CSE 151: Machine learning **Winter 2019**

Homework 1

Submission instructions:

- If a problem asks for a numerical answer, you need only provide this answer. There is no need to show your work, unless you would like to.
- Please type up your solutions.
- Upload the PDF file for your homework to `gradescope` by 6pm on Tuesday January 15.

Part A: Short-answer questions

A random subset of these problems will be graded.

1. *Casting an image into vector form.* A 10×10 greyscale image is mapped to a d-dimensional vector, with one pixel per coordinate. What is d?

2. *The length of a vector.* The Euclidean (or L_2) length of a vector $x \in \mathbb{R}^d$ is

$$\|x\| = \sqrt{\sum_{i=1}^{d} x_i^2},$$

   where x_i is the ith coordinate of x. This is the same as the Euclidean distance between x and the origin. What is the length of the vector which has a 1 in every coordinate? Your answer may be a function of d.

3. *Accuracy of a random classifier.* A particular data set has 4 possible labels, with the following frequencies:

Label	Frequency
A	50%
B	20%
C	20%
D	10%

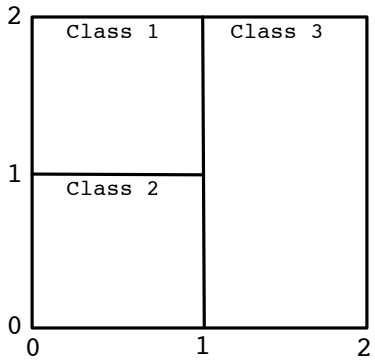
   (a) What is the error rate of a classifier that picks a label (A,B,C,D) at random, each with probability 1/4?

   (b) One very simple type of classifier just returns the same label, always.
   - What label should it return?
   - What will its error rate be?

4. *Decision boundary of the nearest neighbor classifier.* In this problem,

   - The data space is $\mathcal{X} = [0, 2]^2$: each point has two coordinates, and they lie between 0 and 2.
   - The labels are $\mathcal{Y} = \{1, 2, 3\}$.

   The correct labels in different parts of $\mathcal{X}$ are as shown below.



   (a) What is the label of point $(0.5, 0.5)$?

   Now suppose you have a training set consisting of just two points, located at

   $$(0.5, 0.5), \ (0.5, 1.5).$$

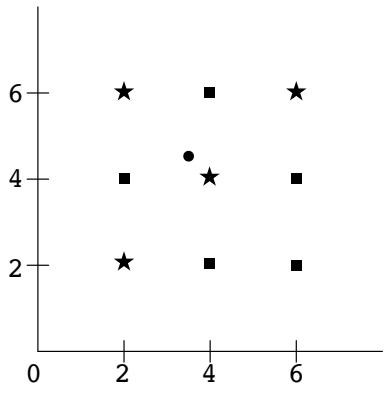
   (b) What label will the nearest neighbor classifier assign to point $(1.5, 0.5)$?

   (c) What label will the nearest neighbor classifier assign to point $(2, 2)$?

   (d) Which label will this classifier never predict?

   (e) Now suppose that when the classifier is used, the test points are uniformly distributed over the square $\mathcal{X}$. What is the error rate of the 1-NN classifier?

5. In the picture below, there are nine training points, each with label either `square` or `star`. These will be used to guess the label of a query point at $(3.5, 4.5)$, indicated by a circle.



   Suppose Euclidean distance is used.

(a) How will the point be classified by 1-NN? The options are `square`, `star`, or `ambiguous`.

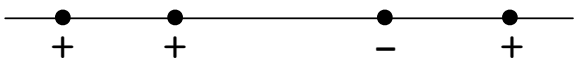
(b) By 3-NN?

(c) By 5-NN?

6. We decide to use 4-fold cross-validation to figure out the right value of k to choose when running k-nearest neighbor on a data set of size 10,000. When checking a particular value of k, we look at four different training sets. What is the size of each of these training sets?

7. An extremal type of cross-validation is *n-fold cross-validation* on a training set of size n. If we want to estimate the error of k-NN, this amounts to classifying each training point by running k-NN on the remaining $n-1$ points, and then looking at the fraction of mistakes made. It is commonly called *leave-one-out cross-validation* (LOOCV).

   Consider the following simple data set of just four points:



   What is the LOOCV error for 1-NN? For 3-NN?

Part B: Programming assignment

This part of the homework will be graded in its entirety.

To begin with:

- Install Python 3 and Jupyter on your computer.
- Obtain `hw1-notebook.zip` from the course website and uncompress it.
- The Jupyter notebook `nn-mnist.ipynb` implements a basic 1-NN classifier for a subset of the MNIST data set. It uses a separate training and test set.
- Go through this notebook, running each segment and taking care to understand exactly what each line is doing.

Now do the following exercises.

1. For test point 100, print its image as well as the image of its nearest neighbor in the training set. Put these images in your writeup. Is this test point classified correctly?

2. The *confusion matrix* for the classifier is a 10×10 matrix N_{ij} with $0 \leq i, j \leq 9$, where N_{ij} is the number of test points whose true label is i but which are classified as j. Thus, if all test points are correctly classified, the off-diagonal entries of the matrix will be zero.

   (a) Compute the matrix N for the 1-NN classifier and print it out.

   (b) Which digit is misclassified most often? Least often?

3. For each digit $0 \leq i \leq 9$: look at all training instances of image i, and compute their mean. This average is a 784-dimensional vector. Use the `show_digit` routine to print out these 10 average-digits.