CSCE 670 - Information Storage and Retrieval

Week 2: TF-IDF; Vector Space Model; BM25

Yu Zhang

yuzhang@tamu.edu

Course Website: https://yuzhang-teaching.github.io/CSCE670-S26.html

Adapted from the slides by Prof. James Caverlee

Recap: Boolean Retrieval

- Our capabilities so far:
  - Boolean keyword queries (AND, OR, NOT)
    - Inverted index
  - Phrase queries ("*x y*")
    - Positional index
  - Proximity queries ("*x NEAR:3 y*")
    - Positional index
  - Wildcard queries ("*x**")
    - Permuterm Index

you don't ever say
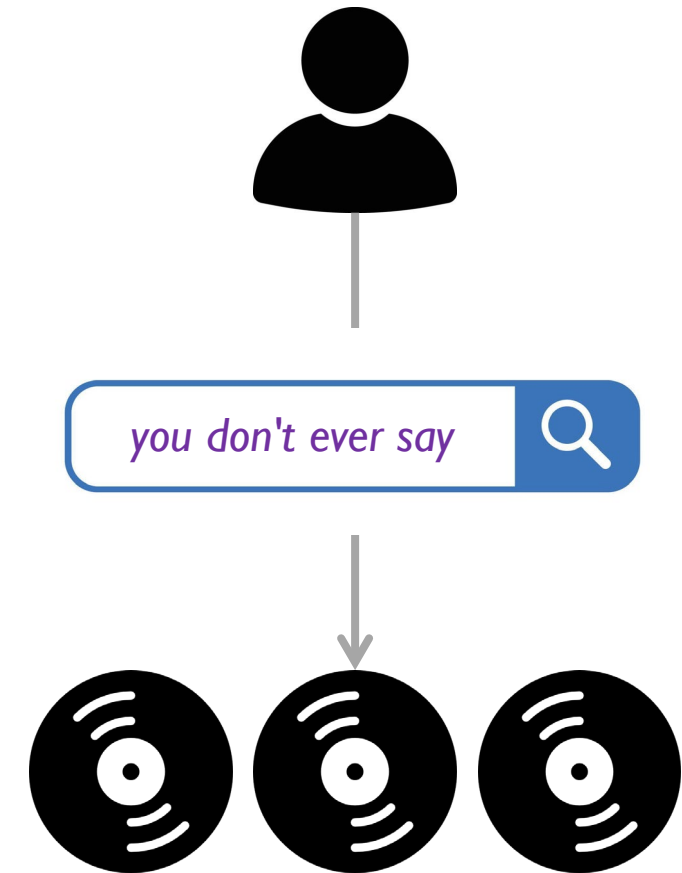
We return a set of matching albums. (100s or 1000s of matches in some cases!)

Ranking

you don't ever say
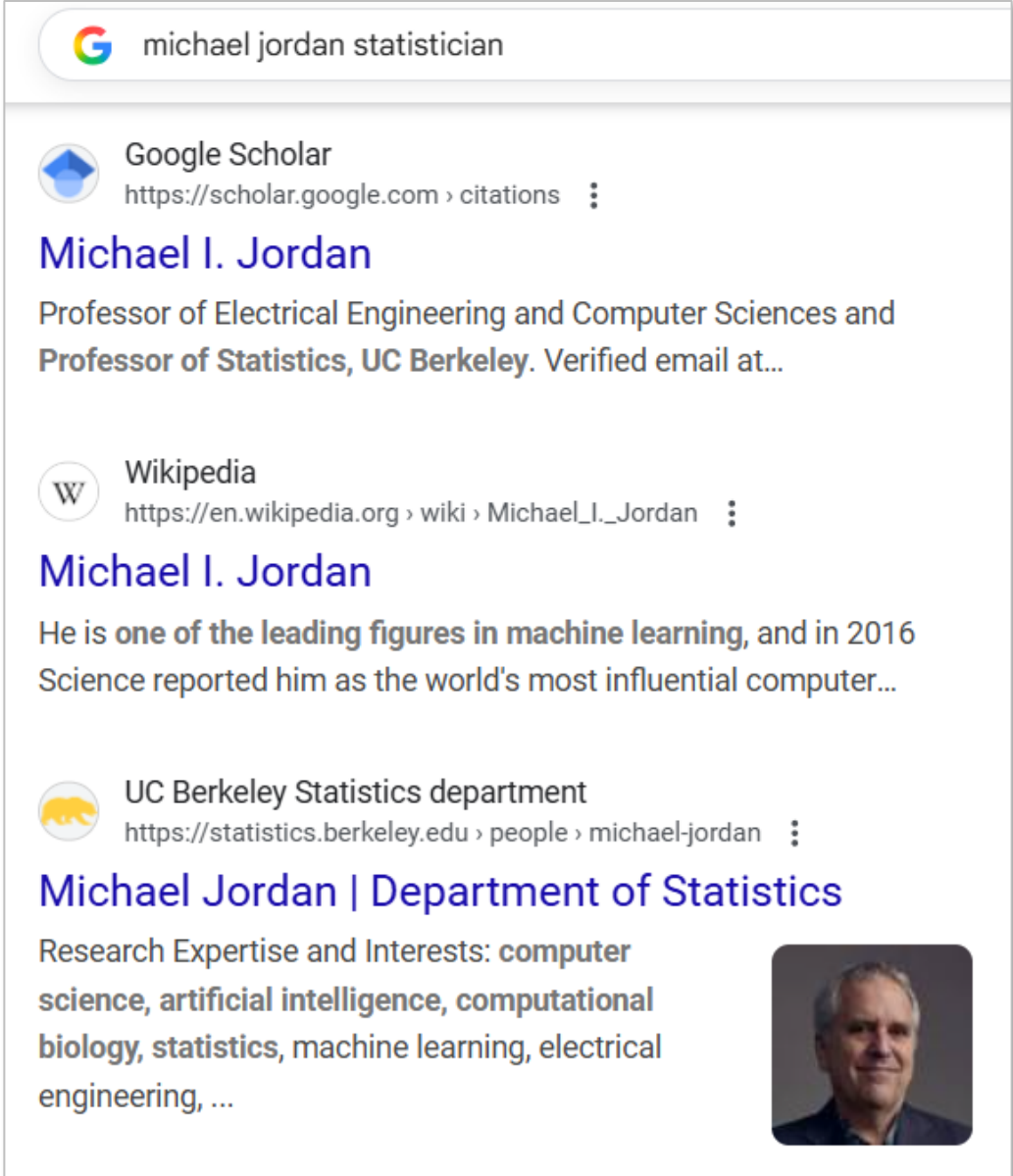
No. 1   No. 2   No. 3   No. 4   No. 5   …
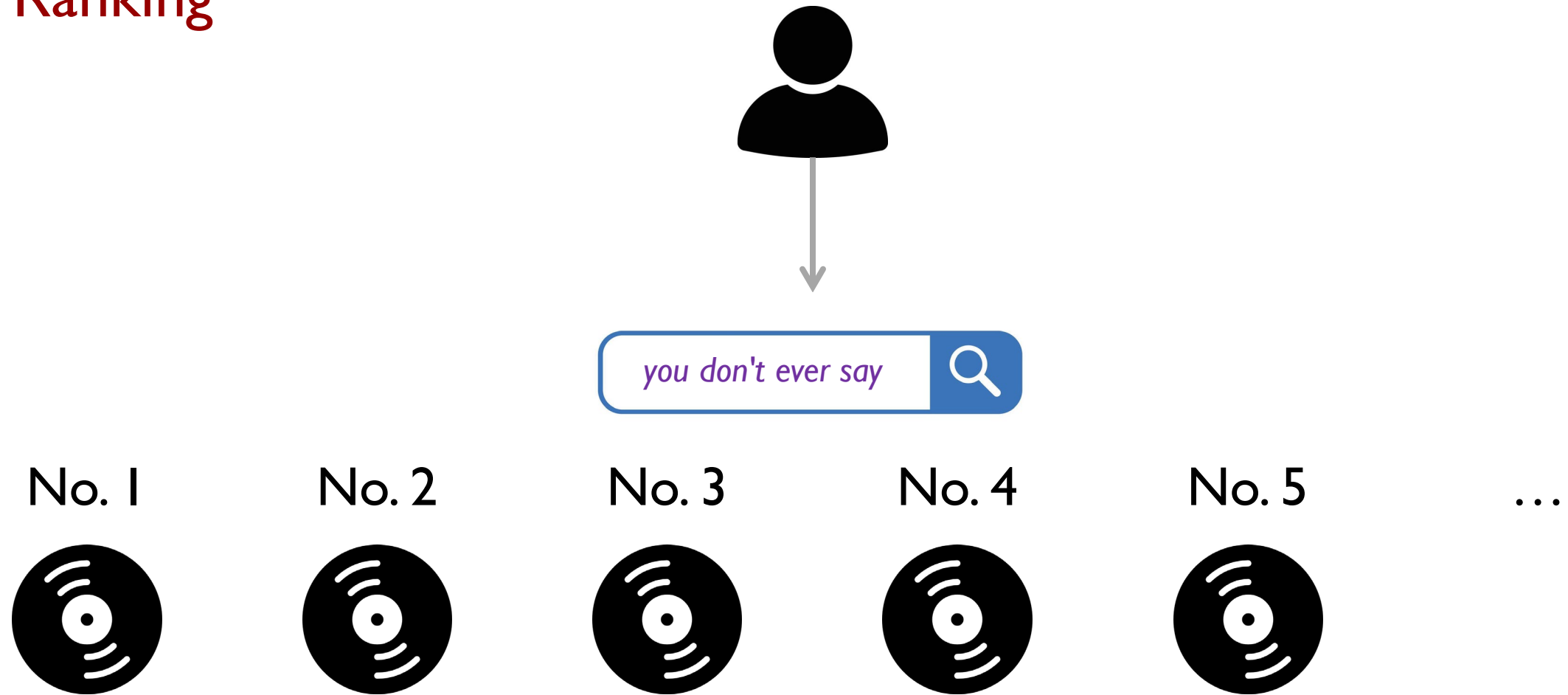
We return a ranked list of albums.

Our Plan: Ranking

- Why is ranking important?

- What factors impact ranking?

- Two foundational text-based approaches
    - TF-IDF
    - BM25

- Two foundational link-based approaches
    - PageRank
    - HITS

- Machine-learned ranking ("learning to rank")

Why is ranking important?

- User Study: eye-tracking and relevance

- Scenario: Participants were asked to answer 10 questions using Google.
  - E.g., "*Find the homepage of Michael Jordan, the statistician.*"

- Eye-Tracking:
  - Record the sequence of eye movements
  - Analyze how users scan the results page of Google

What is Eye-Tracking?

Eye tracking device
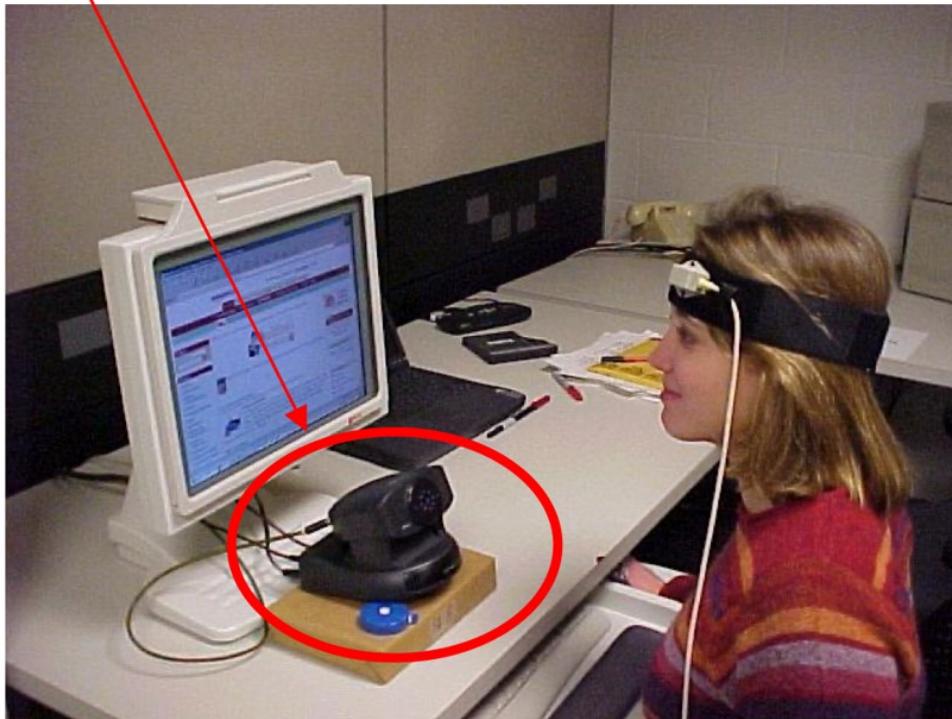
- Device to detect and record where and what people look at
  - Fixations: ~200-300ms, information is acquired
  - Saccades: extremely rapid movements between fixations

Viewing vs. Clicking
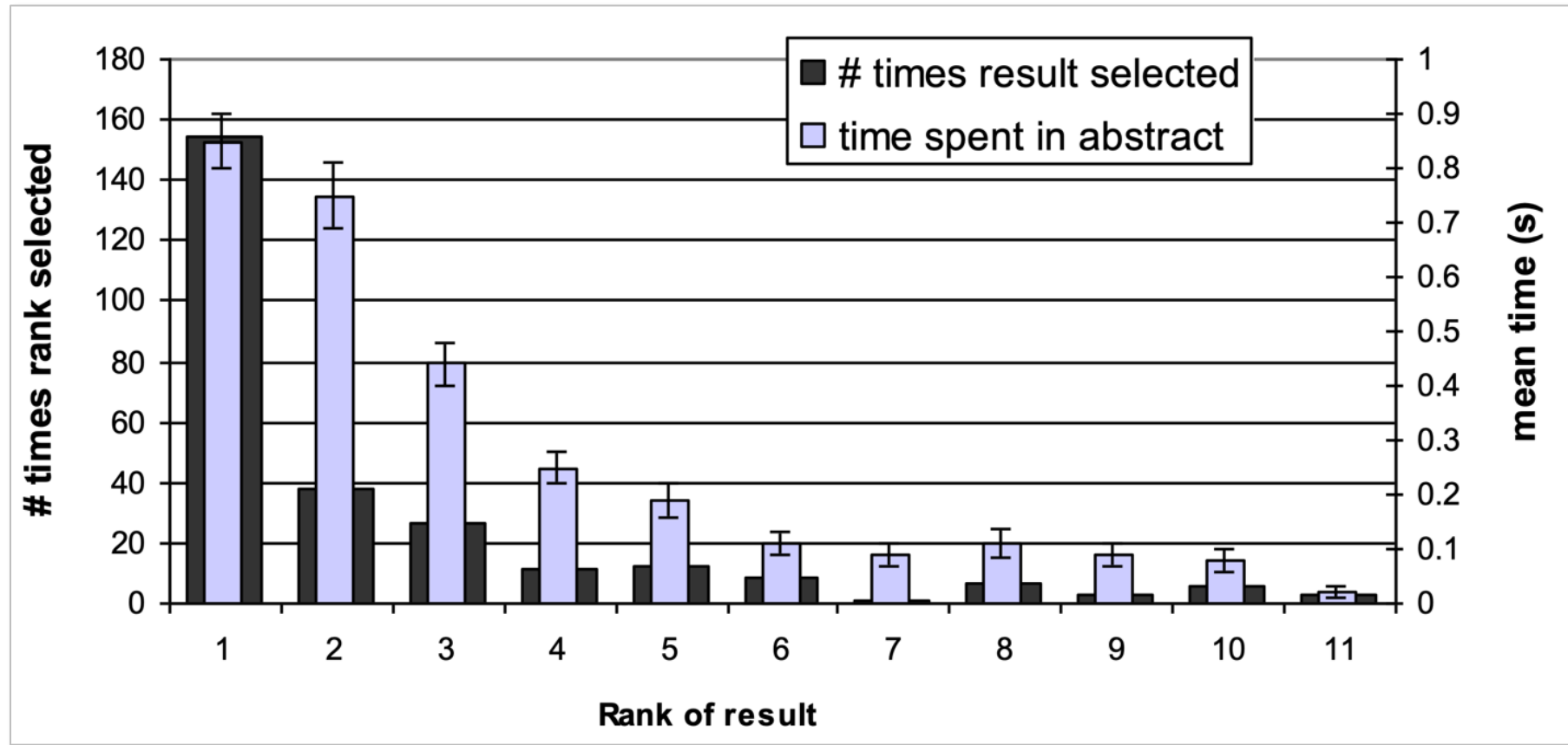
- Users view Documents 1 and 2 more thoroughly / often.
- Users click most frequently on Document 1.

In which order are the results viewed?
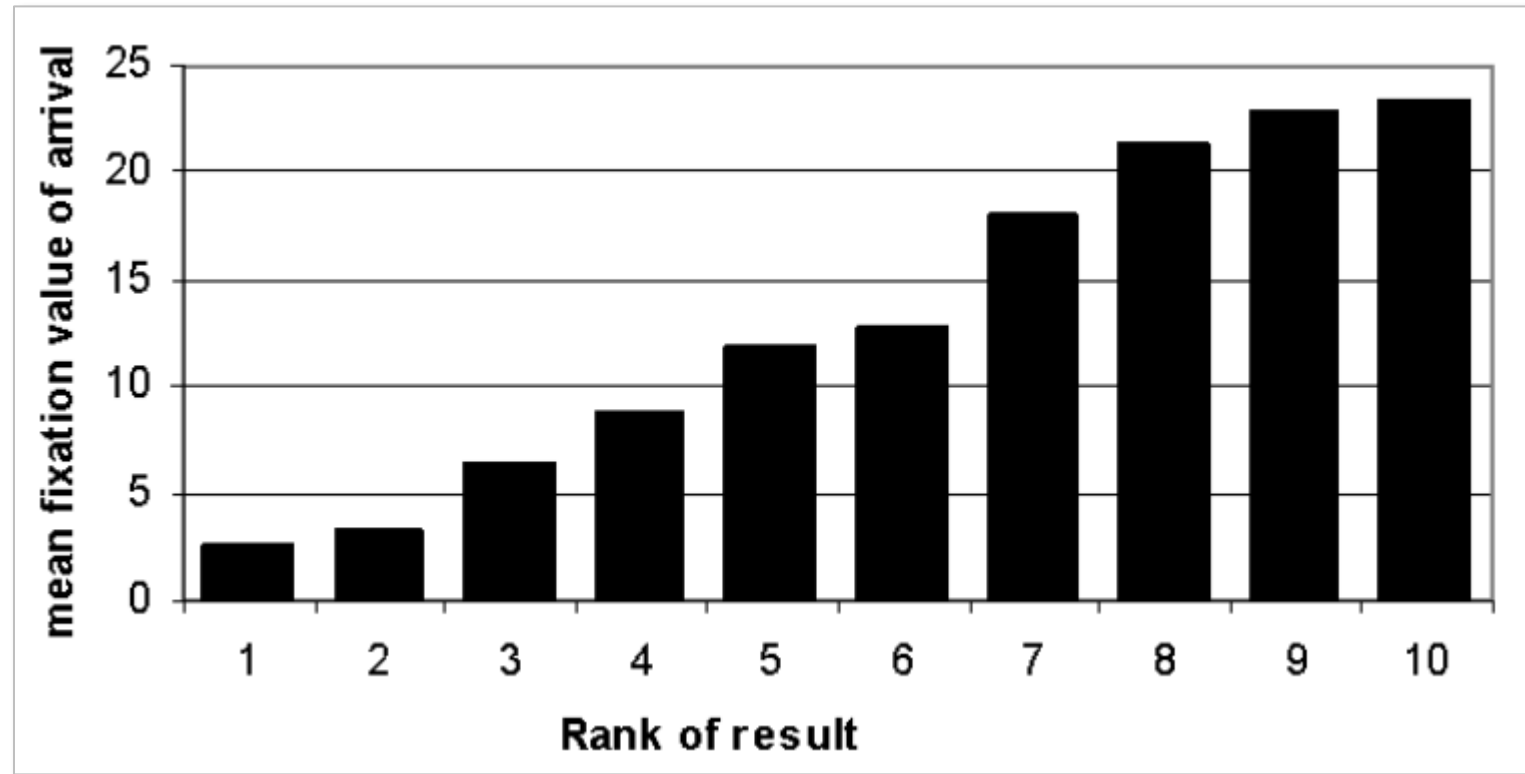
- Users tend to read the results in order.

Presentation Bias

- The top two results returned by Google were switched in order and then presented to the participants.

- Order of presentation influences where users look AND where they click.
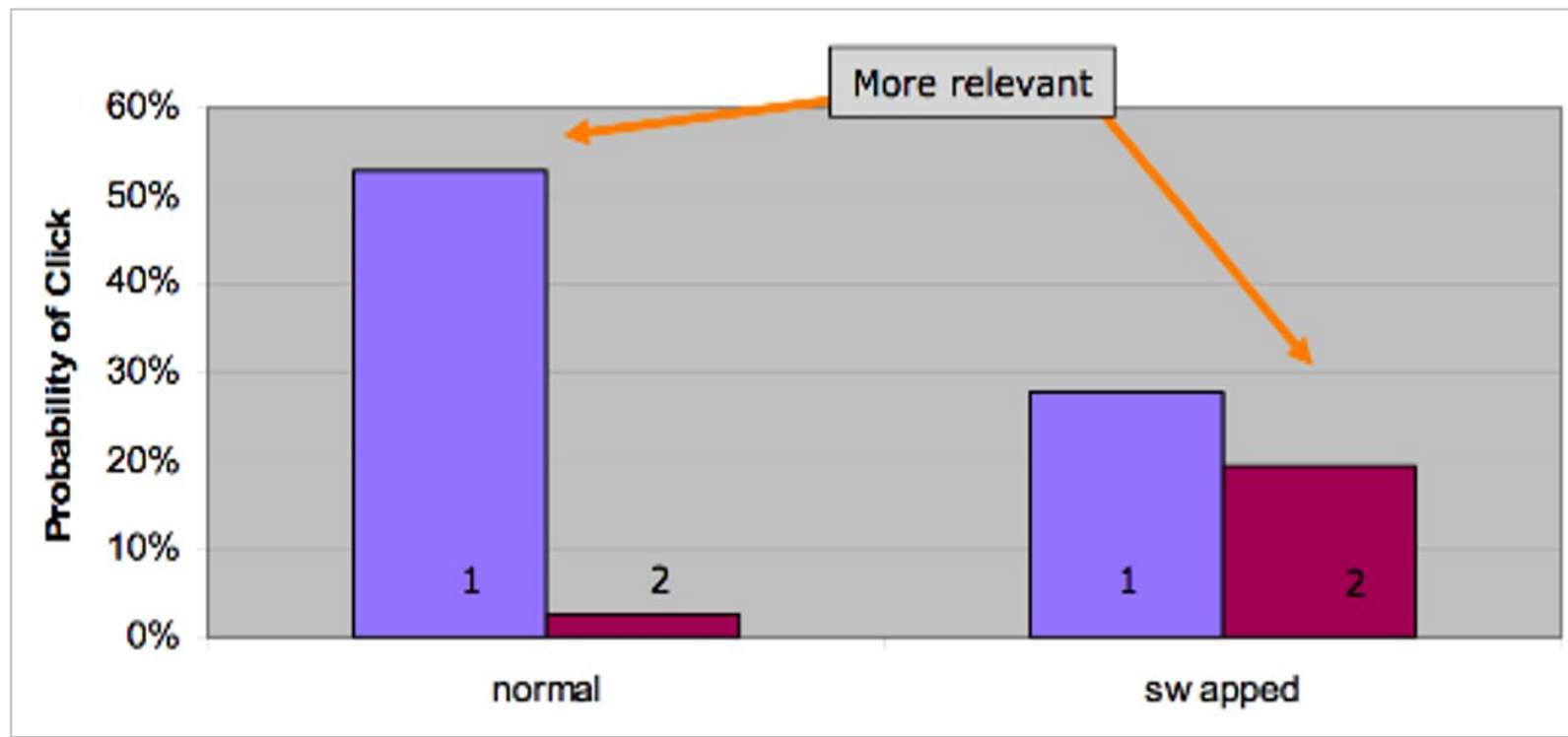
Click-Through Rate (CTR) Per Ranking

Google CTR per ranking

Ranking	CTR
1	28.5%
2	15.7%
3	11.0%
4	8.0%
5	7.2%
6	5.1%
7	4.0%
8	3.2%
9	2.8%
10	2.5%

SISTRIX

https://www.sistrix.com/blog/why-almost-everything-you-knew-about-google-ctr-is-no-longer-valid/
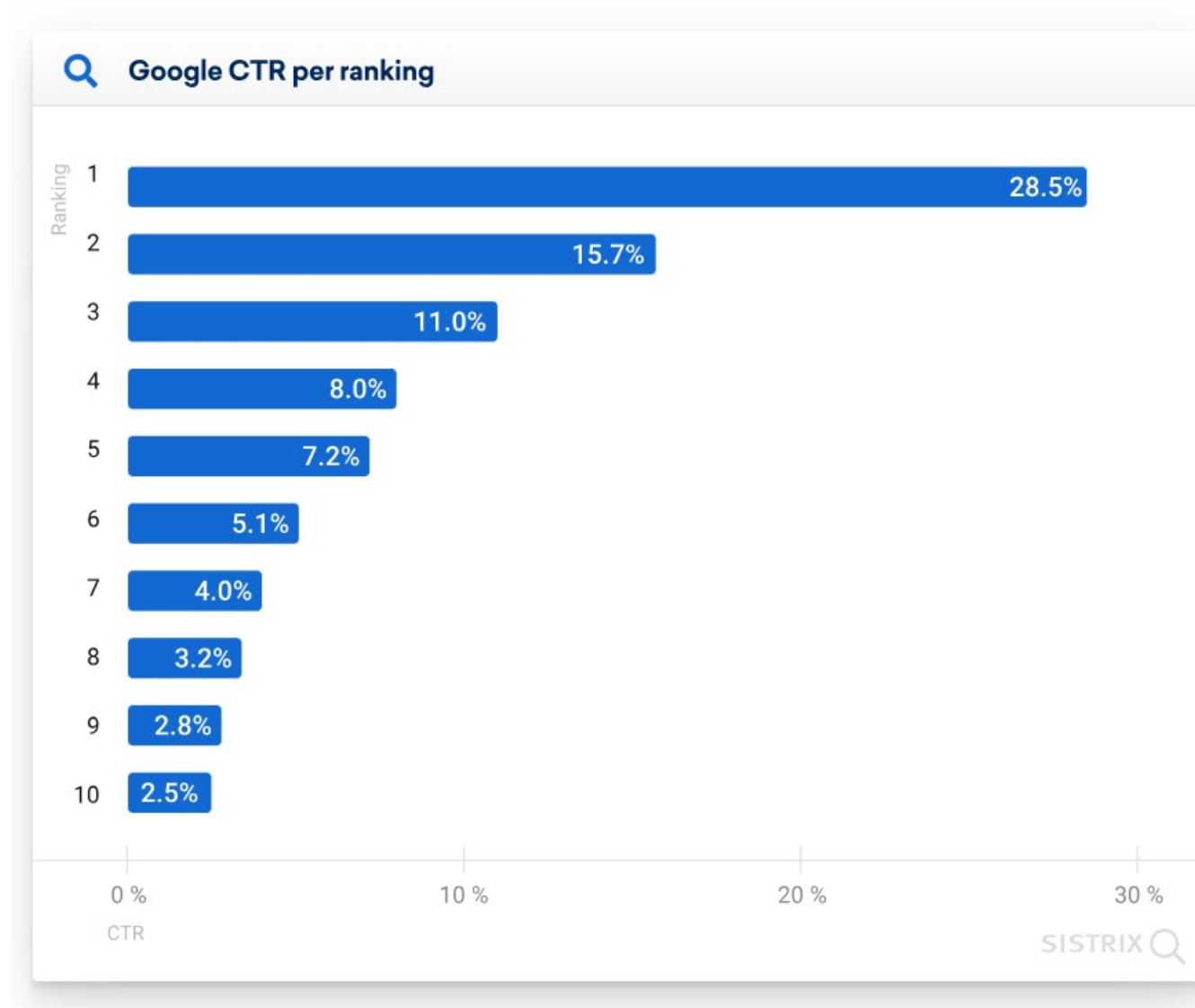
- Accurate ranking increases the likelihood that the documents users click on are actually relevant.

Scoring as the Basis of Ranked Retrieval

- Let's try to build a scoring function:

$$\text{Score}(q, d) \in \mathbb{R}$$

  to score every document d for a particular query q.

- Our hope is to design a scoring function so that the "best" documents (the most relevant, the best at satisfying the user) are scored highest.

TF-IDF

What factors impact scoring?

- Query: "*information retrieval*"

- Document 1: "*Information retrieval is a core area of computer science. Modern information retrieval systems use ranking algorithms to improve search results. Deep learning has also been applied to information retrieval tasks. Classic information retrieval models include TF-IDF and BM25. Evaluation in information retrieval typically involves metrics like MAP and NDCG.*"

- Document 2: "*Natural language processing (NLP) has many applications, such as text classification, machine translation, and information retrieval. These tasks often require a deep understanding of both syntax and semantics. Recent advancements in large language models have significantly improved the performance of NLP systems across many benchmarks.*"

- Which document should be ranked higher? Why?

Term Frequency (TF)

- Score each term t in a document d by the number of times it occurs in the document, denoted as $\text{tf}_{t,d}$

    - Intuition: The more frequently a term appears in a document, the more important it is considered within that document.

- Example

    - Document d: "*zebra any love any zebra*"
    - $\text{tf}_{\text{love},d} = 1$
    - $\text{tf}_{\text{zebra},d} = 2$
    - $\text{tf}_{\text{dream},d} = 0$

Variants of TF

Term frequency	
n (natural)	$\text{tf}_{t,d}$
l (logarithm)	$1 + \log(\text{tf}_{t,d})$
a (augmented)	$0.5 + \dfrac{0.5 \times \text{tf}_{t,d}}{\max_t(\text{tf}_{t,d})}$
b (boolean)	$\begin{cases} 1 & \text{if } \text{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$
L (log ave)	$\dfrac{1 + \log(\text{tf}_{t,d})}{1 + \log(\text{ave}_{t \in d}(\text{tf}_{t,d}))}$

- Why do we need these variants?
- If our scoring function is based solely on TF
  - Any monotonic transformation of TF is equivalent to TF in terms of ranking.
- If TF is only one component of our scoring function, we may want to control its "weight" in the function.
  - Do we want the growth of TF's weight to be equal in the following two cases?
    - $\text{tf}_{t,d}$ increases from 1 to 2
    - $\text{tf}_{t,d}$ increases from 100 to 101
- "diminishing marginal gain"

What factors impact scoring?

- Query: "*mitochondria cell*"

- Document 1: "*The cell structure of an organism varies depending on the type of cell. In multicellular organisms, each cell has a specific function. Cell division plays an important role in growth and repair.*"
  - TF("*cell*", Document 1) = 4; TF("*mitochondria*", Document 1) = 0

- Document 2: "*Mitochondria are known as the powerhouse of the cell. They play a critical role in ATP production and cellular respiration. Damage to mitochondria can lead to metabolic disorders.*"
  - TF("*cell*", Document 2) = 1; TF("*mitochondria*", Document 2) = 2

- Which document should be ranked higher? Why?

Inverse Document Frequency (IDF)

- Score each term in a document by how rare it is across all documents
  - Intuition: The rarer a term is across a collection, the more valuable it is.
  - "*mitochondria*" is more valuable than "*cell*".
  - If a document matches the term "*mitochondria*" once, it should receive a greater reward than matching the term "*cell*" once.

$$\mathrm{idf}_t = \log \frac{|\mathcal{D}|}{|d \in \mathcal{D} : t \in d|}$$

  - $|\mathcal{D}|$: number of documents in the collection
  - $|d \in \mathcal{D} : t \in d|$: number of documents in the collection that contain t

Inverse Document Frequency (IDF)

$$\text{idf}_t = \log \frac{|\mathcal{D}|}{|d \in \mathcal{D}: t \in d|}$$

- Example (We use 10 as the base of the logarithm)
  - Suppose you want to rank 100,000 documents.
  - "*cell*" appears in 1,000 of them
    - $\text{idf}_{\text{cell}} = \log \frac{100000}{1000} = \log 100 = 2$
  - "*mitochondria*" appears in 10 of them
    - $\text{idf}_{\text{mitochondria}} = \log \frac{100000}{10} = \log 10000 = 4$

  - Can you roughly estimate the IDF value of "*a*"?

What factors impact scoring?

- Query: "*TAMU 2026 Spring Break*"

- Document 1: https://registrar.tamu.edu/academic-calendar/spring-2026
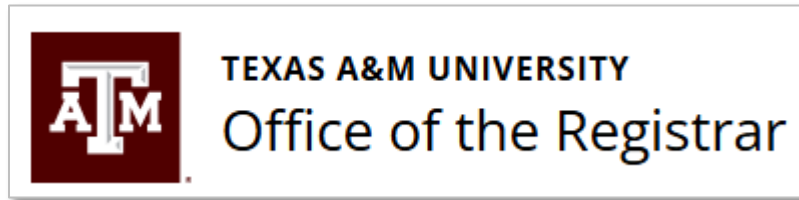
- Document 2: A social media post written by an account with 10 followers mentioning the time of TAMU 2026 Spring Break

- Which document should be ranked higher? Why?

- How can we know the "reputation" of a website?
  - Next week!

Let's first consider the textual factors (TF and IDF) together.

- Given a query q and a document d, we want to calculate $\text{Score}(q, d)$.
- The query q may have one or more terms. For each term t,

$$\text{tfidf}_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

- The overall score is the sum of the TF-IDF scores of all terms.

$$\text{Score}(q, d) = \sum_{t \in q} \text{tfidf}_{t,d}$$

- How should we interpret this formula?
  - A weighted sum of TF
  - The weight is IDF

Example

- Suppose you want to rank 100,000 documents. "*cell*" appears in 1,000 of them; "*mitochondria*" appears in 10 of them. (We use 10 as the base of the logarithm in IDF.)

- Query: "*mitochondria cell*"

- Document 1: "*The cell structure of an organism varies depending on the type of cell. In multicellular organisms, each cell has a specific function. Cell division plays an important role in growth and repair.*"
  - TF("*cell*", Document 1) = 4; IDF("*cell*") = 2
  - TF-IDF("*cell*", Document 1) = 4×2 = 8
  - TF("*mitochondria*", Document 1) = 0; IDF("*mitochondria*") = 4
  - TF-IDF("*mitochondria*", Document 1) = 0×4 = 0
  - TF-IDF(Query, Document 1) = 8+0 = 8

Example

- Suppose you want to rank 100,000 documents. "*cell*" appears in 1,000 of them; "*mitochondria*" appears in 10 of them. (We use 10 as the base of the logarithm in IDF.)

- Query: "*mitochondria cell*"

- Document 2: "*Mitochondria are known as the powerhouse of the cell. They play a critical role in ATP production and cellular respiration. Damage to mitochondria can lead to metabolic disorders.*"

  - TF("*cell*", Document 2) = 1; IDF("*cell*") = 2
  - TF-IDF("*cell*", Document 2) = 1×2 = 2
  - TF("*mitochondria*", Document 2) = 2; IDF("*mitochondria*") = 4
  - TF-IDF("*mitochondria*", Document 1) = 2×4 = 8
  - TF-IDF(Query, Document 1) = 2+8 = 10

Vector Space Model (VSM)

Early History of IR

TF (1957;
by Hans Peter Luhn)

A Statistical Approach to Mechanized Encoding and Searching of Literary Information*

Abstract: Written communication of ideas is carried out on the basis of statistical probability in that a writer chooses that level of subject specificity and that combination of words which he feels will convey the most meaning. Since this process varies among individuals and since similar ideas are therefore relayed at different levels of specificity and by means of different words, the problem of literature searching by machines still presents major difficulties. A statistical approach to this problem will be outlined and the various steps of a system based on this approach will be described. Steps include the statistical analysis of a collection of documents in a field of interest, the establishment of a set of "notions" and the vocabulary by which they are expressed, the compilation of a thesaurus-type dictionary and index, the automatic encoding of documents by machine with the aid of such a dictionary, the encoding of topological notations (such as branched structures), the recording of the coded information, the establishment of a searching pattern for finding pertinent information, and the programming of appropriate machines to carry out a search.

1. Introduction

The essential purpose of literature searching is to find those documents within a collection which have a bearing on a given topic. Many of the systems and devices, such as classifications and subject-heading lists, that have been developed in the past to solve the problems encountered in this searching process are proving inadequate. The need for new solutions is at present being intensified by the rapid growth of literature and the demand for higher levels of searching efficiency.

Specialists in the literature searching field are optimistic about the future application of machine electronic devices in obtaining more satisfactory results. A successful mechanical solution is unlikely, however, if such modern devices are to be viewed merely as agents for *accelerating* systems heretofore fitted to human capabilities. The ultimate benefits of mechanization will be realized only if the characteristics of machines are better understood and are developed which exploit these characteristics to the fullest. Rather than subtilize the artful classificatory schemes now in use, new systems would replace them in large part by mechanical routines based on rather elementary reasoning.

The major technical effort involved in substituting mechanical for intellectual means must, of course, be justified by the improved results obtained. However, if partial mechanical substitution for human effort cannot

be found in automation, there is a real danger that the demand for professional talent will become too great to fill. In view of the foreseeable strain, the most efficient use of talent will have to be made even by automatic systems. The operating requirements of these systems will, above all, have to be well adapted to the degree of education and experience of generally available personnel.

Language difficulties, too, will have to be met. The problems stemming from the mere volumes of literature to be searched are being continually aggravated by the increasing accession of foreign-language documents which rate consideration on an equal level with domestic material. To be of real value, future automatic systems will have to provide a workable means of overcoming the language barrier.

● *Complexity levels of information systems*

The general terms in which the problem of literature searching has been treated might indicate the possibility of a general, or universal, solution. It would be unrealistic to assume that such is practical or desirable. It is quite important to establish some differentiating criteria by which information reference arrays may be distinguished and graded as to their make-up, objectives, and uses. It will then be possible to recognize better the existence of different levels and the necessity of applying appropriately different techniques to their mechanization. The following list of six information systems in order of

IDF (1972;
by Karen Spärck Jones)

A statistical interpretation of term specificity and its application in retrieval

Karen Spärck Jones

Computer Laboratory, University of Cambridge, Cambridge, UK

Abstract: The exhaustivity of document descriptions and the specificity of index terms are usually regarded as independent. It is suggested that specificity should be interpreted statistically, as a function of term use rather than of term meaning. The effects on retrieval of variations in term specificity are examined, experiments with three test collections showing, in particular, that frequently-occurring terms are required for good overall performance. It is argued that terms should be weighted according to collection frequency, so that matches on less frequent, more specific, terms are of greater value than matches on frequent terms. Results for the test collections show that considerable improvements in performance are obtained with this very simple procedure.

Exhaustivity and specificity

We are familiar with the notions of exhaustivity and specificity: exhaustivity is a property of index descriptions, and specificity one of index terms. They are most clearly illustrated by a simple keyword or descriptor system. In this case the exhaustivity of a document description is the coverage of its various topics given by the terms assigned to it; and the specificity of an individual term is the level of detail at which a given concept is represented.

These features of a document retrieval system have been discussed by Cleverdon et al. (1966) and Lancaster (1968), for example, and the effects of variation in either have been noted. For instance, if the exhaustivity of a document description is increased by the assignment of more terms, when the number of terms in the indexing vocabulary is constant, the chance of the document matching a request is increased. The idea of an optimum level of indexing exhaustivity for a given document collection then follows: the average number of descriptors per document should be adjusted so that, hopefully, the chances of requests matching relevant documents are maximized, while too many false drops are avoided. Exhaustivity obviously applies to requests too, and one function of a search strategy is to vary request exhaustivity. I will be mainly concerned here, however, with document descriptions.

Specificity as characterized above is a semantic property of index terms: a term is more or less specific as its meaning is more or less detailed and precise. This is a natural view for anyone concerned with the construction of an entire indexing vocabulary. Some decision has to be made about the discriminating power of individual terms in addition to their descriptive propriety. For example, the index term "beverage" may be as properly used for documents about tea, coffee, and cocoa as the terms "tea", "coffee", and "cocoa". Whether the more

Vector Space Model (1975;
by Gerard Salton)

A Vector Space Model for Automatic Indexing

G. Salton, A. Wong and C. S. Yang
Cornell University

In a document retrieval, or other pattern matching environment where stored entities (documents) are compared with each other or with incoming patterns (search requests), it appears that the best indexing (property) space is one where each entity lies as far away from the others as possible; in these circumstances the value of an indexing system may be expressible as a function of the density of the object space; in particular, retrieval performance may correlate inversely with space density. An approach based on space density computations is used to choose an optimum indexing vocabulary for a collection of documents. Typical evaluation results are shown, demonstrating the usefulness of the model.

General Idea of VSM

- Represent documents and queries as vectors in a high-dimensional space

- A document is more relevant to a query if they are more "similar" in the vector space.
  - We will define "similar" later.

- What are the axes?

- How about each dimension representing a word in the vocabulary?

Fig. 1. Vector representation of document space.
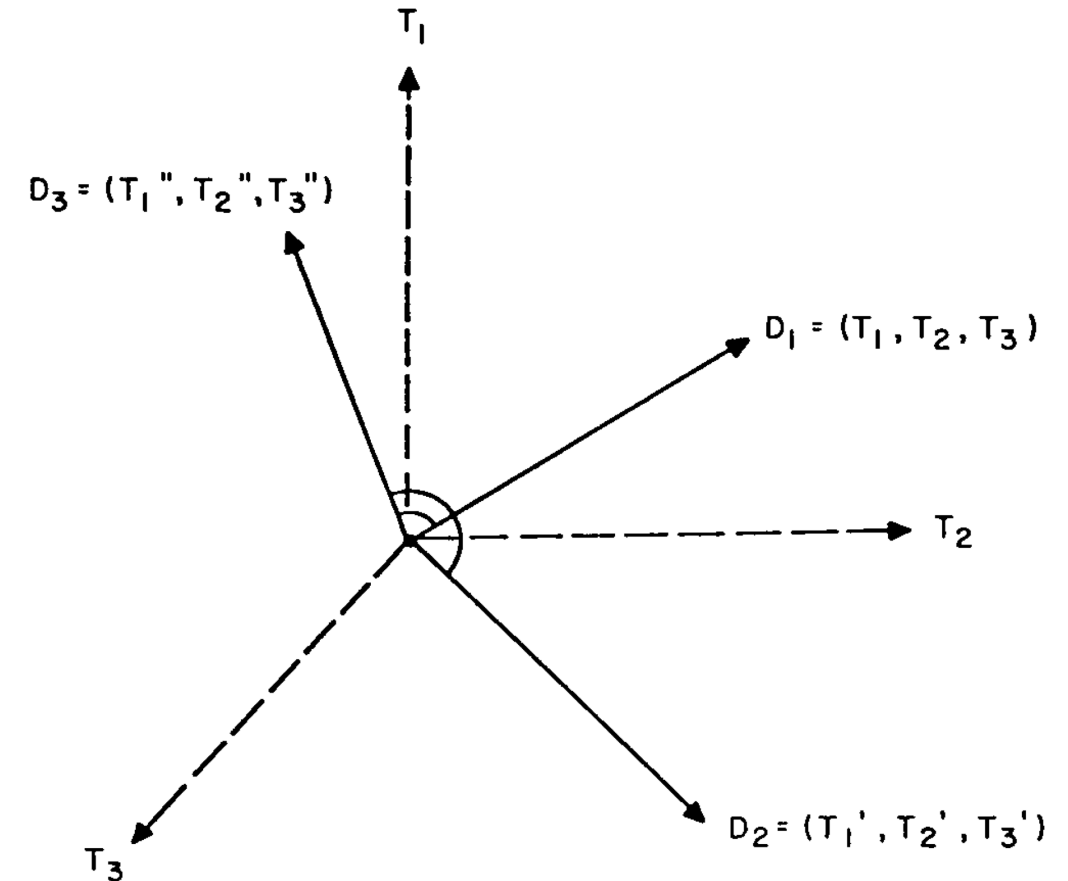
VSM: Example 1

- If the word appears in a query/document, the corresponding entry is 1.
- Otherwise, the corresponding entry is 0.

- Query q: "*any AND zebra*"
- Document d: "*zebra any love any zebra*"

Vocabulary	Vector(q)	Vector(d)
any	1	1
believe	0	0
choose	0	0
…	…	…
love	0	1
starring	0	0
zebra	1	1

- What is the inner product of Vector(q) and Vector(d)?
  - Vector(q) · Vector(d) = 2

- In the setting of Boolean retrieval, d is relevant to this q if and only if Vector(q) · Vector(d) = 2.

VSM: Example 1

- If the word appears in a query/document, the corresponding entry is 1.

- Otherwise, the corresponding entry is 0.

- Query q: "t_1 AND t_2 AND … AND t_N" (There are N words that must appear in d.)

- In the setting of Boolean retrieval, d is relevant to this q if and only if

$$\text{Vector}(q) \cdot \text{Vector}(d) = N$$

VSM: Example 2

- Given a query q, the corresponding entry of word t is $\text{tf}_{t,q}$.

- Given a document d, the corresponding entry of word t is $\text{tf}_{t,d} \times \text{idf}_t$.

- Query q: "*any any zebra*"

- Document d: "*zebra any love any zebra*"

Vocabulary	Vector(q)	Vector(d)
any	2	$\text{tf}_{\text{any},d} \times \text{idf}_{\text{any}}$
believe	0	0
choose	0	0
…	…	…
love	0	$\text{tf}_{\text{love},d} \times \text{idf}_{\text{love}}$
starring	0	0
zebra	1	$\text{tf}_{\text{zebra},d} \times \text{idf}_{\text{zebra}}$

- What is the inner product of Vector(q) and Vector(d)?

VSM: Example 2

- Query q: "*any any zebra*"
- Document d: "*zebra any love any zebra*"
- What is the inner product of Vector(q) and Vector(d)?

Vocabulary	Vector(q)	Vector(d)
any	2	$\text{tf}_{any,d} \times \text{idf}_{any}$
believe	0	0
choose	0	0
…	…	…
love	0	$\text{tf}_{love,d} \times \text{idf}_{love}$
starring	0	0
zebra	1	$\text{tf}_{zebra,d} \times \text{idf}_{zebra}$

$$\text{Vector}(q) \cdot \text{Vector}(d) = 2 \times \text{tf}_{any,d} \times \text{idf}_{any} + 1 \times \text{tf}_{zebra,d} \times \text{idf}_{zebra}$$

$$= \text{tf}_{any,d} \times \text{idf}_{any} + \text{tf}_{any,d} \times \text{idf}_{any} + \text{tf}_{zebra,d} \times \text{idf}_{zebra}$$

$$= \sum_{t \in q}(\text{tf}_{t,d} \times \text{idf}_t)$$

$$= \text{TF-IDF}(q, d)$$

- TF-IDF is a special case of VSM.

Problems with Inner Product?

- Query q:"*any any zebra*"
- Document d_1:"*zebra any love any zebra*"

- Document d_2:"*zebra any love any zebra zebra any love any zebra*"

Vocabulary	Vector(q)	Vector(d_1)	Vector(d_2)
any	2	$\text{tf}_{\text{any},d} \times \text{idf}_{\text{any}}$	$2 \times \text{tf}_{\text{any},d} \times \text{idf}_{\text{any}}$
believe	0	0	0
choose	0	0	0
…	…	…	…
love	0	$\text{tf}_{\text{love},d} \times \text{idf}_{\text{love}}$	$2 \times \text{tf}_{\text{love},d} \times \text{idf}_{\text{love}}$
starring	0	0	0
zebra	1	$\text{tf}_{\text{zebra},d} \times \text{idf}_{\text{zebra}}$	$2 \times \text{tf}_{\text{zebra},d} \times \text{idf}_{\text{zebra}}$

Problems with Inner Product?

- Query q:"*any any zebra*"

- Document d_1:"*zebra any love any zebra*"

- Document d_{100}:"*zebra any love any zebra zebra any love any zebra … (repeat 100 times)*"

Vocabulary	Vector(q)	Vector(d_1)	Vector(d_{100})
any	2	$\text{tf}_{\text{any},d} \times \text{idf}_{\text{any}}$	$100 \times \text{tf}_{\text{any},d} \times \text{idf}_{\text{any}}$
believe	0	0	0
choose	0	0	0
…	…	…	…
love	0	$\text{tf}_{\text{love},d} \times \text{idf}_{\text{love}}$	$100 \times \text{tf}_{\text{love},d} \times \text{idf}_{\text{love}}$
starring	0	0	0
zebra	1	$\text{tf}_{\text{zebra},d} \times \text{idf}_{\text{zebra}}$	$100 \times \text{tf}_{\text{zebra},d} \times \text{idf}_{\text{zebra}}$

Problems with Inner Product?

- $\text{Vector}(d_2) = 2\,\text{Vector}(d_1)$

- $\text{Vector}(q) \cdot \text{Vector}(d_2) = 2\,\text{Vector}(q) \cdot \text{Vector}(d_1)$

- $\text{Vector}(d_{100}) = 100\,\text{Vector}(d_1)$

- $\text{Vector}(q) \cdot \text{Vector}(d_{100}) = 100\,\text{Vector}(q) \cdot \text{Vector}(d_1)$

- …

- We can make the inner product of q and d in the vector space as large as possible by just making d longer!

- How to take the document length factor into account?

Cosine Similarity

- $\mathbf{x} = [x_1, x_2, \ldots, x_N]$
- $\mathbf{y} = [y_1, y_2, \ldots, y_N]$
- $\cos(\mathbf{x}, \mathbf{y}) = \dfrac{x_1 y_1 + x_2 y_2 + \cdots + x_N y_N}{\sqrt{x_1{}^2 + x_2{}^2 + \cdots + x_N{}^2} \times \sqrt{y_1{}^2 + y_2{}^2 + \cdots + y_N{}^2}} = \dfrac{\mathbf{x} \cdot \mathbf{y}}{||\mathbf{x}|| \cdot ||\mathbf{y}||} = \left(\dfrac{\mathbf{x}}{||\mathbf{x}||}\right) \cdot \left(\dfrac{\mathbf{y}}{||\mathbf{y}||}\right)$

- Equivalent to first normalizing the vectors to unit length, and then computing the dot product
  - $\mathbf{x}$, $2\mathbf{x}$, and $100\mathbf{x}$ will be the same vector after length normalization.
- The larger the cosine similarity, the smaller the angle between the two unit vectors (i.e., the more "similar" they are).
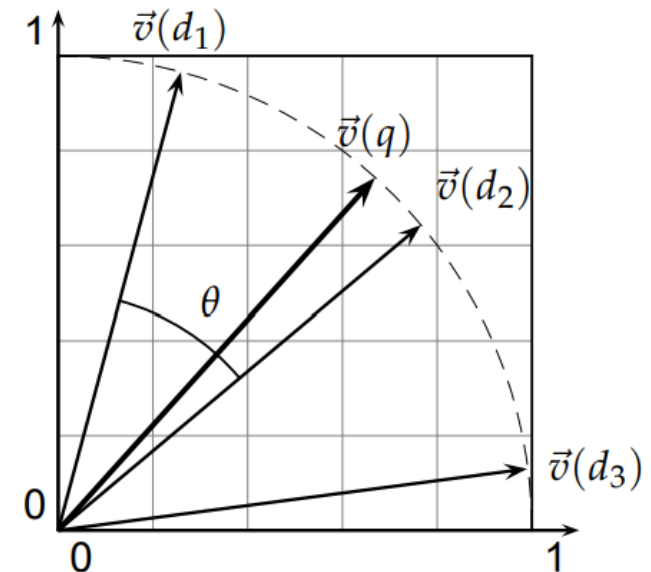
Example

- Query q:"*any any zebra*"
- Document d_1:"*zebra any love any zebra*"

- Let's assume $\text{idf}_{any} = \text{idf}_{love} = 2$ and $\text{idf}_{zebra} = 4$.

Vocabulary	Vector(q)	Vector(d_1)	Vector(d_1)
any	2	$\text{tf}_{any,d} \times \text{idf}_{any}$	$2 \times 2 = 4$
believe	0	0	0
choose	0	0	0
…	…	…	…
love	0	$\text{tf}_{love,d} \times \text{idf}_{love}$	$1 \times 2 = 2$
starring	0	0	0
zebra	1	$\text{tf}_{zebra,d} \times \text{idf}_{zebra}$	$2 \times 4 = 8$

Example

- $\|\text{Vector}(q)\| = \sqrt{2^2 + 1^2} = \sqrt{5}$

- $\|\text{Vector}(d_1)\| = \sqrt{4^2 + 2^2 + 8^2} = \sqrt{84}$

- $\cos(\text{Vector}(q), \text{Vector}(d_1)) = \frac{2\times4+0\times2+1\times8}{\sqrt{5}\times\sqrt{84}} \approx 0.781$

Vocabulary	Vector(q)	Vector(d_1)
any	2	4
believe	0	0
choose	0	0
…	…	…
love	0	2
starring	0	0
zebra	1	8

What's the range of the cosine similarity in VSM?

Our Plan: Ranking

- ☑ Why is ranking important?

- ☑ What factors impact ranking?

- Two foundational text-based approaches
  - ☑ TF-IDF
  - BM25

- Two foundational link-based approaches
  - PageRank
  - HITS

- Machine-learned ranking ("learning to rank")

BM25 (or Okapi BM25)

- **BM** = Best Match

- **25** = the 25th version of the scoring function

- Over time, BM25 has become a default scoring function used broadly across many real-world systems.

- Goal: be sensitive to term frequency and document length while not adding too many parameters

Overview of the Third Text REtrieval Conference (TREC-3)

Donna Harman

National Institute of Standards and Technology
Gaithersburg, MD. 20899

1. Introduction

In November of 1992 the first Text REtrieval Conference (TREC-1) was held at NIST [Harman 1993]. The conference, co-sponsored by ARPA and NIST, brought together information retrieval researchers to discuss their system results on a new large test collection (the TIPSTER collection). This conference became the first in a series of ongoing conferences dedicated to encouraging research in retrieval from large-scale test collections, and to encouraging increased interaction among research groups in industry and academia. From the beginning there has been an almost equal number of universities and companies

description of the test collection being used, and an overview of the results. The papers from the individual groups should be referred to for more details on specific system approaches.

2. The Task and the Participants

The three TREC conferences have all centered around two tasks based on traditional information retrieval modes: a "routing" task and an "adhoc" task. In the routing task it is assumed that the same questions are always being asked, but that new data is being searched. This task is similar to that done by news clipping services or by li-

TREC: https://trec.nist.gov/

IR challenges held each year, featuring expert-annotated data (e.g., relevant query-document pairs) from various domains
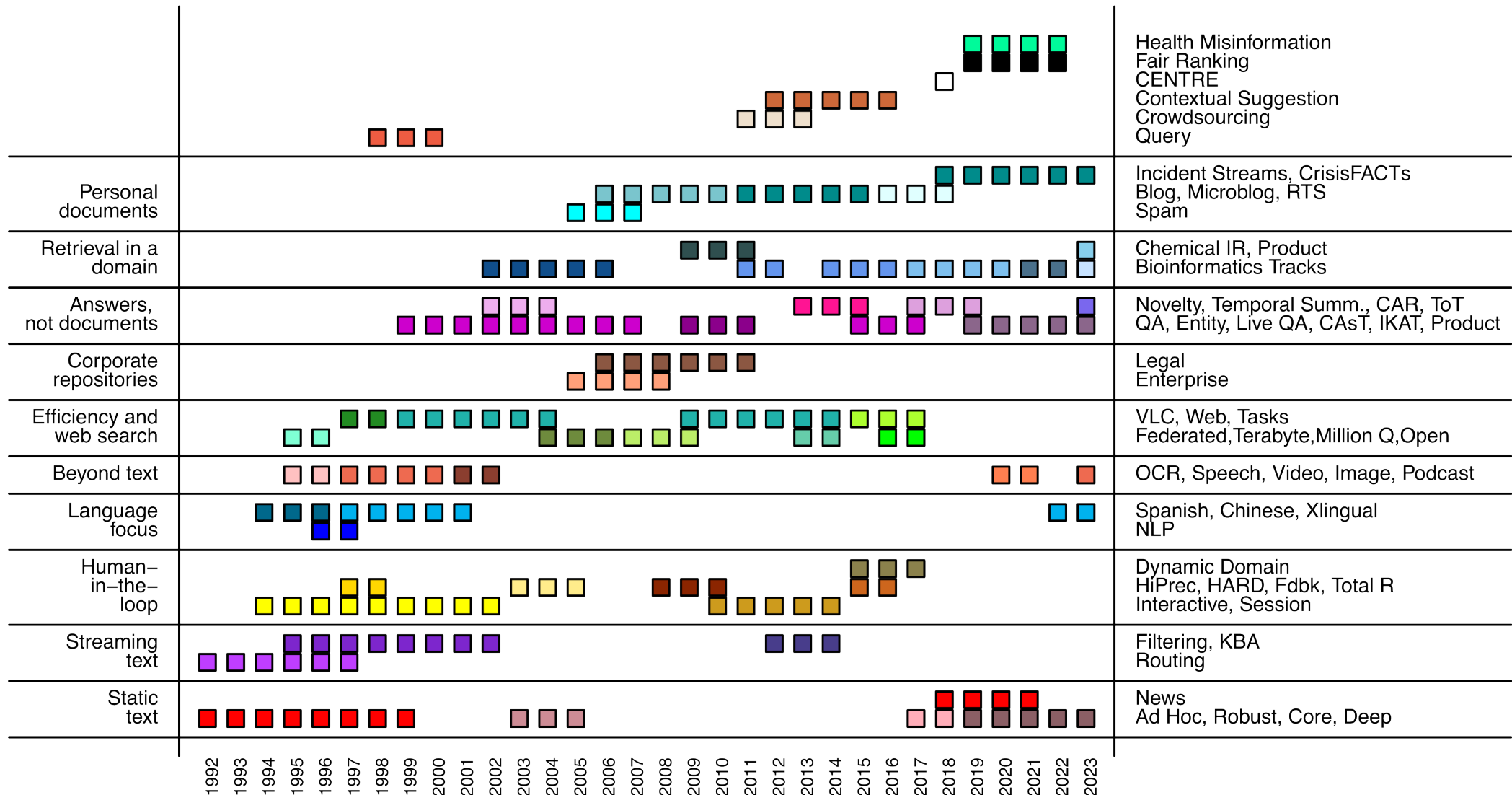
TREC-3

Table 1: TREC-3 Participants (14 companies, 19 universities)

Australian National University	Bellcore
Carnegie Mellon University/CLARITECH	CITRI, Australia
City University, London	Cornell University
Dublin City University	Environment Research Institute of Michigan
Fulcrum	George Mason University
Logicon Operating Systems	Mayo Clinic/Foundation
Mead Data Central	National Security Agency
New York University	NEC Corporation
Queens College	Rutgers University (two groups)
Siemens Corporate Research Inc.	Swiss Federal Institute of Technology (ETH)
TRW/Paracel	Universitaet Dortmund, Germany
University of California - Berkeley	University of Central Florida
University of Massachusetts at Amherst	VPI&SU (Virginia Tech)
University of Minnesota	University of Toronto
Universite de Neuchatel, Switzerland	Verity Inc.
West Publishing Co.	Xerox Palo Alto Research Center

Okapi at TREC–3

S E Robertson   S Walker   S Jones   M M Hancock-Beaulieu

M Gatford

Centre for Interactive Systems Research
Department of Information Science
City University
Northampton Square
London EC1V 0HB
UK

1   Introduction

The sequence of TREC conferences has seen the City University Okapi IR system evolve in several ways. Be-

City at TREC–2

For TREC–2 the simple inverse collection frequency (ICF) term-weighting scheme was elaborated to embody within-document frequency and document length components, as well as within-query frequency, and a large number of weighting functions were investigated. Because of hardware failures few of the runs were ready in time, and City's official results were very poor. How-

What is BM25?
Why are we talking about a method from 1994?

BM25 in (Almost) One Slide

$$\text{BM25}(q, d) = \sum_{t \in q} \text{IDF}(t) \cdot \frac{\text{TF}(t, d) \cdot (k_1 + 1)}{\text{TF}(t, d) + k_1(1 - b + b \cdot \frac{|d|}{\text{avgdl}})}$$

- k_1 controls term frequency scaling
  - $k_1 = 0$: binary model
  - k_1 very large: raw term frequency
- b controls document length normalization
  - $b = 0$: no document length normalization
  - $b = 1$: relative frequency (full document length normalization)
- Typically, k_1 is set between 1.2 and 2; b is set around 0.75

- $|d|$ is the length of d (in words); avgdl = average document length (in words)

The IDF Component in BM25

$$\text{IDF}(t) = \log\left(\frac{N - n(t) + 0.5}{n(t) + 0.5} + 1\right)$$

- $N = |\mathcal{D}|$, number of documents
- $n(t) = |\{d \in \mathcal{D} : t \in d\}|$, number of documents containing the term t

Why are we talking about a method from 1994?

- Even compared to today's large language models, BM25 still delivers strong performance on text retrieval tasks. It is also easy to implement, requires no machine learning training, and does not rely on GPU support.

OPINION

The Neural Hype and Comparisons Against Weak Baselines

Jimmy Lin

David R. Cheriton School of Computer Science, University of Waterloo

Setup

- Two recent IR papers: Paper 1 and Paper 2
  - Several "neural" models proposed/compared in the papers

- "Traditional" (non-neural) ranking models:
  - BM25
  - QL: query likelihood with Dirichlet smoothing
  - RM3 variant of relevance models (pseudo-relevance feedback where top results returned by a QL model are treated as relevant)

- Experiments in Anserini (open-source engine built on Lucene)

Results

Condition	AP	P20
QL	0.2499	0.3556
QL + RM3	0.2865	0.3773
$Neural_1$	0.2815	0.3752
$Neural_2$	0.2801	0.3764
$Neural_3$	0.2856	0.3766
$Neural_3'$	0.2971	0.3948
Anserini: QL	0.2496	0.3543
Anserini: BM25	0.2526	0.3604
Anserini: BM25 + RM3 (independent)	0.2954	0.3885
Anserini: BM25 + RM3 (joint)	0.2973	0.3871

Table 1: Comparison of Anserini results to Paper 1.

Condition	AP	P20
BM25	0.238	0.354
BM25 + Features	0.250	0.367
$Neural_x$	0.258	0.372
$Neural_y$	0.256	0.370
$Neural_x$ + $Neural_y$	0.259	0.373
A + $Neural_y$	0.263	0.380
A + $Neural_y$ + M	0.265	0.380
B + $Neural_y$	0.270	0.383
B + $Neural_y$ + M	0.272	0.386
Anserini: QL	0.2481	0.3517
Anserini: BM25	0.2528	0.3598
Anserini: BM25 + RM3 (independent)	0.2991	0.3901
Anserini: BM25 + RM3 (joint)	0.2956	0.3931

Table 2: Comparison of Anserini results to Paper 2.

How to interpret the BM25 scoring function?
Why is it so complicated?

Generative Model for Documents

- Let's consider a probabilistic interpretation.

- Idea: Words are drawn independently from the vocabulary using a multinomial distribution.

$$p(meet) = 0.002$$
$$p(midnight) = 0.001$$
$$p(the) = 0.015$$
$$\ldots$$
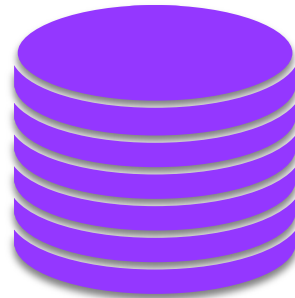
… *meet me at midnight* …

- If we use this model to generate a sentence
  - What is the probability that the first word is "*midnight*"?
  - What is the probability that the second word is "*midnight*", given the first word is "*midnight*"?

Generative Model for Documents

- Given a specific term (e.g., "*midnight*")
  - The distribution of its term frequency (TF) follows a binomial distribution.

p(*midnight*) = 0.001
p(NOT *midnight*) = 0.999

… blah blah blah midnight blah blah blah blah midnight blah blah midnight blah blah …

- Binomial distribution: There is a coin for which the probability of landing heads up in a single toss is 0.001, and tails up is 0.999. What is the probability of getting heads exactly k times in 100 tosses? ($k = 0,1,2, … , 100$)

$$p_k = \binom{100}{k} \times 0.001^k \times 0.999^{100-k}$$

Generative Model for Documents

- When the number of tosses is large (e.g., ≥ 100) and the probability of getting heads is small, a binomial distribution can be approximated by a Poisson distribution.

- Poisson distribution: A call center receives incoming calls at an average rate of λ (i.e., λ calls per hour). The time interval between successive calls is independent of the time of the previous call. Under these conditions, the probability that the call center receives k calls ($k = 0,1,2,\dots$) in one hour is given by:

$$p_k = \frac{\lambda^k e^{-\lambda}}{k!}$$
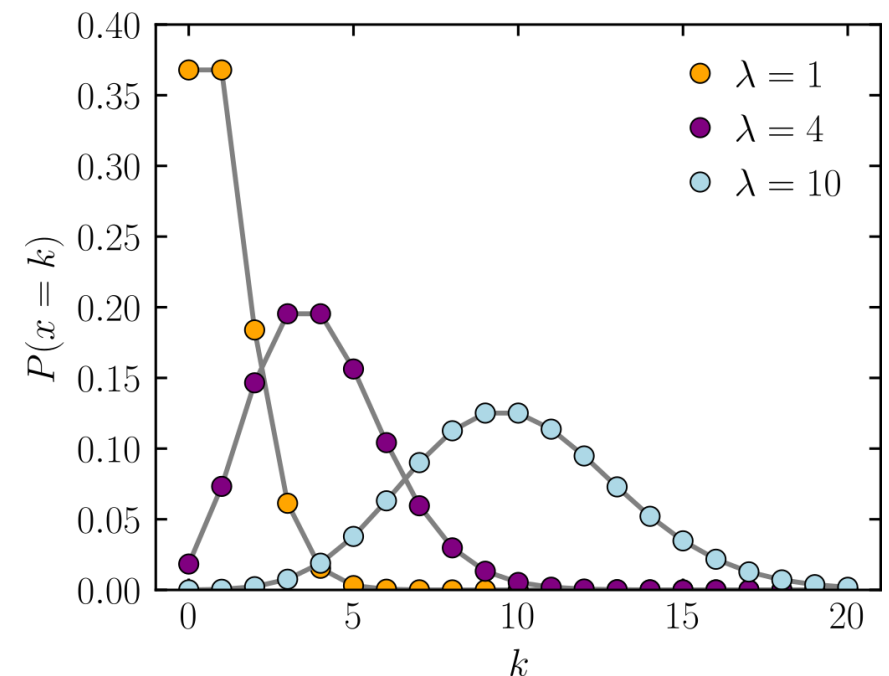
Generative Model for Documents

- Why can we also approximate TF as following a Poisson distribution?

- Assume all documents have the same length (e.g., 100 words). *We will fix this later!*

- Given a word (e.g., "*midnight*")
  - p(*midnight*) = 0.001
  - "*midnight*" should "arrive" at an average rate of $0.001 \times 100 = 0.1$ per document.
  - The number of words between two occurrences of "*midnight*" is independent of the position of the previous "*midnight*".
  - TF = the number of occurrences of "*midnight*" in a document

"One" Poisson Model

- Given 10,000 documents and a word "*information*"
  - For each document, "*information*" has a term frequency.
  - $\text{tf}_1, \text{tf}_2, \ldots, \text{tf}_{10000}$
  - How should these number be distributed?

- Assume "*information*" appears M times in total in these 10,000 documents, then

$$\lambda = \frac{M}{10000}$$

(Definition of λ: the average number of occurrences in a document)

"One" Poisson Model

- $tf_1, tf_2, \ldots, tf_{10000}$ should follow a Poisson distribution with $\lambda = \dfrac{M}{10000}$.
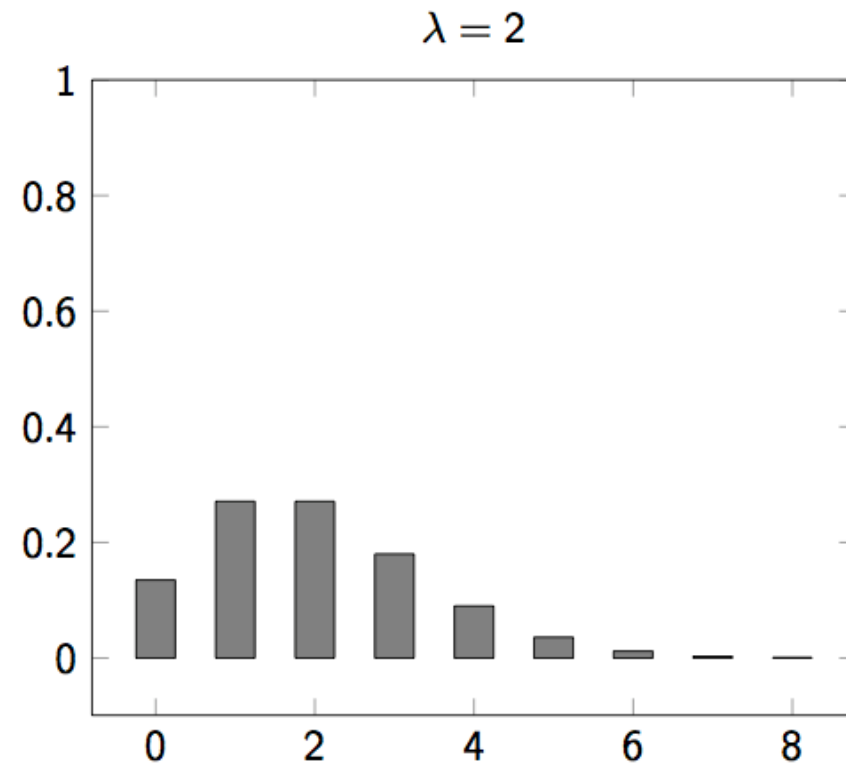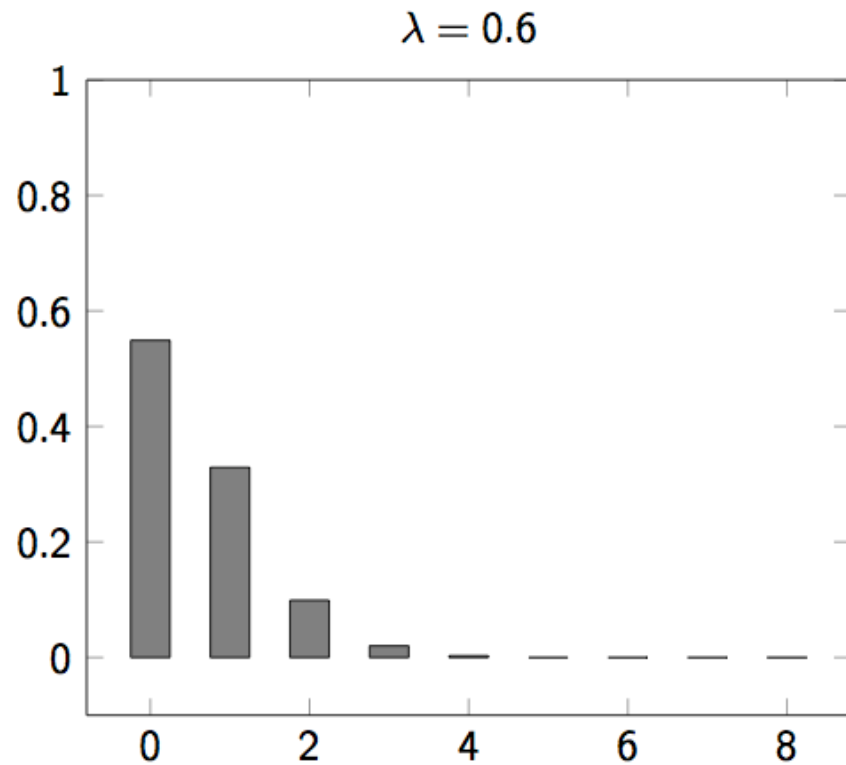
Theory vs. Practice

Table 1. Frequency Distributions for 19 Word Types and Expected Frequencies Assuming a Poisson Distribution with $\lambda = 53/650$

Frequency	Word Type	k	0	1	2	3	4	5	6	7	8	9	10	11	12
51	act		608	35	5	2									
51	actions		617	27	2	0	2	0	2						
54	attitude		610	30	7	2	1								
52	based		600	48	2										
53	body		605	39	4	2									
52	castration		617	22	6	3	1	1							
55	cathexis		619	22	3	2	1	2	0	1					
51	comic		642	3	0	1	0	0	0	0	0	0	1	1	2
53	concerned		601	45	4										
53	conditions		604	39	7										
55	consists		602	41	7										
53	factor		609	32	7	1	1								
52	factors		611	27	11	1									
55	feeling		613	26	7	3	0	0	1						
52	find		602	45	2	1									
54	following		604	39	6	1									
51	force		603	43	4										
51	forces		609	33	6	2									
52	forgetting		629	11	3	2	2	1	1	0	0	0	1		
53	expected, assuming Poisson distribution		599	49	2										

55

Limitations of the "One" Poisson Model

- In the table on the previous slide
    - According to the theory, it is almost impossible for a word to appear ≥ 5 times in a document.
    - In practice, a word can appear 10, 11, or 12 times in a document.
    - Why?

- Each word can be a background word or a topic-specific word in a document.
    - "*information*" may be a topic-specific word in information retrieval papers. (*Discussions revolve around the term "information retrieval".*)
    - "*information*" may be a background word in sports news. (*Discussions typically do not revolve around this word; it is only used when generally needed.*)

- "One" Poisson model gives a reasonable fit for background words but a poor fit for topic-specific words.

"Two" Poisson Model

- $\pi \in [0,1]$: how "relevant" a word t is to a document d
  - $\pi = 0$: totally irrelevant $\rightarrow$ t is a background word in d
  - $\pi = 1$: totally relevant $\rightarrow$ t is a topic-specific word in d
  - $\pi \in (0,1)$: somewhere in between

- When t is a background word, the distribution of its TF follows a Poisson distribution:
$$P(\text{tf} = k | \pi = 0) = \frac{\mu^k e^{-\mu}}{k!}$$

- When t is a topic-specific word, the distribution of its TF follows another Poisson distribution:
$$P(\text{tf} = k | \pi = 1) = \frac{\lambda^k e^{-\lambda}}{k!}$$

"Two" Poisson Model

- $\pi \in (0,1)$: somewhere in between

$$P(\text{tf} = k | \pi) = \pi \frac{\lambda^k e^{-\lambda}}{k!} + (1 - \pi) \frac{\mu^k e^{-\mu}}{k!}$$

- Given 10,000 documents and a word "*information*"

  - How can we know the distribution?

  - Hard! Because π is a hidden variable for each document.

  - Unlike observed variables (e.g, TF) that can be directly calculated

"Two" Poisson Model

- Learning latent variable models from large-scale text data used to be a central problem in IR, NLP, and even machine learning in the broader sense.

[PDF] **Probabilistic latent semantic indexing**

[PDF] acm.org

Full View

T Hofmann - Proceedings of the 22nd annual international ACM ..., 1999 - dl.acm.org

... success in different domains including automatic **indexing** (**Latent Semantic Indexing**, LSI)

[1... approach to LSA and factor analysis { called **Probabilistic Latent** Semantic Analysis (PLSA) { ...

☆ Save   99 Cite   Cited by 7913   Related articles   All 29 versions   »

[PDF] **Latent dirichlet allocation**

[PDF] jmlr.org

Discover Full

DM Blei, AY Ng, MI Jordan - Journal of machine Learning research, 2003 - jmlr.org

We describe **latent Dirichlet allocation** (LDA), a generative probabilistic model for collections

of discrete data such as text corpora. LDA is a three-level hierarchical Bayesian model, in ...

☆ Save   99 Cite   Cited by 57586   Related articles   All 92 versions   Web of Science: 23713   »

Let's make this simpler (?)

- Robertson-Spärck-Jones (RSJ) model:

$$\text{score}(q, d) = \sum_{t \in q,\ \text{TF}(t,d)>0} \log \frac{P(\text{tf} = \text{TF}(t, d)|\pi = 1) \times P(\text{tf} = 0)}{P(\text{tf} = \text{TF}(t, d)) \times P(\text{tf} = 0|\pi = 1)}$$

- If we plug the "One" Poisson model into RSJ and add some "okay" assumptions, we get TF-IDF.

- If we plug the "Two" Poisson model into RSJ, the formula will become complicated and ugly.
  - BUT it is monotonically increasing to $\text{IDF}(t)$ when $\text{TF}(t, d) \rightarrow +\infty$!

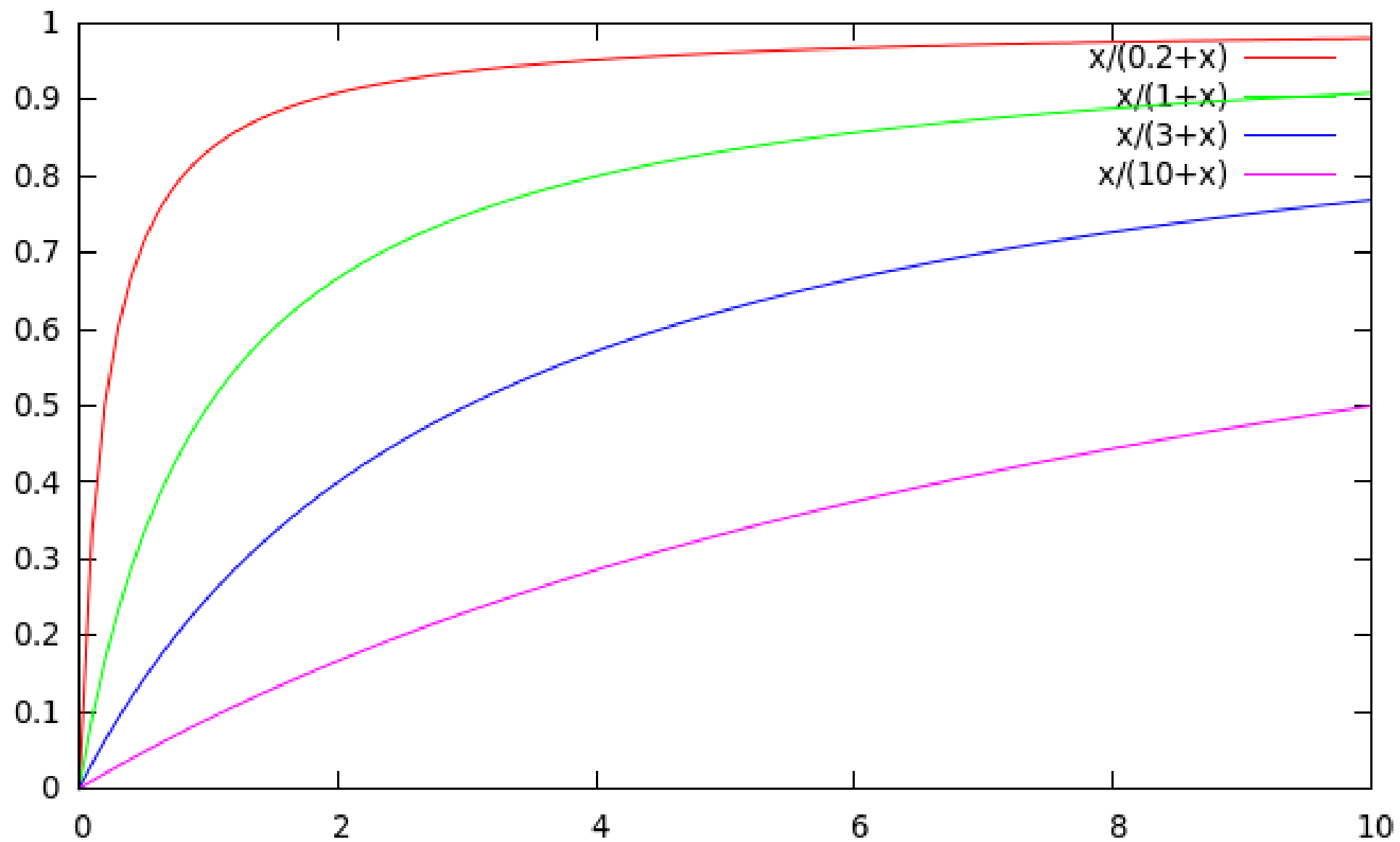# RSJ + "Two" Poisson Model
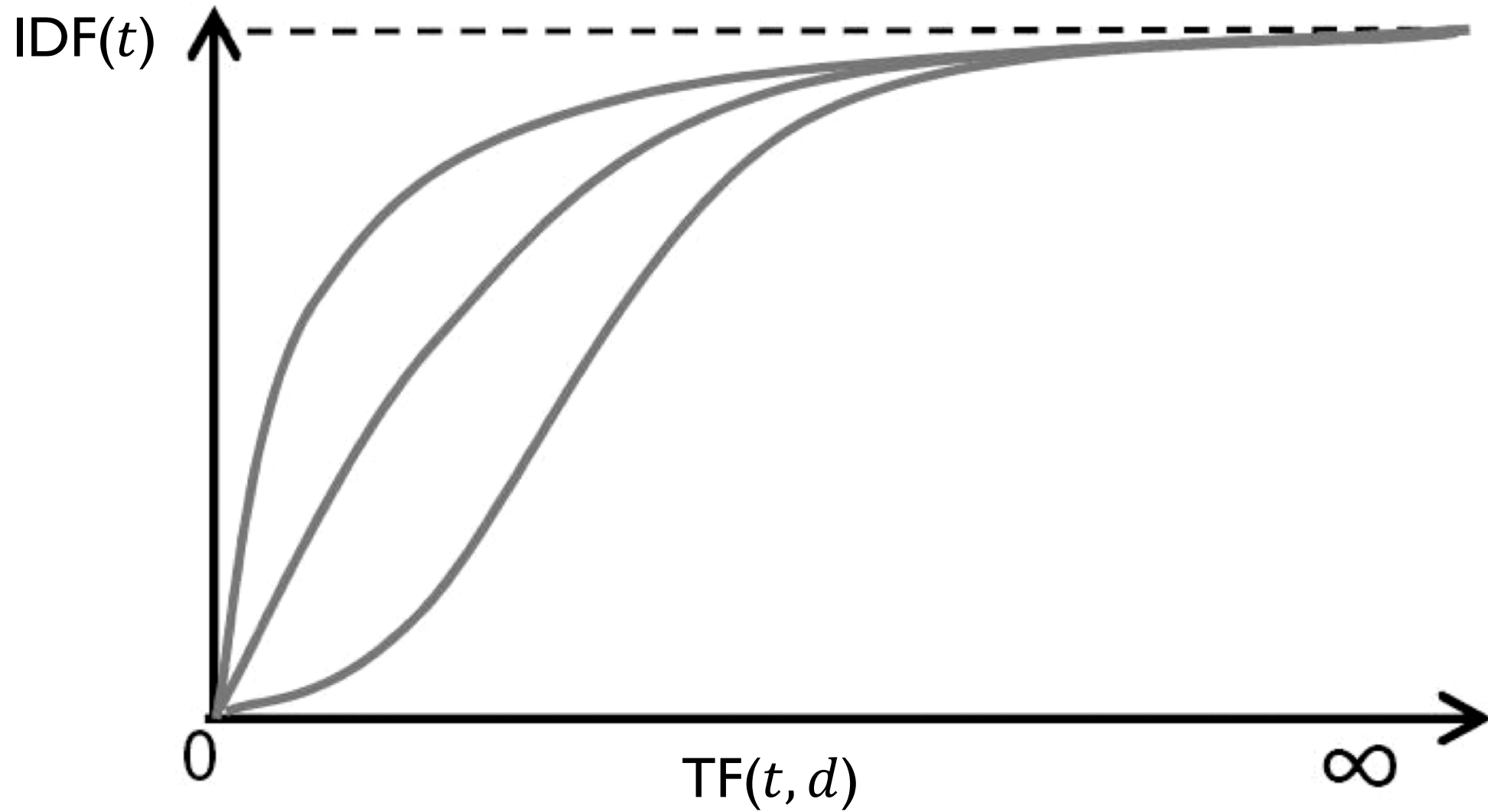
Let's make this simpler!

$$\text{score}(q, d) = g(\text{TF}(t, d)) \times \text{IDF}(t)$$

- g saturates toward a maximum value of 1, which is not true for simple TF-IDF scoring.
  - Think of raw TF or even log-based TF

- Let's approximate g with a simple parametric curve that has the same qualitative properties.

$$\frac{\text{TF}(t, d)}{k_1 + \text{TF}(t, d)}$$

- $k_1 > 0$ is a hyperparameter
- $\frac{\text{TF}(t,d)}{k_1 + \text{TF}(t,d)} = 0$ when $\text{TF}(t, d)$ is 0
- $\frac{\text{TF}(t,d)}{k_1 + \text{TF}(t,d)} = 1$ when $\text{TF}(t, d)$ is ∞

An Early Version of BM25

$$\text{score}(q,d) = \sum_{t \in q} \text{IDF}(t) \cdot \frac{\text{TF}(t,d) \cdot (k_1 + 1)}{\text{TF}(t,d) + k_1}$$

- The $(k_1 + 1)$ factor does not change ranking, but makes $\frac{\text{TF}(t,d) \cdot (k_1+1)}{\text{TF}(t,d)+k_1} = 1$ when $\text{TF}(t,d) = 1$.

- The model is similar to TF-IDF, but the term frequency part is bounded.

Wait! There is also the factor of document length.

- Longer documents are likely to have larger TF values

- Why might documents be longer?
  - Verbosity: suggests observed TF too high
  - Larger Scope: suggests observed TF may be accurate

- A real document collection probably has both effects, so we should apply some kind of "partial" normalization.

- Length normalization factor: $B = 1 - b + b \cdot \dfrac{|d|}{\text{avgdl}}$

  - $|d|$ is the length of d (in words); avgdl = average document length (in words)
  - $b \in [0,1]$ is a hyperparameter

Document Length Normalization

- $B = 1 - b + b \cdot \dfrac{|d|}{\text{avgdl}}$

- What if the length of d is exactly the average document length?
    - $B = 1 - b + b = 1$, no need to normalize
- What if d is longer than average (e.g., $|d| = 2 \times \text{avgdl}$)?
    - $B = 1 - b + 2b = 1 + b > 1$
- What if d is shorter than average (e.g., $|d| = 0.5 \times \text{avgdl}$)?
    - $B = 1 - b + 0.5b = 1 - 0.5b < 1$

- $b = 1$: full document length normalization
- $b = 0$: no document length normalization

Putting it all together …

$$\text{BM25}(q, d) = \sum_{t \in q} \text{IDF}(t) \cdot \frac{\text{TF}(t, d) \cdot (k_1 + 1)}{\text{TF}(t, d) + k_1 (1 - b + b \cdot \frac{|d|}{\text{avgdl}})}$$

$$= B$$

- k_1 controls term frequency scaling
  - $k_1 = 0$: binary model
  - k_1 very large: raw term frequency
- b controls document length normalization
  - $b = 0$: no document length normalization
  - $b = 1$: relative frequency (full document length normalization)
- Typically, k_1 is set between 1.2 and 2; b is set around 0.75

- $|d|$ is the length of d (in words); avgdl = average document length (in words)

BM25 in Homework 2, Quiz 1, and Final

- Homework 2
  - You need to implement the BM25 scoring function.

- Quiz 1 and Final Exam
  - You should be familiar with the binomial generative model and the "One" Poisson model.
  - You should know document length normalization in the BM25 scoring function.
  - You do NOT need to master the derivation of the "Two" Poisson model and the Robertson-Spärck-Jones model.
  - You will NOT be required to manually calculate the BM25 score.

Thank You!

Course Website: https://yuzhang-teaching.github.io/CSCE670-S26.html