At the beginning, there are an equal number of bicycles in each site. Here we make some following assumptions. First, a trip will randomly choose a bicycle from all bicycles and use the location of the bicycle as the start of the trip. Then the trip will randomly select its goal with certain probabilities, which are in proportion to the number of directed edges between the start and all possible goals.

If we define x_i as the ratio of the number of bicycles in the ith sites with the total number of bicycles, denoted by c, and define γ as the number of trips occurring in the unit time, we can write out the differential equations:

$$dx_i = \sum_{j=1}^{n} W_{ij} x_j \frac{\gamma}{c} dt - x_i \frac{\gamma}{c} dt, \qquad i = 1, \dots, n$$

Here, W_{ij} is the transition probability from the jth sites to the ith sites.

These differential equations can be simplified in the form of matrix:

$$\frac{d\boldsymbol{x}}{dt} = \frac{\gamma}{c}(\boldsymbol{W} - \boldsymbol{I})\boldsymbol{x}$$

Obviously, we can solve the differential equations by the eigenvectors and eigenvalues of $\boldsymbol{W} - \boldsymbol{I}$.

However, the above assumptions don't appropriately conform to the fact, because the frequency of the trip from the ith sites site are constant in rough instead of proportional to x_i and those frequencies can differ from each other.

So, we should make some changes to our assumption. We still assume that there are γ trips occurring in the unit time, but they choose their starts randomly with certain probabilities, denoted by $p_i,\ i = 1, \dots, n$, which can be set ahead of time. Now the differential equations change to:

$$dx_i = \sum_{j=1}^{n} W_{ij} p_j \frac{\gamma}{c} dt - p_i \frac{\gamma}{c} dt, \qquad i = 1, \dots, n$$

Note that this is not all. The above differential equations are correct only if $x_i > 0,\ i = 1, \dots, n$. We can use Heaviside function to include more situations.

$$dx_i = \sum_{j=1}^{n} W_{ij} u(x_j) p_j \frac{\gamma}{c} dt - u(x_i) p_i \frac{\gamma}{c} dt, \qquad i = 1, \dots, n$$

These differential equations can be simplified in the form of matrix:

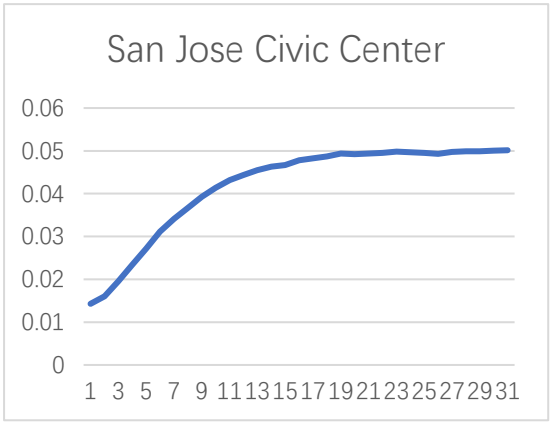
$$\frac{d\boldsymbol{x}}{dt} = \frac{\gamma}{c}(\boldsymbol{W} - \boldsymbol{I})\boldsymbol{U}(\boldsymbol{x})\boldsymbol{p}, \qquad i = 1, \dots, n$$

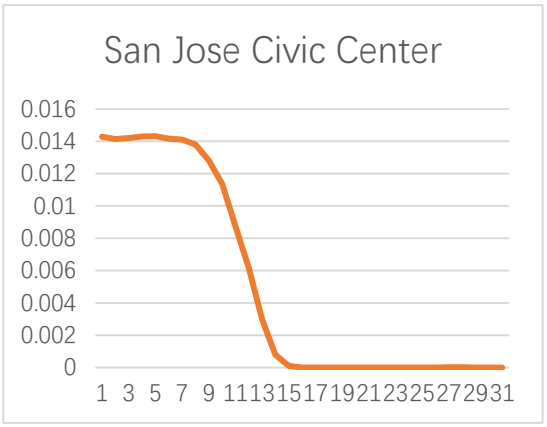
Here, $\boldsymbol{U}(\boldsymbol{x})$ is a diagonal matrix with $u(x_i),\ i = 1, \dots, n$ as its diagonal elements and zero for the rest.

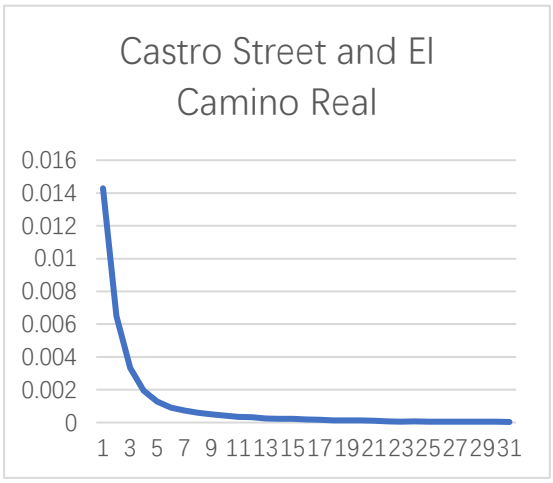
In fact, the solution of the equations is piecewise linear function, but we cannot solve the equations directly.

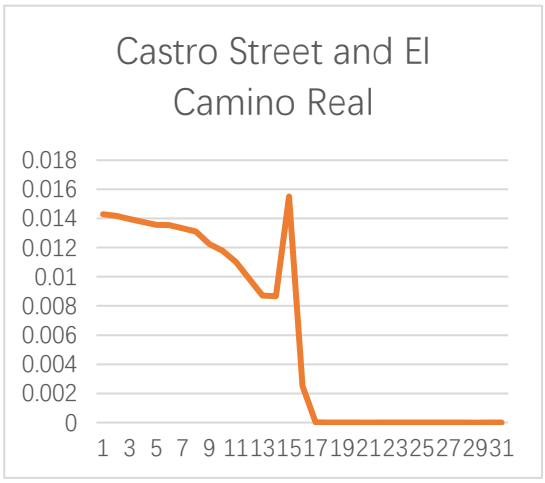
In general, $\boldsymbol{x}$ in the first case will ultimately converge to a certain solution, which is determined by the eigenvectors of $\boldsymbol{W} - \boldsymbol{I}$. However, in the second case, the result is very different from the former. Because of the property of Heaviside function, the variation of $\boldsymbol{x}$ can jump between different values. Sometimes $\boldsymbol{x}$ can hold a dynamic balance fortunately, but $\boldsymbol{x}$ can also hold vibrating.

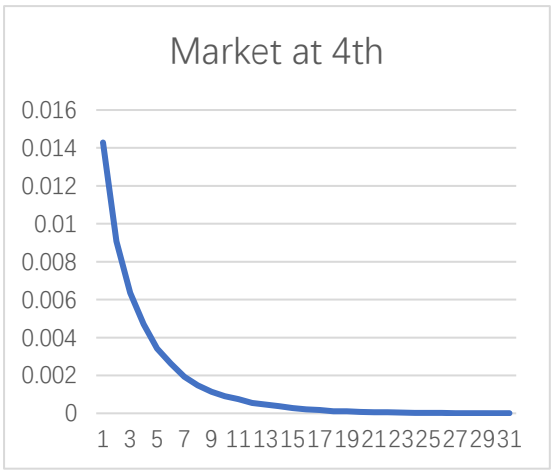
Here are some of our results of simulations. The left column comes from the first case, while the row comes from the second.

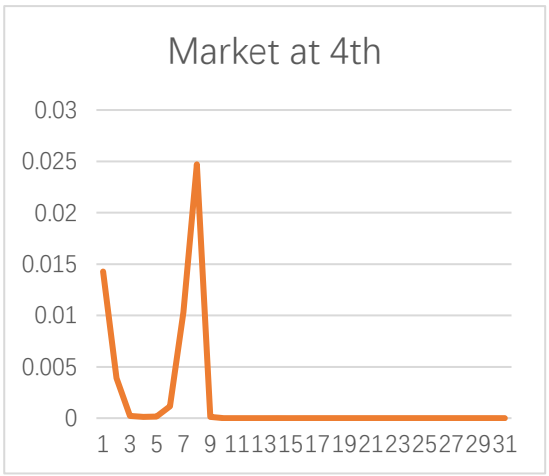


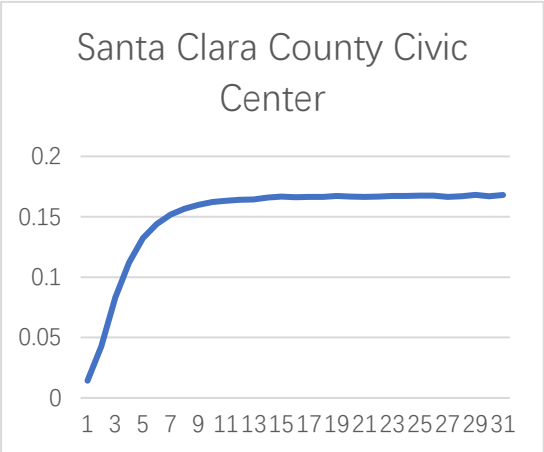


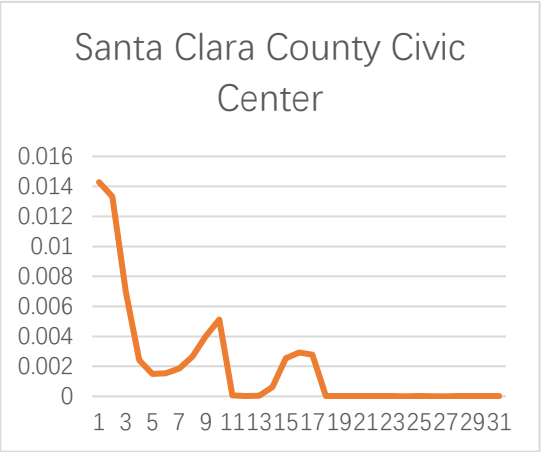


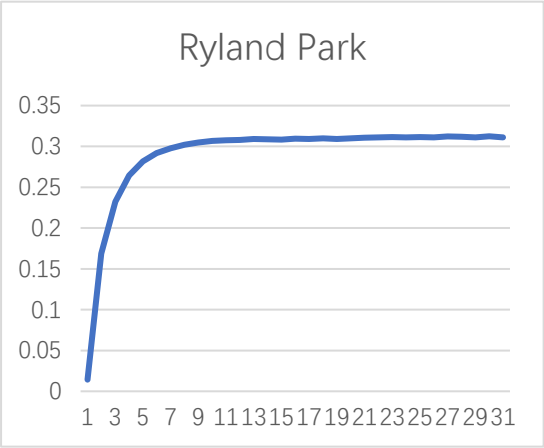


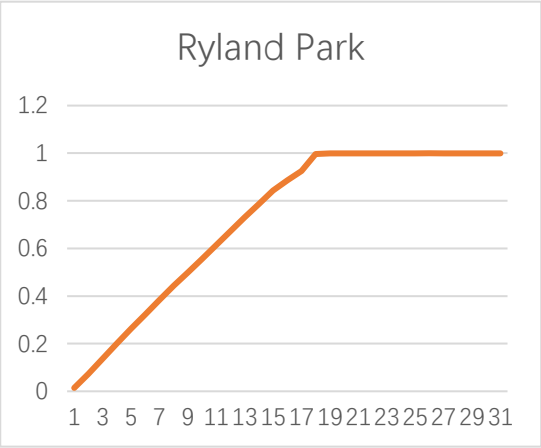












We can see that for the second case, given enough time, nearly all the bicycles concentrate to Ryland Park at the end. This result is surprising because $\boldsymbol{x}$ holds its balance at an extreme point. To some extent, the network degenerates ultimately without more conditions under our assumptions.