

# **Lecture 4 – Solar Radiation**

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# Outline

- The Sun's Structure and Composition
- Sunlight and Spectrum
- The Earth's Revolution and Seasons
- Solar Irradiance & Irradiation
- Optimize Tilt Angle of Solar Collectors

B1 Chap 4; B3 Chap 3

# The Sun: Ultimate Source of Renewable Energy

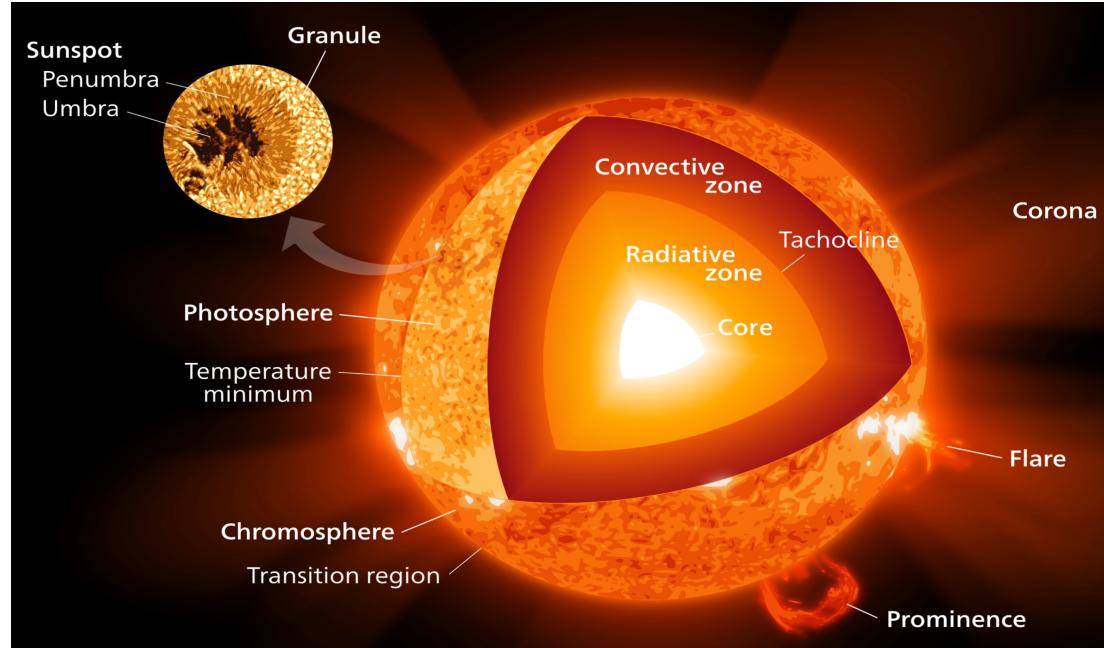


- **Solar** – directly from sunlight
- **Winds** – caused by uneven heating of the atmosphere by the sun, the irregularities of the earth's surface, and rotation of the earth
- **Hydro** – winds and the sun's heat cause water to evaporate → rain/snow and flows downhill into rivers or streams
- **Biomass** – along with rain & snow, sunlight causes plants to grow. The organic matter that makes up plants is known as biomass.
- **Tidal wave** – gravitational pull of Moon & Sun upon the Earth.

# Data of the Sun

Observation data		Physical characteristics
Mean distance from Earth	$1 \text{ au} \approx 1.496 \times 10^8 \text{ km}$ <sup>[1]</sup> 8 min 19 s at light speed	<b>Equatorial radius</b> $695,700 \text{ km}$ , <sup>[7]</sup> $696,342 \text{ km}$ <sup>[8]</sup> $109 \times \text{Earth}$ <sup>[9]</sup>
Visual brightness ( $V$ )	-26.74 <sup>[2]</sup>	<b>Equatorial circumference</b> $4.379 \times 10^6 \text{ km}$ <sup>[9]</sup> $109 \times \text{Earth}$ <sup>[9]</sup>
Absolute magnitude	4.83 <sup>[2]</sup>	<b>Flattening</b> $9 \times 10^{-6}$
Spectral classification	G2V <sup>[3]</sup>	<b>Surface area</b> $6.09 \times 10^{12} \text{ km}^2$ <sup>[9]</sup> $12,000 \times \text{Earth}$ <sup>[9]</sup>
Metallicity	$Z = 0.0122$ <sup>[4]</sup>	<b>Volume</b> $1.41 \times 10^{18} \text{ km}^3$ <sup>[9]</sup> $1,300,000 \times \text{Earth}$
Angular size	31.6–32.7 minutes of arc <sup>[5]</sup>	<b>Mass</b> $1.9885 \times 10^{30} \text{ kg}$ <sup>[2]</sup> $333,000 \times \text{Earth}$ <sup>[2]</sup>
		<b>Average density</b> $1.408 \text{ g/cm}^3$ <sup>[2][9][10]</sup> $0.255 \times \text{Earth}$ <sup>[2][9]</sup>

# Layers of The Sun



- **Core:** the innermost 20-25% of the Sun's radius, where temperature/pressure are sufficient for nuclear fusion to occur.
- **Radiative zone:** between 20-25% to 70% of the radius, energy transfer occurs by means of radiation (photons) rather than by convection.
- **Tachocline:** boundary region between the radiative & convective zones.
- **Convective zone:** between about 70% of the Sun's radius and a point close to the visible surface; the primary means of outward heat transfer.
- **Photosphere:** the deepest part which we can directly observe with visible light.
- **Atmosphere:** a gaseous 'halo' surrounding the Sun, comprising the chromosphere, solar transition region, corona and heliosphere.

# Composition of the Sun

Photospheric composition (by mass)	
Hydrogen	73.46% <sup>[14]</sup>
Helium	24.85%
Oxygen	0.77%
Carbon	0.29%
Iron	0.16%
Neon	0.12%
Nitrogen	0.09%
Silicon	0.07%
Magnesium	0.05%
Sulfur	0.04%

# The Sun's Nuclear Fusion

- The solar energy (both heat and light) originates from a nuclear fusion process.
- This proton-proton fusion process begins with protons (simply a lone hydrogen nucleus) and through a series of steps, these protons fuse together and are turned into helium.
  1. 2 protons fuse. One of them transforms into a neutron via the weak nuclear force, which forms a positron and neutrino (deuterium).
  2. A 3rd proton collides with deuterium resulting in a helium-3 nucleus and a gamma ray (sunlight).
  3. 2 helium-3 nuclei collide, creating a helium-4 nucleus plus 2 extra protons that escape as 2 hydrogen.

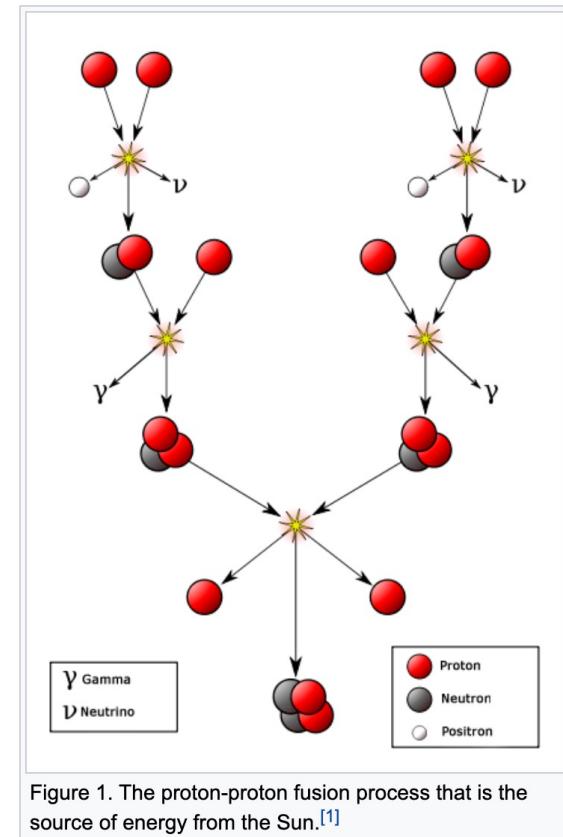
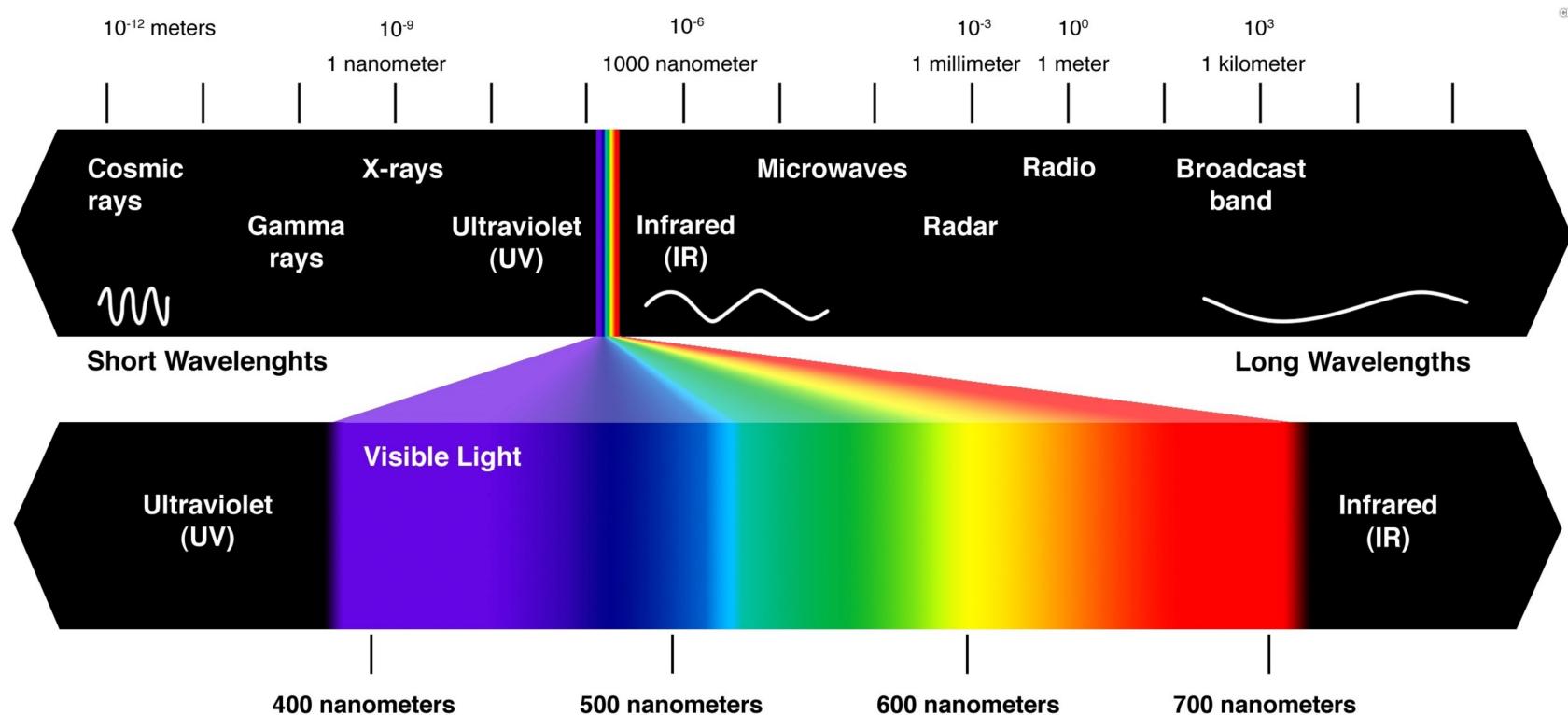


Figure 1. The proton-proton fusion process that is the source of energy from the Sun.<sup>[1]</sup>

- The core is the only part of the Sun that produces any significant amount of heat through fusion (it contributes about 99%).
- The rest of the Sun is heated by energy transferred outward from the core.

# Light Spectrum



$$f = \frac{c}{\lambda}$$

frequency      speed of light ( $3 \times 10^8$  m/s)  
                        wavelength

# Blackbody Radiation

A blackbody is be a perfect emitter and a perfect absorber.

- As a perfect **emitter**: it radiates more energy per unit of surface area than any real object at the same temperature.
- As a perfect **absorber**: it absorbs all radiation that impinges upon it.
- There is no net flow of matter or energy between the body and its environment.

The spectral radiance of a blackbody is given by Planck's law:

$$E_\lambda = \frac{3.74 \times 10^8}{\lambda^5 \left[ \exp\left(\frac{14,400}{\lambda T}\right) - 1 \right]} \quad (4.1)$$

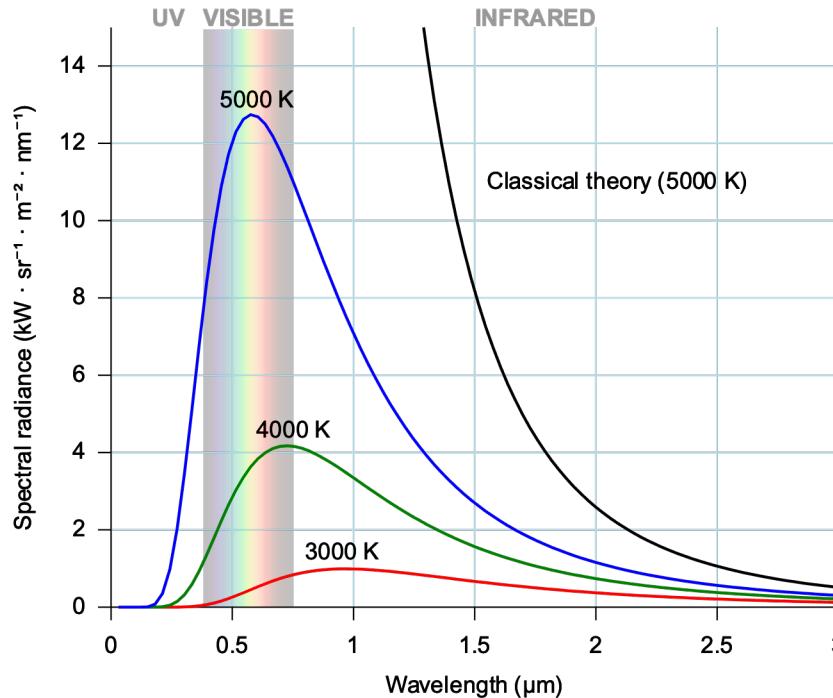
$E_\lambda$ : emissive power intensity per unit area per unit wavelength ( $W/m^2/\mu m$ )

$T$ : absolute temperature of the body ( $K$ )

$\lambda$ : wavelength ( $\mu m$ )

# Planck's Law

- Planck's law describes the electromagnetic radiation emitted by a black body in the thermal equilibrium at a given temperature.
- It was heuristically derived by German physicist Max Planck in 1900. This discovery was a pioneering insight of modern physics and is of fundamental importance to quantum theory.
- The emissive power intensive is a function of wavelength, parameterized by temperature.



# The Spectrums of the Earth & the Sun

- The wavelength at which the intensity reaches its maximum:

$$\lambda_{\max}(\mu\text{m}) = \frac{2898}{T(\text{K})}$$

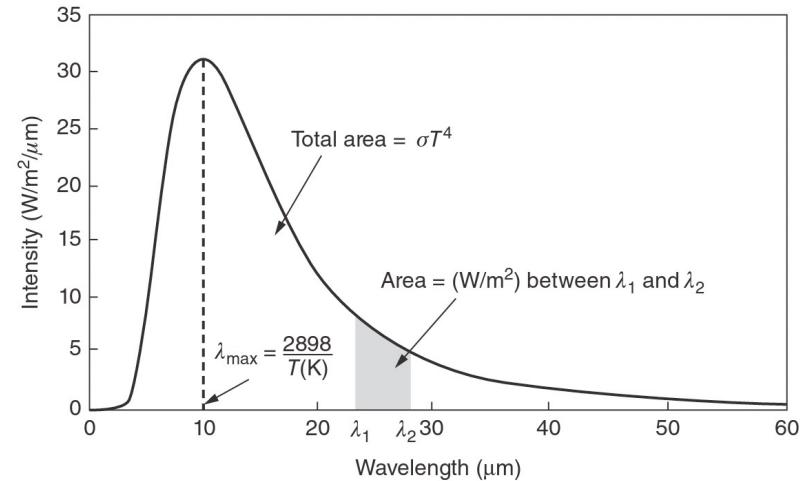


FIGURE 4.1 The spectral emissive power of the earth modeled as a 288 K blackbody.

- The total radiant power emitted:

$$E = \sigma A T^4$$

where  $\sigma$  is the Stefan–Boltzmann constant  $5.67 \times 10^{-8}$ , T is the absolute temperature (K), and A is the surface area (m<sup>2</sup>).

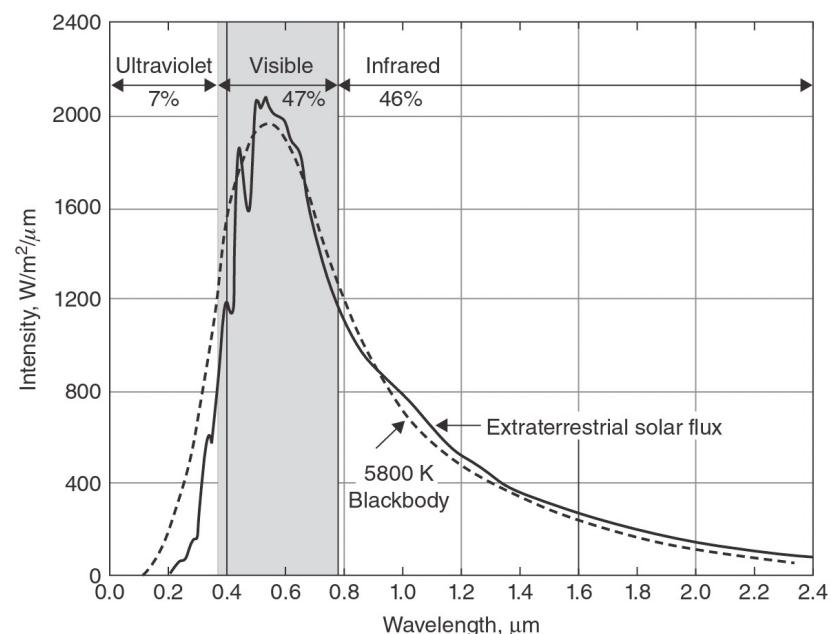


FIGURE 4.2 The extraterrestrial solar spectrum compared with a 5800 K blackbody.

# An Example

**Example 4.1 The Earth's Spectrum.** Consider the earth to be a blackbody with average surface temperature  $15^{\circ}\text{C}$  and area equal to  $5.1 \times 10^{14} \text{ m}^2$ . Find the rate at which energy is radiated by the earth and the wavelength at which maximum power is radiated. Compare this peak wavelength with that for a  $5800 \text{ K}$  blackbody (the sun).

**Solution.** Using Equation 4.2, the earth radiates

$$\begin{aligned} E &= \sigma AT^4 = (5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4) \times (5.1 \times 10^{14} \text{ m}^2) \times (15 + 273 \text{ K})^4 \\ &= 2.0 \times 10^{17} \text{ W} \end{aligned}$$

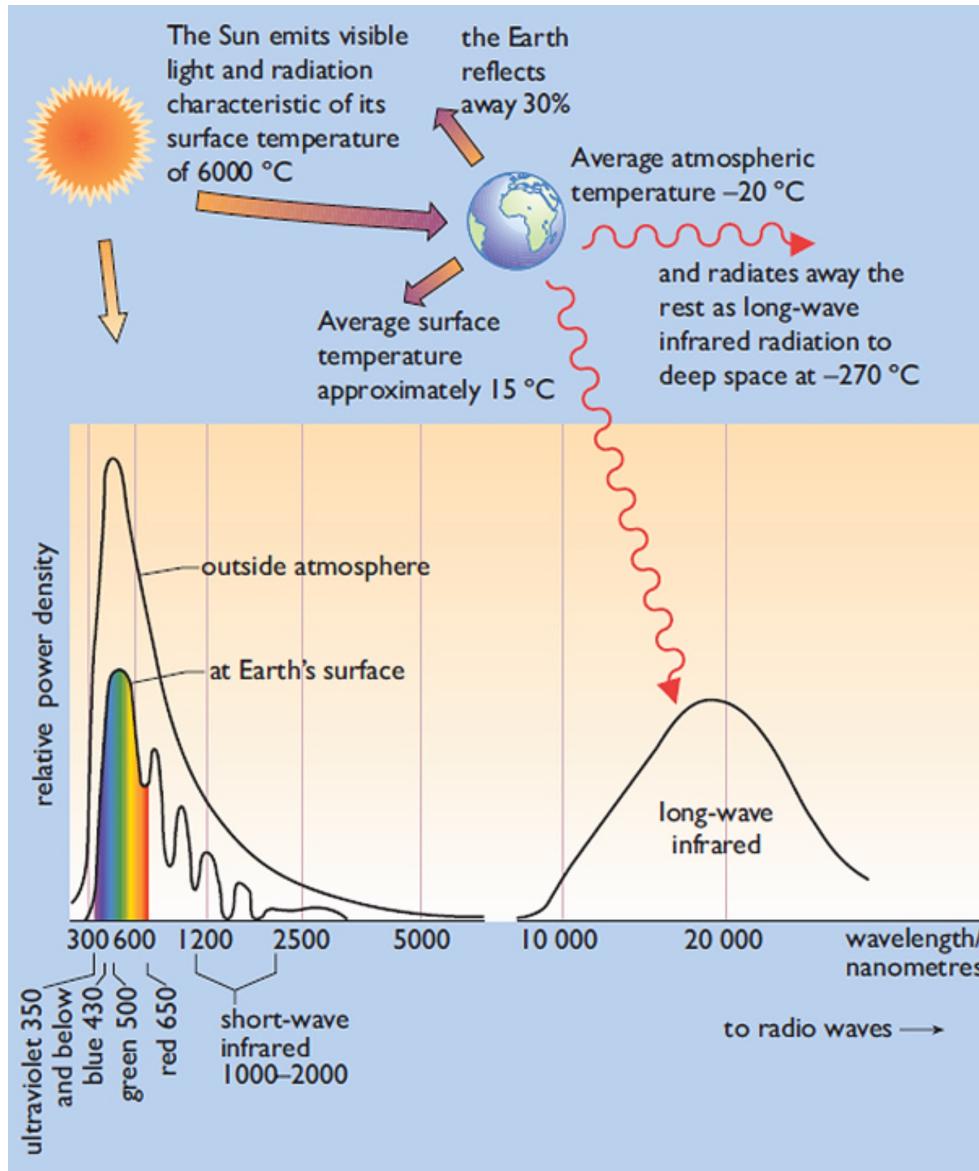
The wavelength at which the maximum power is emitted is given by Equation 4.3:

$$\lambda_{\max}(\text{earth}) = \frac{2898}{T(\text{K})} = \frac{2898}{288} = 10.1 \text{ } \mu\text{m}$$

For the  $5800 \text{ K}$  sun,

$$\lambda_{\max}(\text{sun}) = \frac{2898}{5800} = 0.5 \text{ } \mu\text{m.}$$

# Solar Irradiation

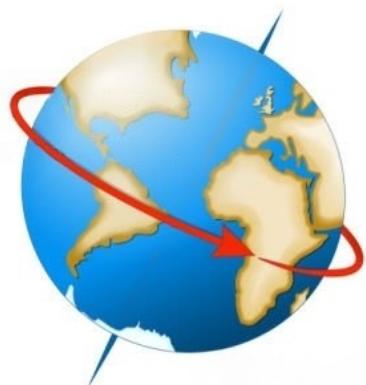


- The Sun is an enormous **nuclear fusion reactor**: hydrogen atoms fuse to form helium at rate of 4 million tons per second.
- This fusion reaction causes the Sun to radiate energy, due to the high surface temperature (6000°C).
- This fusion has been continuing reliably for the past 4 or 5 billion years and is expected to continue for another 4 or 5 billion years.

# The Earth's Rotation and Revolution

**Rotation**

to turn



Takes:  
24 hours or 1 day

Day

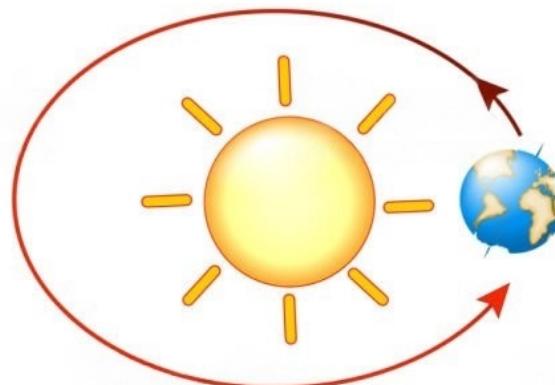


Day

Night

**Revolution**

go around



Takes:  
365 days or 1 year

Seasons



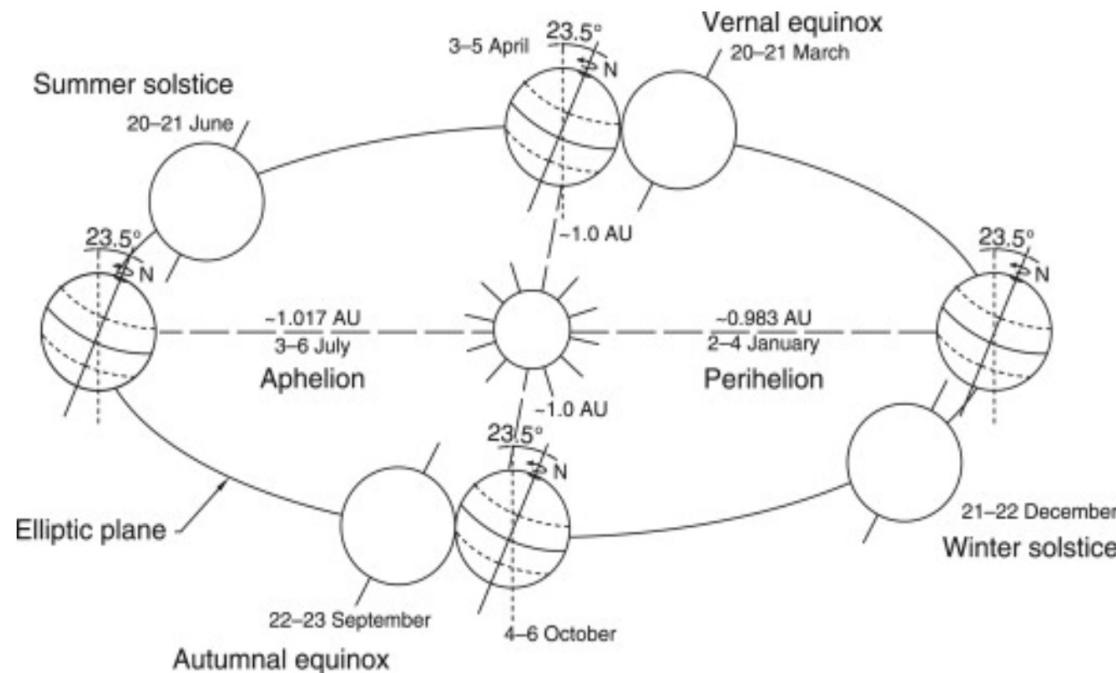
Spring

Summer

Fall

Winter

# Earth to Sun Distance



Earth-to-Sun distance:

$$d = 1.5 \times 10^8 \left\{ 1 + 0.017 \sin \left[ \frac{360(n - 93)}{365} \right] \right\} \text{ km}$$

Mean distance :=  
1astronomical unit (AU)

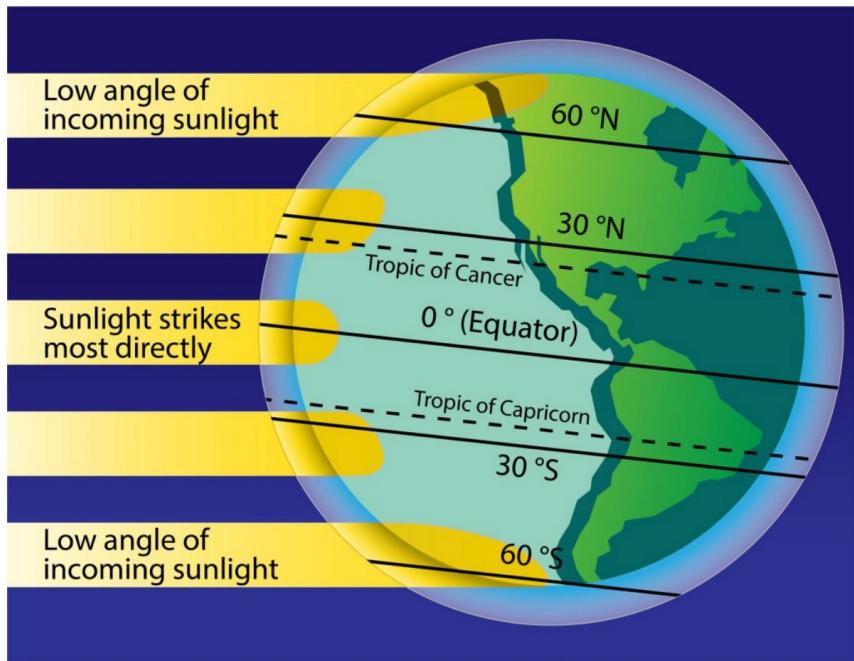
Eccentricity of Earth

n: day #: 1~365

Angle measured  
in degrees

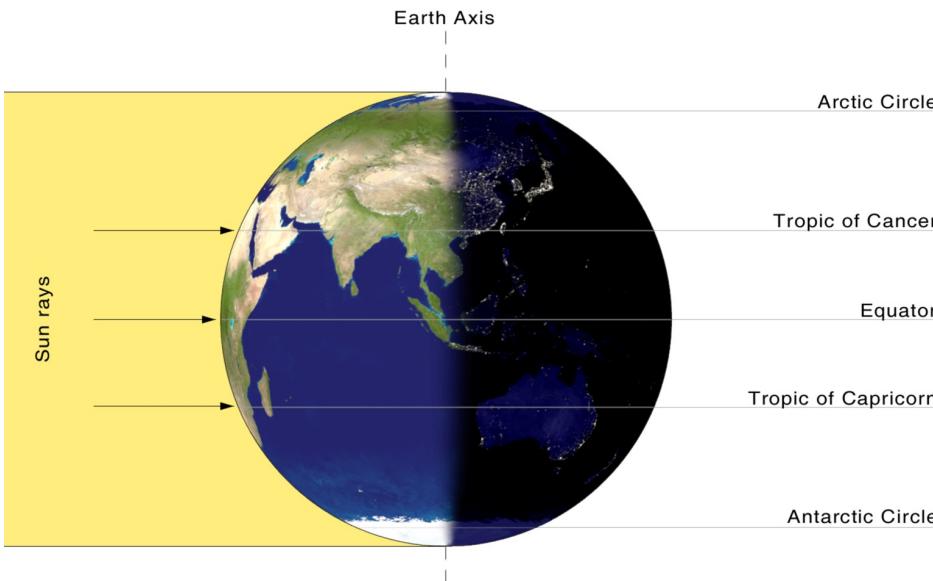
The above formula is derived based on Kepler's first law and an approximation; see details at <https://bit.ly/3jEyHup>

# Unequal Distribution of Solar Radiation

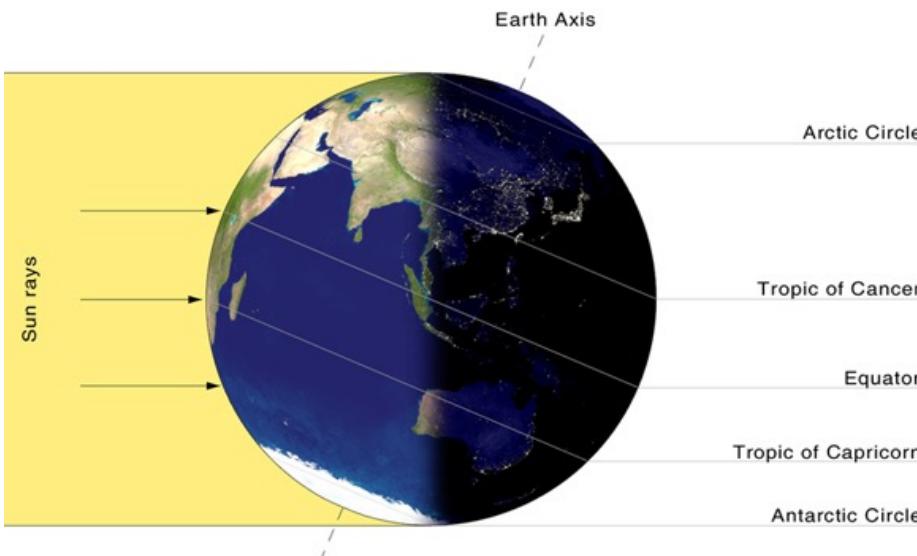


- Because Earth is a sphere, not all part of the Earth receives the same amount of solar radiation.
- Near the equator, the Sun's rays strike the Earth most directly, while at the poles the rays strike at a steep angle.
- This means that less solar radiation is absorbed per square meter of surface area at higher latitudes than at lower latitudes, and that the tropics are warmer than the poles.
- This temperature difference shapes global atmospheric and ocean circulation patterns.

# The Angle of Sun Rays

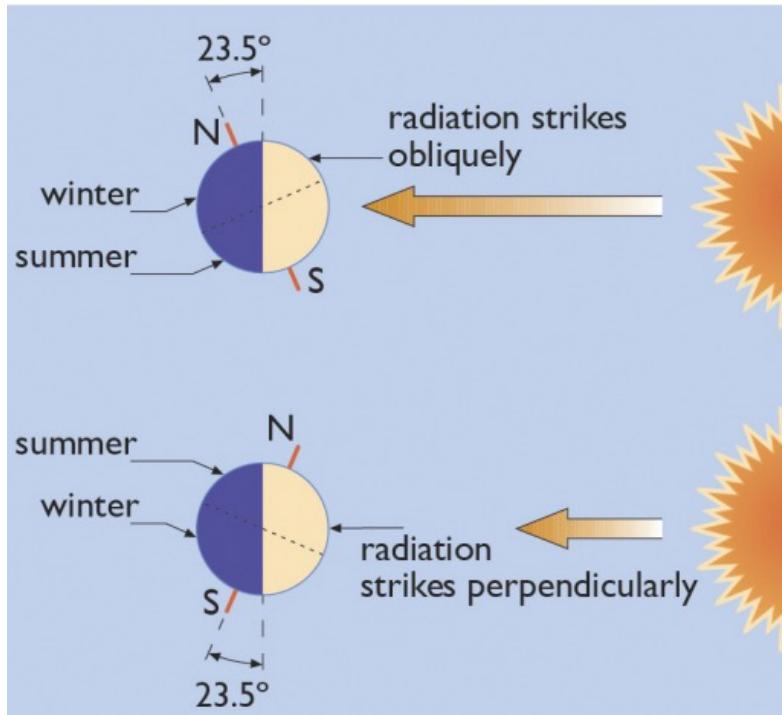


If the Earth if not tilted, the sun rays always strike perpendicularly onto the Equator.



During the Summer time of Southern Hemisphere, the sun rays strike perpendicularly onto the Tropic of Capricorn.

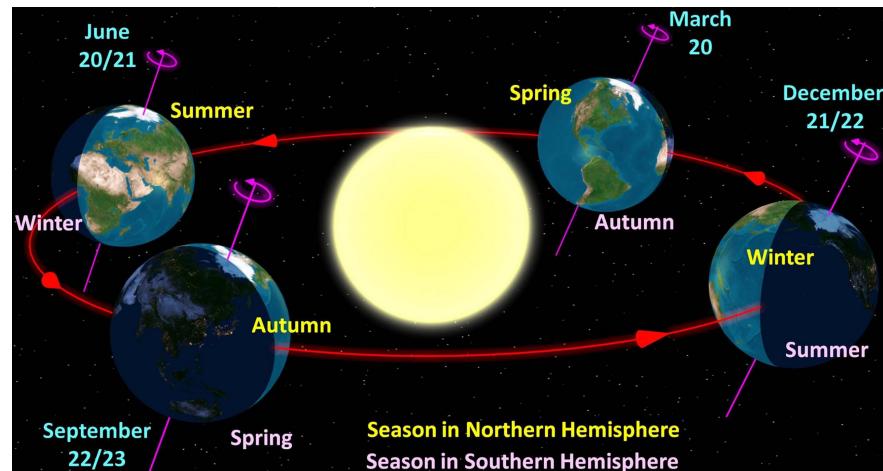
# Different Seasons



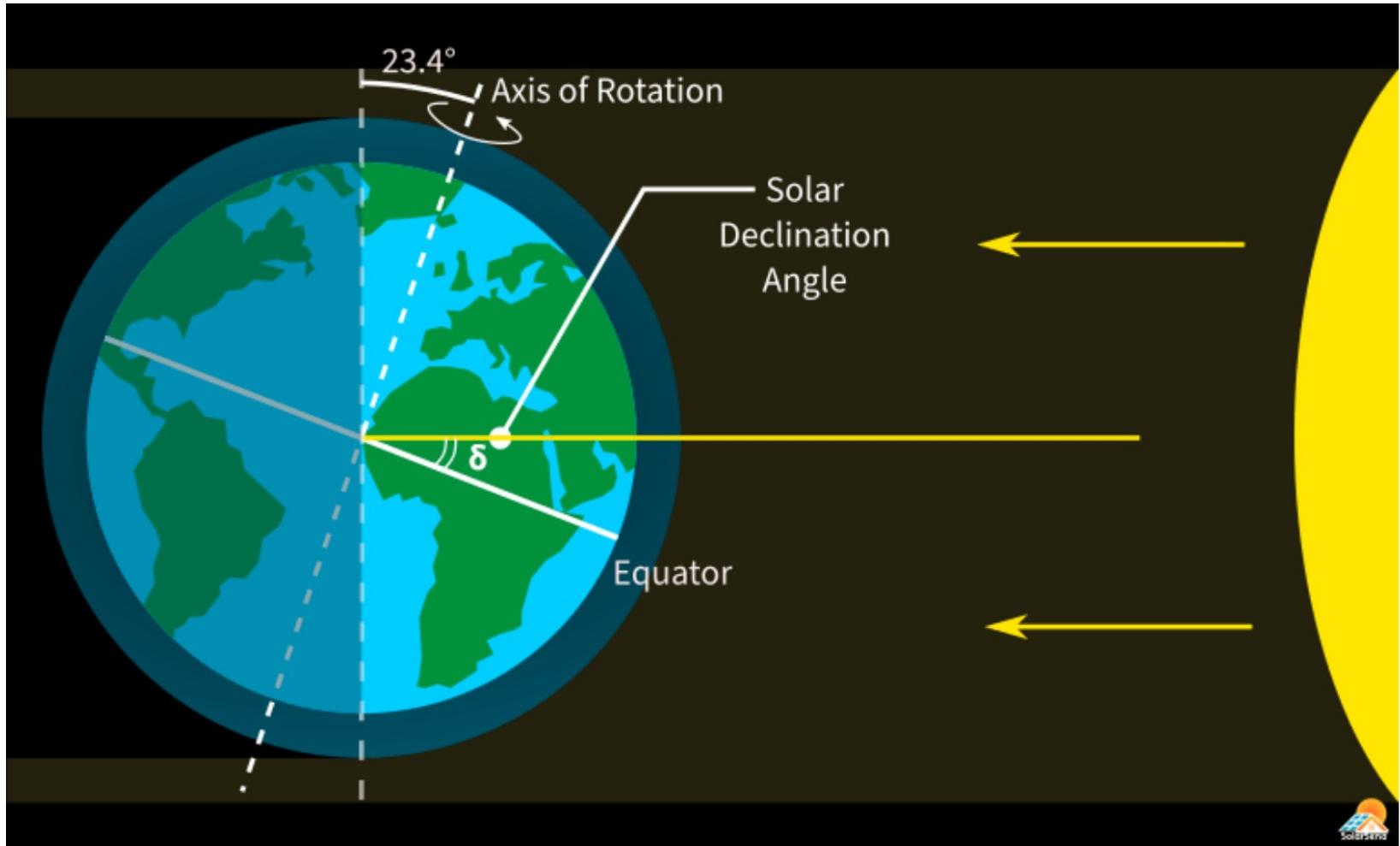
Tilt (at  $23.45^\circ$ ) of the Earth's axis creates different seasons.

The Earth circles the Sun with its polar axis tilted towards the plane of rotation.

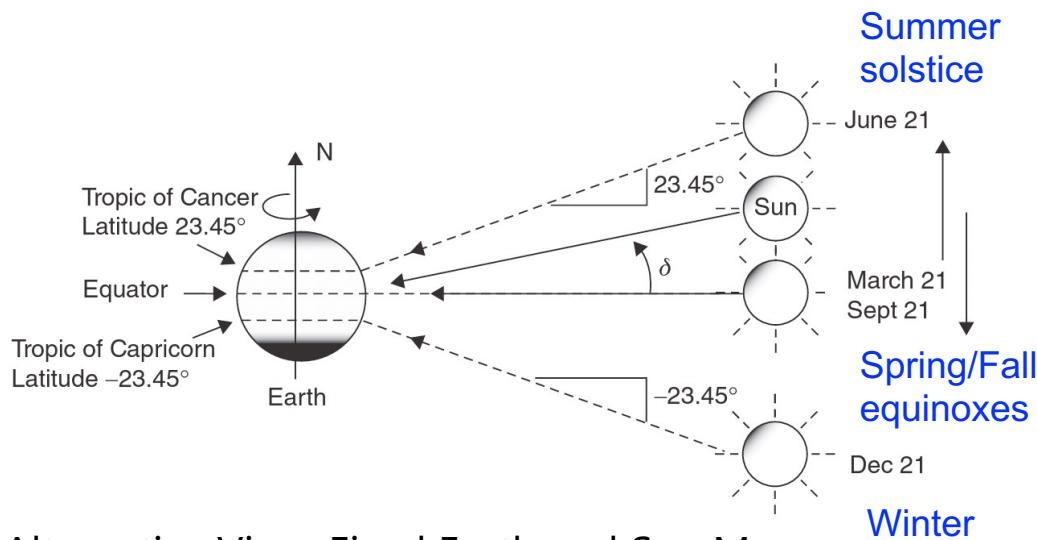
- In Dec., the North Pole is **tilted away** from the Sun, resulting in fewer kilowatt-hours reaching per square meter.
- In June, the North Pole is **tilted towards** the Sun therefore its rays strike the northern hemisphere more perpendicularly and the Sun appears higher in the sky.



# Solar Declination



# Solar Declination (cont'd)



Alternative View: Fixed Earth and Sun Moves

$$\delta = 23.45 \sin \left[ \frac{360}{365} (n - 81) \right]$$

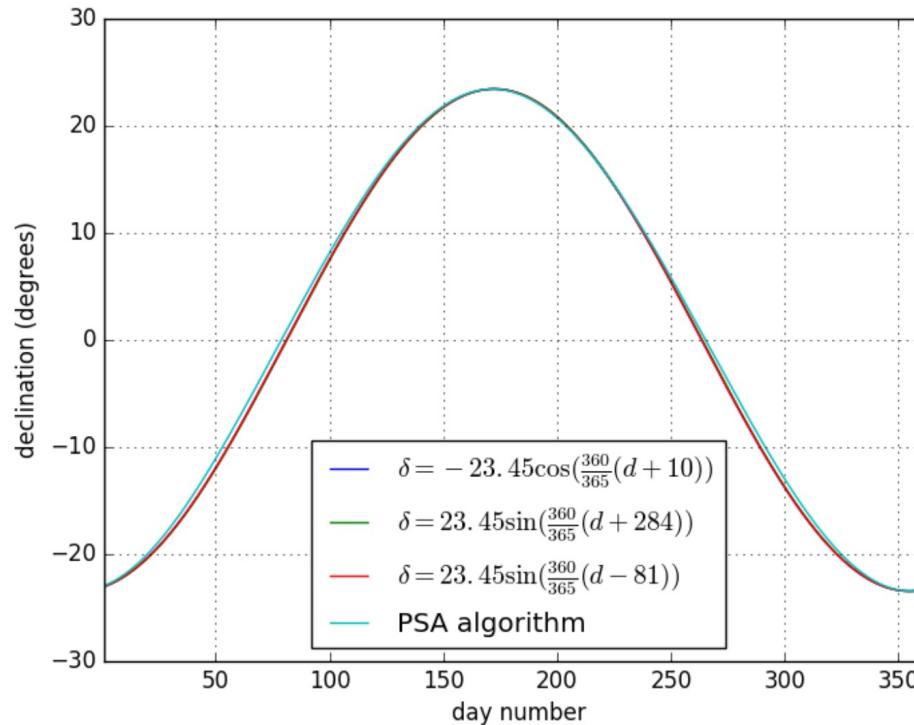
n: day #: 1~365  
angle measured in degrees

Solar declination  $\delta$ : the angle between the Earth-Sun line and the earth's equatorial plane. It describes the latitude of Earth where the sun is directly overhead at noon.

The solar declination angle is an important factor in determining

- 1) the amount of solar radiation that a particular location receives.
- 2) the path of the Sun across the sky and the length of daylight hours at a particular location.

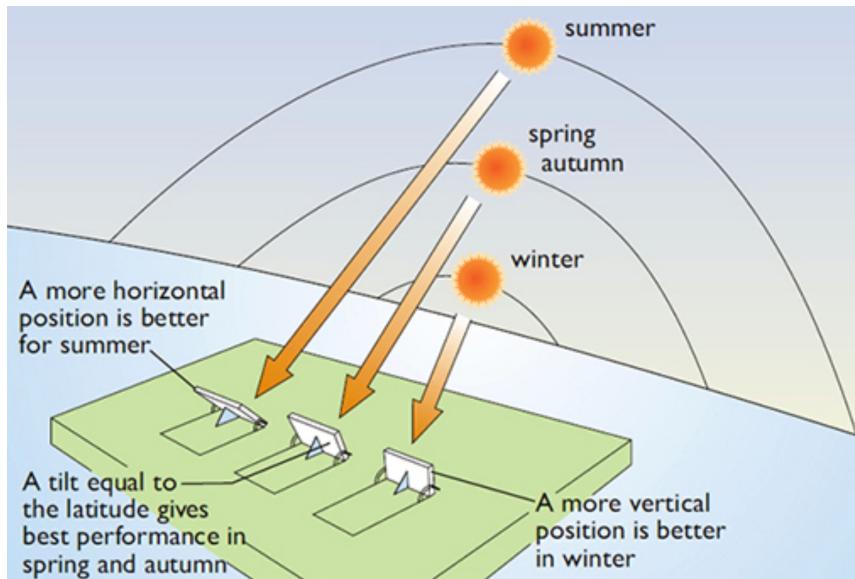
# More Accurate Formula



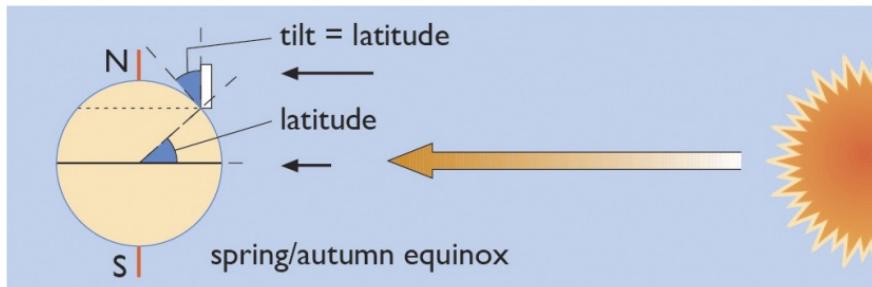
$$\delta = \sin^{-1} \left[ \sin(-23.44^\circ) \cos \left( \frac{360}{365.24} (d + 10) + \frac{360}{\pi} \times 0.0167 \sin \left( \frac{360}{365.24} (d - 2) \right) \right) \right]$$

There are many algorithms for more accurate determinations of declination angle to account for the elliptic and yearly movement of the earth's orbit. These are only needed for concentrators that require more accurate tracking

# Optimizing the Tilt



- In the northern hemisphere, a surface should face south to collect as much radiation as possible.
- Be tilted towards the Sun at an angle depending on the latitude and the time of year.

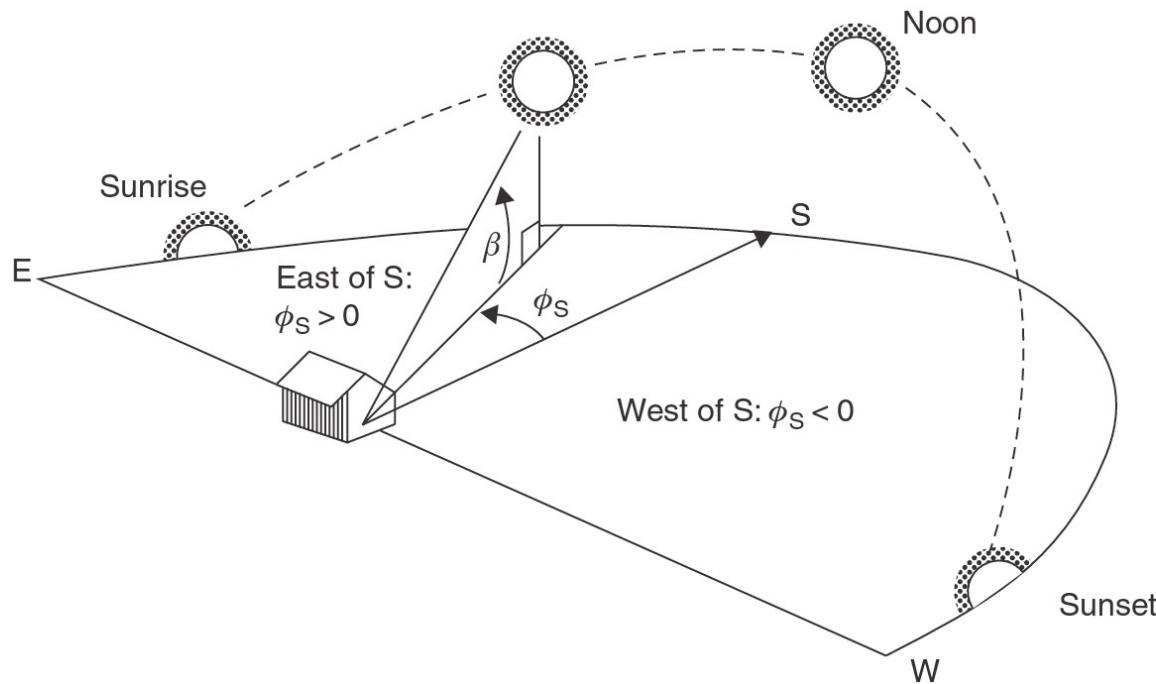


A surface tilted at the latitude angle will be perpendicular to the Sun's rays at mid-day on the Spring or Autumn equinox.

# Position of the Sun



# Tracking the Sun at Any Time



The location of the sun can be described by two angles:

- **Solar altitude (elevation) angle  $\beta$ :** the angle between the sun and the local horizon directly beneath the sun. Its complementary angle is the Solar zenith angle.
- **Solar azimuth angle  $\phi_S$ :** the book uses the south-anticlockwise convention (for the Northern Hemisphere): due south as the reference and positive angles if anticlockwise.

# Computing the Two Solar Angles

$$\sin \beta = \cos L \cos \delta \cos H + \sin L \sin \delta$$

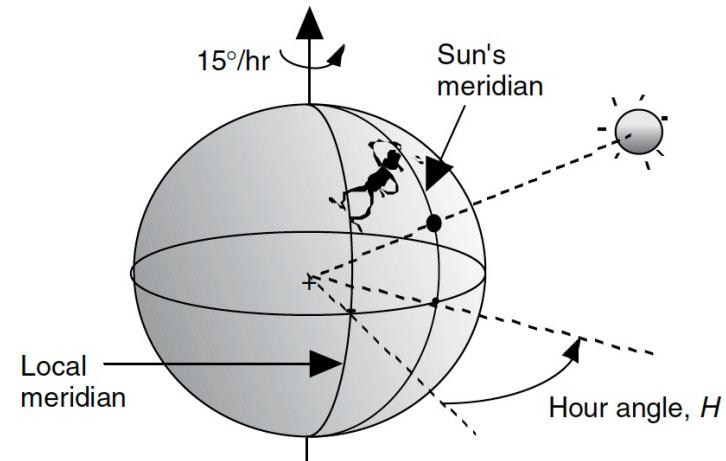
$$\sin \phi_S = \frac{\cos \delta \sin H}{\cos \beta}$$

$$\text{Hour angle } H = \left( \frac{15^\circ}{\text{hour}} \right) \cdot (\text{hours before solar noon})$$

- L: Latitude of the location
- δ: Solar declination
- H: hour angle

**Hour angle:** the number of degrees the Earth must turn before the Sun is directly over the local meridian. H = 0 at solar noon.

**Solar noon:** the moment when the Sun passes a location's meridian (on the line of longitude), reaching its highest position above the horizon and casting the shortest shadow.  
At this time, the Sun appears due south in the Northern Hemisphere. In most cases, it doesn't happen at 12pm sharp.



# Special Cases

$$\sin \beta = \cos L \cos \delta \cos H + \sin L \sin \delta$$

- At solar noon ( $H = 0$ ):

$$\sin \beta = \cos L \cos \delta + \sin L \sin \delta = \cos(L - \delta) \implies \beta_N = 90 - L + \delta$$

- At solar noon ( $H = 0$ ) & Sun is directly overhead:

$$\beta_N = 90 \implies \delta = L$$

$$\sin \beta = \cos L \cos \delta \cos H + \sin L \sin \delta = 0$$

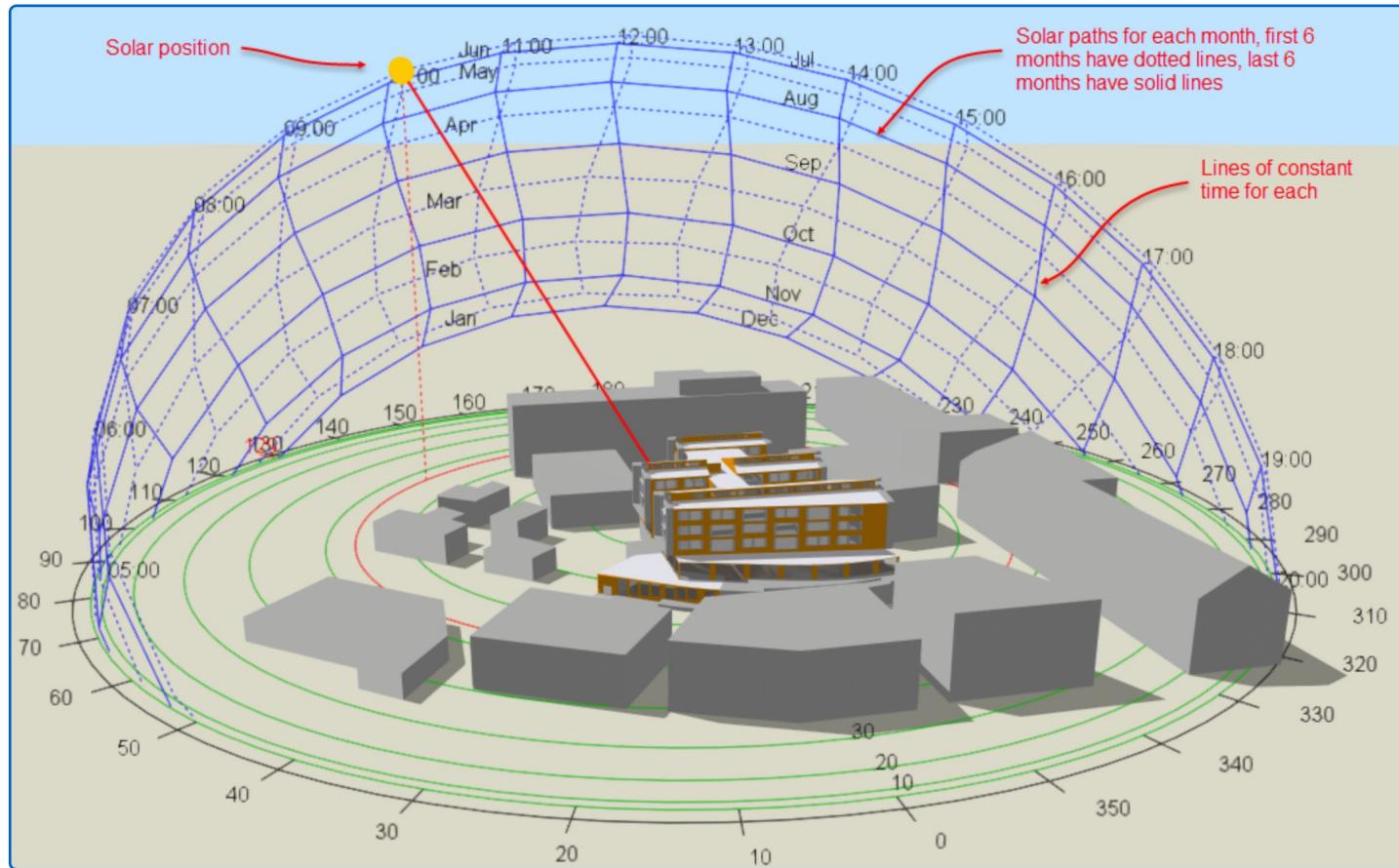
- Solar hour at sunrise/sunset:

$$\cos H = -\frac{\sin L \sin \delta}{\cos L \cos \delta} = -\tan L \tan \delta$$

$$H_{SR} = \cos^{-1}(-\tan L \tan \delta) \quad (+ \text{ for sunrise}) \implies$$

$$\text{Sunrise(geometric)} = 12:00 - \frac{H_{SR}}{15^\circ/h}$$

# Sun Path Diagram



The Sun path diagram helps visualize the paths as it moves through the sky at all times of the year: the green circles are the contour lines for the solar latitude angles. The black circle shows solar azimuth angles (North as reference).

# An Example

**Example 4.2 Tilt Angle of a PV Module.** Find the optimum tilt angle for a south-facing photovoltaic module in Tucson (latitude  $32.1^\circ$ ) at solar noon on March 1.

**Solution.** From Table 4.1, March 1st is the 60th day of the year so the solar declination (Eq. 4.6) is

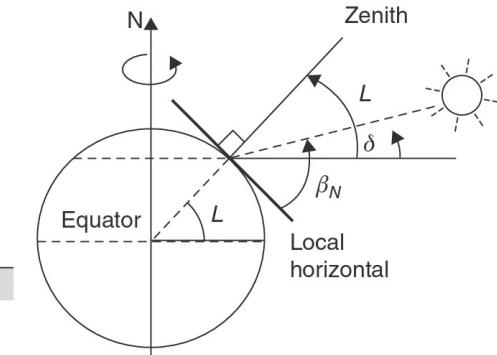
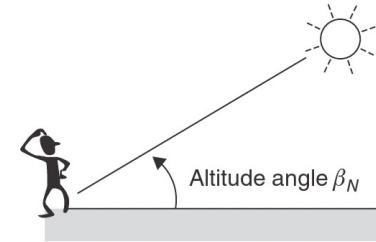
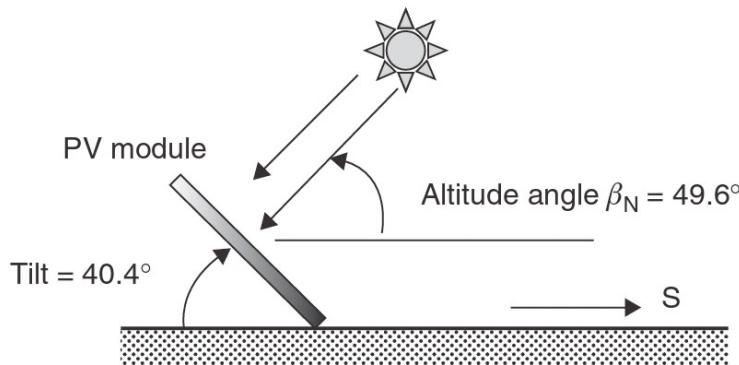
$$\delta = 23.45 \sin \left[ \frac{360}{365} (n - 81) \right] = 23.45^\circ \sin \left[ \frac{360}{365} (60 - 81)^\circ \right] = -8.3^\circ$$

which, from Equation 4.7, makes the altitude angle of the sun equal to

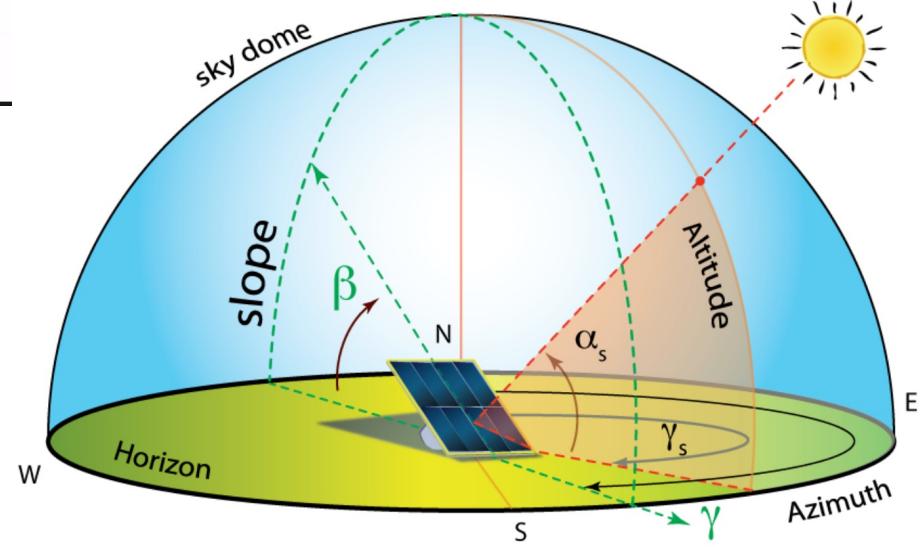
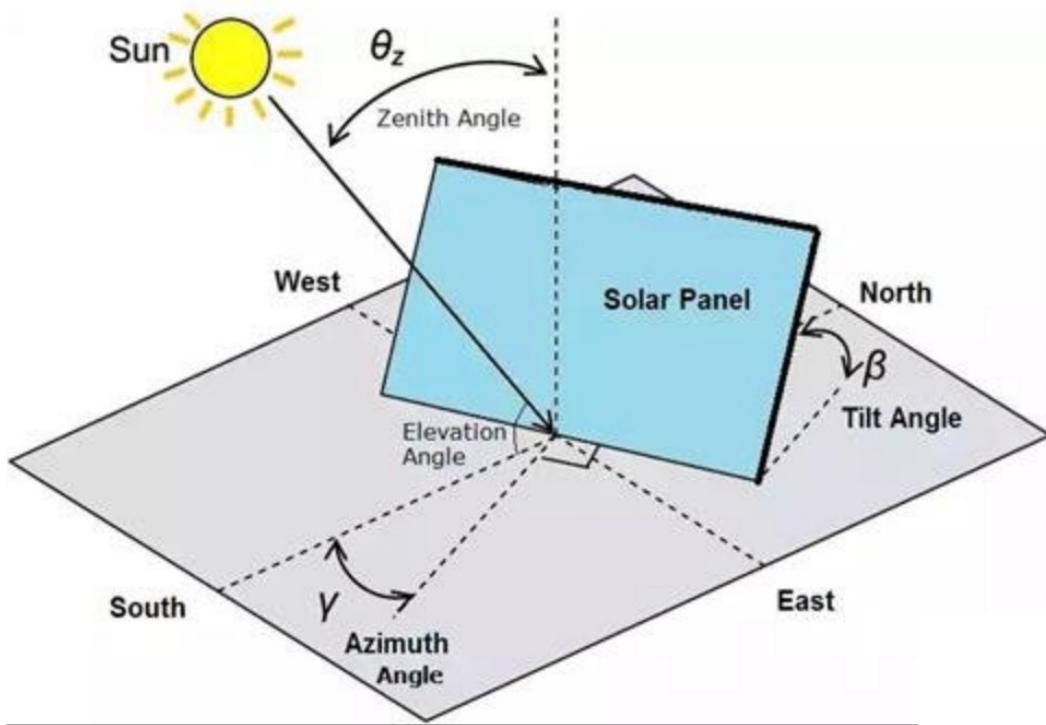
$$\beta_N = 90 - L + \delta = 90 - 32.1 - 8.3 = 49.6^\circ$$

The tilt angle that would make the sun's rays perpendicular to the module at noon would therefore be

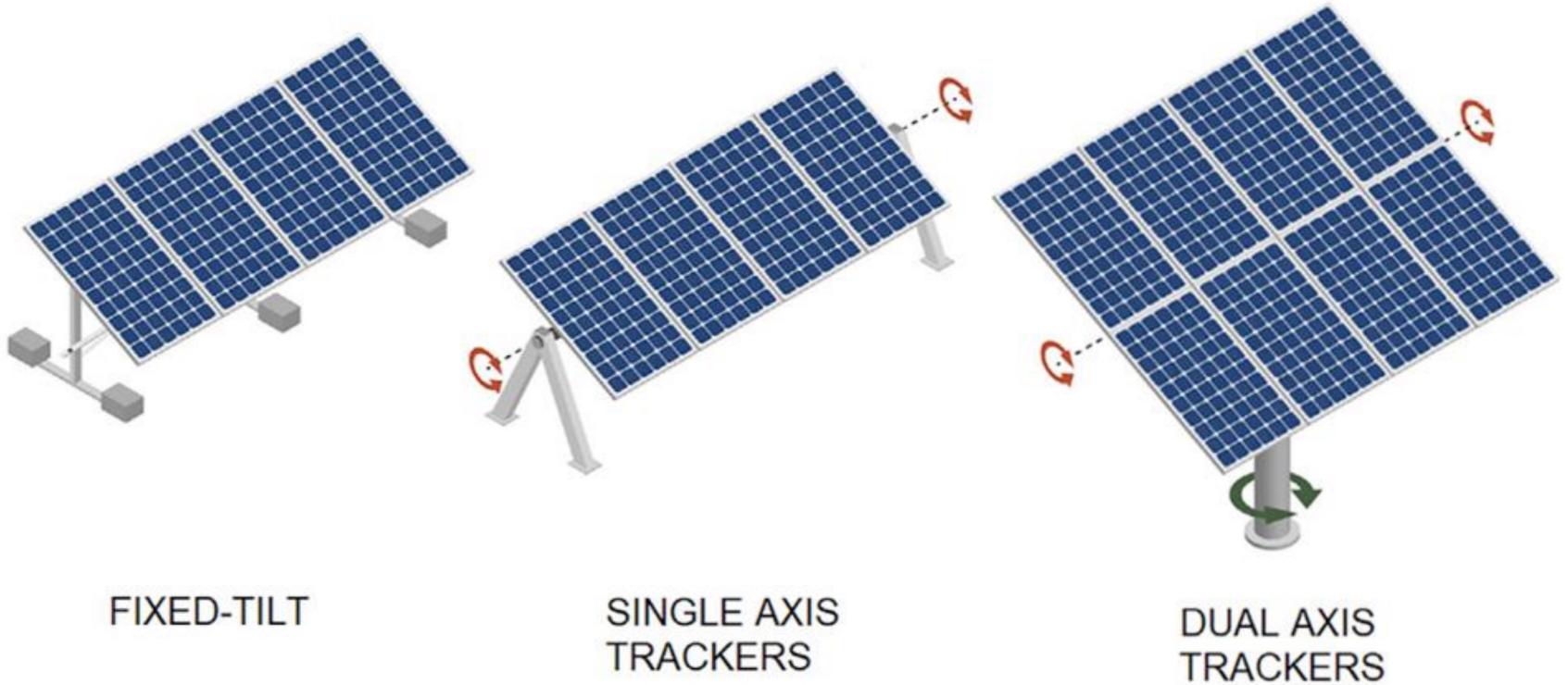
$$\text{Tilt} = 90 - \beta_N = 90 - 49.6 = 40.4^\circ$$



# Solar and Collector Angles in 3-D



# Solar Trackers



# Solar Irradiance & Insolation

- Solar **irradiance**: The power per unit area ( $\text{W/m}^2$ ) received from the Sun in the form of electromagnetic radiation.
- Solar **insolation** (aka solar exposure/irradiation): Solar irradiance integrated over a given period ( $\text{kWh/m}^2$ ).

Important applications:

- 1) prediction of solar energy generation.
- 2) heating/cooling loads of buildings.
- 3) climate modeling and weather forecasting.

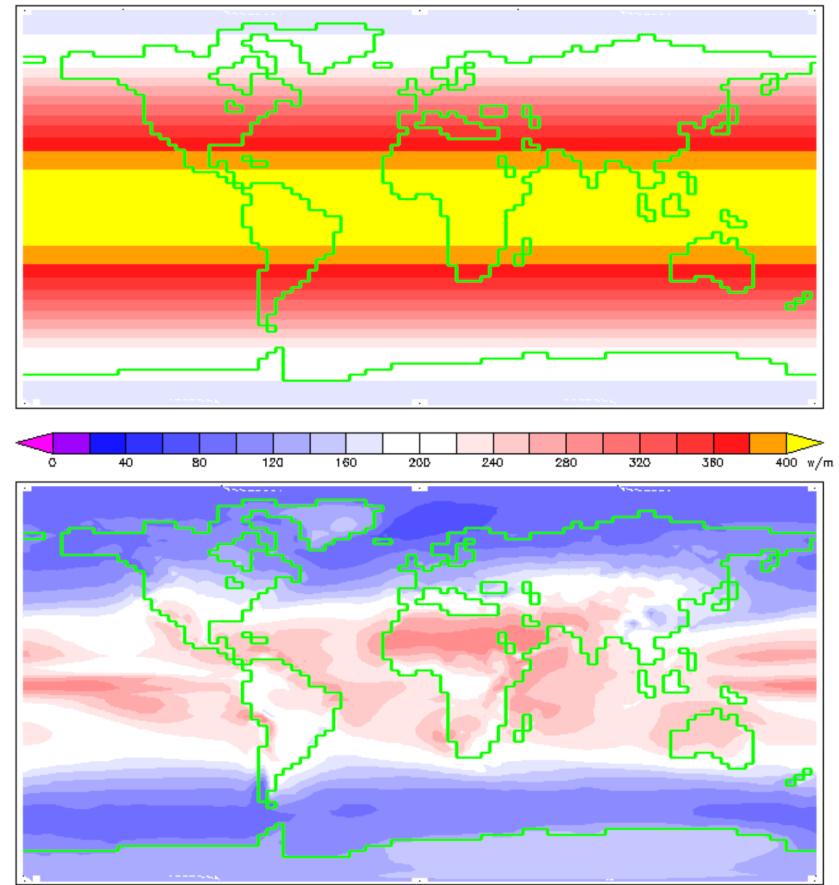
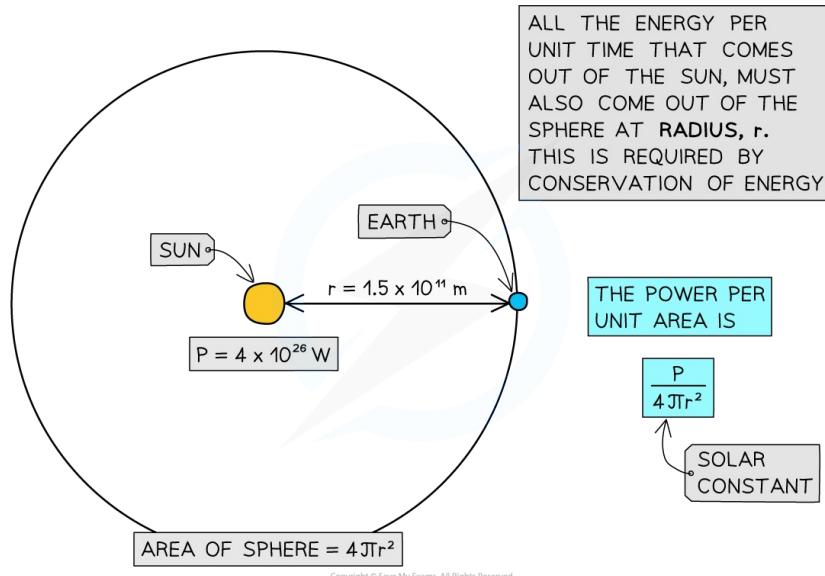


Fig: The shield effect of Earth's atmosphere on solar irradiation.  
Top: at the top of Earth's atmosphere  
Bottom: reaching the Earth's surface

# Solar Constant



The solar constant (SC  $\approx 1361 \text{ W/m}^2$ ) is a flux density measuring mean solar electromagnetic radiation (total solar irradiance) per unit area. It is measured on a surface perpendicular to the rays, 1 AU from the Sun.

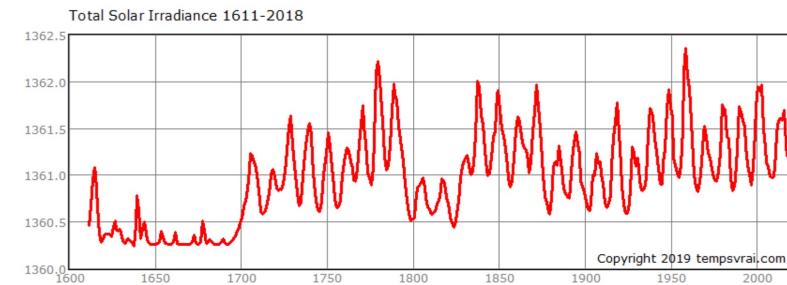
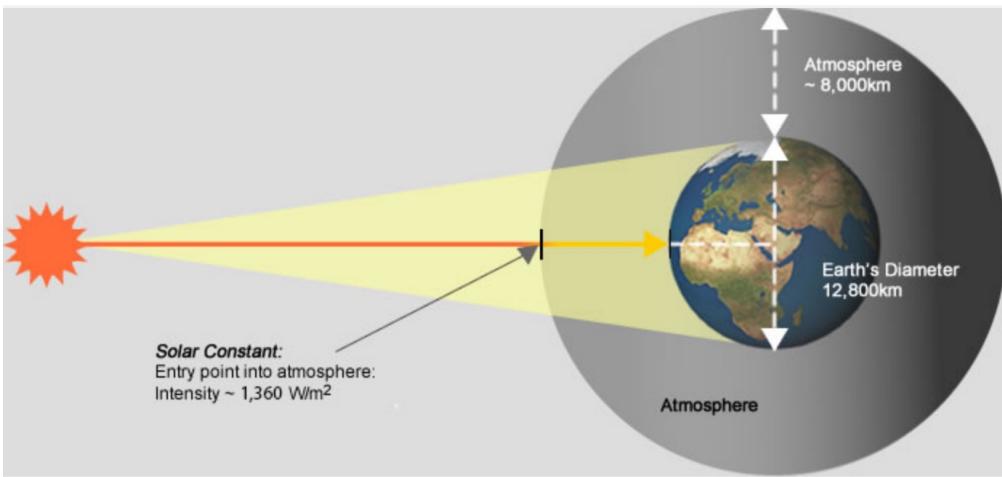
Extraterrestrial (ET) solar irradiance  $I_0$ , which passes perpendicularly through an imaginary surface just outside of the earth's atmosphere, depends on the distance between the Earth and the Sun, which varies with the time of year.

$$I_0 = \text{SC} \cdot \left[ 1 + 0.034 \cos\left(\frac{360n}{365}\right) \right] \quad (\text{W/m}^2)$$

# Total Solar Irradiance

**Total solar irradiance (TSI)** is a measure of the amount of solar radiation received at the top of the earth's atmosphere, averaged over a period of time (usually a year) and over the entire surface of the earth facing the sun.

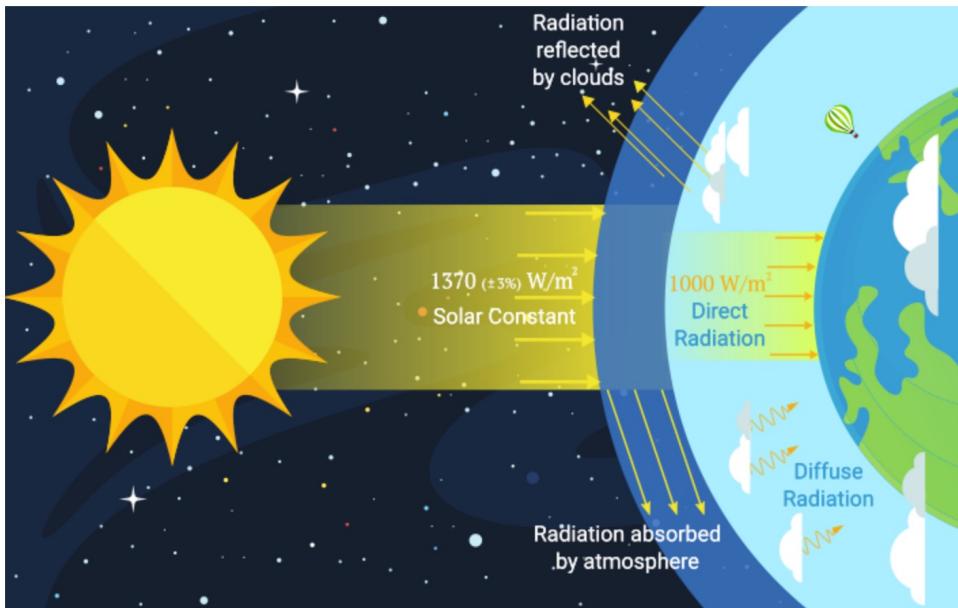
- The average value for TSI is about **1361 W/m<sup>2</sup>**. It has been measured by satellites over the entire solar cycle (about 11 years).
- TSI can vary over shorter time scales due to solar activity and other factors. However, over longer time scales, the average TSI is relatively constant.



This entry point into the atmosphere is called “Air Mass – 0 (AM-0)”, where “0” means no air mass.

# Solar Irradiance (cont'd)

- Solar irradiance varies depending on a number of factors, including the time of day, season, and location.
- **Atmospheric attenuation:** Earth's atmosphere absorbs some of the sun's radiation, particularly in the ultraviolet and infrared portions.
- Clouds, smoke, and air pollution can all reduce the amount of solar irradiance that reaches the surface.
- The average solar irradiance on the Earth's surface is approximately  $1\text{ kW/m}^2$ . The solar irradiance at solar noon can be as high as  $1.4\text{ kW/m}^2$ , while it can drop to as low as  $0.2\text{ kW/m}^2$  during sunrise and sunset.



Beam portion of the radiation reaching the Earth's surface:

$$I_B = A e^{-km}$$

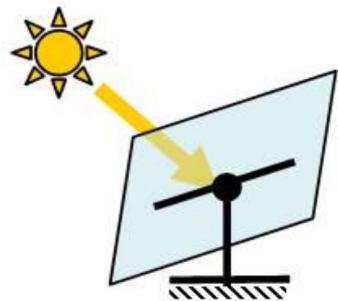
Air mass ratio  $m = \frac{1}{\sin \beta}$

$$A = 1160 + 75 \sin \left[ \frac{360}{365}(n - 275) \right] \quad (\text{W/m}^2)$$

$$k = 0.174 + 0.035 \sin \left[ \frac{360}{365}(n - 100) \right]$$

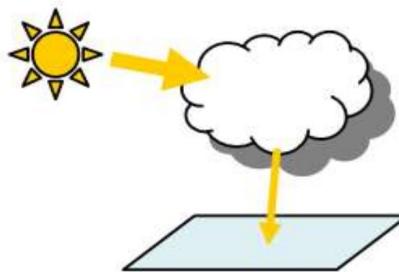
# DNI, DHI and GHI

Direct normal irradiation  
DNI



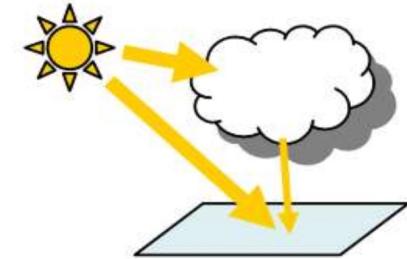
**Direct Normal Irradiance (DNI):** measured at a surface element perpendicular to the Sun

Diffuse horizontal irradiation  
DHI

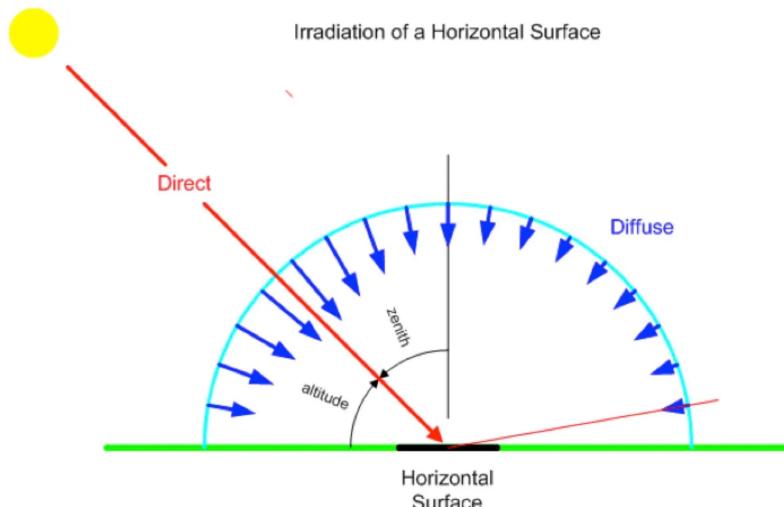


**Diffuse Horizontal Irradiance (DHI):** measured on a horizontal surface with radiation coming from all particles in the sky.

Global horizontal irradiation  
GHI



**Global Horizontal Irradiance (GHI):** the total irradiance on a horizontal surface



GHI is calculated as

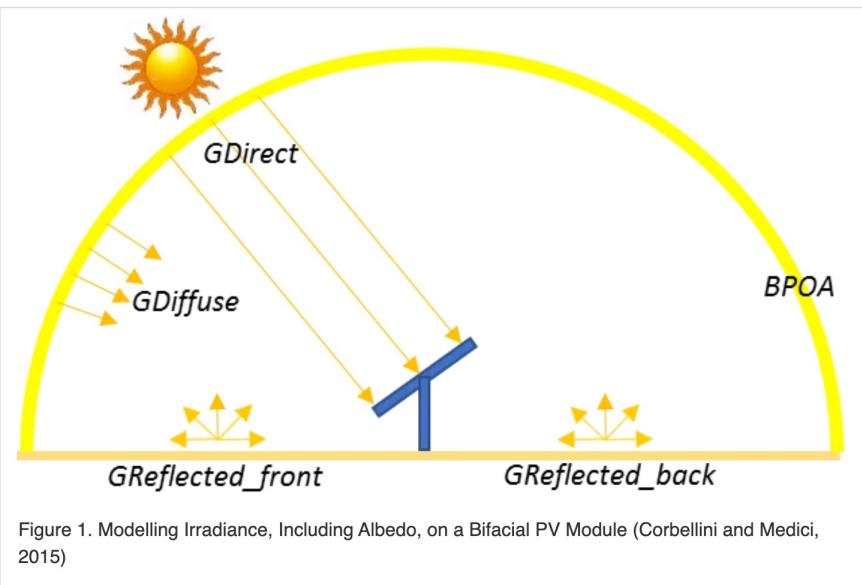
$$GHI = DHI + DNI \times \cos(\theta_z)$$

$\theta_z$  : Solar zenith angle

# Plane of Array (PoA) Irradiance

**POA irradiance (aka Global Tilted Irradiance (GTI)):** Quantify the incident irradiance on a given solar array. It is used extensively in PV modeling and performance analysis.

The bifacial POA can be calculated as follows, where the angle of incidence (AoI) is the angle between the sun's rays and the normal on PV panel.



$$\text{POA}_b = G_{\text{direct}} + G_{\text{diffuse}} + G_{\text{reflected-front}} + G_{\text{reflected-back}}$$

$$G_{\text{direct}} = \text{DNI} \times \cos(\text{AoI})$$

$$G_{\text{diffuse}} = \text{DHI} \times \frac{1 + \cos(\text{tilt})}{2}$$

$$G_{\text{reflected-front}} = \text{GHI} \times \text{Albedo} \times \frac{1 - \cos(\text{tilt})}{2}$$

$$G_{\text{reflected-back}} = \text{GHI} \times \text{Albedo} \times \frac{1 - \cos(\pi - \text{tilt})}{2}$$

## SOLAR RESOURCE MAP DIRECT NORMAL IRRADIATION



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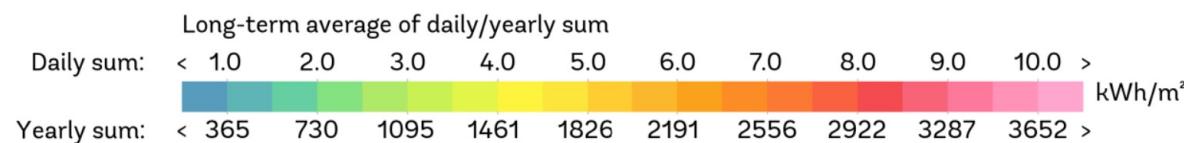
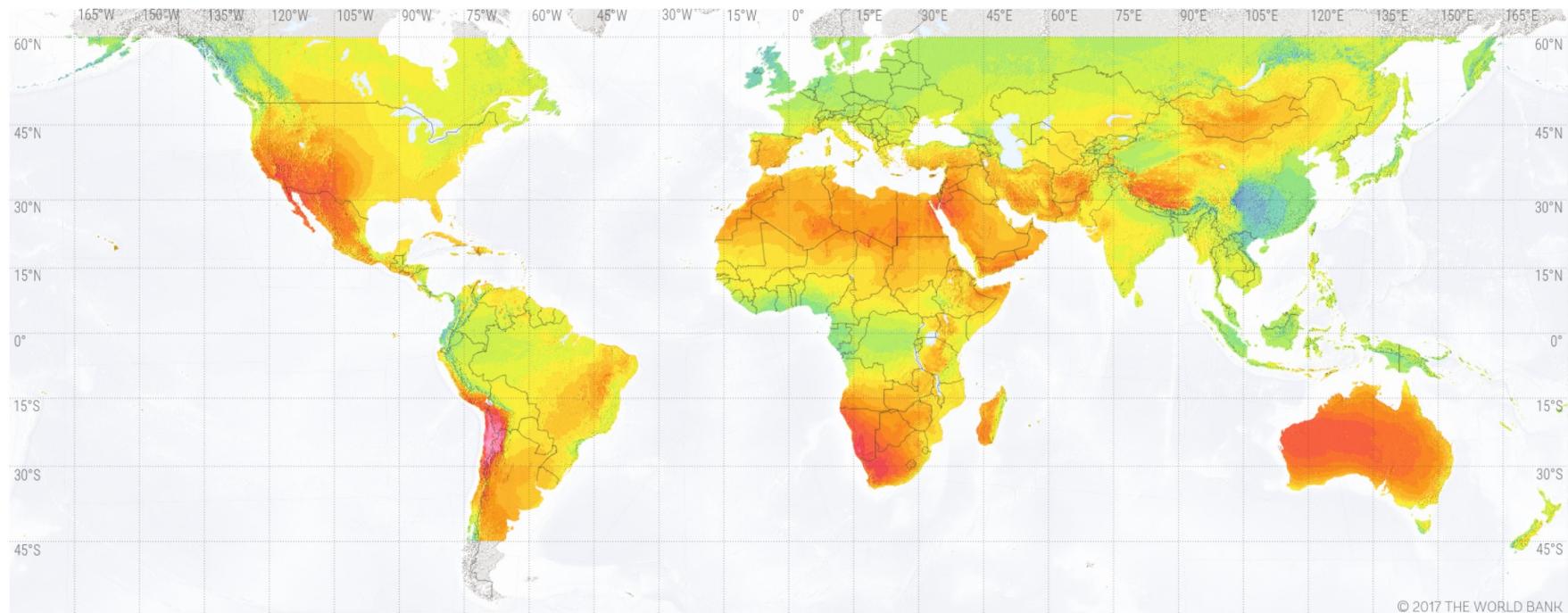
THE WORLD BANK



International Finance Corporation



Energy Sector Management Assistance Program



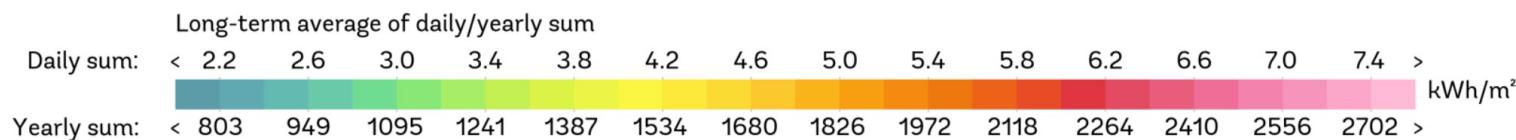
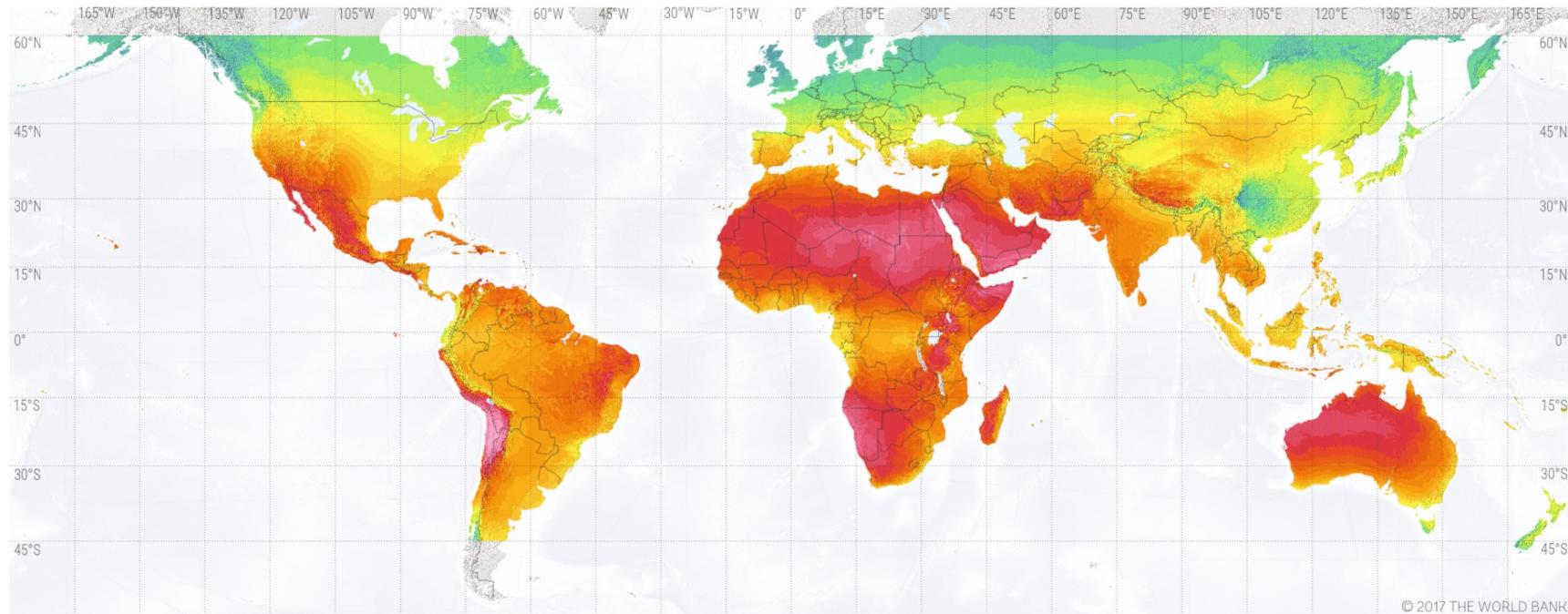
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## SOLAR RESOURCE MAP GLOBAL HORIZONTAL IRRADIATION

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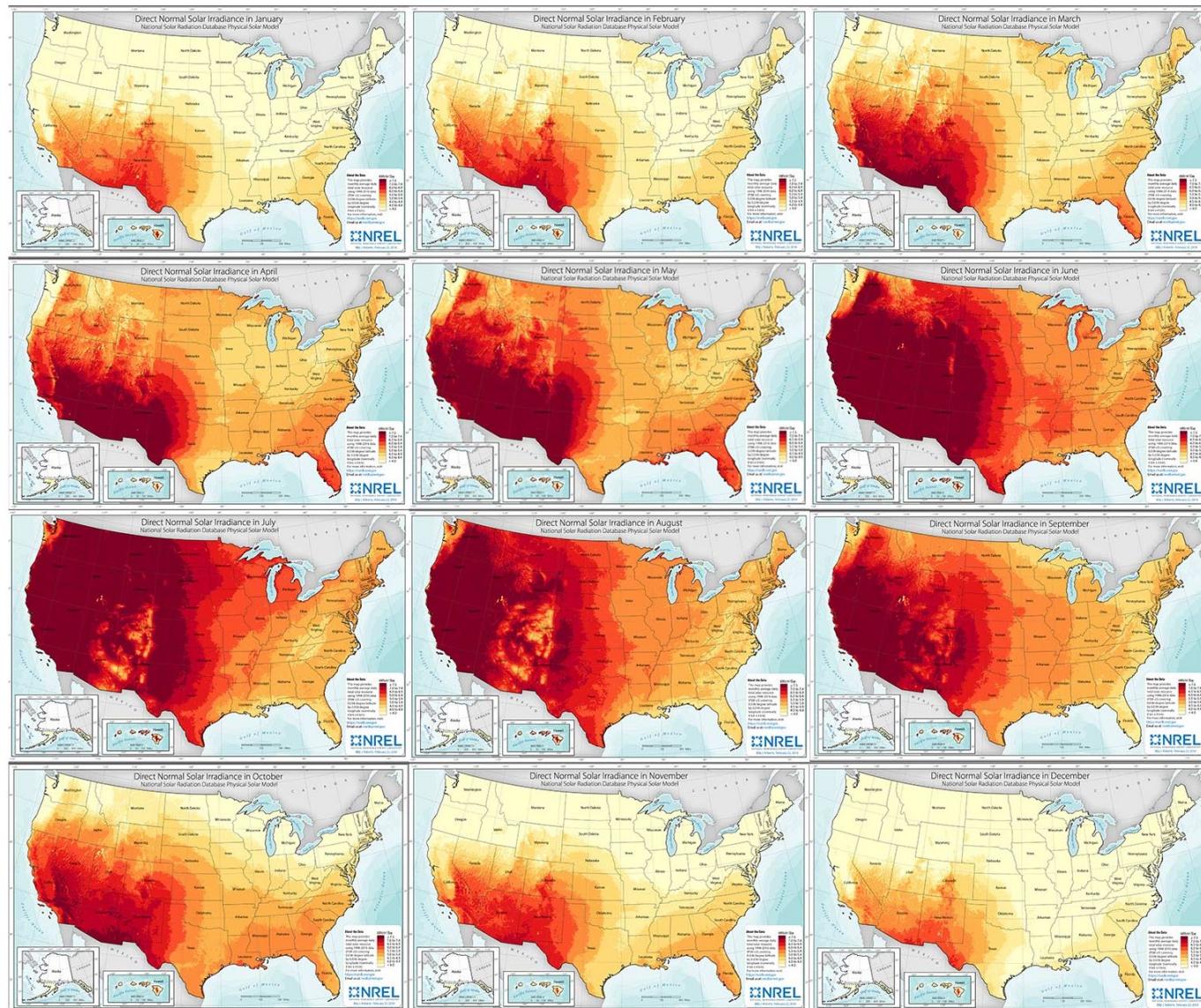
ESMAP  
Energy Sector Management Assistance Program

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# U.S. Monthly Averages of DNI



# U.S. Monthly Averages of GHI

