UM EECS 270 F22
Introduction to Logic Design

18. Algorithmic Two-Level Logic Minimization

Tabular Generation of Prime Implicants (Quine-McCluskey Procedure)

two things are adjacent: If they agree on every position except one.

So we can start from our minterms, compare every min terms with every minterms, and say is it one distance from it? (hamming distance) 看两个minterm有多少bit不一样. 如果是1个或2个以上, 则就不是adjacent. → 但如果variables很多, 则这个方法就很慢.

Trick: two minterms, whose number of "1" is the same, can not be ajacient, because they would have two different at least two positions.

所以我们要做的是

take our minterms and organize them into $\boxed{\text{groups.}}$

based on counting how many "1" or "0" are in the representation of the minterm.

we try to use these two to come up with an algorithm for solving this problem.

- Main Theorems:
  - Adjacency: $x' p + x p = p$ (create "larger" implicants)

    p is larger than $x'p + xp$: P covers both of these smaller implicants.
  - Absorption: $p + x p = p$ (delete subsumed implicants)

    p is larger than xp. So xp is a subset of p.

※我们不能保证不同group的minterm一定adjacient, 但是我们能保证相同group的minterm一定不adjacient.

- Procedure:
  - Arrange product terms in groups such that all terms in one group have the same number of 1s in their binary representation, their Group Index
  - Starting from minterms, arrange groups in ascending-index order
  - Apply adjacency theorem to product terms from adjacent groups only
  - Remove subsumed product terms using absorption theorem
- Product term representation: $p = p_n p_{n\text{-}1} \ldots p_2 p_1$ where

$$\boxed{p_i = \begin{cases} 0 & \text{if ith variable appears complemented} \\ 1 & \text{if ith variable appears uncomplemented} \\ - & \text{if ith variable does not appear} \end{cases}}$$

data structure,

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

group index

How many "1"

$\boxed{\text{index}}$	minterm	A	B	C	D	E	implicant	A	B	C	D	E
1	1	0	0	0	0	1						
2	3	$\boxed{0}$	0	0	$\boxed{1}$	1						
	17	$\boxed{1}$	0	0	$\boxed{0}$	1						
3	19	1	0	0	1	1						
4	15	0	1	1	1	1						
	29	1	1	1	0	1						
5	31	1	1	1	1	1						

adjacient?

Yes.

adjacient?

since

can't be ajacient

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	17	1	0	0	0	1
3	19	1	0	0	1	1
4	15	0	1	1	1	1
	29	1	1	1	0	1
5	31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1

they agree on A, B, C, E
they differ only in D. (D or D')
adjacient rule. → So D gets dropped.

Now, minterm 1 & minterm 3 can not be prime implicant
Because they're included in something larger
which is $\boxed{1,3}$

QM Example
Generation of Prime Implicants

$$F = \sum\nolimits_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	19	1	0	0	1	1
4	15	0	1	1	1	1
	29	1	1	1	0	1
5	31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	19	1	0	0	1	1
4	15	0	1	1	1	1
	29	1	1	1	0	1
5	31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	√ 19	1	0	0	1	1
4	15	0	1	1	1	1
	29	1	1	1	0	1
5	31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1
3,19	-	0	0	1	1

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	√ 19	1	0	0	1	1
4	15	0	1	1	1	1
	29	1	1	1	0	1
5	31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1
3,19	-	0	0	1	1
17,19	1	0	0	-	1

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	√ 19	1	0	0	1	1
4	15	0	1	1	1	1
	29	1	1	1	0	1
5	31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1
3,19	-	0	0	1	1
17,19	1	0	0	-	1

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E} (1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	√ 19	1	0	0	1	1
4	√ 15	0	1	1	1	1
	29	1	1	1	0	1
5	√ 31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1
3,19	-	0	0	1	1
17,19	1	0	0	-	1
15,31	-	1	1	1	1

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

None of the minterm can be prime.

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	√ 19	1	0	0	1	1
4	√ 15	0	1	1	1	1
	√ 29	1	1	1	0	1
5	√ 31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1
3,19	-	0	0	1	1
17,19	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

index	minterm	A	B	C	D	E
1	√ 1	0	0	0	0	1
2	√ 3	0	0	0	1	1
	√ 17	1	0	0	0	1
3	√ 19	1	0	0	1	1
4	√ 15	0	1	1	1	1
	√ 29	1	1	1	0	1
5	√ 31	1	1	1	1	1

implicant	A	B	C	D	E
1,3	0	0	0	-	1
1,17	-	0	0	0	1
3,19	-	0	0	1	1
17,19	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

QM Example
Generation of Prime Implicants

"–" 也要匹配.

missing variables.
product including the

$$F = \sum_{A,B,C,D,E} (1,3,15,17,19,29,31)$$

Now, "–" becomes important: To merge you have to agree between the two

implicant	A	B	C	D	E
$\boxed{1,3}$	0	0	0	-	1
1,17	-	0	0	0	1
$\boxed{3,19}$	-	0	0	1	1
$\boxed{17,19}$	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

merge? Yes

merge? No, because even if they agree on B.C.E. However, in "1.3", D is missing. in "3.19" A is missing.

implicant	A	B	C	D	E

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

implicant	A	B	C	D	E
√ 1,3	0	0	0	-	1
1,17	-	0	0	0	1
3,19	-	0	0	1	1
√17,19	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

implicant	A	B	C	D	E
1,3,17,19	-	0	0	-	1

it covers 4 minterms, since there are two dashes.

1.3. 和 17.19 不是 prime implicants.

因为 somthing larger covers them.

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

implicant	A	B	C	D	E
√ 1,3	0	0	0	-	1
√ 1,17	-	0	0	0	1
√ 3,19	-	0	0	1	1
√ 17,19	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

implicant	A	B	C	D	E
1,3,17,19	-	0	0	-	1
~~1,17,3,19~~	~~-~~	~~0~~	~~0~~	~~-~~	~~1~~

So we don't need it, it's redundant.

they are the same thing.

So when you're programming this, you have to keep track of the order in which you combine things.

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E} (1,3,15,17,19,29,31)$$

implicant	A	B	C	D	E
√ 1,3	0	0	0	-	1
√ 1,17	-	0	0	0	1
√ 3,19	-	0	0	1	1
√ 17,19	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

implicant	A	B	C	D	E
1,3,17,19	-	0	0	-	1
~~1,17,3,19~~	~~-~~	~~0~~	~~0~~	~~-~~	~~1~~

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E} (1,3,15,17,19,29,31)$$

implicant	A	B	C	D	E
√ 1,3	0	0	0	-	1
√ 1,17	-	0	0	0	1
√ 3,19	-	0	0	1	1
√ 17,19	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

implicant	A	B	C	D	E
1,3,17,19	-	0	0	-	1

The thing has not been checked :

final set of prime implements for the function

Given them name P_1, P_2, P_3

next pages. →

QM Example
Generation of Prime Implicants

$$F = \sum_{A,B,C,D,E} (1,3,15,17,19,29,31)$$

implicant	A	B	C	D	E
√ 1,3	0	0	0	-	1
√ 1,17	-	0	0	0	1
√ 3,19	-	0	0	1	1
√ 17,19	1	0	0	-	1
15,31	-	1	1	1	1
29,31	1	1	1	-	1

implicant	A	B	C	D	E
1,3,17,19	-	0	0	-	1

QM Example
Generation of Prime Implicants

$$F=\sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

implicant	A	B	C	D	E
√ 1,3	0	0	0	-	1
√ 1,17	-	0	0	0	1
√ 3,19	-	0	0	1	1
√17,19	1	0	0	-	1
P_1: 15,31	-	1	1	1	1
P_2: 29,31	1	1	1	-	1

implicant	A	B	C	D	E
P_3:1,3,17,19	-	0	0	-	1

this function has three prime implicants.

$$P_1 = BCDE \quad P_2 = ABCE \quad P_3 = B'C'E$$

Next thing we want to do: covering problem.

QM Example
Minimization by Set Covering

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

creat a matrix.

each column is a prime implicant.

"1" means: that column includes that row.

PIs (Variables)

Minterms (Constraints)

	P_1 $BCDE$	P_2 $ABCE$	P_3 $B'C'E$
1			1
3			1
15	1		
17			1
19			1
29		1	
31	1	1	

P_1 : 15, 31.
P_2 : 29, 31.
P_3 : 1, 3, 17, 19

Remember our objective is to make sure every minterm of the function is included in at least one prime implicant.

every row is a minterm.

our goal:

we want to include all of the minterms in our solution using the fewest number of prime implicants.

Cost:

How many literals they have.
How big the "and" gate is

P_1/P_2 : 4 variables $\longrightarrow$ 4 input "and" gate.
P_3 : 3 variables $\longrightarrow$ 3 input "and" gate.

QM Example
Minimization by Set Covering

$$F = \sum\nolimits_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_1 $BCDE$	P_2 $ABCE$	P_3 $B'C'E$
1			1
3			1
15	1		
17			1
19			1
29		1	
31	1	1	

QM Example
Minimization by Set Covering

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_1 $BCDE$	P_2 $ABCE$	P_3 $B'C'E$
1			1
3			1
15	1		
17			1
19			1
29		1	
31	1	1	

$$F = B'C'E +$$

P_3 is essential

since it's the only one that will cover minterm 1.

Next step:

once I included P_3, I can remove all other minterms that it covers. 因为 they don't need to be covered multiple times.

QM Example
Minimization by Set Covering

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_1 $BCDE$	P_2 $ABCE$	P_3 $B'C'E$
15	1		
29		1	
31	1	1	

$$F = B'C'E +$$

QM Example
Minimization by Set Covering

$$F = \sum\nolimits_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_1 $BCDE$	P_2 $ABCE$
15	1	
29		1
31	1	1

$$F = B'C'E +$$

QM Example
Minimization by Set Covering

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_1 $BCDE$	P_2 $ABCE$
15	1	
29		1
31	1	1

P_1 is essential for minterm 15.

$$F = B'C'E + BCDE$$

QM Example
Minimization by Set Covering

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_1 $BCDE$	P_2 $ABCE$
29		1

$$F = B'C'E + BCDE$$

QM Example
Minimization by Set Covering

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_2
	$ABCE$
29	1

$$F = B'C'E + BCDE$$

QM Example
Minimization by Set Covering

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_2 $ABCE$
29	1

P_2 is essential to minterm 29.

$$F = B'C'E + BCDE + ABCE$$

QM Example
Covering Function

$$F=\sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

"0" means I don't need it in the solution.

"2" means I need it in the solution.

	P_1 $BCDE$	P_2 $ABCE$	P_3 $B'C'E$	
1			1	(P_3)
3			1	(P_3)
15	1			(P_1)
17			1	(P_3)
19			1	(P_3)
29		1		(P_2)
31	1	1		(P_1+P_2)

It's the only choice to cover that row

choices to cover that row.

can be covered in one of the two ways.

Covering Function =

take them together in POS then it called

QM Example
Covering Function

这种情况下只有一种 covering function；但有时候会有多个.
然后我们对每个 solution 的 cost 进行分析
↓
最终找出 minimum solution.

$$F = \sum_{A,B,C,D,E}(1,3,15,17,19,29,31)$$

	P_1 $BCDE$	P_2 $ABCE$	P_3 $B'C'E$	
1			1	(P_3)
3			1	(P_3)
15	1			(P_1)
17			1	(P_3)
19			1	(P_3)
29		1		(P_2)
31	1	1		(P_1+P_2)

$$\text{Covering Function} = (P_3)\,(P_1)\,(P_2)\,(P_1+P_2) = P_1P_2P_3$$

A Large Example

$$f(x_1, x_2, x_3, x_4, x_5, x_6) = \prod (4, 8, 16, 17, 32, 34)$$

Prime implicants (PIs) generated by the tabular Q-M procedure

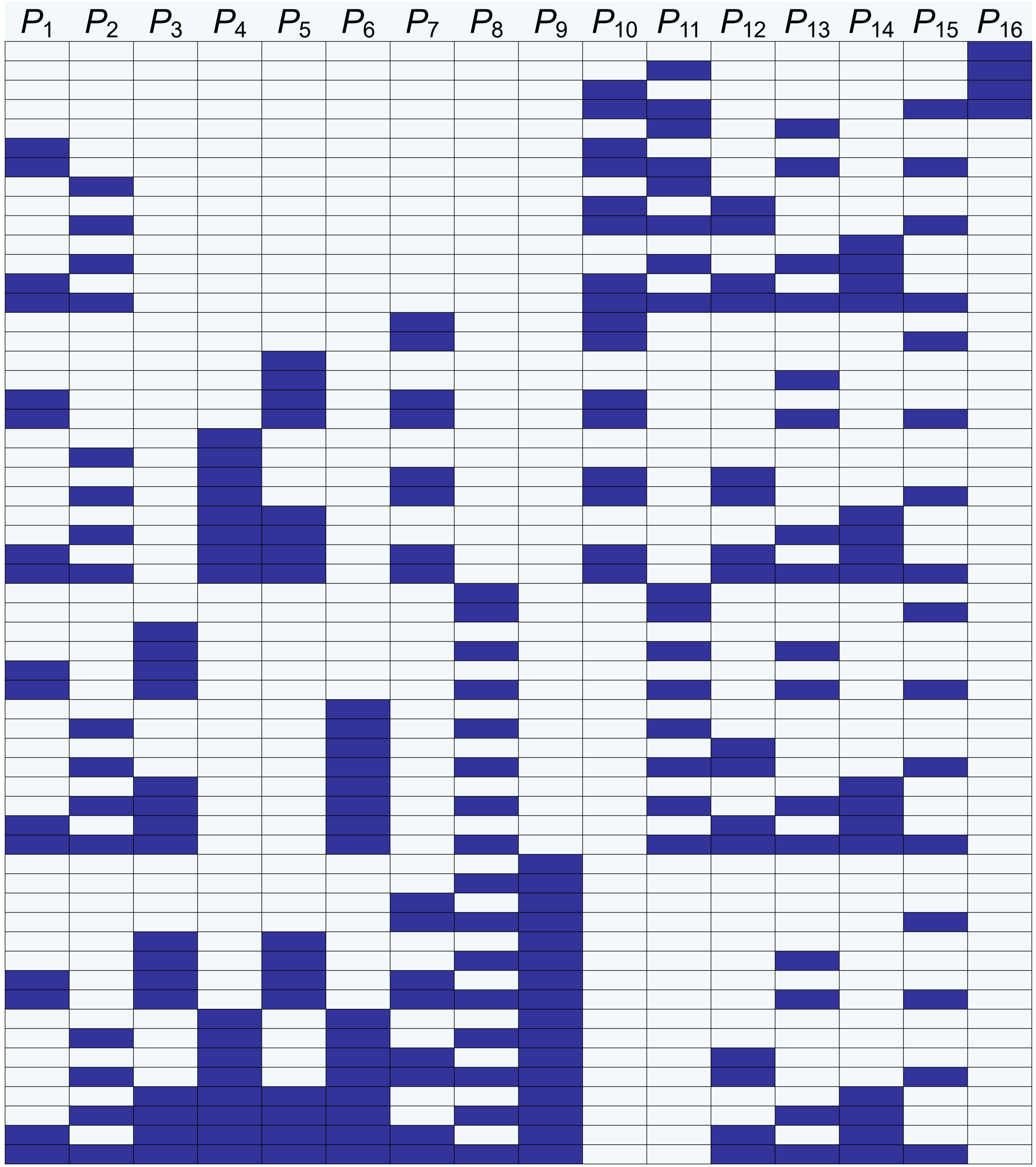
cost 2

$$\begin{array}{ll} P_1 = x_4 x_5 & P_9 = x_1 x_2 \\ P_2 = x_3 x_6 & P_{10} = x_1' x_5 \\ P_3 = x_1 x_4 & P_{11} = x_2' x_6 \\ P_4 = x_2 x_3 & P_{12} = x_3 x_5 \\ P_5 = x_2 x_4 & P_{13} = x_4 x_6 \\ P_6 = x_1 x_3 & P_{14} = x_3 x_4 \\ P_7 = x_2 x_5 & P_{15} = x_5 x_6 \\ P_8 = x_1 x_6 & \boxed{P_{16} = x_1' x_2' x_3' x_4'} \end{array}$$

cost 4

However, we don't need all of them

Next step: find the smallest subset of them that will actually implement the function find the prime implecant

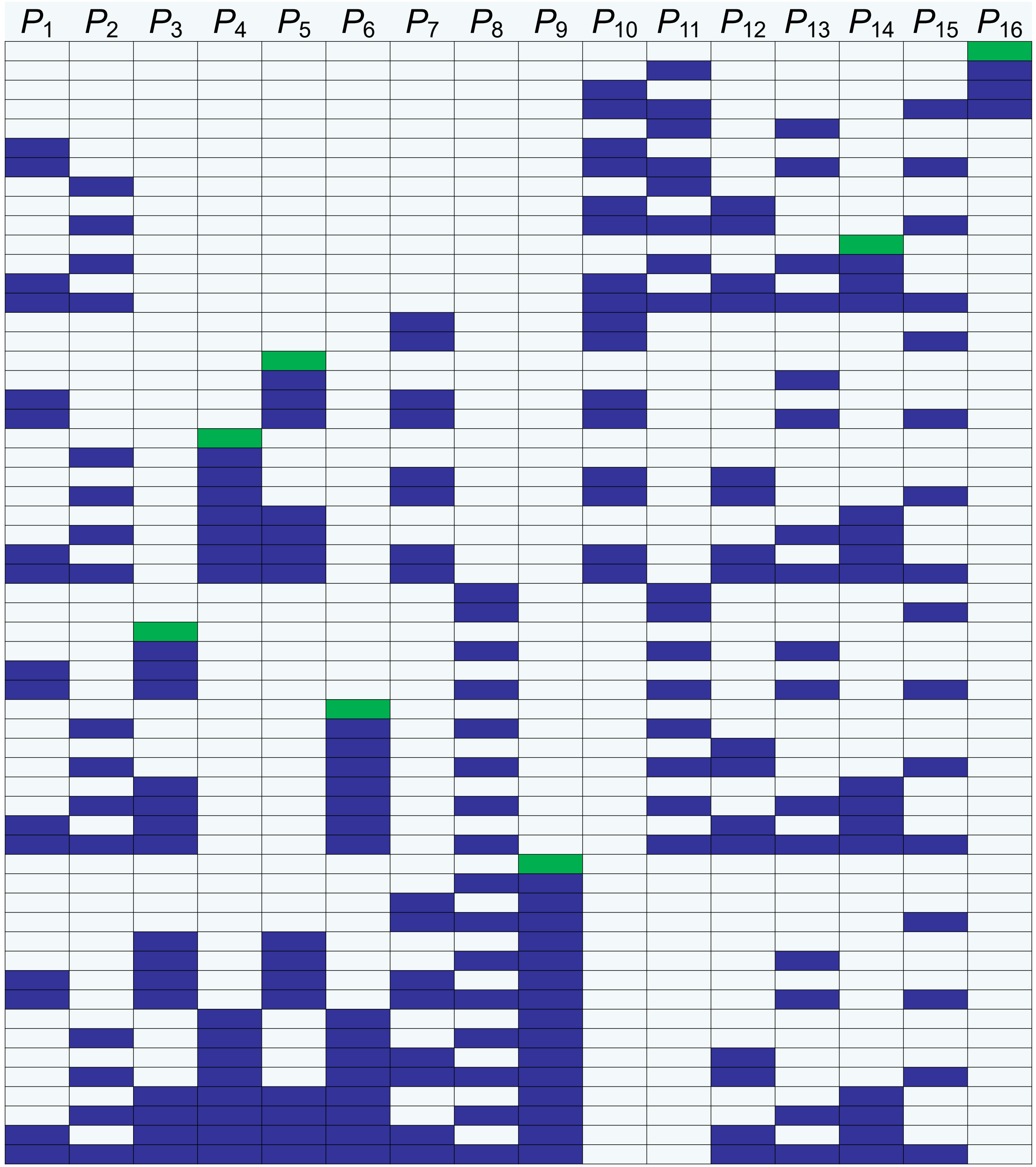


Covering Matrix:
58 minterms
16 PIs

Empty cells: 0
Shaded cells: 1

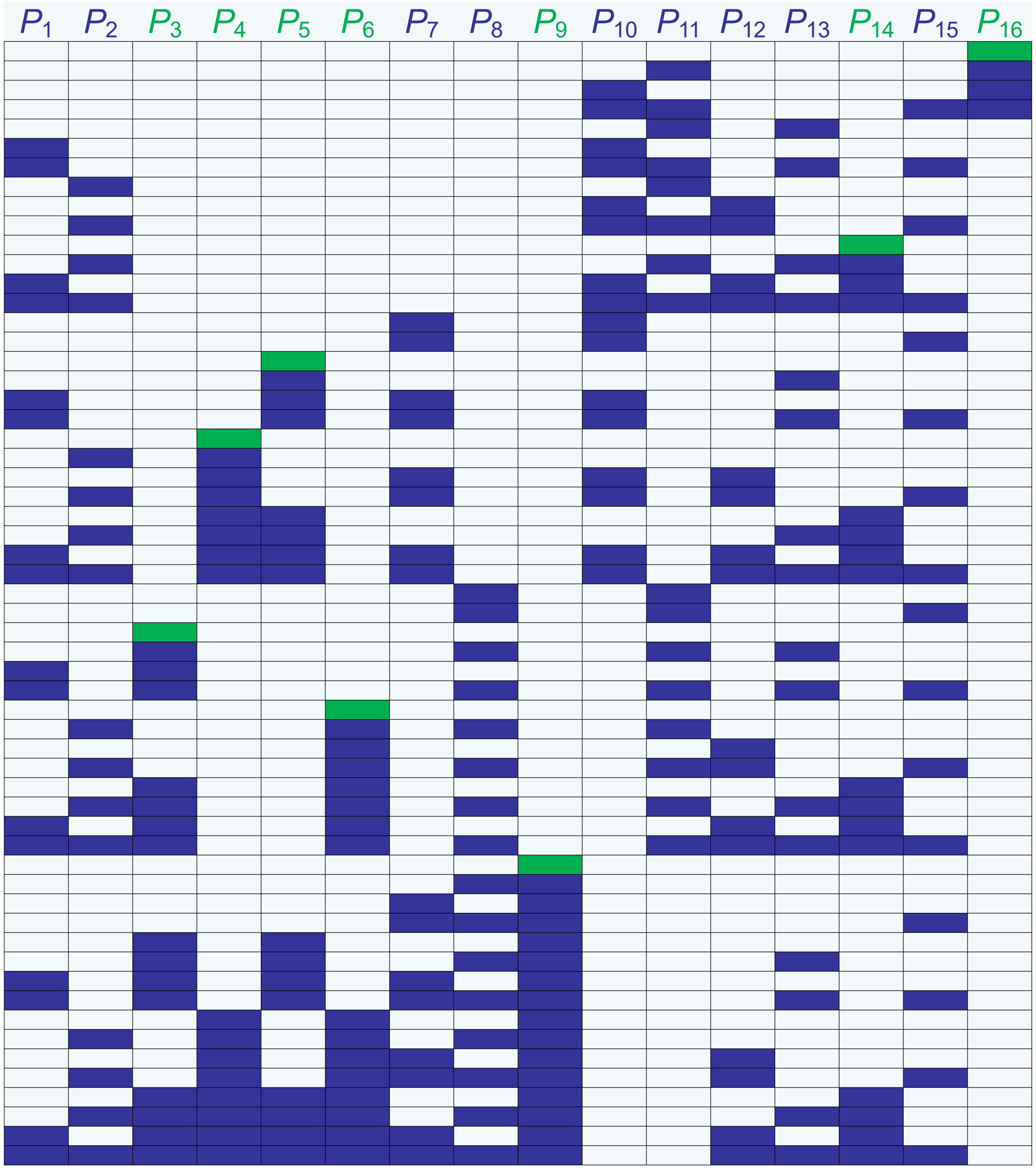
Identify Essential PIs (EPIs)

Green cells identify distiguished minterms

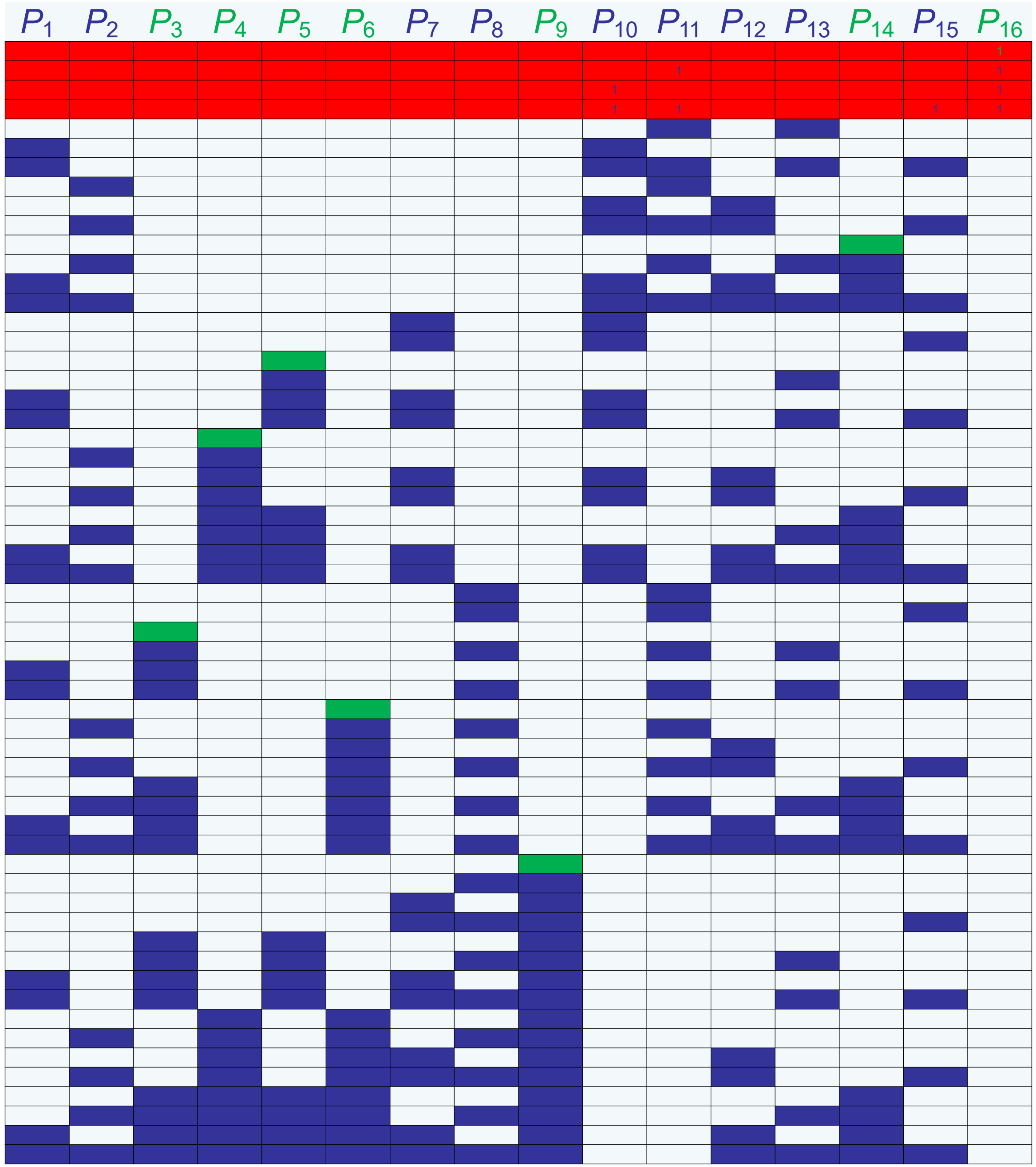


Remove EPIs and the minterms they cover

Partial Solution: P_{16}



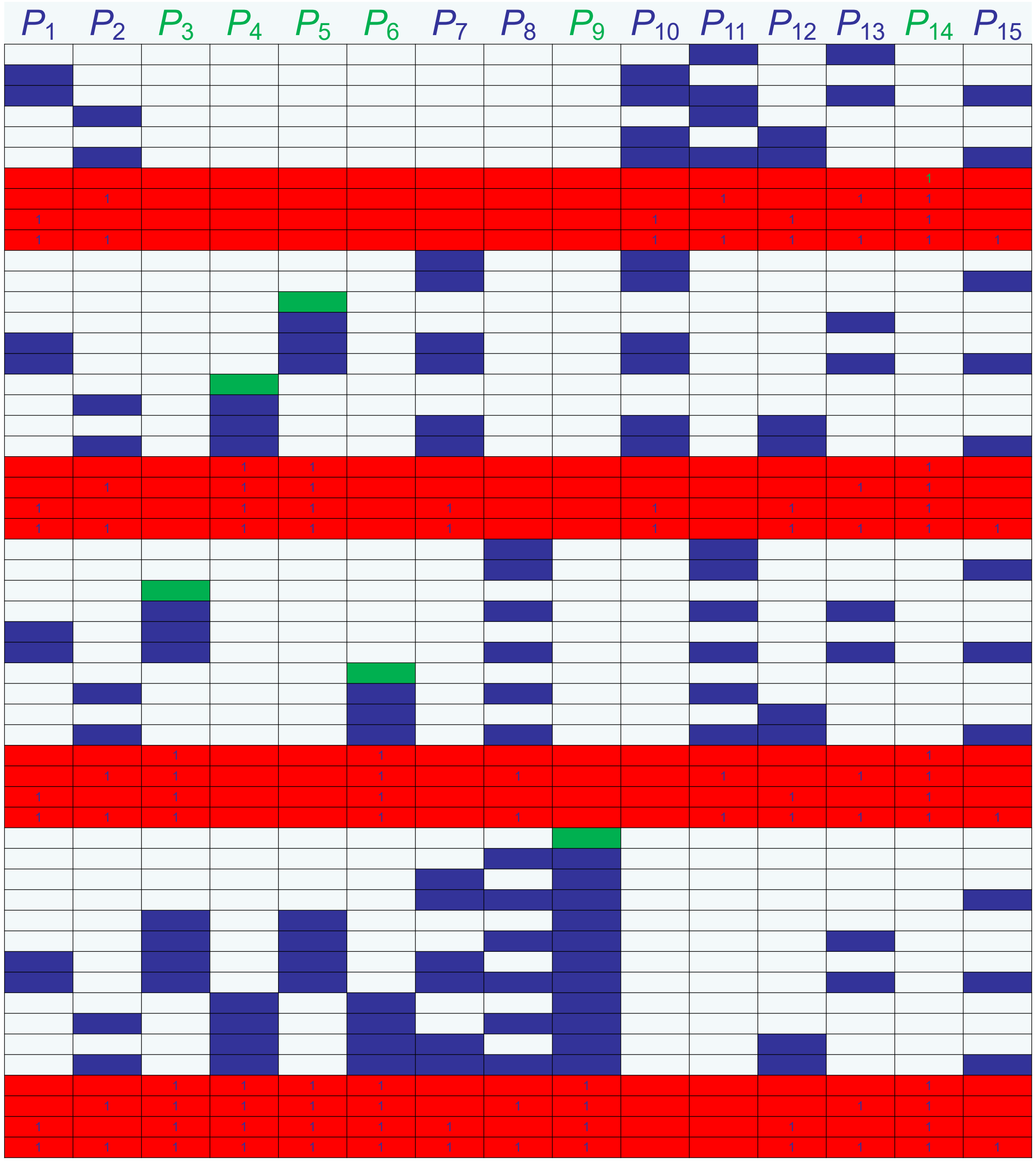
Remove EPIs and the minterms they cover



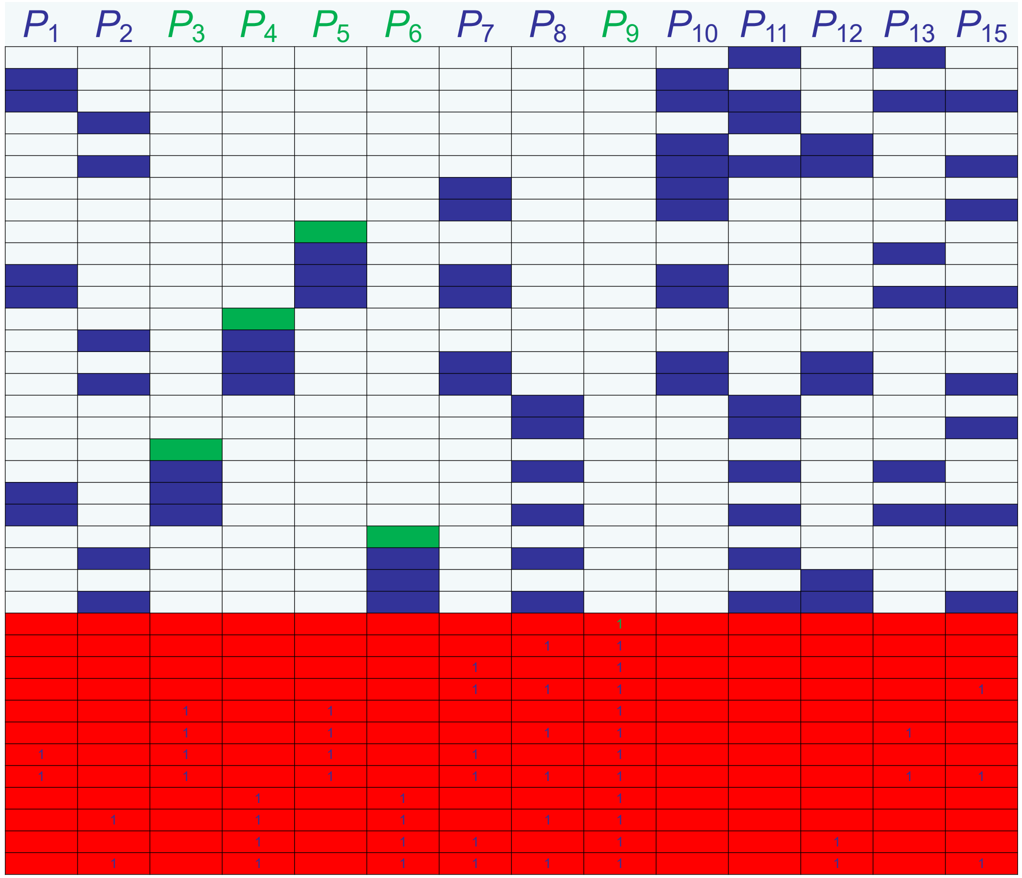
Partial Solution: P_{16}

Remove EPIs and the minterms they cover

Partial Solution:
P_{16}
P_{14}



Remove EPIs and the minterms they cover



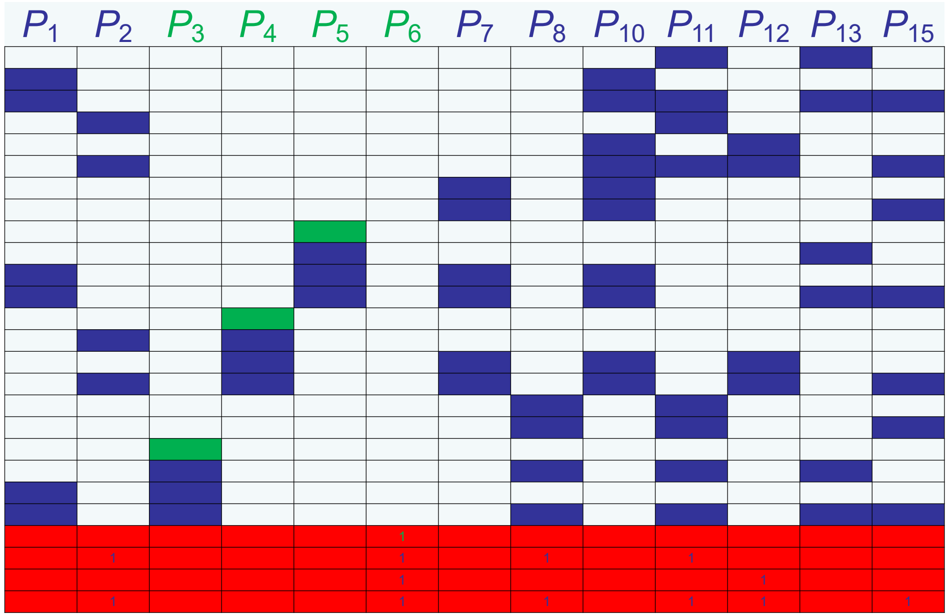
Partial Solution:

P_{16}

P_{14}

P_9

Remove EPIs and the minterms they cover



Partial Solution:

P_{16}
P_{14}
P_9
P_6

P_1	P_2	P_3	P_4	P_5	P_7	P_8	P_{10}	P_{11}	P_{12}	P_{13}	P_{15}
				1							
				1						1	
1				1	1		1				
1				1	1		1			1	1

Remove EPIs and the minterms they cover

Partial Solution:

P_{16}

P_{14}

P_9

P_6

P_5

P_1	P_2	P_3	P_4	P_7	P_8	P_{10}	P_{11}	P_{12}	P_{13}	P_{15}
			1							
	1		1							
			1	1		1		1		
	1		1	1		1		1		1

Remove EPIs and the minterms they cover

Partial Solution:

P_{16}

P_{14}

P_9

P_6

P_5

P_4

P_1	P_2	P_3	P_7	P_8	P_{10}	P_{11}	P_{12}	P_{13}	P_{15}
		1							
		1		1		1		1	
1		1							
1		1		1		1		1	1

Remove EPIs and the minterms they cover

Partial Solution:
P_{16}
P_{14}
P_9
P_6
P_5
P_4
P_3

Cyclic Matrix:
10 rows
9 cols

两者中的一个，因为他们cover了最多的row. 之前的例子并没有遇到这种情况.
在该情景下没有essential prime implicant,
每行都有两种及以上的选择.
Procedure : branch and bound

P_1	P_2	P_7	P_8	P_{10}	P_{11}	P_{12}	P_{13}	P_{15}
					■		■	
■				■				
■				■	■		■	■
	■				■			
				■		■		
	■			■	■	■		■
		■		■				
		■		■				■
			■		■			
			■		■			■

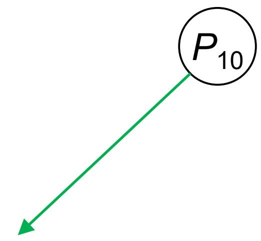
Partial Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 16

选择P_{10}不一定能给出 best solution. 但我们可以试一下.

P_1	P_2	P_7	P_8	P_{10}	P_{11}	P_{12}	P_{13}	P_{15}
					1		1	
1				1				
1				1	1		1	1
	1				1			
				1		1		
	1			1	1	1		1
		1		1				
		1		1				1
			1		1			
			1		1			1



Partial Solution: | P_1 | P_2 | P_3^* | P_4^* | P_5^* | P_6^* | P_7 | P_8 | P_9^* | P_{10} | P_{11} | P_{12} | P_{13} | P_{14}^* | P_{15} | P_{16}^* | Cost: 16

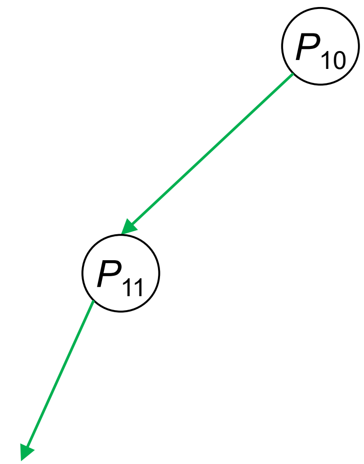
P_1	P_2	P_7	P_8	P_{11}	P_{12}	P_{13}	P_{15}
				1		1	
	1			1			
			1	1			
			1	1			1

P_{10}

Partial Solution:	P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*	Cost: 18

Cyclic Matrix:
4 rows
5 cols

P_2	P_8	P_{11}	P_{13}	P_{15}
		1	1	
1		1		
	1	1		
	1	1		1



Partial Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 20

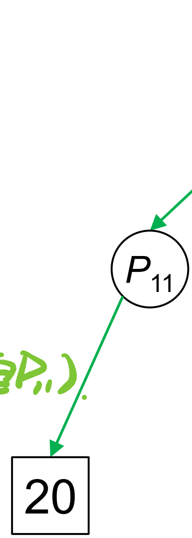
P_2 P_8 P_{11} P_{13} P_{15}

回去确认我们是否选了最合适的.

最后一个决定是选 P_{11}. 若不选 P_{11}, 删去他选别的会怎样?

P_2 P_8 P_{13} 都是essential prime implicant (若去掉 P_{11}).

next page →



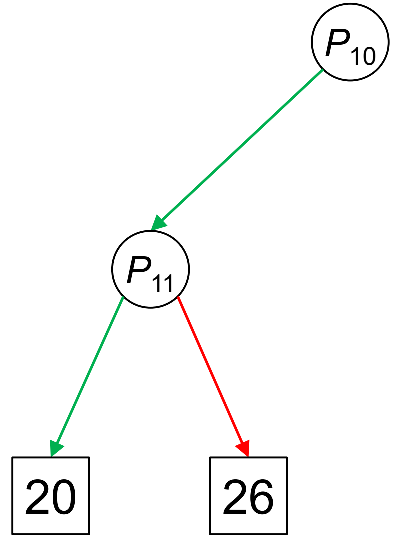
Partial Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 20

P_2	P_8	P_{13}	P_{15}
		1	
1			
	1		
	1		1

P_{15} is not essential

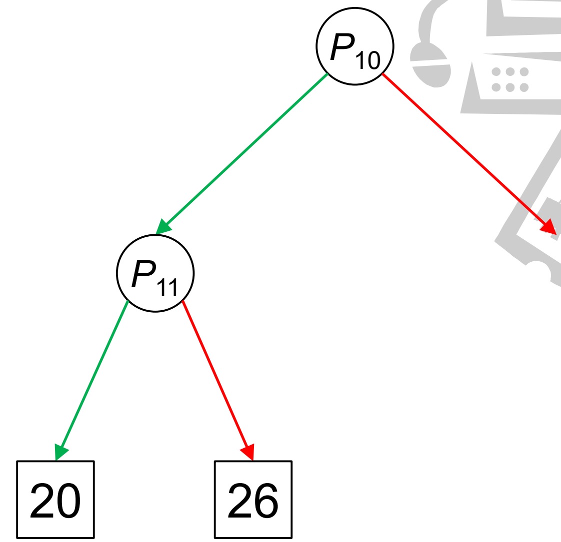


Partial Solution:
Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

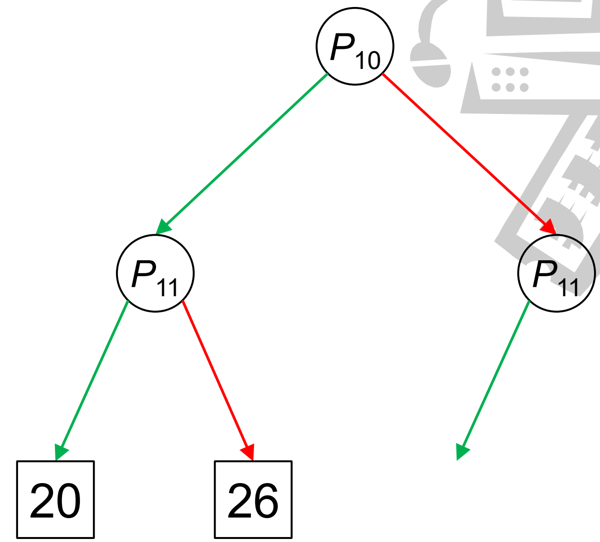
Cost: 26
Cost: 28

P_1	P_2	P_7	P_8	P_{10}	P_{11}	P_{12}	P_{13}	P_{15}
					1		1	
1				1				
1				1	1		1	1
	1				1			
				1		1		
	1			1	1	1		1
		1		1				
		1		1				1
			1		1			
			1		1			1



Partial Solution: P_1 P_2 P_3^* P_4^* P_5^* P_6^* P_7 P_8 P_9^* P_{10} P_{11} P_{12} P_{13} P_{14}^* P_{15} P_{16}^* Cost: 16

P_1	P_2	P_7	P_8	P_{11}	P_{12}	P_{13}	P_{15}
				1		1	
1							
1				1		1	1
	1			1			
					1		
	1			1	1		1
		1					
		1					1
			1	1			
			1	1			1

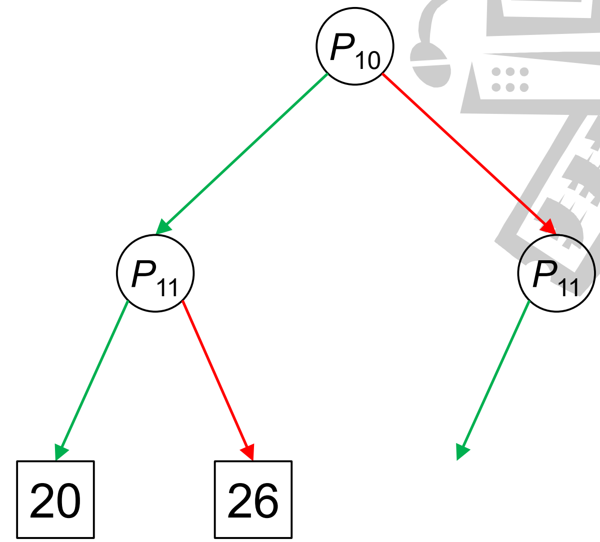


Partial Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 18

P_1	P_2	P_7	P_8	P_{12}	P_{13}	P_{15}
1						
				1		
		1				
		1				1

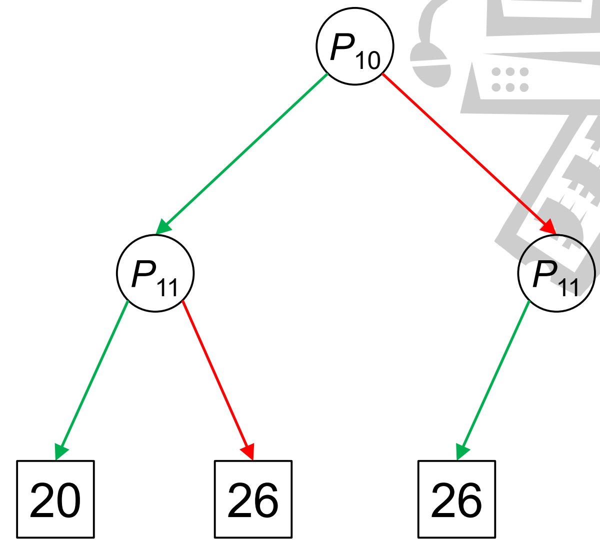


Partial Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 18

P_1	P_7	P_{12}	P_{15}
1			
		1	
	1		
	1		1

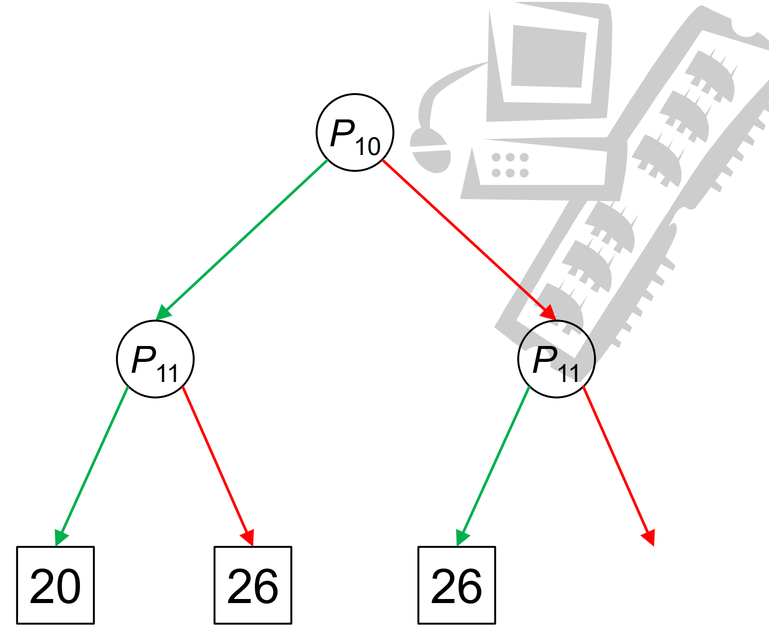


Partial Solution:
Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 26
Cost: 18

P_1	P_2	P_7	P_8	P_{11}	P_{12}	P_{13}	P_{15}
				1		1	
1							
1				1		1	1
	1			1			
					1		
	1			1	1		1
		1					
		1					1
			1	1			
			1	1			1

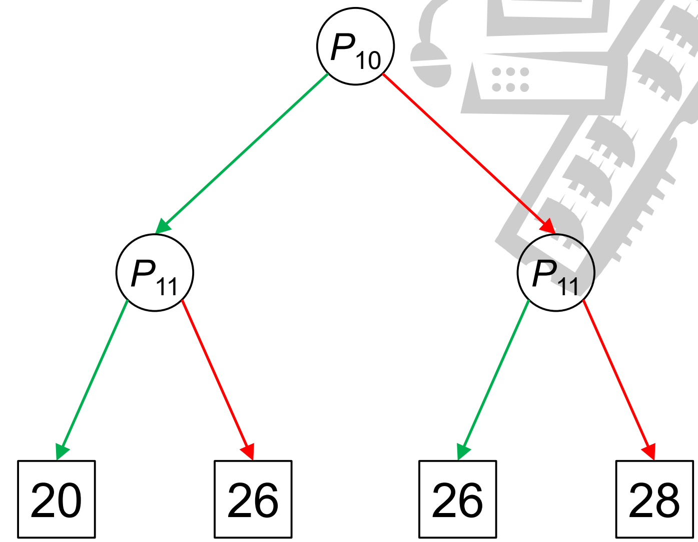


Partial Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 16

P_1	P_2	P_7	P_8	P_{12}	P_{13}	P_{15}
					1	
1						
1					1	1
	1					
				1		
	1			1		1
		1				
		1				1
			1			
			1			1



Partial Solution:

P_1	P_2	P_3^*	P_4^*	P_5^*	P_6^*	P_7	P_8	P_9^*	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}^*	P_{15}	P_{16}^*

Cost: 26

Simple Solver (espresso)