Assignment 3

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COL 774: Machine Learning

Contents

1	Dec	cision '	Irees, Random Forests and Gradient Boosted Trees	2
	1.1	Datas	et 1: Mammographic mass lesion severity prediction	2
		1.1.1	Decision Tree Construction and Visualization	2
		1.1.2	Decision Tree Grid Search	3
		1.1.3	Decision Tree Post Pruning (Cost Complexity Pruning)	4
		1.1.4	Random Forests	8
		1.1.5	Missing Data Imputation	G
		1.1.6	Gradient Boosted Trees	10
	1.2	Datas	et 2: Drug rating prediction	10
		1.2.1	Decision Tree Construction	10
		1.2.2	Decision Tree Grid Search	11
		1.2.3	Decision Tree Post Pruning (Cost Complexity Pruning)	11
		1.2.4	Random Forests	12
		1.2.5	Gradient Boosted Trees (XGBoost)	12
		1.2.6	GBM (Gradient Boosted Machines)	12
		1.2.7	Training with Varying amount of data	

2	Net	ıral Networks	15
	2.1	Implementing Neural Networks	15
	2.2	Training Neural Network with constant learning rate	15
	2.3	Training Neural Network with adaptive learning rate	20
	2.4	Sigmoid and ReLu as activation Functions	24
	2.5	Varying the number of hidden layers	26
	2.6	Training the network using BCE objective function	33
	2.7	Training the network using MLPClassifier	34

1 Decision Trees, Random Forests and Gradient Boosted Trees

We will be using Naïve Bayes approach to classify the text in one of the two categories i.e. review being negative and positive. Here, Our assumption is occurrence of a word in a review is conditionally independent of occurrence of another word. All precision, recall and f1 scores are on test data for different features taken for training the model.

1.1 Dataset 1: Mammographic mass lesion severity prediction

Mammography is the most effective method for breast cancer screening available today. We will work on a mammography dataset available from the UCI repository. The dataset contains 5 features:

- (a) BI-RADS assessment
- (b) Age
- (c) Shape
- (d) Margin
- (e) Density

The Decision Tree is implemented using this dataset to predict the type of brease cancer.

1.1.1 Decision Tree Construction and Visualization

The missing data in this part is just ignored and deafault parameters are used. The observations made are:

- 1. Depth of decision tree obtained are: 20
- 2. No. of leaves in decision tree obtained are: 119
- 3. Accuracy over Training data set = 0.9692307692307692
- 4. Accuracy over Validation data set = 0.8016528925619835

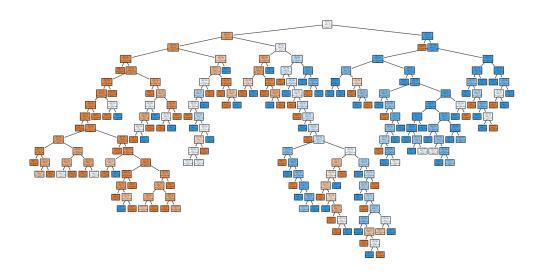


Figure 1: Decision Tree

On the training set, the model clearly shows an overfit, which leads to excellent training accuracy but subpar performance on the test set.

1.1.2 Decision Tree Grid Search

Here in this part the grid search over the space of parameters including max depth, min samples split and min samples leaf is performed. The set of parameters of over which grid search is performed are:

• $n_{\text{estimators}} : [10,20,40,50],$

• subsample : [0.1,0.2,0.3,0.4,0.5,0.6],

• $\max_{depth} : [4,5,6,7,8,9,10]$

The observations made are:

1. Depth of decision tree obtained are: 5

2. No. of leaves in decision tree obtained are: 18

3. Accuracy over Training data set after Grid search = 0.8637362637362638

4. Accuracy over Validation data set after Grid search = 0.8760330578512396

5. Accuracy over Test data set after Grid search = 0.7865612648221344

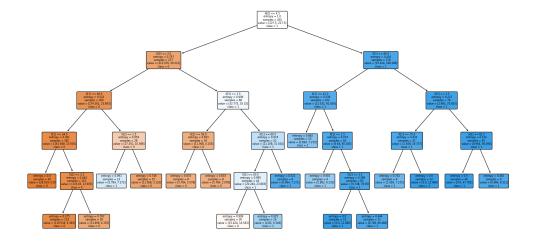


Figure 2: Decision Tree

By varying and adjusting the parameters of decision tree classifier, we have prevented overfitting to some extent.

1.1.3 Decision Tree Post Pruning (Cost Complexity Pruning)

Minimal cost complexity pruning is implemented using DecisionTreeClassifier.cost_complexity_pruning_path. Pruning was performed on the default decision tree using scikit-learn to obtain a range of ccp alpha parameters and corresponding impurities. The variation in parameters with ccp alpha was observed and plotted. The last ccp alpha value corresponding to a trivial single-node tree was included just for mentioning the existence of single node.

- 1. Depth of decision tree obtained are: 3
- 2. No. of leaves in decision tree obtained are: 4
- 3. Accuracy over Training data set after Pruning = 0.8395604395604396
- 4. Accuracy over Validation data set after Pruning = 0.909090909090909091
- 5. Accuracy over Test data set after Pruning = 0.7905138339920948

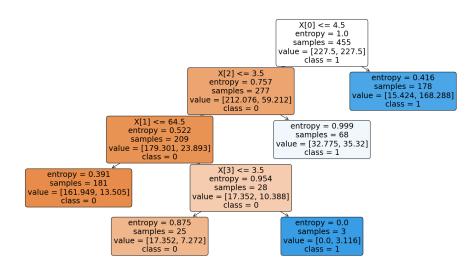


Figure 3: Decision Tree

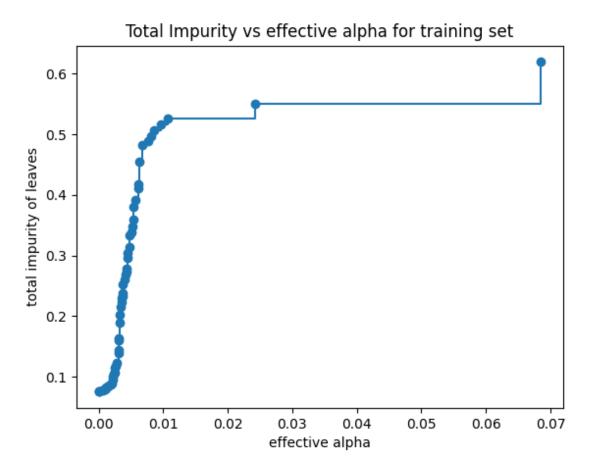


Figure 4: Impurity Vs Alpha

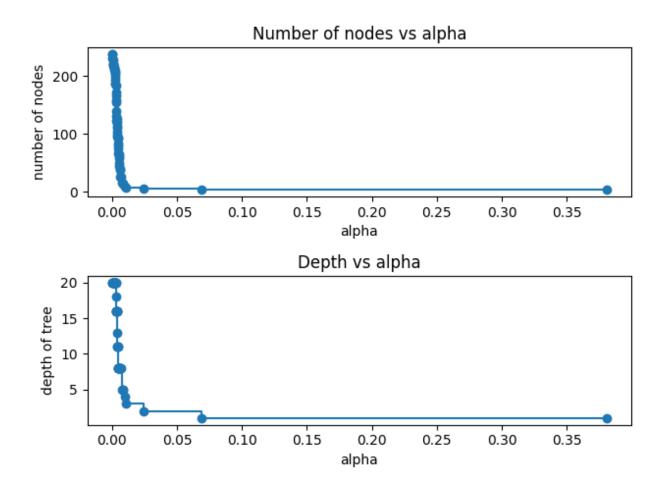


Figure 5: Nodes and Depth Vs Alpha



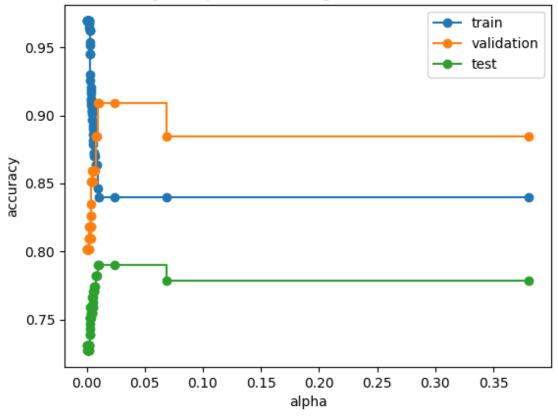


Figure 6: Accuracy

Figure 6 shows the increase in validation and test set accuracies along with decline in the training set accuracies. Here, pruning prevents overfitting and as a result the training set accuracy decreases. But, at higher values of ccp alphas the model underfits and training, validation, test set accuracies decreases.

1.1.4 Random Forests

The Random Forest is implemented after performing grid search over :

- (a) n estimators
- (b) max features
- (c) min sample split

And, the out of bag accuracy is used here. The Accuracy over the best classifier build using grid search over random forest is:

- 1. Out of Bad Accuracy is = 0.8351648351648352
- 2. Accuracy over Training data set after using random forest = 0.8857142857142857
- 3. Accuracy over Validation data set after using random forest = 0.9173553719008265

4. Accuracy over Test data set after using random forest = 0.802608695652174

The random forest slightly increases the accuracy by a small amount on all three data sets.

1.1.5 Missing Data Imputation

The Accuracy over the best classifier build using grid search over Decision Tree is:

- 1. Imputation used over training data for decision tree is: median
- 2. Depth of decision tree obtained are: 18
- 3. No. of leaves in decision tree obtained are: 155
- 4. Accuracy over Training data set = 0.9608938547486033
- 5. Accuracy over Validation data set = 0.8347107438016529
- 6. Accuracy over Test data set = 0.7470355731225297

The above results shows that with imputation by median, the model learns more about the trend of the data and tries to conserve the distribution and hence performs significantly better than the case where these data points were ignored.

- 1. Imputation used over training data for decision tree is: mode
- 2. Depth of decision tree obtained are: 16
- 3. No. of leaves in decision tree obtained are: 155
- 4. Accuracy over Training data set = 0.9590316573556797
- 5. Accuracy over Validation data set = 0.8264462809917356
- 6. Accuracy over Test data set = 0.7509881422924901

Here also, with imputation the model learns more about the data and performs better than ignoring the data points with missing attributes. The mode imputation results in better performance on the test and validation dataset.

The Accuracy over the best classifier build using grid search over Random Forest is:

- 1. Accuracy over Training data set after using imputation in Random Forest as median = 0.88268156424581
- 2. Accuracy over Validation data set after using imputation in Random Forest as median = 0.9090909090909091
- 3. Accuracy over Test data set after using imputation in Random Forest as median = 0.7984189723320159
- 1. Accuracy over Training data set after using imputation in Random Forest as mode = 0.8752327746741154
- 2. Accuracy over Validation data set after using imputation in Random Forest as mode = 0.9090909090909091
- 3. Accuracy over Test data set after using imputation in Random Forest as mode = 0.8023715415019763

Here, not much difference is made by imputation in both cases namely median and mode.

1.1.6 Gradient Boosted Trees

XGBoost (Extreme Gradient Boosting) is functional gradient boosting based approach where an ensemble of "weak learners" (decision trees in our case) is used with the goal to construct a model with less bias, and better predictive performance. I have used the XGBoost classifier and experimenting with different parameter values (in the given range):

- (a) n estimators (10 to 50 in range of 10)
- (b) subsample (0.1 to 0.6 in range of 0.1)
- (c) max depth(4 to 10 in range of 1)

The Accuracy obtained is:

- 1. Accuracy over Training data set after using xgb = 0.8747252747252747
- 2. Accuracy over Validation data set after using xgb = 0.9256198347107438
- 3. Accuracy over Test data set after using xgb = 0.782608695652174
- 1. Accuracy over Training data set after using imputation in Random Forest as mode = 0.8752327746741154
- 2. Accuracy over Validation data set after using imputation in Random Forest as mode = 0.9090909090909091
- 3. Accuracy over Test data set after using imputation in Random Forest as mode = 0.8023715415019763

1.2 Dataset 2: Drug rating prediction

Thus dataset related to prediction of rating associated medical drugs, where the attributes are primarily textual. Any dataset with text attributes requires conversion of text to numerical attributes for computations to be performed. In this dataset there are two text attributes, namely condition and review. The review attribute contains reviews about drugs collected from patients with specified conditions. We are going to use these two text attributes to predict the rating of a drug. Here, CountVectorizer is used and punctuations and stopwords have been removed.

1.2.1 Decision Tree Construction

The Observations made are:

- 1. Time to train the Decision Tree: 164.48403000831604
- 2. Depth of decision tree obtained are: 108
- 3. No. of leaves in decision tree obtained are: 45268
- 4. Accuracy over Training data set = 1.0
- 5. Accuracy over Validation data set = 0.5603546260513753
- 6. Accuracy over Test data set = 0.5565599077483911

1.2.2 Decision Tree Grid Search

Here in this part the grid search over the space of parameters including max depth, min samples split and min samples leaf is performed. The set of parameters of over which grid search is performed are:

- n_estimators : [100],
- max_features : [1],
- min_samples_split : [2],
- max_leaf_nodes : [None]

The observations made are:

- 1. Time to train the Decision Tree using grid search: 883.0011687278748
- 2. Depth of decision tree obtained are: 108
- 3. No. of leaves in decision tree obtained are: 45312
- 4. Accuracy over Training data set after Grid search = 1.0
- 5. Accuracy over Validation data set after Grid search = 0.5604579553204241
- 6. Accuracy over Test data set after Grid search = 0.5584942156753339

Here, the grid search is made over the small dataset because it was taking lots and lots of time to grid search over a range of parameters.

1.2.3 Decision Tree Post Pruning (Cost Complexity Pruning)

Pruning was performed on the default decision tree using scikit-learn to obtain a range of ccp_alpha parameters and corresponding impurities. No. of ccp_alpha obtained were 32k. The variation in parameters with ccp_alpha was observed and plotted. The last ccp_alpha value corresponding to a trivial single-node tree was excluded for analysis. For the selection of the best model, every 100th value in ccp_alpha was used to train and validate considering the huge number of values.

The best classifier was selected by performance on validation set and the following results were obtained.

- 1. The ccp_alpha obtained is: 0.0
- 2. Depth of decision tree obtained are: 108
- 3. No. of leaves in decision tree obtained are: 45312
- 4. Accuracy over Training data set after Grid search = 1.0
- 5. Accuracy over Validation data set after Grid search = 0.5704278543261246
- 6. Accuracy over Test data set after Grid search = 0.5693974656758163

1.2.4 Random Forests

In this part, a grid search was performed on the hyperparameters n_estimators, min_samples_split and max_features in order to train random forest.

The parameters and accuracies obtained is:

1. n_estimators: 450

2. max_features: 0.8

3. min_sample_split : 2

4. The out of bag accuracy is: 65.2035749574238162

5. Accuracy over Training data set after Grid search = 1.0

6. Accuracy over Validation data set after Grid search = 0.6494551836124622

7. Accuracy over Test data set after Grid search = 0.6471878656758163

When using a random forest instead of a single decision tree, we see a considerable improvement in test accuracy since random forests have less volatility and are thus more resilient. The training accuracy still shows overfitting, which gradient boosting may help to reduce.

1.2.5 Gradient Boosted Trees (XGBoost)

Extreme gradient boosting based estimators were constructued using the *xgoost* package and grid search was performed to tune it's hyperparameters.

The parameters and accuracies obtained is:

1. n_estimators: 450

 $2. \text{ max_depth} : 40$

3. min_sample_split : 2

4. Accuracy over Training data set after Grid search = 0.7789541967642131

5. Accuracy over Validation data set after Grid search = 0.5273314397721642

6. Accuracy over Test data set after Grid search = 0.5341263645122973

The training, test and validation set accuracies decreases because the no. of trees are less.

1.2.6 GBM (Gradient Boosted Machines)

Gradient boosted machines based estimators were constructued using the *lightgbm* package and grid search was performed to tune it's hyperparameters.

The parameters and accuracies obtained is:

1. n_estimators: 2000

- 2. max_depth: 40
- 3. subsamples: 0.4
- 4. Accuracy over Training data set after Grid search = 0.9856419676425406
- 5. Accuracy over Validation data set after Grid search = 0.6491883246014102
- 6. Accuracy over Test data set after Grid search = 0.6413071624533165

LightGBM performs the best among all the preceding models because to the scalability to huge datasets of gradient boosted machines, with a noticeable decrease in overfit on training set as compared to the random forest model in section.

The decision tree models train noticeably quicker than the other models outlined above, with pruning and parameter fixing requiring almost the same amounts of time while taking somewhat longer than the default decision tree. The multi-estimator models that perform the slowest with parameter fixing are random forest and XGBoost, respectively. LightGBM performs better on the test data and achieves training speeds that are comparable to those of a decision tree.

1.2.7 Training with Varying amount of data

- 1. The training data set size is 20000.
- 2. Part A: Training Accuracy: 1.0, Test Accuracy: 0.32, Validation Accuracy: 0.32
- 3. Part B: Training Accuracy: 1.0, Test Accuracy: 0.32, Validation Accuracy: 0.32
- 4. Part C: Training Accuracy: 1.0, Test Accuracy: 0.32, Validation Accuracy: 0.32
- 5. Part D: Training Accuracy: 1.0, Test Accuracy: 0.32, Validation Accuracy: 0.32
- 6. Part E: Training Accuracy: 0.98, Test Accuracy: 0.28, Validation Accuracy: 0.28
- 7. Part F: Training Accuracy: 1.0, Test Accuracy: 0.33, Validation Accuracy: 0.32
- 1. The training data set size is 40000.
- 2. Part A: Training Accuracy: 1.0, Test Accuracy: 0.38, Validation Accuracy: 0.37
- 3. Part B: Training Accuracy: 1.0, Test Accuracy: 0.38, Validation Accuracy: 0.39
- 4. Part C: Training Accuracy: 1.0, Test Accuracy: 0.38, Validation Accuracy: 0.38
- 5. Part D: Training Accuracy: 1.0, Test Accuracy: 0.38, Validation Accuracy: 0.37
- 6. Part E: Training Accuracy: 0.94, Test Accuracy: 0.36, Validation Accuracy: 0.36
- 7. Part F: Training Accuracy: 0.99, Test Accuracy: 0.39, Validation Accuracy: 0.38
- 1. The training data set size is 60000.
- 2. Part A: Training Accuracy: 1.0, Test Accuracy: 0.41, Validation Accuracy: 0.41

- 3. Part B: Training Accuracy: 1.0, Test Accuracy: 0.42, Validation Accuracy: 0.42
- 4. Part C: Training Accuracy: 1.0, Test Accuracy: 0.41, Validation Accuracy: 0.40
- 5. Part D: Training Accuracy: 1.0, Test Accuracy: 0.40, Validation Accuracy: 0.41
- 6. Part E: Training Accuracy: 0.83, Test Accuracy: 0.43, Validation Accuracy: 0.43
- 7. Part F: Training Accuracy: 0.98, Test Accuracy: 0.45, Validation Accuracy: 0.45
- 1. The training data set size is 80000.
- 2. Part A: Training Accuracy: 1.0, Test Accuracy: 0.49, Validation Accuracy: 0.49
- 3. Part B: Training Accuracy: 1.0, Test Accuracy: 0.50, Validation Accuracy: 0.51
- 4. Part C: Training Accuracy: 1.0, Test Accuracy: 0.50, Validation Accuracy: 0.50
- 5. Part D: Training Accuracy: 1.0, Test Accuracy: 0.51, Validation Accuracy: 0.50
- 6. Part E: Training Accuracy: 0.78, Test Accuracy: 0.53, Validation Accuracy: 0.52
- 7. Part F: Training Accuracy: 0.97, Test Accuracy: 0.54, Validation Accuracy: 0.54
- 1. The training data set size is 100000.
- 2. Part A: Training Accuracy: 1.0, Test Accuracy: 0.55, Validation Accuracy: 0.55
- 3. Part B: Training Accuracy: 1.0, Test Accuracy: 0.55, Validation Accuracy: 0.56
- 4. Part C: Training Accuracy: 1.0, Test Accuracy: 0.55, Validation Accuracy: 0.55
- 5. Part D: Training Accuracy: 1.0, Test Accuracy: 0.54, Validation Accuracy: 0.54
- 6. Part E: Training Accuracy: 0.75, Test Accuracy: 0.55, Validation Accuracy: 0.55
- 7. Part F: Training Accuracy: 0.97, Test Accuracy: 0.56, Validation Accuracy: 0.55
- 1. The training data set size is 120000.
- 2. Part A: Training Accuracy: 1.0, Test Accuracy: 0.51, Validation Accuracy: 0.51
- 3. Part B: Training Accuracy: 1.0, Test Accuracy: 0.52, Validation Accuracy: 0.51
- 4. Part C: Training Accuracy: 1.0, Test Accuracy: 0.52, Validation Accuracy: 0.52
- 5. Part D: Training Accuracy: 1.0, Test Accuracy: 0.52, Validation Accuracy: 0.52
- 6. Part E: Training Accuracy: 0.84, Test Accuracy: 0.51, Validation Accuracy: 0.51
- 7. Part F: Training Accuracy: 0.97, Test Accuracy: 0.52, Validation Accuracy: 0.51
- 1. The training data set size is 140000.

2. Part A: Training Accuracy: 1.0, Test Accuracy: 0.52, Validation Accuracy: 0.52

3. Part B: Training Accuracy: 1.0, Test Accuracy: 0.51, Validation Accuracy: 0.52

4. Part C: Training Accuracy: 1.0, Test Accuracy: 0.50, Validation Accuracy: 0.50

5. Part D: Training Accuracy: 1.0, Test Accuracy: 0.51, Validation Accuracy: 0.52

6. Part E: Training Accuracy: 0.83, Test Accuracy: 0.51, Validation Accuracy: 0.50

7. Part F: Training Accuracy: 0.99, Test Accuracy: 0.51, Validation Accuracy: 0.50

With the aforementioned facts, it is obvious that the models with the mentioned sample exhibit the same pattern. Generally speaking, every model performs better on the test data as it receives more information with the amount of data. The apparent trade-off is the length of time required to train on huge datasets, which reduces it to the processing power a person possesses.

2 Neural Networks

The dataset used to build and train the neural networks is Fashion MNIST dataset. This dataset consists of 28×28 gray-scale images belonging to Zalando's 10 article classes. The first 784 columns of the data correspond to the pixel values. The last column is the target label. The data before training/testing needs is normalised(by division with the highest pixel value). In this problem I have used what is referred to as one-hot encoding of the output labels. Each y vector will have exactly one entry as being one which corresponds to the actual class label and all others will be zero.

2.1 Implementing Neural Networks

The neural network is trained using mini-batch Stochastic Gradient Descent (mini-batch SGD) algorithm the loss function is the mean squared error over the mini-batch. The inferred class label is simply the label having the highest probability as output by the network. Given a total of m examples, and M samples in each batch, the loss corresponding to batch number b can be described as:

$$J^{b}(\theta) = \frac{1}{2M} \sum_{i=(b-1)M}^{bM} \sum_{l=1}^{r} (y_{l}^{(i)} - o_{l}^{(i)})^{2}$$

$$\tag{1}$$

2.2 Training Neural Network with constant learning rate

The parameters for neural network training is:

1. Mini Batch Size: 100

2. No. of features: 784

3. Hidden Layers with no. of nodes: [5,10,15,20,25]

4. No. of target classes: 10

5. Learning Rate: 0.1

- 6. Stopping Criteria (α): 1e-8
- 7. Maximum Iterations after which gradient descent stops: 1000

The Accuracy and Confusion Matrix for is:

- 1. No. of hidden layer units = 5
- 2. Time to train the neural network = 161.96771264076233
- 3. Accuracy over Training data set = 0.86015
- 4. Accuracy over Test data set = 0.8278
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 4891 & 14 & 104 & 279 & 49 & 10 & 519 & 45 & 74 & 15 \\ 18 & 5702 & 81 & 139 & 20 & 3 & 28 & 4 & 5 & 0 \\ 96 & 20 & 4667 & 51 & 722 & 0 & 395 & 13 & 31 & 5 \\ 135 & 159 & 78 & 5170 & 188 & 7 & 205 & 41 & 14 & 3 \\ 17 & 66 & 463 & 123 & 4727 & 2 & 568 & 13 & 21 & 0 \\ 3 & 0 & 0 & 9 & 1 & 5694 & 3 & 129 & 37 & 124 \\ 773 & 18 & 507 & 165 & 671 & 5 & 3703 & 29 & 110 & 19 \\ 2 & 0 & 0 & 1 & 1 & 138 & 5 & 5667 & 17 & 169 \\ 17 & 2 & 24 & 32 & 43 & 39 & 108 & 23 & 5689 & 23 \\ 2 & 0 & 0 & 1 & 0 & 87 & 9 & 199 & 3 & 5699 \end{bmatrix}$$

$$\begin{bmatrix} 764 & 2 & 25 & 56 & 11 & 2 & 112 & 9 & 17 & 2 \\ 3 & 941 & 10 & 32 & 7 & 0 & 4 & 1 & 2 & 0 \\ 26 & 6 & 731 & 10 & 137 & 0 & 80 & 2 & 6 & 2 \\ 33 & 29 & 25 & 822 & 33 & 0 & 45 & 8 & 5 & 0 \\ 1 & 11 & 100 & 26 & 755 & 0 & 101 & 3 & 3 & 0 \\ 2 & 1 & 0 & 1 & 0 & 914 & 0 & 33 & 15 & 34 \\ 130 & 2 & 104 & 40 & 132 & 0 & 545 & 10 & 29 & 8 \\ 1 & 0 & 0 & 0 & 0 & 28 & 1 & 944 & 1 & 25 \\ 1 & 1 & 8 & 10 & 5 & 6 & 24 & 4 & 933 & 8 \\ 0 & 0 & 0 & 0 & 0 & 25 & 2 & 41 & 3 & 929 \end{bmatrix}$$

- 1. No. of hidden layer units = 10
- 2. Time to train the neural network = 212.68221139907837
- 4. Accuracy over Test data set = 0.8533

5. Confusion Matrix over Training data set =

F 5183	14	90	204	28	5	402	0	72	2	
26	5786	34	118	19	3	10	1	3	0	
69	3	5096	53	336	1	401	0	40	1	
157	41	59	5431	179	1	113	0	19	0	
22	8	566	148	4753	0	472	0	31	0	(4)
4	1	0	4	1	5833	1	107	10	39	(4)
753	13	537	182	250	1	4143	0	119	2	
0	0	0	0	0	101	0	5701	13	185	
19	1	25	22	14	8	62	10	5833	6	
L 1	1	0	2	0	39	0	132	0	5825	

$$\begin{bmatrix} 821 & 2 & 16 & 42 & 8 & 1 & 95 & 0 & 15 & 0 \\ 7 & 950 & 9 & 24 & 8 & 0 & 1 & 0 & 1 & 0 \\ 15 & 1 & 797 & 11 & 73 & 1 & 91 & 0 & 11 & 0 \\ 25 & 13 & 17 & 864 & 43 & 0 & 32 & 0 & 4 & 2 \\ 1 & 1 & 119 & 36 & 731 & 0 & 104 & 0 & 8 & 0 \\ 0 & 0 & 0 & 1 & 0 & 927 & 0 & 44 & 3 & 25 \\ 145 & 2 & 110 & 37 & 71 & 1 & 603 & 0 & 30 & 1 \\ 0 & 0 & 0 & 0 & 0 & 31 & 0 & 929 & 0 & 40 \\ 3 & 0 & 6 & 6 & 5 & 3 & 10 & 4 & 961 & 2 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 & 34 & 1 & 950 \end{bmatrix}$$

- 1. No. of hidden layer units = 15
- 2. Time to train the neural network = 247.7032928466797
- 4. Accuracy over Test data set = 0.8631
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5064 & 3 & 64 & 176 & 28 & 3 & 602 & 0 & 59 & 1 \\ 16 & 5842 & 14 & 87 & 18 & 0 & 17 & 0 & 5 & 1 \\ 48 & 4 & 4969 & 42 & 539 & 1 & 373 & 0 & 24 & 0 \\ 120 & 28 & 37 & 5448 & 203 & 0 & 137 & 0 & 26 & 1 \\ 14 & 5 & 292 & 133 & 5249 & 0 & 288 & 0 & 19 & 0 \\ 3 & 2 & 3 & 0 & 1 & 5857 & 2 & 102 & 10 & 20 \\ 523 & 16 & 305 & 149 & 357 & 1 & 4587 & 0 & 60 & 2 \\ 0 & 0 & 0 & 3 & 0 & 92 & 0 & 5785 & 10 & 110 \\ 16 & 2 & 21 & 23 & 23 & 3 & 47 & 13 & 5847 & 5 \\ 0 & 1 & 0 & 0 & 0 & 32 & 0 & 146 & 5 & 5816 \end{bmatrix}$$

$$\begin{bmatrix} 800 & 2 & 20 & 34 & 9 & 3 & 121 & 0 & 11 & 0 \\ 3 & 958 & 2 & 24 & 6 & 0 & 6 & 0 & 1 & 0 \\ 18 & 2 & 757 & 9 & 117 & 0 & 90 & 0 & 7 & 0 \\ 24 & 14 & 12 & 864 & 41 & 0 & 34 & 0 & 10 & 1 \\ 0 & 1 & 83 & 34 & 797 & 1 & 76 & 0 & 8 & 0 \\ 2 & 0 & 0 & 0 & 0 & 945 & 0 & 34 & 4 & 15 \\ 112 & 1 & 70 & 37 & 97 & 0 & 661 & 0 & 22 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30 & 0 & 944 & 0 & 26 \\ 4 & 1 & 4 & 6 & 6 & 4 & 12 & 4 & 958 & 1 \\ 0 & 0 & 0 & 0 & 0 & 15 & 1 & 37 & 0 & 947 \end{bmatrix}$$
 (7)

- 1. No. of hidden layer units = 20
- 2. Time to train the neural network = 266.08308243751526
- 4. Accuracy over Test data set = 0.868
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5283 & 10 & 73 & 141 & 21 & 3 & 425 & 1 & 43 & 0 \\ 15 & 5870 & 14 & 81 & 5 & 2 & 11 & 0 & 2 & 0 \\ 67 & 4 & 5004 & 48 & 509 & 2 & 349 & 0 & 17 & 0 \\ 113 & 34 & 34 & 5534 & 145 & 0 & 127 & 1 & 11 & 1 \\ 17 & 6 & 352 & 130 & 5187 & 0 & 292 & 0 & 15 & 1 \\ 1 & 0 & 1 & 0 & 0 & 5883 & 3 & 81 & 11 & 20 \\ 541 & 18 & 280 & 122 & 307 & 3 & 4685 & 2 & 40 & 2 \\ 0 & 0 & 0 & 0 & 0 & 63 & 1 & 5797 & 6 & 133 \\ 17 & 3 & 20 & 16 & 17 & 2 & 50 & 11 & 5857 & 7 \\ 1 & 0 & 0 & 0 & 0 & 27 & 0 & 116 & 1 & 5855 \end{bmatrix}$$

6. Confusion Matrix over Test data set =

$$\begin{bmatrix} 825 & 1 & 13 & 34 & 6 & 1 & 105 & 0 & 15 & 0 \\ 5 & 958 & 4 & 23 & 5 & 0 & 4 & 0 & 1 & 0 \\ 17 & 3 & 776 & 12 & 100 & 3 & 83 & 1 & 5 & 0 \\ 30 & 10 & 14 & 874 & 31 & 1 & 38 & 0 & 2 & 0 \\ 1 & 2 & 93 & 35 & 784 & 0 & 80 & 0 & 5 & 0 \\ 1 & 1 & 0 & 0 & 0 & 943 & 1 & 29 & 2 & 23 \\ 123 & 2 & 86 & 26 & 83 & 0 & 660 & 3 & 16 & 1 \\ 0 & 0 & 0 & 0 & 0 & 24 & 0 & 946 & 0 & 30 \\ 3 & 1 & 5 & 6 & 4 & 2 & 12 & 4 & 963 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & 38 & 1 & 951 \end{bmatrix}$$

1. No. of hidden layer units = 25

- 2. Time to train the neural network = 295.7713224887848
- 4. Accuracy over Test data set = 0.8732
- 5. Confusion Matrix over Training data set =

$\lceil 5475 \rceil$	7	77	137	17	3	239	2	42	1 7	
16	5876	8	79	9	1	9	0	2	0	
83	2	5209	53	422	1	211	0	18	1	
136	17	29	5535	176	0	95	0	11	1	
12	7	342	117	5310	1	189	0	22	0	(10)
3	1	3	0	1	5900	2	61	8	21	(10)
542	13	280	131	340	1	4648	1	43	1	
0	0	1	0	0	49	0	5818	7	125	
22	2	32	15	19	10	37	13	5848	2	
L 1	1	0	0	0	22	0	107	5	5864	

$$\begin{bmatrix} 831 & 1 & 16 & 36 & 5 & 2 & 97 & 0 & 11 & 1 \\ 4 & 957 & 6 & 24 & 3 & 0 & 5 & 0 & 1 & 0 \\ 22 & 2 & 796 & 15 & 93 & 0 & 67 & 0 & 4 & 1 \\ 28 & 13 & 13 & 879 & 33 & 1 & 26 & 0 & 7 & 0 \\ 0 & 0 & 99 & 33 & 815 & 0 & 46 & 0 & 7 & 0 \\ 0 & 0 & 0 & 1 & 0 & 953 & 0 & 26 & 1 & 19 \\ 146 & 2 & 92 & 30 & 86 & 0 & 628 & 1 & 14 & 1 \\ 0 & 0 & 0 & 0 & 0 & 27 & 0 & 945 & 0 & 28 \\ 4 & 1 & 10 & 6 & 5 & 1 & 2 & 6 & 965 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 32 & 1 & 963 \end{bmatrix}$$

$$(11)$$

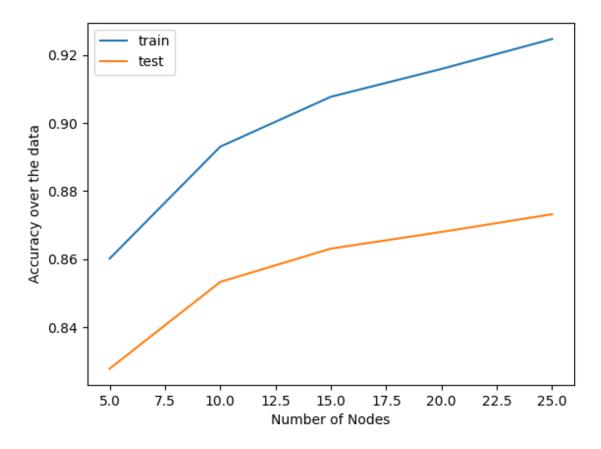


Figure 7: No. of Nodes vs Accuracy

2.3 Training Neural Network with adaptive learning rate

The parameters for neural network training is :

- 1. Mini Batch Size: 100
- 2. No. of features: 784
- 3. Hidden Layers with no. of nodes : [5,10,15,20,25]
- 4. No. of target classes: 10
- 5. Learning Rate: Adaptive
- 6. Stopping Criteria (α) : 1e-8
- 7. Maximum Iterations after which gradient descent stops: 1000

The Accuracy and Confusion Matrix for is:

1. No. of hidden layer units = 5

- 2. Time to train the neural network = 175.3580424785614
- 3. Accuracy over Training data set = 0.8551
- 4. Accuracy over Test data set = 0.8227
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 4530 & 29 & 144 & 191 & 114 & 9 & 819 & 17 & 136 & 11 \\ 16 & 5678 & 94 & 133 & 30 & 2 & 27 & 2 & 12 & 6 \\ 147 & 6 & 4445 & 32 & 872 & 2 & 437 & 0 & 59 & 0 \\ 99 & 74 & 92 & 4986 & 321 & 7 & 336 & 34 & 38 & 13 \\ 12 & 5 & 457 & 75 & 5068 & 2 & 352 & 0 & 29 & 0 \\ 5 & 0 & 2 & 1 & 2 & 5662 & 2 & 215 & 32 & 79 \\ 756 & 9 & 553 & 90 & 671 & 0 & 3796 & 9 & 112 & 4 \\ 1 & 0 & 1 & 0 & 1 & 125 & 1 & 5668 & 13 & 190 \\ 42 & 3 & 24 & 21 & 49 & 9 & 71 & 31 & 5745 & 5 \\ 3 & 2 & 0 & 0 & 0 & 45 & 1 & 218 & 3 & 5728 \end{bmatrix}$$

$$\begin{bmatrix} 737 & 4 & 22 & 37 & 15 & 3 & 140 & 7 & 33 & 2 \\ 4 & 941 & 17 & 29 & 4 & 0 & 2 & 0 & 2 & 1 \\ 42 & 2 & 705 & 10 & 161 & 0 & 77 & 0 & 3 & 0 \\ 19 & 25 & 16 & 784 & 63 & 0 & 79 & 3 & 10 & 1 \\ 1 & 2 & 105 & 16 & 791 & 0 & 75 & 0 & 10 & 0 \\ 1 & 0 & 1 & 1 & 0 & 906 & 0 & 51 & 14 & 26 \\ 118 & 1 & 112 & 24 & 142 & 0 & 569 & 1 & 31 & 2 \\ 0 & 0 & 0 & 0 & 0 & 29 & 0 & 931 & 1 & 39 \\ 13 & 0 & 3 & 4 & 15 & 3 & 17 & 11 & 934 & 0 \\ 3 & 0 & 0 & 0 & 0 & 12 & 1 & 55 & 0 & 929 \end{bmatrix}$$
 (13)

- 1. No. of hidden layer units = 10
- 2. Time to train the neural network = 221.94053149223328
- 4. Accuracy over Test data set = 0.8496
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 808 & 3 & 15 & 56 & 9 & 0 & 94 & 1 & 13 & 1 \\ 4 & 957 & 5 & 27 & 4 & 0 & 3 & 0 & 0 & 0 \\ 19 & 3 & 769 & 12 & 121 & 1 & 69 & 2 & 4 & 0 \\ 32 & 13 & 13 & 874 & 35 & 0 & 26 & 0 & 6 & 1 \\ 0 & 2 & 114 & 39 & 769 & 3 & 62 & 0 & 11 & 0 \\ 0 & 0 & 1 & 1 & 0 & 924 & 0 & 46 & 5 & 23 \\ 139 & 2 & 123 & 54 & 110 & 2 & 552 & 0 & 17 & 1 \\ 0 & 0 & 0 & 0 & 0 & 28 & 0 & 945 & 0 & 27 \\ 1 & 1 & 8 & 5 & 8 & 4 & 19 & 5 & 948 & 1 \\ 0 & 0 & 0 & 0 & 0 & 12 & 1 & 37 & 0 & 950 \end{bmatrix}$$
 (15)

- 1. No. of hidden layer units = 15
- 2. Time to train the neural network = 264.362580537796
- 4. Accuracy over Test data set = 0.8609
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5211 & 11 & 74 & 136 & 30 & 5 & 480 & 1 & 50 & 2 \\ 17 & 5851 & 18 & 82 & 10 & 1 & 17 & 0 & 4 & 0 \\ 80 & 4 & 4908 & 46 & 568 & 3 & 347 & 0 & 44 & 0 \\ 143 & 22 & 32 & 5466 & 177 & 0 & 138 & 1 & 20 & 1 \\ 13 & 12 & 321 & 123 & 5175 & 3 & 327 & 1 & 24 & 1 \\ 3 & 0 & 0 & 0 & 0 & 5856 & 5 & 95 & 14 & 27 \\ 590 & 13 & 360 & 129 & 371 & 1 & 4451 & 1 & 83 & 1 \\ 0 & 0 & 0 & 2 & 0 & 61 & 0 & 5830 & 9 & 98 \\ 19 & 2 & 13 & 25 & 19 & 10 & 59 & 16 & 5834 & 3 \\ 0 & 0 & 0 & 0 & 29 & 0 & 161 & 4 & 5806 \end{bmatrix}$$

6. Confusion Matrix over Test data set =

$$\begin{bmatrix} 811 & 6 & 13 & 32 & 5 & 3 & 114 & 1 & 15 & 0 \\ 3 & 958 & 4 & 22 & 6 & 0 & 4 & 0 & 3 & 0 \\ 28 & 2 & 749 & 10 & 113 & 2 & 85 & 0 & 11 & 0 \\ 30 & 9 & 11 & 862 & 37 & 2 & 36 & 1 & 9 & 3 \\ 1 & 1 & 91 & 33 & 787 & 2 & 74 & 0 & 11 & 0 \\ 1 & 1 & 0 & 0 & 0 & 939 & 0 & 36 & 3 & 20 \\ 129 & 1 & 77 & 36 & 88 & 2 & 646 & 0 & 21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 26 & 0 & 950 & 0 & 24 \\ 3 & 1 & 2 & 5 & 5 & 3 & 17 & 5 & 959 & 0 \\ 0 & 0 & 1 & 0 & 0 & 8 & 2 & 41 & 0 & 948 \end{bmatrix}$$
 (17)

1. No. of hidden layer units = 20

- 2. Time to train the neural network = 283.87608575820923
- 3. Accuracy over Training data set = 0.9173
- 4. Accuracy over Test data set = 0.866
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5387 & 4 & 82 & 128 & 18 & 6 & 322 & 0 & 52 & 1 \\ 15 & 5850 & 14 & 94 & 8 & 0 & 14 & 0 & 5 & 0 \\ 81 & 5 & 5244 & 47 & 387 & 1 & 215 & 1 & 18 & 1 \\ 124 & 22 & 55 & 5535 & 148 & 0 & 97 & 0 & 19 & 0 \\ 15 & 10 & 433 & 153 & 5184 & 0 & 181 & 0 & 22 & 2 \\ 4 & 1 & 0 & 1 & 0 & 5887 & 2 & 70 & 11 & 24 \\ 599 & 11 & 457 & 127 & 305 & 0 & 4451 & 2 & 46 & 2 \\ 0 & 0 & 0 & 0 & 0 & 60 & 0 & 5821 & 11 & 108 \\ 21 & 2 & 29 & 18 & 17 & 2 & 45 & 20 & 5841 & 5 \\ 1 & 1 & 0 & 0 & 0 & 25 & 0 & 134 & 1 & 5838 \end{bmatrix}$$
 (18)

$$\begin{bmatrix} 819 & 2 & 15 & 37 & 5 & 1 & 106 & 0 & 15 & 0 \\ 3 & 958 & 1 & 27 & 6 & 0 & 4 & 0 & 1 & 0 \\ 23 & 2 & 819 & 10 & 84 & 1 & 52 & 1 & 8 & 0 \\ 26 & 8 & 18 & 885 & 25 & 0 & 31 & 1 & 6 & 0 \\ 0 & 1 & 119 & 43 & 779 & 0 & 53 & 0 & 5 & 0 \\ 0 & 1 & 0 & 0 & 0 & 943 & 0 & 34 & 4 & 18 \\ 146 & 0 & 116 & 32 & 88 & 0 & 599 & 0 & 19 & 0 \\ 0 & 0 & 0 & 0 & 0 & 28 & 0 & 943 & 0 & 29 \\ 4 & 1 & 3 & 7 & 5 & 2 & 15 & 4 & 959 & 0 \\ 0 & 1 & 0 & 0 & 0 & 11 & 0 & 31 & 1 & 956 \end{bmatrix}$$

- 1. No. of hidden layer units = 25
- 2. Time to train the neural network = 350.8760907649994
- 4. Accuracy over Test data set = 0.8741
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5399 & 5 & 90 & 130 & 20 & 3 & 294 & 0 & 59 & 0 \\ 17 & 5872 & 11 & 81 & 7 & 0 & 10 & 0 & 2 & 0 \\ 73 & 3 & 5281 & 57 & 373 & 2 & 186 & 1 & 24 & 0 \\ 115 & 24 & 39 & 5586 & 143 & 0 & 79 & 0 & 14 & 0 \\ 18 & 5 & 357 & 135 & 5267 & 1 & 197 & 0 & 20 & 0 \\ 1 & 0 & 0 & 1 & 0 & 5905 & 1 & 68 & 9 & 15 \\ 515 & 18 & 347 & 125 & 285 & 4 & 4662 & 0 & 44 & 0 \\ 0 & 0 & 0 & 0 & 0 & 53 & 1 & 5825 & 10 & 111 \\ 15 & 4 & 31 & 21 & 15 & 2 & 32 & 14 & 5864 & 2 \\ 0 & 0 & 0 & 1 & 0 & 22 & 0 & 116 & 3 & 5858 \end{bmatrix}$$
 (20)

$$\begin{bmatrix} 832 & 2 & 16 & 37 & 3 & 3 & 91 & 0 & 16 & 0 \\ 4 & 962 & 2 & 25 & 3 & 0 & 3 & 0 & 1 & 0 \\ 18 & 1 & 805 & 15 & 97 & 2 & 56 & 0 & 6 & 0 \\ 32 & 7 & 17 & 892 & 28 & 1 & 19 & 0 & 4 & 0 \\ 0 & 2 & 107 & 39 & 792 & 0 & 54 & 0 & 6 & 0 \\ 1 & 0 & 0 & 1 & 1 & 945 & 0 & 28 & 2 & 22 \\ 128 & 0 & 105 & 34 & 70 & 1 & 644 & 0 & 18 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20 & 0 & 949 & 1 & 30 \\ 0 & 1 & 7 & 5 & 1 & 5 & 11 & 4 & 966 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11 & 0 & 34 & 1 & 954 \end{bmatrix}$$
 (21)

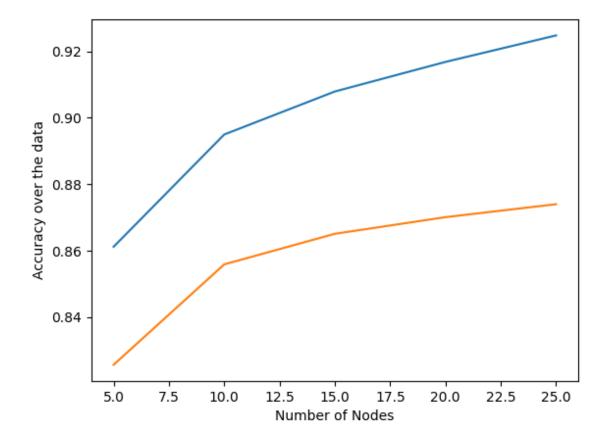


Figure 8: No. of Nodes vs Accuracy

2.4 Sigmoid and ReLu as activation Functions

Using Sigmoid as Activation Function, the accuracy and confusion matrices are:

1. No. of hidden layer units = [100, 100]

- 2. Time to train the neural network = 904.3589193820953
- 3. Accuracy over Training data set = 0.9696
- 4. Accuracy over Test data set = 0.8848
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5790 & 8 & 60 & 38 & 10 & 3 & 58 & 0 & 31 & 2 \\ 17 & 5922 & 5 & 36 & 6 & 1 & 6 & 1 & 4 & 2 \\ 66 & 1 & 5701 & 31 & 122 & 2 & 67 & 0 & 10 & 0 \\ 48 & 5 & 40 & 5806 & 53 & 0 & 38 & 0 & 10 & 0 \\ 25 & 5 & 139 & 74 & 5686 & 1 & 58 & 0 & 12 & 0 \\ 0 & 1 & 1 & 1 & 0 & 5970 & 0 & 20 & 4 & 3 \\ 172 & 12 & 138 & 69 & 93 & 0 & 5502 & 2 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11 & 0 & 5942 & 3 & 44 \\ 14 & 3 & 17 & 14 & 13 & 1 & 5 & 6 & 5922 & 5 \\ 0 & 0 & 0 & 2 & 0 & 8 & 0 & 55 & 0 & 5935 \end{bmatrix}$$

$$\begin{bmatrix} 855 & 1 & 17 & 17 & 4 & 4 & 91 & 0 & 11 & 0 \\ 8 & 968 & 0 & 14 & 2 & 0 & 7 & 0 & 1 & 0 \\ 17 & 2 & 834 & 12 & 76 & 0 & 56 & 0 & 3 & 0 \\ 32 & 6 & 23 & 889 & 28 & 0 & 18 & 0 & 4 & 0 \\ 1 & 1 & 104 & 34 & 793 & 1 & 63 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 954 & 0 & 26 & 1 & 18 \\ 135 & 3 & 80 & 24 & 67 & 0 & 678 & 0 & 13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 22 & 0 & 955 & 0 & 23 \\ 6 & 0 & 5 & 5 & 3 & 2 & 11 & 5 & 963 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & 30 & 1 & 959 \end{bmatrix}$$

Using ReLu as Activation Function, the accuracy and confusion matrices are:

- 1. No. of hidden layer units = [100, 100]
- 2. Time to train the neural network = 899.413823712596
- 4. Accuracy over Test data set = 0.8827

5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5831 & 6 & 31 & 28 & 16 & 2 & 69 & 0 & 17 & 0 \\ 7 & 5949 & 3 & 14 & 9 & 0 & 17 & 0 & 1 & 0 \\ 22 & 3 & 5810 & 32 & 47 & 0 & 73 & 0 & 12 & 1 \\ 17 & 4 & 19 & 5887 & 16 & 0 & 50 & 0 & 7 & 0 \\ 8 & 4 & 30 & 24 & 5886 & 2 & 41 & 0 & 5 & 0 \\ 0 & 0 & 1 & 2 & 0 & 5975 & 0 & 14 & 5 & 3 \\ 55 & 13 & 69 & 36 & 60 & 1 & 5747 & 4 & 14 & 1 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 5952 & 4 & 35 \\ 4 & 3 & 9 & 3 & 9 & 3 & 15 & 14 & 5937 & 3 \\ 0 & 1 & 0 & 0 & 0 & 3 & 0 & 26 & 1 & 5969 \end{bmatrix}$$

6. Confusion Matrix over Test data set =

$$\begin{bmatrix} 807 & 1 & 19 & 26 & 5 & 2 & 132 & 0 & 8 & 0 \\ 5 & 968 & 3 & 12 & 6 & 1 & 4 & 0 & 1 & 0 \\ 21 & 3 & 809 & 9 & 79 & 2 & 71 & 0 & 5 & 1 \\ 23 & 8 & 11 & 884 & 34 & 1 & 35 & 0 & 4 & 0 \\ 1 & 1 & 83 & 33 & 820 & 1 & 58 & 0 & 3 & 0 \\ 2 & 0 & 0 & 1 & 0 & 948 & 2 & 31 & 3 & 13 \\ 116 & 2 & 70 & 20 & 65 & 0 & 718 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 & 0 & 19 & 0 & 962 & 0 & 19 \\ 4 & 0 & 5 & 8 & 8 & 3 & 9 & 7 & 956 & 0 \\ 0 & 0 & 0 & 1 & 0 & 10 & 1 & 33 & 0 & 955 \end{bmatrix}$$

2.5 Varying the number of hidden layers

Varying the number of hidden layers increases the size of the set of hypothesis functions that neural network can approximate. The number of units per hidden layer is 50. The no. of hidden layers are varied from 2 to 5.

Using sigmoid as activation function:

- 1. No. of hidden layer units = [50, 50]
- 2. Time to train the neural network = 585.0355925559998
- 4. Accuracy over Test data set = 0.8778
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5598 & 8 & 65 & 65 & 21 & 2 & 211 & 0 & 28 & 2 \\ 13 & 5909 & 2 & 50 & 13 & 1 & 9 & 0 & 2 & 1 \\ 62 & 3 & 5377 & 40 & 321 & 1 & 181 & 0 & 14 & 1 \\ 37 & 6 & 32 & 5735 & 91 & 0 & 90 & 1 & 8 & 0 \\ 11 & 3 & 114 & 94 & 5630 & 0 & 137 & 0 & 11 & 0 \\ 1 & 1 & 0 & 1 & 0 & 5972 & 2 & 18 & 2 & 3 \\ 193 & 14 & 106 & 84 & 153 & 1 & 5430 & 2 & 17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 & 0 & 5948 & 4 & 36 \\ 15 & 4 & 11 & 13 & 20 & 2 & 16 & 11 & 5904 & 4 \\ 0 & 0 & 1 & 0 & 0 & 10 & 0 & 58 & 3 & 5928 \end{bmatrix}$$
 (26)

$$\begin{bmatrix} 818 & 1 & 15 & 30 & 4 & 3 & 118 & 0 & 11 & 0 \\ 6 & 961 & 6 & 22 & 3 & 0 & 1 & 0 & 1 & 0 \\ 15 & 2 & 762 & 13 & 120 & 1 & 86 & 0 & 1 & 0 \\ 29 & 5 & 19 & 883 & 29 & 0 & 32 & 0 & 3 & 0 \\ 1 & 2 & 71 & 32 & 828 & 0 & 60 & 1 & 5 & 0 \\ 0 & 0 & 0 & 2 & 0 & 950 & 0 & 26 & 2 & 20 \\ 110 & 1 & 70 & 28 & 77 & 0 & 703 & 0 & 11 & 0 \\ 0 & 0 & 0 & 0 & 0 & 24 & 0 & 955 & 0 & 21 \\ 3 & 0 & 4 & 7 & 3 & 2 & 10 & 6 & 965 & 0 \\ 0 & 0 & 0 & 0 & 0 & 13 & 1 & 33 & 0 & 953 \end{bmatrix}$$

- 1. No. of hidden layer units = [50, 50, 50]
- 2. Time to train the neural network = 732.591703414917
- 4. Accuracy over Test data set = 0.8744
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5729 & 4 & 95 & 52 & 33 & 3 & 58 & 0 & 25 & 1 \\ 19 & 5917 & 12 & 33 & 15 & 0 & 3 & 0 & 1 & 0 \\ 63 & 7 & 5642 & 30 & 194 & 2 & 50 & 1 & 9 & 2 \\ 58 & 12 & 44 & 5708 & 146 & 0 & 25 & 0 & 4 & 3 \\ 16 & 8 & 200 & 51 & 5675 & 0 & 42 & 0 & 6 & 2 \\ 4 & 0 & 0 & 2 & 0 & 5957 & 1 & 25 & 3 & 8 \\ 329 & 9 & 339 & 47 & 283 & 3 & 4974 & 0 & 14 & 2 \\ 1 & 0 & 0 & 0 & 0 & 10 & 0 & 5941 & 0 & 48 \\ 25 & 6 & 27 & 7 & 22 & 1 & 7 & 1 & 5900 & 4 \\ 0 & 0 & 0 & 0 & 1 & 7 & 0 & 29 & 2 & 5961 \end{bmatrix}$$
 (28)

6. Confusion Matrix over Test data set =

$$\begin{bmatrix} 847 & 3 & 21 & 29 & 9 & 0 & 81 & 0 & 10 & 0 \\ 11 & 970 & 2 & 12 & 2 & 0 & 2 & 0 & 1 & 0 \\ 20 & 3 & 828 & 12 & 92 & 0 & 43 & 0 & 1 & 1 \\ 43 & 9 & 20 & 859 & 45 & 1 & 19 & 0 & 4 & 0 \\ 0 & 2 & 118 & 23 & 820 & 0 & 33 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 944 & 0 & 30 & 2 & 24 \\ 150 & 1 & 101 & 26 & 99 & 0 & 613 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 18 & 0 & 945 & 0 & 37 \\ 4 & 1 & 6 & 3 & 8 & 7 & 10 & 4 & 956 & 1 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 28 & 1 & 962 \end{bmatrix}$$
 (29)

1. No. of hidden layer units = [50, 50, 50, 50]

- 2. Time to train the neural network = 816.7182228565216
- 4. Accuracy over Test data set = 0.8663
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5720 & 9 & 86 & 54 & 14 & 16 & 84 & 0 & 17 & 0 \\ 12 & 5911 & 20 & 37 & 5 & 3 & 7 & 1 & 4 & 0 \\ 64 & 3 & 5692 & 29 & 124 & 5 & 76 & 0 & 7 & 0 \\ 37 & 4 & 61 & 5802 & 51 & 1 & 36 & 0 & 8 & 0 \\ 18 & 10 & 459 & 70 & 5376 & 3 & 59 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 5976 & 1 & 16 & 4 & 2 \\ 422 & 11 & 526 & 108 & 126 & 7 & 4792 & 0 & 7 & 1 \\ 0 & 0 & 0 & 0 & 0 & 34 & 0 & 5939 & 4 & 23 \\ 27 & 5 & 39 & 12 & 8 & 8 & 7 & 7 & 5879 & 8 \\ 0 & 2 & 1 & 0 & 0 & 9 & 0 & 37 & 0 & 5951 \end{bmatrix}$$

$$\begin{bmatrix} 848 & 1 & 26 & 31 & 6 & 4 & 78 & 0 & 6 & 0 \\ 5 & 966 & 4 & 16 & 3 & 1 & 5 & 0 & 0 & 0 \\ 17 & 0 & 874 & 11 & 52 & 2 & 42 & 0 & 2 & 0 \\ 37 & 12 & 22 & 870 & 35 & 1 & 21 & 0 & 2 & 0 \\ 1 & 1 & 172 & 36 & 733 & 0 & 53 & 0 & 4 & 0 \\ 1 & 0 & 0 & 0 & 0 & 948 & 0 & 31 & 2 & 18 \\ 159 & 1 & 124 & 39 & 71 & 2 & 589 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 32 & 0 & 941 & 0 & 27 \\ 2 & 0 & 8 & 6 & 8 & 10 & 6 & 4 & 954 & 2 \\ 0 & 0 & 0 & 0 & 0 & 17 & 1 & 42 & 0 & 940 \end{bmatrix}$$

- 1. No. of hidden layer units = [50, 50, 50, 50, 50]
- 2. Time to train the neural network = 921.581799030304
- 4. Accuracy over Test data set = 0.8676
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5670 & 7 & 33 & 75 & 33 & 17 & 125 & 0 & 40 & 0 \\ 56 & 5783 & 40 & 64 & 12 & 2 & 39 & 1 & 2 & 1 \\ 44 & 2 & 5571 & 47 & 191 & 4 & 127 & 0 & 14 & 0 \\ 61 & 2 & 33 & 5740 & 70 & 5 & 75 & 0 & 13 & 1 \\ 29 & 10 & 144 & 122 & 5542 & 2 & 145 & 0 & 6 & 0 \\ 1 & 1 & 1 & 0 & 1 & 5965 & 1 & 20 & 7 & 3 \\ 226 & 7 & 102 & 69 & 126 & 2 & 5452 & 0 & 15 & 1 \\ 0 & 0 & 0 & 0 & 0 & 13 & 0 & 5940 & 12 & 35 \\ 18 & 3 & 29 & 18 & 20 & 20 & 40 & 3 & 5846 & 3 \\ 0 & 0 & 2 & 1 & 0 & 7 & 0 & 29 & 3 & 5958 \end{bmatrix}$$

$$\begin{bmatrix} 805 & 1 & 23 & 36 & 6 & 1 & 120 & 0 & 8 & 0 \\ 7 & 944 & 7 & 29 & 5 & 0 & 7 & 0 & 1 & 0 \\ 21 & 0 & 800 & 8 & 84 & 0 & 86 & 0 & 1 & 0 \\ 38 & 16 & 8 & 862 & 26 & 1 & 46 & 0 & 3 & 0 \\ 5 & 0 & 98 & 30 & 782 & 0 & 75 & 0 & 10 & 0 \\ 0 & 1 & 0 & 0 & 0 & 941 & 0 & 36 & 8 & 14 \\ 120 & 0 & 68 & 33 & 84 & 2 & 683 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 21 & 0 & 954 & 0 & 25 \\ 0 & 0 & 4 & 6 & 9 & 12 & 12 & 3 & 951 & 3 \\ 0 & 0 & 0 & 1 & 0 & 11 & 0 & 33 & 1 & 954 \end{bmatrix}$$

Using ReLu as activation function

- 1. No. of hidden layer units = [50, 50]
- 2. Time to train the neural network = 601.0700619220734
- 4. Accuracy over Test data set = 0.8689
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5402 & 12 & 30 & 47 & 23 & 2 & 445 & 1 & 38 & 0 \\ 6 & 5905 & 6 & 50 & 9 & 6 & 11 & 1 & 5 & 1 \\ 18 & 2 & 5227 & 28 & 269 & 1 & 446 & 1 & 8 & 0 \\ 12 & 5 & 22 & 5746 & 109 & 0 & 97 & 2 & 7 & 0 \\ 3 & 5 & 96 & 53 & 5630 & 1 & 197 & 0 & 14 & 1 \\ 0 & 2 & 1 & 1 & 0 & 5967 & 2 & 17 & 4 & 6 \\ 96 & 9 & 79 & 65 & 135 & 0 & 5584 & 0 & 28 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 5921 & 8 & 66 \\ 5 & 5 & 10 & 10 & 24 & 1 & 27 & 10 & 5904 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 27 & 0 & 5968 \end{bmatrix}$$

$$\begin{bmatrix} 773 & 3 & 9 & 23 & 8 & 2 & 171 & 0 & 11 & 0 \\ 2 & 966 & 4 & 20 & 3 & 0 & 3 & 0 & 2 & 0 \\ 12 & 2 & 747 & 9 & 105 & 2 & 119 & 1 & 3 & 0 \\ 24 & 17 & 8 & 856 & 49 & 2 & 37 & 1 & 5 & 1 \\ 4 & 1 & 75 & 33 & 803 & 0 & 79 & 0 & 4 & 1 \\ 0 & 0 & 0 & 2 & 0 & 940 & 1 & 31 & 6 & 20 \\ 84 & 4 & 54 & 21 & 65 & 2 & 755 & 0 & 14 & 1 \\ 0 & 0 & 0 & 0 & 0 & 18 & 0 & 944 & 0 & 38 \\ 4 & 3 & 8 & 5 & 7 & 4 & 13 & 6 & 950 & 0 \\ 1 & 0 & 0 & 1 & 0 & 11 & 1 & 31 & 0 & 955 \end{bmatrix}$$

- 1. No. of hidden layer units = [50, 50, 50]
- 2. Time to train the neural network = 722.5276672840118
- 3. Accuracy over Training data set = 0.8903
- 4. Accuracy over Test data set = 0.8004
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5836 & 6 & 30 & 30 & 15 & 2 & 56 & 0 & 25 & 0 \\ 14 & 5935 & 3 & 26 & 11 & 1 & 6 & 1 & 3 & 0 \\ 41 & 3 & 5706 & 20 & 124 & 3 & 85 & 3 & 15 & 0 \\ 42 & 12 & 18 & 5840 & 48 & 1 & 34 & 0 & 5 & 0 \\ 13 & 7 & 144 & 41 & 5718 & 0 & 67 & 0 & 10 & 0 \\ 2 & 3 & 0 & 1 & 0 & 5976 & 0 & 17 & 1 & 0 \\ 171 & 13 & 83 & 36 & 79 & 2 & 5589 & 1 & 26 & 0 \\ 0 & 0 & 0 & 0 & 1 & 15 & 0 & 5977 & 5 & 2 \\ 11 & 2 & 9 & 7 & 12 & 4 & 11 & 5 & 5939 & 0 \\ 23 & 5 & 30 & 10 & 443 & 1851 & 21 & 2287 & 428 & 902 \end{bmatrix}$$

$$(36)$$

$$\begin{bmatrix} 850 & 3 & 14 & 25 & 1 & 0 & 96 & 2 & 9 & 0 \\ 9 & 965 & 4 & 13 & 6 & 0 & 3 & 0 & 0 & 0 \\ 22 & 0 & 798 & 8 & 79 & 0 & 84 & 1 & 8 & 0 \\ 35 & 6 & 11 & 876 & 39 & 0 & 27 & 0 & 6 & 0 \\ 4 & 2 & 107 & 30 & 792 & 0 & 60 & 0 & 5 & 0 \\ 2 & 0 & 0 & 1 & 1 & 961 & 1 & 30 & 4 & 0 \\ 148 & 5 & 65 & 23 & 80 & 0 & 668 & 0 & 11 & 0 \\ 0 & 0 & 1 & 0 & 0 & 27 & 0 & 971 & 1 & 0 \\ 3 & 1 & 8 & 10 & 2 & 1 & 9 & 7 & 959 & 0 \\ 5 & 1 & 4 & 1 & 72 & 309 & 6 & 375 & 63 & 164 \end{bmatrix}$$

- 1. No. of hidden layer units = [50, 50, 50, 50]
- 2. Time to train the neural network = 821.6736297607422
- 4. Accuracy over Test data set = 0.8735

5. Confusion Matrix over Training data set =

F 5761	6	31	40	13	1	114	1	32	1	
7	5922	6	39	6	0	17	0	2	1	
26	5	5711	27	163	0	58	0	10	0	
18	9	14	5866	36	1	50	0	6	0	
12	4	73	36	5804	0	61	0	8	2	(38)
0	1	0	1	0	5966	0	21	2	9	(30)
99	7	79	35	139	3	5619	0	17	2	
0	0	1	1	0	4	0	5965	2	27	
11	5	11	14	15	1	9	3	5931	0	
0	0	0	0	0	1	0	32	1	5966	

$$\begin{bmatrix} 806 & 1 & 16 & 27 & 5 & 1 & 131 & 2 & 11 & 0 \\ 8 & 959 & 3 & 21 & 5 & 0 & 3 & 0 & 0 & 1 \\ 17 & 0 & 782 & 18 & 103 & 1 & 74 & 0 & 5 & 0 \\ 33 & 10 & 14 & 857 & 35 & 0 & 45 & 0 & 4 & 2 \\ 2 & 0 & 85 & 36 & 819 & 2 & 52 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 939 & 1 & 34 & 4 & 22 \\ 107 & 2 & 66 & 23 & 89 & 1 & 699 & 0 & 13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20 & 0 & 959 & 1 & 20 \\ 4 & 1 & 3 & 13 & 5 & 6 & 8 & 5 & 955 & 0 \\ 0 & 0 & 1 & 1 & 0 & 8 & 0 & 29 & 1 & 960 \end{bmatrix}$$

- 1. No. of hidden layer units = [50, 50, 50, 50, 50]
- 2. Time to train the neural network = 861.6063125133514
- 3. Accuracy over Training data set = 0.9791
- 4. Accuracy over Test data set = 0.8803
- 5. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5879 & 7 & 30 & 39 & 4 & 2 & 28 & 1 & 9 & 1 \\ 9 & 5945 & 8 & 27 & 5 & 2 & 2 & 0 & 2 & 0 \\ 43 & 4 & 5780 & 33 & 88 & 1 & 40 & 0 & 11 & 0 \\ 39 & 1 & 14 & 5888 & 30 & 1 & 21 & 1 & 5 & 0 \\ 18 & 5 & 90 & 55 & 5789 & 0 & 39 & 0 & 4 & 0 \\ 0 & 0 & 3 & 1 & 1 & 5973 & 1 & 13 & 3 & 5 \\ 171 & 10 & 67 & 53 & 53 & 2 & 5634 & 1 & 6 & 3 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 5959 & 3 & 32 \\ 12 & 2 & 13 & 13 & 4 & 1 & 9 & 0 & 5945 & 1 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 37 & 0 & 5954 \end{bmatrix}$$

[853	2	15	23	3	1	95	0	8	0 7		
3	968	2	18	6	0	2	0	0	1		
22	1	812	15	76	1	66	1	6	0		
33	9	11	886	27	0	27	0	7	0		
2	3	81	41	809	1	57	2	4	0	(1	1 \
2	0	0	3	1	947	0	29	2	16	(4	1)
144	2	73	30	79	1	658	0	12	1		
0	0	0	0	0	24	0	949	1	26		
13	1	5	6	7	3	4	2	958	1		
	0	0	0	0	12	1	24	0	963		

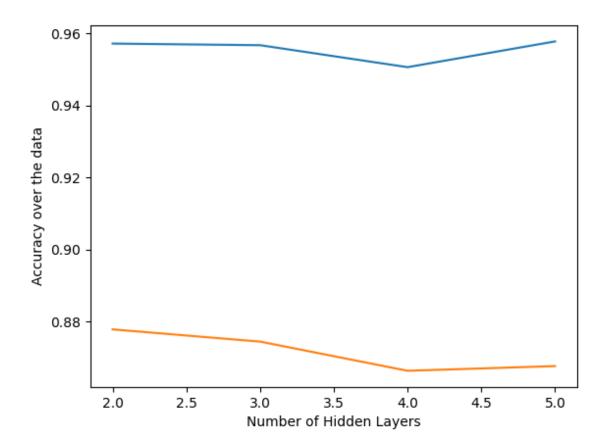


Figure 9: No. of Hidden Layers vs Accuracy (Sigmoid)

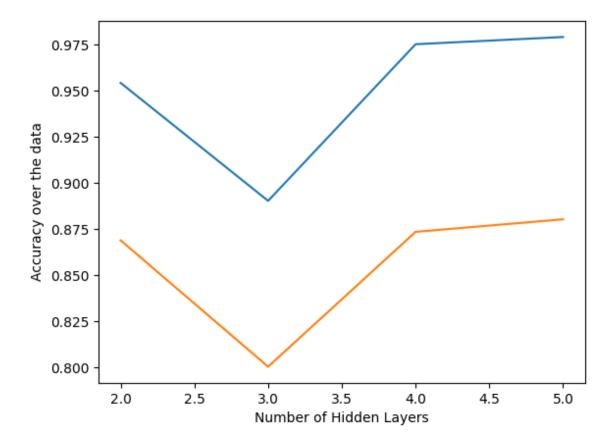


Figure 10: No. of Hidden Layers vs Accuracy (ReLu)

2.6 Training the network using BCE objective function

Considering the backpropagation was programmed to implement a stochastic gradient descent on a desired cost function J, we can formulate the following:

Let the output of neurons be \mathcal{O}_i and ψ be the activation function.

$$\delta_j = \frac{\partial J}{\partial \mathcal{O}_j} \frac{\partial \mathcal{O}_j}{\partial net_j} = \begin{cases} \frac{\partial J(\mathcal{O}_j, y)}{\partial \mathcal{O}_j} \frac{\partial \psi(net_j)}{\partial net_j} & \text{if } j \text{ is and output neuron} \\ (\sum_{l \in L} w_{jl} \delta_l) \frac{\partial \psi(net_j)}{\partial net_j} & \text{if } j \text{ is and inner neuron} \end{cases}$$

The only change to implement gradient descent on binary cross entropy loss will be in the δ_j calculation at the output layer, where $\partial J(\mathcal{O}_j, y)/\partial \mathcal{O}_j$ will now be evaluated as follows.

$$J(\mathcal{O}, y) = -\sum_{j=1}^{C} y_{j} log(\mathcal{O}_{j}) + (1 - y_{j}) log(1 - \mathcal{O}_{j})$$

$$\frac{\partial J(\mathcal{O}, y)}{\partial \mathcal{O}_{j}} = \frac{\mathcal{O}_{j} - y_{j}}{\mathcal{O}_{j}(1 - \mathcal{O}_{j})}$$

$$\frac{\partial J(\mathcal{O}_{j}, y)}{\partial \mathcal{O}_{j}} \frac{\partial \psi(net_{j})}{\partial net_{j}} = \mathcal{O}_{j} - y_{j}$$

$$(42)$$

- 1. No. of hidden layer units = [50, 50, 50]
- 2. Accuracy over Training data set = 0.9637
- 3. Accuracy over Test data set = 0.8827
- 4. Confusion Matrix over Training data set =

$$\begin{bmatrix} 5871 & 9 & 34 & 39 & 4 & 2 & 28 & 1 & 9 & 1 \\ 9 & 5951 & 4 & 23 & 5 & 2 & 2 & 0 & 1 & 0 \\ 41 & 3 & 5784 & 33 & 83 & 5 & 40 & 0 & 11 & 0 \\ 37 & 3 & 9 & 5888 & 30 & 1 & 22 & 4 & 7 & 0 \\ 17 & 5 & 89 & 55 & 5793 & 0 & 37 & 0 & 3 & 0 \\ 0 & 0 & 2 & 1 & 1 & 5975 & 1 & 13 & 3 & 4 \\ 169 & 9 & 69 & 53 & 54 & 3 & 5631 & 1 & 7 & 4 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 & 5961 & 3 & 31 \\ 11 & 2 & 14 & 13 & 3 & 2 & 10 & 0 & 5941 & 3 \\ 0 & 1 & 0 & 0 & 1 & 7 & 0 & 35 & 0 & 5959 \end{bmatrix}$$

$$\begin{bmatrix} 851 & 2 & 15 & 25 & 3 & 1 & 95 & 0 & 8 & 0 \\ 3 & 969 & 2 & 18 & 6 & 0 & 2 & 0 & 0 & 2 \\ 23 & 1 & 809 & 15 & 76 & 2 & 67 & 1 & 6 & 0 \\ 33 & 9 & 11 & 889 & 27 & 0 & 25 & 0 & 6 & 0 \\ 1 & 3 & 80 & 41 & 811 & 1 & 57 & 2 & 4 & 0 \\ 2 & 0 & 1 & 3 & 2 & 941 & 0 & 231 & 3 & 16 \\ 144 & 2 & 73 & 30 & 79 & 1 & 659 & 0 & 13 & 1 \\ 0 & 0 & 0 & 0 & 0 & 25 & 0 & 947 & 1 & 27 \\ 12 & 1 & 5 & 6 & 7 & 3 & 4 & 2 & 959 & 1 \\ 0 & 0 & 0 & 0 & 0 & 11 & 1 & 23 & 0 & 965 \end{bmatrix}$$

2.7 Training the network using MLPClassifier

The MLPClassifier from scikit-learn library to implement a neural network with 2 hidden layers with 100 units each. The activation function used ReLu and MLPClassifier by default uses binary cross entropy loss. The accuracies are:

- 1. No. of hidden layer units = [100,100]
- 2. Accuracy over Training data set = 0.9886
- 3. Accuracy over Test data set = 0.8925

4. Confusion Matrix over Training data set =

T5871	9	34	39	4	2	28	1	9	1	
9	5951	4	23	5	2	2	0	1	0	
41	3	5784	33	83	5	40	0	11	0	
37	3	9	5888	30	1	22	4	7	0	
17	5	89	55	5793	0	37	0	3	0	(45)
0	0	2	1	1	5975	1	13	3	4	(45)
169	9	69	53	54	3	5631	1	7	4	
0	0	0	0	0	5	0	5961	3	31	
11	2	14	13	3	2	10	0	5941	3	
0	1	0	0	1	7	0	35	0	5959	

Γ851	2	15	25	3	1	95	0	8	0]	
3	969	2	18	6	0	2	0	0	2	
23	1	809	15	76	2	67	1	6	0	
33	9	11	889	27	0	25	0	6	0	
1	3	80	41	811	1	57	2	4	0	(16)
2	0	1	3	2	941	0	231	3	16	(46)
144	2	73	30	79	1	659	0	13	1	
0	0	0	0	0	25	0	947	1	27	
12	1	5	6	7	3	4	2	959	1	
[0	0	0	0	0	11	1	23	0	965	