Assignment 3

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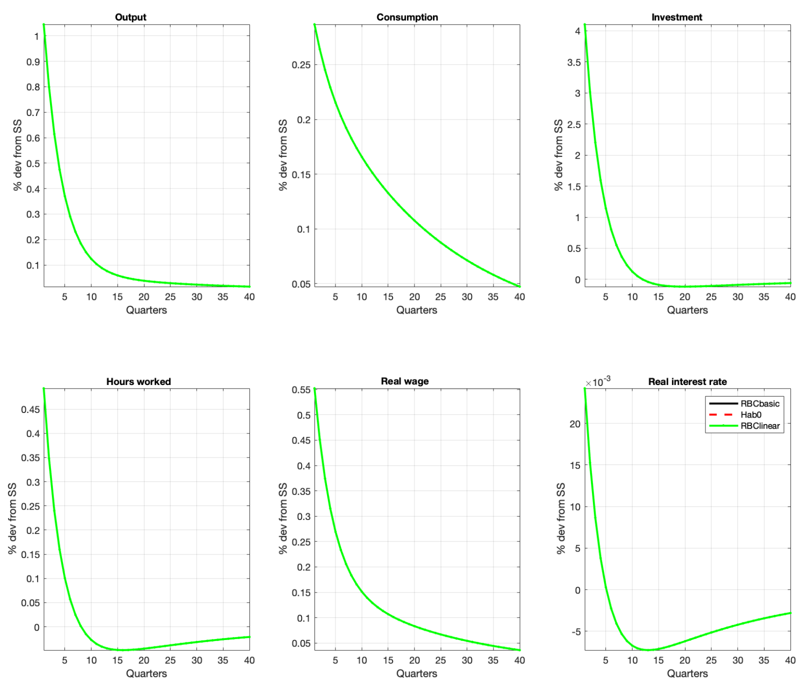
List of things done

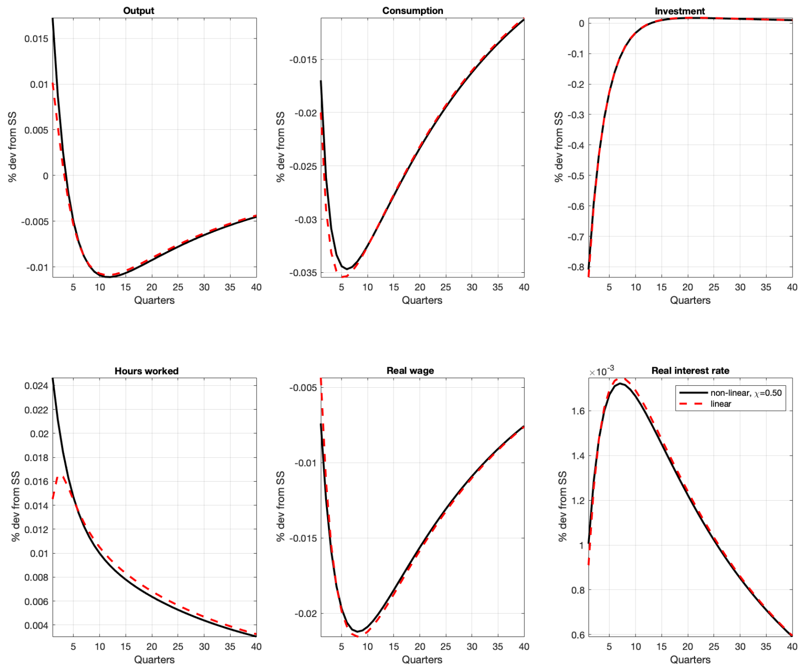
1. Ran "RBC_basic.mod" and "RBClinear_basic.mod" provided.
2. Corrected parameter values in "RBC_hab.mod" to match parameter values
    - Parameterizations are different between "RBC_basic.mod" in this module and that of the previous week...
    - Set $\phi_X = 0$, corresponding to zero adjustment costs.
3. Ran "RBC_hab.mod" for habit parameters $\chi \in \{0, 0.5, 0.99\}$.
4. Linearized RBC model with External Habitat and created "RBClinear_hab.mod".
5. Ran "RBClinear_hab.mod" for habit parameters $\chi \in \{0, 0.5, 0.99\}$.

Linearization and the basic RBC model

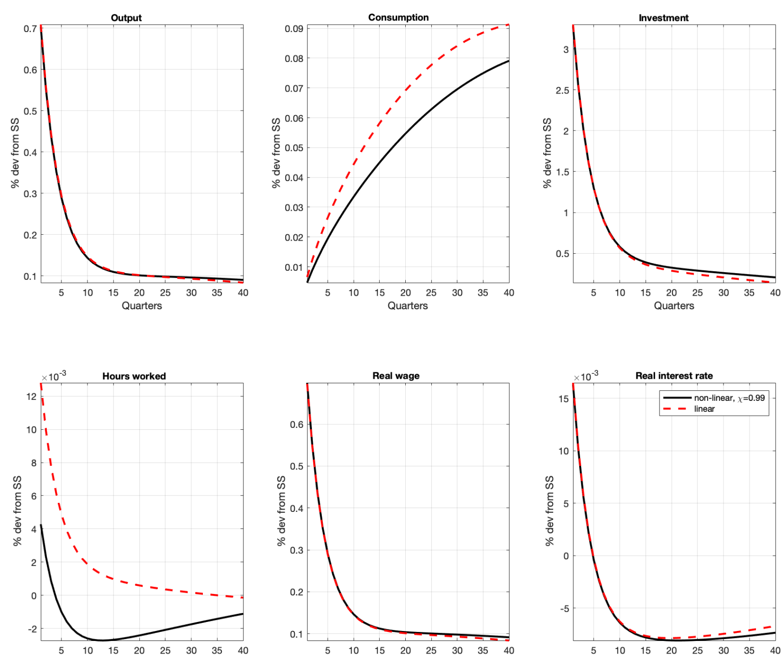


This figure plots the TFP shock IRF results for the basic nonlinearized RBC model (black), nonlinearized external habitat RBC model with habitat parameter set at zero (red), and the linearized RBC model (green). From parameterization of the external habitat model, we clearly see that when $\chi = 0$, it becomes the standard RBC model. As such, it is no surprise that Hab0 and RBCbasic yield identical IRFs. Because the lines coincide, we conclude that the linearized (RBClinear) model does very well at approximating the nonlinear model. Intuitively, this makes sense because there are no external sources of friction, so the errors of approximation would be minimal.

Comparing External Habit Models



These two figures compare the nonlinear and linearized model with external habit; TOP panel presents the (TFP shock) IRFs to the case when $\chi = 0.5$; Bottom panel presents the (TFP shock) IRFS to the case when $\chi = 0.99$. Generally, the linearized model can capture most of the information conveyed in the nonlinear model, albeit with reduced curvature. More curvature is introduced to the external habit model as the magnitude of the friction increases. Thereby, the linearized version of the model performs slightly worse than in the case with lower curvature.



Linearize RBC model with JR preferences

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Suppose we are to linearize a standard RBC model with Jaimovich-Rebello (JR) Preferences. When there are no capital adjustment costs, the only changes that need to be implemented are marginal utility of consumption deviations and marginal utility of leisure deviations. Here, I write out the steps in excruciating detail.

Given the JR preference:

$$U(C_t, L_t) = \frac{[(C_t - \chi C_{t-1})^{1-\varrho}(1-H_t)^{\varrho}]^{1-\sigma_c} - 1}{1-\sigma_c}$$

we have marginal utility of consumption (C_t) and marginal utility of labor supply (H_t):

$$U_{C,t} = (1-\varrho)[C_t - \chi C_{t-1}]^{(1-\varrho)(1-\sigma_c)-1}[1-H_t]^{\varrho(1-\sigma_c)} \tag{1}$$

$$U_{H,t} = -\varrho[C_t - \chi C_{t-1}]^{(1-\varrho)(1-\sigma_c)}[1-H_t]^{\varrho(1-\sigma_c)-1} \tag{2}$$

1 Linearize Marginal Utility of Consumption

Take ln of equation (1):

$$\ln U_{C,t} = \ln(1-\varrho) + [(1-\varrho)(1-\sigma_c)-1]\ln(C_t - \chi C_{t-1}) + \varrho(1-\sigma_c)\ln(1-H_t)$$

$$\ln U_C = \ln(1-\varrho) + [(1-\varrho)(1-\sigma_c)-1]\ln(C - \chi C) + \varrho(1-\sigma_c)\ln(1-H)$$

Taking the difference yields the log-difference ($u_{c,t} \triangleq \ln U_{C,t} - \ln U_C$):

$$\begin{aligned}\ln U_{C,t} - \ln U_C = [(1-\varrho)(1-\sigma_c)-1]&[\ln(C_t - \chi C_{t-1}) - \ln(C - \chi C)] \\ &+ [\varrho(1-\sigma_c)][\ln(1-H_t) - \ln(1-H)].\end{aligned}$$

Define $f(C_t, C_{t-1}) \triangleq \ln(C_t - \chi C_{t-1})$. Then by first order Taylor approximation, we have

$$
\begin{aligned}
f(C_t, C_{t-1}) &\approx f(C, C) + \frac{\partial f}{\partial C_t}\Big|_{C,C}(C_t - C) + \frac{\partial f}{\partial C_t - 1}\Big|_{C,C}(C_{t-1} - C) \\
&= \ln(C - \chi C) + \frac{1}{C(1-\chi)}(C_t - C) + \frac{-\chi}{C(1-\chi)}(C_{t-1} - C) \\
&= \ln(C - \chi C) + \frac{1}{C(1-\chi)}(C_t - C)\frac{C}{C} + \frac{-\chi}{C(1-\chi)}(C_{t-1} - C)\frac{C}{C}
\end{aligned}
$$

$$
\Longrightarrow [\ln(C_t - \chi C_{t-1}) - \ln(C - \chi C)] = \frac{c_t - \chi c_{t-1}}{1-\chi}.
$$

Similarly, define $g(H_t) \triangleq \ln(1 - H_t)$. Then, by first order Taylor Approximation, we have

$$
\begin{aligned}
g(H_t) &\approx g(H) + \frac{\partial g}{\partial H_t}\Big|_{H}(H_t - H) \\
&= \ln(1 - H) - \frac{1}{1-H}(H_t - H) \\
&= \ln(1 - H) - \frac{1}{1-H}(H_t - H)\frac{H}{H}
\end{aligned}
$$

$$
\Longrightarrow [\ln(1 - H_t) - \ln(1 - H)] = -\frac{H}{1-H}h_t.
$$

$$
\boxed{u_{c,t} = [(1-\varrho)(1-\sigma_c) - 1]\tfrac{c_t - \chi c_{t-1}}{1-\chi} - [\varrho(1-\sigma_c)]\tfrac{H}{1-H}h_t.}
$$

2 Linearize Marginal Utility of Leisure

Take natural of equation (2), using the fact that $U_{H,t} = -U_{L,t}$:

$$
\begin{aligned}
\ln U_{L,t} &= \ln \varrho + [(1-\varrho)(1-\sigma_c)]\ln(C_t - \chi C_{t-1}) + [\varrho(1-\sigma_c) - 1]\ln(1 - H_t) \\
\ln U_L &= \ln \varrho + [(1-\varrho)(1-\sigma_c)]\ln(C - \chi C) + [\varrho(1-\sigma_c) - 1]\ln(1 - H)
\end{aligned}
$$

Taking the difference yields the log-difference ($u_{h,t} \triangleq \ln U_{H,t} - \ln U_H$):

$$
\begin{aligned}
\ln U_{L,t} - \ln U_L = [(1-\varrho)(1-\sigma_c)]&[\ln(C_t - \chi C_{t-1}) - \ln(C - \chi C)] \\
&+ [\varrho(1-\sigma_c) - 1][\ln(1 - H_t) - \ln(1 - H)].
\end{aligned}
$$

Using same method as before and substituting the colored terms, we get the log-linearized expression for marginal utility of leisure:

$$
u_{l,t} = [(1-\varrho)(1-\sigma_c)]\frac{c_t - \chi c_{t-1}}{1-\chi} - [\varrho(1-\sigma_c) - 1]\frac{H}{1-H}h_t
$$

$$
\boxed{u_{l,t} = u_{C,t} + \tfrac{c_t - \chi c_{t-1}}{1-\chi} + \tfrac{H}{1-H}h_t.}
$$