Descriptors

EECS 442 – Jeong Joon Park

Winter 2024, University of Michigan

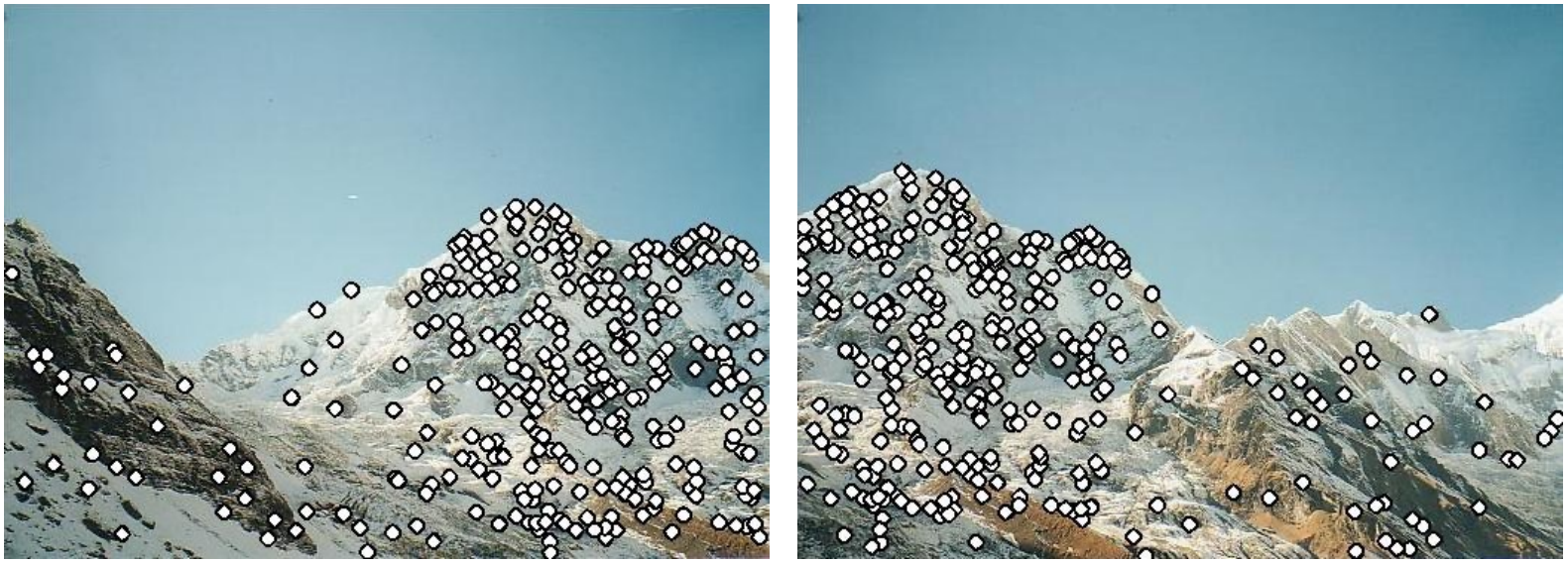
Administrivia

- Questions to the Instructor (me)
  - Best used by asking conceptual questions
  - Detailed HW-related questions: TAs. Why?
    - TAs make, edit HWs, prepare solutions, etc.

Administrivia

- Project
- We will post details on Wednesday
- Will provide sample topics

Recap: Motivation

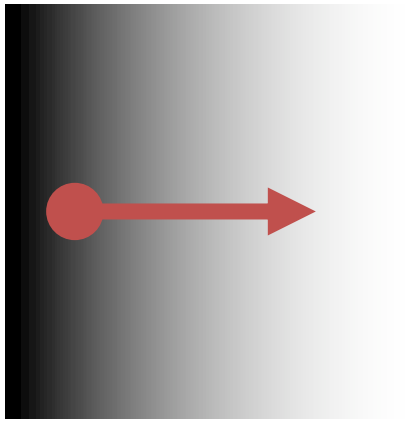


1: find corners+features

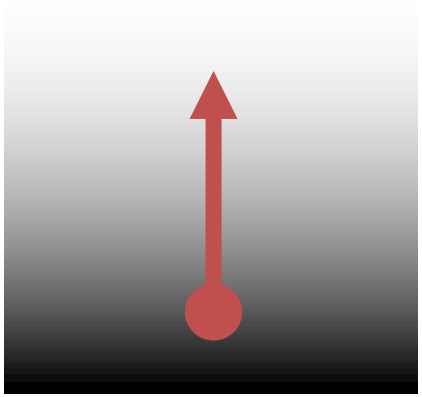
Image credit: M. Brown

Last Time – Gradients

Image gradients – treat image like function of x,y – gives edges, corners, etc.



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



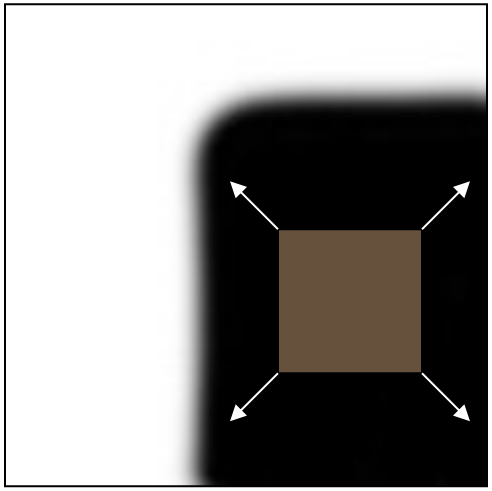
$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

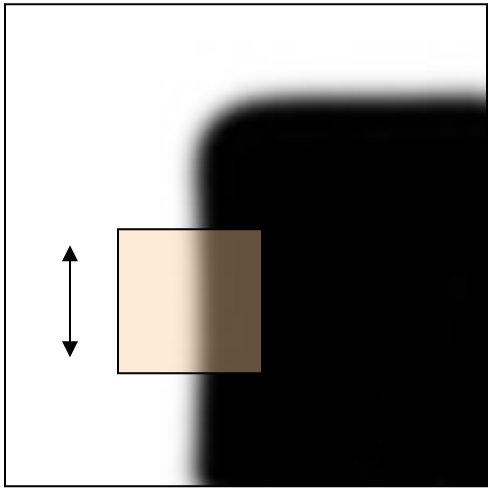
Figure credit: S. Seitz

Last Time – Corner Detection

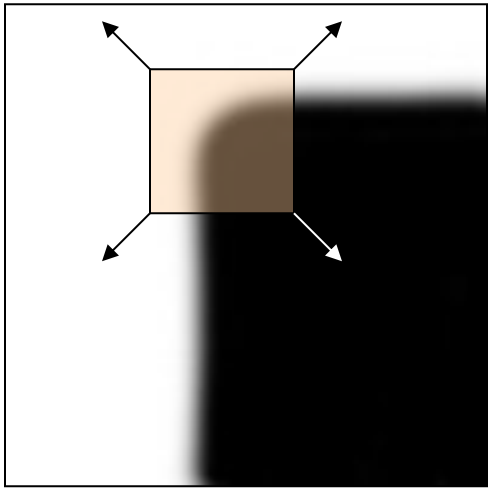
Can localize the location, or any shift → big intensity change.



"flat" region: no change in all directions



"edge": no change along the edge direction

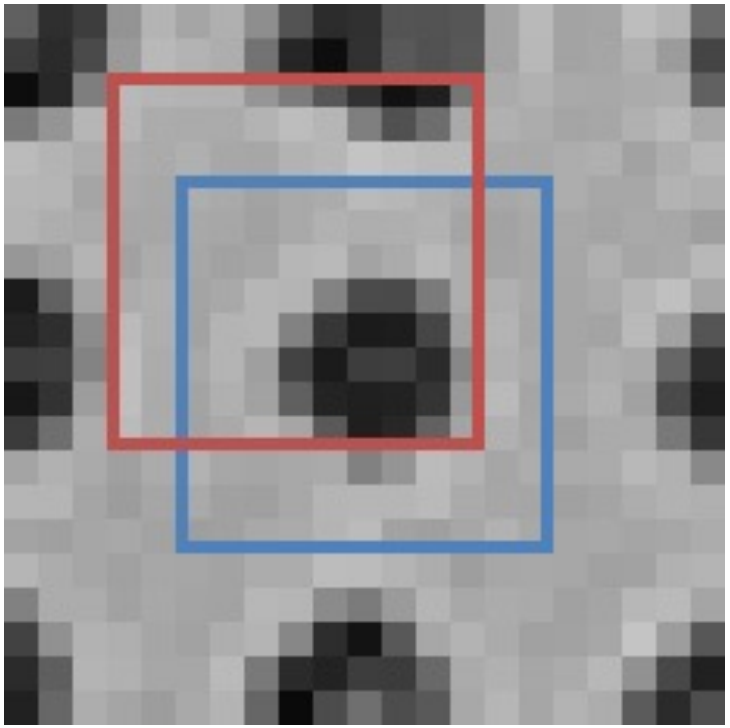


"corner": significant change in all directions

Diagram credit: S. Lazebnik

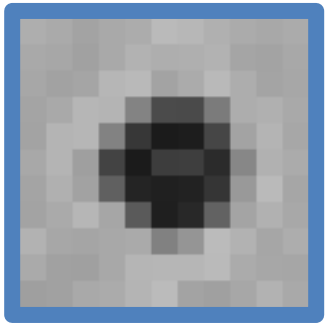
Last Time – Corner Detection

Zoom-In at x,y



Window with and w/o Offset

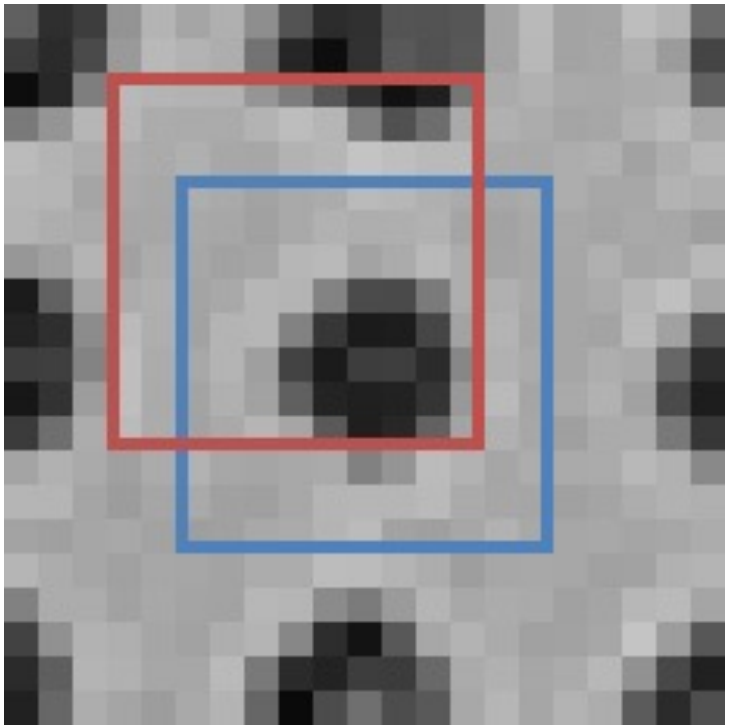
“Window”
At x+u, y+v
Here: u=-2,v=-3



“Window”
At x, y

Last Time – Corner Detection

Zoom-In at x,y



Error (Sum Sqs) for u,v offset

$$E(u,v) = \sum_{(x,y)\in W} (I[x+u, y+v] - I[x,y])^2$$

$$\left(\quad - \quad \right)^2$$

Formalizing Corner Detection

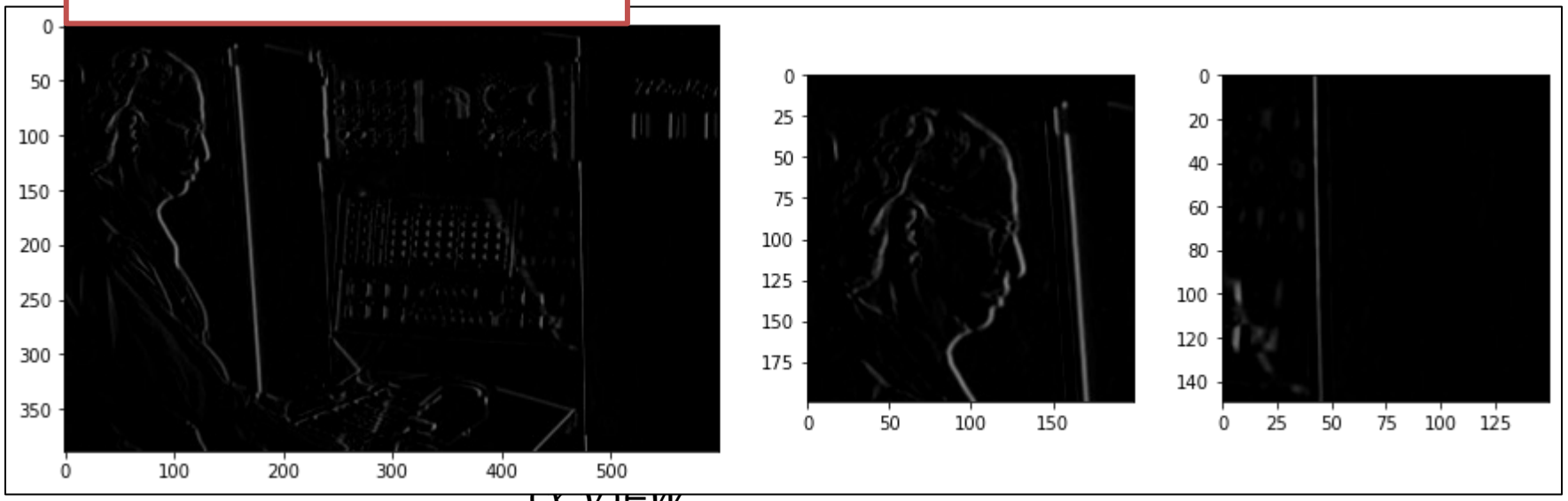
By linearizing image, we can approximate E(u,v) with quadratic function of u and v

$$E(u,v) \approx \sum_{(x,y)\in W} \left(I_x^2 u^2 + 2I_x I_y uv + I_y^2 v^2\right)$$

$$= [u,v]\boldsymbol{M}[u,v]^T$$

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y\in W} I_x^2 & \sum_{x,y\in W} I_x I_y \\ \sum_{x,y\in W} I_x I_y & \sum_{x,y\in W} I_y^2 \end{bmatrix}$$

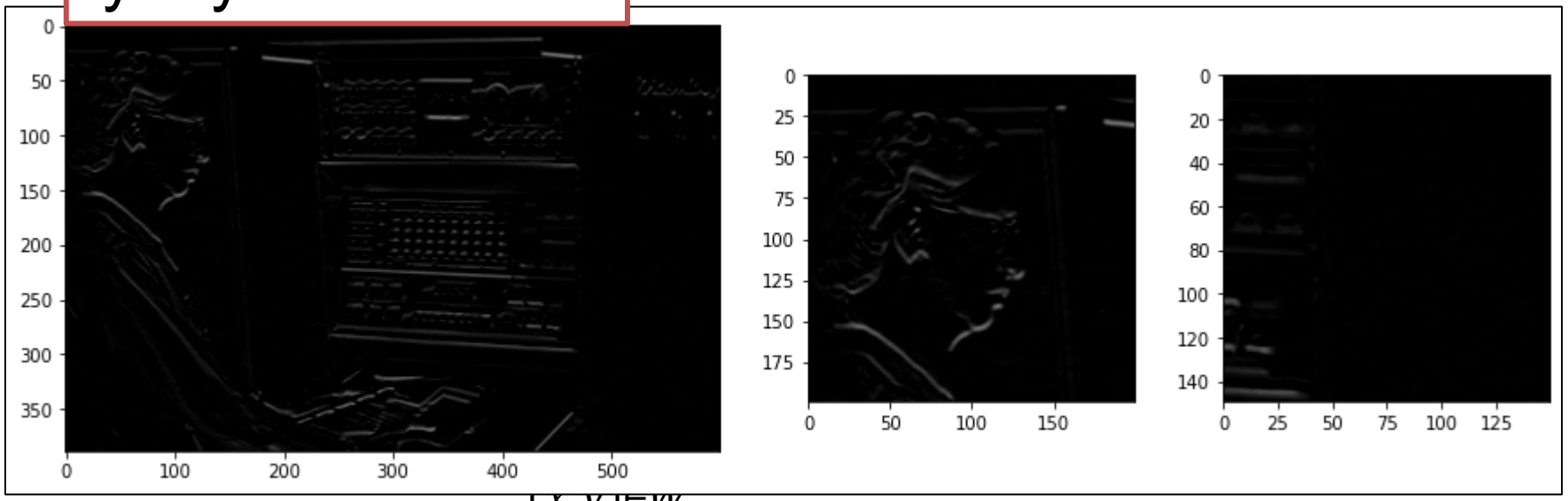
Ix = x derivative



$$\sum_{x,y \in W} = [u, v]\boldsymbol{M}[u, v]^T$$

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix}$$

Iy = y derivative



$$= [u, v]\boldsymbol{M}[u, v]^T$$

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix}$$

Sum goes over all the pixels in window W:

$$\sum_{x,y \in W} I_x^2 = \sum_{x,y \in W} \left(I_x[y,x]\right)^2$$

i.e., sum of squares of x gradients in window

$$= [u,v]\boldsymbol{M}[u,v]^T$$

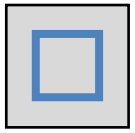
$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix}$$

Intuitively what is M?

Pretend gradients are *either* vertical or horizontal at a pixel (so Ix Iy = 0)

Obviously Wrong!

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} \approx \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

a,b both small: flat  $\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

One big, other small: edge  $\begin{bmatrix} 50 & 0 \\ 0 & 0.1 \end{bmatrix}$ or $\begin{bmatrix} 0.1 & 0 \\ 0 & 50 \end{bmatrix}$

a,b both big: corner  $\begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$

Intuitively what is M?

Pretend gradients are *either* vertical or horizontal at a pixel (so Ix Iy = 0)

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} \approx ? \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

a,b both small: flat

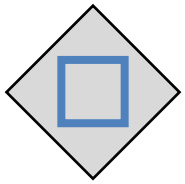


Image might be rotated by rotation θ!

One big, other small: edge

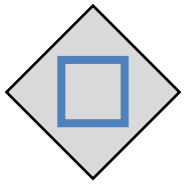
a,b both big: corner

Intuitively what is M?

Pretend gradients are *either* vertical or horizontal at a pixel (so Ix Iy = 0)

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

a,b both small: flat



If image rotated by rotation θ / matrix **V**

One big, other small: edge

M will look like

$$\boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

a,b both big: corner

So What Now?

Can calculate M at pixel, by summing nearby gradients, but need access to a and b.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

Given **M**, can decompose it into eigenvectors **V** and eigenvalues λ_1, λ_2 with $\mathbf{M} = \boldsymbol{V}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$.

Really slow. Why?

So What Now?

Can calculate M at pixel, by summing nearby gradients, but need access to a and b.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

Instead: compute quantity R from **M**

$$R = \det(\boldsymbol{M}) - \alpha \operatorname{trace}(\boldsymbol{M})^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

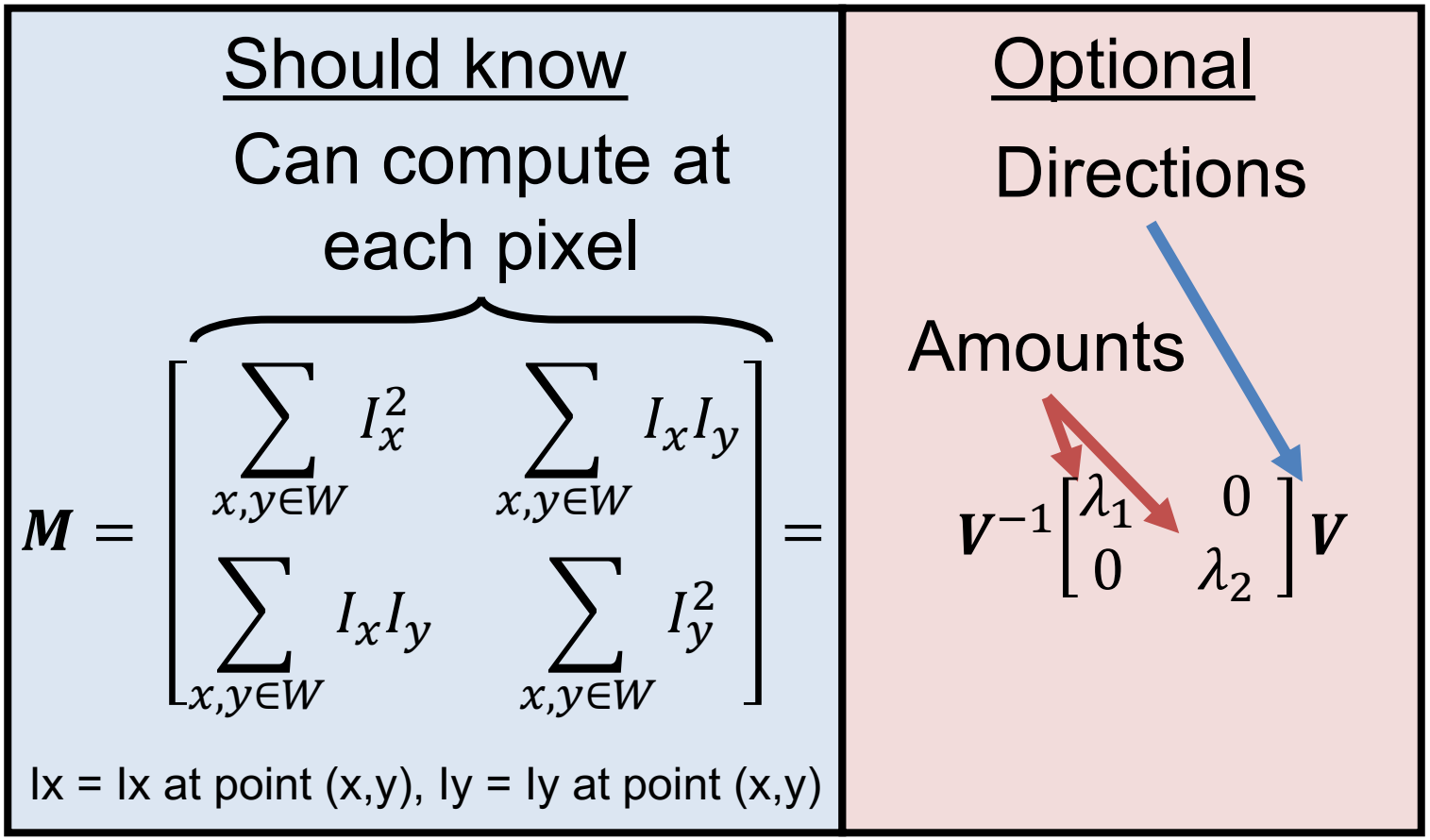
Easy fast formula for 2x2

Fast – sum the diagonal

Empirical value, usually 0.04-0.06

The tl;dr

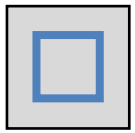
TL;DR: Taylor expansion for error E(u,v). All terms in equation are sums of image gradients and in **M**



Putting It Together

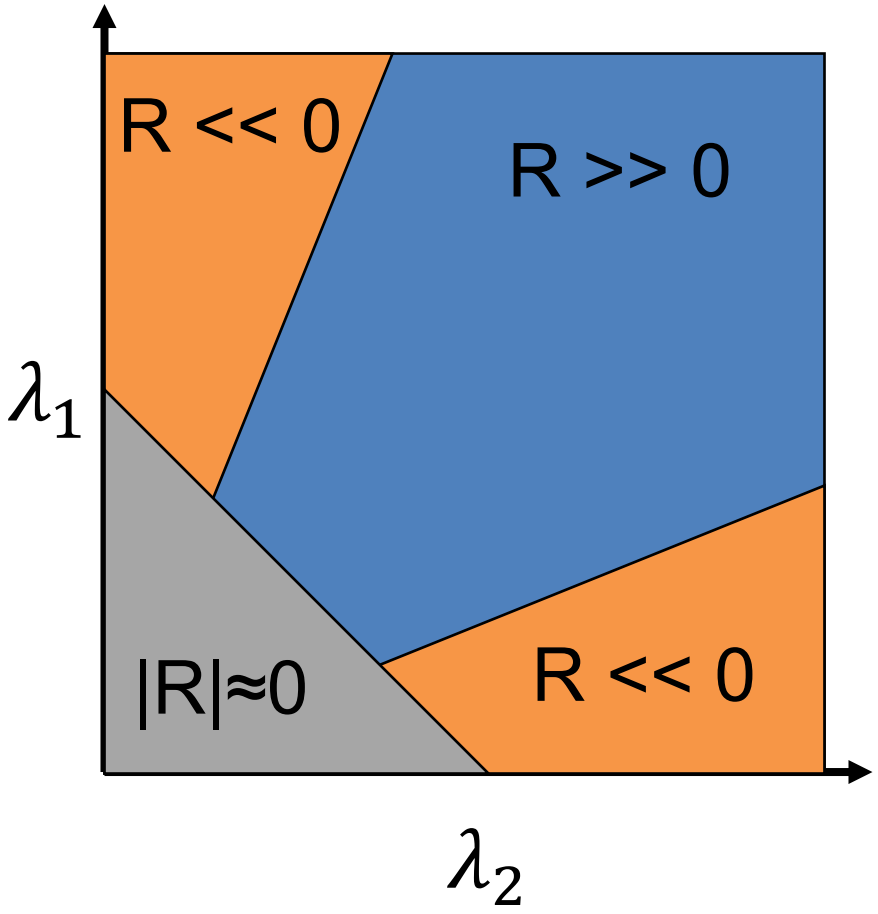
$$R = \det(\boldsymbol{M}) - \alpha \operatorname{trace}(\boldsymbol{M})^2 = \lambda_1\lambda_2 - \alpha\,(\lambda_1 + \lambda_2)^2$$

det, trace are *fast*

flat  $\lambda_1, \lambda_2 \approx 0$

edge  $\lambda_1 \gg \lambda_2 \gg 0$
$\lambda_2 \gg \lambda_1 \gg 0$

corner  $\lambda_1 \approx \lambda_2 \gg 0$



Remake of standard diagram from S. Lazebnik from original Harris paper.

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} w(x,y) I_x^2 & \sum_{x,y \in W} w(x,y) I_x I_y \\ \sum_{x,y \in W} w(x,y) I_x I_y & \sum_{x,y \in W} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R

$$R = \det(\boldsymbol{M}) - \alpha\, trace(\boldsymbol{M})^2$$
$$= \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
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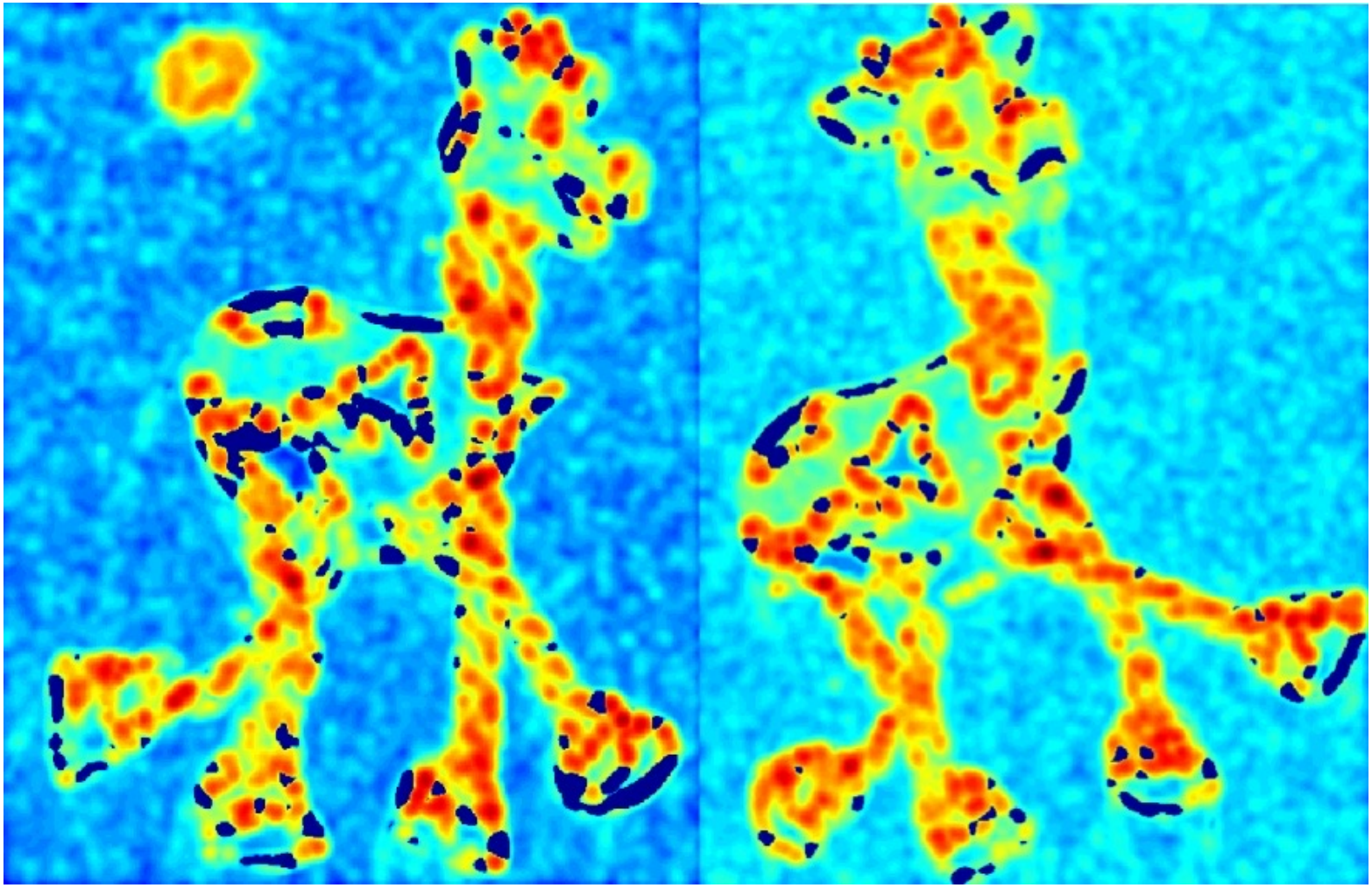
Slide credit: S. Lazebnik

Computing R

Slide credit: S. Lazebnik

Computing R



Slide credit: S. Lazebnik

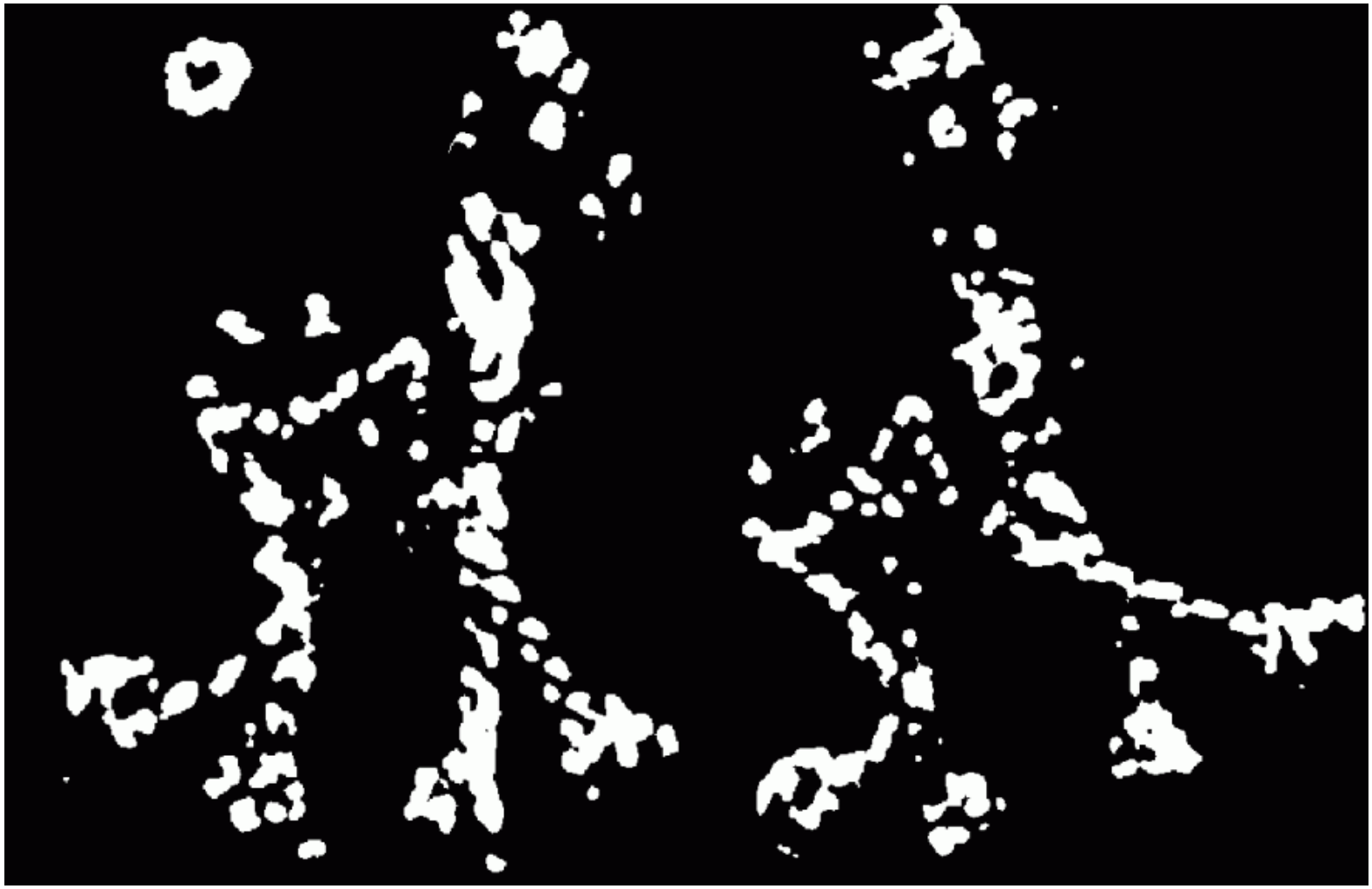
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded R



Slide credit: S. Lazebnik

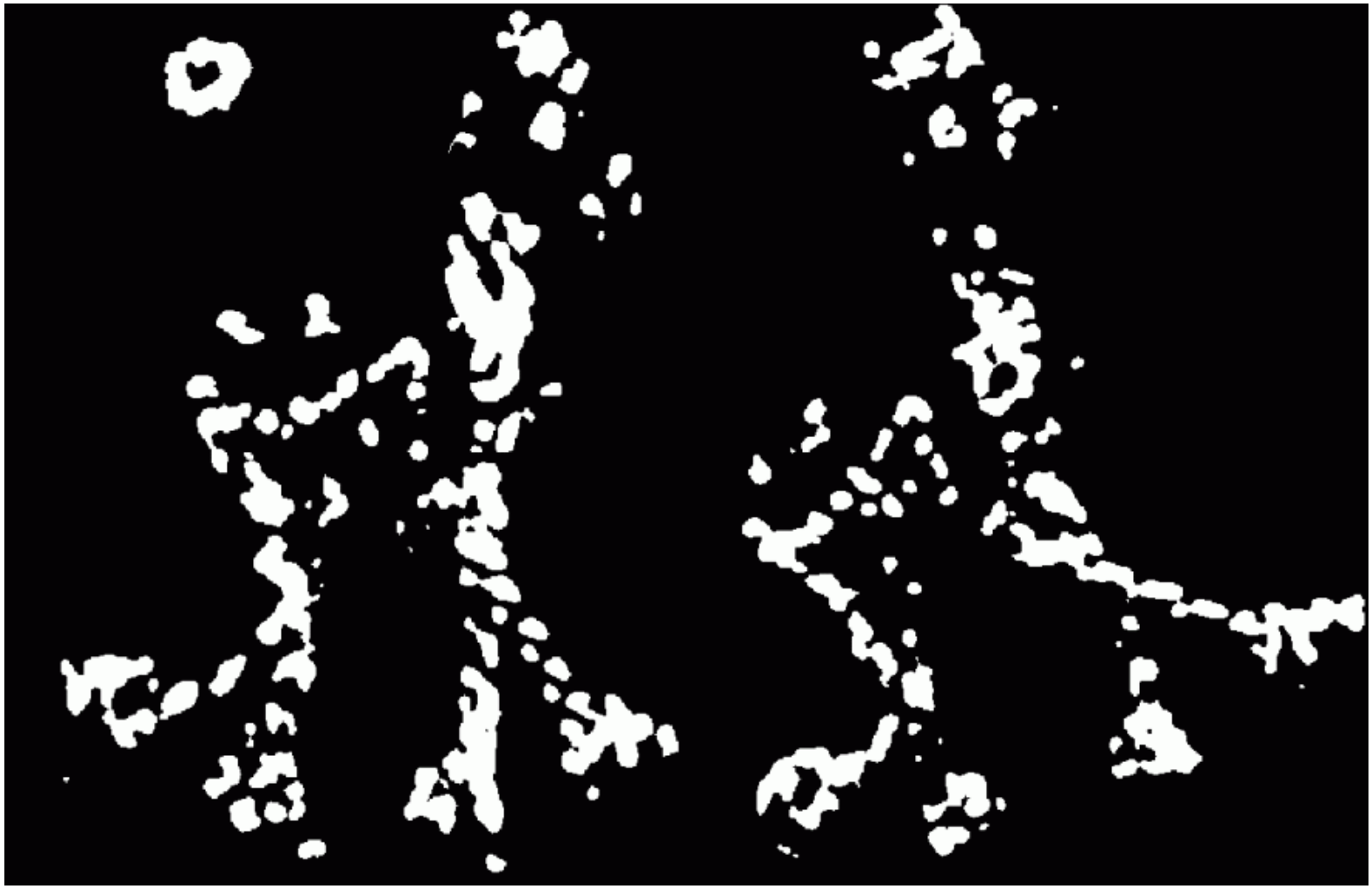
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R
5. Take only local maxima (called non-maxima suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded



Slide credit: S. Lazebnik

Final Results

Slide credit: S. Lazebnik

Desirable Properties

If our detectors are repeatable, they should be:

- **Invariant** to some things: image is transformed and corners remain the same
- **Covariant/equivariant** with some things: image is transformed and corners transform with it.

Slide credit: S. Lazebnik

Recall Motivating Problem

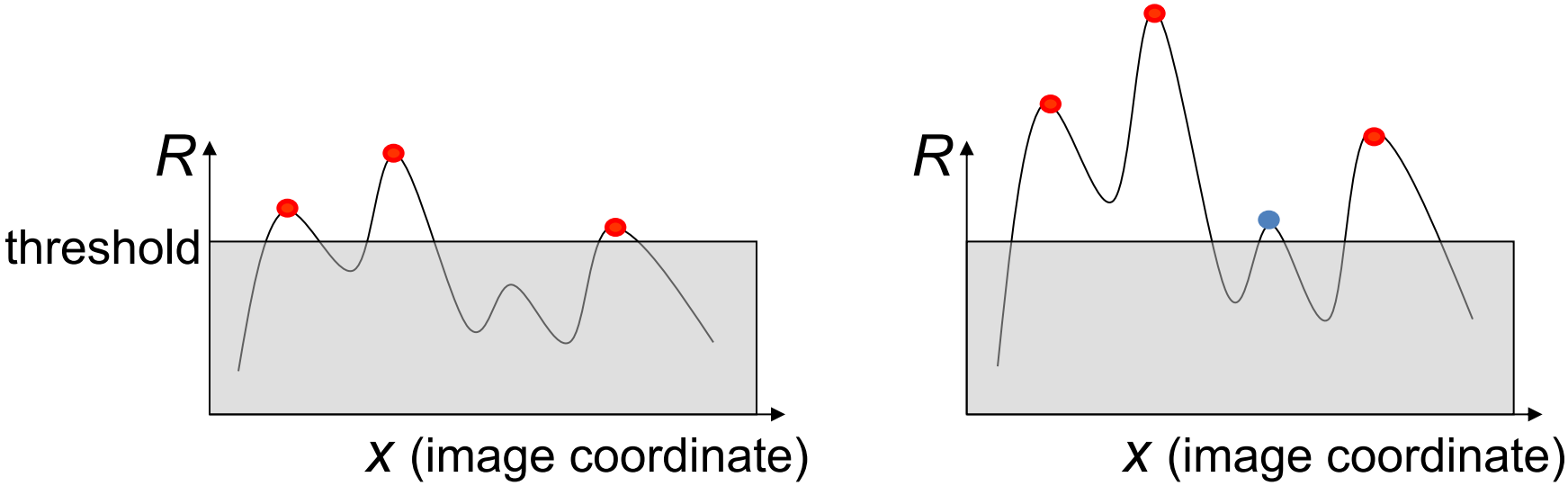
Images may be different in lighting and geometry

Affine Intensity Change

$$I_{new} = aI_{old} + b$$

M only depends on derivatives, so *b* is irrelevant

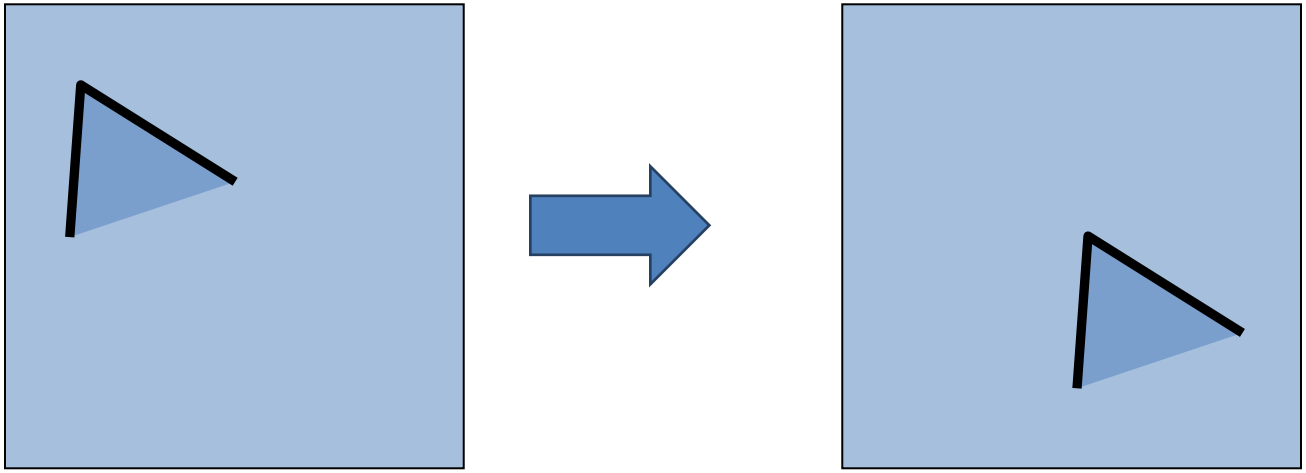
But *a* scales derivatives and there's a threshold



Partially invariant to affine intensity changes

Slide credit: S. Lazebnik

Image Translation

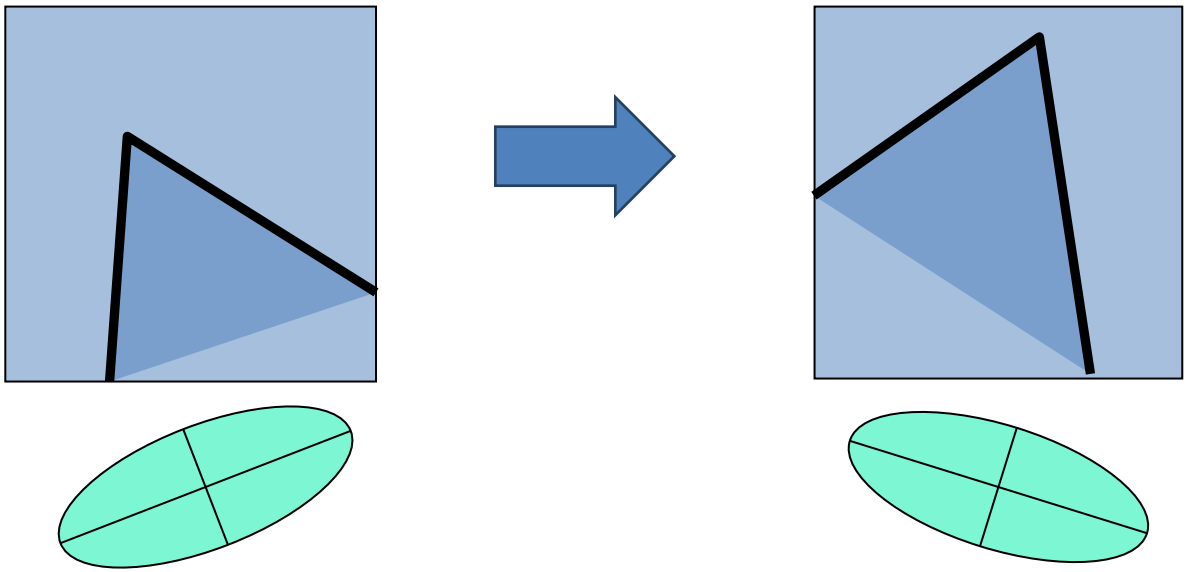


All done with convolution. Convolution is translation equivariant.

Equivariant with translation

Slide credit: S. Lazebnik

Image Rotation

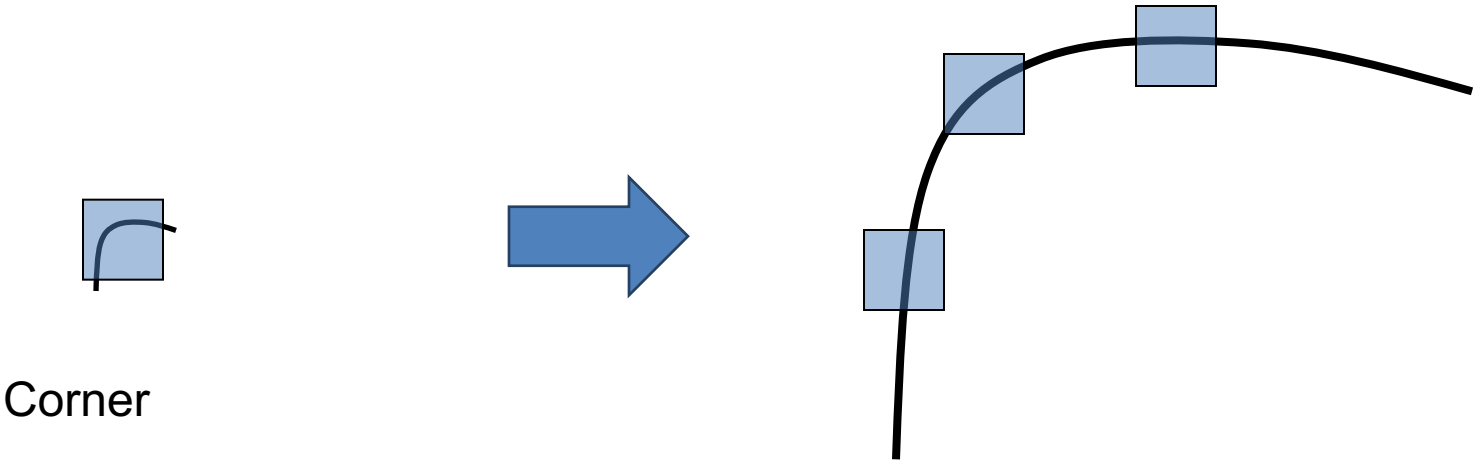


Rotations just cause the corner rotation matrix to change. Eigenvalues remain the same.

Equivariant with rotation

Slide credit: S. Lazebnik

Image Scaling



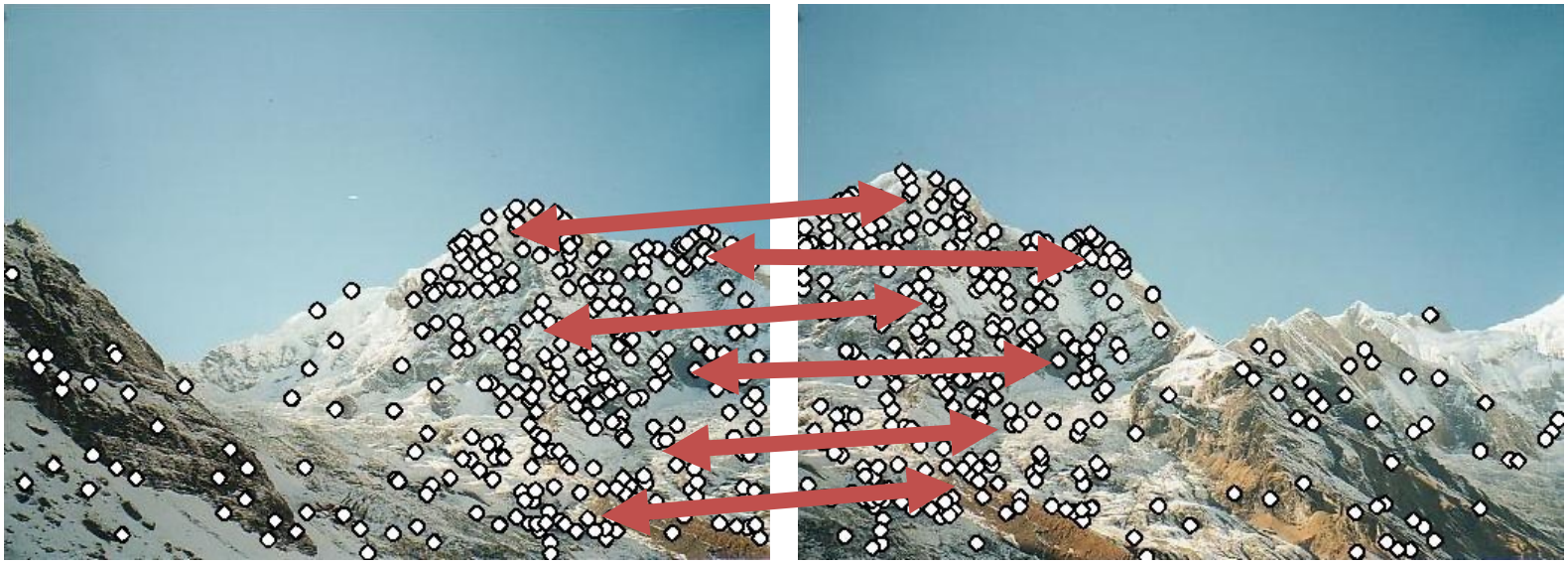
Corner in one scale → Edges in another scale

Not equivariant with scaling

How do we fix this?

Slide credit: S. Lazebnik

Recap: Motivation



1: find corners+features

2: match based on local image data

How?

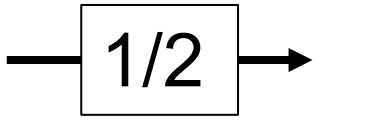
Image credit: M. Brown

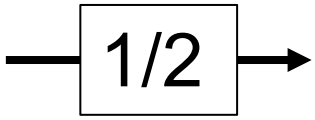
Today

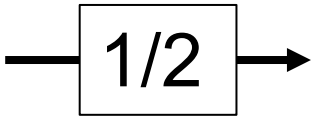
- Fixing scaling by making detectors in both location **and scale**
- Enabling matching between features by **describing regions**

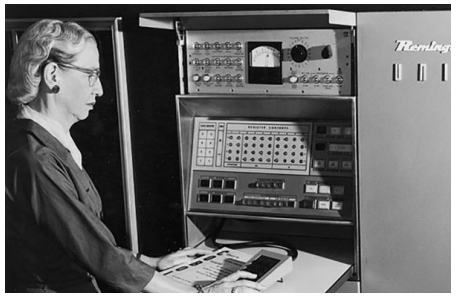
Key Idea: Scale Space

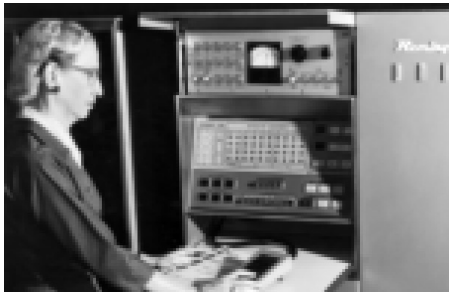
Left to right: each image is half-sized
Upsampled with big pixels below

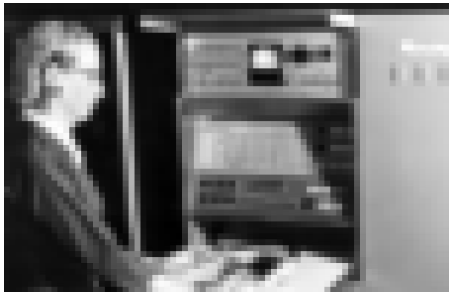


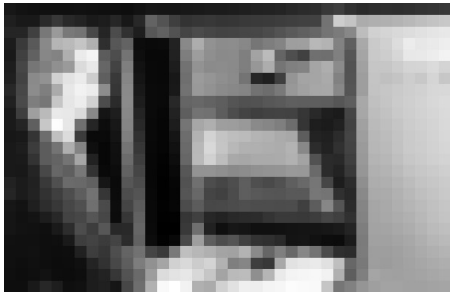










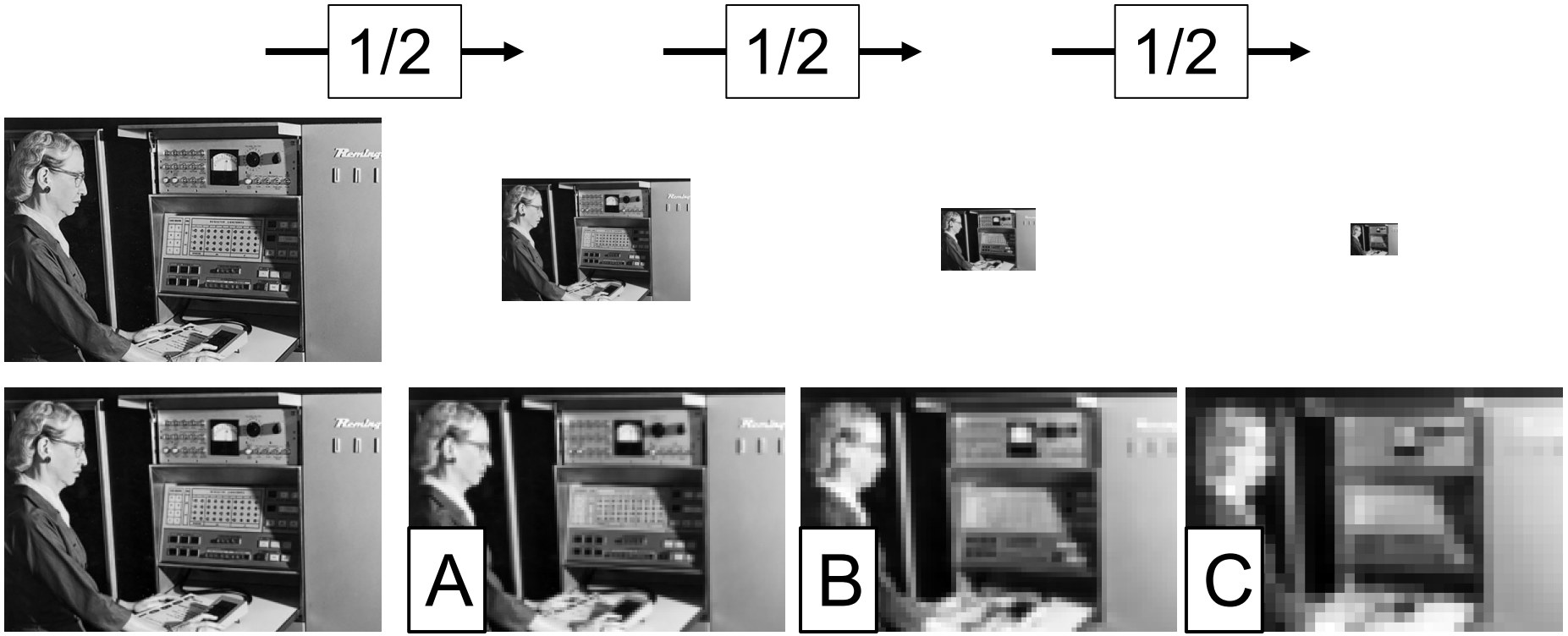


Note: I'm also slightly blurring to prevent aliasing (https://en.wikipedia.org/wiki/Aliasing)

Key Idea: Scale Space

Left to right: each image is half-sized

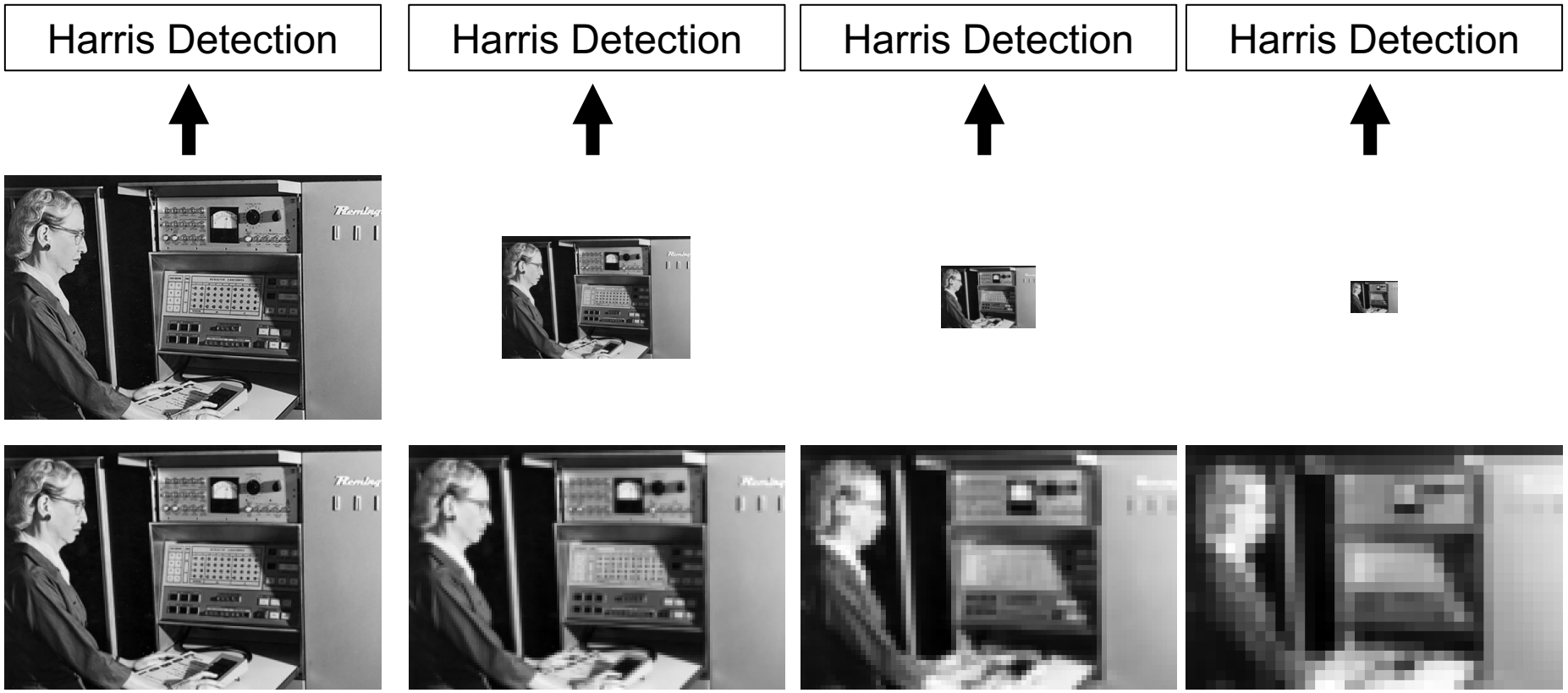
If I apply a KxK filter, how much of information of the original image does it see in each image?



Note: I'm also slightly blurring to prevent aliasing (https://en.wikipedia.org/wiki/Aliasing)

Solution to Scales

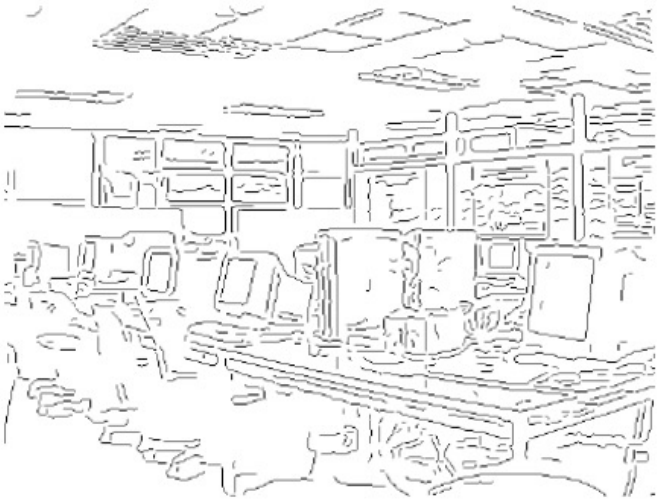
Try them all!



See: Multi-Image Matching using Multi-Scale Oriented Patches, Brown et al. CVPR 2005

What about the Internal Structures?

Edges & Corners convey boundary information



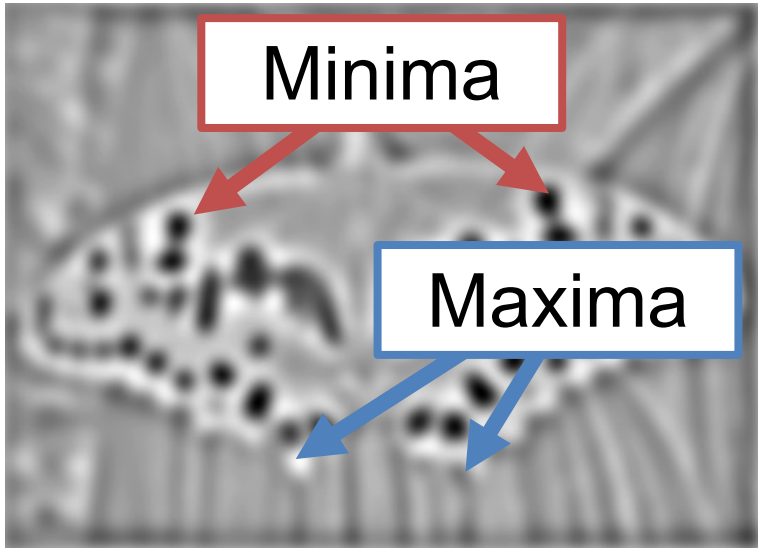
What about interior texture of the object?

Blob Detection

Another detector: Detect and describe “Regions”

*

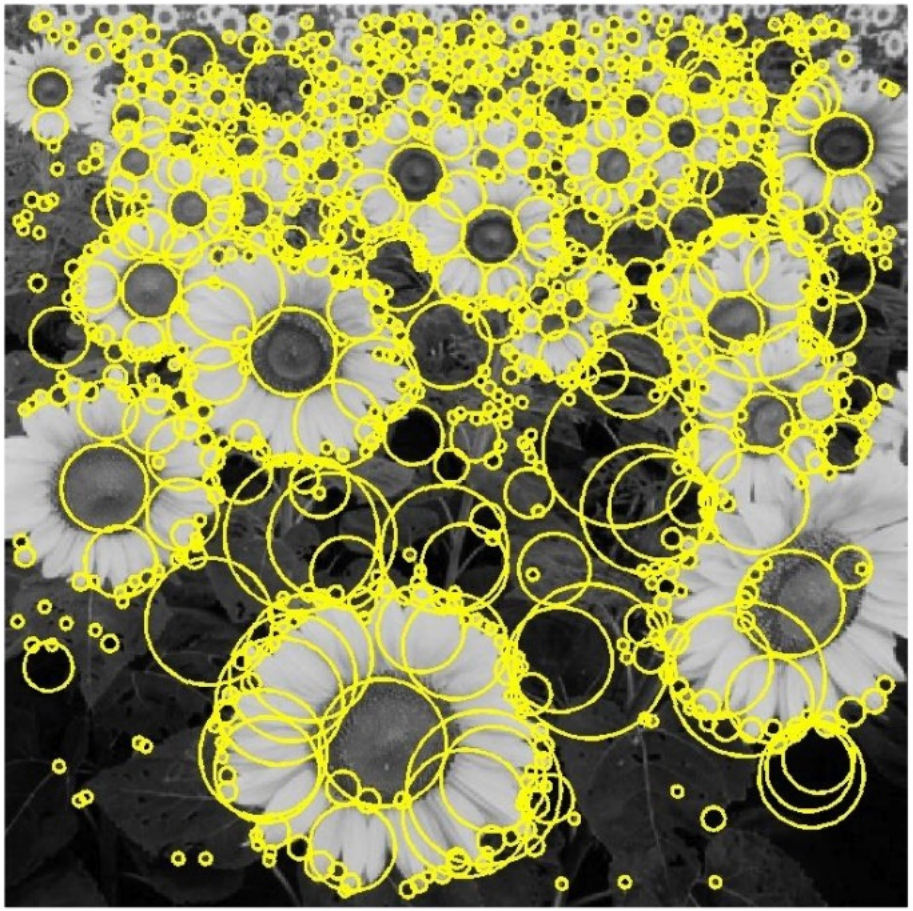
=



Find maxima ***and minima*** of blob filter response in ***scale and space***

Slide credit: N. Snavely

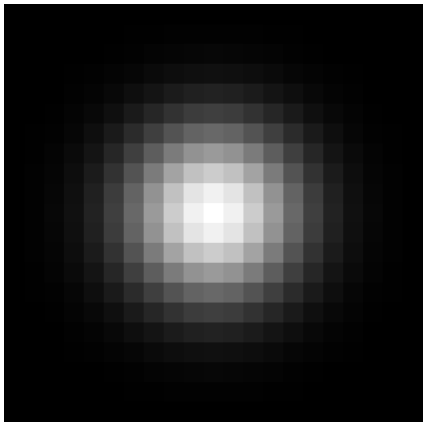
Blob Detection with Scale Selection



Slide credit: N. Snavely

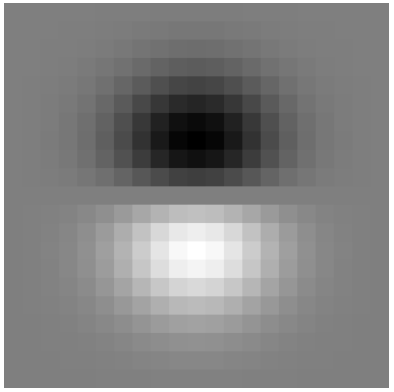
Gaussian Derivatives

Gaussian

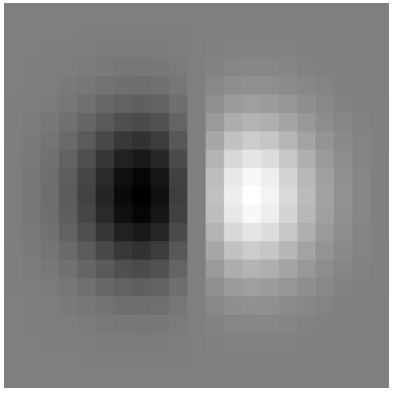


1st Deriv

$\frac{\partial}{\partial y} g$

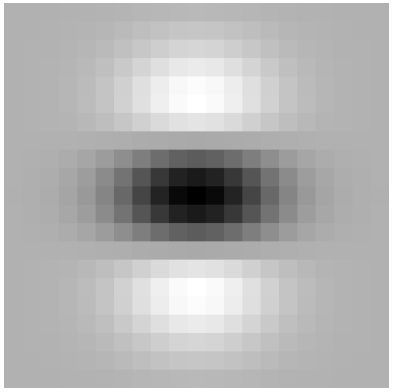


$\frac{\partial}{\partial x} g$

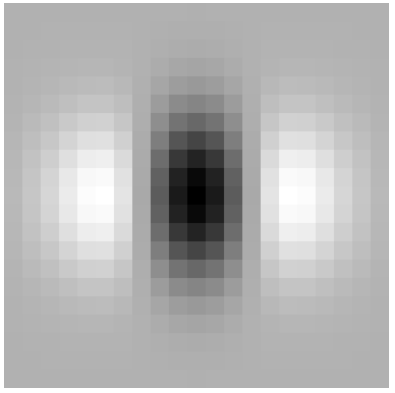


2nd Deriv

$\frac{\partial^2}{\partial^2 y} g$

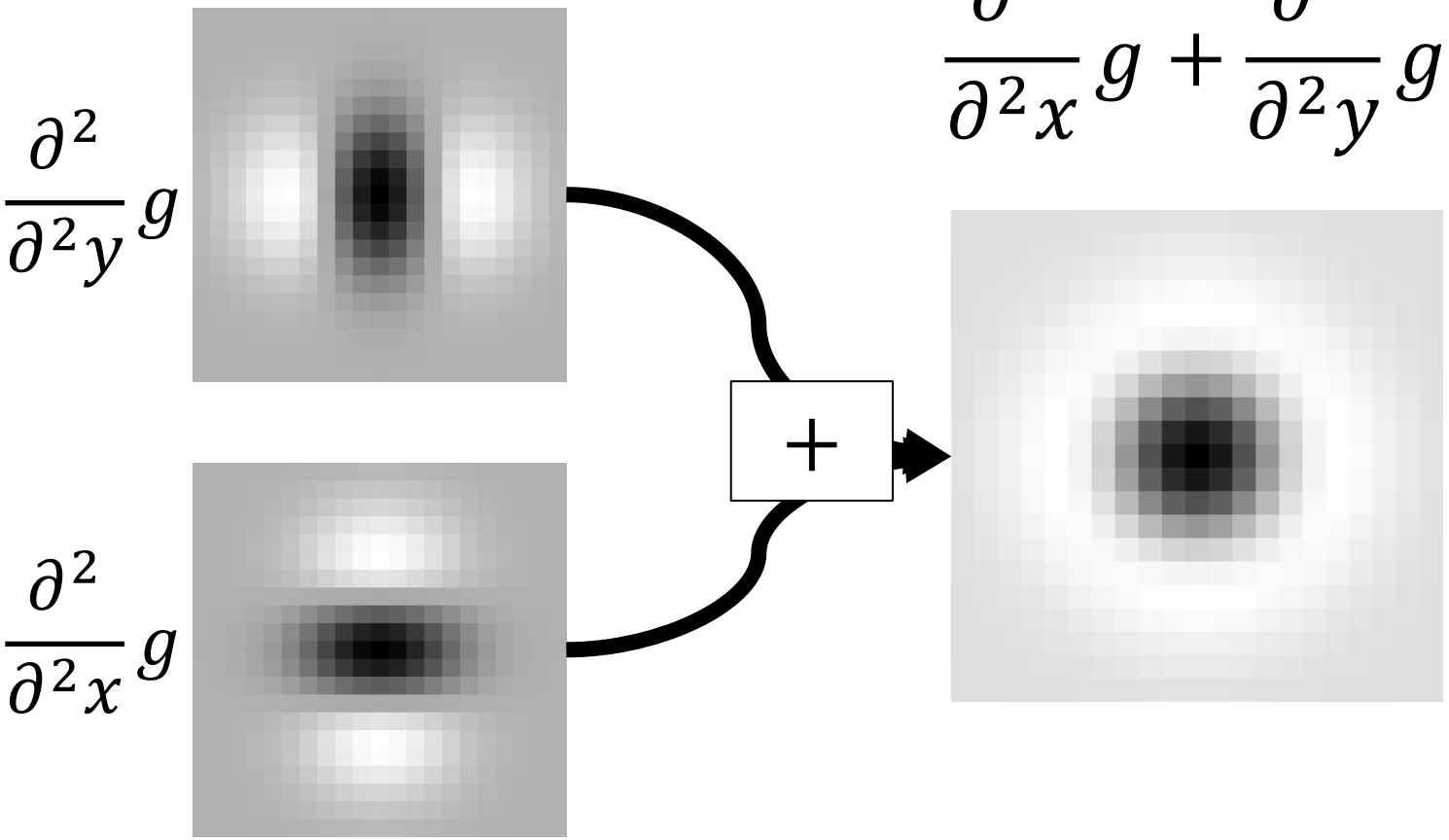


$\frac{\partial^2}{\partial^2 x} g$



Laplacian of Gaussian (LoG)

$$\frac{\partial^2}{\partial^2 x} g + \frac{\partial^2}{\partial^2 y} g$$

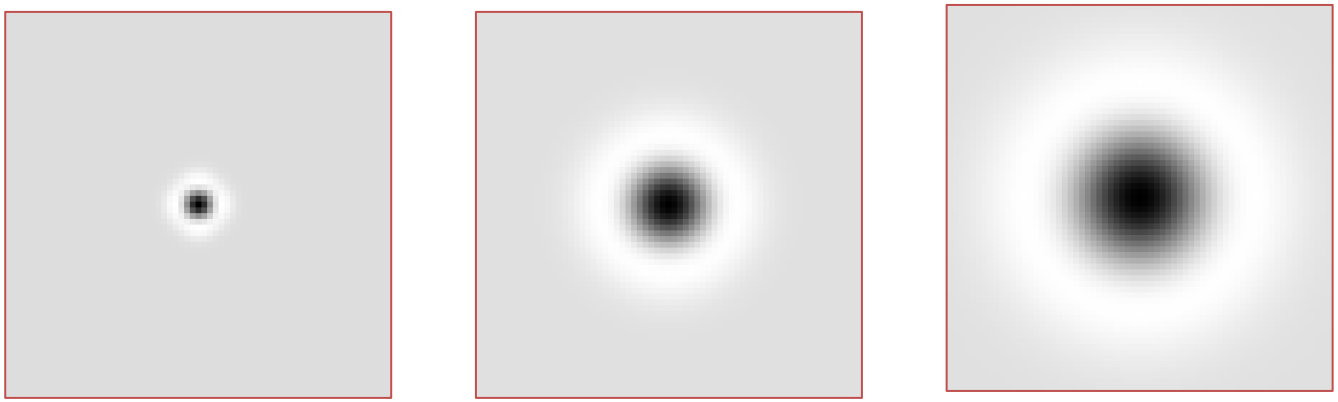


Slight detail: for technical reasons, you need to scale the Laplacian of Gaussian if you want to compare across sigmas.

$$\nabla^2_{norm} = \sigma^2 \left(\frac{\partial^2}{\partial x^2} g + \frac{\partial^2}{\partial^2 y} g \right)$$

Size of Laplacian of Gaussian

$\sigma = 2$ $\sigma = 6$ $\sigma = 10$

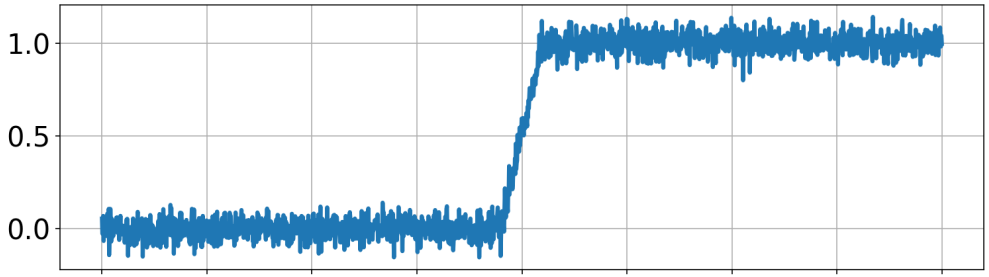


Slight detail: for technical reasons, you need to scale the Laplacian of Gaussian if you want to compare across sigmas.

$$\nabla^2_{norm} = \sigma^2 \left(\frac{\partial^2}{\partial x^2} g + \frac{\partial^2}{\partial^2 y} g \right)$$

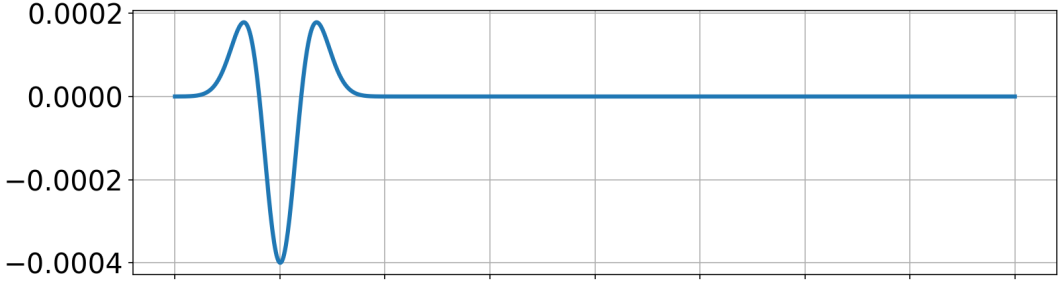
Edge Detection with LoG

f



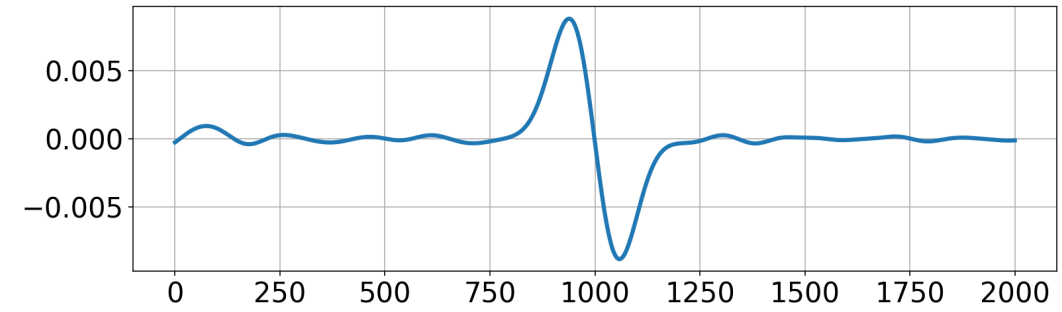
Edge

$$\frac{\partial^2}{\partial^2 x} g$$



Laplacian Of Gaussian (LoG)

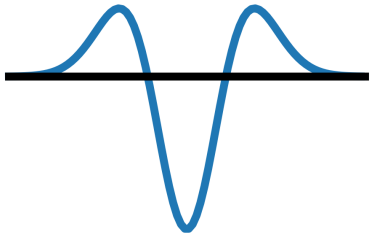
$$f * \frac{\partial^2}{\partial^2 x} g$$



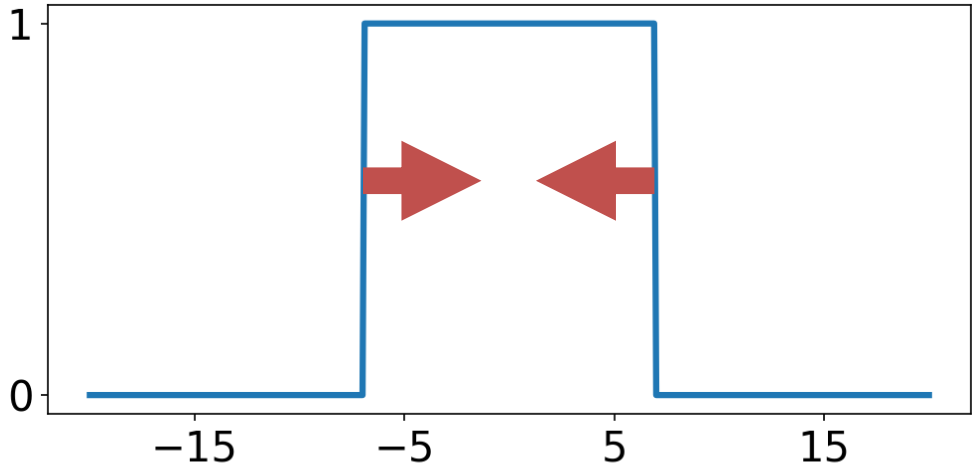
Edge = Zero-crossing Riple

Modern remake of classic S. Seitz slide

Blob Detection with LoG

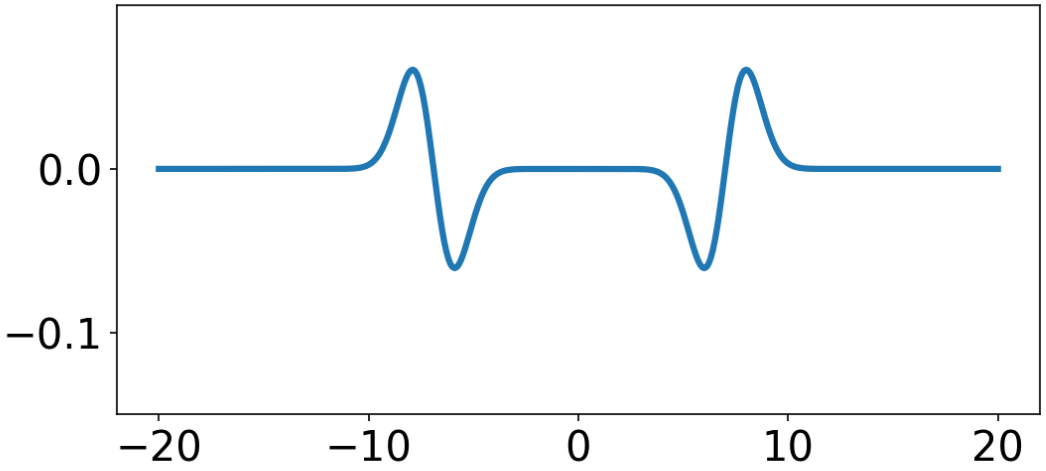


f



Edges

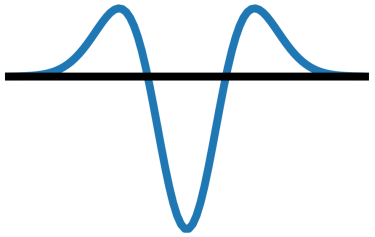
$$f * \frac{\partial^2}{\partial^2 x} g$$



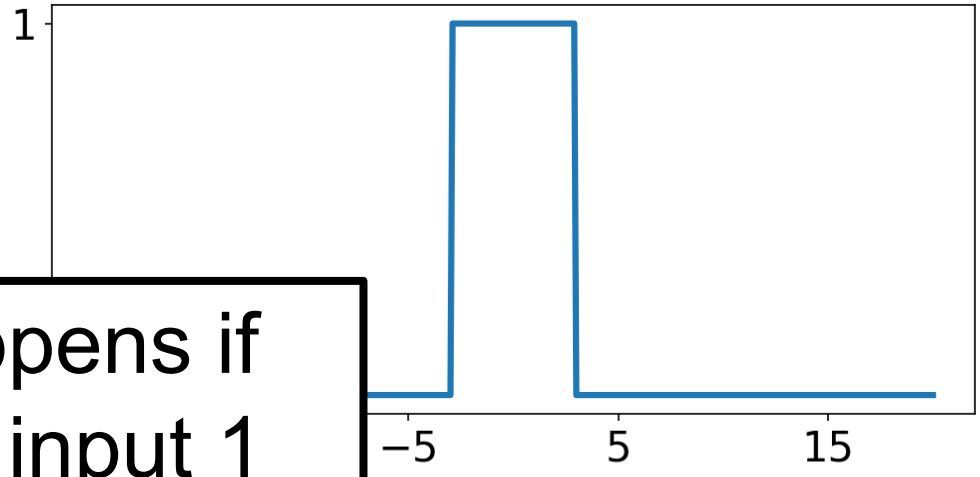
Blob * LoG =
Zero-crossings
Superposition
of Riples

Modern remake of classic S. Seitz slide

Blob Detection with LoG



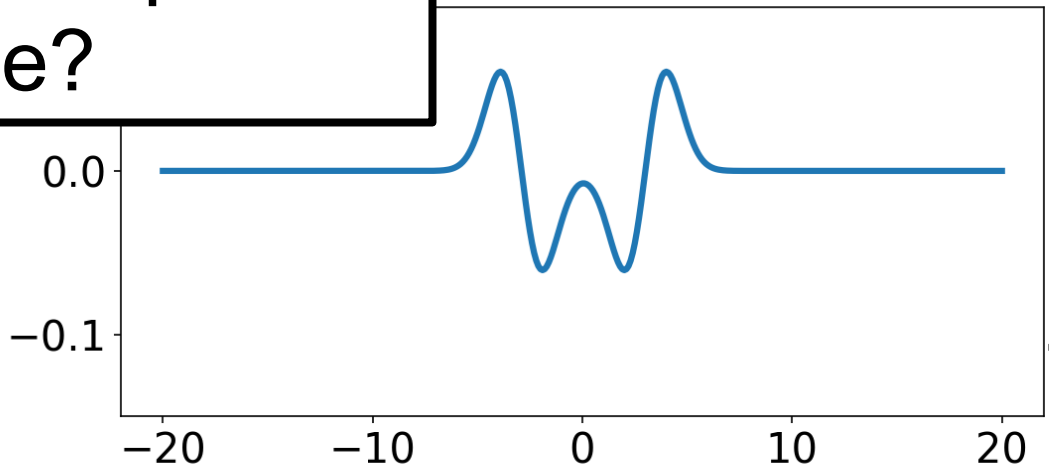
f



Edges

What happens if we make input 1 unit wide?

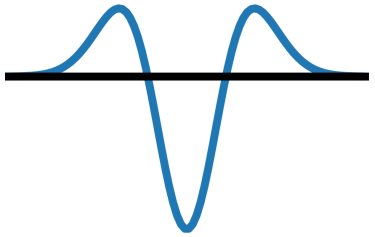
$$f * \frac{\partial^2}{\partial^2 x} g$$



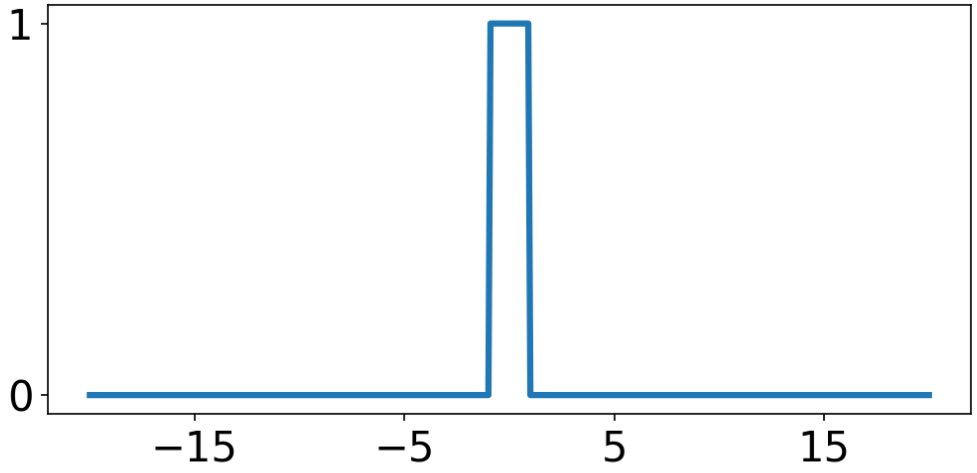
Edges * LoG = Zero-crossings Superposition of Riples

Modern remake of classic S. Seitz slide

Blob Detection with LoG

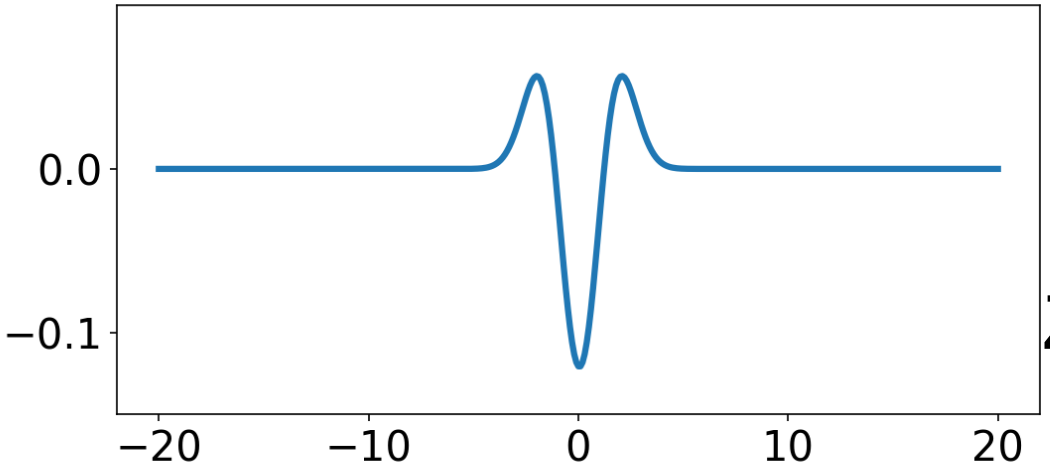


f



Edge

$$f * \frac{\partial^2}{\partial^2 x} g$$



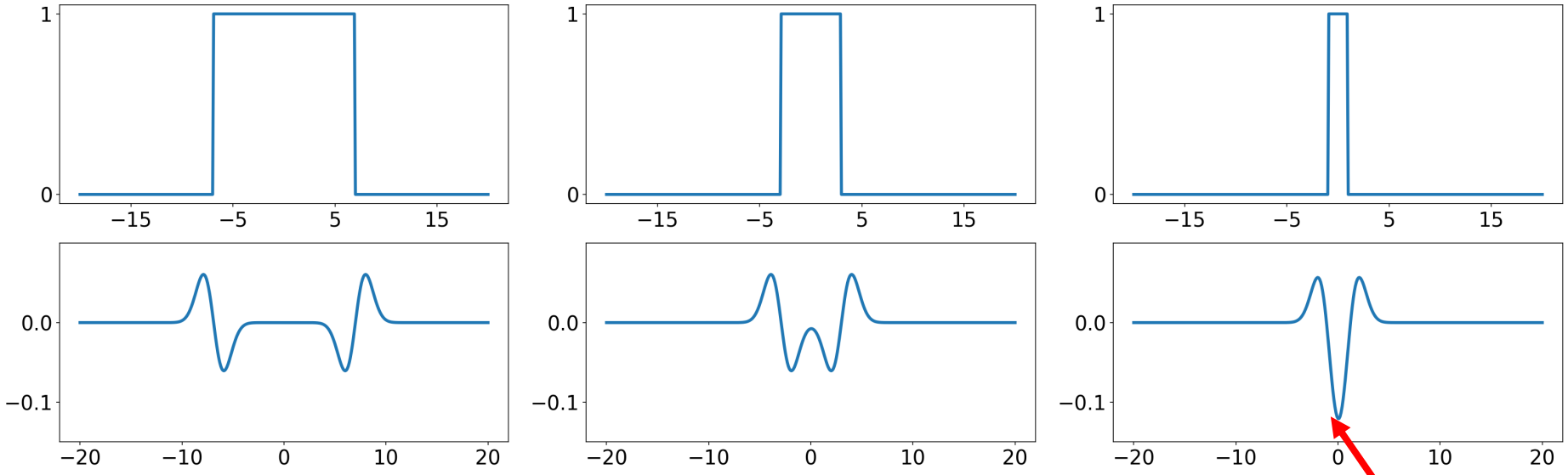
Edge * LoG = Zero-crossings Constructive Interference

Modern remake of classic S. Seitz slide

Blob Detection with LoG

Each LoG kernel has a specific scale that responds highly

Original Signal

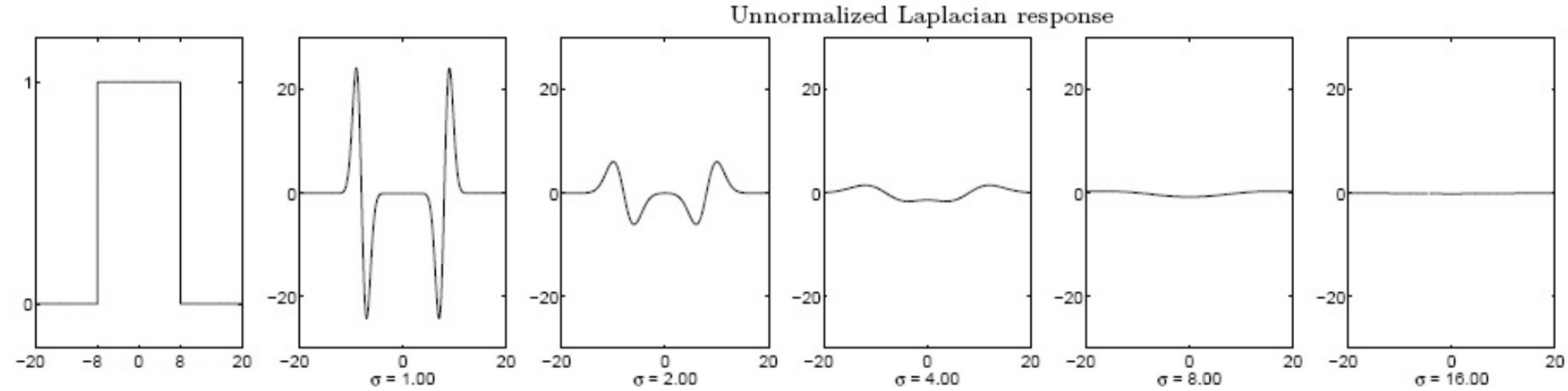


Convolved with Laplacian ($\sigma = 1$)

Maximum

Scale Selection

- Convolve with Laplacians at several scales and look for the maximum response
- However, Laplacian response decays as scale increases: energy dispersed widely



Original Signal (Radius=8)

Increasing σ →

Slide credit: S. Lazebnik

Scale Normalization

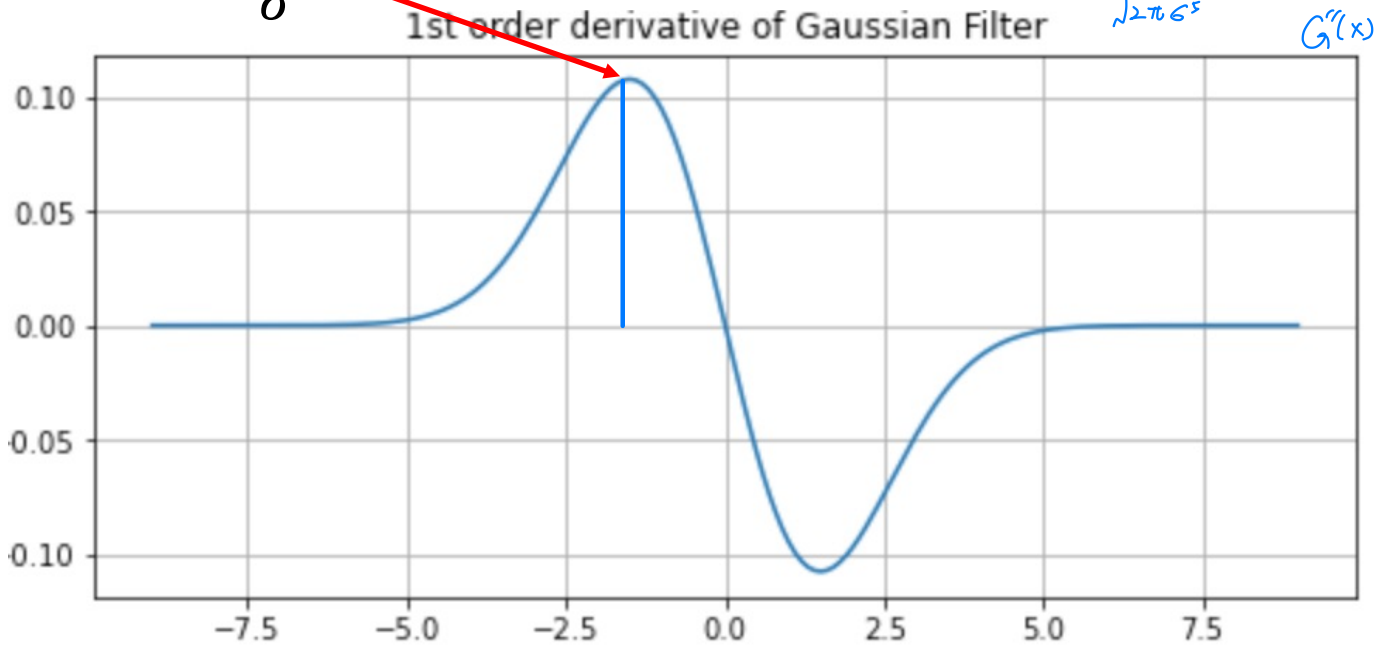
- The response of a derivative of Gaussian filter to an edge decreases as σ increases

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{x^2}{2\sigma^2}\right)}$$

$$G'(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{x^2}{2\sigma^2}} \cdot \left(-\frac{x}{2\sigma^2}\right) = \frac{-x}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2}{2\sigma^2}}$$

$G''(x)$

Peak height $\propto \frac{1}{\sigma}$

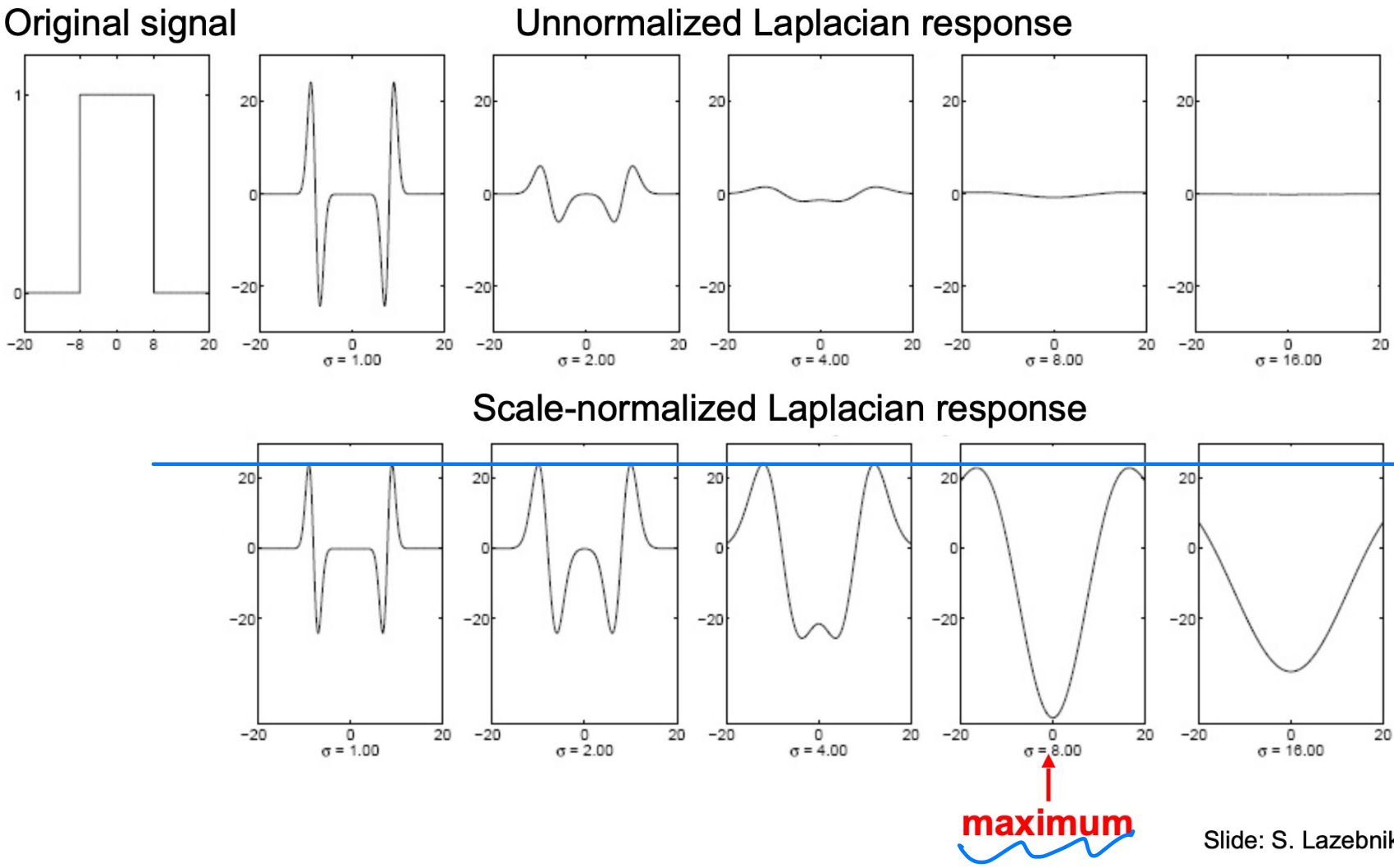


$-\sigma \quad +\sigma$

Scale Normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases
- Peak height $\propto \frac{1}{\sigma}$
- To keep response the same (scale-invariant), must multiply Gaussian derivative by σ
- Laplacian is the second Gaussian derivative, so it must be multiplied by σ^2

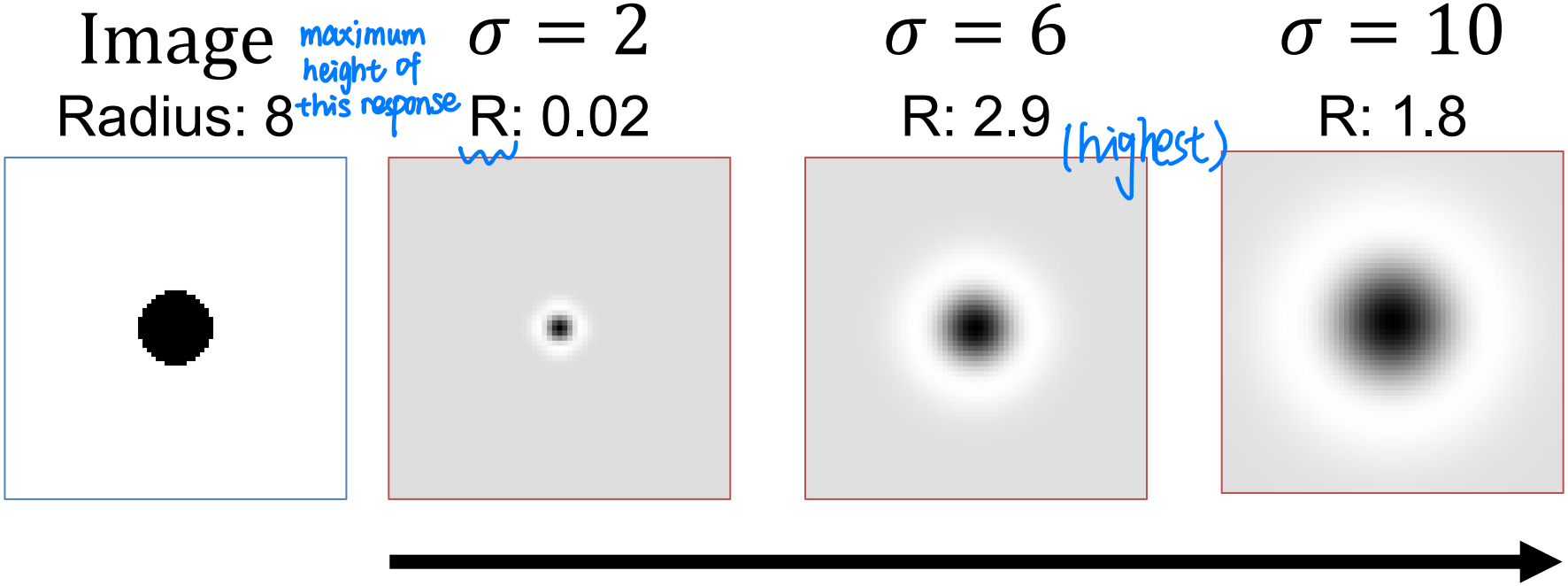
Scale Normalization



Slide: S. Lazebnik

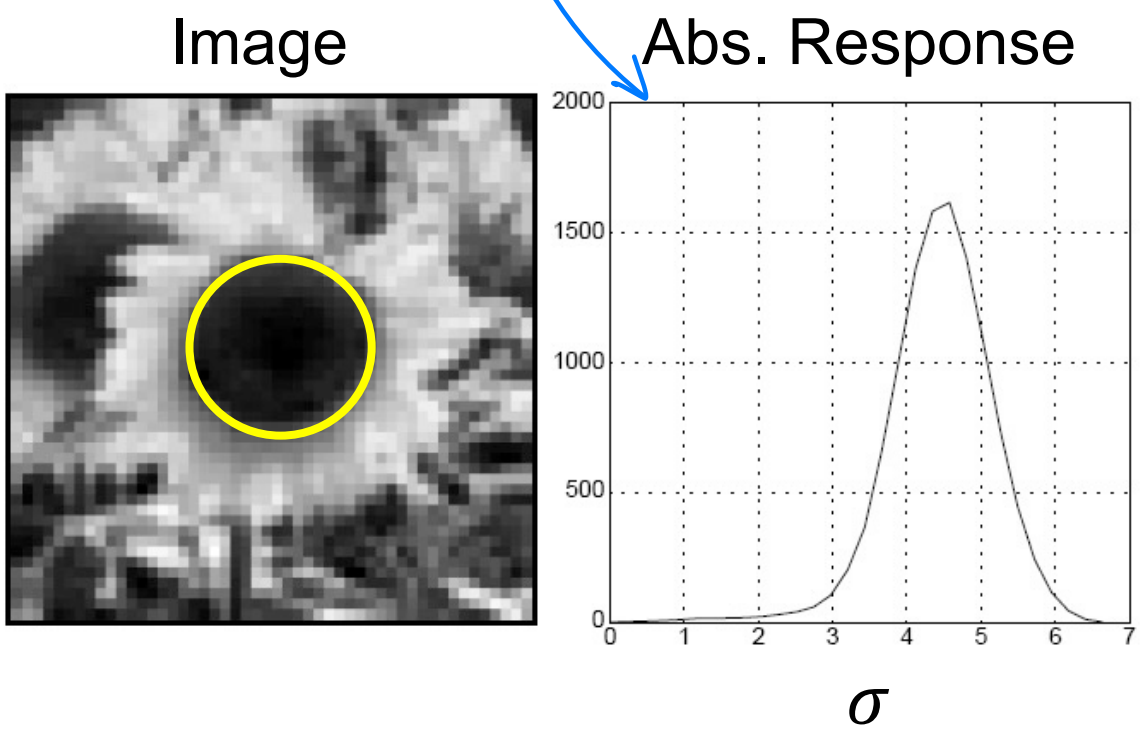
Scale Selection

Given binary circle and Laplacian filter of scale σ, we can compute the response as a function of the scale.



Characteristic Scale

Characteristic scale of a blob is the scale that produces the maximum response



Slide credit: S. Lazebnik. For more, see: T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* **30** (2): pp 77--116.

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

Slide credit: S. Lazebnik

Scale-space blob detector: Example

Slide credit: S. Lazebnik

Scale-space blob detector: Example

sigma = 11.9912

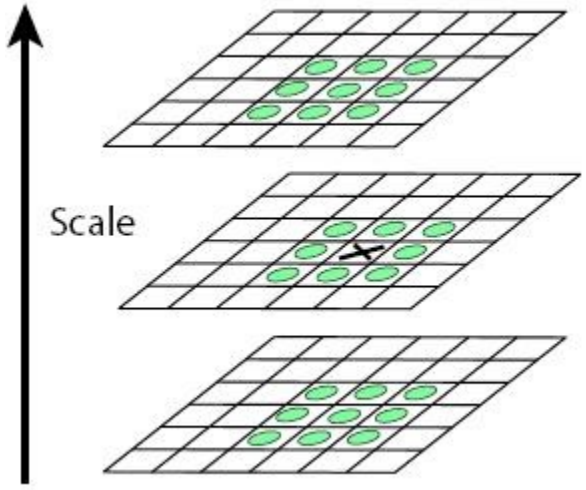
Slide credit: S. Lazebnik

Scale-space blob detector: Example

Slide credit: S. Lazebnik

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find local maxima of squared Laplacian response in scale-space



Slide credit: S. Lazebnik

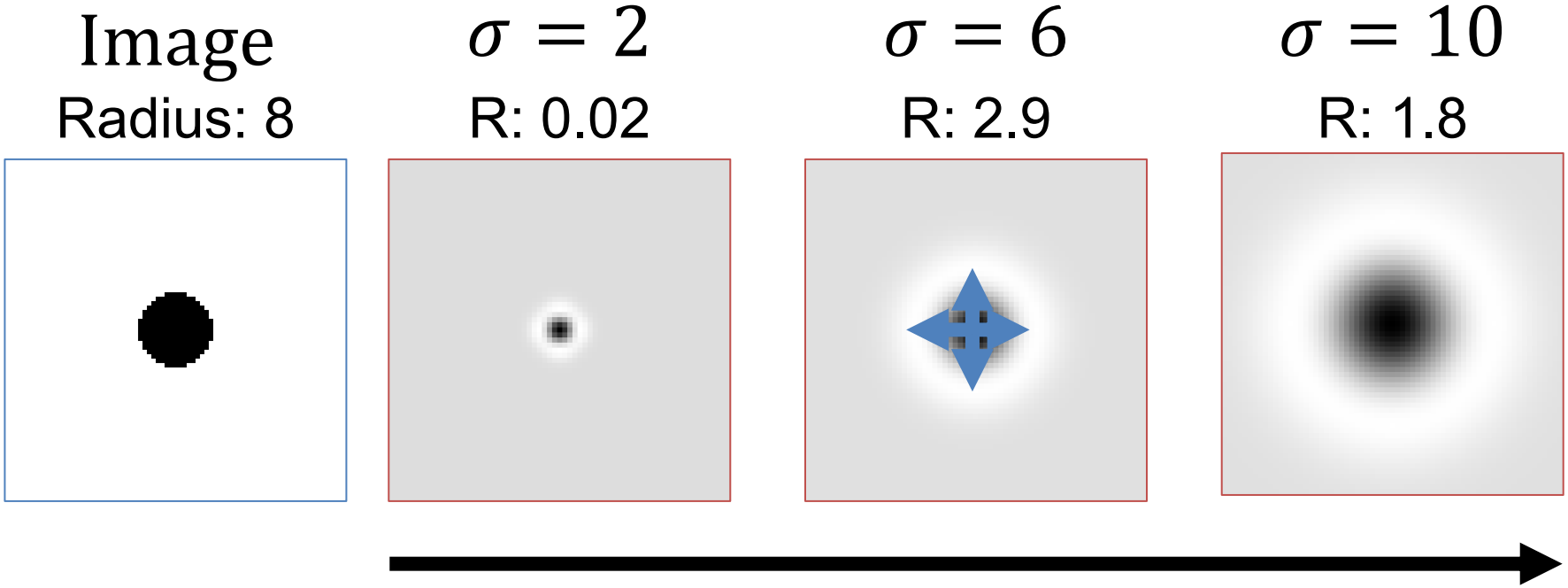
Finding Spatial Maxima

Point i,j is maxima (minima if you flip sign) in image I if it's bigger than all neighbors (8)

```
for y=range(i-1,i+1+1):
        for x in range(j-1,j+1+1):
                if y == i and x== j: continue
                #below has to be true
                I[y,x] < I[i,j]
```

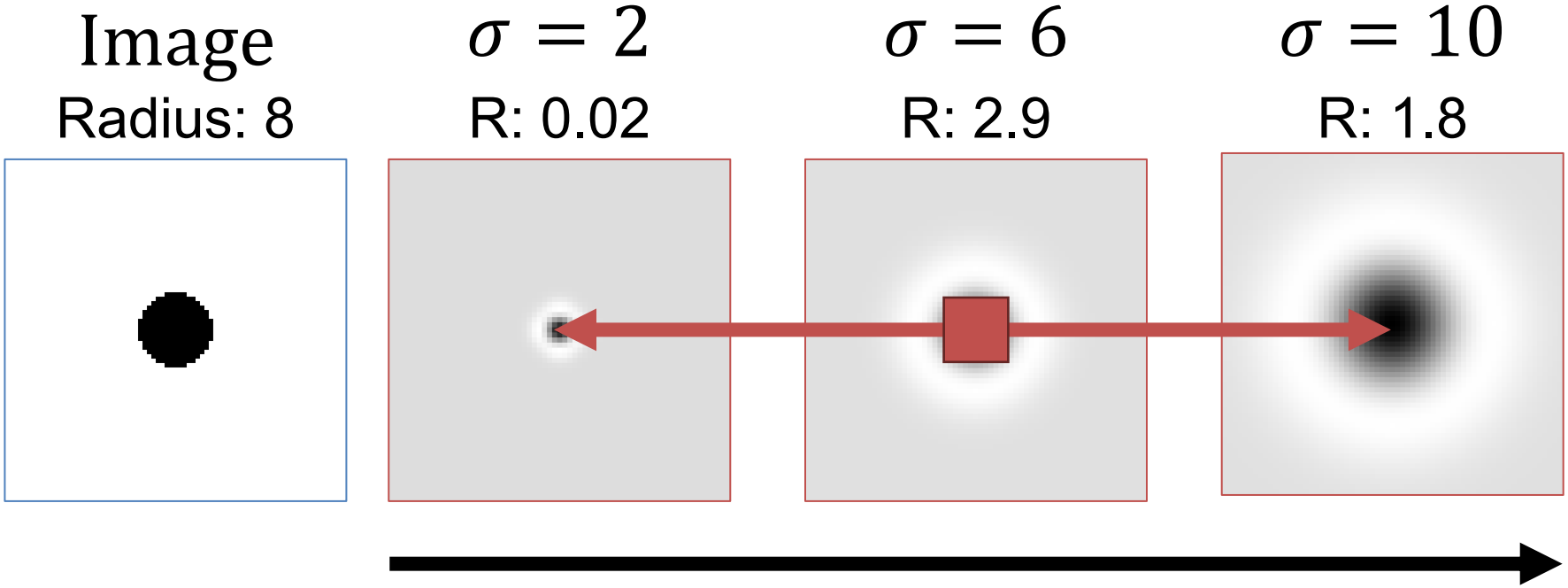
Scale Space

Blue lines are image-space neighbors (should be just one pixel over but that's impossible to draw)



Scale Space

Red lines are the scale-space neighbors

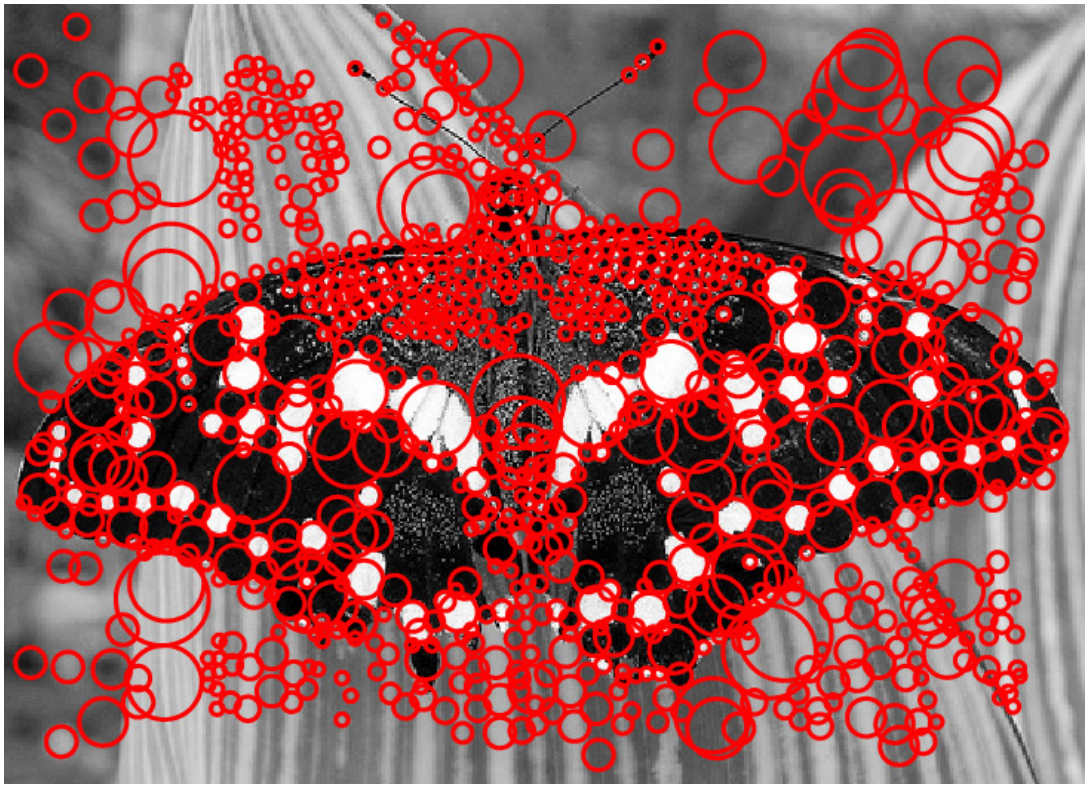


Finding Scale-Space Maxima

Suppose I[:,:,k] is image at scale k. Point i,j,k is maxima (minima if you flip sign) in image I if:

```
for y=range(i-1,i+1+1):
        for x in range(j-1,j+1+1):
                for c in range(k-1,k+1+1):
                        if y == i and x== j and c==k:
                                continue
                #below has to be true
                I[y,x,c] < I[i,j,k]
```

Scale-space blob detector: Example



Slide credit: S. Lazebnik

Efficient implementation

- Approximating the Laplacian with a difference of Gaussians:

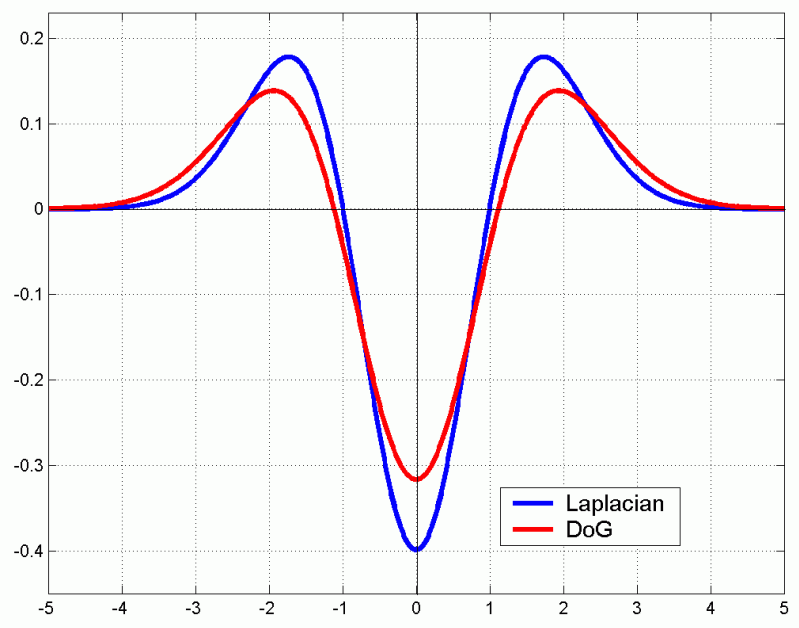
$$L = \sigma^2 (G_{xx}(x,y,\sigma) + G_{yy}(x,y,\sigma))$$

(Laplacian)

$$DoG = G(x,y,k\sigma) - G(x,y,\sigma)$$

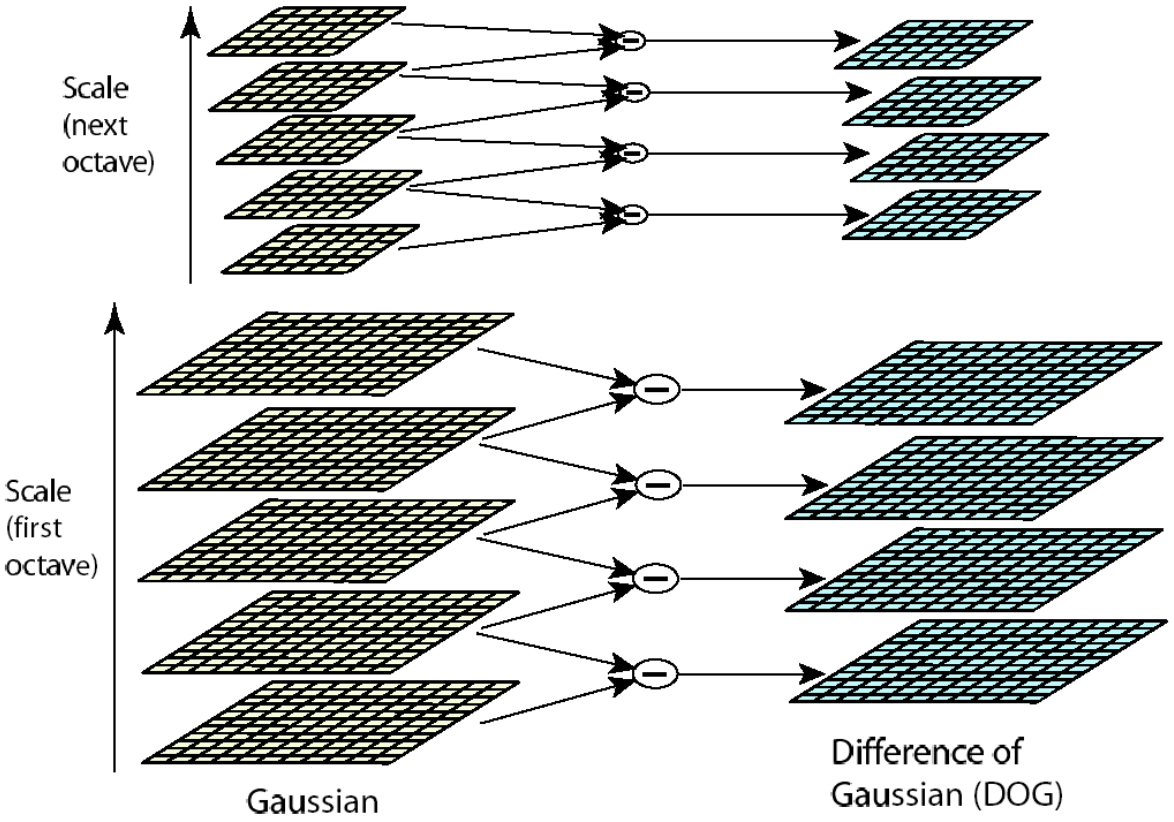
(Difference of Gaussians)

k is empirically determined



Slide credit: S. Lazebnik

Efficient implementation



David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

Slide credit: S. Lazebnik

Problem 1 Solved

- How do we deal with scales: try them all
- **Why is this efficient?**

Vast majority of effort is in the first and second scales

$$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{4^i} \ldots = \frac{4}{3}$$

Problem 2 – Describing Features

Image – 40

1/2 size, rot. 45°
Lightened+40

Full Image

100x100 crop at Glasses

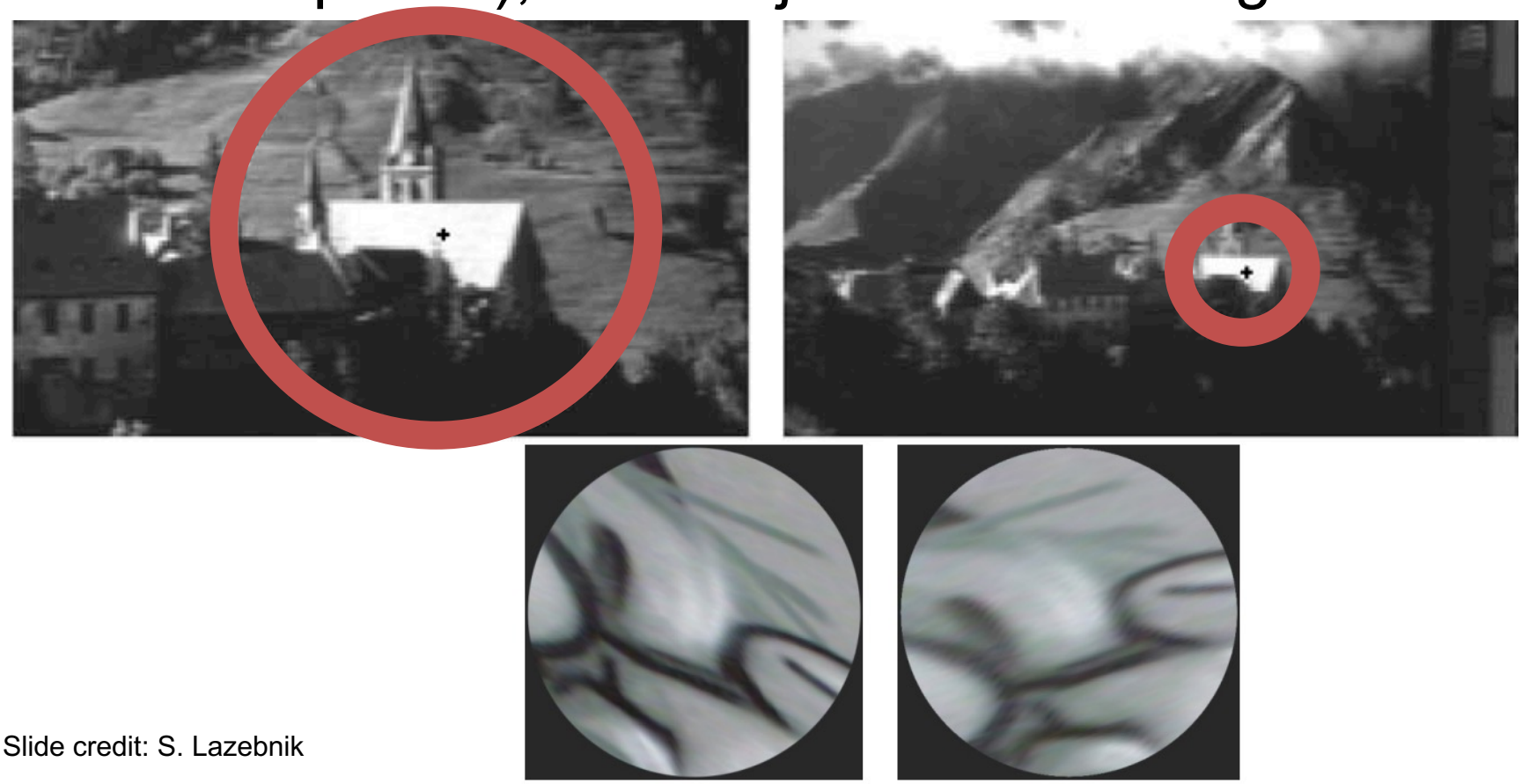
Problem 2 – Describing Features

Once we've found a corner/blobs, we can't just use the nearby patch. What about:

1. Scale?
2. Rotation?
3. Additive light?

Handling Scale

Given characteristic scale (maximum Laplacian response), we can just rescale image

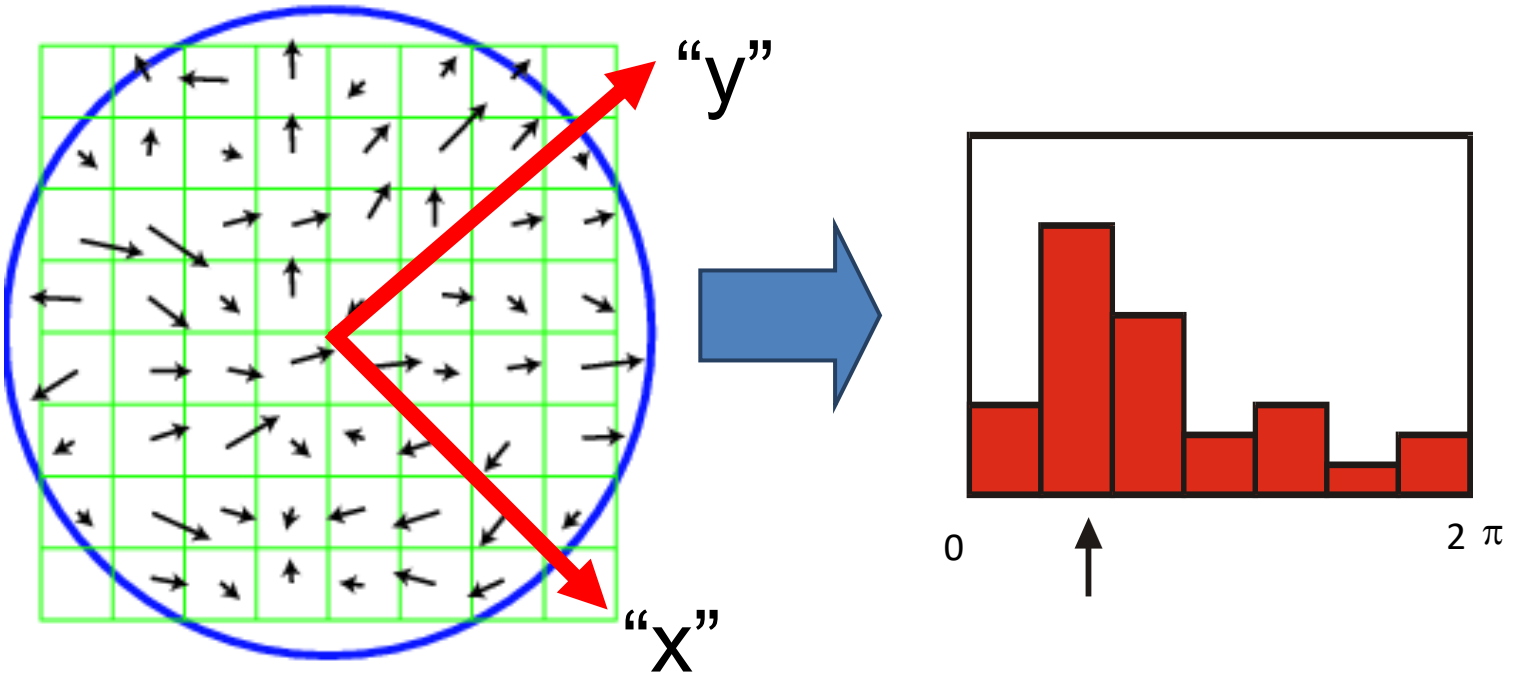


Slide credit: S. Lazebnik

Handling Rotation

Create histogram of local gradient directions in the patch

Given window, can compute “dominant orientation”
and then rotate image

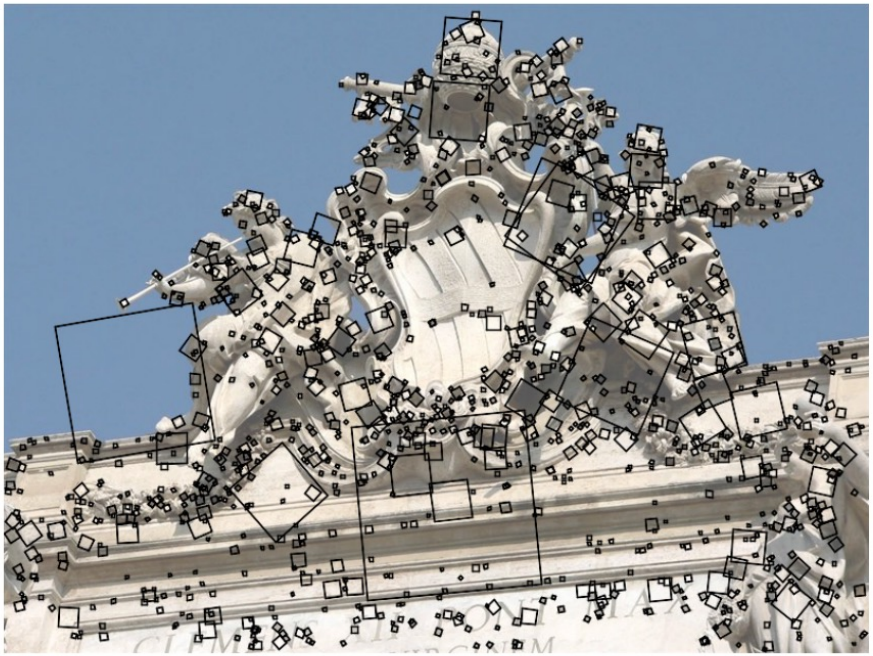


Slide credit: S. Lazebnik

Rectangles at different sizes, sizes tell you the detection of of blond at certain size. So that is the patch that we are interested in. The rectangles are rotated, which tells you the domination of these blobs.

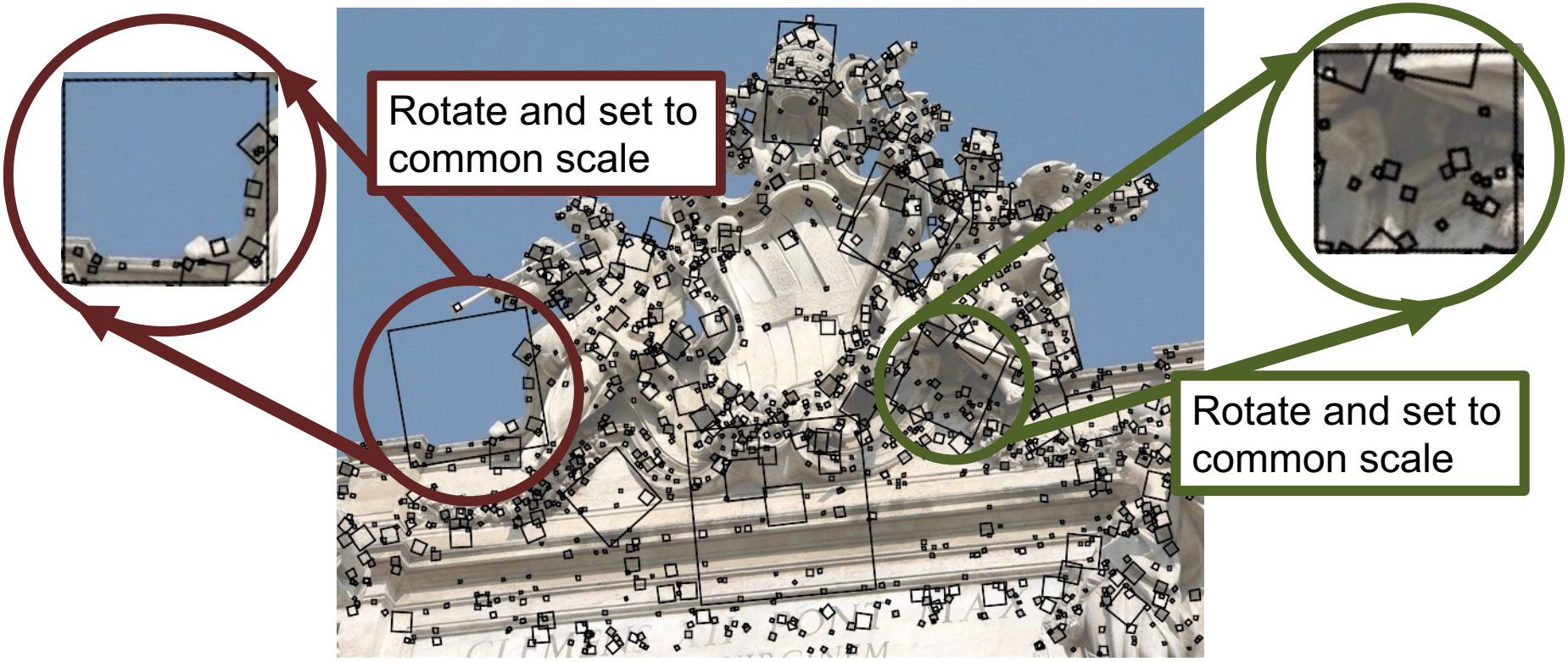
Scale and Rotation

SIFT features at characteristic scales and dominant orientations



Picture credit: S. Lazebnik. Paper: David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

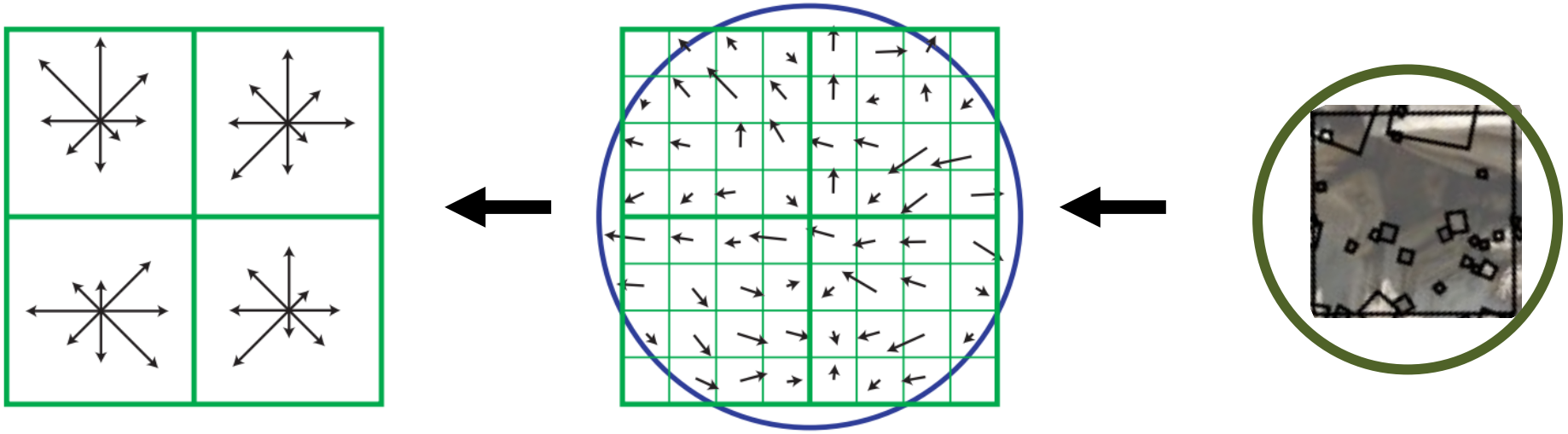
Scale and Rotation



Picture credit: S. Lazebnik. Paper: David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

SIFT Descriptors

All these arrows tell you the gradient direction of each pixel.



1. Compute gradients
2. Build histogram (2x2 shown, 4x4 in practice)
3. (4x4) cells * 8 orientations = 128-dim descriptor

Gradients ignore global illumination changes

Figure from David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.

SIFT Descriptors

- In principle: build a histogram of the gradients
- In reality: quite complicated
  - Gaussian weighting: smooth response
  - Normalization: reduces illumination effects
  - Clamping
  - Tons of more stuff

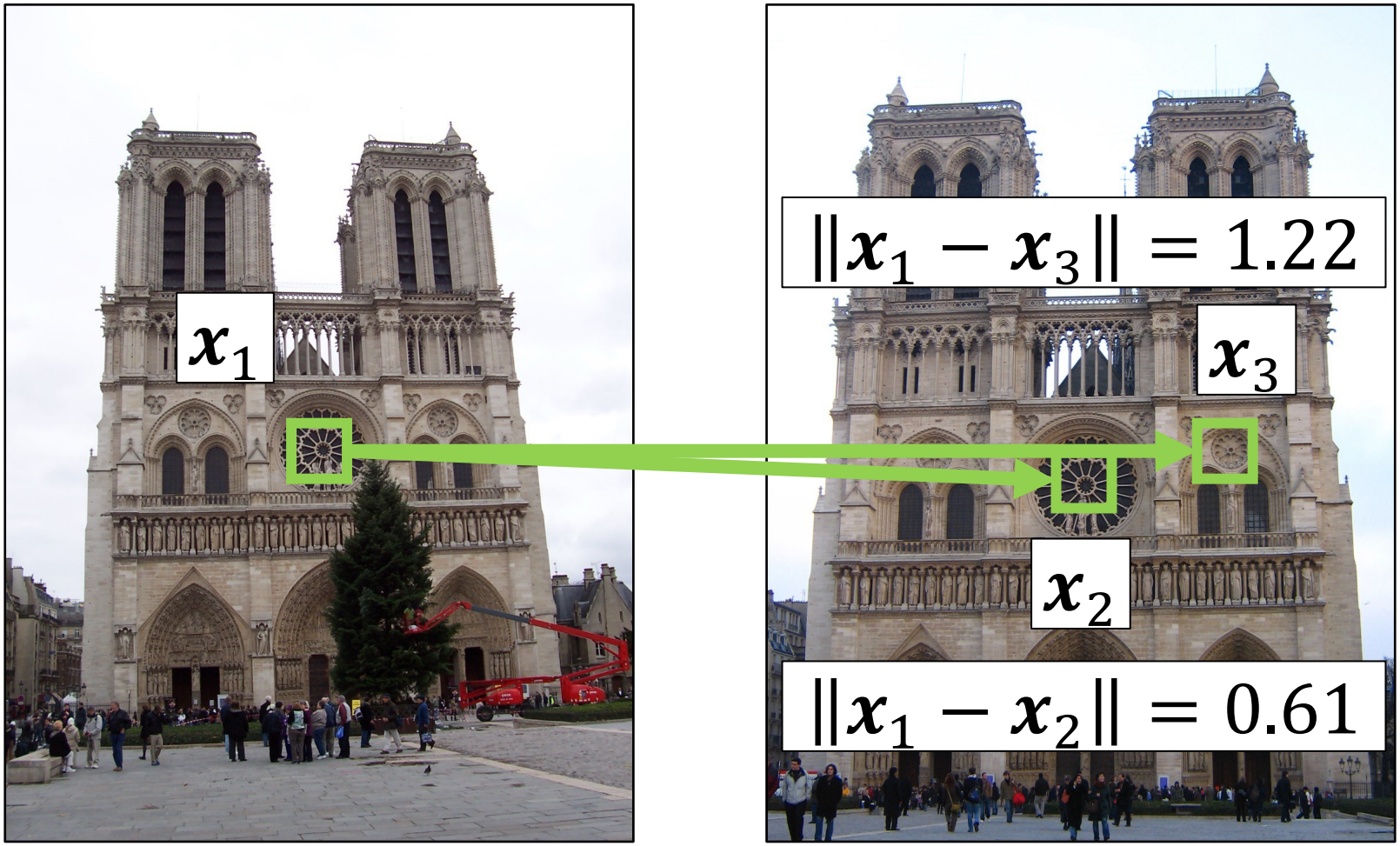
Feature Descriptors

Think of feature as some non-linear filter that maps each pixel to a 128-dimensional feature

128D vector **x**

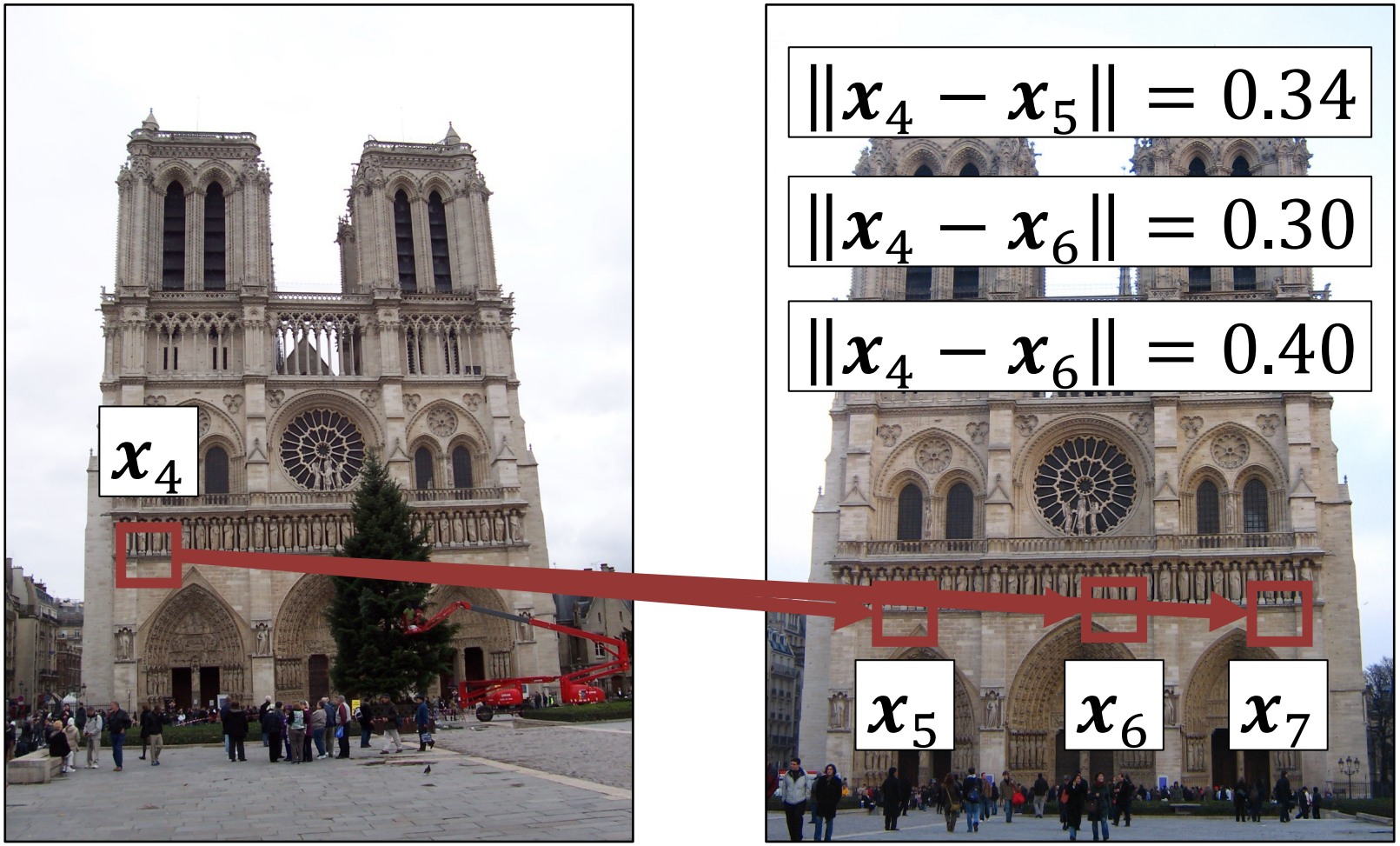
Photo credit: N. Snavely

Instance Matching



Example credit: J. Hays

Instance Matching



Example credit: J. Hays

2nd Nearest Neighbor Trick

- Given a feature x_q, nearest neighbor to x is a good match, but distances can't be thresholded.
- Instead, find nearest neighbor (x_{1NN}) and second nearest neighbor (x_{2NN}). This ratio is a good test for matches:

$$r = \frac{\|\boldsymbol{x}_q - \boldsymbol{x}_{1NN}\|}{\|\boldsymbol{x}_q - \boldsymbol{x}_{2NN}\|}$$

Properties of SIFT

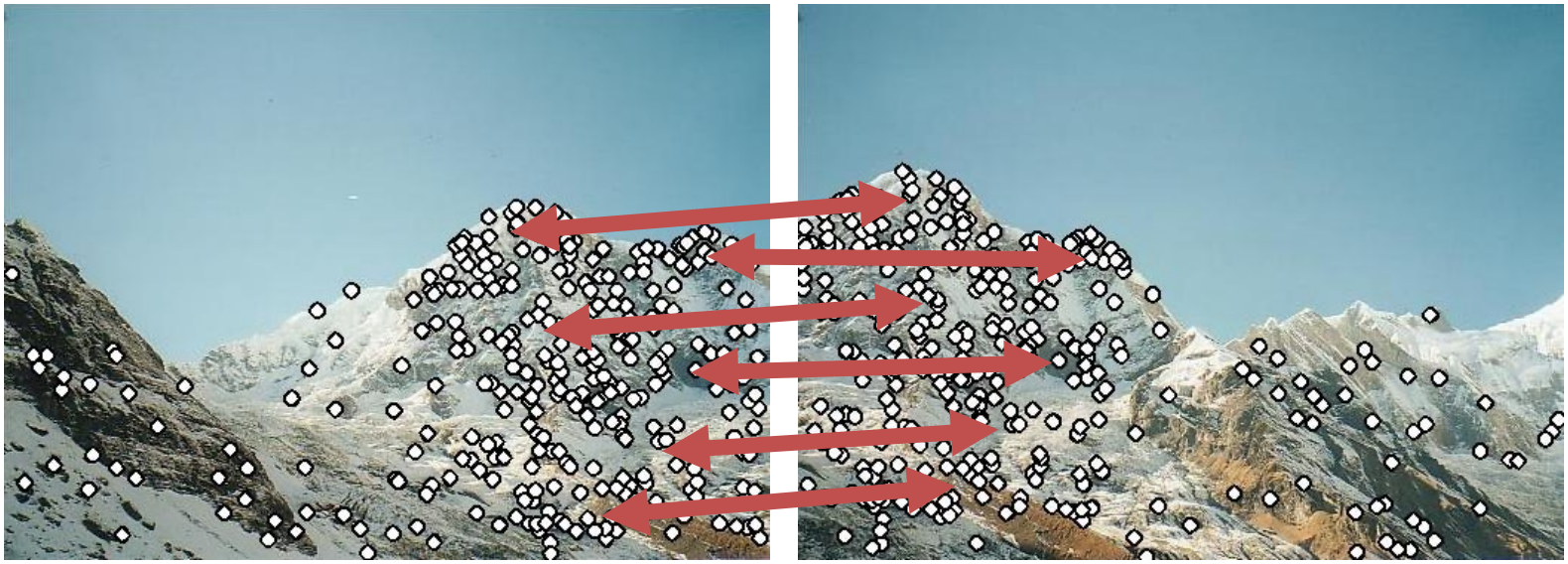
- Can handle: up to ~60 degree out-of-plane rotation, changes of illumination (night vs day)
- Fast, efficient, code available (but was patented)

Slide credit: N. Snavely

Summing Up

1. Convolve image with scale-normalized Laplacian at several scales
2. Find local maxima of squared Laplacian response in scale-space
3. Compute gradients
4. Compute SIFT Descriptors (Histogram of Grad)
5. Match between features

So Far; What's Next?



1: find corners+features

2: match based on local image data

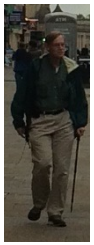
3: next time: compute transforms from matches

Extra Reading for the Curious

Aside: This Trick is Common

Given a 50x16 person detector, how do I detect:
(a) 250x80 (b) 150x48 (c) 100x32 (d) 25x8 people?

Sample people from image



Aside: This Trick is Common

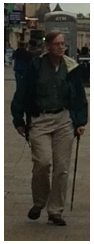
Detecting all the people
The red box is a fixed size

Sample people from image

Aside: This Trick is Common

Detecting all the people
The red box is a fixed size

Sample people from image



Aside: This Trick is Common

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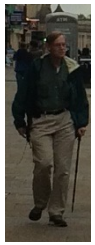
Sample people from image



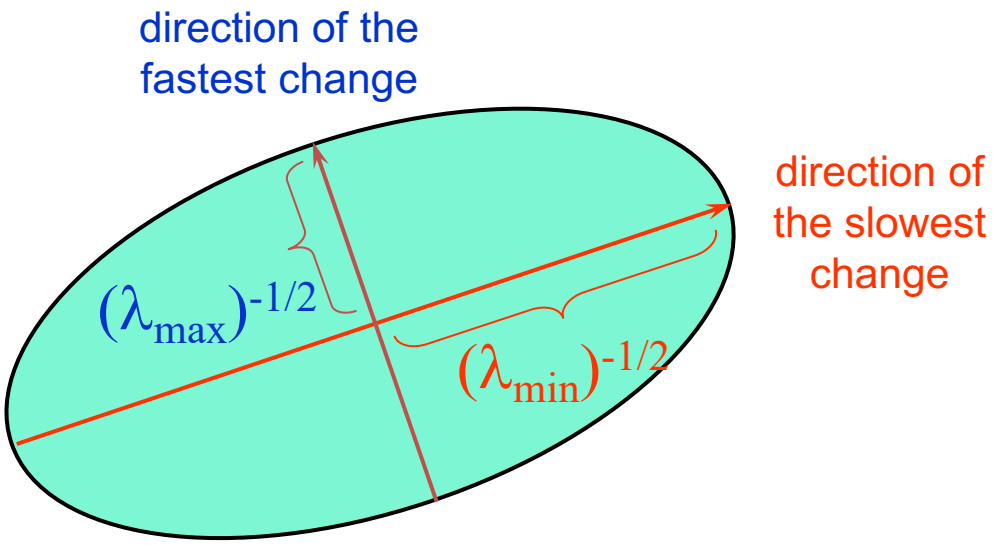
Affine adaptation

Consider the second moment matrix of the window containing the blob:

$$M \square \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \square R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

Recall:

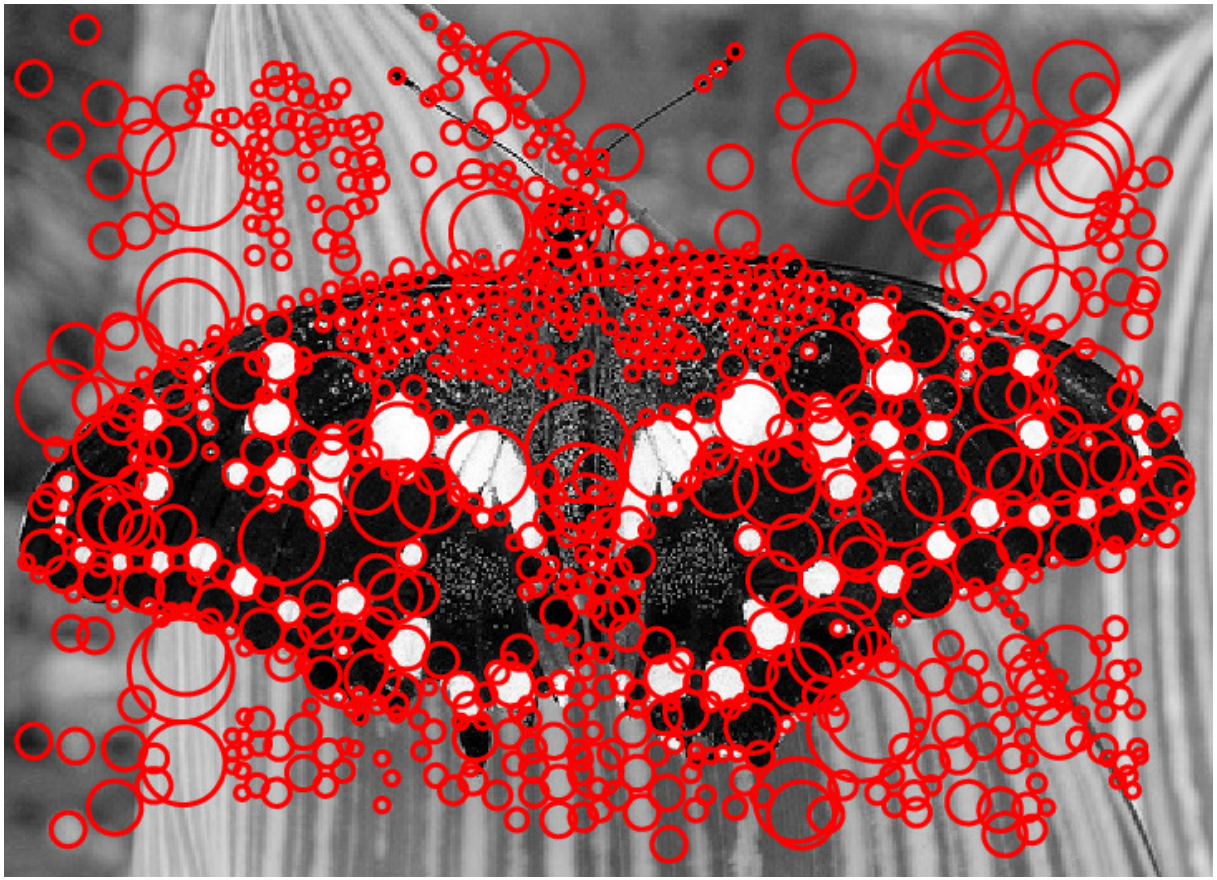
$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} \square \text{const}$$



This ellipse visualizes the "characteristic shape" of the window

Slide: S. Lazebnik

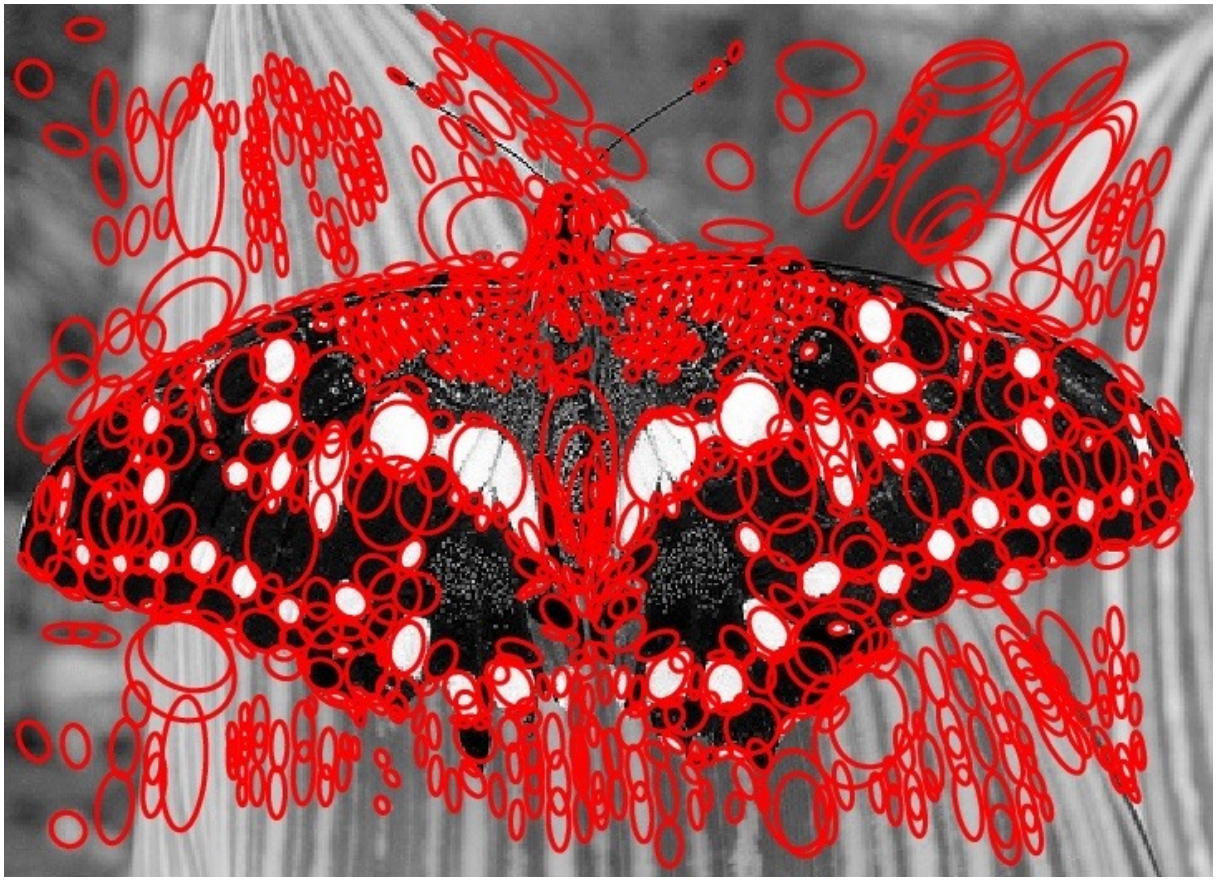
Affine adaptation example



Scale-invariant regions (blobs)

Slide: S. Lazebnik

Affine adaptation example



Affine-adapted blobs

Slide: S. Lazebnik

2nd Nearest Neighbor Trick

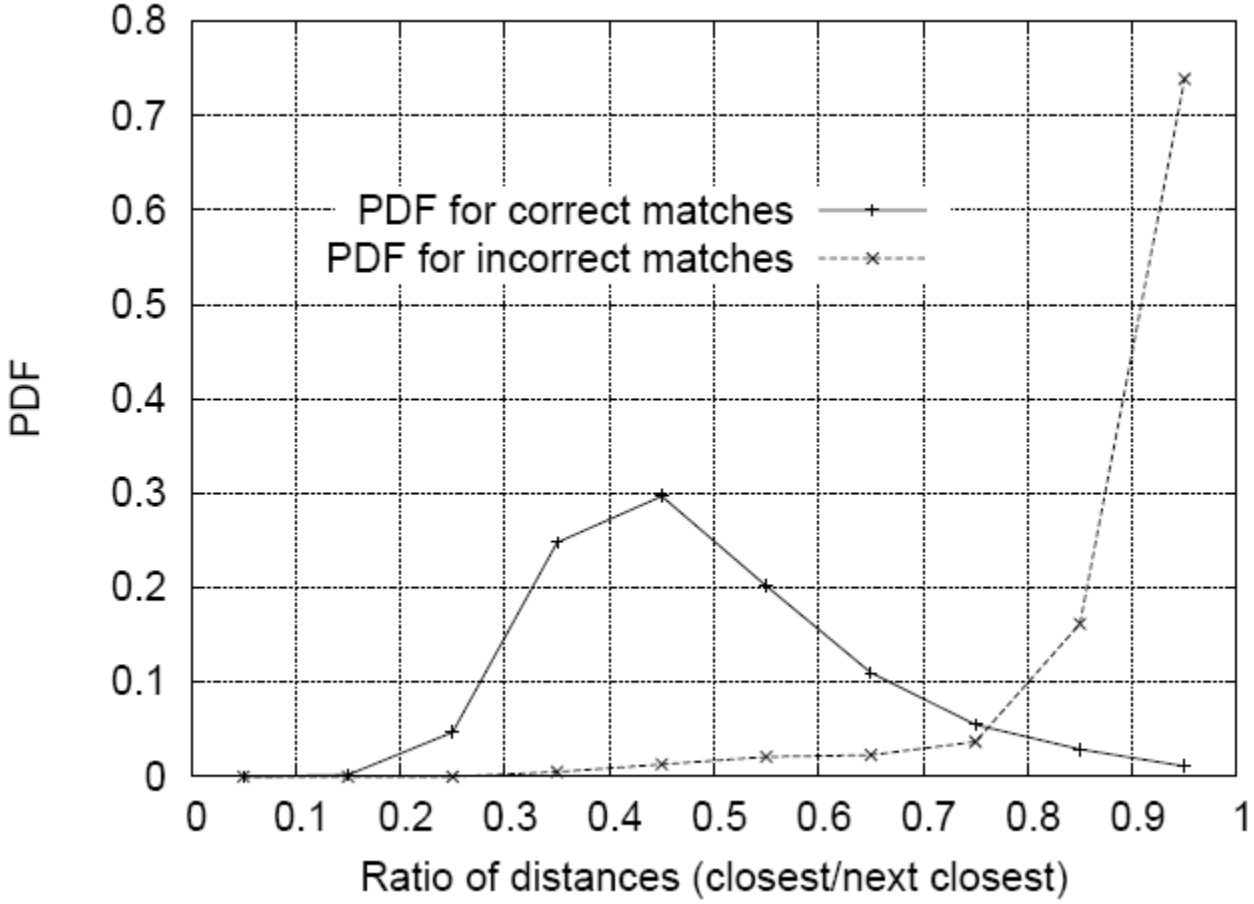


Figure from David G. Lowe. **"Distinctive image features from scale-invariant keypoints."** *IJCV* 60 (2), pp. 91-110, 2004.