UNIVERSITY OF CALIFORNIA
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Power System State Estimation via Conic Relaxations: Algorithms and Performance

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Outline

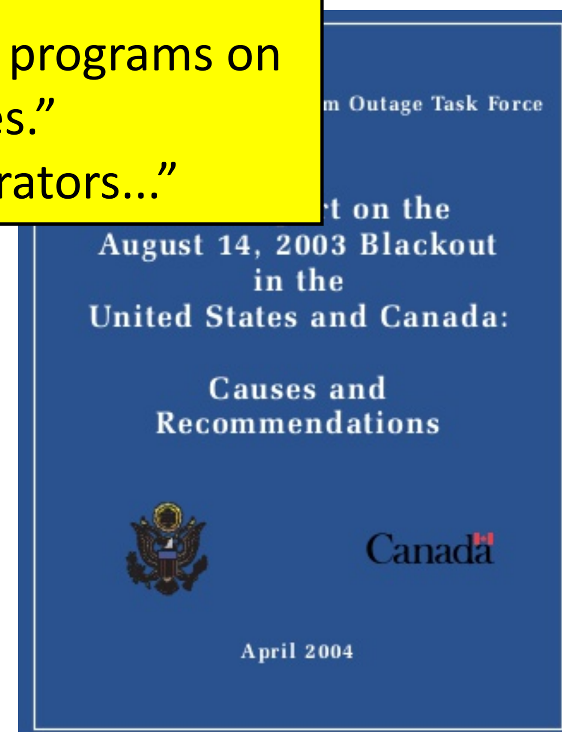
- System modeling and problem formulation
- Convex relaxations for power flow (PF) analysis
  - Design of the objective function
  - Exact recovery of the PF solution
- Convex relaxations for power system state estimation
  - Penalized semidefinite relaxation
  - Theoretically guaranteed performance bound
- Concluding remarks

Context

Time: August 14, 2003
Location: Midwest/Northeast US & Ontario, CAN

Costs: 50 million people, 61,800 MWs of load lost. $10 billion economic loss in the US and Canada's GDP was down 0.7%.

Recommendations:
1. "U.S. DOE should expand its research programs on ***reliability-related*** tools and technologies."
2. "Adopt better ***real-time tools*** for operators..."



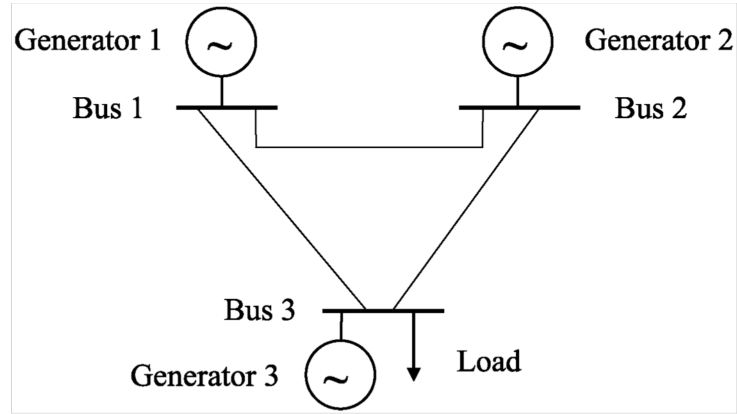
"**Key phase events:** MISO's state estimator (SE) software solution was compromised..." "The failure of its SE contributed to the lack of situational awareness."

System Modeling

- Represent power grid by a connected graph

$$\mathcal{G} = (\mathcal{N}, \mathcal{L})$$

set of buses   set of power lines



Time domain

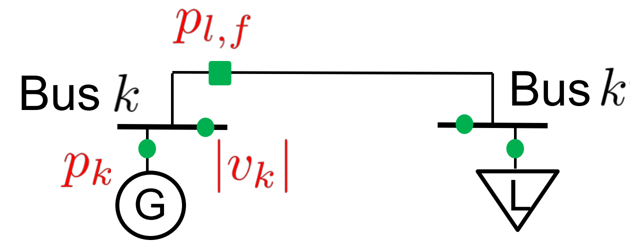
$$v(t) = V_{\max}\cos(\omega t + \theta_v)$$

Phasor domain

$$v = \frac{V_{\max}}{\sqrt{2}} e^{\mathrm{j}\theta_v}$$

- Complex voltage: $\mathbf{v} = [v_1, \ldots, v_n]^\top \in \mathbb{C}^n$
- Nodal current injection: $\mathbf{i} = \mathbf{Y}\mathbf{v}$
- Net injected complex power: $\mathbf{p} + \mathbf{q}\mathrm{j} = \mathrm{diag}(\mathbf{v}\mathbf{i}^*)$

Nodal and Line Quantities



- Voltage magnitude and nodal power injections:

$$|v_k|^2 = \mathrm{Tr}(\mathbf{E}_k \mathbf{v}\mathbf{v}^*), \; p_k = \mathrm{Tr}(\mathbf{Y}_{k,p} \mathbf{v}\mathbf{v}^*), \; q_k = \mathrm{Tr}(\mathbf{Y}_{k,q} \mathbf{v}\mathbf{v}^*)$$

where $\mathbf{E}_k := \mathbf{e}_k \mathbf{e}_k^\top, \; \mathbf{Y}_{k,p} := \frac{1}{2}(\mathbf{Y}^*\mathbf{E}_k + \mathbf{E}_k\mathbf{Y}), \; \mathbf{Y}_{k,q} := \frac{\mathrm{j}}{2}(\mathbf{E}_k\mathbf{Y} - \mathbf{Y}^*\mathbf{E}_k)$

- Branch active and reactive powers:

$$\begin{aligned} p_{l,f} &= \mathrm{Tr}(\mathbf{Y}_{l,p_f} \mathbf{v}\mathbf{v}^*), \quad & p_{l,t} &= \mathrm{Tr}(\mathbf{Y}_{l,p_t} \mathbf{v}\mathbf{v}^*) \\ q_{l,f} &= \mathrm{Tr}(\mathbf{Y}_{l,q_f} \mathbf{v}\mathbf{v}^*), \quad & q_{l,t} &= \mathrm{Tr}(\mathbf{Y}_{l,q_t} \mathbf{v}\mathbf{v}^*) \end{aligned}$$

- All quantities are quadratic functions of complex voltage $\mathbf{v}$

$$\boxed{\mathbf{v} = \text{state of the system}}$$

Problem Statement

Power system state estimation (PSSE):

Given noisy measurements $z_j = \mathrm{Tr}(\mathbf{M}_j \mathbf{v}\mathbf{v}^*) + \eta_j,\ j = 1, 2, \ldots, m,$
estimate the complex voltage $\mathbf{v}$.

❑ Functionality of PSSE:

- Provides real-time power system conditions
- Constitutes the core of online security analysis
- Provides diagnostics for modeling and maintenance

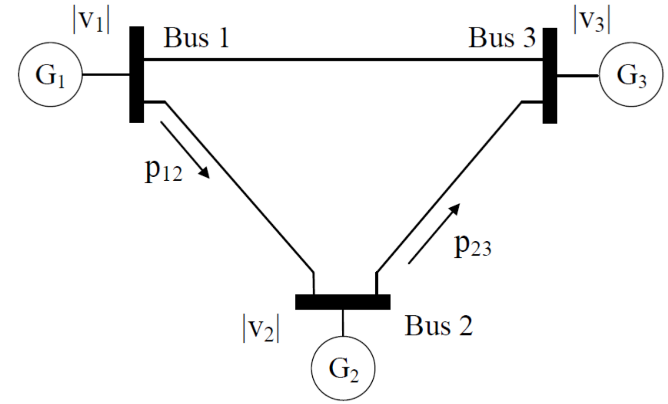
❑ Measurements:

- From the supervisory control and data acquisition (SCADA) system and phasor measurement units (PMUs)
- Corrupted by noise; missing or grossly inaccurate (outliers/bad data)

Related Work

- *PF analysis*
  - NP-hard for both T&D networks [Bienstock-Verma'15], [Lehmann et al'16]
  - Newton-Raphson method and fast decoupled load flow (FDLF)
  - Other techniques: Holomorphic embedding LF and numerical polynomial homotopy continuation (NPHC) [Trias'12], [Li'03], [Mehta et al'15]
  - Semidefinite relaxations (SDR) [Madani-Lavaei-Baldick'15]
- *PSSE*
  - Modeling and implementation: [Schweppe et al'70]
  - Gauss-Newton methods [Abur-Gomez'04] [Caro-Conejo-Minguez'09]
  - SDR: [Zhu-Giannakis'11,14], [Weng-Ilic, et al'12,13,15]
- *Our contributions:*
  - Conditions to guarantee an exact SDR for PF problem
  - Theoretical performance bound quantifying the SDR solution for PSSE

Motivating Example



- Direct calculation of power flows

$$p_{st} = \mathrm{Re}(v_s(v_s - v_t)^* y_{st}^*) \quad \Rightarrow$$

$$\begin{cases} \measuredangle v_1 - \measuredangle v_2 = \arccos\left(\dfrac{p_{12} - |v_1|^2 \mathrm{Re}(y_{12})}{|v_1||v_2||y_{12}|}\right) + \measuredangle y_{12} \\ \measuredangle v_2 - \measuredangle v_3 = \arccos\left(\dfrac{p_{23} - |v_2|^2 \mathrm{Re}(y_{23})}{|v_2||v_3||y_{23}|}\right) + \measuredangle y_{23} \end{cases} \quad \Rightarrow$$

Find unique $\measuredangle v_1, \measuredangle v_2, \measuredangle v_3$ $\quad [0 < (\measuredangle v_s - \measuredangle v_t) - \measuredangle y_{st} < 180^\circ \text{ and } \measuredangle v_{\mathrm{ref}} = 0]$

- Direct calculation is NOT applicable for noisy measurements
  - Optimization framework to estimate the voltage $\mathbf{v}$?
  - Convexity of the formulation?
  - How good is the performance of the convexification?

Power Flow (PF) Problem

- PF problem: PSSE with noiseless measurements $(\eta_j = 0, \forall j)$

$$\begin{aligned} &\text{find} \quad \mathbf{v} \in \mathbb{C}^n \\ &\text{s.t.} \quad \operatorname{Tr}(\mathbf{M}_j \mathbf{v}\mathbf{v}^*) = z_j, \ \forall j \in \mathcal{M} \end{aligned}$$

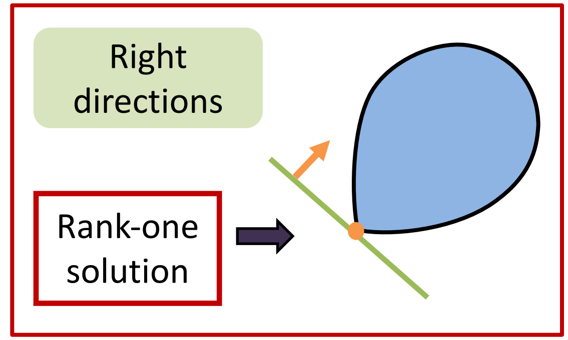
Standard PF	p_k	q_k	$\lvert v_k \rvert$	$\measuredangle v_k$
PV bus	X	?	X	?
PQ bus	X	X	?	?
REF bus	?	?	X	X

- All measurements are nodal quantities
- Number of equations = number of unknowns $(2n-1)$

Semidefinite Relaxation

- Our approach: Design a linear objective $\mathrm{Tr}(\mathbf{M}_0\mathbf{X})$ with $\mathbf{X} := \mathbf{v}\mathbf{v}^*$

$$
\begin{aligned}
\underset{\mathbf{X}\in\mathbb{H}^n}{\text{minimize}} \quad & \mathrm{Tr}(\mathbf{M}_0\mathbf{X}) \\
\text{subject to} \quad & \mathrm{Tr}(\mathbf{M}_j\mathbf{X}) = z_j,\ j \in \mathcal{M} \\
& \mathbf{X} \succeq \mathbf{0},\ \ \cancel{\mathrm{rank}(\mathbf{X}) = 1}
\end{aligned}
$$



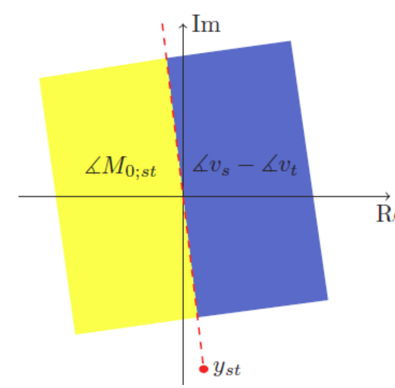
Question: *When the SDP relaxation is exact to recover* $\mathbf{v}$ *?*

- Assumptions

A1) Available measurements: $\begin{cases} |v_k|^2, \forall k \in \mathcal{N} \\ p_{l,f}\ \ (p_{l,t}), \forall l \in \mathcal{L}_{\mathrm{ST}} \end{cases}$

A2) Angle conditions:

$$
\begin{aligned}
-180^\circ < \measuredangle M_{0;st} - \measuredangle y_{st} < 0, \quad & \forall (s,t) \in \mathcal{L}_{\mathrm{ST}}, \\
0 < (\measuredangle v_s - \measuredangle v_t) - \measuredangle y_{st} < 180^\circ, \quad & \forall (s,t) \in \mathcal{L}_{\mathrm{ST}}, \\
(\measuredangle v_s - \measuredangle v_t) - \measuredangle M_{0;st} \neq 0 \ \text{ or } \ 180^\circ, \quad & \forall (s,t) \in \mathcal{L}_{\mathrm{ST}}
\end{aligned}
$$



Exact Recovery

> ***Theorem 1***
>
> *Under assumptions A1-A2, the SDP relaxation recovers the voltage vector* $\mathbf{v}$.

- Proof sketch:
  - Dual SDP

$$
\begin{aligned}
\underset{\boldsymbol{\mu} \in \mathbb{R}^m}{\text{maximize}} \quad & -\mathbf{z}^\top \boldsymbol{\mu} \\
\text{subject to} \quad & \mathbf{H}(\boldsymbol{\mu}) := \mathbf{M}_0 + \sum_{j=1}^{m} \mu_j \mathbf{M}_j \succeq \mathbf{0}
\end{aligned}
$$

  - To show the existence of a dual certificate $\boldsymbol{\mu}$ satisfying

$$
\mathbf{H}(\boldsymbol{\mu}) \succeq 0, \quad \mathbf{H}(\boldsymbol{\mu})\mathbf{v} = 0, \quad \operatorname{rank}(\mathbf{H}(\boldsymbol{\mu})) = n - 1
$$

Proof of Theorem 1

$$\mathbf{H} = \mathbf{M}_0 + \sum_{j=1}^{m} \mu_j \mathbf{M}_j = \sum_{j=n+1}^{m} \overbrace{\left(\mathbf{M}_{0,j} + \frac{1}{m-n} \sum_{k=1}^{n} \mu_k \mathbf{E}_k + \mu_j \mathbf{M}_j\right)}^{\mathbf{H}_j}$$

- Explore the special structure of matrix $\mathbf{Y}_{l,pf}$

$$\mathbf{Y}_{l,pf}(s,t) = \mathbf{Y}^*_{l,pf}(t,s) = -\frac{y_{st}}{2}, \quad \mathbf{Y}_{l,pf}(s,s) = \mathrm{Re}(y_{st})$$

Define $\tilde{\mu}_s := \frac{1}{m-n}\mu_s + \mu_s \mathrm{Re}(y_{st})$, $\tilde{\mu}_t := \frac{1}{m-n}\mu_t$

$$\tilde{\mathbf{H}}_j := \begin{bmatrix} \tilde{\mu}_s & m_{st,j} - \frac{\mu_j}{2} y_{st} \\ m^*_{st,j} - \frac{\mu_j}{2} y^*_{st} & \tilde{\mu}_t \end{bmatrix}$$

$\mathbf{H}_j \mathbf{v} = \mathbf{0} \Rightarrow \tilde{\mathbf{H}}_j \check{\mathbf{v}} = \mathbf{0} \Rightarrow$

$$\boxed{\begin{aligned} m^*_{st,j} - \frac{\mu_j}{2} y^*_{st} &= -\frac{\tilde{\mu}_t v_t}{v_s} \\ \tilde{\mu}_s &= \tilde{\mu}_t \frac{|v_t|^2}{|v_s|^2} \end{aligned}} \Rightarrow \tilde{\mathbf{H}}_j = \tilde{\mu}_t \begin{bmatrix} \frac{|v_t|^2}{|v_s|^2} & -\frac{v_t^*}{v_s^*} \\ -\frac{v_t}{v_s} & 1 \end{bmatrix}$$

rank-1

Proof of Theorem 1 (cont'd)

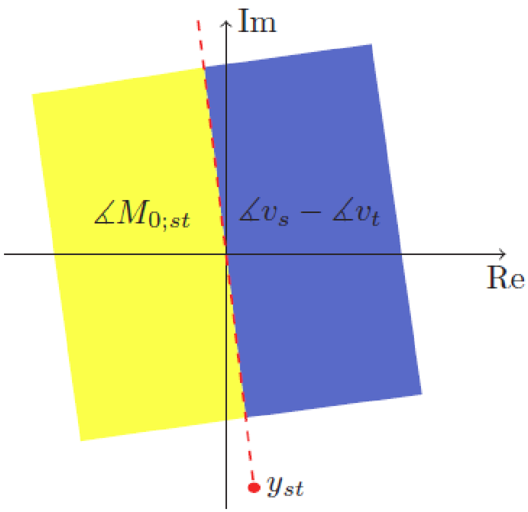
- To prove $\mathrm{rank}(\mathbf{H}) = n-1 \Leftarrow \dim(\mathrm{null}(\mathbf{H})) = 1 \Leftarrow \mathbf{x} = r\mathbf{v},\ \forall \mathbf{x} \in \mathrm{null}(\mathbf{H})$

$$\left.\begin{array}{l} \mathbf{H}_j\mathbf{x} = \mathbf{0} \Rightarrow \frac{x_s}{x_t} = \frac{v_s}{v_t} \text{ for } j\text{-th measurement over line } (s,t) \\ \mathbf{H}_{j'}\mathbf{x} = \mathbf{0} \Rightarrow \frac{x_t}{x_a} = \frac{v_t}{v_a} \text{ for } j'\text{-th measurement over line } (t,a) \end{array}\right\} \Rightarrow$$

$$\frac{x_s}{v_s} = \frac{x_t}{v_t} = \frac{x_a}{v_a} = r$$

- Line measurements over the connected subgraph $\mathcal{G}_s$, repeat the argument for all the buses.

- Angle conditions: $\tilde{\mu}_t = -(m^*_{st,j} - \frac{\mu_j}{2} y^*_{st})\frac{v_s}{v_t} > 0$



$$\Rightarrow \boxed{\begin{array}{rl} -180^\circ < \measuredangle M_{0;st} - \measuredangle y_{st} < 0, & \forall (s,t) \in \mathcal{L}_\mathcal{T}, \\ 0 < (\measuredangle v_s - \measuredangle v_t) - \measuredangle y_{st} < 180^\circ, & \forall (s,t) \in \mathcal{L}_\mathcal{T}, \\ (\measuredangle v_s - \measuredangle v_t) - \measuredangle M_{0;st} \neq 0 \text{ or } 180^\circ, & \forall (s,t) \in \mathcal{L}_\mathcal{T} \end{array}}$$

SOCP Relaxation

$$\begin{aligned} \underset{\mathbf{X} \in \mathbb{H}^N}{\text{minimize}} \quad & \mathrm{Tr}(\mathbf{M}_0 \mathbf{X}) \\ \text{subject to} \quad & X_{k,k} = |v_k|^2, \quad \forall k \in \mathcal{N} \\ & \mathrm{Tr}(\mathbf{Y}_{l,p_f} \mathbf{X}) = p_{l,f}, \quad \forall l \in \mathcal{L}_{\mathrm{ST}} \\ & \begin{bmatrix} X_{s,s} & X_{s,t} \\ X_{t,s} & X_{t,t} \end{bmatrix} \succeq \mathbf{0}, \quad \forall (s,t) \in \mathcal{L}_{\mathrm{ST}} \end{aligned}$$

Theorem 2

Under assumptions A1-A2, the SOCP relaxation recovers the voltage $\mathbf{v}$.

- ❑ Additional measurements

Corollary 1

Under assumptions A1-A2, the SDP and SOCP relaxations with additional constraints of power injection and line measurements both recover the voltage $\mathbf{v}$.

Power System State Estimation

- Penalized SDP:

$$\begin{aligned} \underset{\mathbf{X}\in\mathbb{H}^n,\boldsymbol{\nu}\in\mathbb{R}^m}{\text{minimize}} \quad & \rho f(\boldsymbol{\nu}) + \mathrm{Tr}(\mathbf{M}_0\mathbf{X}) \\ \text{subject to} \quad & \mathrm{Tr}(\mathbf{M}_j\mathbf{X}) + \nu_j = z_j, \quad \forall j \in \mathcal{M} \\ & \mathbf{X} \succeq \mathbf{0} \end{aligned}$$

- Data fitting cost: $f_{\mathrm{WLAV}}(\boldsymbol{\nu}) = \sum_{j=1}^{m} |\nu_j|/\sigma_j, \quad f_{\mathrm{WLS}}(\boldsymbol{\nu}) = \sum_{j=1}^{m} \nu_j^2/\sigma_j^2$

Theorem 3

If $\rho \geq \max_{j\in\mathcal{M}} |\sigma_j \hat{\mu}_j|$, *then there exists a scalar* $\beta > 0$ *such that*

$$\zeta := \frac{\|\mathbf{X}^{\mathrm{opt}} - \beta\mathbf{v}\mathbf{v}^*\|_F}{\sqrt{n \times \mathrm{Tr}(\mathbf{X}^{\mathrm{opt}})}} \leq 2\sqrt{\frac{\rho \times f_{\mathrm{WLAV}}(\boldsymbol{\eta})}{n\lambda}},$$

where λ *is the second smallest eigenvalue of* $\mathbf{H}(\hat{\boldsymbol{\mu}})$.

Estimation Error

- Define the root-mean-square error:

$$\zeta := \frac{\|\mathbf{X}^{\mathrm{opt}} - \beta \mathbf{v}\mathbf{v}^*\|_F}{\sqrt{n \times \mathrm{Tr}(\mathbf{X}^{\mathrm{opt}})}} \le 2 \overbrace{\sqrt{\frac{\rho}{n\lambda}}}^{\omega(\mathcal{G}')} \sqrt{f_{\mathrm{WLAV}}(\boldsymbol{\eta})}$$

Corollary 2

Under assumptions of Theorem 3, the tail probability of the estimation error ζ *is upper bounded as* $\mathbb{P}(\zeta > t) \le \mathrm{e}^{-\gamma m}$ *for every* $t > 0,$ *where* $\gamma = \frac{t^4 \lambda^2}{32\kappa^2 \rho^2} - \ln 2.$

- Effect of more measurements

Theorem 4

Consider two choices of the graph $\mathcal{G}'$, *denoted as* $\mathcal{G}_1'$ *and* $\mathcal{G}_2'$, *such that* $\mathcal{G}_1'$ *is a subgraph of* $\mathcal{G}_2'$. *Then, the relation* $\omega(\mathcal{G}_2') \le \omega(\mathcal{G}_1')$ *holds.*

Proof of Theorem 3

- Penalized SDP: $$\min_{\mathbf{X}\succeq\mathbf{0}} \operatorname{Tr}(\mathbf{M}_0\mathbf{X})+\rho\sum_{j=1}^{m}\sigma_j^{-1}\left|\operatorname{Tr}\left(\mathbf{M}_j(\mathbf{X}-\mathbf{v}\mathbf{v}^*)\right)-\eta_j\right|$$

$$\begin{cases} \operatorname{Tr}\left(\mathbf{M}_0(\mathbf{X}^{\text{opt}}-\mathbf{v}\mathbf{v}^*)\right)+\rho\sum_{j=1}^{m}\sigma_j^{-1}\left|\operatorname{Tr}\left(\mathbf{M}_j(\mathbf{X}^{\text{opt}}-\mathbf{v}\mathbf{v}^*)\right)\right| \le 2\rho f_{\text{WLAV}}(\boldsymbol{\eta}) \\ \mathbf{M}_0=\mathbf{H}(\hat{\boldsymbol{\mu}})-\sum_{j=1}^{m}\hat{\mu}_j\mathbf{M}_j,\ \mathbf{H}(\hat{\boldsymbol{\mu}})\mathbf{v}=\mathbf{0} \end{cases}$$

$\Rightarrow$ $\operatorname{Tr}(\mathbf{H}(\hat{\boldsymbol{\mu}})\mathbf{X}^{\text{opt}})\le 2\rho f_{\text{WLAV}}(\boldsymbol{\eta})$, if $\rho\ge\max_{j\in\mathcal{M}}|\sigma_j\hat{\mu}_j|$

$$\begin{cases} \check{\mathbf{X}}:=\begin{bmatrix}\tilde{\mathbf{X}} & \tilde{\mathbf{x}}\\ \tilde{\mathbf{x}}^* & \alpha\end{bmatrix}=\mathbf{U}^*\mathbf{X}^{\text{opt}}\mathbf{U}\ \Rightarrow\ 2\rho f_{\text{WLAV}}(\boldsymbol{\eta})\ge\operatorname{Tr}(\boldsymbol{\Lambda}\check{\mathbf{X}})\ge\lambda\operatorname{Tr}(\tilde{\mathbf{X}}) \\ \tilde{\mathbf{X}}-\alpha^{-1}\tilde{\mathbf{x}}\tilde{\mathbf{x}}^*\succeq\mathbf{0}\ \Rightarrow\ \|\tilde{\mathbf{x}}\|_2^2\le\alpha\operatorname{Tr}(\tilde{\mathbf{X}})=\operatorname{Tr}(\mathbf{X}^{\text{opt}})\operatorname{Tr}(\tilde{\mathbf{X}})-\operatorname{Tr}^2(\tilde{\mathbf{X}}) \end{cases}$$

$$\mathbf{X}^{\text{opt}}=\begin{bmatrix}\tilde{\mathbf{U}} & \tilde{\mathbf{v}}\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{X}} & \tilde{\mathbf{x}}\\ \tilde{\mathbf{x}}^* & \alpha\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{U}}^*\\ \tilde{\mathbf{v}}^*\end{bmatrix}=\tilde{\mathbf{U}}\tilde{\mathbf{X}}\tilde{\mathbf{U}}^*+\tilde{\mathbf{v}}\tilde{\mathbf{x}}^*\tilde{\mathbf{U}}^*+\tilde{\mathbf{U}}\tilde{\mathbf{x}}\tilde{\mathbf{v}}^*+\alpha\tilde{\mathbf{v}}\tilde{\mathbf{v}}^*$$

$\Rightarrow$ $$\|\mathbf{X}^{\text{opt}}-\alpha\tilde{\mathbf{v}}\tilde{\mathbf{v}}^*\|_F^2=\|\tilde{\mathbf{X}}\|_F^2+2\|\tilde{\mathbf{x}}\|_2^2\le\frac{4\rho f_{\text{WLAV}}(\boldsymbol{\eta})}{\lambda}\operatorname{Tr}(\mathbf{X}^{\text{opt}})$$

Rank-1 Approximation

- Given $\mathbf{X}^{\text{opt}}$ of the penalized SDP , we can recover $\mathbf{v} \in \mathbb{C}^n$ as follows:

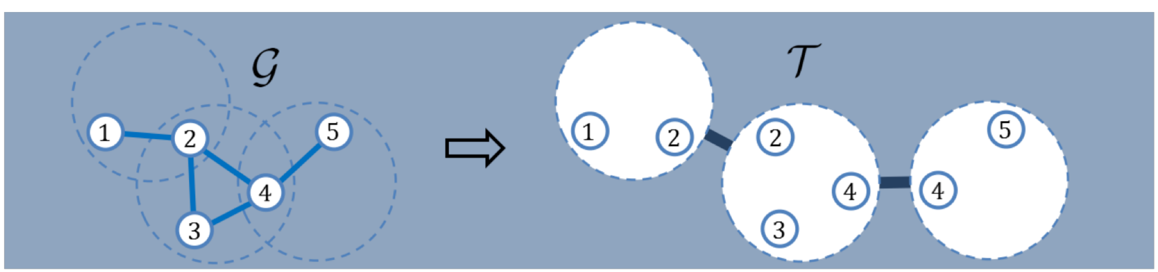
S1) set the voltage magnitude $|\hat{v}_k| = \sqrt{\mathbf{X}^{\text{opt}}_{k,k}}, \text{ for } k = 1, 2, \ldots, n$

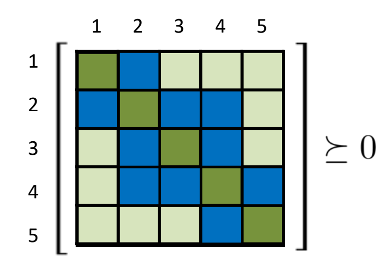
S2) set the voltage angles by solving a linear program:

$$\begin{aligned} \measuredangle \hat{\mathbf{v}} = \underset{\measuredangle \mathbf{v} \in [-\pi,\pi]^N}{\arg\min} & \sum_{(s,t)\in\mathcal{L}} |\measuredangle \mathbf{X}^{\text{opt}}_{s,t} - \measuredangle v_s + \measuredangle v_t| \\ \text{s. t.} \qquad & \measuredangle v_{\text{ref}} = 0 \end{aligned}$$

- Complexity reduction: tree decomposition

Full-scale to decomposed SDP

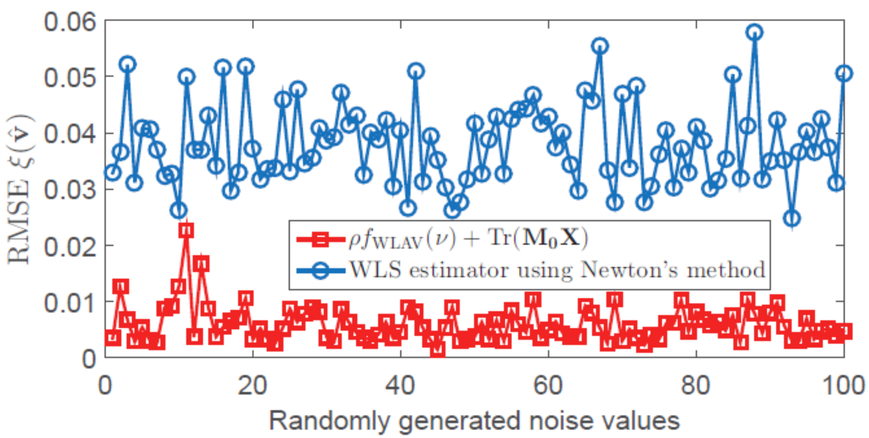




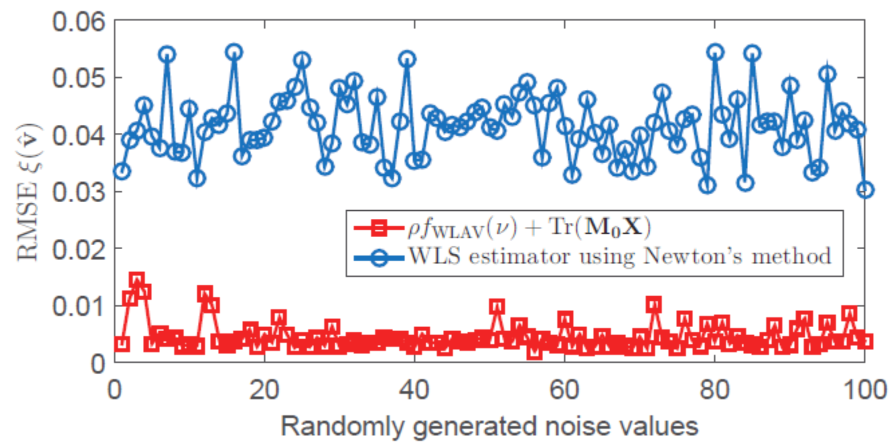
R. Madani, M. Ashraphijuo, and J. Lavaei, “Promises of Conic Relaxation for Contingency-Constrained Optimal Power Flow Problem,” IEEE Trans. Power Systems, 2016.

Penalized SDP vis-à-vis Newton's Method

- Performance metric: $\mathrm{RMSE} = \|\hat{\mathbf{v}} - \mathbf{v}_o\| / \sqrt{n}$
  - Available measurements: $\{|v_k|^2\}_{k\in\mathcal{N}}$ and $\{p_{l,f}, p_{l,t}\}_{l\in\mathcal{L}}$
  - Bad data: 20% of the line measurements



(a) IEEE 57-bus system



(b) IEEE 118-bus system

Comparison of Different Regularizers

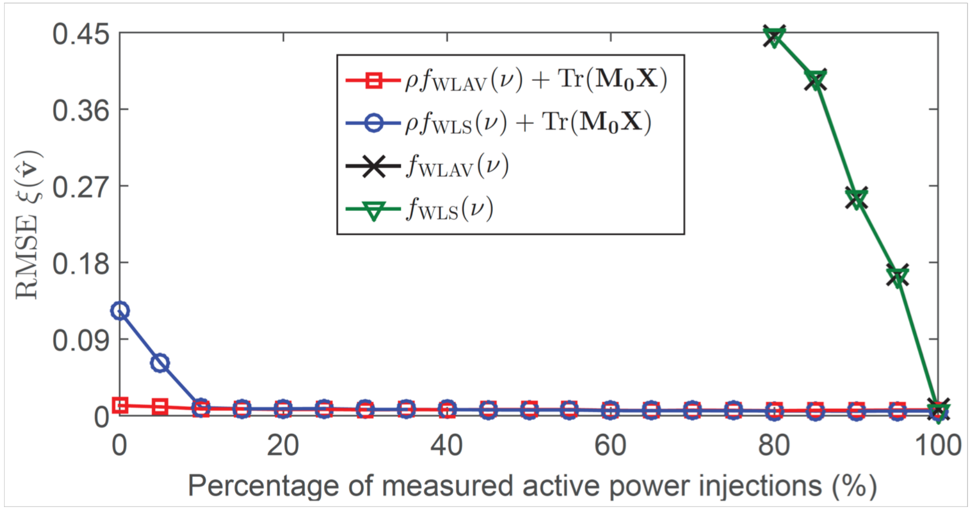
- IEEE benchmark systems
  - Various noise levels: $\sigma_j = c \times |\bar{v}_k|^2,\ 1.5c \times \bar{p}_k,\ \text{or } 2c \times \bar{p}_{l,f}$
  - Bad data: 10% of the measurements
  - 50 Monte-Carlo simulations
- Performance measure: $\xi(\hat{\mathbf{v}}) = \|\hat{\mathbf{v}} - \mathbf{v}_o\| / \sqrt{n}$

Methods	$\rho f(\boldsymbol{\nu}) + \mathrm{Tr}(\mathbf{M}_0\mathbf{X})$		$\rho f(\boldsymbol{\nu}) + \|\mathbf{X}\|_*$		$\rho f(\boldsymbol{\nu})$	
	WLAV	WLS	WLAV	WLS	WLAV	WLS
9-bus	0.0648	0.1293	1.2744	1.1483	1.1619	1.1633
14-bus	0.1307	0.1784	1.1320	1.3871	1.4233	1.4215
30-bus	0.2055	0.2543	1.4236	1.4306	1.4269	1.4268
39-bus	0.1324	0.1239	1.1317	1.3135	1.2764	1.2757
57-bus	0.2343	0.2809	1.2981	1.3004	1.3235	1.3098
118-bus	0.1136	0.1641	1.3620	1.3272	1.3445	1.3577

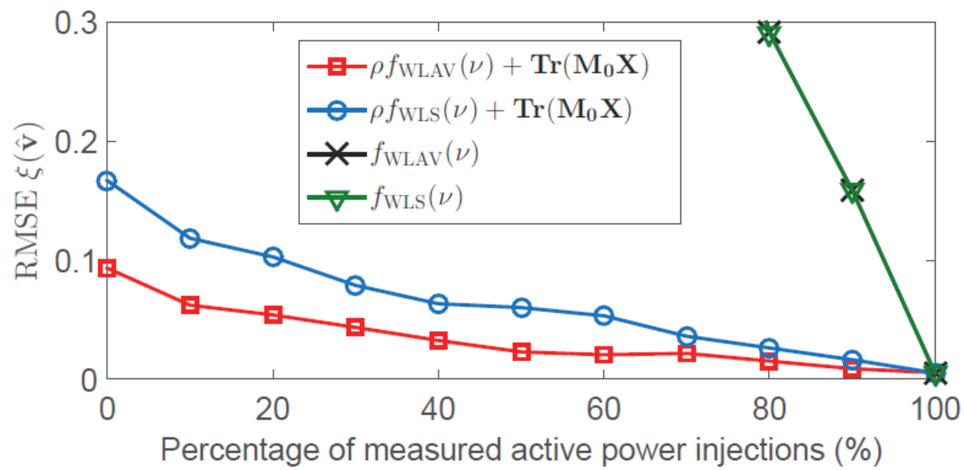
- Proposed SDP approach has the smallest estimation error

Effect of Additional Active Power Injections

- PEGASE 1354-bus: SDP relaxation with tree decomposition



- PEGASE 9241-bus: SOCP relaxation



Simulation Time

- Modeling tool and solver: CVX+SDPT3

Cases	Solver time	Total time
9-bus	0.89s	1.58s
14-bus	1.23s	2.54s
30-bus	1.33s	3.21s
39-bus	1.56s	3.28s
57-bus	1.97s	4.09s
118-bus	2.38s	5.63s
1354-bus	4.55s	9.48s
2869-bus	13.17s	24.44s
9241-bus	58.00s	109.14s

Summary

❑ Takeaway points

- Framework of conic relaxations for PSSE
- Conditions of exact recovery of the PF solution via SDP/SOCP relaxations
- Estimation error bound of the SDR optimal solution for PSSE

❑ Ongoing work

- Robust PSSE: uncertain system parameters; e.g., admittance values
- Dynamic and online PSSE