# CSCE 670 - Information Storage and Retrieval

# Lecture 12: Recommender Systems (Matrix Factorization)

Yu Zhang

yuzhang@tamu.edu

October 2, 2025

Course Website: https://yuzhang-teaching.github.io/CSCE670-F25.html

Adapted from the slides by Prof. Jure Leskovec (Stanford)

# Recap: (Item-Item) Collaborative Filtering

- Some Users have rated some Items (e.g., CDs, movies).
- Derive unknown User-Item ratings from those of "similar" Items

- Step 1: For item $i$, find other similar Items $\mathcal{N}$ (e.g., using the Pearson Correlation Coefficient)
- Step 2: Estimate rating for item $i$

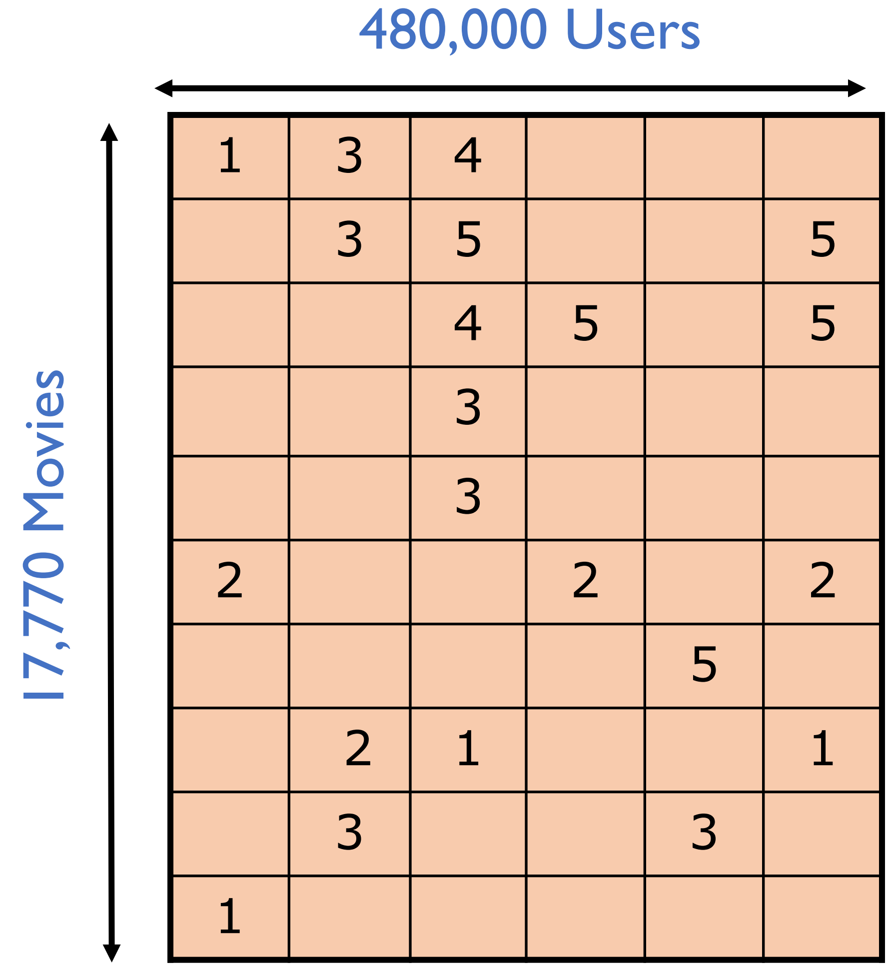
$$U_{xi} = \frac{\sum_{j \in \mathcal{N}} sim(i,j) \cdot U_{xj}}{\sum_{j \in \mathcal{N}} sim(i,j)}$$

$sim(i,j)$: Pearson Correlation Coefficient between item $i$ and item $j$

# The Netflix Prize

- Training data
  - 100 million ratings, 480,000 users, 17,770 movies
  - 6 years of data: 2000-2005
- Test data
  - Last few ratings of each user (2.8 million)
  - Evaluation criterion: RMSE
    - $\sqrt{\frac{1}{M}\sum_{x,i}\left(U_{xi} - U^*_{xi}\right)^2}$ where $U_{xi}$ is predicted, and $U^*_{xi}$ is the true rating of $x$ on $i$; $M$ is the number of testing samples
    - Netflix's system RMSE: 0.9514
- Competition
  - 2,700+ teams
  - $1 million prize for 10% improvement on Netflix

# Recap: RMSE

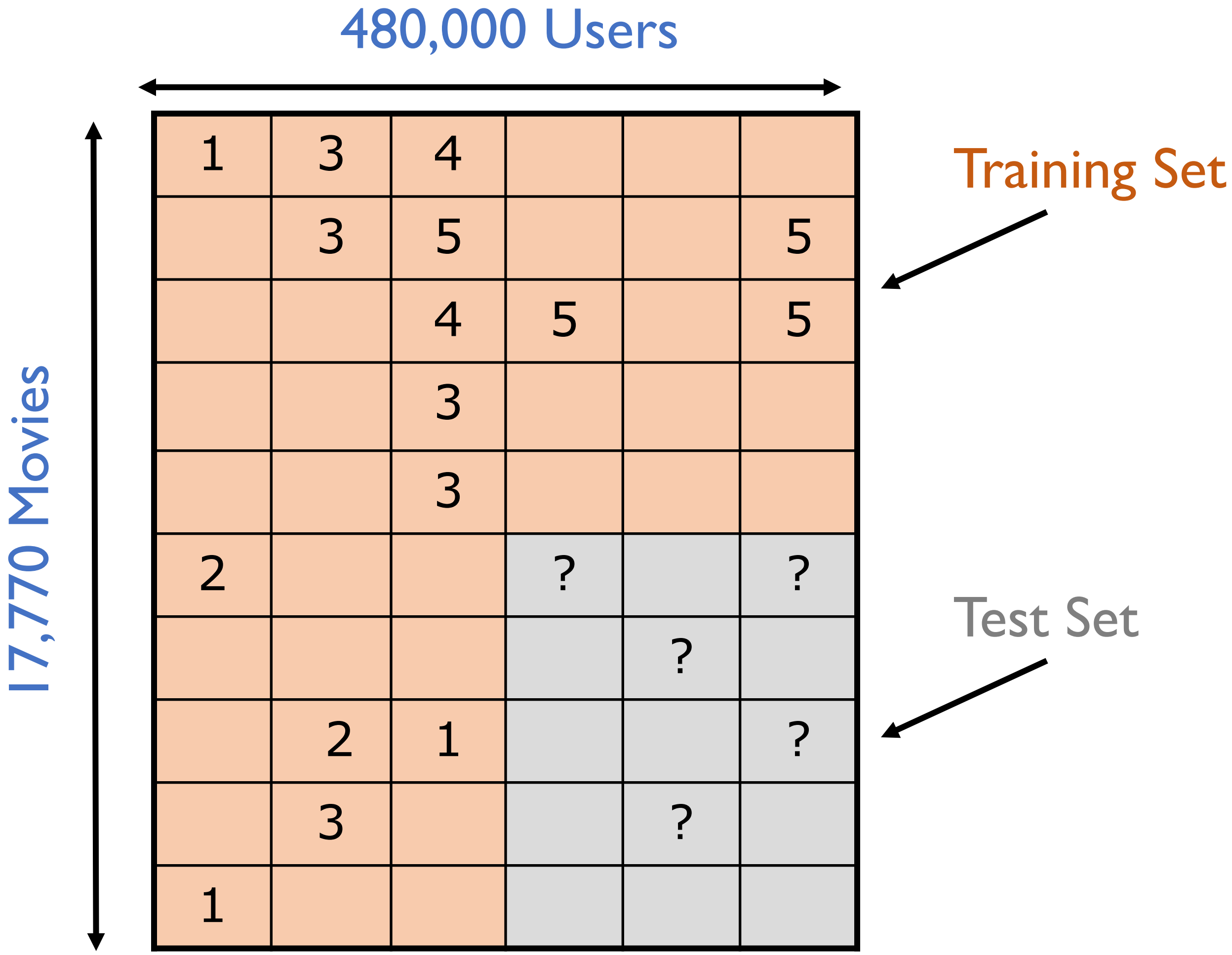


480,000 Users

17,770 Movies

| | | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 4 | | | |
| | 3 | 5 | | | 5 |
| | | 4 | 5 | | 5 |
| | | 3 | | | |
| | | 3 | | | |
| 2 | | | 2 | | 2 |
| | | | | 5 | |
| | 2 | 1 | | | 1 |
| | 3 | | | 3 | |
| 1 | | | | | |

# Recap: RMSE



480,000 Users

17,770 Movies

| | | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 4 | | | |
| | 3 | 5 | | | 5 |
| | | 4 | 5 | | 5 |
| | | 3 | | | |
| | | 3 | | | |
| 2 | | | ? | | ? |
| | | | | ? | |
| | 2 | 1 | | | ? |
| | 3 | | | ? | |
| 1 | | | | | |

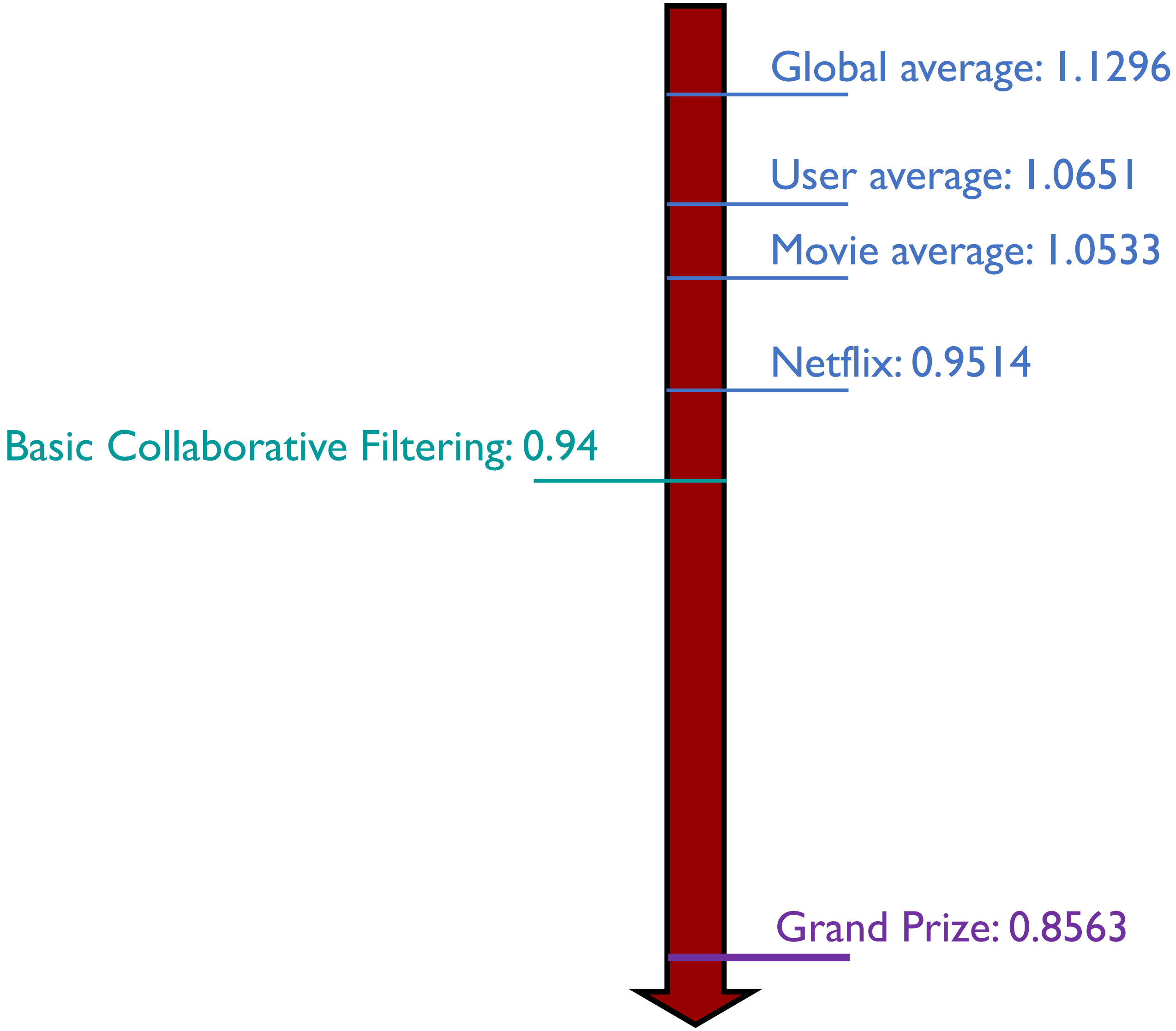
Training Set

Test Set

RMSE:

$$\sqrt{\frac{1}{M}\sum_{x,i}\left(U_{xi}-U_{xi}^{*}\right)^{2}}$$

# Performance of Various Models



Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative Filtering: 0.94

Grand Prize: 0.8563

# Recap: Modeling Deviations

- Basic Collaborative Filtering:

$$U_{xi} = \frac{\sum_{j \in \mathcal{N}} sim(i,j) \cdot U_{xj}}{\sum_{j \in \mathcal{N}} sim(i,j)}$$

- In practice,

$$U_{xi} = b_{xi} + \frac{\sum_{j \in \mathcal{N}} sim(i,j) \cdot (U_{xj} - b_{xj})}{\sum_{j \in \mathcal{N}} sim(i,j)}$$

  - $b_{xi}$: baseline estimate for $U_{xi}$ $(b_{xi} = \mu + b_x + b_i)$
  - $\mu$: overall mean movie rating
  - $b_x$: rating deviation of user $x$, which is the (avg. rating given by user $x$) – $\mu$
  - $b_i$: rating deviation of item $i$, which is the (avg. rating given to item $i$) – $\mu$

# One Step Further: Learning the Weight

$$U_{xi} = b_{xi} + w_{ij}(U_{xj} - b_{xj})$$

- $w_{ij}$ is learned from training data
    - We allow $\sum_{j\in\mathcal{N}} w_{ij} \neq 1$.
- $w_{ij}$ models the interaction between pairs of movies.
    - It does not depend on user $x$.

- What is the objective?
    - RMSE! $\sqrt{\frac{1}{M}\sum_{x,i}\left(U_{xi} - U_{xi}^{*}\right)^{2}}$
    - Or equivalently: $\sum_{x,i}(U_{xi} - U_{xi}^{*})^{2}$

# Recommendations via Optimization

$$J(w) = \sum_{x,i} (U_{xi} - U_{xi}^*)^2 = \sum_{x,i} \left( \left[ b_{xi} + \sum_{j \in \mathcal{N}} w_{ij} \left( U_{xj} - b_{xj} \right) \right] - U_{xi}^* \right)^2$$

- How to find the values of $w_{ij}$?
  - Gradient descent!

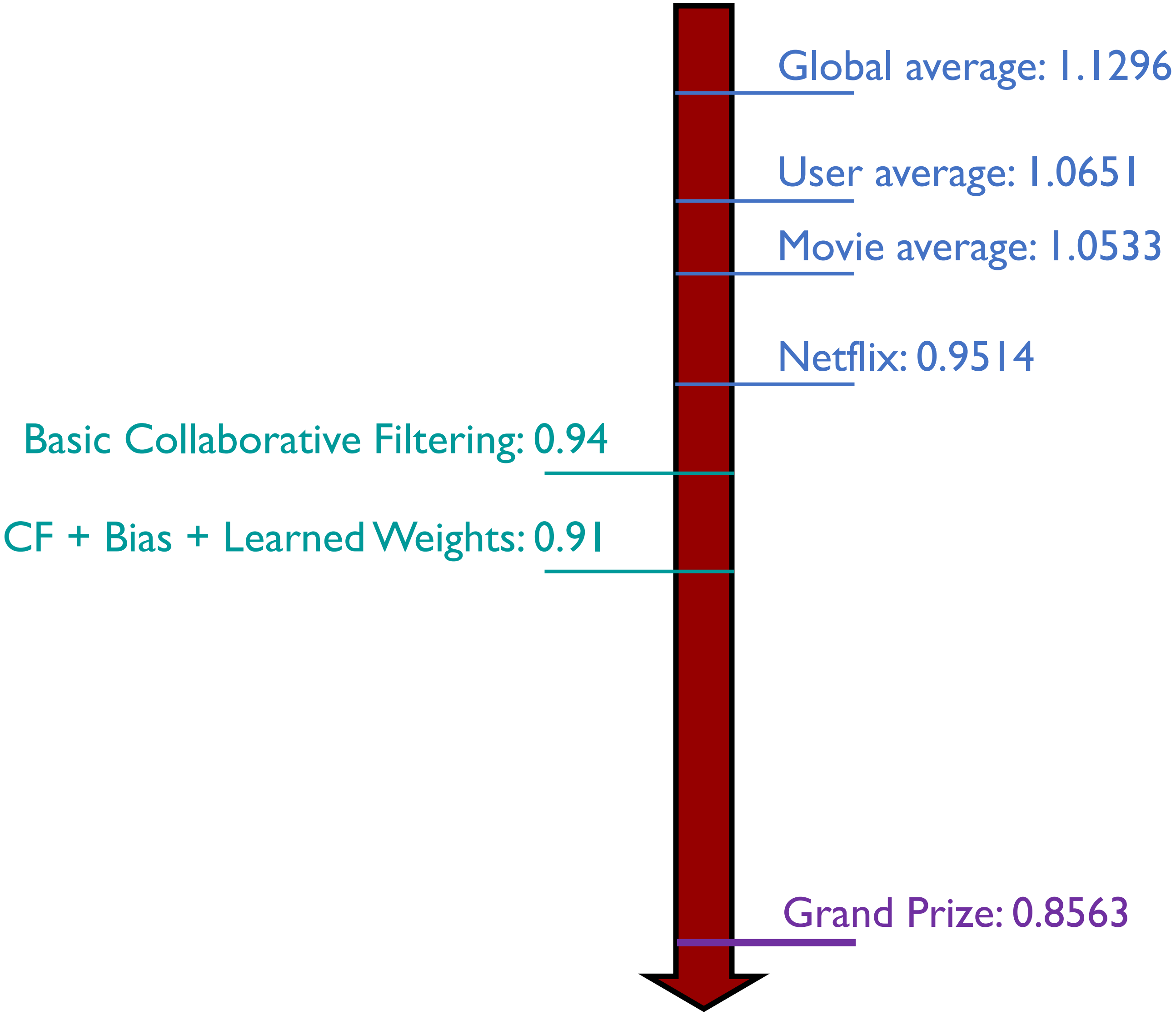
$$\frac{\partial J}{\partial w_{ij}} = 2 \sum_{x,i} \left( \left[ b_{xi} + \sum_{j \in \mathcal{N}} w_{ij} \left( U_{xj} - b_{xj} \right) \right] - U_{xi}^* \right) (U_{xj} - b_{xj})$$

(for $j \in \mathcal{N}$)

$$\frac{\partial J}{\partial w_{ij}} = 0$$

(for $j \notin \mathcal{N}$)

# Performance of Various Models



Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

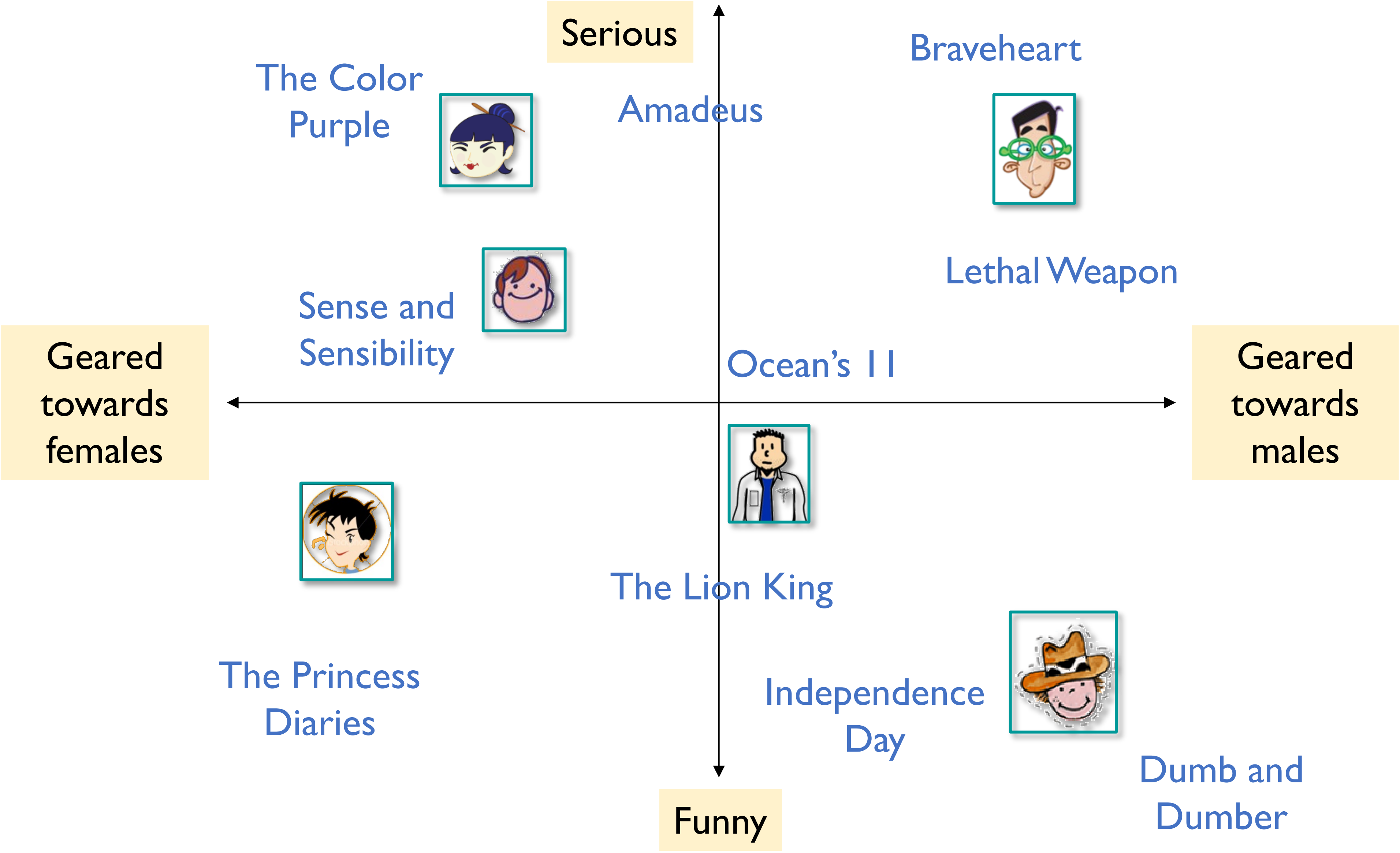
Basic Collaborative Filtering: 0.94

CF + Bias + Learned Weights: 0.91

Grand Prize: 0.8563

# Latent-Factor Models
(Matrix Factorization)

# There are certain latent factors that influence users’ ratings.



Serious

The Color Purple

Amadeus

Braveheart

Lethal Weapon

Sense and Sensibility

Geared towards females

Ocean’s 11

Geared towards males

The Lion King

The Princess Diaries

Independence Day

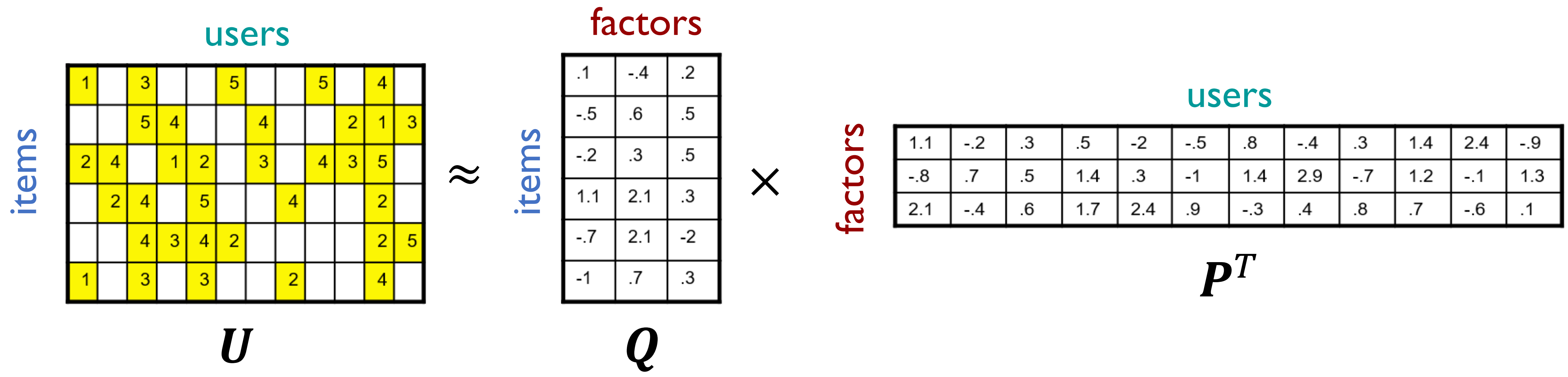
Dumb and Dumber

Funny

# Latent-Factor Models

- $U \approx QP^T$

The number of factors is small.
In other words, $Q$ and $P$ are "thin".



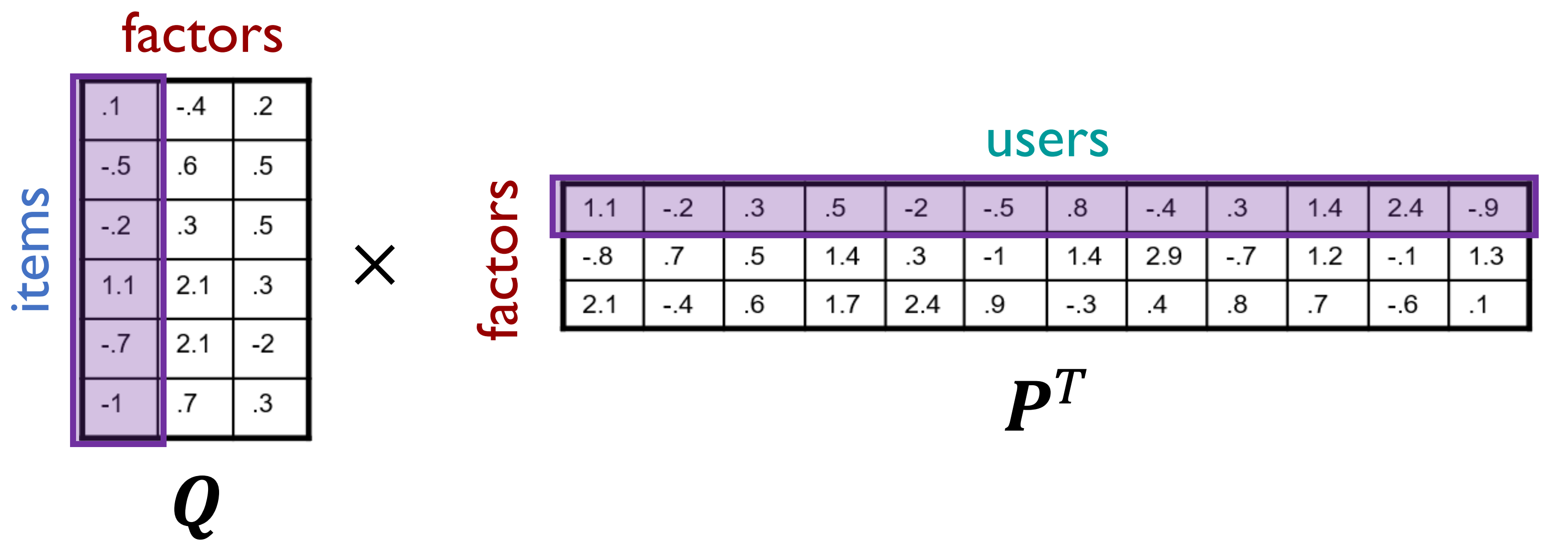
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 3 | | | 5 | | | 5 | | 4 | |
| | | 5 | 4 | | | 4 | | | 2 | 1 | 3 |
| 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | |
| | 2 | 4 | | 5 | | | 4 | | | 2 | |
| | | 4 | 3 | 4 | 2 | | | | | 2 | 5 |
| 1 | | 3 | | 3 | | | 2 | | | 4 | |

| | | |
|---|---|---|
| .1 | -.4 | .2 |
| -.5 | .6 | .5 |
| -.2 | .3 | .5 |
| 1.1 | 2.1 | .3 |
| -.7 | 2.1 | -2 |
| -1 | .7 | .3 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | -.2 | .3 | .5 | -2 | -.5 | .8 | -.4 | .3 | 1.4 | 2.4 | -.9 |
| -.8 | .7 | .5 | 1.4 | .3 | -1 | 1.4 | 2.9 | -.7 | 1.2 | -.1 | 1.3 |
| 2.1 | -.4 | .6 | 1.7 | 2.4 | .9 | -.3 | .4 | .8 | .7 | -.6 | .1 |

- For now, let's assume this is mathematically doable.
  - $U$ has missing entries but let's first ignore that!
  - Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones.

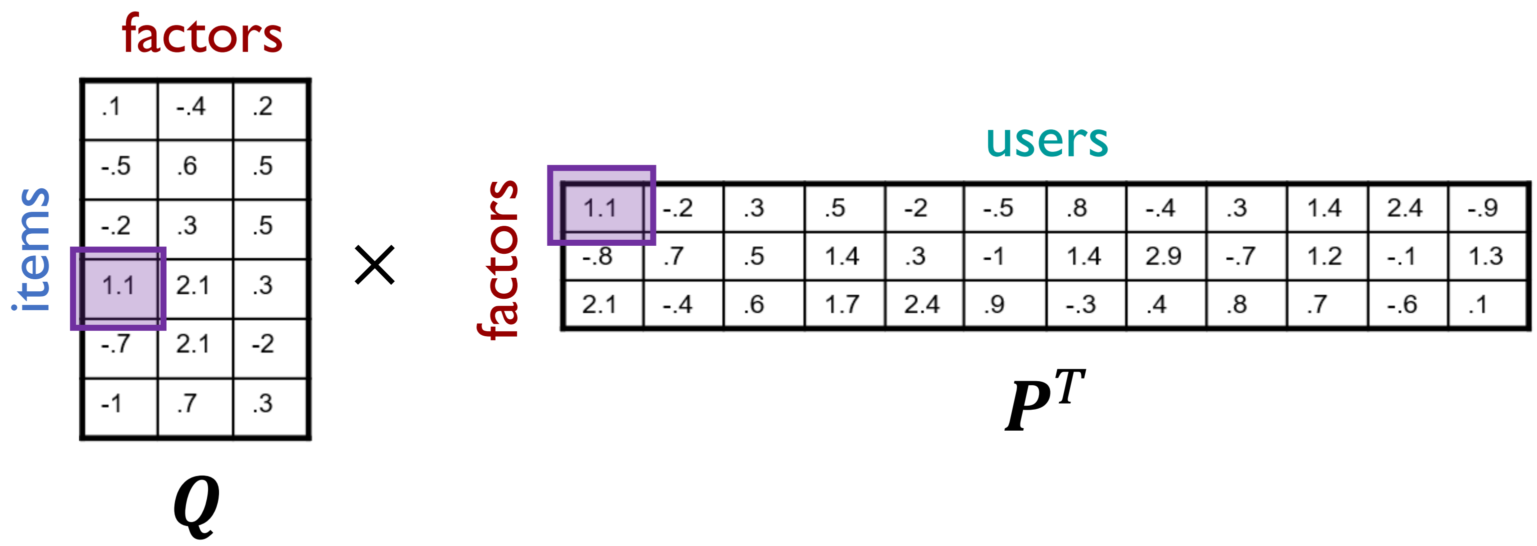
# How to interpret $\boldsymbol{Q}$ and $\boldsymbol{P}$?



$$\boldsymbol{Q} = \begin{array}{c|ccc} & & \text{factors} & \\ \hline \text{items} & .1 & -.4 & .2 \\ & -.5 & .6 & .5 \\ & -.2 & .3 & .5 \\ & 1.1 & 2.1 & .3 \\ & -.7 & 2.1 & -2 \\ & -1 & .7 & .3 \end{array} \times \boldsymbol{P}^T = \begin{array}{c|cccccccccccc} & & & & & & \text{users} & & & & & & \\ \hline \text{factors} & 1.1 & -.2 & .3 & .5 & -2 & -.5 & .8 & -.4 & .3 & 1.4 & 2.4 & -.9 \\ & -.8 & .7 & .5 & 1.4 & .3 & -1 & 1.4 & 2.9 & -.7 & 1.2 & -.1 & 1.3 \\ & 2.1 & -.4 & .6 & 1.7 & 2.4 & .9 & -.3 & .4 & .8 & .7 & -.6 & .1 \end{array}$$

- Let’s assume that the first factor is *the level of seriousness*.

# How to interpret $Q$ and $P$?



| | | |
|---|---|---|
| .1 | -.4 | .2 |
| -.5 | .6 | .5 |
| -.2 | .3 | .5 |
| 1.1 | 2.1 | .3 |
| -.7 | 2.1 | -2 |
| -1 | .7 | .3 |

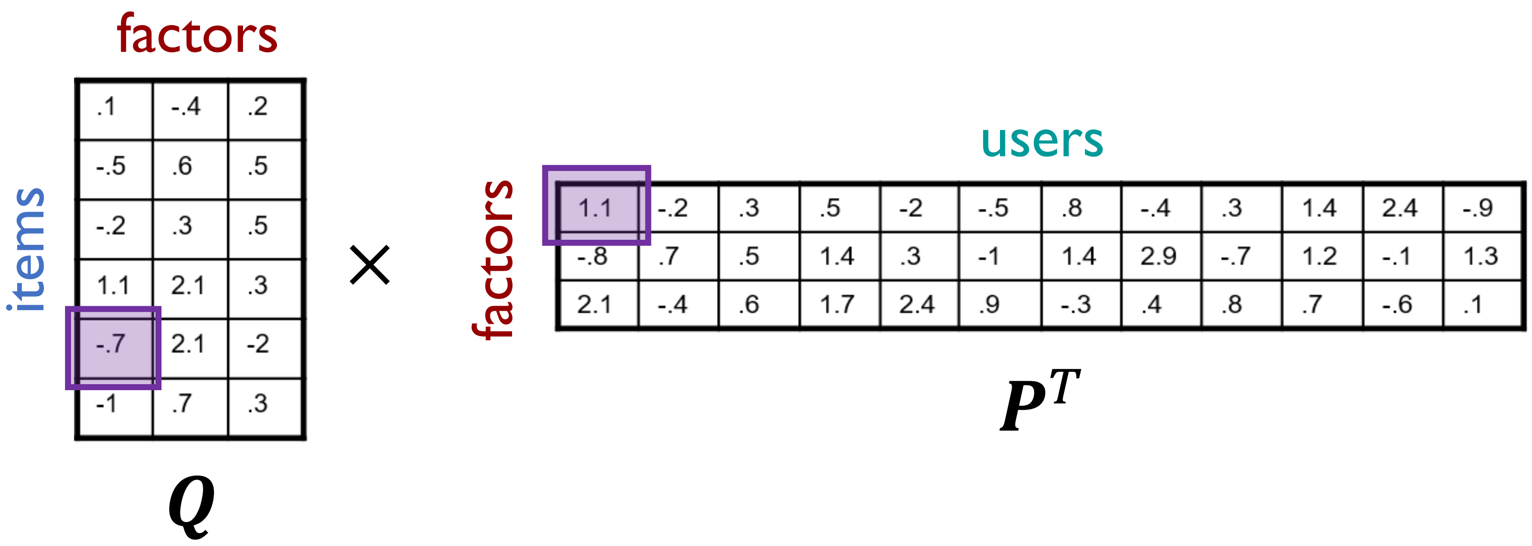
$Q$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | -.2 | .3 | .5 | -2 | -.5 | .8 | -.4 | .3 | 1.4 | 2.4 | -.9 |
| -.8 | .7 | .5 | 1.4 | .3 | -1 | 1.4 | 2.9 | -.7 | 1.2 | -.1 | 1.3 |
| 2.1 | -.4 | .6 | 1.7 | 2.4 | .9 | -.3 | .4 | .8 | .7 | -.6 | .1 |

$P^T$

- Let's assume that the first factor is *the level of seriousness*.
  - The seriousness of User 1 is 1.1
  - The seriousness of Movie 4 is 1.1
  - So, User 1 may like Movie 4

# How to interpret $\boldsymbol{Q}$ and $\boldsymbol{P}$?



$$\boldsymbol{Q} = \begin{bmatrix} .1 & -.4 & .2 \\ -.5 & .6 & .5 \\ -.2 & .3 & .5 \\ 1.1 & 2.1 & .3 \\ \boxed{-.7} & 2.1 & -2 \\ -1 & .7 & .3 \end{bmatrix} \times \boldsymbol{P}^T = \begin{bmatrix} \boxed{1.1} & -.2 & .3 & .5 & -2 & -.5 & .8 & -.4 & .3 & 1.4 & 2.4 & -.9 \\ -.8 & .7 & .5 & 1.4 & .3 & -1 & 1.4 & 2.9 & -.7 & 1.2 & -.1 & 1.3 \\ 2.1 & -.4 & .6 & 1.7 & 2.4 & .9 & -.3 & .4 & .8 & .7 & -.6 & .1 \end{bmatrix}$$

- Let’s assume that the first factor is *the level of seriousness*.
  - The seriousness of User 1 is 1.1
  - The seriousness of Movie 5 is -0.7
  - So, User 1 may NOT like Movie 5

# Of course, we need to consider all the factors.

- Ratings as “sum of products”

users

items

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 3 | | | 5 | | | 5 | | 4 | |
| | | 5 | 4 | | | 4 | | | 2 | 1 | 3 |
| 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | |
| | 2 | 4 | | 5 | | | 4 | | | 2 | |
| | | 4 | 3 | 4 | 2 | | | | | 2 | 5 |
| 1 | | 3 | | 3 | | | 2 | | | 4 | |

$\boldsymbol{U}$

$\approx$

factors

items

| | | |
|---|---|---|
| .1 | -.4 | .2 |
| -.5 | .6 | .5 |
| -.2 | .3 | .5 |
| 1.1 | 2.1 | .3 |
| -.7 | 2.1 | -2 |
| -1 | .7 | .3 |

$\boldsymbol{Q}$

$\times$

users

factors

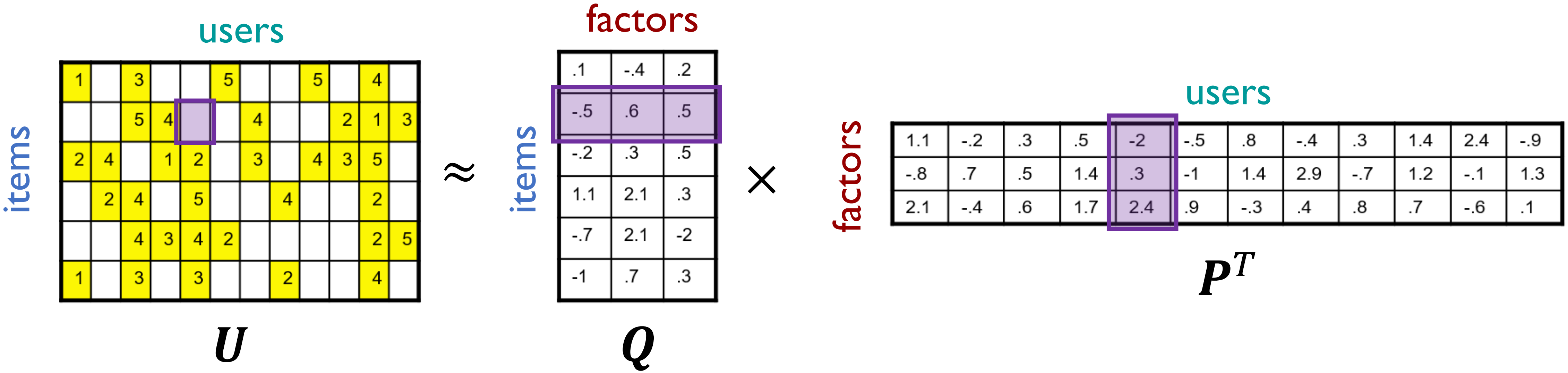
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | -.2 | .3 | .5 | -2 | -.5 | .8 | -.4 | .3 | 1.4 | 2.4 | -.9 |
| -.8 | .7 | .5 | 1.4 | .3 | -1 | 1.4 | 2.9 | -.7 | 1.2 | -.1 | 1.3 |
| 2.1 | -.4 | .6 | 1.7 | 2.4 | .9 | -.3 | .4 | .8 | .7 | -.6 | .1 |

$\boldsymbol{P}^T$

$$U_{xi} = \sum_{\phi:\text{ all factors}} Q_{i\phi} \cdot P_{x\phi}$$

# Estimating the Missing Rating

- Ratings as "sum of products"



|  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 |  | 3 |  |  | 5 |  |  | 5 |  | 4 |  |
|  |  | 5 | 4 |  |  | 4 |  |  | 2 | 1 | 3 |
| 2 | 4 |  | 1 | 2 |  | 3 |  | 4 | 3 | 5 |  |
|  | 2 | 4 |  | 5 |  |  | 4 |  |  | 2 |  |
|  |  | 4 | 3 | 4 | 2 |  |  |  |  | 2 | 5 |
| 1 |  | 3 |  | 3 |  |  | 2 |  |  | 4 |  |

|  |  |  |
|---|---|---|
| .1 | -.4 | .2 |
| -.5 | .6 | .5 |
| -.2 | .3 | .5 |
| 1.1 | 2.1 | .3 |
| -.7 | 2.1 | -2 |
| -1 | .7 | .3 |

|  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | -.2 | .3 | .5 | -2 | -.5 | .8 | -.4 | .3 | 1.4 | 2.4 | -.9 |
| -.8 | .7 | .5 | 1.4 | .3 | -1 | 1.4 | 2.9 | -.7 | 1.2 | -.1 | 1.3 |
| 2.1 | -.4 | .6 | 1.7 | 2.4 | .9 | -.3 | .4 | .8 | .7 | -.6 | .1 |

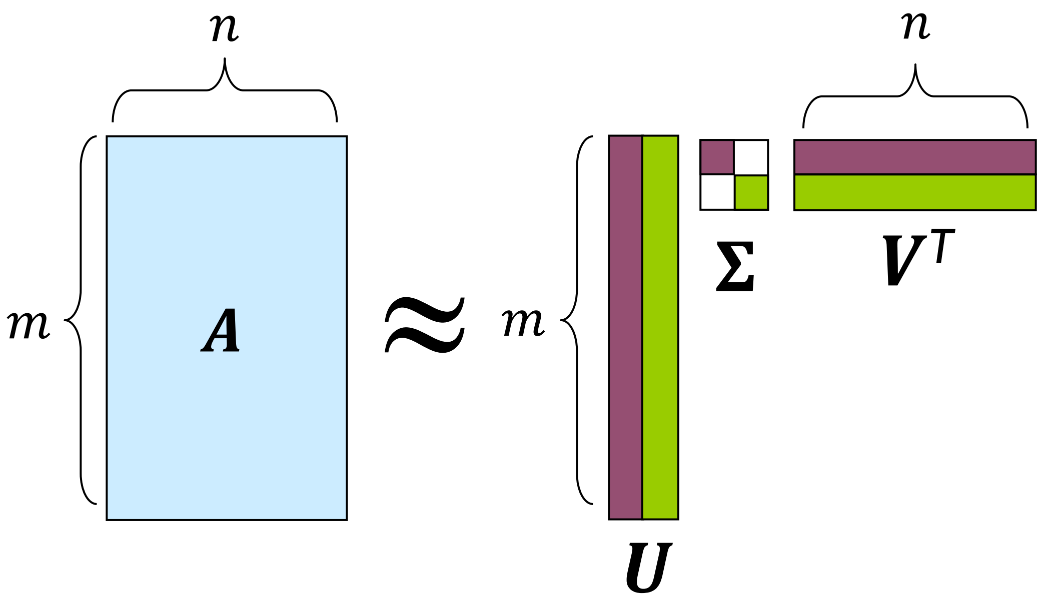
$$U_{xi} = \sum_{\phi\text{: all factors}} Q_{i\phi} \cdot P_{x\phi} = (-0.5) \times (-2) + 0.6 \times 0.3 + 0.5 \times 2.4 = 2.38$$

# How to find $\boldsymbol{Q}$ and $\boldsymbol{P}$?

# Singular Value Decomposition (SVD)

$$\boldsymbol{A} \approx \boldsymbol{U\Sigma V}^T$$

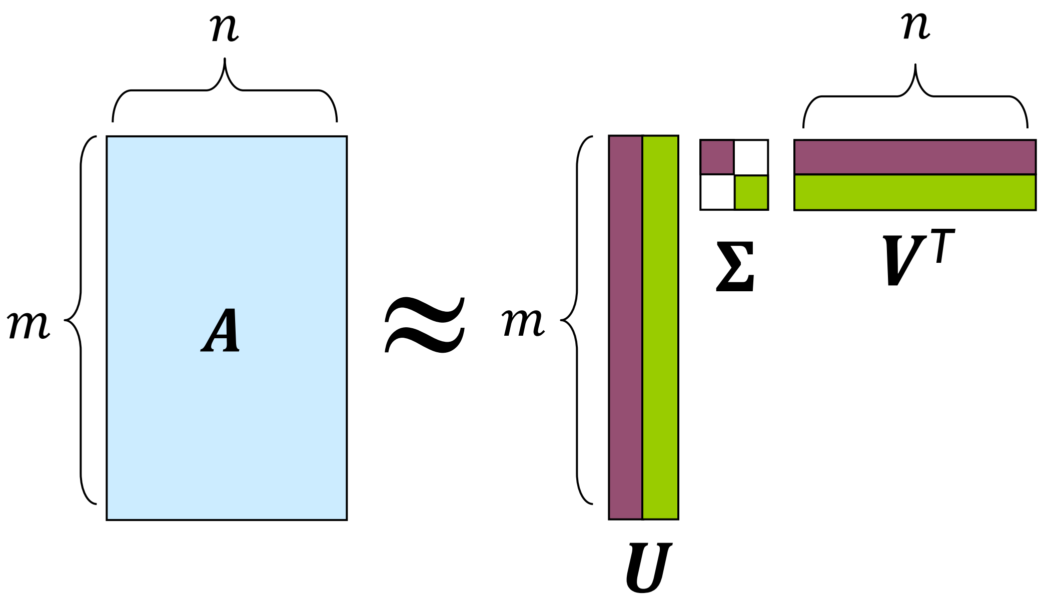
- Input matrix: $\boldsymbol{A}$
- Step 1: Compute $\boldsymbol{A}^T\boldsymbol{A}$
- Step 2: Find the eigenvalues of eigenvectors of $\boldsymbol{A}^T\boldsymbol{A}$
  - Eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$
  - Eigenvectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n$
- Step 3: Consider the largest $k$ eigenvalues and their corresponding eigenvectors only. (The choice of k depends on how closely you wish to approximate)
  - $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_k]$



# Singular Value Decomposition (SVD)

$$\boldsymbol{A} \approx \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$$

- Step 2: Find the eigenvalues of eigenvectors of $\boldsymbol{A}^T\boldsymbol{A}$
  - Eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$
  - Eigenvectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_n$
- Step 3: Consider the largest $k$ eigenvalues and their corresponding eigenvectors only.
  - $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_k]$
  - $\boldsymbol{\Sigma} = \mathrm{diag}\{\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_k}\}$
- Step 4: $\boldsymbol{U} = \boldsymbol{A}\boldsymbol{V}\boldsymbol{\Sigma}^{-1}$
  - Or you can do Steps 1-3 again for $\boldsymbol{A}\boldsymbol{A}^T$ (rather than $\boldsymbol{A}^T\boldsymbol{A}$) to get $\boldsymbol{U}$



# SVD is good, but …

- SVD gives the minimum reconstruction error if we know all entries in $\boldsymbol{A}$.

$$\min_{\boldsymbol{U},\boldsymbol{\Sigma},\boldsymbol{V}} \sum_{i,j} \left(A_{ij} - [\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T]_{ij}\right)^2$$

  - Exactly our objective!
  - Using SVD for our matrix factorization task?

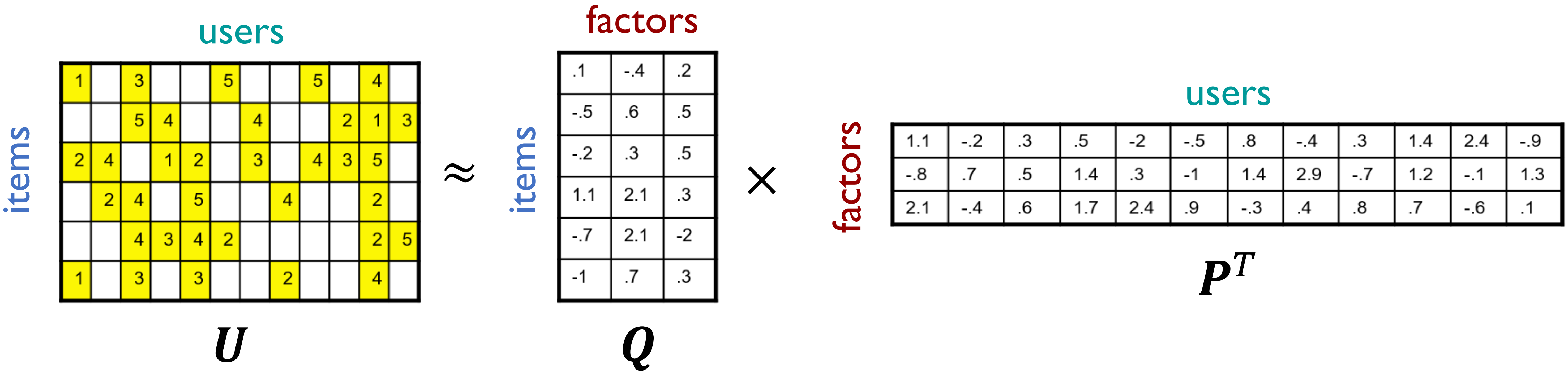
| Latent Factor Model | User-Item Matrix: $\boldsymbol{U}$ | User-Factor Matrix: $\boldsymbol{Q}$ | Factor-Item Matrix: $\boldsymbol{P}^T$ |
|---|---|---|---|
| SVD | Input Matrix: $\boldsymbol{A}$ | $\boldsymbol{U}$ | $\boldsymbol{\Sigma}\boldsymbol{V}^T$ |

- BUT, our user-item matrix $\boldsymbol{U}$ has missing values!
  - How to interpret missing values? (as 0? a bad idea)
  - Does the property of minimum reconstruction error still hold if there are missing values? (we don’t know)

# Factorizing a Matrix with Missing Values

$$\min_{\boldsymbol{Q},\boldsymbol{P}} \sum_{(x,i)\text{ known}} (U_{xi} - [\boldsymbol{Q}\boldsymbol{P}^T]_{xi})^2 = \sum_{(x,i)\text{ known}} (U_{xi} - \boldsymbol{q}_i \boldsymbol{p}_x^T)^2$$

- $\boldsymbol{q}_i$ (item vector): the row corresponding to item $i$ in $\boldsymbol{Q}$
- $\boldsymbol{p}_x^T$ (user vector): the column corresponding to user $x$ in $\boldsymbol{P}^T$



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | 3 | | | 5 | | | 5 | | 4 | |
| | | 5 | 4 | | | 4 | | | 2 | 1 | 3 |
| 2 | 4 | | 1 | 2 | | 3 | | 4 | 3 | 5 | |
| | 2 | 4 | | 5 | | | 4 | | | 2 | |
| | | 4 | 3 | 4 | 2 | | | | | 2 | 5 |
| 1 | | 3 | | 3 | | | 2 | | | 4 | |

$\boldsymbol{U}$

| | | |
|---|---|---|
| .1 | -.4 | .2 |
| -.5 | .6 | .5 |
| -.2 | .3 | .5 |
| 1.1 | 2.1 | .3 |
| -.7 | 2.1 | -2 |
| -1 | .7 | .3 |

$\boldsymbol{Q}$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | -.2 | .3 | .5 | -2 | -.5 | .8 | -.4 | .3 | 1.4 | 2.4 | -.9 |
| -.8 | .7 | .5 | 1.4 | .3 | -1 | 1.4 | 2.9 | -.7 | 1.2 | -.1 | 1.3 |
| 2.1 | -.4 | .6 | 1.7 | 2.4 | .9 | -.3 | .4 | .8 | .7 | -.6 | .1 |

$\boldsymbol{P}^T$

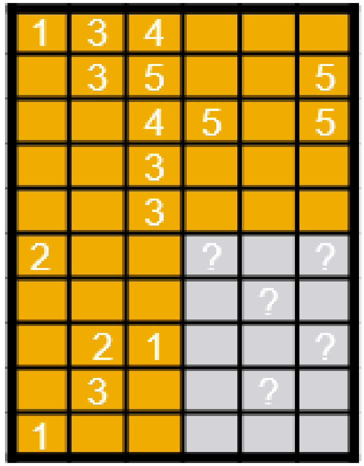
# Overfitting

$$\min_{\boldsymbol{Q},\boldsymbol{P}} \sum_{(x,i)\text{ known}} (U_{xi} - \boldsymbol{q}_i \boldsymbol{p}_x^T)^2$$

- $\boldsymbol{q}_i$ (item vector): the row corresponding to item $i$ in $\boldsymbol{Q}$
- $\boldsymbol{p}_x^T$ (user vector): the column corresponding to user $x$ in $\boldsymbol{P}^T$

- No closed form solution.
- All item vectors and user vectors are parameters to be learned!

- Overfitting: With too much freedom (too many free parameters) the model starts fitting noise in the training data, thus not generalizing well to unseen test data.



| | | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 4 | | | |
| | 3 | 5 | | | 5 |
| | | 4 | 5 | | 5 |
| | | 3 | | | |
| | | 3 | | | |
| 2 | | | ? | | ? |
| | | | | ? | |
| | 2 | 1 | | | ? |
| | 3 | | | ? | |
| 1 | | | | | |

# Regularization

- Model parameters can be "complicated" where there are sufficient training data
- Model parameters should be "simple" where training data are scarce

$$\min_{\boldsymbol{Q},\boldsymbol{P}} \sum_{(x,i)\ \text{known}} (U_{xi} - \boldsymbol{q}_i \boldsymbol{p}_x^T)^2 + \left[ c_1 \sum_x \|\boldsymbol{p}_x\|^2 + c_2 \sum_i \|\boldsymbol{q}_i\|^2 \right]$$

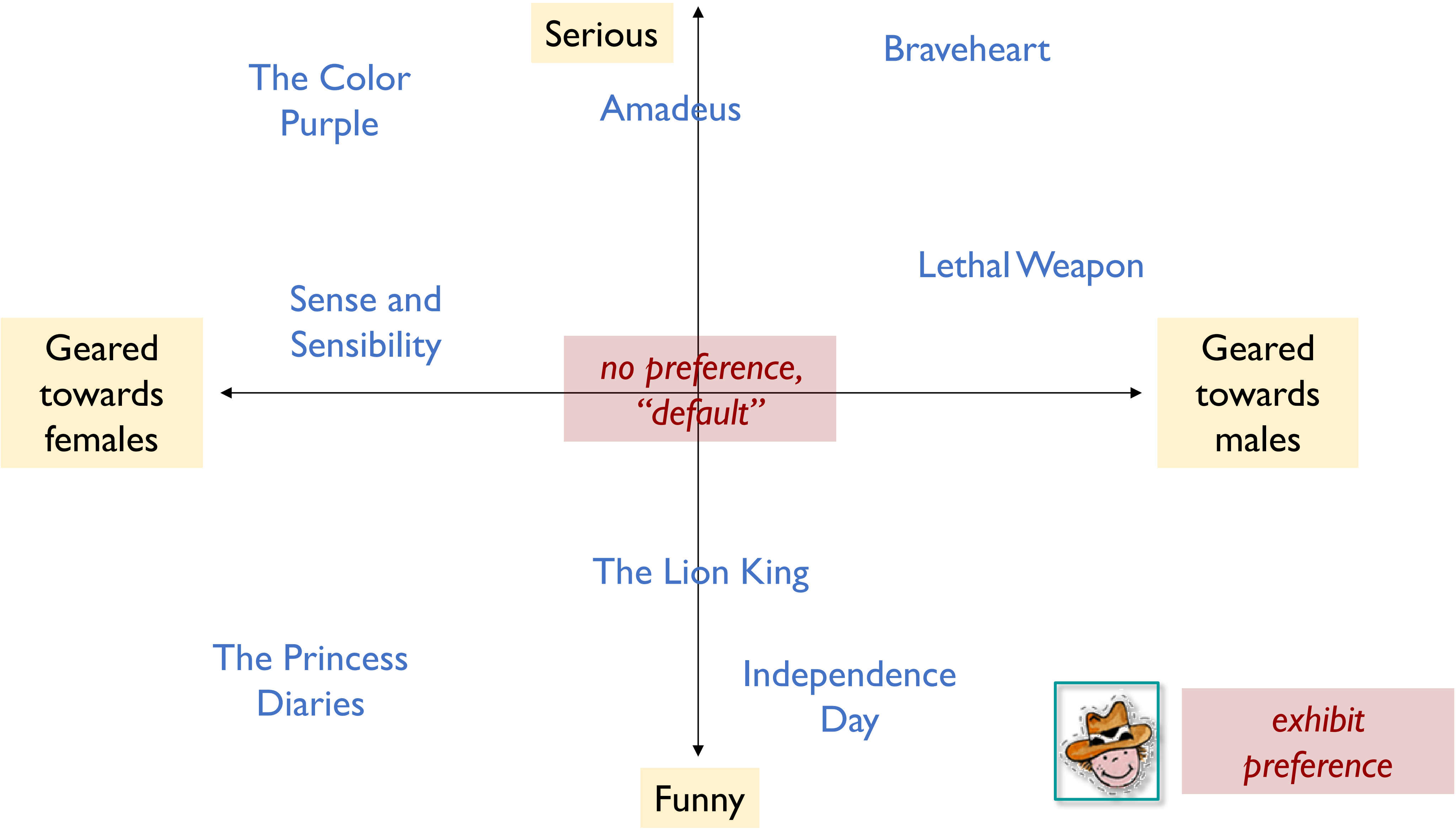
Original Objective

Regularization Term

($c_1$ and $c_2$ are hyperparameters)

- How to understand the Regularization Term?

# The Effect of Regularization



Serious

Braveheart

The Color Purple

Amadeus

Lethal Weapon

Sense and Sensibility

Geared towards females

*no preference, "default"*

Geared towards males

The Lion King

The Princess Diaries

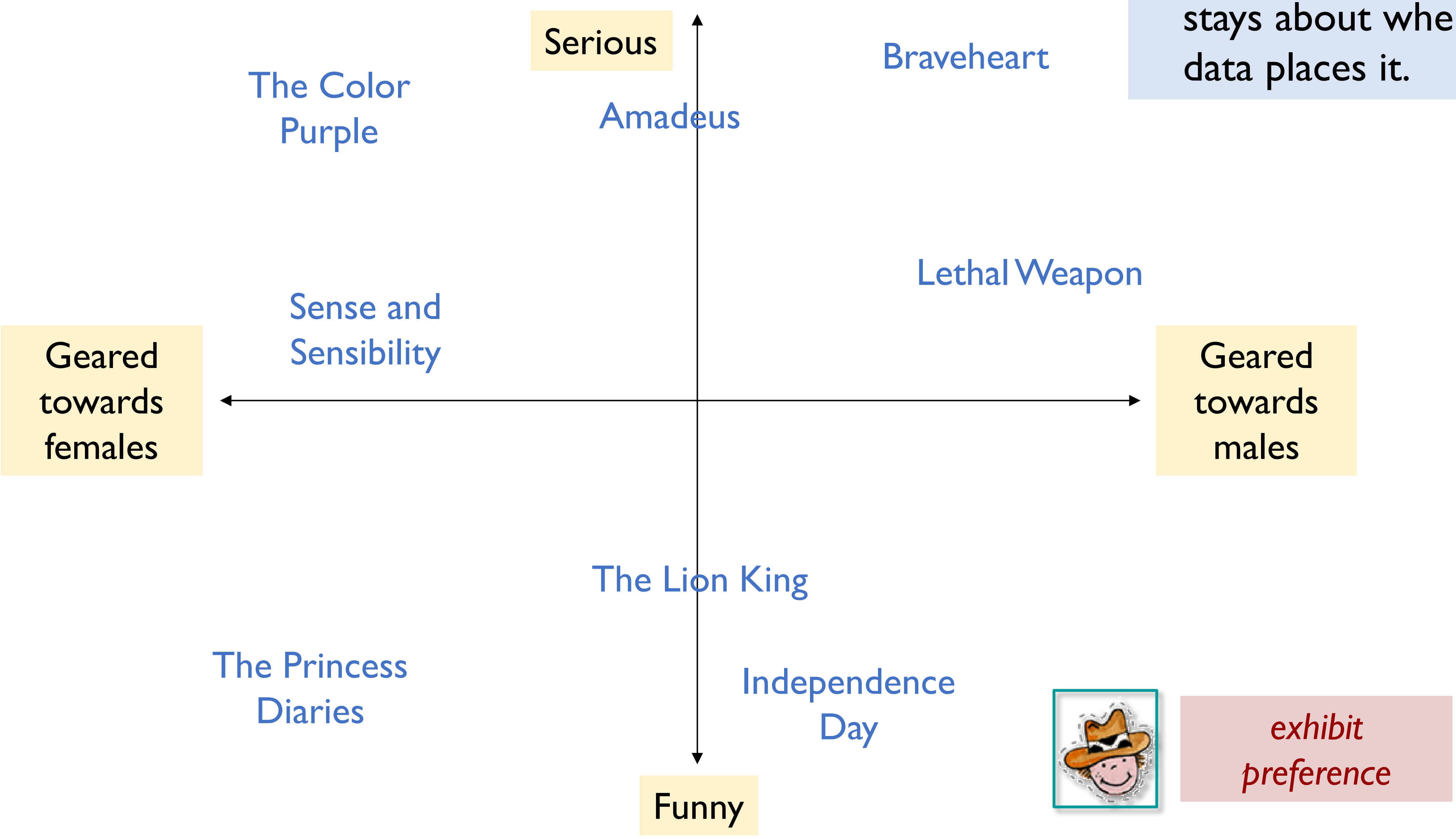
Independence Day

Funny

*exhibit preference*

# The Effect of Regularization

- If the user has rated hundreds of movies, it stays about where the data places it.



Serious

Funny

Geared towards females

Geared towards males

The Color Purple

Amadeus

Braveheart

Lethal Weapon

Sense and Sensibility

The Lion King

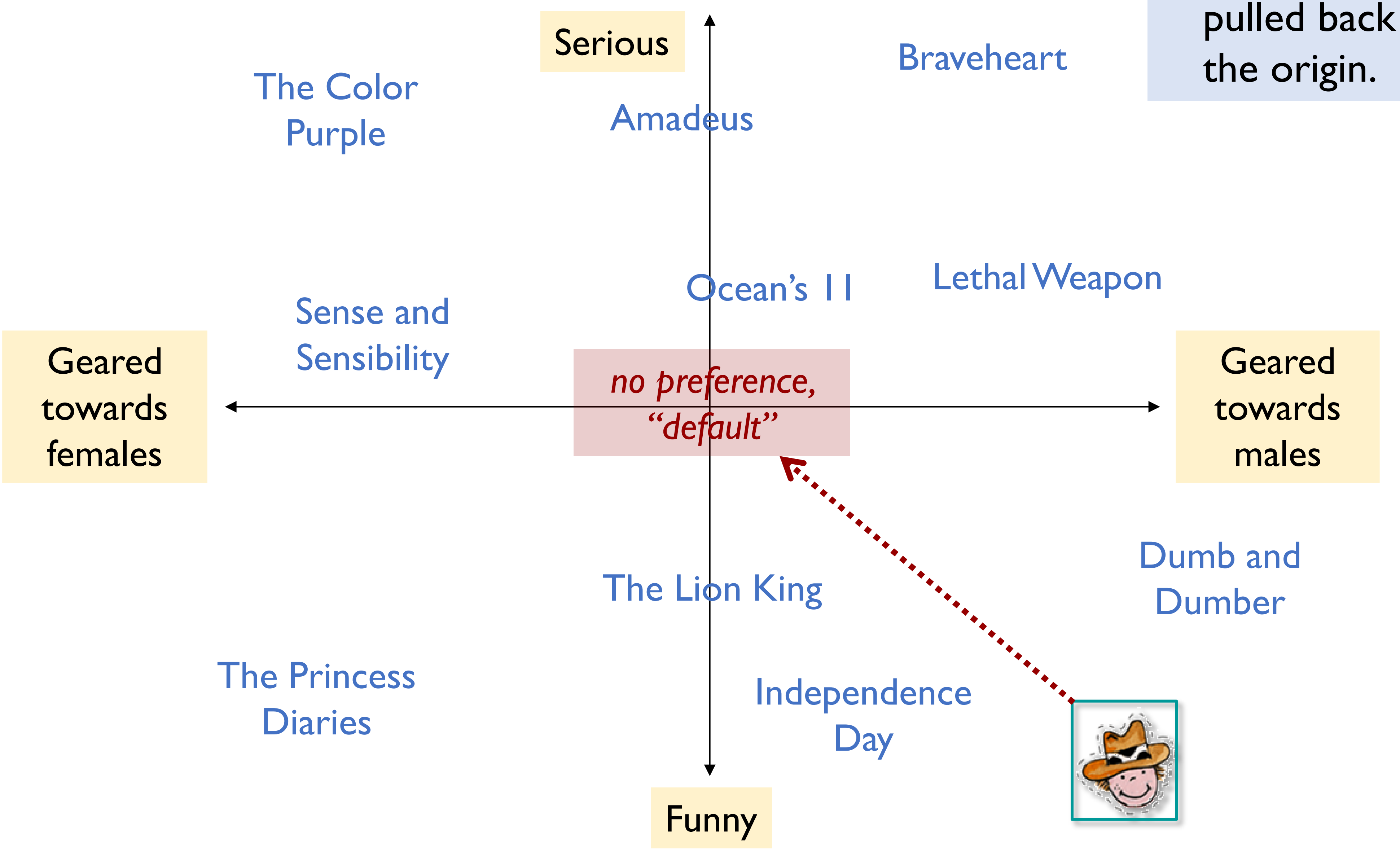
The Princess Diaries

Independence Day

*exhibit preference*

# The Effect of Regularization

- If the user has rated only a handful, it is pulled back towards the origin.



Serious

Funny

Geared towards females

Geared towards males

The Color Purple

Amadeus

Braveheart

Sense and Sensibility

Ocean's 11

Lethal Weapon

*no preference, "default"*

The Lion King

Dumb and Dumber

The Princess Diaries

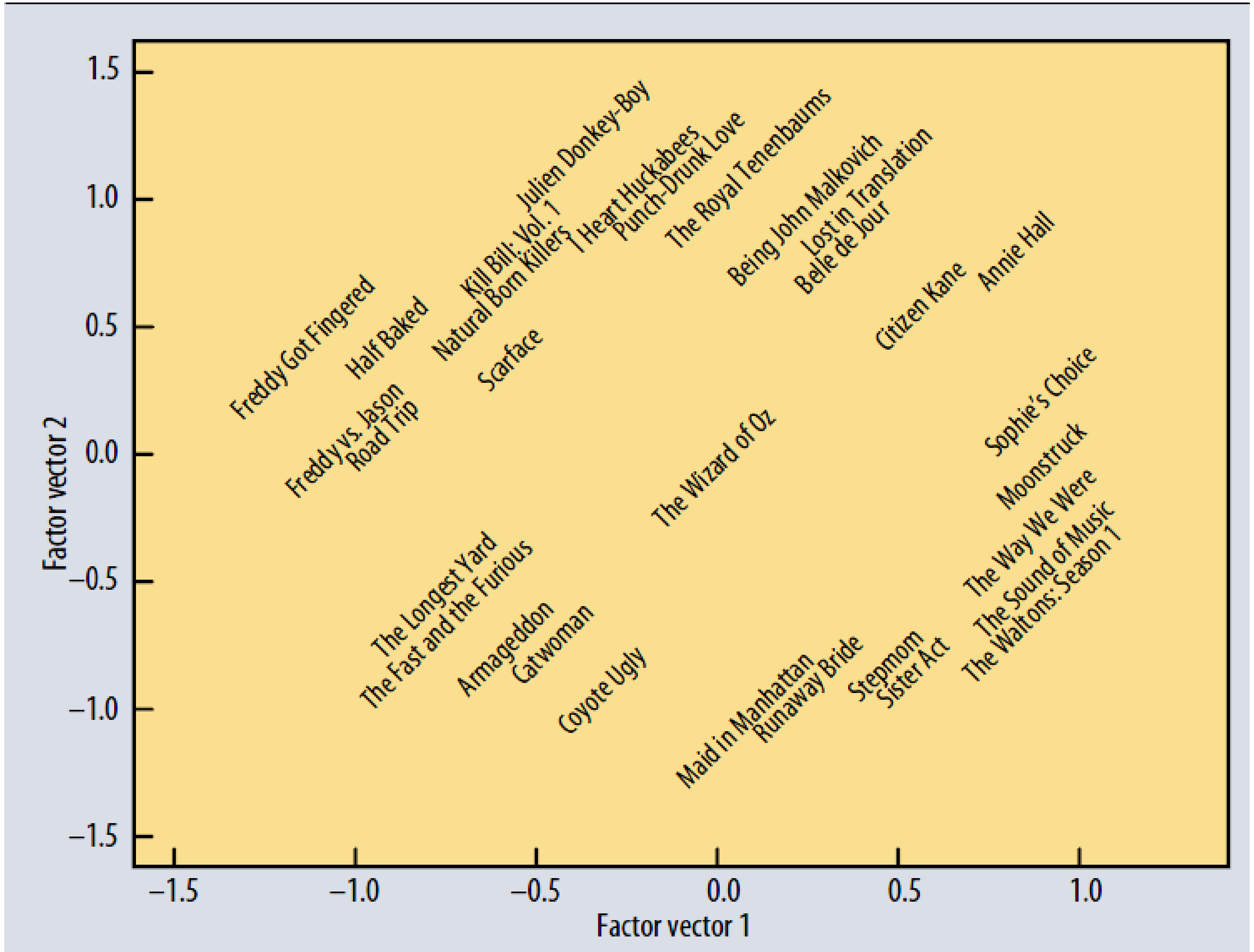
Independence Day

# Gradient Descent

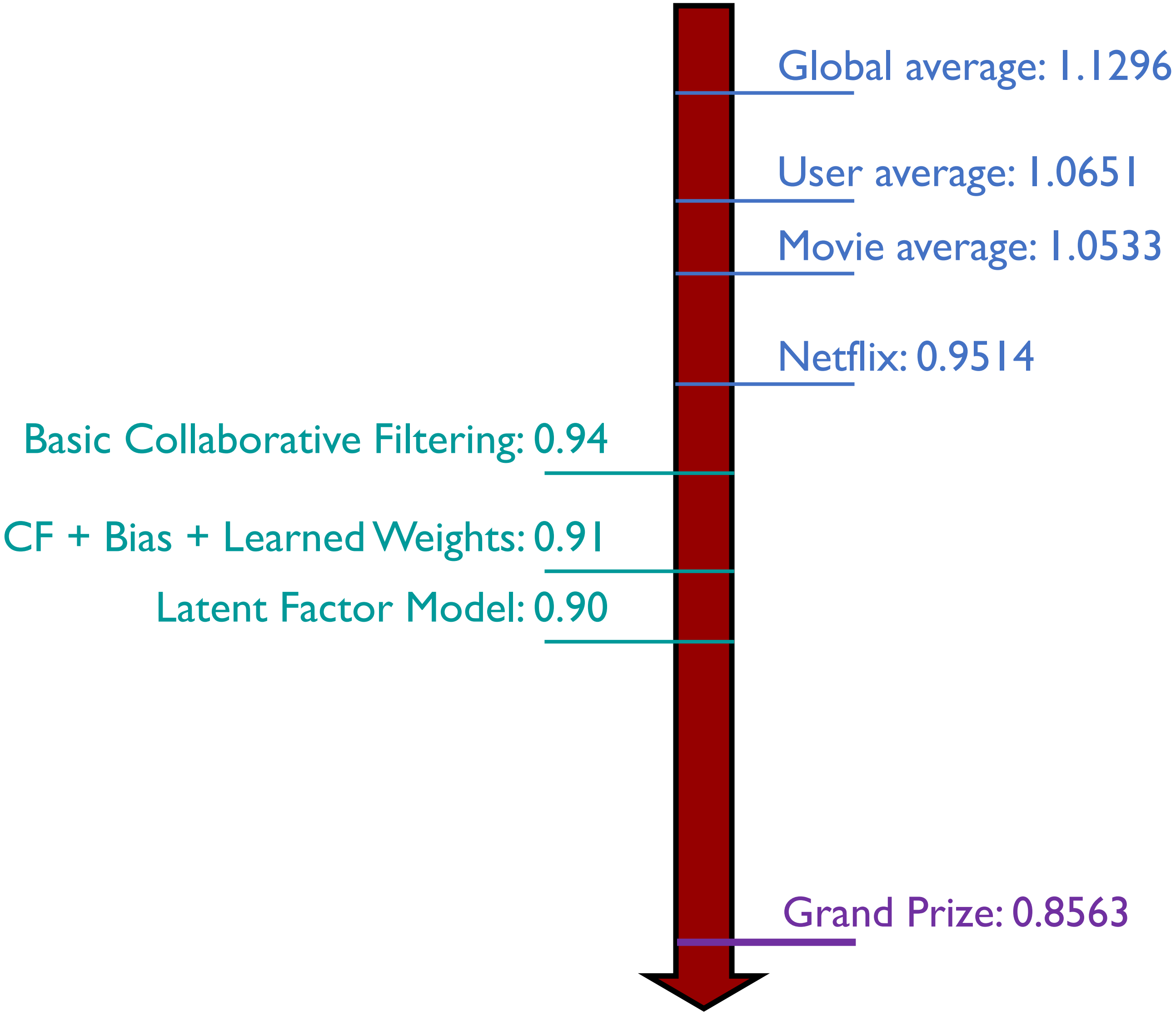
$$\min_{\boldsymbol{Q},\boldsymbol{P}} J = \sum_{(x,i)\text{ known}} (U_{xi} - \boldsymbol{q}_i \boldsymbol{p}_x^T)^2 + \left[ c_1 \sum_x \|\boldsymbol{p}_x\|^2 + c_2 \sum_i \|\boldsymbol{q}_i\|^2 \right]$$

- Step 1: Initialize $\boldsymbol{Q}$ and $\boldsymbol{P}$ using SVD (pretend missing ratings are 0)
- Step 2: Gradient descent
  - $P_{x\phi} = P_{x\phi} - \eta \frac{\partial J}{\partial P_{x\phi}}$
    - $\frac{\partial J}{\partial P_{x\phi}} = \sum_{(x,i)\text{ known}} \left(-2(U_{xi} - \boldsymbol{q}_i \boldsymbol{p}_x^T) Q_{i\phi} + 2c_1 P_{x\phi}\right)$
  - $Q_{i\phi} = Q_{i\phi} - \eta \frac{\partial J}{\partial Q_{i\phi}}$
    - $\frac{\partial J}{\partial Q_{i\phi}} = \sum_{(x,i)\text{ known}} \left(-2(U_{xi} - \boldsymbol{q}_i \boldsymbol{p}_x^T) P_{x\phi} + 2c_2 Q_{i\phi}\right)$

# Learned Item Vectors in the Latent Factor Space



# Performance of Various Models



Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative Filtering: 0.94

CF + Bias + Learned Weights: 0.91

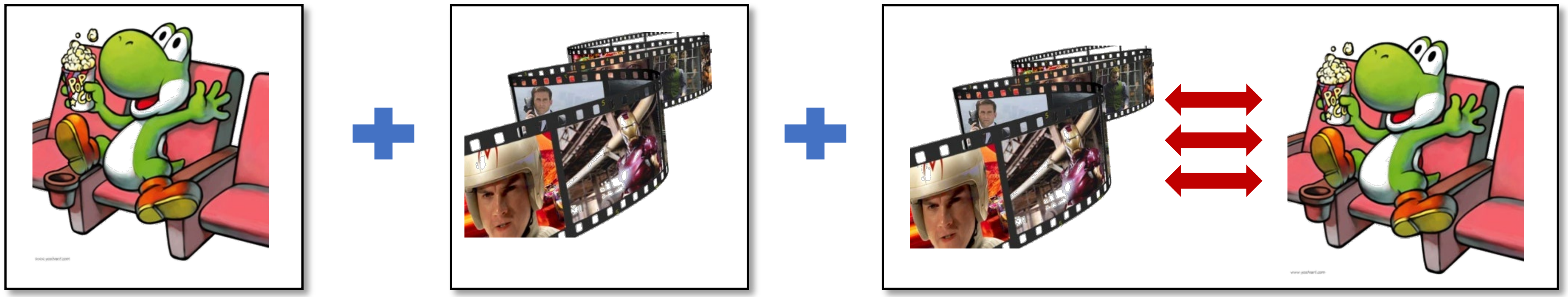
Latent Factor Model: 0.90

Grand Prize: 0.8563

# Extending Latent Factor Models to Include Bias

# Bias, Again

- Basic Latent Factor Model: $U_{xi} = \boldsymbol{q}_i \boldsymbol{p}_x^T$
- Latent Factor Model with Bias: $U_{xi} = \mu + b_x + b_i + \boldsymbol{q}_i \boldsymbol{p}_x^T$
  - $\mu$: overall mean movie rating
  - $b_x$: rating deviation of user $x$
  - $b_i$: rating deviation of item $i$



user bias

movie bias

user-movie interaction

# Bias, Again

- Latent Factor Model with Bias:

$$U_{xi} = \mu + b_x + b_i + \boldsymbol{q}_i \boldsymbol{p}_x^T$$

  - $\mu$: overall mean movie rating
    - E.g., $\mu = 2.7$
  - $b_x$: rating deviation of user $x$ (*to be learned*)
    - E.g., Bob is a critical reviewer. Based on the training data, his rating will be 0.7 star lower than the mean ⇒ $b_x = -0.7$.
  - $b_i$: rating deviation of item $i$ (*to be learned*)
    - E.g., Star Wars will get a mean rating of 0.5 higher than the average ⇒ $b_i = 0.5$
  - $\boldsymbol{q}_i$ and $\boldsymbol{p}_x$: vector of user $x$ and item $i$ in the latent factor space (*to be learned*)
    - E.g., based on the genre, Bob likes Star Wars ⇒ $\boldsymbol{q}_i \boldsymbol{p}_x^T = 0.3$
  - $U_{xi} = 2.7 - 0.7 + 0.5 + 0.3 = 2.8$

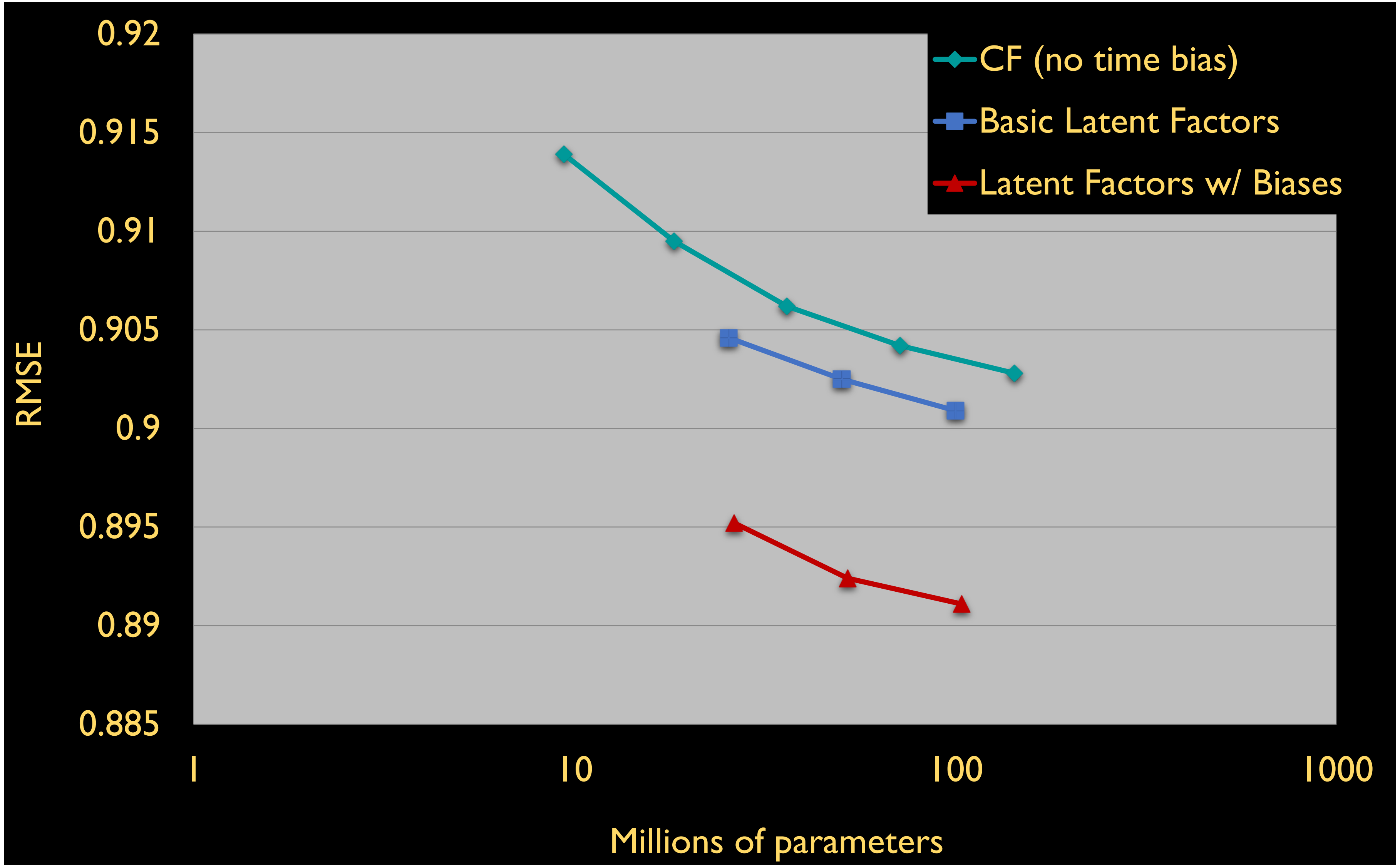
# Fitting the New Model

$$\min_{\boldsymbol{Q},\boldsymbol{P},b_x,b_i} J = \sum_{(x,i)\text{ known}} (U_{xi} - (\mu + b_x + b_i + \boldsymbol{q}_i\boldsymbol{p}_x^T))^2$$
$$+ \left[c_1 \sum_x \|\boldsymbol{p}_x\|^2 + c_2 \sum_i \|\boldsymbol{q}_i\|^2 + c_3 \sum_x \|b_x\|^2 + c_4 \sum_i \|b_i\|^2\right]$$

- Both biases $b_x, b_i$ as well as interactions $\boldsymbol{q}_i, \boldsymbol{p}_x$ are treated as parameters to be learned via gradient descent
  - $P_{x\phi} = P_{x\phi} - \eta \frac{\partial J}{\partial P_{x\phi}}$, $\quad Q_{i\phi} = Q_{i\phi} - \eta \frac{\partial J}{\partial Q_{i\phi}}$
  - $b_x = b_x - \eta \frac{\partial J}{\partial b_x}$, $\quad b_i = b_i - \eta \frac{\partial J}{\partial b_i}$

# Performance of Various Models

- Which hyperparameter determines the number of parameters?
  - Number of factors



# Performance of Various Models

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative Filtering: 0.94

CF + Bias + Learned Weights: 0.91

Latent Factor Model: 0.90

Latent Factor Model + Bias: 0.89

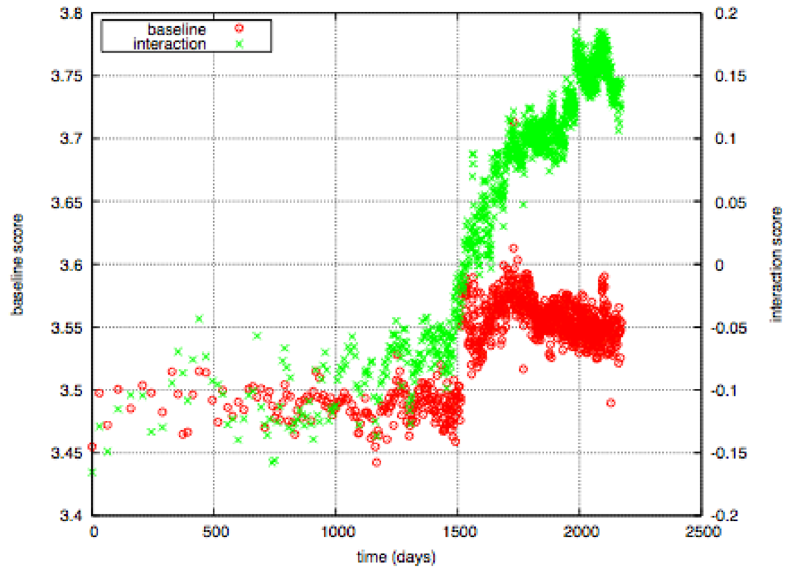
Grand Prize: 0.8563

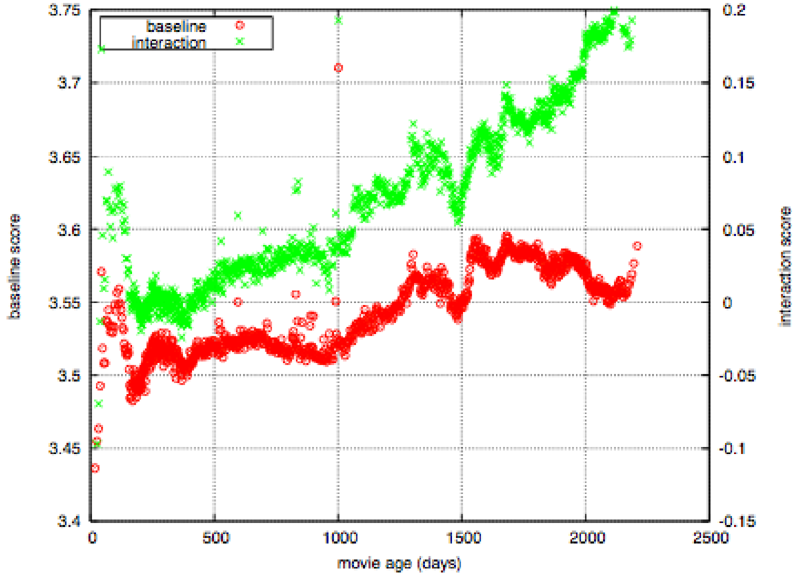
# Extended Content: The Netflix Challenge 2006-2009
(will not appear in quizzes or the exam)

# Temporal Biases Of Users [Koren, KDD 2009]

- A sudden surge in the average movie rating observed in early 2004.
  - Possible reasons:
    - Improvements in Netflix
    - GUI improvements
    - Meaning of rating changed

- For the rating of a single movie, its age is an important factor.
  - Users prefer the newest movies
  - For not that new movies, people believe even older movies are just inherently better



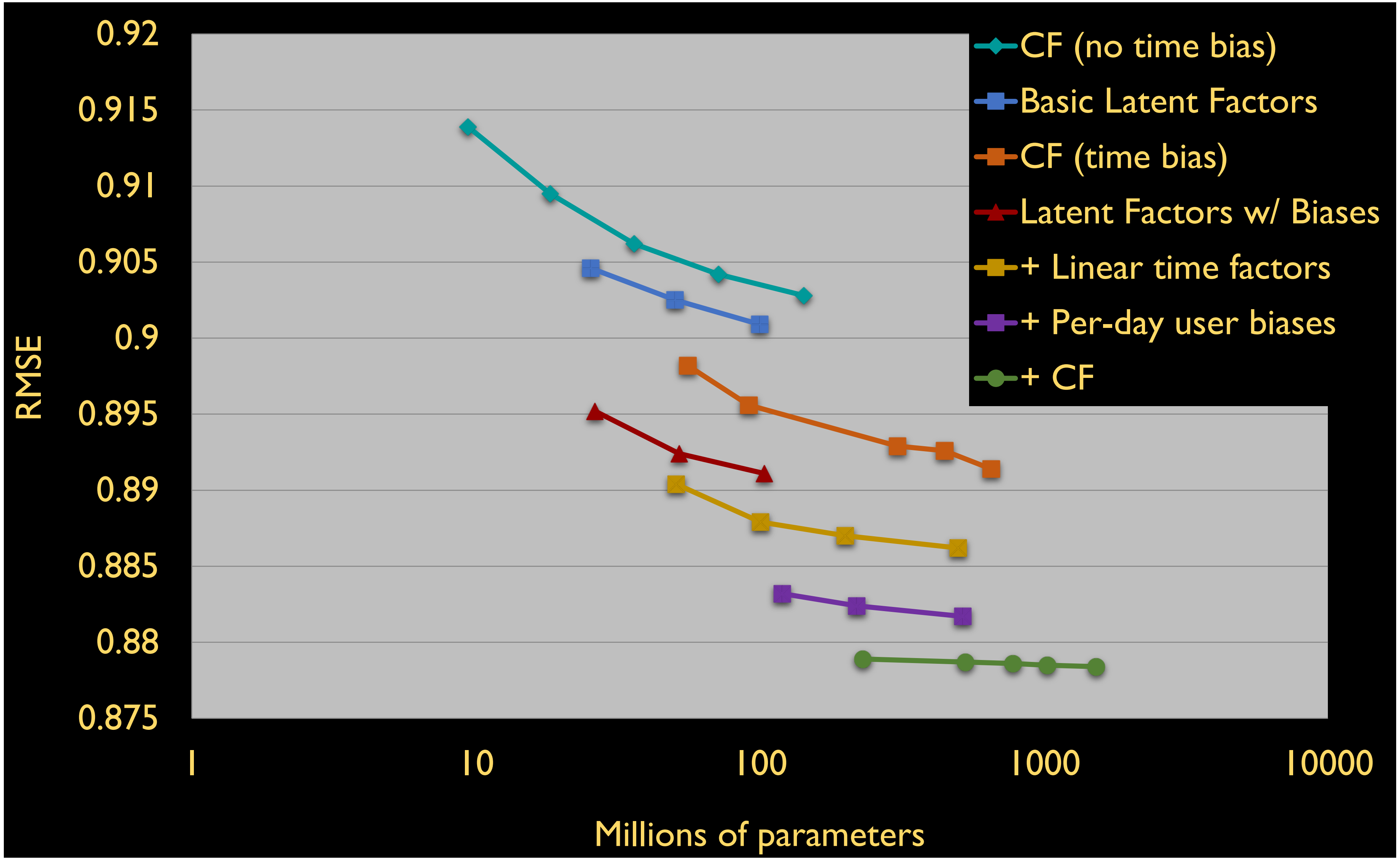


# Temporal Biases and Factors

- Latent Factor Model with Constant Bias: $U_{xi} = \mu + b_x + b_i + \boldsymbol{q}_i \boldsymbol{p}_x^T$
- Latent Factor Model with Temporal Bias: $U_{xi} = \mu + b_x(t) + b_i(t) + \boldsymbol{q}_i \boldsymbol{p}_x^T$
    - Make parameters $b_x$ and $b_i$ to depend on time
        - Parameterize time-dependence by linear trends
        - Each bin corresponds to 10 consecutive weeks
            - $b_i(t) = b_i + b_{i,\mathrm{Bin}(t)}$

- One can further add temporal dependence to user/item vectors
    - $\boldsymbol{p}_x(t)$: user preference vector on day $t$

# Adding Temporal Effects



# Performance of Various Models

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative Filtering: 0.94

CF + Bias + Learned Weights: 0.91

Latent Factor Model: 0.90

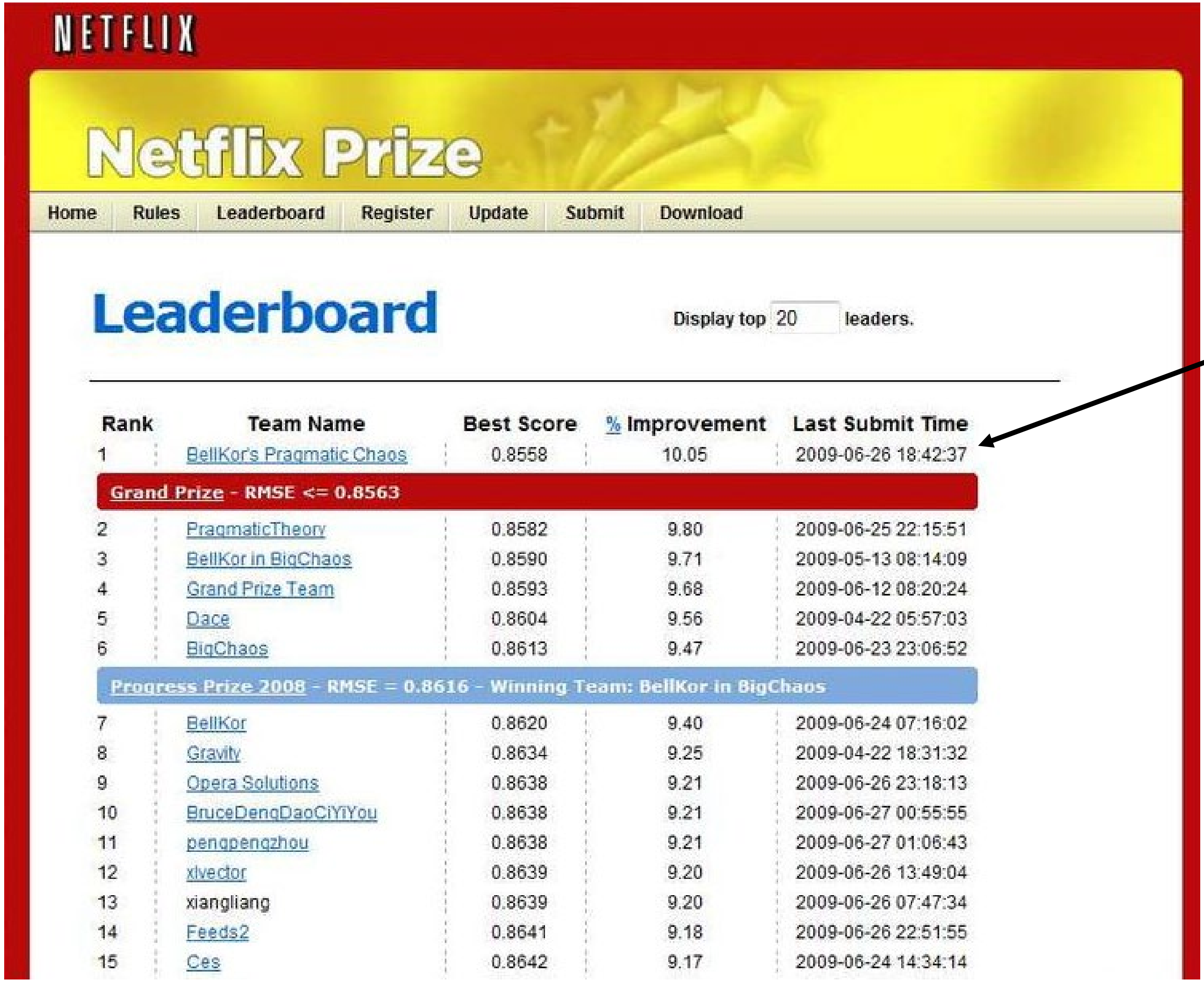
Latent Factor Model + Bias: 0.89

Latent Factor Model + Bias + Time: 0.876

*Still no prize!*
*Getting desperate.*

Grand Prize: 0.8563

# BellKor Recommender System: Winner of the Netflix Challenge



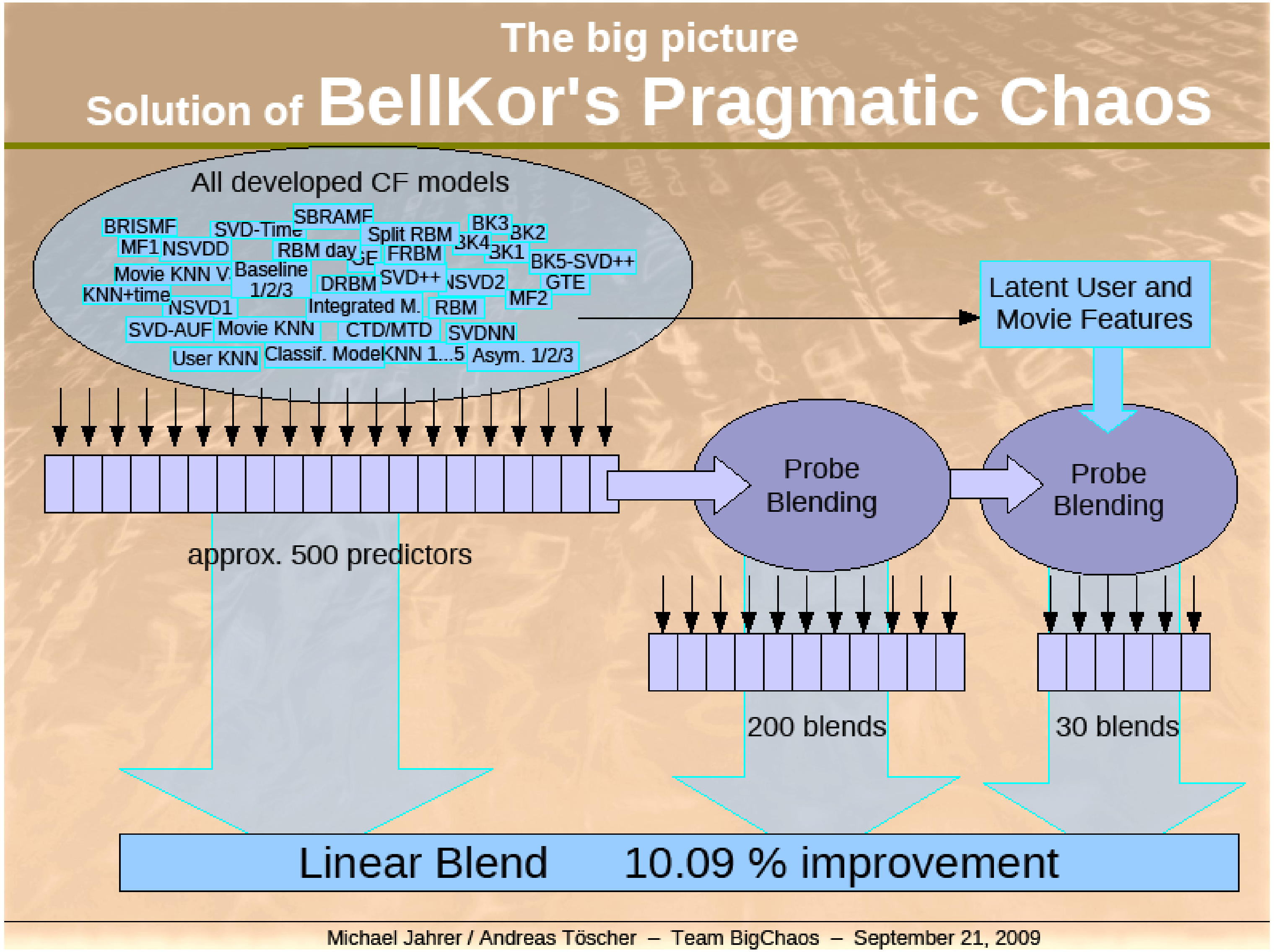
## Leaderboard

Display top 20 leaders.

| Rank | Team Name | Best Score | % Improvement | Last Submit Time |
|---|---|---|---|---|
| 1 | BellKor's Pragmatic Chaos | 0.8558 | 10.05 | 2009-06-26 18:42:37 |
| **Grand Prize - RMSE <= 0.8563** | | | | |
| 2 | PragmaticTheory | 0.8582 | 9.80 | 2009-06-25 22:15:51 |
| 3 | BellKor in BigChaos | 0.8590 | 9.71 | 2009-05-13 08:14:09 |
| 4 | Grand Prize Team | 0.8593 | 9.68 | 2009-06-12 08:20:24 |
| 5 | Dace | 0.8604 | 9.56 | 2009-04-22 05:57:03 |
| 6 | BigChaos | 0.8613 | 9.47 | 2009-06-23 23:06:52 |
| **Progress Prize 2008 - RMSE = 0.8616 - Winning Team: BellKor in BigChaos** | | | | |
| 7 | BellKor | 0.8620 | 9.40 | 2009-06-24 07:16:02 |
| 8 | Gravity | 0.8634 | 9.25 | 2009-04-22 18:31:32 |
| 9 | Opera Solutions | 0.8638 | 9.21 | 2009-06-26 23:18:13 |
| 10 | BruceDengDaoCiYiYou | 0.8638 | 9.21 | 2009-06-27 00:55:55 |
| 11 | pengpengzhou | 0.8638 | 9.21 | 2009-06-27 01:06:43 |
| 12 | xlvector | 0.8639 | 9.20 | 2009-06-26 13:49:04 |
| 13 | xiangliang | 0.8639 | 9.20 | 2009-06-26 07:47:34 |
| 14 | Feeds2 | 0.8641 | 9.18 | 2009-06-26 22:51:55 |
| 15 | Ces | 0.8642 | 9.17 | 2009-06-24 14:34:14 |

June 26, 2009
RMSE = 0.8558

# A “Kitchen Sink” Approach



The big picture

Solution of BellKor's Pragmatic Chaos

All developed CF models

BRISMF
SVD-Time
SBRAMF
Split RBM
BK3
BK2
MF1
NSVDD
RBM day
3K4
3K1
GE
FRBM
BK5-SVD++
Movie KNN V.
Baseline 1/2/3
DRBM
SVD++
NSVD2
GTE
KNN+time
NSVD1
Integrated M.
RBM
MF2
SVD-AUF
Movie KNN
CTD/MTD
SVDNN
User KNN
Classif. Model
KNN 1...5
Asym. 1/2/3

Latent User and Movie Features

approx. 500 predictors

Probe Blending

200 blends

Probe Blending

30 blends

Linear Blend 10.09 % improvement

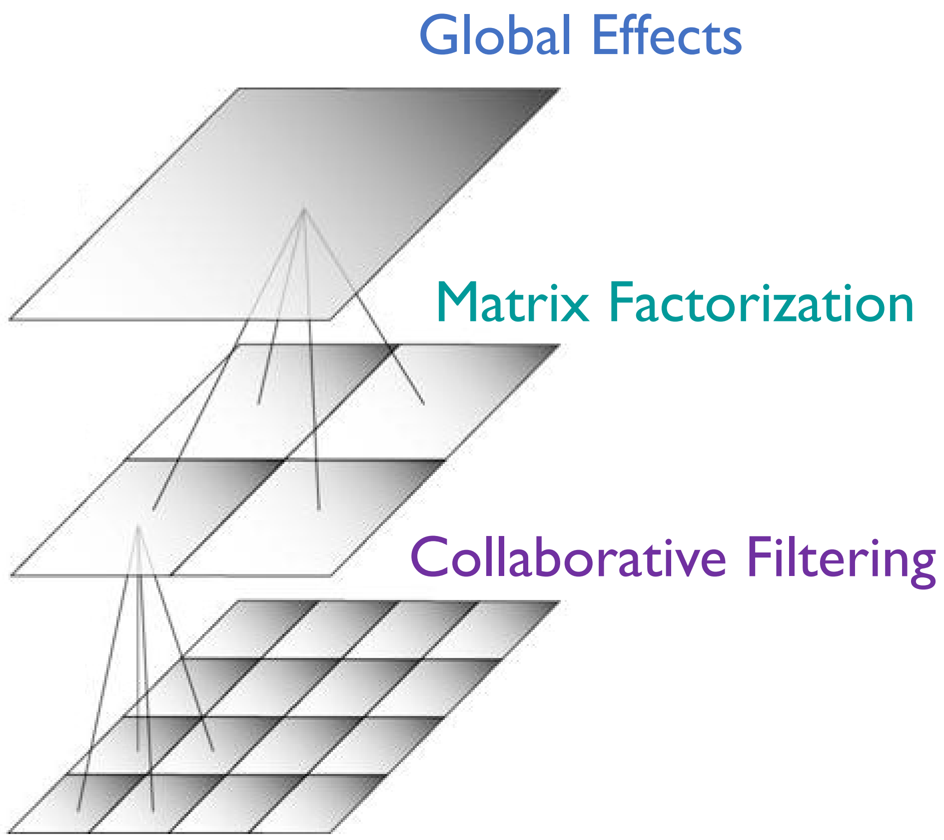
Michael Jahrer / Andreas Töscher – Team BigChaos – September 21, 2009

- For a *research project*, this is a very bad idea (since you don’t know which part works or why).
- To *achieve a certain level of model performance* (and win a prize), this might be an unavoidable path to take.

# BellKor Recommender System: Rough Idea

- Multi-scale modeling of the data: Combine top level, “regional” modeling of the data, with a refined, local view.

- Global:
  - Overall deviations of users/movies
- Matrix Factorization:
  - Addressing “regional” effects
- Collaborative Filtering:
  - Extract local patterns



# Next Lecture

- Finish the story of the Netflix Prize
- Quiz 2!
  - All policies are the same as Quiz 1 (number of questions, time limit, grading, etc.)
  - Scope:
    - Lecture 8 (Statistical Significance Test in IR Evaluation)
    - Lectures 9 & 10 (Learning to Rank)
    - Lecture 11 (Collaborative Filtering)
    - Lecture 12 (Matrix Factorization)
    - Homework 1

Thank You!

Course Website: https://yuzhang-teaching.github.io/CSCE670-F25.html