{3}=\frac{28}{3}$.

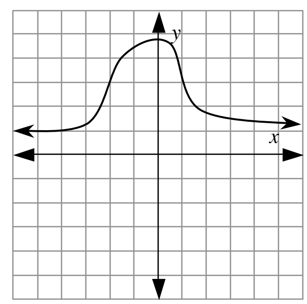
108. $f(2.5)\approx 0.449$ and $f(3.5)\approx 0.814 : A(f, 1\le x\le 4)\approx 0.821+0.449+0.814\approx 2.084$ un^2

109. a. The midpoint rectangles give the best approximation because they leave out small pieces of area under the curve but include other small pieces above the cure, which gives a better overall estimate than left rectangles which miss larger areas, or right rectangles which include larger areas which they should no include.

b. No, the graph could be decreasing.

110. a. Start off increasing slowly, then increase quickly to a maximum. Now decrease quickly to a minimum. Then increase slowly to a slightly lower maximum, then decrease to the same elevation you started at.

b. Increasing, decreasing, level, less steep, more steep, etc.



111. See sample graph at right. Answers vary, but examine the slope.

112.

$f(x)=\frac{1}{x}$

D: $\{x : x\ne 0\}$, R: $\{y : y\ne 0\}$

x	–3	–2	–1	0	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	–1	und	1	$\frac{1}{2}$	$\frac{1}{3}$

Δy $-\frac{1}{6}$ $-\frac{1}{2}$ und und $-\frac{1}{2}$ $-\frac{1}{6}$

The slope goes from slightly negative, to very negative. Then from very negative to slightly negative.

$f(x)=\frac{1}{x^2}$

D: $\{x : x\ne 0\}$, R: $\{y : y > 0\}$

x	–3	–2	–1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{4}$	1	und	1	$\frac{1}{4}$	$\frac{1}{9}$

Δy $\frac{5}{36}$ $\frac{3}{4}$ und und $-\frac{3}{4}$ $-\frac{5}{36}$

The slope goes from slightly positive, to very positive. Then from very negative to slightly negative.

$f(x)=\sin x$

D: $(-\infty,\infty)$, R: $[-1,1]$

x	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
y	1	0	–1	0	1	0	–1

Δy –1 –1 1 1 –1 –1

The slope cycles between –1 and 1.

$f(x)=\cos x$

D: $(-\infty,\infty)$, R: $[-1,1]$

x	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
y	0	–1	0	1	0	-1	0

Δy –1 1 1 –1 –1 1

The slope cycles between –1 and 1.

$f(x) = (0.5)^x$

D: $(-\infty, \infty)$, R: $(0, \infty)$

x	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Δy $\quad -4 \quad -2 \quad -1 \quad -\frac{1}{2} \quad -\frac{1}{4} \quad -\frac{1}{8}$

The slope starts out very negative and becomes closer and closer to zero.

$f(x) = 2^x$

D: $(-\infty, \infty)$, R: $(0, \infty)$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Δy $\quad \frac{1}{8} \quad \frac{1}{4} \quad \frac{1}{2} \quad 1 \quad 2 \quad 4$

The slope starts out almost 0, then continues to increase,

$f(x) = \log x$

D: $(0, \infty)$, R: $(-\infty, \infty)$

x	1	2	3	4	5
y	0	0.3	0.48	0.6	0.7

Δy $\quad 0.3 \quad 0.18 \quad 0.12 \quad 0.1$

The slope is always positive and always decreasing.

$f(x) = \log_2 x$

D: $(0, \infty)$, R: $(-\infty, \infty)$

x	1	2	3	4	5
y	0	1	1.58	2	2.32

Δy $\quad 1 \quad 0.58 \quad 0.42 \quad 0.32$

The slope is always positive and always decreasing.

$f(x) = \sqrt{x}$

D: $[0, \infty)$, R: $[0, \infty)$

x	0	1	2	3	4	5
y	0	1	1.41	1.72	2	2.23

Δy $\quad 1 \quad 0.41 \quad 0.31 \quad 0.28 \quad 0.23$

The slope is always positive and always decreasing.

$f(x) = \sqrt[3]{x}$

D: $(-\infty, \infty)$, R: $(-\infty, \infty)$

x	-3	-2	-1	0	1	2	3
y	-1.44	-1.26	-1	0	1	1.26	1.44

Δy $\quad 0.18 \quad 0.26 \quad 1 \quad 1 \quad 0.26 \quad 0.18$

The slope is always positive, starts off flat and increases until $x = 0$, where it begins to decrease.

113. First part: $y = -a(x-2)^2 + 8$ and goes through (0, 6):

$$y = -a(x-2)^2 + 8$$
$$6 = -a(0-2)^2 + 8$$
$$6 = -4a + 8$$
$$4a = 2$$
$$a = \tfrac{1}{2}$$
$$y = -\tfrac{1}{2}(x-2)^2 + 8$$

Second part: $y = a\sqrt{x-5} + 3.5$ (cubed root also would work) and goes through (9, 6):

$$y = a\sqrt{x-5} + 3.5$$
$$6 = a\sqrt{9-5} + 3.5$$
$$2.5 = a\sqrt{4}$$
$$a = \tfrac{2.5}{2} = \tfrac{5}{4}$$
$$y = \tfrac{5}{4}\sqrt{x-5} + 3.5$$

So $y = \begin{cases} -\frac{1}{2}(x-2)^2 + 8, & 0 \le x \le 5 \\ \frac{5}{4}\sqrt{x-5} + 3.5, & x > 5 \end{cases}$

114. a. $25 - x^2 \geq 0,\ 25 \geq x^2,\ D\text{: } [-5, 5]$

b. $x + 5 > 0,\ x > -5,\ D\text{: } (-5, \infty)$

c. $x^2 - x - 12 \neq 0,\ (x-4)(x+3) \neq 0,\ D\text{: } \{x : x \neq 4 \text{ and } x \neq -3\}$

d. $x + 2 \geq 0,\ x \geq -2,$ and $x^2 - 4 \neq 0,\ x^2 \neq 4,\ x \neq 2, -2,\ D: \{x : x > -2 \text{ and } x \neq 2\}$

115. $$\left[\left(\frac{x^{-1}+x^2}{x}\right) - x + x^{-2}\right]^{-2} = \left(\frac{1+x^3}{x^2} - \frac{x^3}{x^2} + \frac{1}{x^2}\right)^{-2} = \left(\frac{1+x^3-x^3+1}{x^2}\right)^{-2} = \left(\frac{2}{x^2}\right)^{-2} = \left(\frac{x^2}{2}\right)^2 = \frac{x^4}{4}$$

116. a.
$$\begin{aligned} \tan x \csc x &= 2 \\ \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} &= 2 \\ \frac{1}{\cos x} &= 2 \\ \cos x &= \tfrac{1}{2} \\ x &= \tfrac{\pi}{3}, \tfrac{5\pi}{3} \end{aligned}$$

b.
$$\begin{aligned} \sin x \cos x &= \tfrac{1}{4} \\ 2 \sin x \cos x &= \tfrac{1}{2} \\ \sin 2x &= \tfrac{1}{2} \\ 2x &= \tfrac{\pi}{6}, \tfrac{5\pi}{6} \\ x &= \tfrac{\pi}{12}, \tfrac{13\pi}{12}, \tfrac{5\pi}{12}, \tfrac{17\pi}{12} \end{aligned}$$

c.
$$\begin{aligned} 2\sin^2 x - \cos x - 1 &= 0 \\ 2\left(1 - \cos^2 x\right) - \cos x - 1 &= 0 \\ -2\cos^2 x - \cos x + 1 &= 0 \\ (-2\cos x + 1)(\cos x + 1) &= 0 \\ \cos x &= \tfrac{1}{2}, -1 \\ x &= \tfrac{\pi}{3}, \tfrac{5\pi}{3}, \pi \end{aligned}$$

d.
$$\begin{aligned} \tan x + \cot x &= -2 \\ \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} &= -2 \\ \sin^2 x + \cos^2 x &= -2\cos x \sin x \\ 1 &= -\sin 2x \\ -1 &= \sin 2x \\ 2x &= \tfrac{3\pi}{2} \\ x &= \tfrac{3\pi}{4}, \tfrac{7\pi}{4} \end{aligned}$$

117. Possible functions:

a. $y = \frac{x-3}{(x-3)(x+1)}$ b. $y = \frac{x+4}{x(x+4)(x-5)}$ c. $y = \frac{1}{x} + 3x - 1$

118. a. $D\text{: } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],\ R\text{: } [-1, 1]$

b. $D\text{: } [-1, 1],\ R\text{: } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

c. $\cos x\text{: } D\text{: } [0, \pi],\ R\text{: } [-1, 1];\ \cos^{-1} x : D\text{: } [-1, 1],\ R\text{: } [0, \pi]$

119. If $g(x)$ is even then $g(-a) = g(a)$. Thus, $g(x)$ fails the horizontal line test and $g^{-1}(x)$ is not a function unless we restrict the domain of g.

120. a. This is a hemisphere with radius 5. b. $V = \frac{1}{2} \cdot \frac{4}{3} \pi r^3 = \frac{2}{3} \pi \cdot 5^3 = \frac{250\pi}{3}$ un^3

121. The midpoint approximation is better in this case.
Midpoint: $A(f, 2 \le x \le 8) \approx 2 \cdot f(3) + 2 \cdot f(5) + 2 \cdot f(7)$

$$= 2(-0.25)3(-6) + 2(-0.25)5(-4) + 2(-0.25)7(-1) = 9 + 10 + 7 = 26 \text{ un}^2$$

Trapezoid: $A(f, 2 \le x \le 8) \approx (f(2) + f(4)) + (f(4)f(4)) + (f(6) + f(8))$

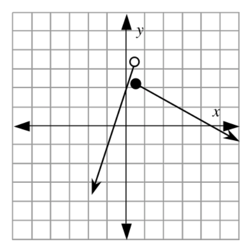
$$= -0.25(2)(-7) - 0.25(4)(-5) - \cdots - 0.25(8)(-1) = 24.5 \text{ un}^2$$

124. a. All walks have the same speed, but the first two would be walking away from the motion detector and the last would be walking towards the motion detector. All walks begin 5 feet from the motion detector.

b. They both start away from the motion detector, walk to the detector, then away from the detector. The slopes are different, so the speed of walking for x^3 is greater than that of x^2.

c. Ara has the greatest speed at the beginning. Adelyn has the greatest speed at the end.

125. See graph at right. Note: Graph is scaled by 2's.



a. Because the curve is composed of two linear pieces, the exact area can be computed using two trapezoids.

b. $A(f(x), -1 \le x \le 5) = \frac{1+7}{2} \cdot 2 + \frac{5+3}{2} \cdot 4 = 8 + 16 = 24$ un^2

126. a. $D = (-1, 2]$, $R = (-2, 2]$ $\quad y = \begin{cases} -1.5x - 0.5, & -1 < x < 1 \\ 3x - 4, & 1 \le x \le 2 \end{cases}$

b. $D = (-\infty, -1) \cup (0, \infty)$, $R = (0, \infty)$ $\quad y = \begin{cases} (2^{3/2} - 1 - x)^{2/3}, & x < -1 \\ \frac{1}{x}, & x > 0 \end{cases}$

Answers will vary in defining the function.

127. $A(f, -2 \le x \le 3) \approx \frac{1}{2} f(-2) + \frac{1}{2} f(-1.5) + \cdots + \frac{1}{2} f(2.5)$

$$= \tfrac{1}{2}(-2)^2 - (-2) - 6) + \tfrac{1}{2}((-1.5)^2 - (-1.5) - 6) + ... + \tfrac{1}{2}((2.5)^2 - 2.5 - 6)$$

$$= \tfrac{1}{2}(0) + \tfrac{1}{2}(-2.25) + \tfrac{1}{2}(-4) + \tfrac{1}{2}(-5.25) + \cdots + \tfrac{1}{2}(-4) + \tfrac{1}{2}(-2.25) = -20.625 = -20\tfrac{5}{8} \text{ un}^2$$

128. The original function is quadratic. The differences of the differences are constant.

129. a. $x^2 - x \neq 0, x(x-1) \neq 0, x \neq 0, 1 : D = \{x : x \neq 0 \text{ and } x \neq 1\}$

b. $\frac{1}{x^2 - x} = \frac{1}{2}, x^2 - x = 2, x^2 - x - 2 + 0, (x-2)(x+1) = 0, x = -1, 2$

c. Part (b) demonstrates that $g^{-1}(0.5)$ has two solutions.

130. a. $f(3) = 3^2 = 9$

b. $f(-3) = (-3)^2 = 9$

c. $g(9) = \sqrt{9} = 3$

d. $g(f(3)) = g(9) = 3$

e. $g(f(6)) = g(6^2) = \sqrt{6^2} = 6$

f. $g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

131. a. The function values are 1, 5, 9, and 13. The change is constant (it is 4 each time).

b. The areas are 0, 3, 10, and 21. Their change is quadratic, though students may only see that they are nonlinear, growing by consecutive multiples of 2.

132. a. Increases when $x < -3$ and $x > 2$, decreases when $-3 < x < 2$.

b. Increases when $-1 < x < 1$, decreases when $x < -1$ and $x > 1$.

c. No, the graph curves in the interval.

133. The speed of walk 1 is the reciprocal of walk 2.

134. a. *i*: $9 < t < 12$; When the slope is negative.

ii. $3 < t < 6$ and $t > 12$; When the slope is zero.

iii. Answers vary, $t = 10$ is a good answer. When the slope is steepest (pos. or neg.).

iv. Answers vary, $t = 7.5$ is a good answer. When the slope is decreasing.

b. Diamonique increases her speed, moving away from the detector at a constant rate for 3 seconds. She then stays at a constant velocity for the next 3 seconds. She again increases her speed for 2 seconds, then stays at a constant velocity for 2 seconds. She then decreases her velocity at a fairly constant rate for 2 seconds. At the end she is moving a constant rate. She is walking away from the detector the entire time.

135. a. Fredo's graph represents the position of the athlete during the race and should have time on the *x*-axis and distance on the *y*-axis. Frieda's graph represents the velocity of the athlete and should have time on the *x*-axis and velocity on the *y*-axis.

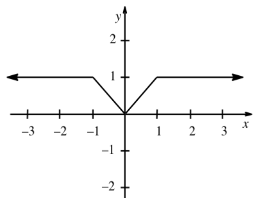
b. Both graphs show the race lasting 18 units of time. The slope of Fredo's line gives you the rate of the athlete. Since $90 \div 18 = 5$, the rate of the athlete in Fredo's graph matches the rate graphed in Frieda's graph. The area under Frieda's line is 90, which confirms the length of the race shown in Fredo's graph.

136. a. 12 mph, –36 mph, and –6 mph (approximately)

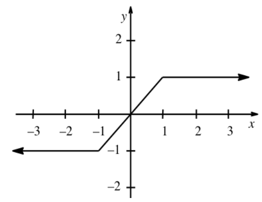
b. Her velocity is the slope of her distance function.

c. About 8 miles.

138. a. $A(f, -3 \le x \le 3) = 2 + \frac{1}{2} + \frac{1}{2} + 2 = 5$



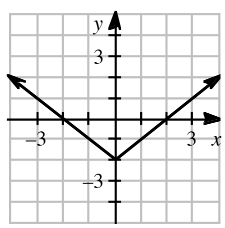
b. $A(f, -3 \le x \le 3) = 0$ because of symmetry.



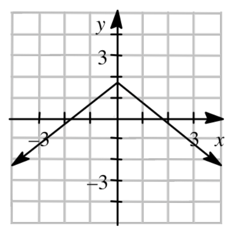
139. At $x = 2$: $\sqrt{x+2} - 1 = a(x+1)^2$

$$\sqrt{4} - 1 = a(3)^2$$
$$1 = 9a$$
$$\frac{1}{9} = a$$

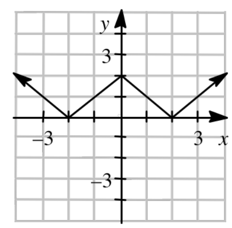
140. a.



b.



c.



141. a. $x + 2 \ne 0, x \ne -2 : D = \{x : x \ne -2\}$

b. $x - 4 \ge 0, x \ge 4 : D = [4, \infty)$

c. $g(x)$ must be defined and $\ne -2$, so $x \ge 4$ and $\sqrt{x-4} \ne -2$ so simply $D = [4, \infty)$.

d. $g(f(x)) = \sqrt{\frac{1}{x+2} - 4} : x \ne -2$ and $\frac{1}{x+2} - 4 \ge 0$

$$\frac{1}{x+2} - \frac{4x+8}{x+2} \ge 0$$
$$\frac{-7-4x}{x+2} \ge 0$$

The numerator and denominator must either both be positive or both be negative. Check points on either side of the zeros ($x = -\frac{7}{4}, -2$).

At $x = -3$, $k(-3) = \sqrt{\frac{-7-4(-3)}{-3+2}} = \sqrt{-5}$, which is not possible.

At $x = -\frac{15}{8}$, $k\left(-\frac{15}{8}\right) = \sqrt{\frac{7-4\left(-\frac{15}{8}\right)}{-\frac{15}{8}+2}} = \sqrt{\frac{29/2}{1/8}} = \sqrt{4} = 2$

At $x = -1$, $k(-1) = \sqrt{\frac{-7-4(-1)}{-1+2}} = \sqrt{-3}$, which is not possible.

Therefore $D = \left\{x : -2 < x \le -\frac{7}{4}\right\}$.

142. a. $f(x)=\frac{x-3}{x^2+4x-21}=\frac{x-3}{(x+7)(x-3)}=\frac{1}{x+7}$, except at $x=3$ there is a hole.
$x=-7$ is a vertical asymptote.
From end behavior, as $x\to-\infty$ and as $x\to\infty, y\to 0,$ so $y=0$ is a horiz. asymptote.

b. $g(x)=\frac{x^{4/3}}{x^2-2x}=\frac{x\sqrt[3]{x}}{x(x-2)}=\frac{\sqrt[3]{x}}{x-2}$, except at $x=0$ there is a hole.
$x=2$ is a vertical asymptote.
End behavior: as $x\to-\infty$ and as $x\to\infty, y\to 0,$ so $y=0$ is a horizontal asymptote.

143. Examples: sleep per day, allowance per month, miles per gallon.

144. $f(x)$ is quadratic, for example, $f(x)=2x^2$.

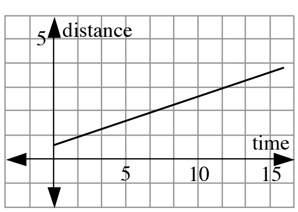
145. $A(f,-2\le x\le 4)\approx f(-1.5)+f(-0.5)+f(0.5)+f(2.5)+f(3.5)$
$=\left(\frac{1}{4}(-1.5)^3-\frac{1}{2}(1.5)^2-(-1.5)+3\right)+\cdots+\left(\frac{1}{4}(3.5)^3-\frac{1}{2}(3.5)^2-3.5+3\right)=14.875\ \text{un}^2$

146. a. As $x\to-\infty, y\to 0$. As $x\to\infty, y\to\infty$. As $x\to 0^{-1}, y\to\infty$. As $x\to 0^{+}, y\to\infty$.

b. As $x\to-\infty, y\to 0$. As $x\to\infty, y\to\infty$. As $x\to 0^{-}, y\to\approx 0.69$.
As $x\to 0^{+}, y\to\approx 0.69$.

147. a. *i.* About 15s *ii.* About 5m *iii.* About 3m
iv. Yes, at $t\approx 5$ s and $t\approx 14.5$ s. *v.* No, she turned around twice.

b. Part (*ii*) asks about total distance traveled while part (*iii*) asks about displacement.

148. a.


b. About 0.2m/sec

c. Her average velocity is the slope of the graph of her direct route.

150. Agnalia is correct. Motion implies continuity.

151. a. Answers vary, but you will notice that the bug changes directions and speeds.

b. *i.* $\frac{8-0}{16-0} = \frac{1}{2}$ cm/sec *ii.* $\frac{2-0}{2-0} = 1$ cm/sec

iii. $\frac{4-9}{11-8} = -\frac{5}{3}$ cm/sec *iv.* $\frac{4-4}{15-5} = 0$ cm/sec

c. Yes, explanations will vary—some students will observe that this must happen at least three times.

d. Yes, explanations will vary—some students will observe that this must happen at least twice.

152. a. It's an ice cream cone: a hemisphere on top of a cone, except sideways.

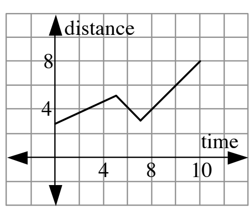
b. $V = \frac{1}{2} \cdot \frac{4}{3} \pi r^3 + \frac{1}{3} \pi r^2 h, r = \frac{10}{2}$ (30°-60°-90° triangle) and $h = \frac{10}{2}\sqrt{3}$:

$V = \frac{2}{3}\pi \cdot 5^3 + \frac{1}{3}\pi 5^2 (5\sqrt{3}) = \frac{250\pi}{3} + \frac{125\sqrt{3}\pi}{3} \approx 488.524 \text{ un}^3$

153. Take care to account for the width of the rectangles and/or trapezoids.

154. a. $\frac{\sqrt{3}}{2}$ b. $\frac{\sqrt{3}}{3}$ c. $\frac{2\sqrt{3}}{3}$ d. 2

155. See sample graph below. He must retrace his steps for 2 feet.



156. a. The area must be computed: it is about $\frac{1}{2} \cdot 40 + \frac{1}{2} \cdot 55 + 1 \cdot 60 + \frac{1}{4} \cdot 30 = 115$ miles.

b. average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{115 \text{ miles}}{2\frac{1}{4} \text{ hours}} = \frac{115}{9/4}$ mph $= 51\frac{1}{9}$ mph

157. Rearrange: $9y = 4x - 12, y = \frac{4x}{9} - \frac{12}{9}$ has a slope of $\frac{4}{9}$. $y = \frac{4}{9}(x-6) - 7$

158. a. $\frac{1}{2}$ b. $-\frac{\sqrt{2}}{2}$ c. $\sqrt{3}$ d. 2

159. $h = 2r$ or $r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$ un^3

160. a. Since $f(-1) = 0$, the rectangle has a height of 0.

b. $A(y, -2 \le x \le 2) = -7 + 0 + 1 + 2 = -4$ un^2

161. The points on the parabola are (1, 0), (2, 3), (3, 4), (4, 3), and (5, 0).

$A = \frac{1}{2}\left((0+3) + (3+4) + (4+3) + (3+0)\right)(1) = 10$ un^2

This is an underestimate.

162. a. ≈ 5 m/min.

b. Area is in units of the base times units of the height: $\text{min} \cdot \frac{\text{m}}{\text{min}} = \text{m}$, meters.

c. Area under a velocity curve is total displacement.

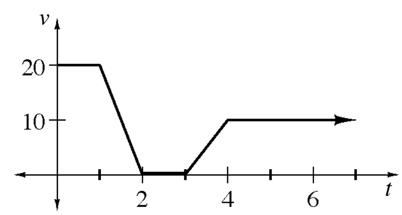
d. This is the area, or about $20 + \frac{20+0}{2} + \frac{0+10}{2} = 20 + 10 + 5 = 35$ m.

e. $\frac{35}{4}$ m/min

f. A rectangle.

g. Twice, there are two points of intersection.

h. See graph at right.



Examine the area under the curve.

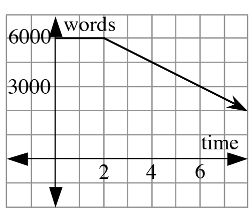
$A(0 \le t \le 4) = 20 + 10 + 0 + 5 = 35$m

Dijin still needs to travel 65m after $t = 4$.

$A(4 \le t \le x) = 10x = 65\text{m} \;\Rightarrow\; x = 6.5$ min

Therefore the total time is 4 + 6.5 = 10.5 minutes to class.

163. a.



b. We can find the area under the functions:

$6000 \cdot 2 + \frac{6000+7000-500(5)}{2} \cdot 3$

$= 12000 + 15750 = 27750$ words

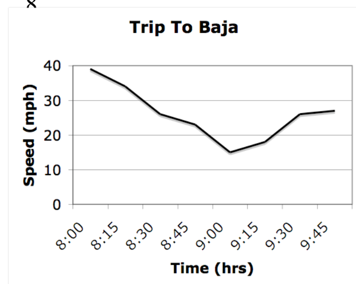
164. Rate can be multiplied by time to get total change, if the rate is constant. In general total change is found by taking the area under the rate function.

165. a. See graph at right.

The trapezoid method works best, using speed $= 27$ at 10:00.

Rectangle width $= \frac{1}{4}$ hr. $A \approx \frac{39+34}{8} + \frac{34+26}{8} + \cdots + \frac{26+27}{8} + \frac{27+27}{8} \approx 50.5$ miles

Using left rectangles yields 52 miles.

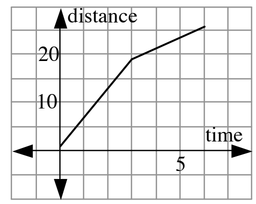


b. Approximately $22 + 51 = 73$ miles

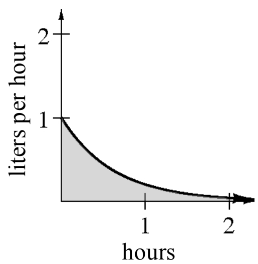
166. See graph below right.

To calculate total time $= \frac{t}{2} \cdot 5 + \frac{t}{2} \cdot 3 = 26 - 2,\ \frac{8}{2}t = 24,\ t = \frac{24}{4} = 6$

For example: The important points are (0, 2),(3, 17), and (6, 26). At $t = 3$ his position must be $2 + 3 \cdot 5 = 17$.



167. a.



b. The number of liters that leaked out between 0 and 1 hours.

The units are $\frac{\text{liters}}{\text{hour}} \cdot \text{hours} = \text{liters}$.

168. $y = 7(x-3) - 2$

169. The amount of change.

170. For the first part: $y = a(x+2)^2 - 3$

Substitute (2, 7): $7 = a(2+2)^2 - 3$

$10 = 16a$

$\frac{5}{8} = a,\ y = \frac{5}{8}(x+2)^2 - 3$

Second part: $y = a|x-4|$

Substitute (2, 7): $7 = a|2-4| = 2a$,

$a = \frac{7}{2},\ y = \frac{7}{2}|x-4|$

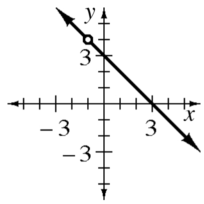
Answer: $y = \begin{cases} \frac{5}{8}(x+2)^2 - 3, & x < 2 \\ \frac{7}{2}|x-4|, & x \geq 2 \end{cases}$

171. As $x \to -\infty, y \to \infty$. As $x \to \infty, y \to -\infty$. As $x \to -\frac{1}{2}^-, y \to \frac{5}{2}$. As $x \to -\frac{1}{2}^+, y \to \frac{5}{2}$.

172. It is a cylinder next to a cone: $V = \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 = \pi(2^2)\cdot 2 + \frac{1}{3}\pi(2^2)(2) = \frac{32\pi}{3}$

173. $h(x) = f(g(x)) = f(-x+2) = \frac{(-x+2)^2 - 2(-x+2) - 3}{(-x+2)-3} = \frac{x^2 - 4x + 4 + 2x - 4 - 3}{-x-1} = -\frac{x^2 - 2x - 3}{x+1}$

$= -\frac{(x-3)(x+1)}{x+1} = -x + 3$ unless $x = -1$

a. 

$D = \{x : x \neq -1\}$

b. $b(x) = -x + 3$

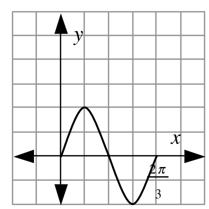
174. Total miles traveled in the Coronado, calculated by area under the graph of velocity:

$20 \cdot \frac{1}{2} + 40 \cdot + 70 \cdot \frac{1}{2} + 60 \cdot 1 + 30 \cdot 1 = 10 + 40 + 35 + 60 + 30 = 175$ miles

For D.O.: $60 \cdot 1 + 40 \cdot \frac{1}{2} + 70 \cdot 1 + 50 \cdot \frac{1}{2} + 30 \cdot 1 = 60 + 20 + 70 + 25 + 30 = 205$ miles

Coronado gas mileage is $\frac{175 \text{ miles}}{7.955 \text{ gallons}} \approx 22$ mpg, vs. $\frac{205 \text{ miles}}{8.542 \text{gallons}} \approx 24$ mpg for the D.O., so she should choose the D.O. Sensation.

175. See graph at right.



It is a sine curve with amplitude = 2 and period = $\frac{2\pi}{3}$.

Maximum at $\left(\frac{\pi}{6}, 2\right)$, minimum at $\left(\frac{\pi}{2}, -2\right)$

176. a. For the first minute it remains the same. During the second minute it steadily decreases. For the third second it remains the same, and for the fourth second it steadily and slowly increases.

b. Acceleration is the change in velocity over time, or $\frac{\text{m/min}}{\text{min}} = \frac{\text{m}}{\text{min}^2}$.

c. $a(t) = \begin{cases} 0 & \text{for } 0 \le t \le 1 \\ -20 & \text{for } 1 < t \le 2 \\ 0 & \text{for } 2 < t \le 3 \\ 10 & \text{for } 3 < t \le 4 \end{cases}$

d. It does not change when $a(t) = 0$.

e. On [3, 4] acceleration is positive because his velocity is increasing.

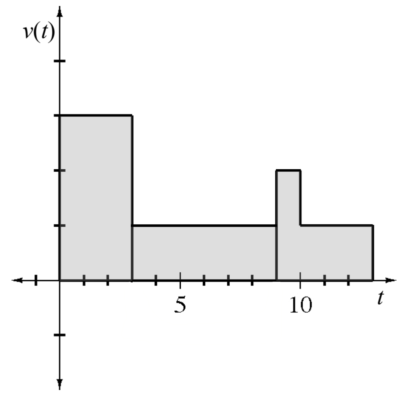
f. How his velocity is changing.

177. a. Points that have positive y-vlaues: A, B

b. Speed is the absolute value of velocity, so the point with the greatest $|y|$: E

c. Acceleration is the slope of a velocity graph, so points where the slope is negative: C, D

d. Points where the y-value is zero: C, G



178. a. $A = 3(3) + 6(-1) + 1(2) + 3(1) = 8$ miles

b. See graph at right.
The graph should reflect negative area on $3 < t < 9$ over the x-axis.

c. A, C, or D

d. D; If she starts 3 miles from home and her displacement is 8 miles, she will be 11 miles from home at $t = 13$.

179. a. For example: The bug starts at rest, and for the first minute it quickly accelerates. It maintains a high speed, decelerates, maintains a lower speed, then decelerates and stops. After a few minutes it accelerates in the opposite direction and maintains a low speed in that direction, and then slows to a stop.

b. The bug is then traveling in the negative direction.

c. We estimate the area:
$\frac{1+5}{2} + 5 \cdot 2 + \frac{5+3.5}{2} + \frac{3.5+2}{2} + 2 + \frac{3+.5}{2} \approx 3 + 10 + 4.25 + 2.75 + 2 + 1.25 \approx 24$ ft

d. We need to estimate total area: subtract the second area from the first:
$24 - (\frac{0+1.5}{2} + \frac{1.5+2}{2} + \frac{2+0}{2}) = 24 - (0.75 + 1.75 + 1) = 24 - 3.5 \approx 20.5$ ft

e. Add areas: $24 + 3.5 \approx 27.5$ ft

f. When the bug never travels in the negative direction (backward).

g. Carl is correct, we would need information about the initial position to answer this question.

h. The bug's total displacement was 20 feet, so –7 + 20 = 13: (13, 0).

180. a. Traveling upward but slowing down.

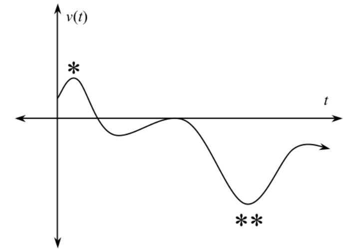
b. Traveling downward but slowing down.

c. Traveling upward (not slowing down or speeding up).

d. At rest, but being pulled downward.

e. Negative velocity, positive acceleration.

181. See graph at right.



a. The global maximum point represents the greatest velocity.

b. Reflect the negative values above the x-axis.

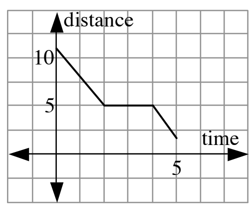
c. The global maximum point of the speed graph. On the velocity graph it is the global minimum.

d. Speed is the absolute value of velocity.

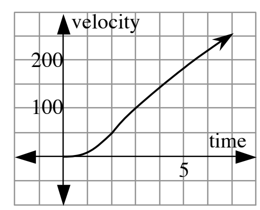
182. a. $V = \frac{1}{4}\cdot(9^2) + \frac{1}{4}(8.75^2) + \cdots + \frac{1}{4}(7.5^2) + \frac{1}{4}(7.25^2) = 132.6875$ or $132\frac{11}{16}$ in.^{3}

b. Starting from the bottom with radius changing by 0.5 in:
$V = \frac{1}{4}\pi(2)^2 + \frac{1}{4}\pi(2.5)^2 + \cdots + \frac{1}{4}\pi(5)^2 + \frac{1}{4}\pi(5.5)^2 = \frac{123\pi}{4} = 30.75\pi$ in.^{3}

183. For example:



He should end at $12 + 5\cdot(-2) = 2$.
The important points are (0, 12) and (5, 2).



184. a. See graph at right.

b. $12\ \frac{\text{ft}}{\text{s}} = v(1)$

c. Trapezoid or midpoint methods are okay, we need to estimate area.
Trapezoid: $\frac{0+12}{2} + \frac{12+48}{2} + \frac{48+95}{2} = 6 + 30 + 71.5 = 107.5$ ft.
Midpoint: $12(.5)^2 + 12(1.5)^2 + (-2.5^2 + 52(2.5) - 52) = 3 + 27 + 71.75 = 101.75$ ft

d. 50 + answer to part (c) = 157.5 ft

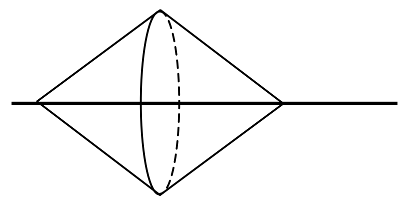
185. Since $x^2 - 4 = 0$ when $x = \pm 2$, we can have three regions: $f(x) = \begin{cases} x^2 - 3, & x < -2 \\ -x^2 + 5, & -2 \le x < 2 \\ x^2 - 3, & x \ge 2 \end{cases}$
$D = (-\infty, \infty)$ $R = [1, \infty)$

186. This can be found using polynomial division: $\frac{x^3+3x^2-4x-1}{x^2-1} = x + 3 + \frac{-3x+2}{x^2-1}$, so $b(x) = x + 3$.
The term $\frac{-3x+2}{x^2+1}$ will be insignificant for large positive or negative x.
As $x \to -\infty, y \to -\infty$. As $x \to \infty, y \to \infty$. Notice asymptotes as $x = \pm 1$.
As $x \to -1^-, y \to \infty$. As $x \to -1^+, y \to -\infty$. As $x \to 1^-, y \to \infty$. As $x \to 1^+, y \to -\infty$.

187. a.
$$\sin 2x = \sin x$$
$$2\sin x\cos x = \sin x$$
$$2\cos x = 1 \text{ or } \sin x = 0,\ x = 0,\ \pi$$
$$\cos x = \tfrac{1}{2},\ x = \tfrac{\pi}{3}, \tfrac{5\pi}{3}, 0, \pi$$

b.
$$\sin\left(x+\pi\right) + \cos\left(x - \tfrac{\pi}{2}\right) = 1$$
$$-\sin x + \cos x \cos\tfrac{\pi}{2} + \sin x \sin\tfrac{\pi}{2} = 1$$
$$-\sin x + \sin x = 1$$
$$0 = 1$$
No solution

c.
$$\tfrac{\cos x}{\sin x} - \tfrac{\sin x}{\cos x} = 2\sqrt{3}$$
$$\cos^2 x - \sin^2 x = 2\sqrt{3}\sin x\cos x$$
$$\cos 2x = \sqrt{3}\sin 2x$$
$$1 = \sqrt{3}\,\tfrac{\sin 2x}{\cos 2x}$$
$$\tfrac{1}{\sqrt{3}} = \tan 2x$$
$$2x = \tfrac{\pi}{6}, \tfrac{7\pi}{6}$$
$$x = \tfrac{\pi}{12}, \tfrac{13\pi}{12}, \tfrac{7\pi}{12}, \tfrac{19\pi}{12}$$



188. a. It is a double cone, i.e., two cones with the same base.

b. $V = \frac{1}{3}bh + \frac{1}{3}bh = \frac{2}{3}\pi r^2 \cdot 12$. $20^2 = 12^2 + r^2$, so $r = \sqrt{20^2 - 12^2} = 16$

$V = \frac{2}{3}\pi\left(16^2\right)\cdot 12 = 2\pi\left(256\right)\cdot 4 = 2048\pi \approx 6433.98\ \text{un}^3$

189. a. $g(-5) = (3(-5)-1)^2 = (-16)^2 = 256$

b. $g(a+1) = (3(a+1)-1)^2 = (3a+2)^2 = 9a^2 + 12a + 4$

c.

$49 = (3x-1)^2$	$7 = 3x - 1$	$-7 = 3x - 1$
$\pm 7 = 3x - 1$	$8 = 3x$	$-6 = 3x$
	$\frac{8}{3} = x$	$-2 = x$

d.
$$x = (3y-1)^2$$
$$\pm\sqrt{x} = 3y - 1$$
$$1 \pm \sqrt{x} = 3y$$
$$\tfrac{1\pm\sqrt{x}}{3} = y = g^{-1}(x)$$

190. Fredo's graph represents velocity, while Freida's represents distance. Basically the graphs nearly match up, except that where Fredo's graph is flat, Freida's graph should be straight, but instead Freida's graph appears to be a smooth curve.

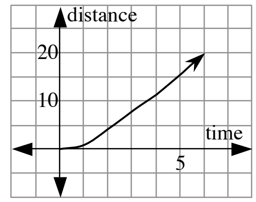
192. a. $v(t)=\begin{cases}2t, & 0\le t<2\\ 4, & t\ge 2\end{cases}$

b.

t	0	0.5	1	1.5	2	3	4	5	6
$d(t)$	0	0.25	1	2.25	4	8	12	16	20

c. See graph at right.

d. $d(t)=\begin{cases}t^2, & 0\le t<1\\ 4t-4, & 2\le t\end{cases}$



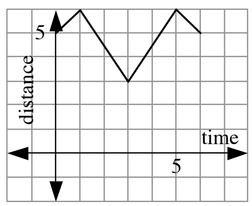
193. a. The point with a maximum y-value: A

b. Speed is the absolute value of velocity, so the point with the greatest $|y|$: C

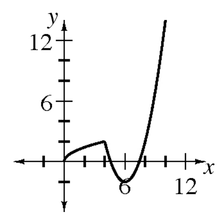
c. The point where the velocity graph changes form positive values to negative values: B

d. Acceleration is the slope of a velocity graph, so points where the slope is positive: D, E

194. Sample graph:



195. See graph at right. The graph starts with a large slope which gradually decreases until $x=4$ when it abruptly becomes large and negative. Gradually the slope levels out as x reaches 6 and then increases thereafter, becoming larger and larger (and positive).



196. a. $f(x)=3x-5,\ g(x)=4-x^2,\ h(x)=2\cos x$

b. *i.* $f(g(h(\pi)))=f(g(2\cos(\pi))=f(g(-2))=f(4-(-2)^2)=f(0)=3(0)-5=-5$

ii. $g^{-1}(4)\Rightarrow 4=4-x^2\ \Rightarrow\ x=0 \qquad h(g^{-1}(4))=h(0)=2\cos(0)=2$

iii. $f^{-1}(h(\pi))=f^{-1}(-2)\ \Rightarrow\ -2=3x-5\ \Rightarrow\ 3=3x\ \Rightarrow\ x=1$

197. a. The rate is always positive, and increases each year.

b. $\frac{2796-2300}{4}=\frac{496}{4}=124$ people/year.

c. For example, we could take the average rate between 1997 and 1999: $\frac{2796-2536}{2}=\frac{260}{2}=130$ people/year.

199. There are two identical cones: $V = \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h = \frac{2}{3}\pi 5^2 \cdot 5 = \frac{250\pi}{3}$ un^3

200. For example, $g(x) = \frac{x+2}{3}$, $f(x) = \sqrt{x} + 6$, or $g(x) = x + \frac{2}{3}$, $f(x) = \sqrt{3x} + 6$, or $g(x) = \frac{x^2+2}{3}$, $f(x) = x + 6$.

201. $\frac{x}{5.5} = \frac{x+d}{20}$, $20x = 5.5\left(x+6\right)$, $20x = 5.5x + 5.5d$, $14.5x = 5.5d$, $x = \frac{5.5}{14.5}d = \frac{55}{145}d = \frac{11}{29}d$

202. a. $\cos x \neq 0$, $x \neq \frac{\pi}{2} + \pi n$, $D\text{:}\left\{x : x \neq \frac{\pi}{2} + \pi n \text{ for some integer } n\right\}$

b. $x^2 + 1 > 0$, $x^2 > -1$, $D\text{:}\left(-\infty,\infty\right)$

c. $x^2 - x - 6 \neq 0$, $\left(x-3\right)\left(x+2\right) \neq 0$, $D\text{:}\left\{x : x \neq 3 \text{ and } x \neq -2\right\}$

d. $x - 1 > 0$, $x > 1$ and $\mathrm{x}^2 - 16 > 0$, $x^2 > 16$, $x > 4$ or $x < -4$, so the *x*-values which meet both conditions are $x > 4$. $D:\left\{x : x > 4\right\}$

203. a: $g(-x) = f(x)$, i.e., $g(x) = 1 - 2^{-x} = 1 - (2^{-1})^x = 1 - 0.5^x$