Lecture 11 – Wind Energy I

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ECE180J

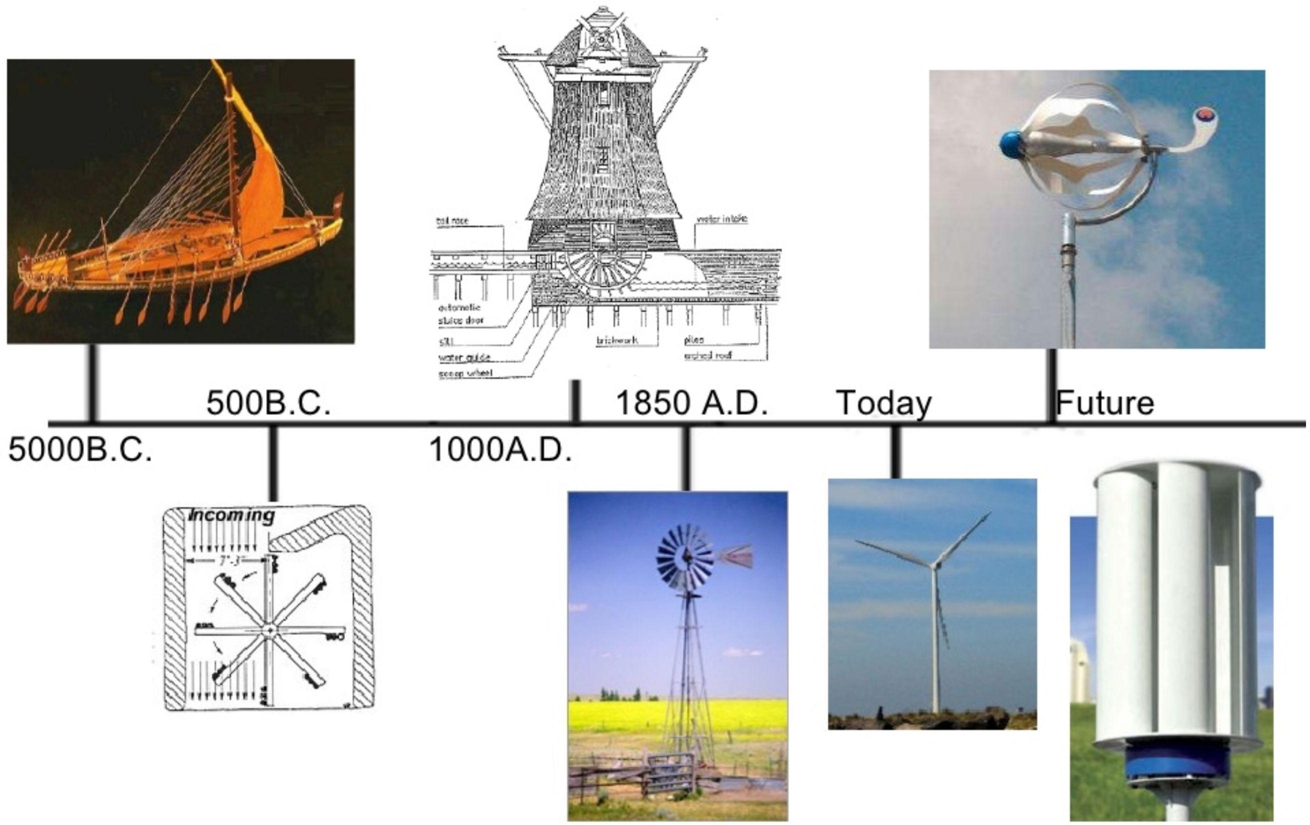
Outline

- History and Development of Wind Energy
- The Origins of Wind
- Air Density: Impact of Temperature and Height
- The Betz Limit

- B3 Chap 8
- B1 Chap 7

History of Wind Power

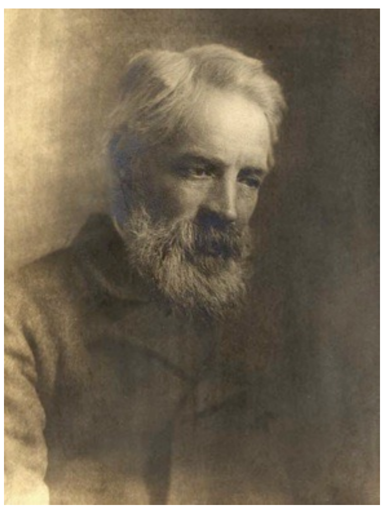
- Wind energy has been used for thousands of years for milling grain, pumping water and other mechanical power applications.



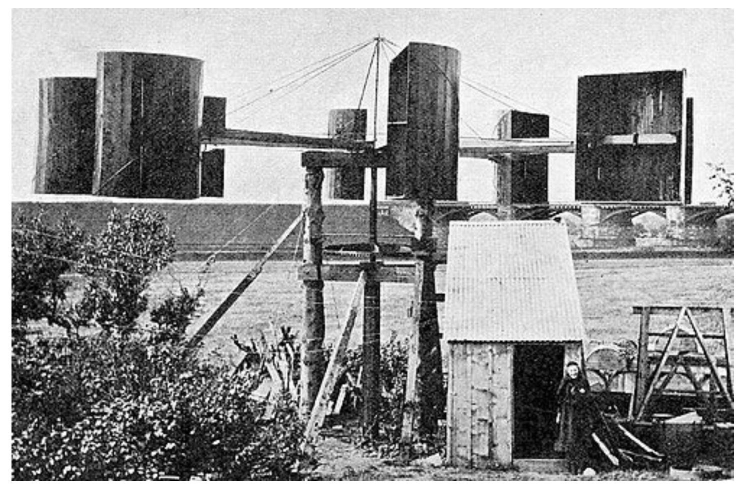
1. Wind energy propelled boats along the Nile River as early as 5000 B.C.
2. Simple windmills in China were pumping water,
3. Vertical-axis windmills were grinding grain in the Middle East.

Pioneers: James Blyth in Scotland

- In 1887, James Blyth built a cloth-sailed wind turbine in his holiday cottage, and used the produced electricity to charge accumulators.
- Stored electricity further powered the lights, which made the cottage become the first house in the world to be powered by wind-generated electricity.



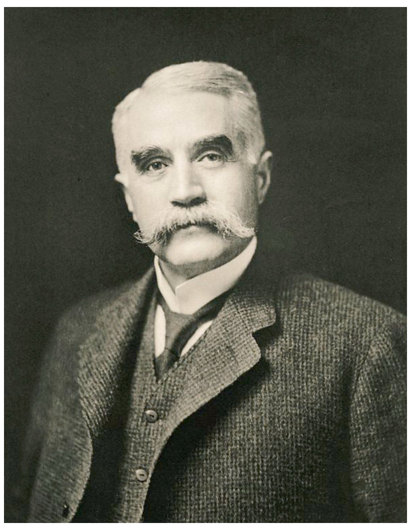
James Blyth (1839-1906): Scottish electrical engineer and a professor



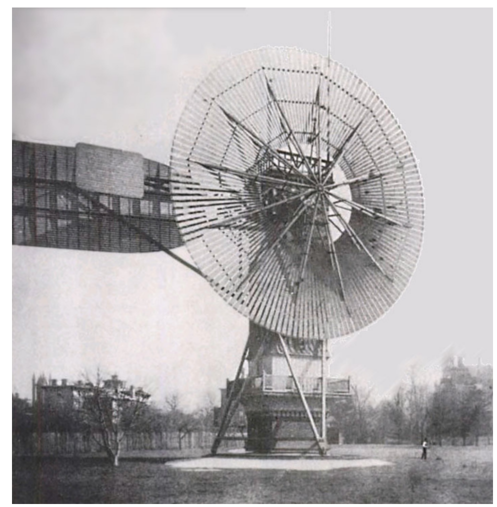
Blyth's windmill at his cottage in Marykirk: a vertical axle and cup-like structures.

Pioneers: Charles Brush in the US

- In 1888, Charles Brush powered his mansion with the world's first automatically operated wind turbine generator.
- It charged the home's 12 batteries. It was the first home in Cleveland to have electricity.
- Over its 20-year life, the turbine never failed to keep the home continuously powered.



Charles Brush (1849 –1929), American engineer, inventor, entrepreneur, & philanthropist.



The world‘s first automatically operated wind turbine was built in 1888. It had a 12kW dynamo.

Pioneers: Poul la Cour in Denmark

- Poul la Cour was a pioneer in the field of aerodynamics and is known for his development of windmills for electricity generation.
- Developed the theory for the ideal windmill blade profile.
- Discovered that the airflows behind/in front of the blade are both important.
- Studied the potential of storing wind energy in hydrogen & oxygen by electrolysis.

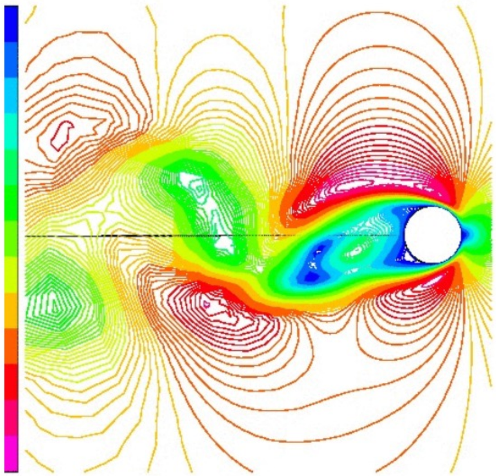
Poul la Cour (1846 – 1908): Danish scientist and inventor

The Test Turbines in Askov - 1897

Future Wind Turbines

Fig: Vortex Bladeless Wind Generator

- Consist of a cylinder fixed vertically with an elastic rod.
- The cylinder oscillates on a wind range, which then generates electricity through an alternator system.



Retrieved from: Duke University

- Fluid mechanics: as the wind passes through a blunt body, the flow is modified and generates a cyclical pattern of vortices.
- Once the frequency of these forces is close enough to body’s structural frequency, the body starts to oscillate and enters into resonance with the wind (a.k.a. Vortex Induced Vibration).

Development of Wind Power

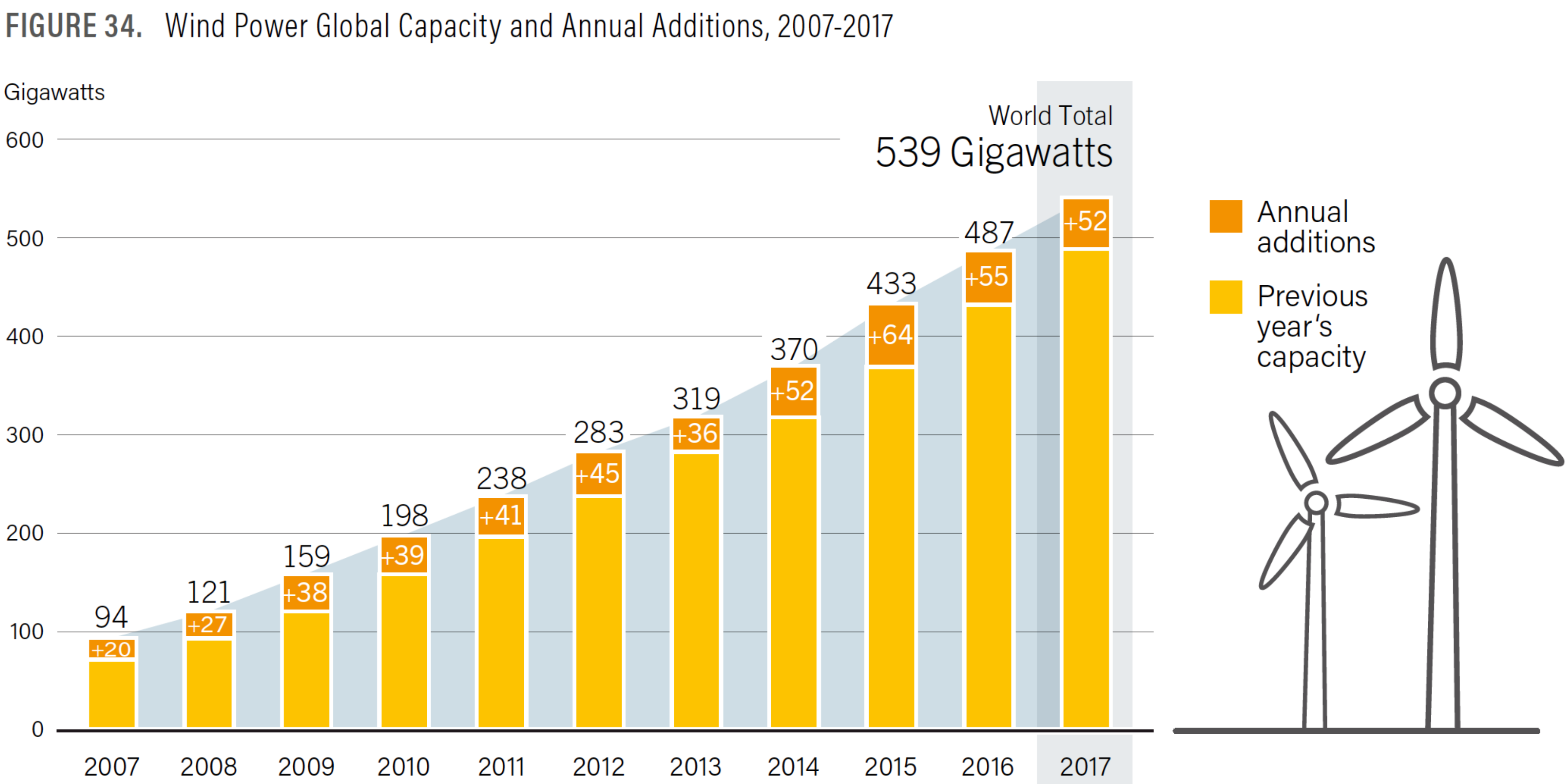
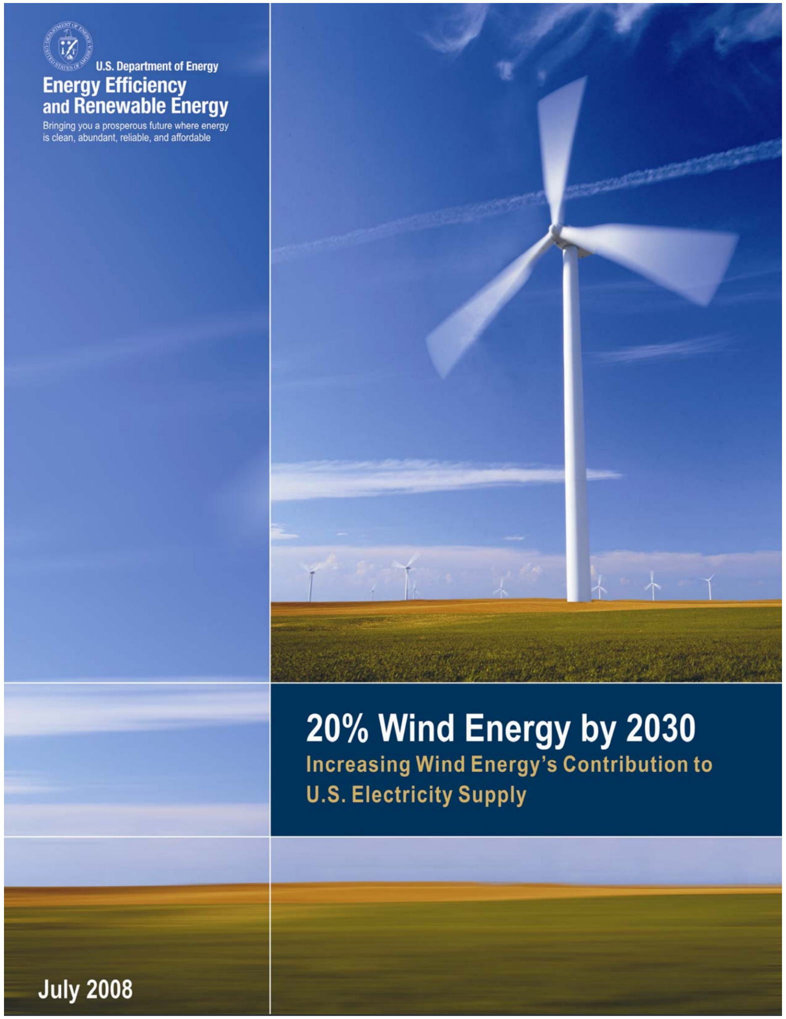


Figure Source: Ren21 Renewables 2017 Global Status Report

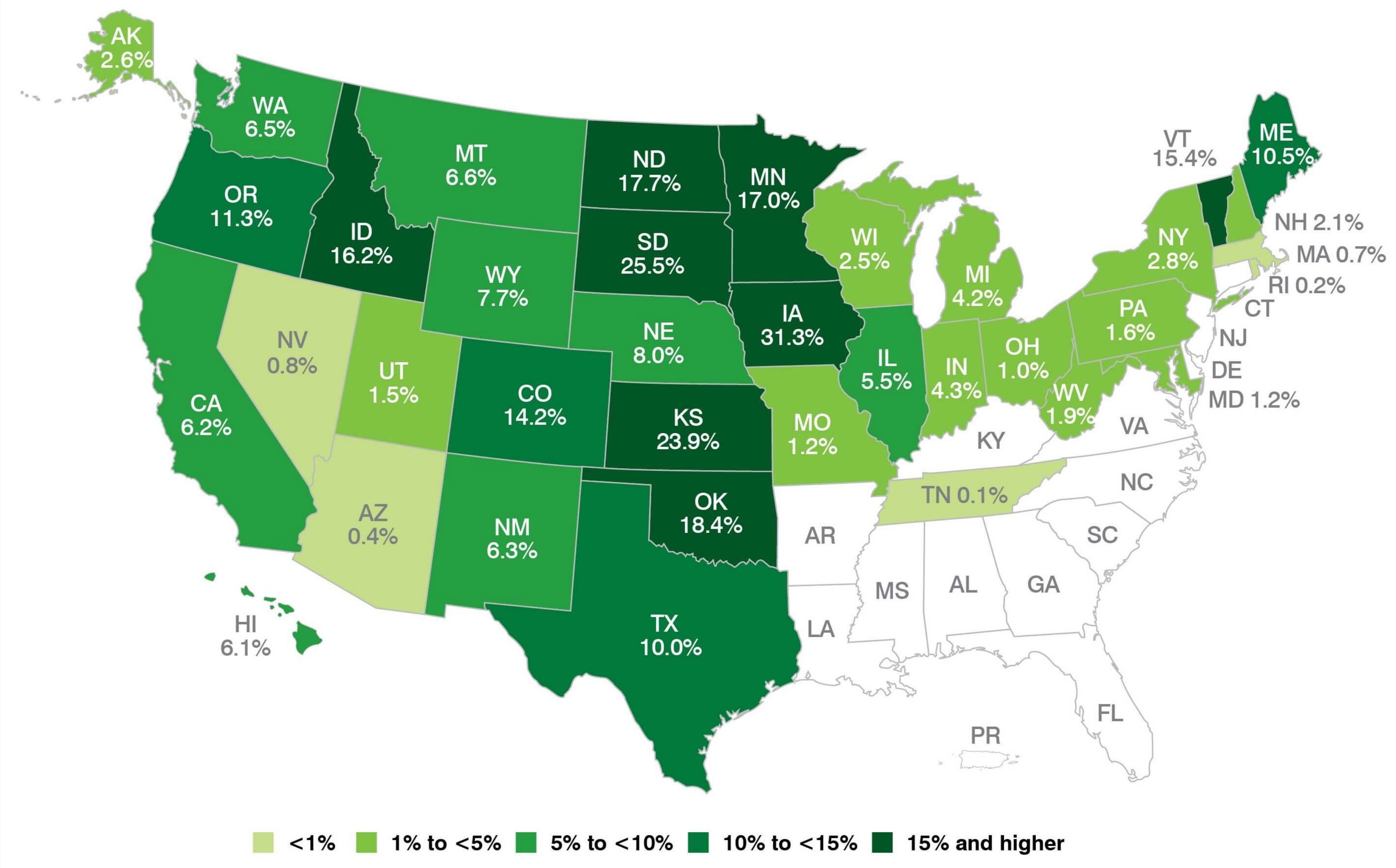
Goal

U.S. DoE: Wind energy supplying 10% of the country's electricity in 2020, 20% in 2030, and 35% in 2050.

1) require enhanced transmission infrastructure, streamlined siting and permitting regimes.
2) require the number of turbine installations to increase from approximately 7000/year in 2017.
3) done reliably for less than 0.5 cents per kWh.
4) not limited by the availability of raw materials.



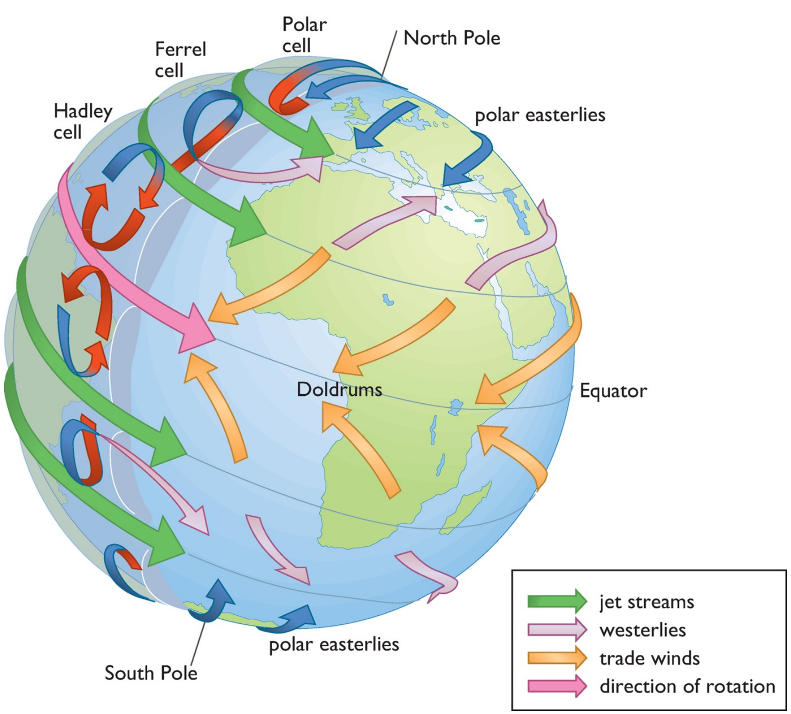
Wind Energy Share of Electricity Generation in 201

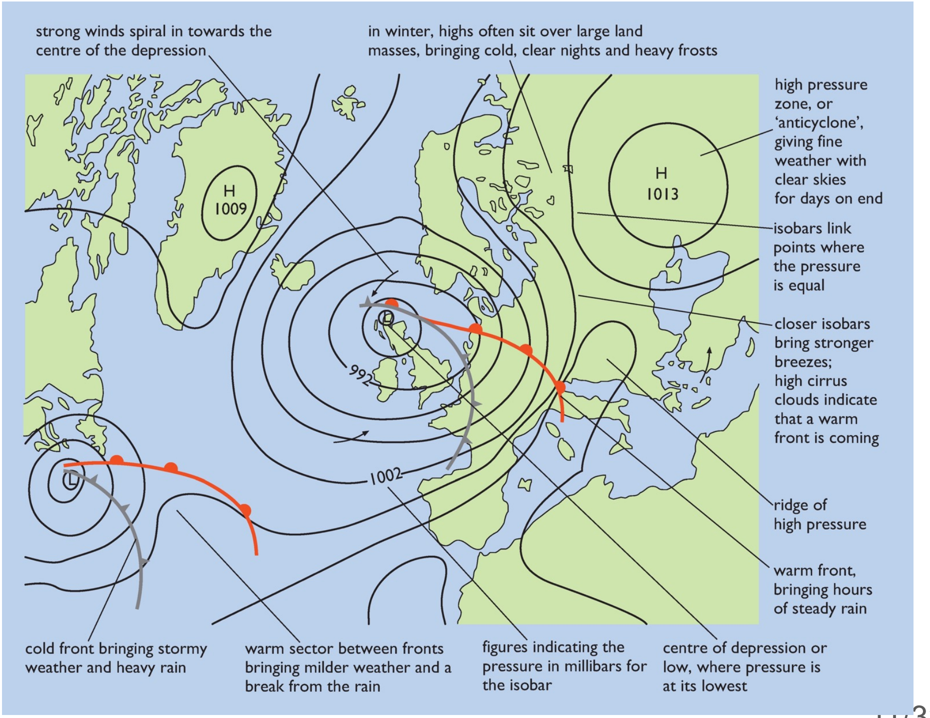


The Origins of Wind

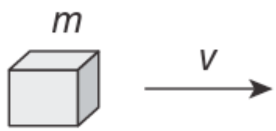
The tropics are warmer than higher latitude regions. Differential solar heating of the Earth's surface causes variations in atmospheric pressure → movements of atmospheric air masses → wind systems.

Weight of the column of air above a specified surface area yields atmospheric pressure (unit: 1 bar = 100,000 Pa).
The average atmospheric pressure at sea level is about 1 bar.





Energy/Power in Wind

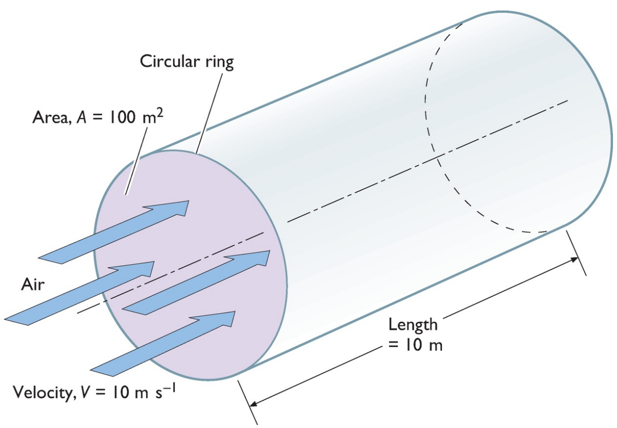


$$KE = \frac{1}{2}mv^2$$

moving air

kinetic energy = half mass x velocity squared = 0.5mV²

kilograms; metres per second (ms⁻¹)



Mass of moving air per second (mass flow rate)
= air density x area x length of cylinder of air passing per second
= air density x area x velocity

$$mass\ flow\ rate\ (mass/time) = \rho AV$$

$$\Longrightarrow \quad P = 0.5\rho AV^3 \qquad Watt = \frac{kg}{m^3} \times m^2 \times \left(\frac{m}{s}\right)^3$$

ρ is the air density (kg/m³); at 15 °C and 1 atm, ρ = 1.225 kg/m³.

1 atm = 1.01325 bar = 101.325 kPa

Cube Law

$$\frac{P}{A} = 0.5\rho V^3$$

Windspeed (m/s)	Windspeed (mph)	Power (W/m^2)
0	0	0
1	2.24	1
2	4.47	5
3	6.71	17
4	8.95	39
5	11.19	77
6	13.42	132
7	15.66	210
8	17.90	314
9	20.13	447
10	22.37	613
11	24.61	815
12	26.84	1058
13	29.08	1346
14	31.32	1681
15	33.56	2067

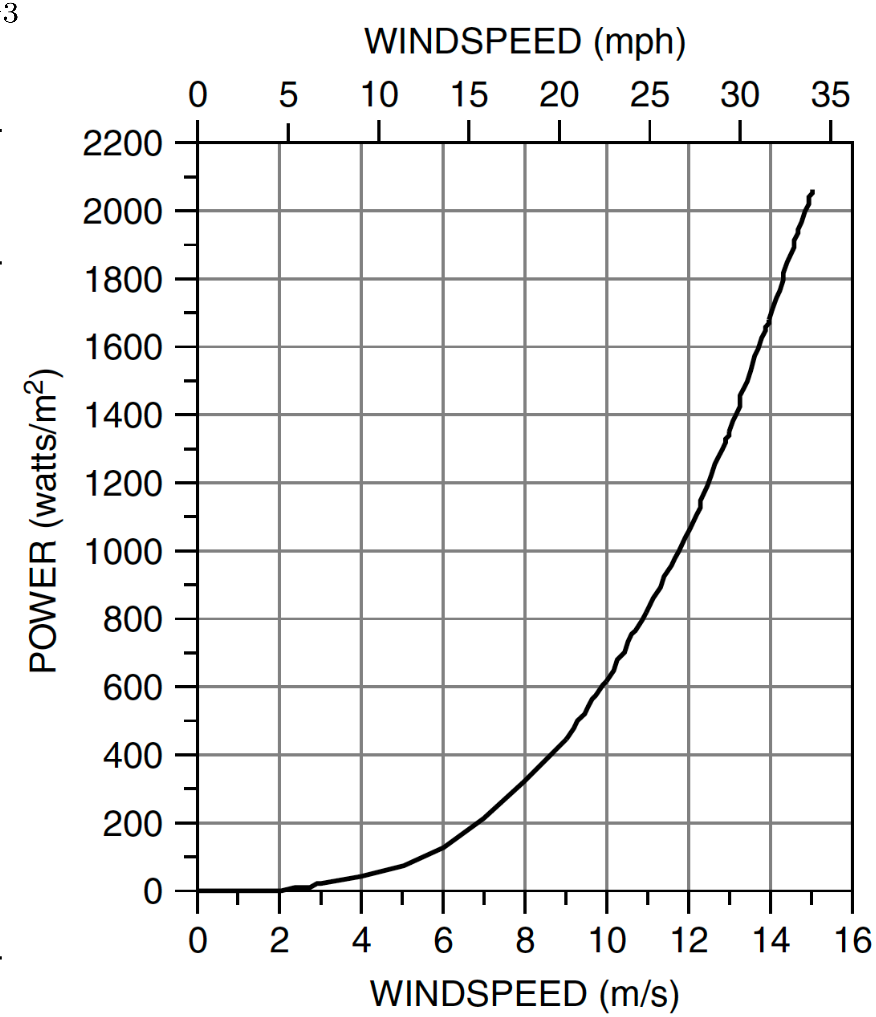


Fig: Power in the wind per square meter of cross section (15 $^\circ$C and 1 atm).

Example

Example 7.1 Don't Just Use Average Wind Speed. Compare the amount of wind energy at 15°C and 1-atm pressure that passes through 1 m^2 of cross-sectional area for the following wind regimes:

a. 100 h of 6 m/s winds (13.4 mph)

b. 50 h at 3 m/s plus 50 h at 9 m/s (i.e., an average wind speed of 6 m/s)

Solution

a. With steady 6 m/s winds, all we have to do is multiply power, given in Equation 7.7, by hours:

$$\begin{aligned}\text{Energy}\,(6\,\text{m/s}) &= \frac{1}{2}\rho A v^3 \Delta t \\ &= \frac{1}{2}\cdot 1.225\,\text{kg/m}^3 \cdot 1\,\text{m}^2 \cdot (6\,\text{m/s})^3 \cdot 100\,\text{h} = 13{,}230\,\text{Wh}\end{aligned}$$

b. With 50 h at 3 m/s

$$\text{Energy}\,(3\,\text{m/s}) = \frac{1}{2}\cdot 1.225\,\text{kg/m}^3 \cdot 1\,\text{m}^2 \cdot (3\,\text{m/s})^3 \cdot 50\,\text{h} = 827\,\text{Wh}$$

And 50 h at 9 m/s contains

$$\text{Energy}\,(9\,\text{m/s}) = \frac{1}{2}\cdot 1.225\,\text{kg/m}^3 \cdot 1\,\text{m}^2 \cdot (9\,\text{m/s})^3 \cdot 50\,\text{h} = 22{,}326\,\text{Wh}$$

for a total of $827 + 22{,}326 = 23{,}153$ Wh

Temperature Correction for Air Density

The ideal gas law: $PV = nRT$

n is the amount of substance of gas (aka. number of moles).

where P is the absolute pressure (atm), V is the volume (m^3), n is the mass (mol), R is the ideal gas constant $= 8.2056 \times 10^{-5}\ m^3 \cdot atm \cdot K^{-1} \cdot mol^{-1}$, and T is the absolute temperature (K), where $K = {}^{\circ}C + 273.15$.

If we let M.W. stand for the molecular weight of the gas (g/mol), we can write the following expression for air density, ρ:

$$\rho\ (\mathrm{kg/m^3}) = \frac{n(\mathrm{mol}) \cdot \mathrm{M.W.\ (g/mol)} \cdot 10^{-3}\ (\mathrm{kg/g})}{V\ (\mathrm{m^3})}$$

$$n = \frac{PV}{RT}$$

$$\Longrightarrow \quad \rho = \frac{P \times \mathrm{M.W.} \times 10^{-3}}{RT} = \frac{p(\mathrm{atm})}{T(\mathrm{K})} \times \frac{28.97\ \mathrm{g/mol} \times 10^{-3}\ \mathrm{kg/g}}{8.2056 \times 10^{-5}\ \mathrm{m^3 \cdot atm/(K \cdot mol)}}$$

$$\Longrightarrow \quad \boxed{\rho(\mathrm{kg/m^3}) = \frac{353.1\ p(\mathrm{atm})}{T(\mathrm{K})}}$$

Temperature Correction for Air Density (cont'd)

Since we are working with air, we can easily figure out its equivalent molecular weight by looking at its constituent molecules, which are mostly nitrogen (78.08%), oxygen (20.95%), a little bit of argon (0.93%), carbon dioxide (0.039%), and so forth. Using the constituent molecular weights ($N_2 = 28.02$, $O_2 = 32.00$, $Ar = 39.95$, $CO_2 = 44.01$) we find the equivalent molecular weight of air to be

$$\begin{aligned}\text{M.W.(air)} &= 0.781 \times 28.02 + 0.2095 \times 32.00 \\ &\quad + 0.0093 \times 39.95 + 0.00039 \times 44.01 = 28.97\end{aligned}$$

Q: Find the density of air at 1 atm and 30 ^{o}C

Solution.

$$\rho = \frac{1 \text{ atm} \times 28.97 \text{ g/mol} \times 10^{-3} \text{ kg/g}}{8.2056 \times 10^{-5} \text{m}^3 \cdot \text{atm}/(\text{K} \cdot \text{mol}) \times (273.15 + 30) \text{ K}} = 1.165 \text{ kg/m}^3$$

Altitude Correction for Air Density

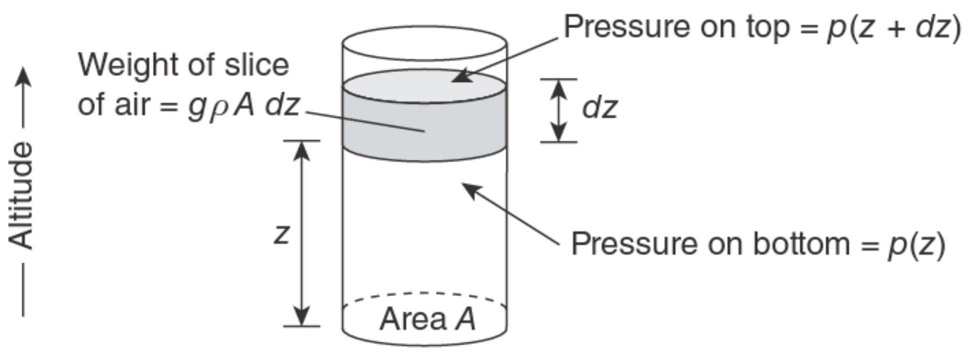


Fig: A column of air in static equilibrium used to determine the relationship between air pressure and altitude.

$$p(z) = p(z + \mathrm{d}z) + \frac{g\rho A \mathrm{d}z}{A}$$

Express the incremental pressure $\mathrm{d}p$ for an incremental change in elevation $\mathrm{d}z$

$$\mathrm{d}p = p(z + \mathrm{d}z) - p(z) = -g\rho \mathrm{d}z$$

$$\text{That is, } \frac{\mathrm{d}p}{\mathrm{d}z} = -\rho g$$

Altitude Correction for Air Density (cont'd)

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho g = -\frac{353.1}{\mathrm{T}}\left(\frac{\mathrm{kg}}{\mathrm{m}^3}\right) \times \left(9.806\frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{\mathrm{atm}}{101{,}325\,\mathrm{Pa}} \times \frac{1\,\mathrm{Pa}}{\mathrm{N/m}^2} \times \frac{1\mathrm{N}}{\mathrm{kg}\cdot\mathrm{m/s}^2}\right)\cdot p$$

$$= -\left(\frac{0.0342}{T}\right)p$$

$$\Rightarrow \qquad p = p_0 \exp\left(-0.0342z/T\right)$$

where p_0 is the reference pressure of 1 atm.

Finally, we get a useful density correction for both temperature and altitude:

$$\boxed{\rho(\mathrm{kg/m^3}) = \frac{353.1 \exp\left(-0.0342\, z/T\right)}{T}}$$

where T is in kelvins (K) and z is in meters.

Quiz

$$\boxed{\rho(\text{kg/m}^3) = \frac{353.1 \exp\left(-0.0342\, z/T\right)}{T}}$$

Example 7.2 Combined Temperature and Altitude Correction. Find the power density (W/m^2) in 10 m/s winds at an elevation of 2000 m (6562 ft) and a temperature of 25°C (298.15 K). Compare that to the power density under standard 1-atm and 15°C conditions.

Solution. From Equation 7.17

$$\rho = \frac{353.1 \exp\left(-0.0342 \times 2000/298.15\right)}{298.15} = 0.9415\,\text{kg/m}^3$$

$$P/A = \frac{1}{2}\rho v^3 = 0.5 \times 0.9415 \times 10^3 = 470.8\,\text{W/m}^2$$

Under standard conditions

$$P/A = \frac{1}{2}\rho v^3 = 0.5 \times 1.225 \times 10^3 = 612.5\,\text{W/m}^2$$

That is a 23% decrease in power density at the higher elevation and temperature.

Impact of Tower Height

- In the first few hundred meters above the ground, wind speed is greatly affected by the friction that the air experiences as it moves across the earth's surface.
- Smooth surfaces, such as a calm sea, offer very little resistance, and the variation of speed with elevation is modest.
- At the other extreme, surface winds are slowed considerably by high irregularities such as forests and buildings.

$$\boxed{\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}}$$

where v is the windspeed at height H, v_0 is the windspeed at height H_0 (often a reference height of 10 m), and α is the friction coefficient.

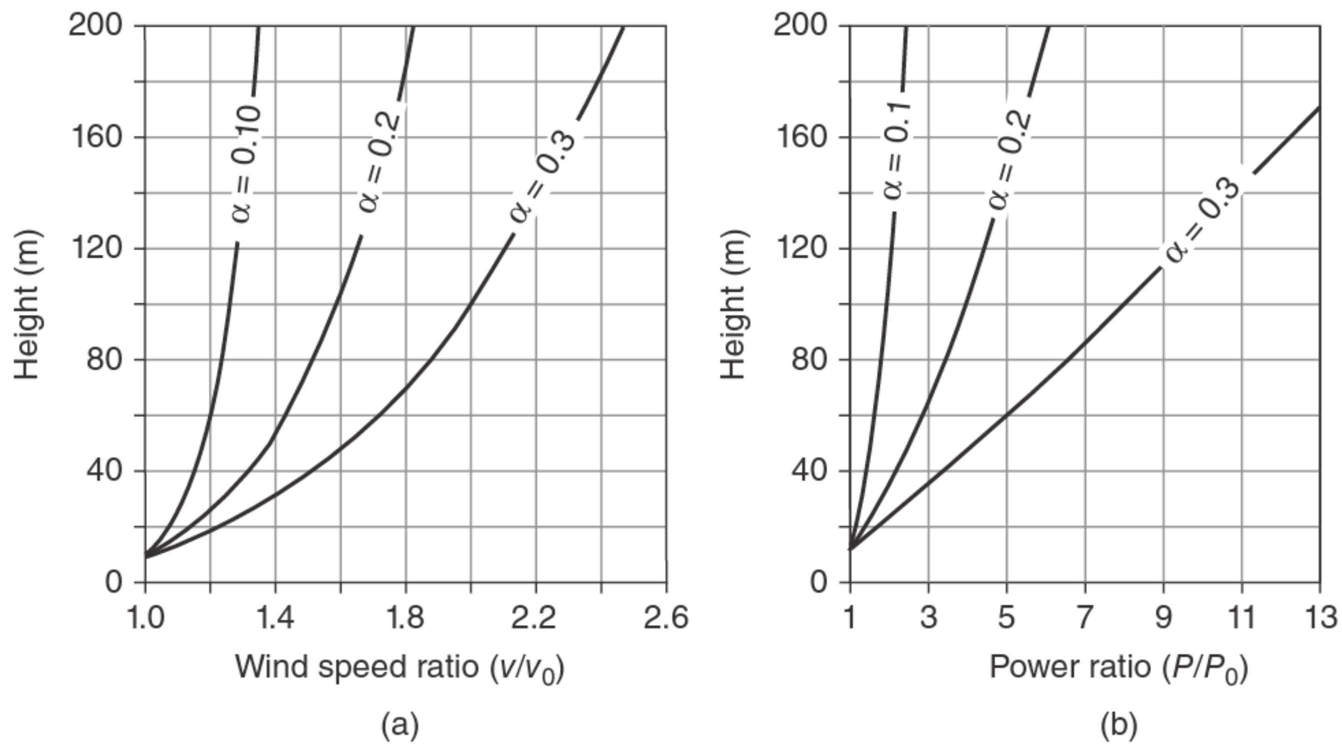
Impact of Tower Height (cont'd)

α is a function of the terrain over which the wind blows.

Terrain Characteristics	Friction Coefficient α
Smooth hard ground, calm water	0.10
Tall grass on level ground	0.15
High crops, hedges and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40

Oftentimes, for rough approximations in somewhat open terrain a value of 1/7 (the "one-seventh" rule-of-thumb) is used for α .

Impact of Tower Height (cont'd)



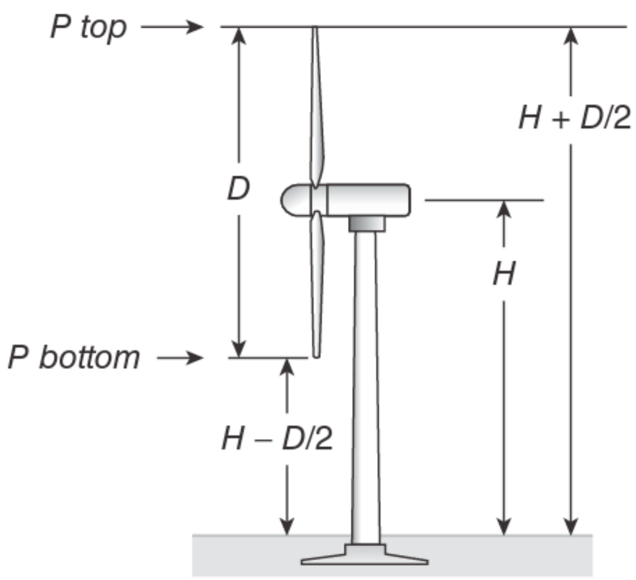
Increasing (a) windspeed and (b) power ratios with height for various friction coefficients α using a reference height of 10 m

$$\left(\frac{P}{P_0}\right) = \left(\frac{1/2\rho A v^3}{1/2\rho A v_0^3}\right) = \left(\frac{v}{v_0}\right)^3 = \left(\frac{H}{H_0}\right)^{3\alpha}$$

Quiz

Example 7.3 Impact of Tower Height on Rotor Stress. A wind turbine with a 50-m rotor diameter is to be mounted on either a 50-m tower or an 80-m tower. Assume the usual 1/7th rule of thumb for the shear-friction coefficient.

a. Compare the wind power density at each hub height.
b. For each height, compare the ratio of the power density at the highest point that the tip of a rotor blade reaches to the lowest point to which it falls.



a. Using Equation 7.20 with 50-m and 100-m hub heights:

$$\frac{P}{P_0} = \left(\frac{H}{H_0}\right)^{3\alpha} = \left(\frac{80}{50}\right)^{3\times 1/7} = 1.22$$

so there is 22% more power available at the taller hub height.

b. For the 50-m blade at a 50-m hub height, the tip reaches 75 m at its highest point and 25 m at its lowest:

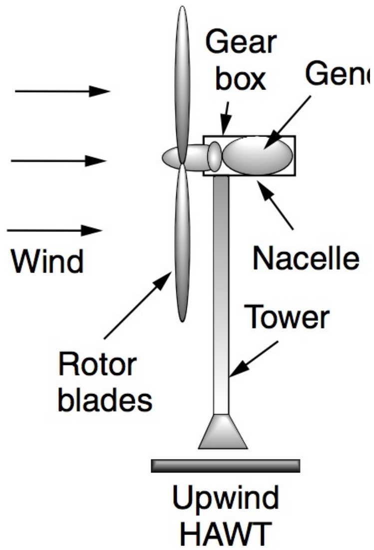
$$\frac{P}{P_0} = \left(\frac{75}{25}\right)^{3\times 1/7} = 1.60$$

The power in the wind at the top of the rotor swing is 60% higher than it is when the tip reaches its lowest point.

At the 80-m hub height, the power ratio from the highest to the lowest point will be

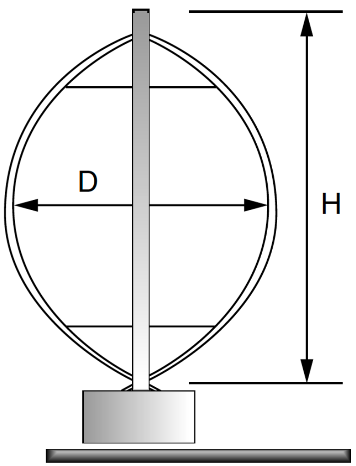
$$\frac{P}{P_0} = \left(\frac{105}{55}\right)^{3\times 1/7} = 1.32$$

Effect of Swept Area



$$A = (\pi/4)D^2$$

- Doubling the diameter increases the power available by a factor of four.
- Turbine cost increases roughly in proportion to blade diameter, but power is proportional to diameter squared.
- Bigger machines have proven to be more cost effective.

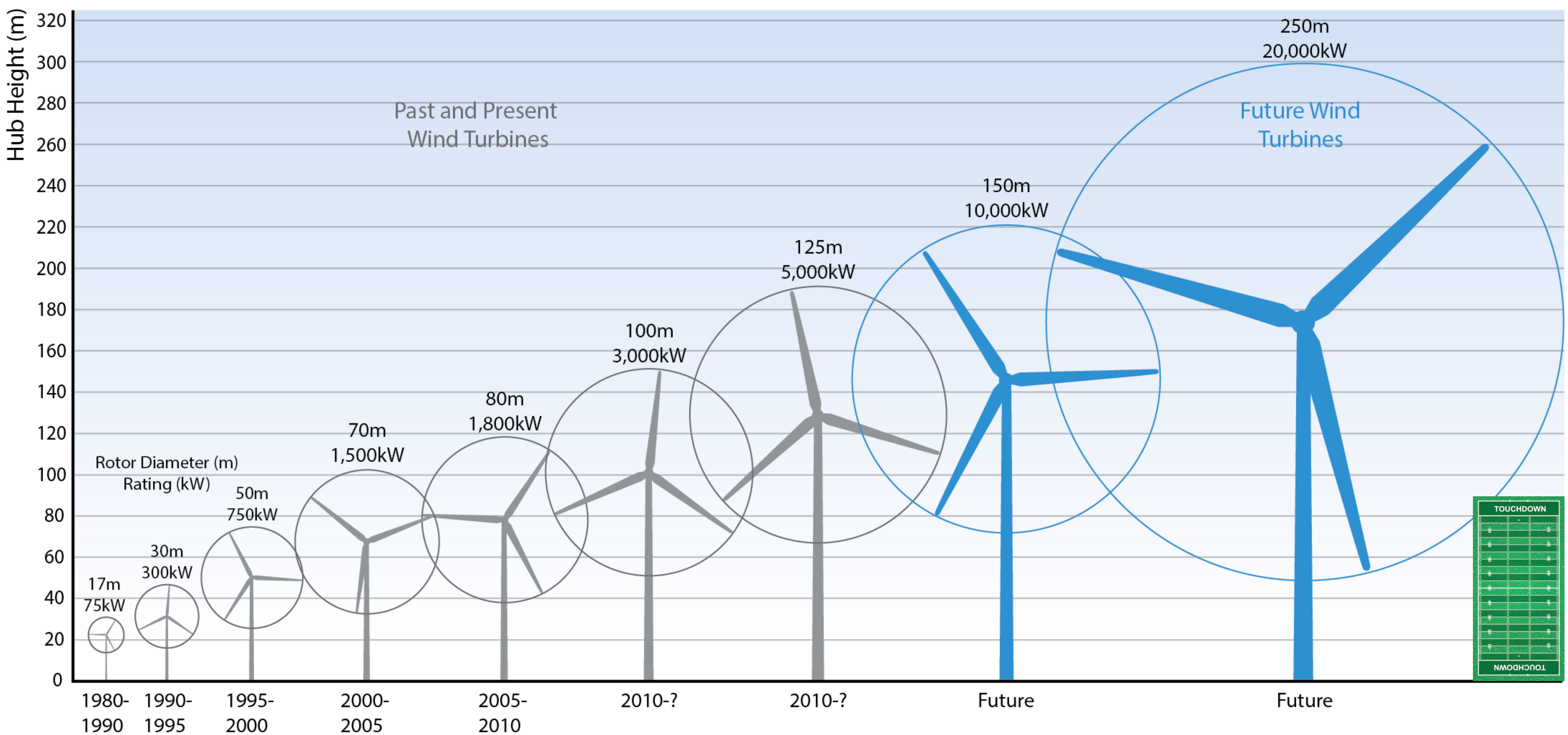


Vertical-axis Darrieus rotor

$$A \cong \frac{2}{3} D \cdot H$$

Development of Wind Power (Cont'd)

[Wind Farm Growth Through the Years]



Wind Power Conversion Efficiency

Power in the wind passing through a wind turbine rotor is proportional to:

- density of the air
- area of the rotor
- cube of the wind velocity

The power contained in the wind is not in practice the amount of power that can be extracted by a wind turbine because losses are incurred in the energy extraction/conversion process.

Maximum Rotor Efficiency

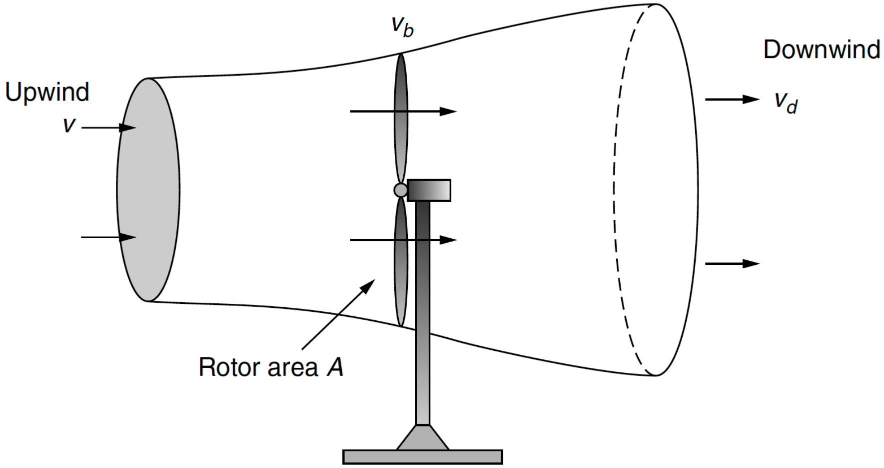


Fig: Approaching wind slows and expands as a portion of its kinetic energy is extracted by the wind turbine. The envelope forms a "*stream tube*".

The power extracted by the blades P_b is equal to the difference in kinetic energy between the upwind and downwind air flows

$$P_b = \frac{1}{2}\dot{m}(v^2 - v_d^2)$$

Mass flow rate is the mass of a substance which passes per unit of time (kg/s):

$$\dot{m} = \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t} = \frac{\mathrm{d}m}{\mathrm{d}t}$$

The mass flow rate of air within the stream tube is everywhere the same.

$$\dot{m} = \rho A v_b$$

Betz Efficiency

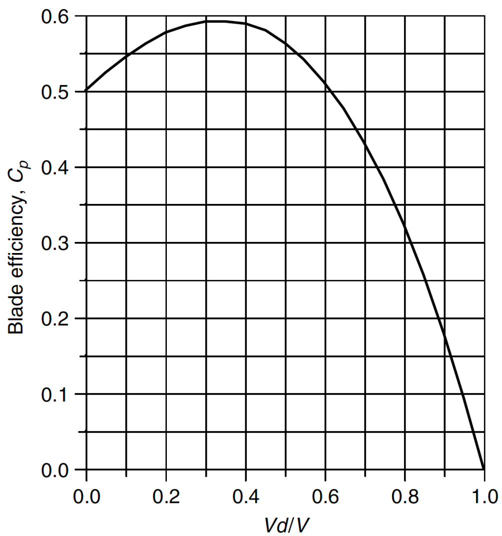
$$Assume\ v_b = \frac{1}{2}(v + v_d) \Rightarrow \qquad P_b = \frac{1}{2}\rho A\left(\frac{v+v_d}{2}\right)(v^2 - v_d^2)$$

The ratio of downstream-to-upstream windspeed $\lambda = \left(\frac{v_d}{v}\right)$

$$P_b = \frac{1}{2}\rho A\left(\frac{v+\lambda v}{2}\right)(v^2 - \lambda^2 v^2) = \underbrace{\frac{1}{2}\rho A v^3}_{\text{Power in the wind}} \cdot \underbrace{\left[\frac{1}{2}(1+\lambda)(1-\lambda^2)\right]}_{\text{Fraction extracted}}$$

$$\text{Rotor efficiency} = C_P = \tfrac{1}{2}(1+\lambda)(1-\lambda^2)$$

Established By German physicist Albert Betz'1919



$$\frac{dC_p}{d\lambda} = \frac{1}{2}[(1+\lambda)(-2\lambda) + (1-\lambda^2)] = 0$$

which has solution

$$\lambda = \frac{v_d}{v} = \frac{1}{3}$$

$$\boxed{\text{Maximum rotor efficiency} = \frac{1}{2}\left(1+\frac{1}{3}\right)\left(1-\frac{1}{3^2}\right) = 59.3\%}$$

Tip-Speed-Ratio

- If the rotor turns too slowly, the efficiency drops off since the blades are letting too much wind pass by unaffected.
- If the rotor turns too fast, efficiency is reduced as the turbulence caused by one blade increasingly affects the blade that follows.

tangential speed

$$\boxed{\text{Tip-Speed-Ratio (TSR)} = \frac{\text{Rotor tip speed}}{\text{Wind speed}} = \frac{\text{rpm} \times \pi D}{60\ v}}$$

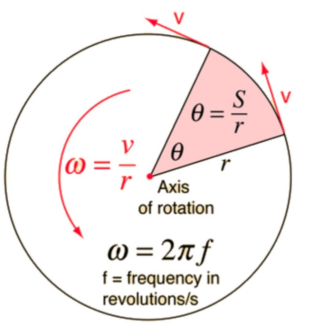
where rpm is the rotor speed, revolutions per minute; D is the rotor diameter (m); and v is the wind speed (m/s) upwind of the turbine.

- Revolutions per minute (rpm) is the number of turns in one minute. It is a unit of rotational speed or the frequency of rotation around a fixed axis.

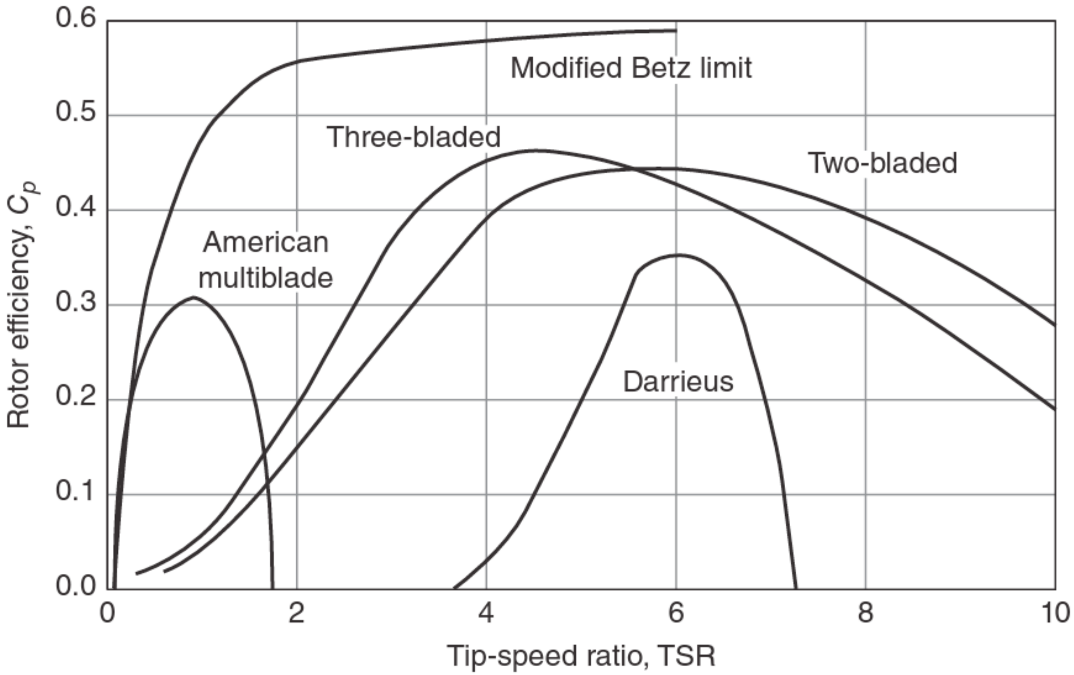
$$1\ \text{rad/s} \quad = \quad \frac{1}{2\pi}\ \text{Hz} \quad = \quad \frac{60}{2\pi}\ \text{rpm}$$

angular speed

rpm: rotational speed



Rotor Types vs TSR



Rotors with fewer blades reach their optimum efficiency at higher rotational speeds.

- American multiblade spins slowly, maximum efficiency about 30%.
- The two- and three-blade rotors spin much faster, maximum efficiencies of roughly 40–50%.
- The “ideal efficiency” line reflects the fact that a slowly turning rotor does not intercept all of the wind, which reduces the maximum possible efficiency to something below the Betz limit.

Example

Example 6.7 How Fast Does a Big Wind Turbine Turn? A 40-m, three-bladed wind turbine produces 600 kW at a windspeed of 14 m/s. Air density is the standard 1.225 kg/m^3. Under these conditions,

a. At what rpm does the rotor turn when it operates with a TSR of 4.0?
b. What is the tip speed of the rotor?
c. If the generator needs to turn at 1800 rpm, what gear ratio is needed to match the rotor speed to the generator speed?
d. What is the efficiency of the complete wind turbine (blades, gear box, generator) under these conditions?

Solution

a. Using (6.27),

$$\text{rpm} = \frac{\text{TSR} \times 60\ v}{\pi D} = \frac{4 \times 60\ \text{s/min} \times 14\ \text{m/s}}{40\pi\ \text{m/rev}} = 26.7\ \text{rev/min}$$

That's about 2.2 seconds per revolution ... pretty slow!

b. The tip of each blade is moving at

$$\text{Tip speed} = \frac{26.7\ \text{rev/min} \times \pi 40\ \text{m/rev}}{60\ \text{s/ min}} = 55.9\ \text{m/s}$$

Notice that even though 2.2 s/rev sounds slow; the tip of the blade is moving at a rapid 55.9 m/s, or 125 mph.

Example (cont'd)

c. If the generator needs to spin at 1800 rpm, then the gear box in the nacelle must increase the rotor shaft speed by a factor of

$$\text{Gear ratio} = \frac{\text{Generator rpm}}{\text{Rotor rpm}} = \frac{1800}{26.7} = 67.4$$

d. From (6.4) the power in the wind is

$$P_w = \frac{1}{2}\rho A {v_w}^3 = \frac{1}{2} \times 1.225 \times \frac{\pi}{4} \times 40^2 \times 14^3 = 2112 \text{ kW}$$

so the overall efficiency of the wind turbine, from wind to electricity, is

$$\text{Overall efficiency} = \frac{600 \text{ kW}}{2112 \text{ kW}} = 0.284 = 28.4\%$$