Lecture 12 – Wind Energy II

Dr. Yu Zhang

ECE, UCSC

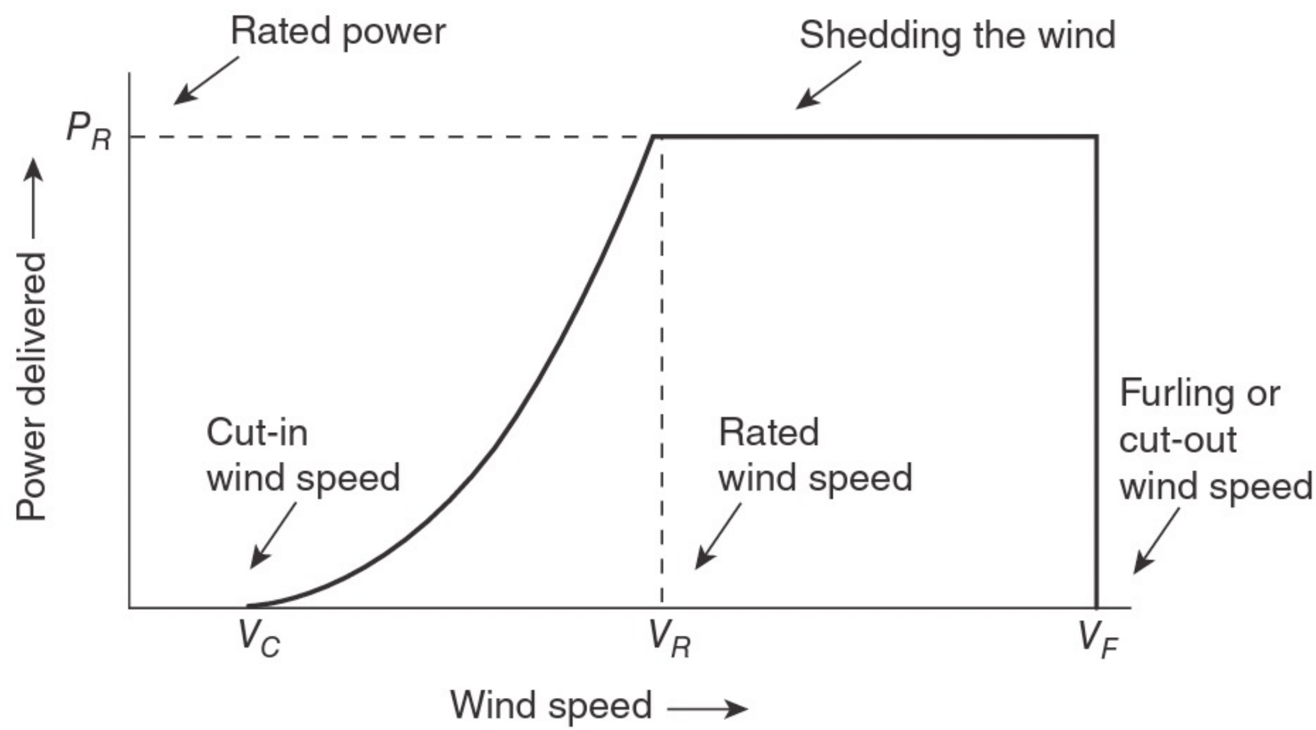
ECE180J

Outline

- Speed-to-Power Mapping
- Wind Histogram and PDF
- Weibull and Rayleigh Distributions
- Discretizing the PDF

- B3 Chap 8
- B1 Chap 7

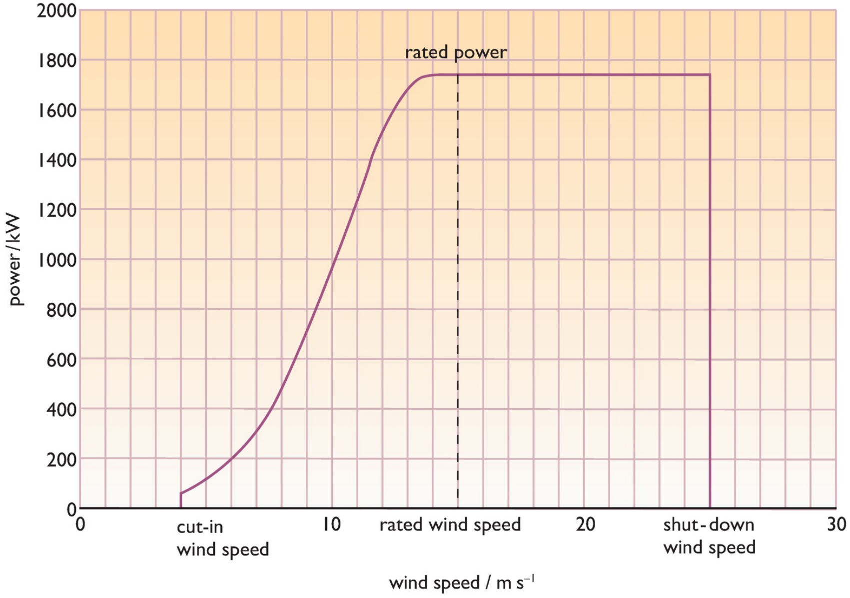
Idealized Wind Turbine Power Curve



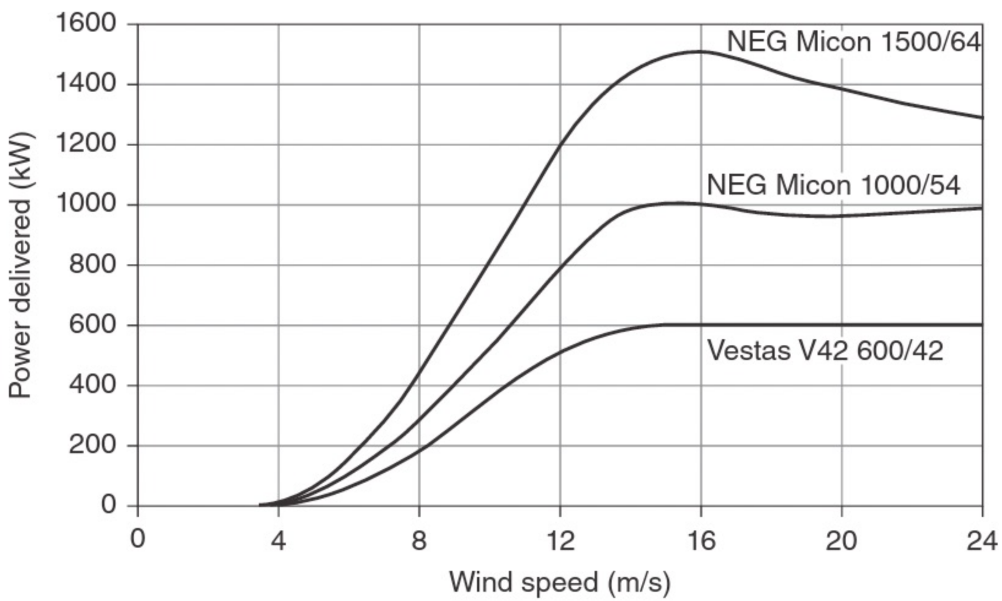
- V_C: the minimum needed to generate net power.
- V_R: the generator delivers as much as power as the it's designed for.
- V_F: machine is shut down for too strong wind (mechanical brakes lock the rotor shaft in place).

Real Examples

- Cut-in, rated, and shut-down wind speeds depend on turbine models. Typically V_C (4-8m/s), V_R (12-16m/s), V_F (24-28m/s).



Rated power 1500 kW & rotor diameter 64 m



Matching Generators and Rotors

Example 7.4 Matching Generators and Rotors. An 82-m, 1.65-MW, fixed-speed wind turbine has a rated wind speed of 13 m/s. It is connected through a gearbox to a 4-pole, 60-Hz synchronous generator.

a. What gear ratio should the gearbox have if the turbine is designed to turn at 14.4 rpm?
b. What is the tip speed ratio when the winds are blowing at the rated wind speed?
c. What is the overall efficiency of the machine (including rotor, gearbox, generator, etc.) at its rated wind speed?
d. The power curve for this machine indicates that it will deliver half of its rated output in 8 m/s winds. What is its efficiency and TSR at that wind speed?
e. What would be the TSR with 8 m/s winds if the generator could switch from four poles to six?

Solution

a. From Equation 7.1 the generator shaft spins at

$$N = \frac{120f}{p} = \frac{120 \times 60}{4} = 1800\,\text{rpm}$$

So the gear ratio should be

$$\text{Gear ratio} = \frac{\text{Generator speed}}{\text{Rotor speed}} = \frac{1800\,\text{rpm}}{14.4\,\text{rpm}} = 125$$

Matching Generators and Rotors (cont'd)

b. The TSR at the 13 m/s rated wind speed:

$$\text{TSR} = \frac{82\pi \text{ m/rev} \times 14.4 \text{ rev/min}}{13 \text{ m/s} \times 60 \text{ s/min}} = 4.76$$

c. Overall efficiency at rated wind speed:
Assuming the standard 1.225 kg/m^3 air density, the power in the wind at 13 m/s is

$$P_\text{w} = \frac{1}{2}\rho A v_\text{w}^3 = \frac{1}{2} \times 1.225 \times \frac{\pi}{4} \times 82^2 \times 13^3 = 7106 \times 10^3 \text{ W}$$

So the overall efficiency of this 1.65 MW machine is

$$\text{Overall efficiency} = \frac{1650 \text{ kW}}{7106 \text{ kW}} = 23.2\%$$

d. Efficiency and TSR at 8 m/s:

$$P_\text{w} = \frac{1}{2} \times 1.225 \times \frac{\pi}{4} \times 82^2 \times 8^3 = 1656 \times 10^3 \text{ W}$$

$$\text{Overall effciency} = \frac{0.5 \times 1650 \text{ kW}}{1656 \text{ kW}} = 49.8\%$$

$$\text{TSR} = \frac{82\pi \text{ m/rev} \times 14.4 \text{ rev/min}}{8 \text{ m/s} \times 60 \text{ s/min}} = 7.7$$

Matching Generators and Rotors (cont'd)

e. Switching from four- to six-pole operation:

$$N = \frac{120f}{p} = \frac{120 \times 60}{6} = 1200\,\text{rpm}$$

With our 125 gear ratio, the turbine would now spin at

$$\text{Rotor} = 1200\,\text{rpm}/125 = 9.6\,\text{rpm}$$

And TSR would be

$$\text{TSR} = \frac{82\pi\ \text{m/rev} \times 9.6\,\text{rev/min}}{8\,\text{m/s} \times 60\,\text{s/min}} = 5.1$$

This looks closer to the optimum range of TSRs.

Average Wind Speed

$$P_{\text{avg}} = \left(\frac{1}{2}\rho A v^3\right)_{\text{avg}} = \frac{1}{2}\rho A \left(v^3\right)_{\text{avg}} \neq \frac{1}{2}\rho A (v_{avg})^3$$

$$v_{\text{avg}} = \frac{\sum_i [v_i \cdot (\text{hours at } v_i)]}{\sum \text{hours}} = \sum_i [v_i \cdot (\text{fraction of hours at } v_i)] = \sum_i \left[v_i \cdot \text{probability}\,(v = v_i)\right]$$

EX: During a 10h period, there're 3h of no wind, 3h at 5mph, and 4h at 10mph.

$$v_{\text{avg}} = \frac{\text{Miles of wind}}{\text{Total hours}} = \frac{3\,\text{h} \cdot 0\,\text{mile/h} + 3\,\text{h} \cdot 5\,\text{mile/h} + 4\,\text{h} \cdot 10\,\text{mile/h}}{3 + 3 + 4\,\text{h}}$$

$$= \left(\frac{3\,\text{h}}{10\,\text{h}}\right) \times 0\,\text{mph} + \left(\frac{3\,\text{h}}{10\,\text{h}}\right) \times 5\,\text{mph} + \left(\frac{4\,\text{h}}{10\,\text{h}}\right) \times 10\,\text{mph} = 5.5\,\text{mph}$$

$$\left(v^3\right)_{\text{avg}} = \frac{\sum_i \left[v_i^3 \cdot (\text{hours at } \mathrm{v_i})\right]}{\sum \text{hours}} = \sum_i \left[v_i^3 \cdot (\text{fraction of hours at } v_i)\right] = \sum_i \left[v_i^3 \cdot \text{probability}\,(v = v_i)\right]$$

Wind Histogram

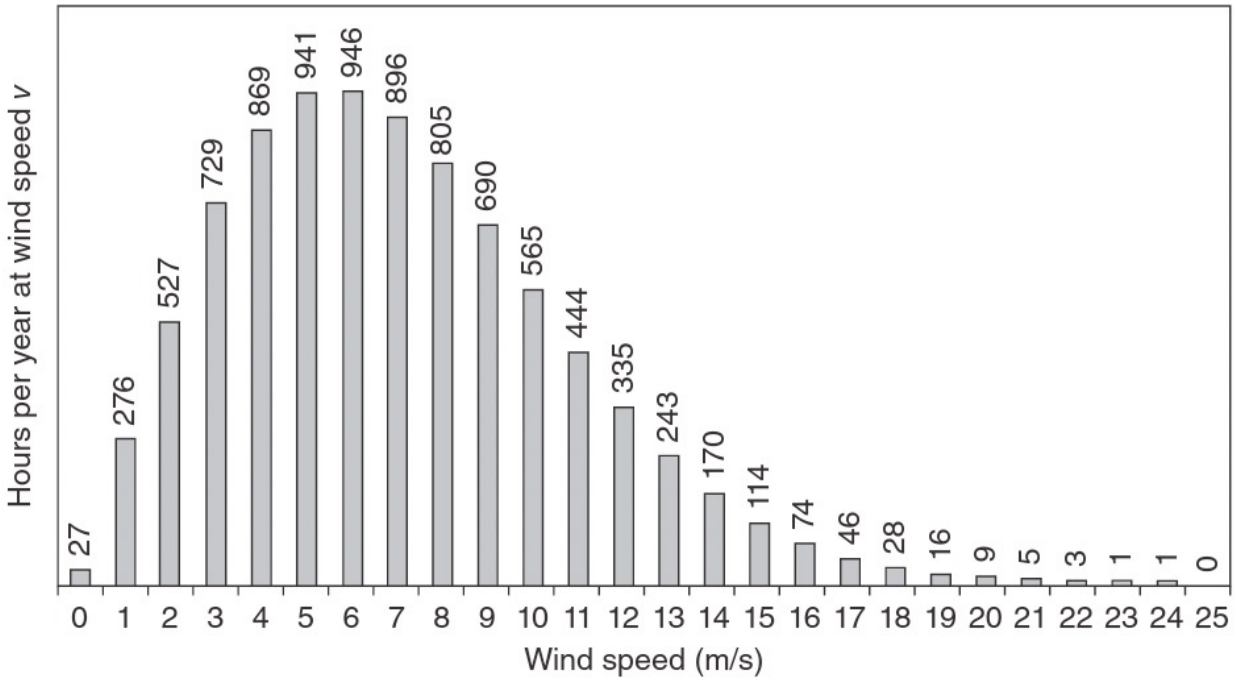


FIGURE 7.24 An example of a wind histogram showing hours that the wind blows in each wind speed bracket.

Example 7.5 Average Power in the Wind. Using the data given in Figure 7.24, find the average wind speed and the average power in the wind (W/m^2).

Assume the standard air density of 1.225 kg/m^3. Compare the result with that which would be obtained if the average power were miscalculated using just the average wind speed.

Example Solution

Solution. We need to set up a spreadsheet to determine average wind speed v and the average value of v^3. Let us do a sample calculation of one line of a spreadsheet using 805 h/yr at 8 m/s:

$$\text{Fraction of the hours at 8 m/s} = \frac{805\ \text{h/yr}}{24\ \text{h/d} \times 365\ \text{d/yr}} = 0.0919$$

$$[v_8 \cdot (\text{Fraction of the hours at 8 m/s})] = 8 \times 0.0919 = 0.735$$

$$\left[(v_8)^3 \cdot (\text{Fraction of the hours at 8 m/s})\right] = 8^3 \times 0.0919 = 47.05$$

Wind Speed v_i (m/s)	Hours per Year at v_i	Fraction of Hours at v_i	v_i × Fraction of Hours at v_i	$(v_i)^3$ × Fraction of Hours at v_i
0	27	0.0031	0.000	0.00
1	276	0.0315	0.032	0.03
2	527	0.0602	0.120	0.48
3	729	0.0832	0.250	2.25
4	869	0.0992	0.397	6.35
5	941	0.1074	0.537	13.43
6	946	0.1080	0.648	23.33
7	896	0.1023	0.716	35.08
8	805	0.0919	0.735	47.05
9	690	0.0788	0.709	57.42
10	565	0.0645	0.645	64.50
.	.	.	.	.
.	.	.	.	.
22	3	0.0003	0.008	3.65
23	1	0.0001	0.003	1.39
24	1	0.0001	0.003	1.58
25	0	0.0000	0.000	0.00
Total:	8760	1.000	7.0	653.26

Example Solution (cont'd)

The average wind speed is

$$v_{\text{avg}} = \sum_i [v_i \text{ (Fraction of hours at } v_i)] = 7.0\,\text{m/s}$$

The average value of v^3 is

$$\left(v^3\right)_{\text{avg}} = \sum_i \left[(v_i)^3 \cdot \text{(Fraction of hours at } v_i)\right] = 653.24$$

The average power in the wind on a per unit of area basis is

$$P_{\text{avg}}/A = \frac{1}{2}\rho\left(v^3\right)_{\text{avg}} = 0.5 \times 1.225 \times 653.24 = 400\,\text{W/m}^2$$

If we had miscalculated average power in the wind by using the *average* wind speed of 7 m/s in Equation 7.7 we would have found:

$$P_{\text{avg}}/A(\text{WRONG}) = \frac{1}{2}\rho\left(v_{\text{avg}}\right)^3 = 0.5 \times 1.225 \times 7.0^3 = 210\,\text{W/m}^2$$

From Histogram to PDF

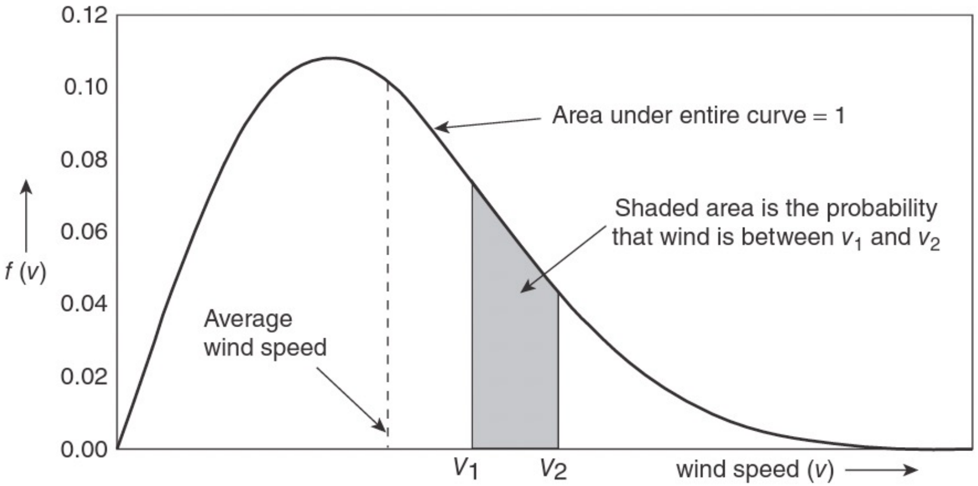


FIGURE 7.25 A wind speed probability density function (pdf).

$f(v) =$ wind speed probability density function

$$\text{Probability}\,(v_1 \le v \le v_2) = \int_{v1}^{v2} f(v)dv$$

$$\text{Probability}\,(0 \le v \le \infty) = \int_0^{\infty} f(v)dv = 1$$

The number of hours per year that wind blows between any two speeds:

$$\text{Hours/yr}\,(v_1 \le v \le v_2) = 8760 \int_{v_1}^{v_2} f(v)dv$$

$$v_{\text{avg}} = \int_0^{\infty} v \cdot f(v)dv$$

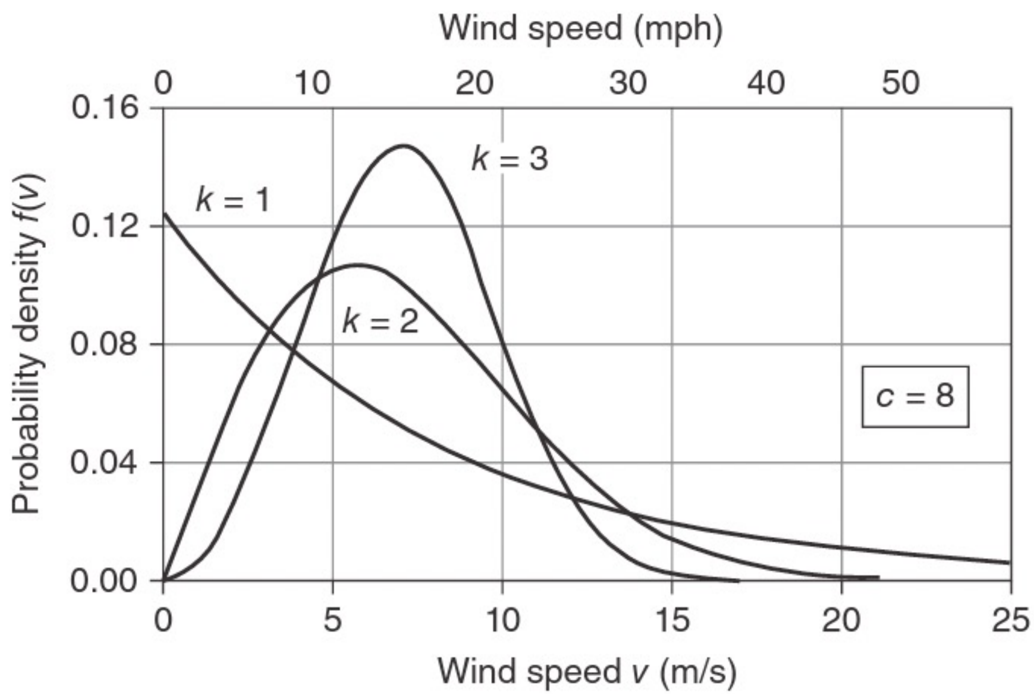
$$\left(v^3\right)_{\text{avg}} = \int_0^{\infty} v^3 f(v)dv$$

Wind Speed – Weibull Distribution

- Weibull distribution – probability density function (pdf)

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right]$$

where $k>0$ is the shape parameter, and $c>0$ is the scale parameter.



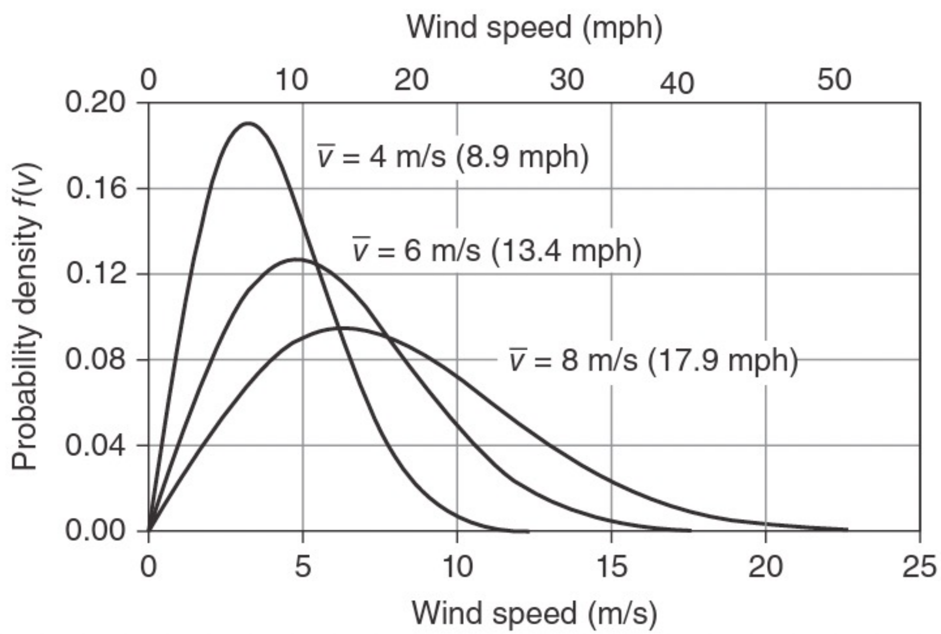
Wind Speed – Rayleigh Distribution

- Rayleigh distribution – a special case of Weibull: k=2

$$f(v) = \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right]$$

$$\bar{v} = \int_0^\infty v \cdot f(v)\mathrm{d}v = \int_0^\infty 2\left(\frac{v}{c}\right)^2 \exp\left[-\left(\frac{v}{c}\right)^2\right] = \frac{\sqrt{\pi}}{2}c$$

$$\Rightarrow \quad f(v) = \frac{\pi v}{2\bar{v}^2} \exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]$$



Average Power for Wind with Rayleigh Distribution

$$(v^3)_{\text{avg}} = \int_0^\infty v^3 \cdot f(v)dv = \int_0^\infty v^3 \cdot \frac{\pi v}{2\bar{v}^2} \exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]dv$$

$$= \frac{6}{\pi} \cdot \bar{v}^3 \approx 1.91\bar{v}^3$$

$$\bar{P} = \frac{6}{\pi} \cdot \left(\frac{1}{2}\rho A \bar{v}^3\right) \text{ (Rayleigh assumptions)}$$

Example 7.6 Average Power in the Wind. A 10-m-high anemometer finds the average wind speed at that height to be 6 m/s. Estimate the average power in the wind at a height of 50 m. Assume Rayleigh statistics, a standard friction coefficient $\alpha = 1/7$, and standard air density $\rho = 1.225\ \text{kg/m}^3$. Repeat for a taller 80-m height.

Example Solution

Solution. We first adjust the winds at 10 m to those expected at 50 m using Equation 7.18:

$$\bar{v}_{50} = \bar{v}_{10}\left(\frac{H_{50}}{H_{10}}\right)^{\alpha} = 6\left(\frac{50}{10}\right)^{1/7} = 7.55\,\text{m/s}$$

So, using Equation 7.49, per unit of area the average power in the wind would be

$$\bar{P} = \frac{6}{\pi} \times \left(\frac{1}{2}\rho A \bar{v}^3\right) = \frac{6}{\pi}\left(\frac{1}{2} \times 1.225 \times 7.55^3\right) = 504\,\text{W/m}^2$$

Let us take a slightly different approach to finding the power at 80 m. From Equation 7.20 we can write

$$\bar{P}_{10} = \frac{6}{\pi} \cdot \frac{1}{2} \cdot 1.225 \cdot 6^3 = 252.67\,\text{W/m}^2$$

$$\bar{P}_{80} = \bar{P}_{10}\left(\frac{H_{80}}{H_{10}}\right)^{3\alpha} = 252.67\left(\frac{80}{10}\right)^{3\times 1/7} = 616\,\text{W/m}^2$$

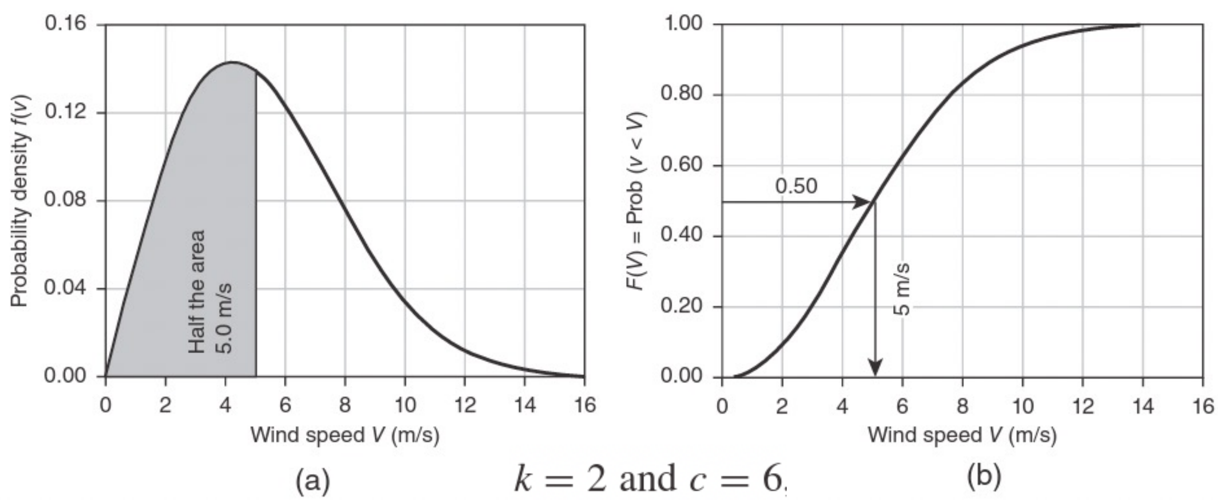
Wind Speed CDF

$$F(V) = \int_0^V f(v)dv = \text{prob}(v \le V) = \int_0^V \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] dv$$

$$\text{change of variable: } x = \left(\frac{v}{c}\right)^k \Rightarrow dx = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} dv$$

$$\Rightarrow F(V) = \int_0^{\left(\frac{V}{c}\right)^k} e^{-x}\, dx = 1 - \exp\left[-\left(\frac{V}{c}\right)^k\right] \quad \text{(Weibull)}$$

$$Set\ k = 2, c = 2\bar{v}/\sqrt{\pi} \Rightarrow \ F(V) = \text{prob}(v \le V) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{V}{\bar{v}}\right)^2\right] \text{(Rayleigh)}$$



$k = 2$ and $c = 6$.

Example

Example 7.7 Linking a Power Curve to Rayleigh Statistics. A 2.1-MW Suzlon S97 wind turbine has a cut-in wind speed $v_c = 3.5$ m/s, rated wind speed $v_R = 11$ m/s, and a furling wind speed of $v_F = 20$ m/s. If this machine is located in Rayleigh winds with an average wind speed of 7 m/s, find the following:

a. How many hours per year is the wind below the cut-in wind speed?
b. How many hours per year will the turbine be shut down due to excessive winds?
c. How many kWh/yr will be generated when the machine is running at rated power?
d. If this turbine in these winds has an overall capacity factor of 38%, what fraction of its annual energy delivered will be provided by winds below the 11 m/s rated wind speed?

Solution

a. Using Equation 7.56, the probability that the wind speed is below cut-in 3.5 m/s is

$$F(V_c) = \text{prob}(v \leq V_c) = 1 - \exp\left[-\frac{\pi}{4}\left(\frac{V_c}{\bar{v}}\right)^2\right]$$

$$= 1 - \exp\left[-\frac{\pi}{4}\left(\frac{3.5}{7}\right)^2\right] = 0.178$$

In a year with 8760 h (365 × 24), the number of hours the wind will be less than 3.5 m/s is

$$\text{Hours}(v \leq 3.5\,\text{m/s}) = 8760\,\text{h/yr} \times 0.178 = 1562\,\text{h/yr}$$

Example (cont'd)

b. Using (7.58), the hours when the wind is higher than $V_F = 20$ m/s will be

$$\text{Hours}(v \geq V_F) = 8760 \cdot \exp\left[-\frac{\pi}{4}\left(\frac{V_F}{\bar{v}}\right)^2\right]$$

$$= 8760 \cdot \exp\left[-\frac{\pi}{4}\left(\frac{20}{7}\right)^2\right] = 14.4\,\text{h/yr}$$

c. Assuming its power curve is flat above V_R, the turbine will deliver 2100 kW any time the wind is between $V_R = 11$ m/s and $V_F = 20$ m/s. The number of hours that the wind is higher than 11 m/s is

$$\text{Hours}(v \geq 11) = 8760 \cdot \exp\left[-\frac{\pi}{4}\left(\frac{11}{7}\right)^2\right] = 1260\,\text{h/yr}$$

So, the number of hours per year that the winds blow between 11 m/s and 20 m/s is $1260 - 14 = 1246$ h/yr. The energy the turbine delivers from those winds will be

$$\text{Energy}(V_R \leq v \leq V_F) = 2100\,\text{kW} \times 1246\,\text{h/yr} = 2.62 \times 10^6\,\text{kWh/yr}$$

Example (cont'd)

With a 38% CF, the turbine will deliver

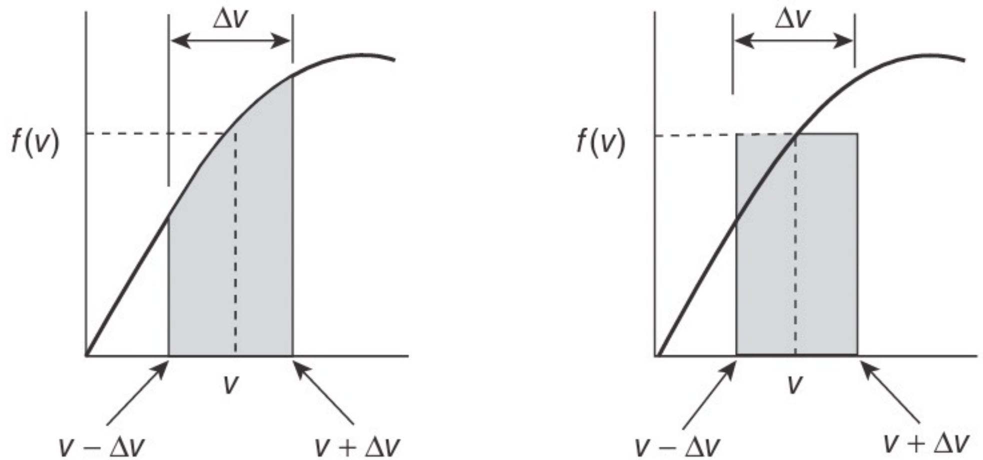
$$2100\,\text{kW} \times 8760\,\text{h/yr} \times 0.38 = 6.99 \times 10^6\,\text{kWh/yr}$$

Of that, we expect 2.62 million kWh/yr to be generated in winds above the rated wind speed, so the fraction delivered at winds below that will be

$$\%\text{ of energy from } v < 11\,\text{m/s} = \frac{6.99 - 2.62}{6.99} = 0.625 = 62.5\%$$

Discretizing PDF

What is the probability that the wind blows at (around) some specified speed?



(a) Area $= \int_{v-\Delta v}^{v+\Delta v} f(v)\,\Delta v$

(b) Area $= f(v)\,\Delta v$

$$\boxed{\text{prob}(v - \Delta v/2 \leq V \leq v + \Delta v/2) = \int_{v-\Delta v/2}^{v+\Delta v/2} f(v)\mathrm{d}v \approx f(v)\Delta v}$$

Example (Correctness of Discretizing)

Example 7.8 Discretizing $f(v)$. For a wind site with Rayleigh winds having average speed $\bar{v} = 8$ m/s, what is the probability that the wind would blow between 6.5 and 7.5 m/s? How does this compare to the pdf evaluated at 7 m/s?

Solution. Using Equation 7.59, we obtain

$$\text{prob}(v > 6.5) = \exp\left[-\frac{\pi}{4}\left(\frac{6.5}{8}\right)^2\right] = 0.59542$$

$$\text{prob}(v > 7.5) = \exp\left[-\frac{\pi}{4}\left(\frac{7.5}{8}\right)^2\right] = 0.50143$$

So, using the correct area under the pdf, the probability that the wind is between 6.5 and 7.5 m/s is

$$\text{prob}(6.5 < v < 7.5) = 0.59542 - 0.50143 = 0.09400$$

Now try the simplification suggested in Figure 7.31b. Using the Rayleigh pdf (Eq. 7.46), the probability density function at 7 m/s is

$$f(v) = \frac{\pi v}{2\bar{v}^2} \exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]$$

Using a 1 m/s increment around $v = 7$ m/s, the rectangular approximation is

$$\text{prob}(6.5 < v < 7.5) = 1 \times \frac{\pi \cdot 7}{2 \cdot 8^2} \exp\left[-\frac{\pi}{4}\left(\frac{7}{8}\right)^2\right] = 0.09416$$