Convolutional Neural Nets

EECS 442 – Jeong Joon Park

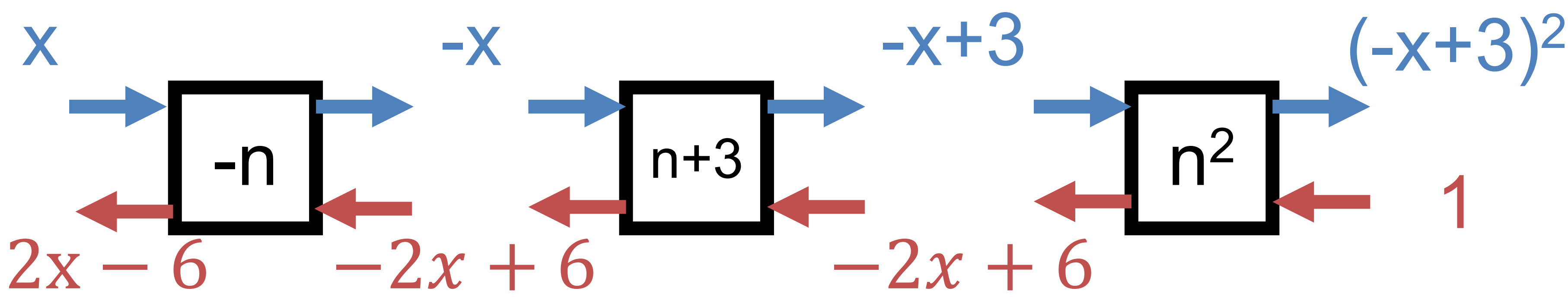
Winter 2024, University of Michigan

Administrivia

- HW3 Due this Wednesday

Previously – Backpropagation

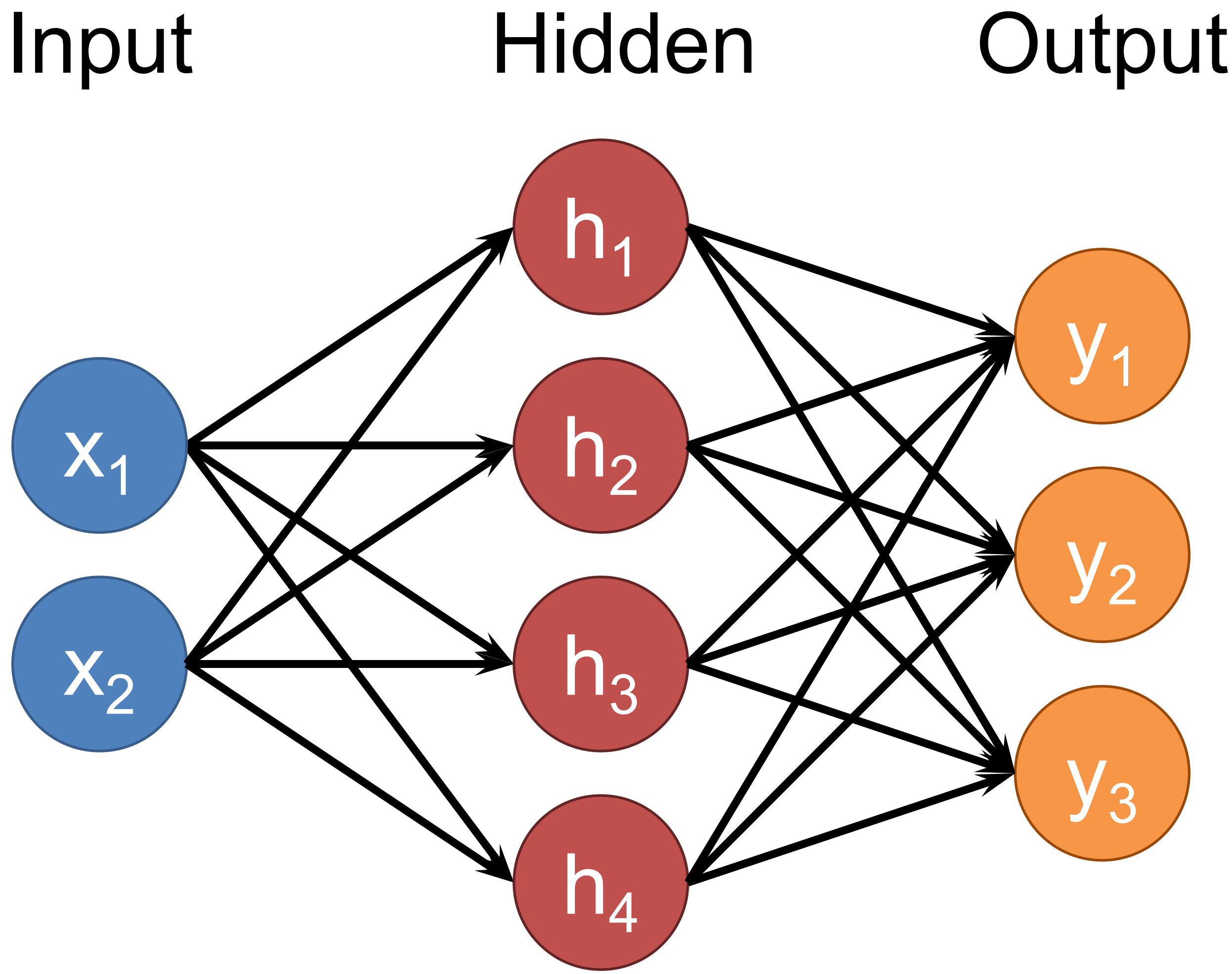
$$f(x) = (-x+3)^2$$



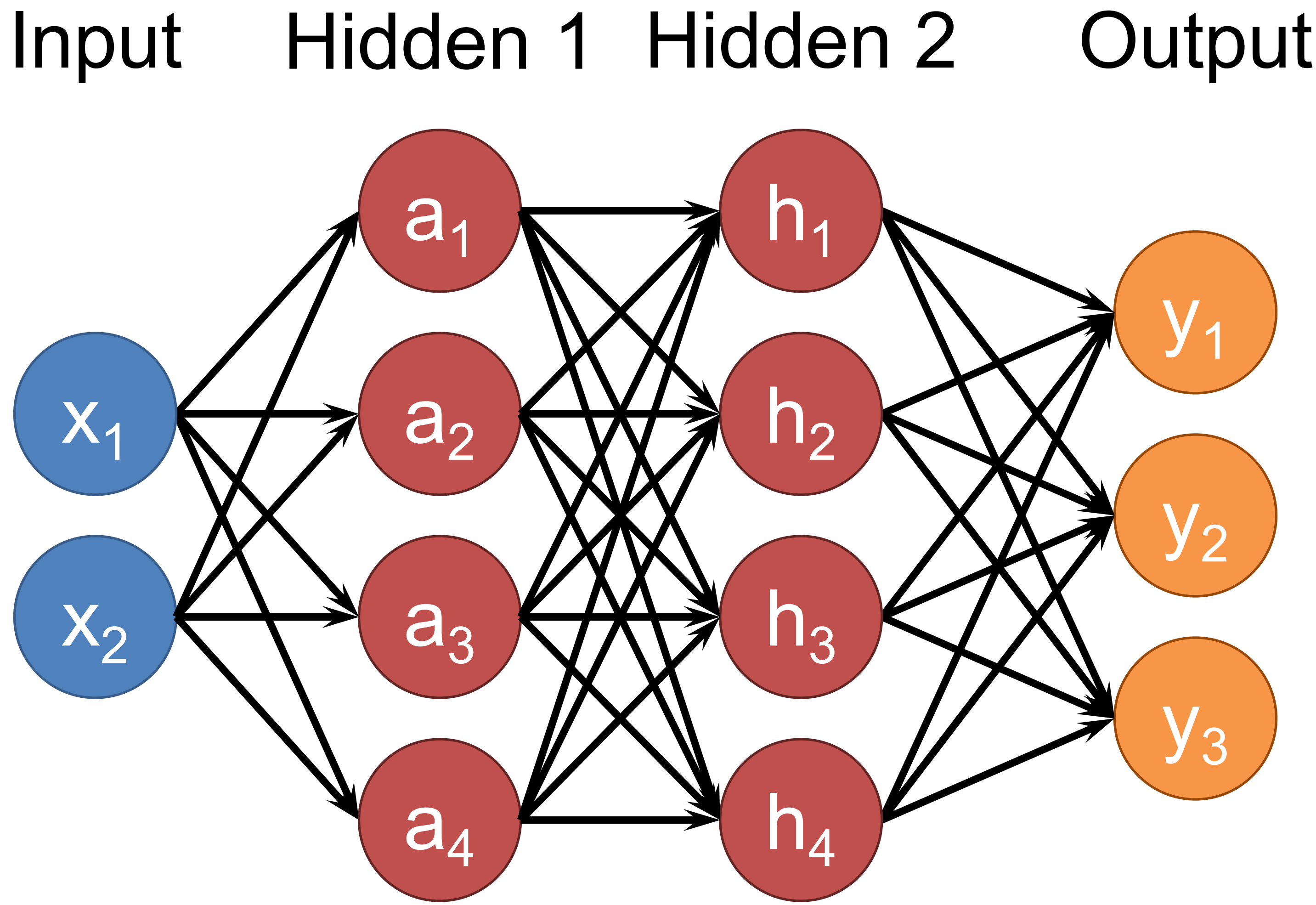
Forward pass: compute function

Backward pass: compute derivative of all parts of the function

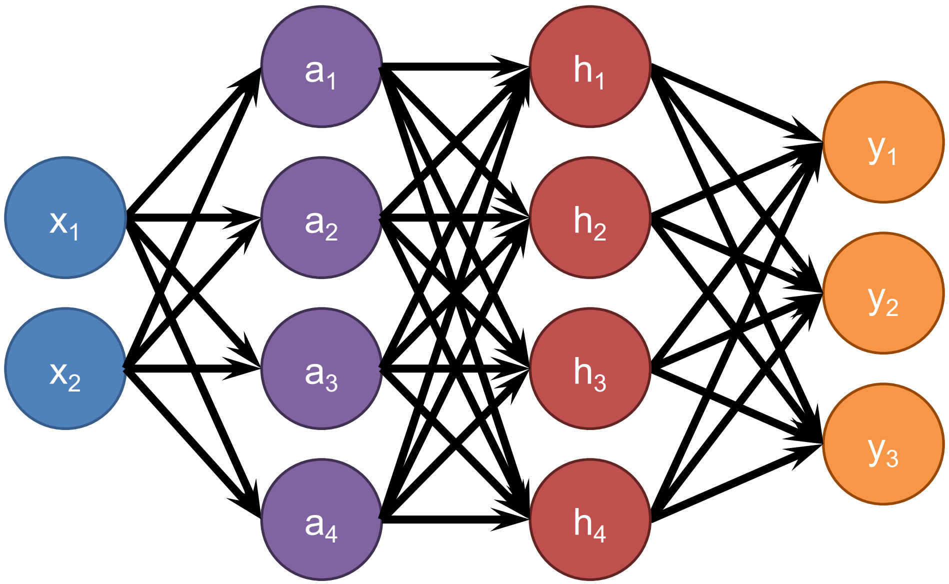
Setting Up A Neural Net



Setting Up A Neural Net

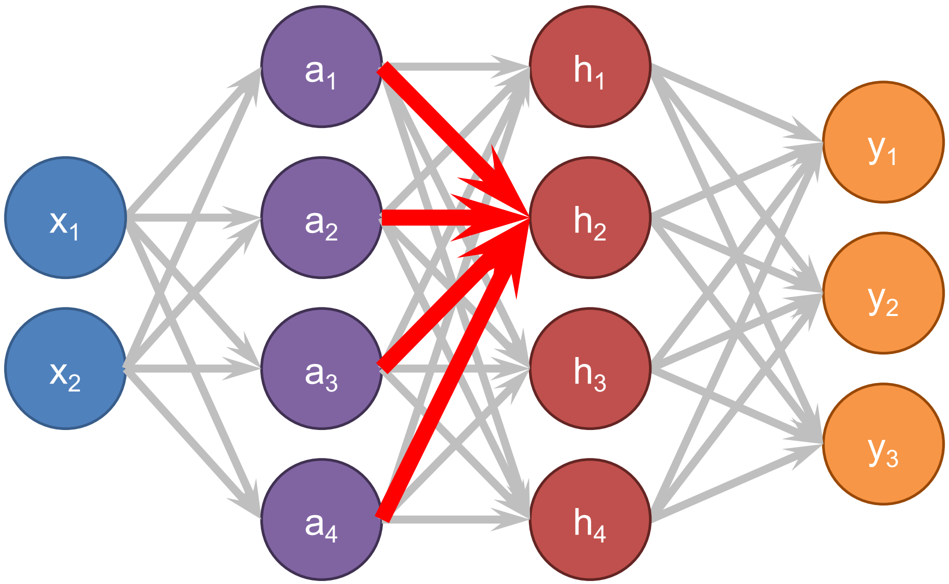


Fully Connected Network



Each neuron connects to each neuron in the previous layer
Fully-Connected Net

Fully Connected Network



$\boldsymbol{a}$ All layer *a* values

$\boldsymbol{w_i}, b_i$ Neuron i weights, bias

f Activation function

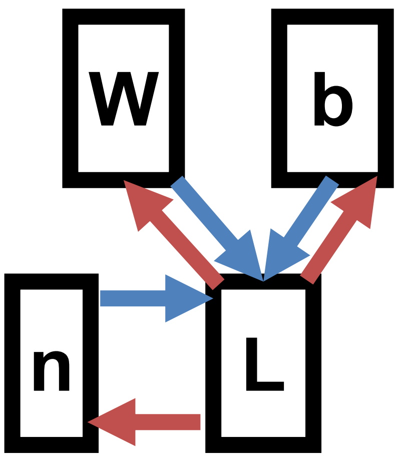
$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{a} + \boldsymbol{b})$$

$$\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = f\left(\begin{bmatrix} \leftarrow w_1^T \rightarrow \\ \leftarrow w_2^T \rightarrow \\ \leftarrow w_3^T \rightarrow \\ \leftarrow w_4^T \rightarrow \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \right)$$

Fully Connected Network

Define New Block: “Linear Layer”

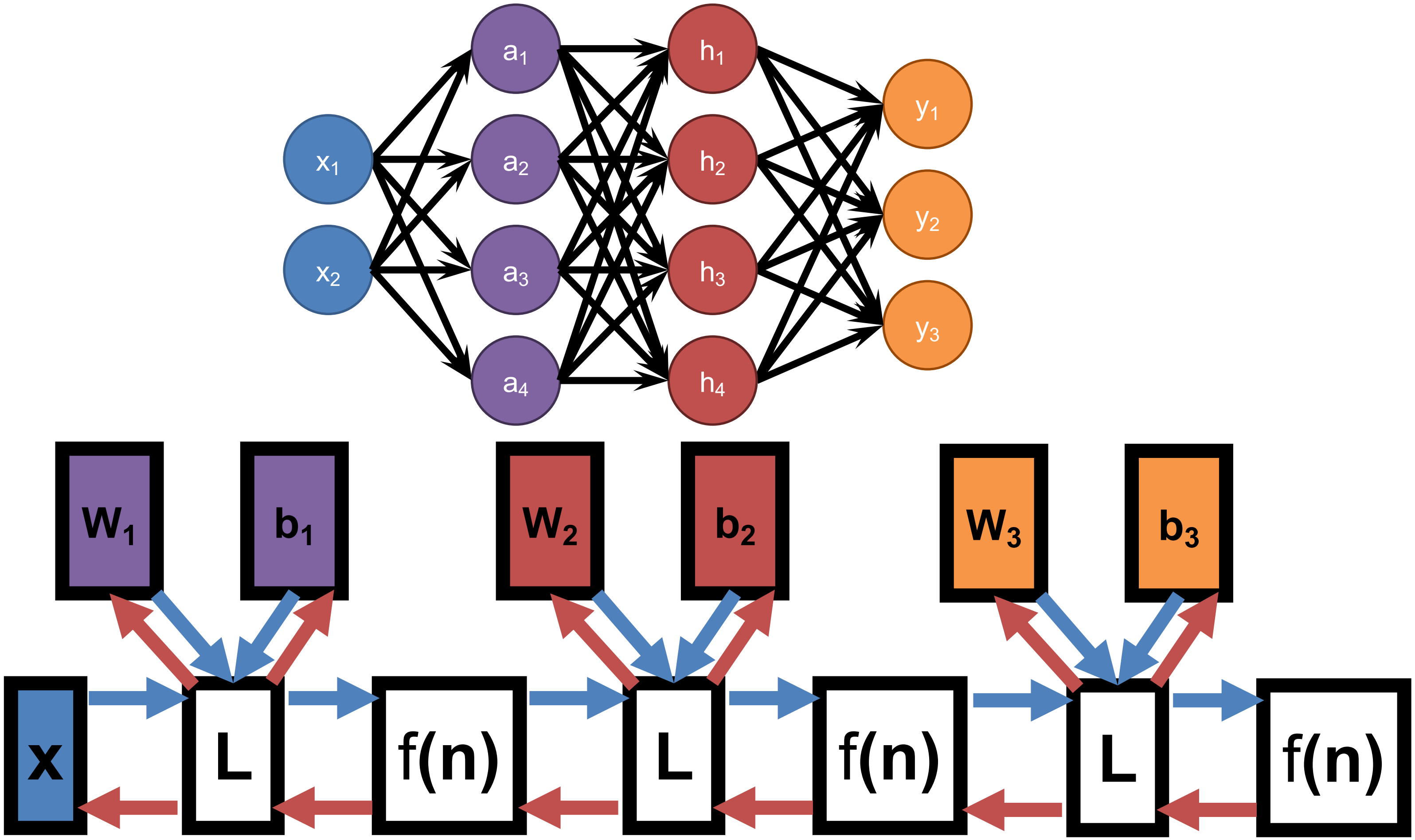
(It’s Technically Affine)



$$L(\boldsymbol{n}) = \boldsymbol{W}\boldsymbol{n} + \boldsymbol{b}$$

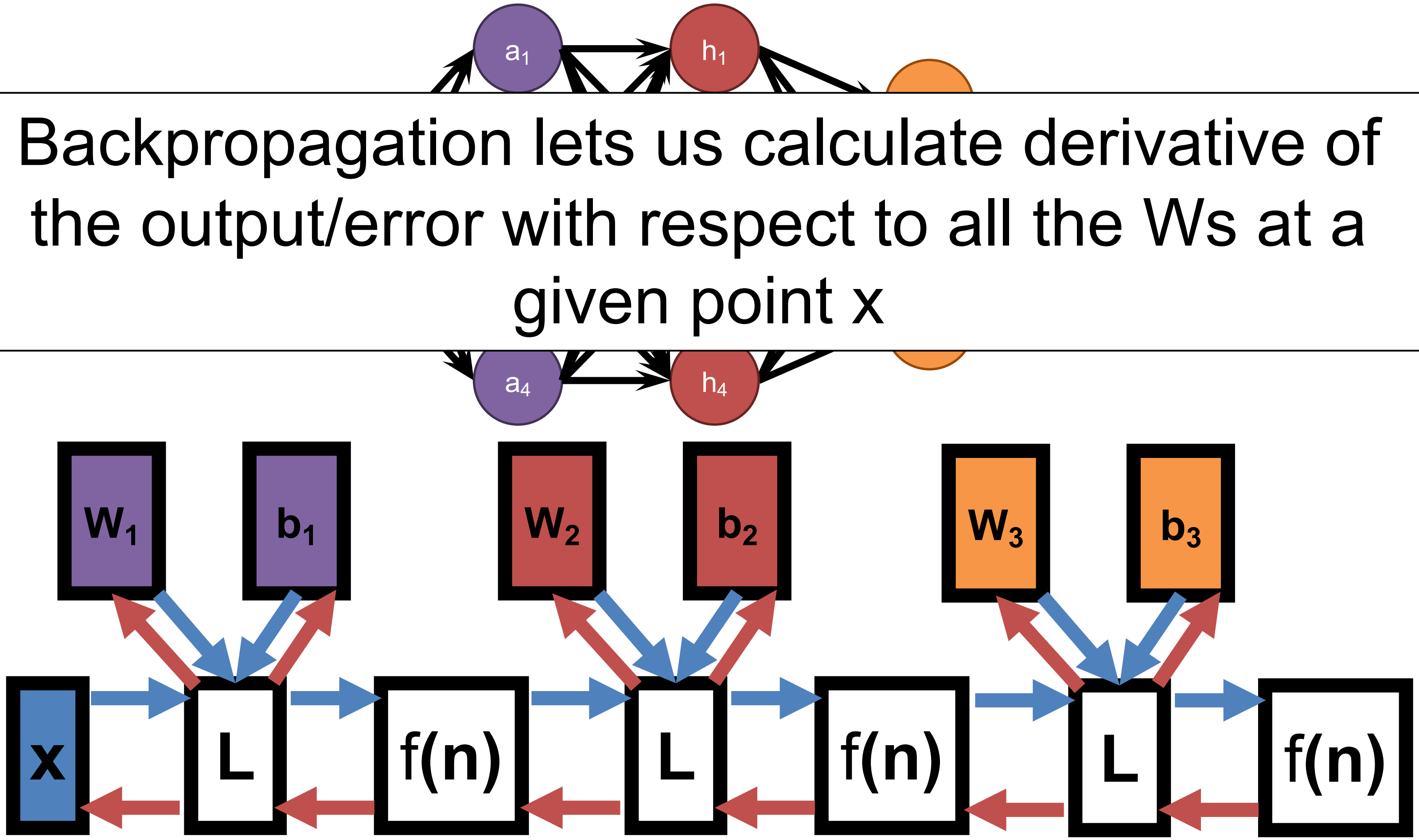
Can get gradient with respect to all the inputs

Fully Connected Network



Fully Connected Network

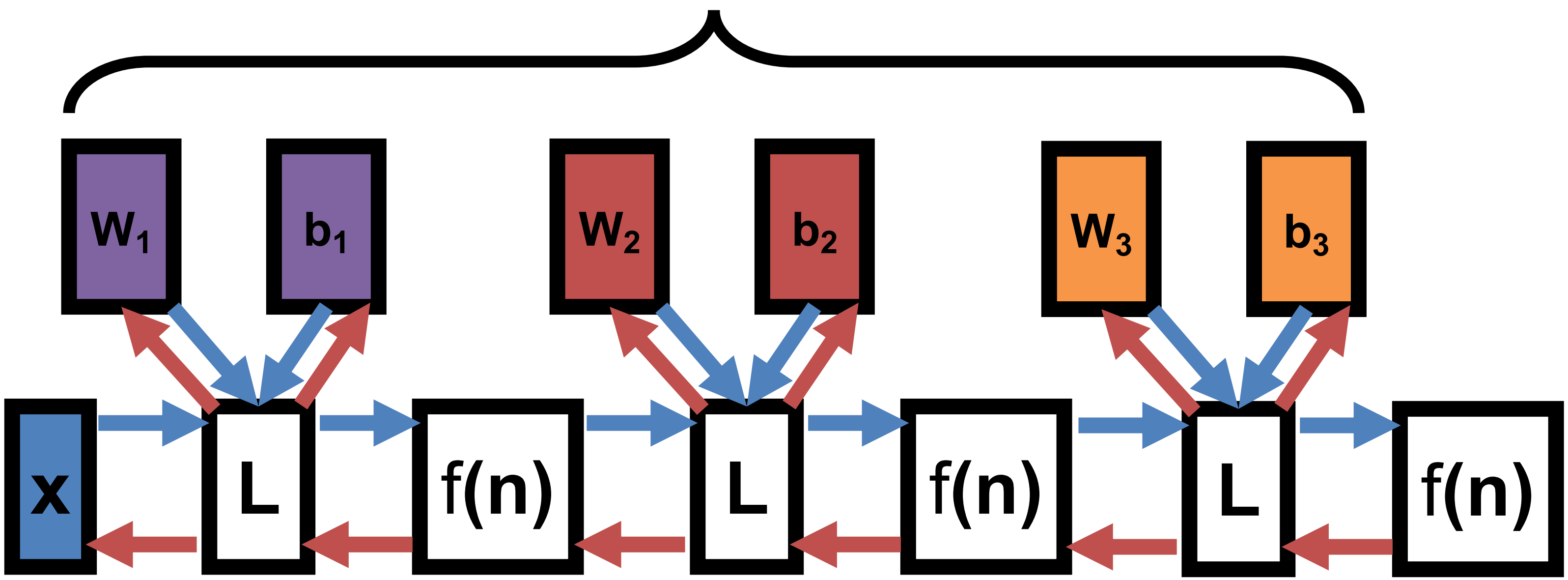
Backpropagation lets us calculate derivative of the output/error with respect to all the Ws at a given point x



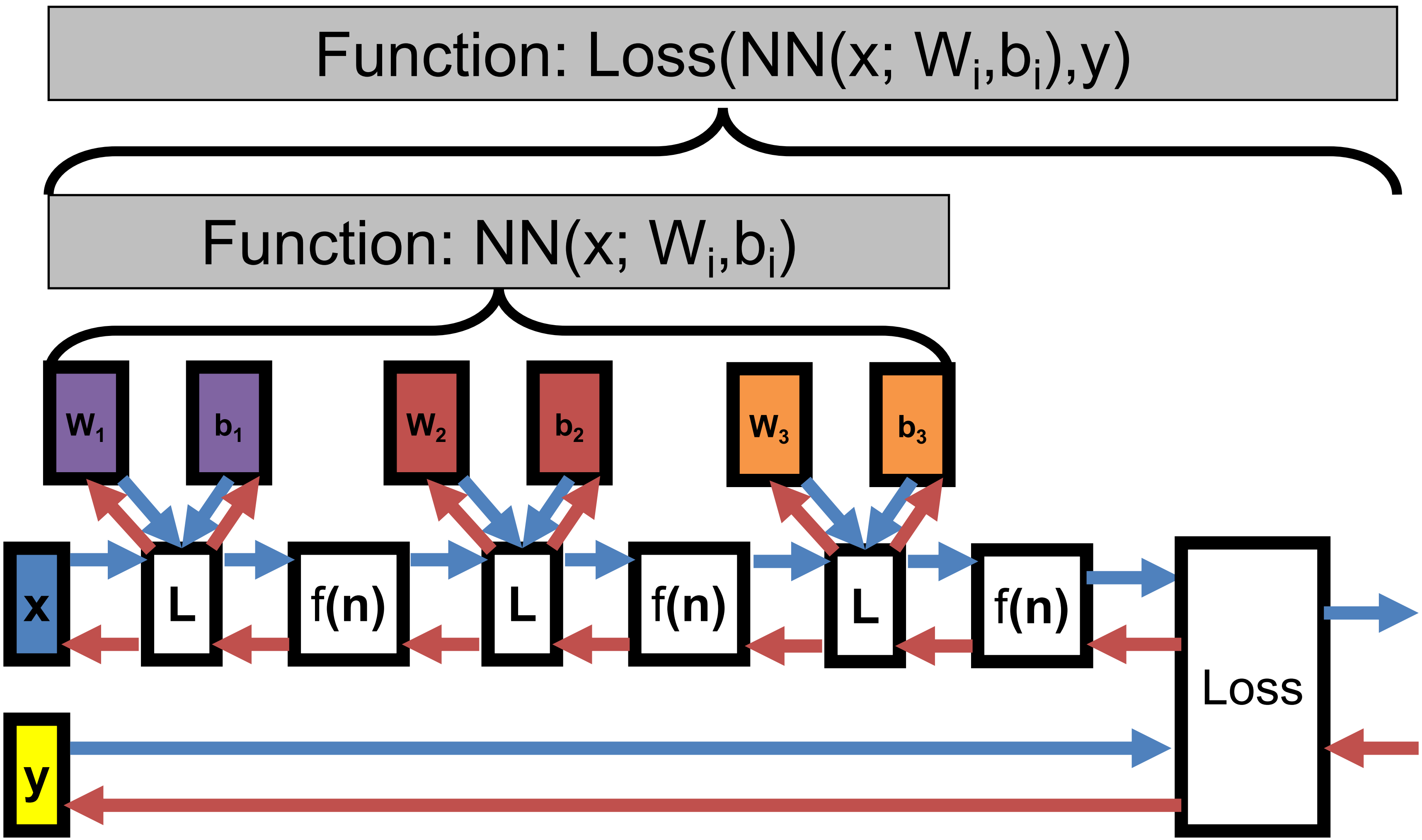
Putting It All Together – 1

Function: $NN(x; W_i, b_i)$

Parameterized by $W = \{W_i, b_i\}$



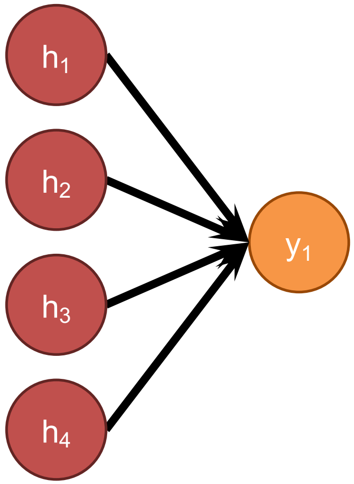
Putting It All Together



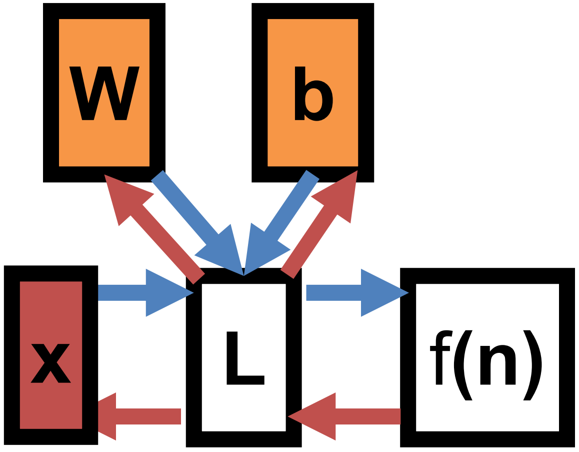
Putting It All Together

```
W = initializeWeights()
for i in range(numIterations):
        #sample a batch
        batch = random.subset(0,#datapoints,K)
        batchX, batchY = dataX[batch], dataY[batch]
        #compute gradient with batch
        gradW = backprop(Loss(NN(batchX,W),batchY))
        #update W with gradient step or use momentum
        W += -stepsize*gradW
return W
```

What Can We Represent?

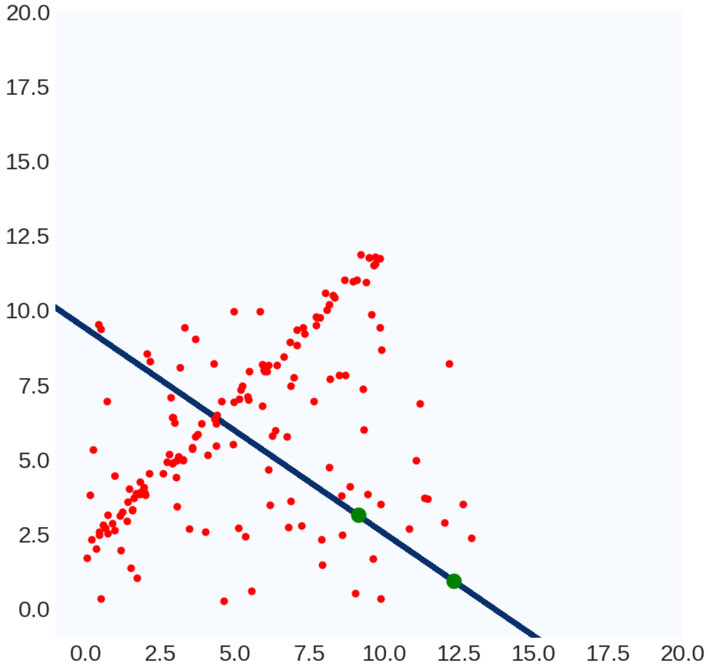


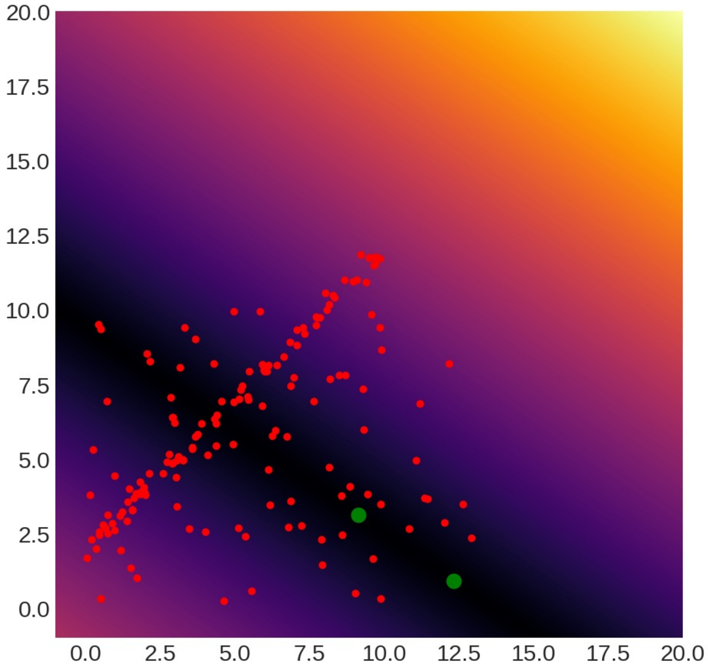
$$L(\boldsymbol{n}) = \boldsymbol{W}\boldsymbol{n} + \boldsymbol{b}$$



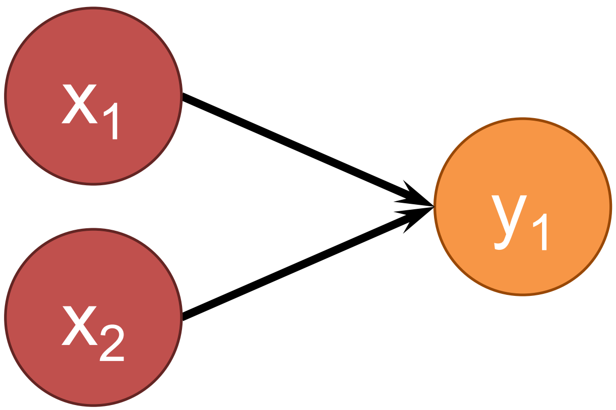
What Can We Represent

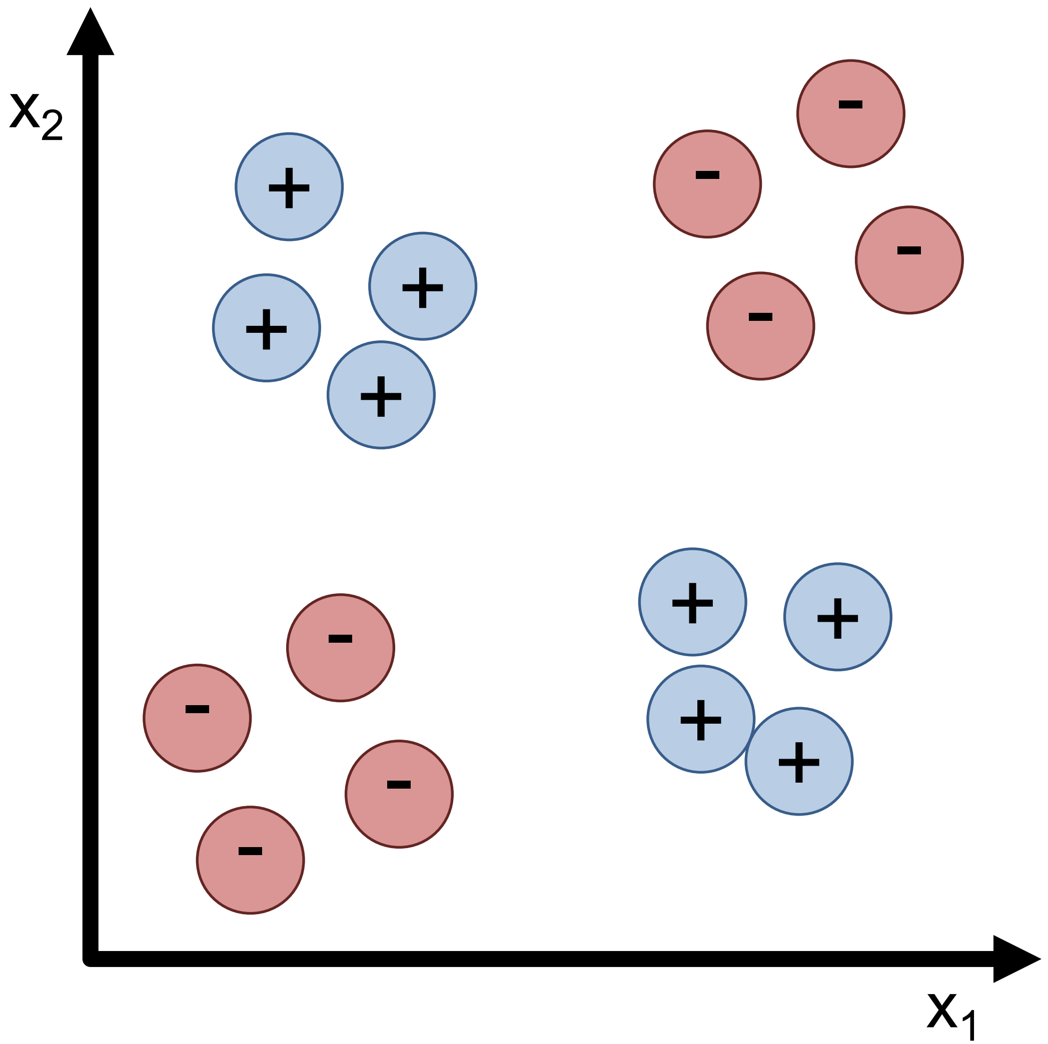
- Recall: ax+by+z is
  - proportional to **signed** distance to line
  - equal to signed distance if you normalize
- Generalization to N-D: hyperplane $\mathbf{w}^T\mathbf{x}$+b



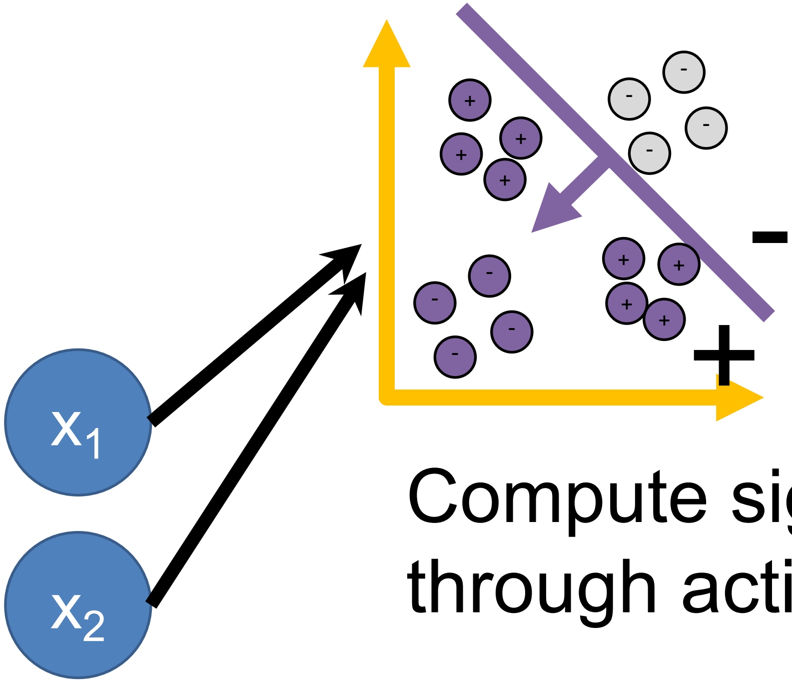


Can We Train a Network To Do It?



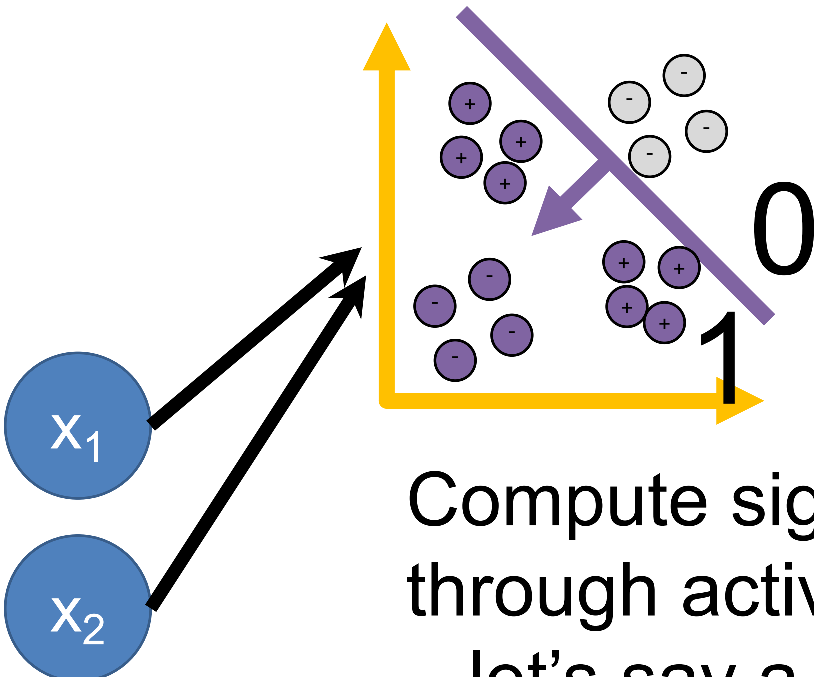


Can We Train a Network To Do It?



Compute signed distance; pass through activation, e.g., ReLU

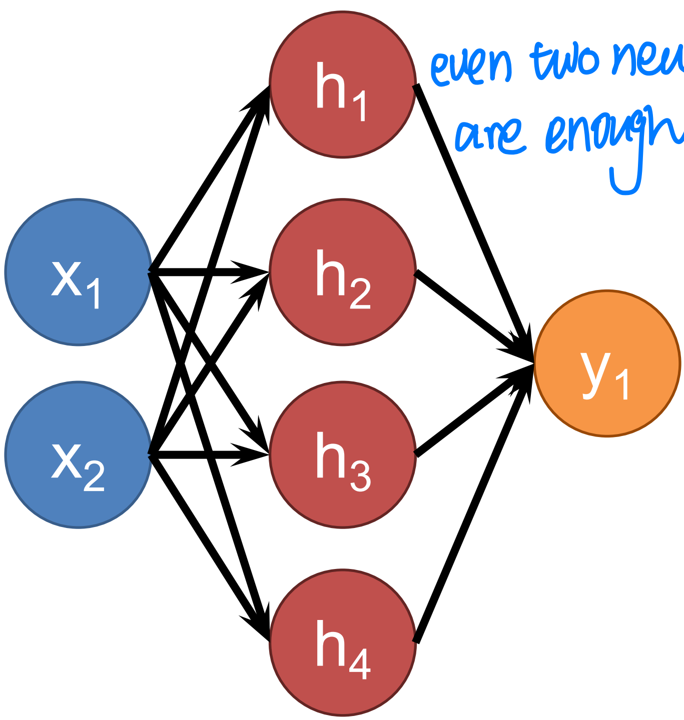
Can We Train a Network To Do It?



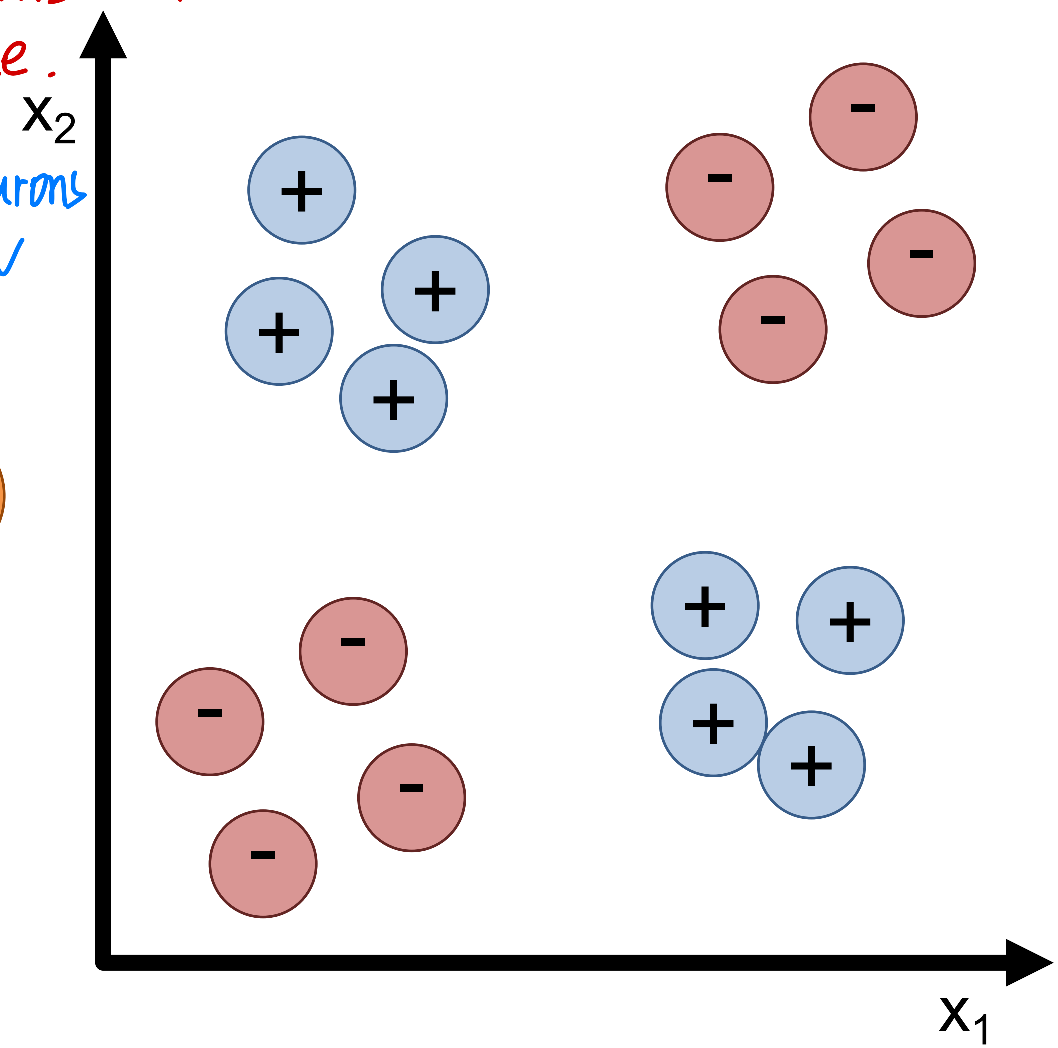
Compute signed distance; pass through activation
– let's say a step function (0 or 1)

Can We Train a Network To Do It?

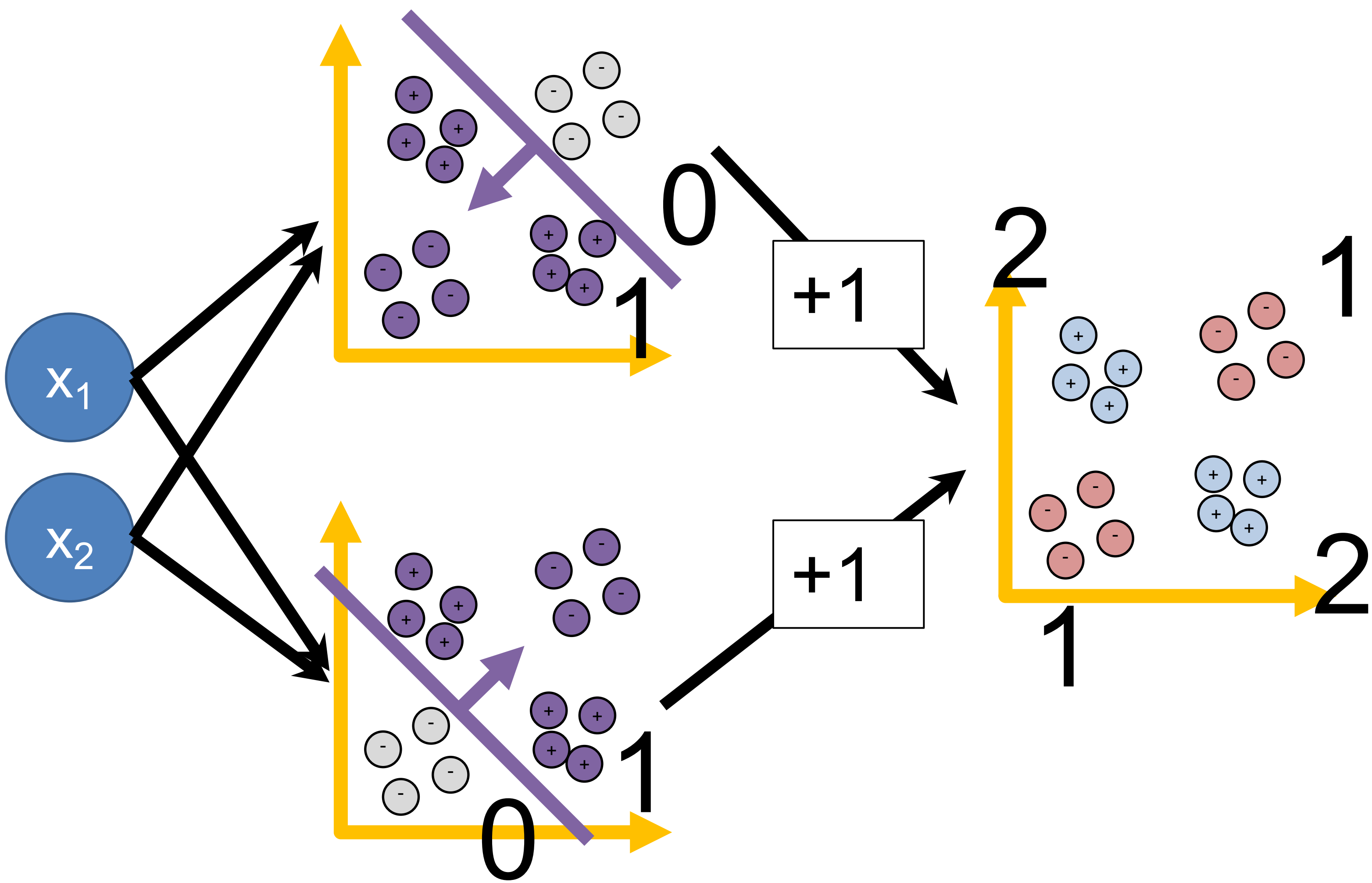
4 neurons in the hidden layers
And each neurons represents a line in half space.



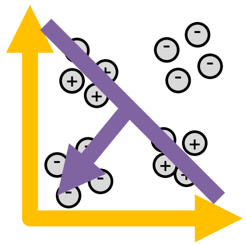
even two neurons are enough



Can We Train a Network To Do It?

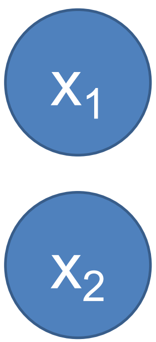


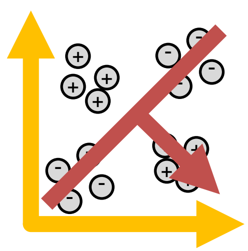
Can We Train a Network To Do It?



$\max(\mathbf{w_1}^T\mathbf{x}+b,0)$

$\max(\mathbf{w_1}^T\mathbf{x}+b,0)=$ Distance to line defined by $\mathbf{w}_1$

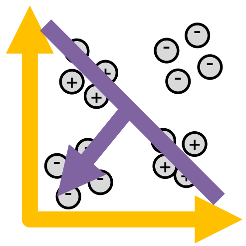




$\max(\mathbf{w_2}^T\mathbf{x}+b,0)$

$\max(\mathbf{w_2}^T\mathbf{x}+b,0) =$ Distance to line defined by $\mathbf{w}_2$

Can We Train a Network To Do It?



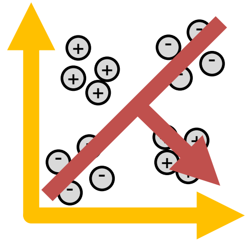
Distance to $\mathbf{w}_1$

x_1

x_2

Next layer computes:

max($\mathbf{w}_1$ Distance - $\mathbf{w}_2$ Distance,0)



Distance to $\mathbf{w}_2$

Can We Train a Network To Do It?

- Three Neurons Demo

Can We Train a Network To Do It?

use neurons to divide your space with half-spaces (possitive value / zero)

Result: feedforward neural networks with a finite number of neurons in a hidden layer can approximate any continuous function with a bounded domain

Universal Approximation Theorem.

Cybenko (1989) for neural networks with sigmoids; Hornik (1991) more generally

In practice, doesn't give a practical guarantee.

Why?

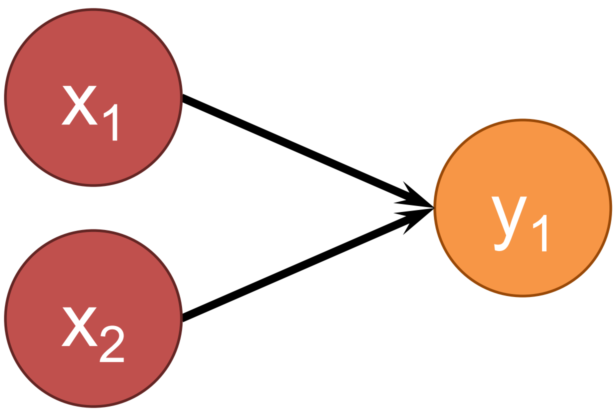
Developing Intuitions

There is no royal road to geometry. – Euclid

- Best way: play with data, be skeptical of everything you do, be skeptical of everything you are told
- Remember: this is linear algebra, not magic
- Technique: How would you set the weights by hand if you were forced to be a deep net

Parameters

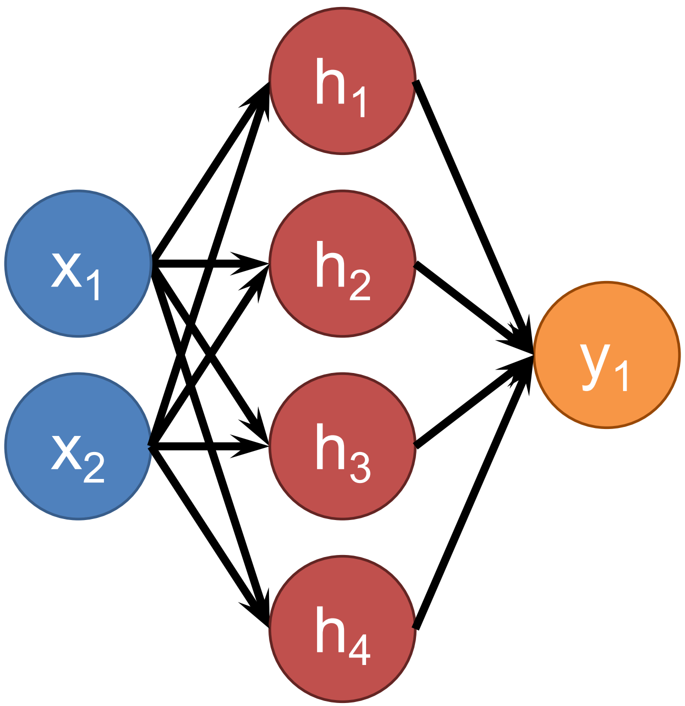
How many parameters does this network have?



Weights: 1x2
Parameters: 3 (bias!)

Parameters

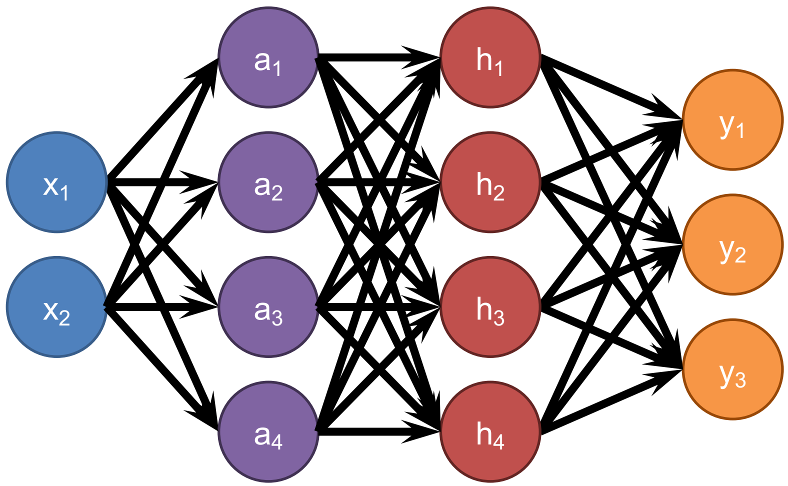
How many parameters does this network have?



Weights: 1x4+4x2 = 12
Parameters: 12+5 = 17

Parameters

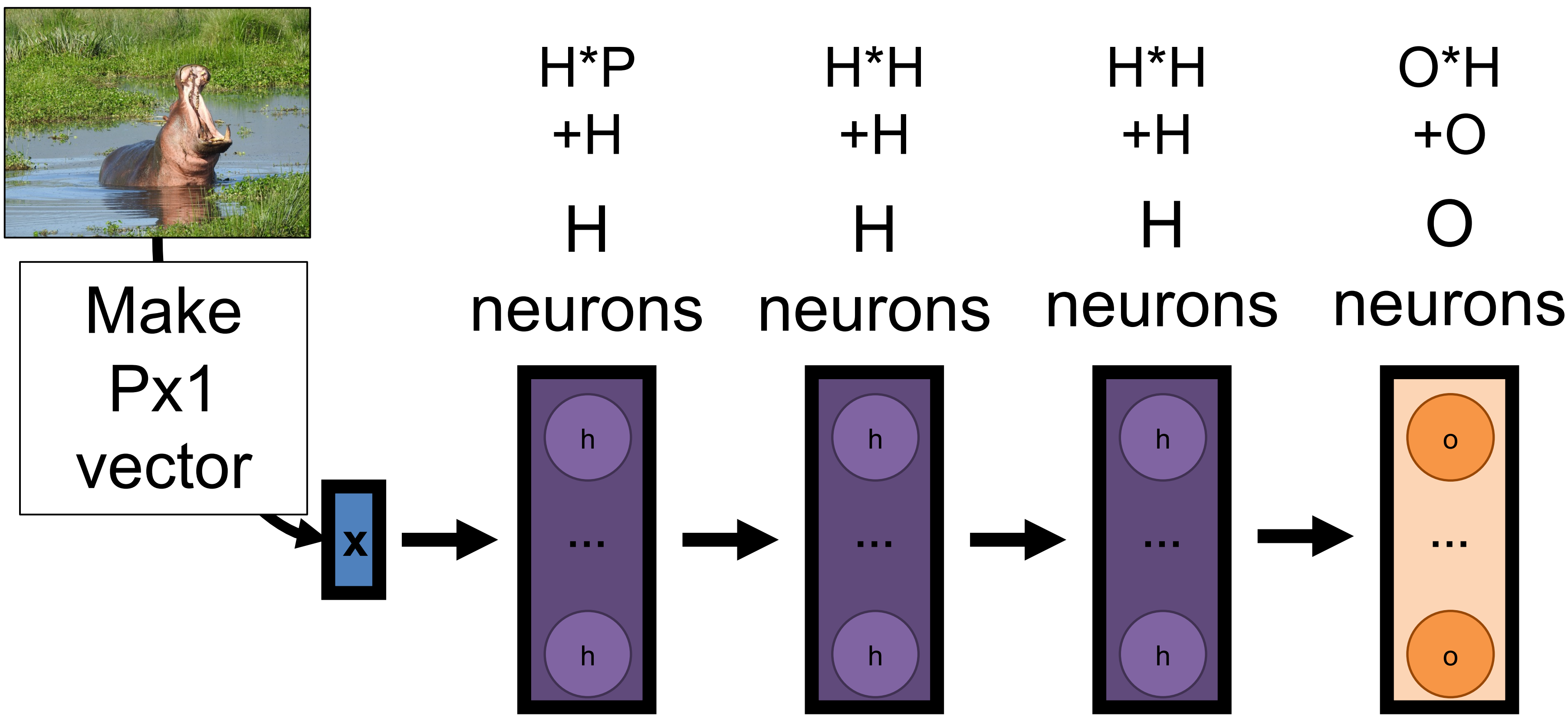
How many parameters does this network have?



Weights: 3x4+4x4+4x2 = 36

Parameters: 36+11 = 47

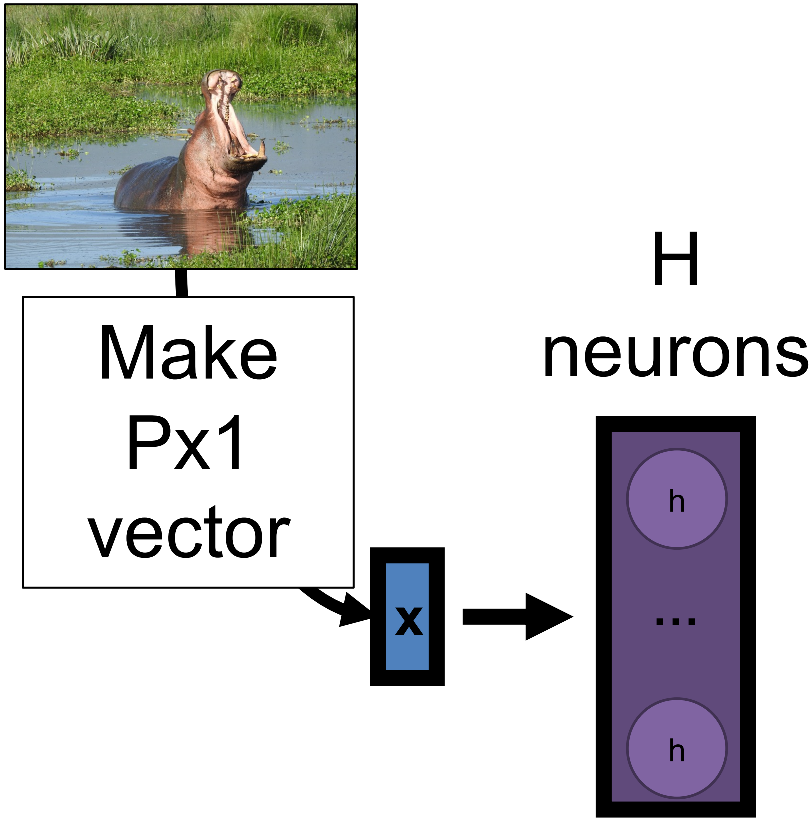
Parameters



P: 285x350 picture **(terrible!)**, H: 1000, O: 3

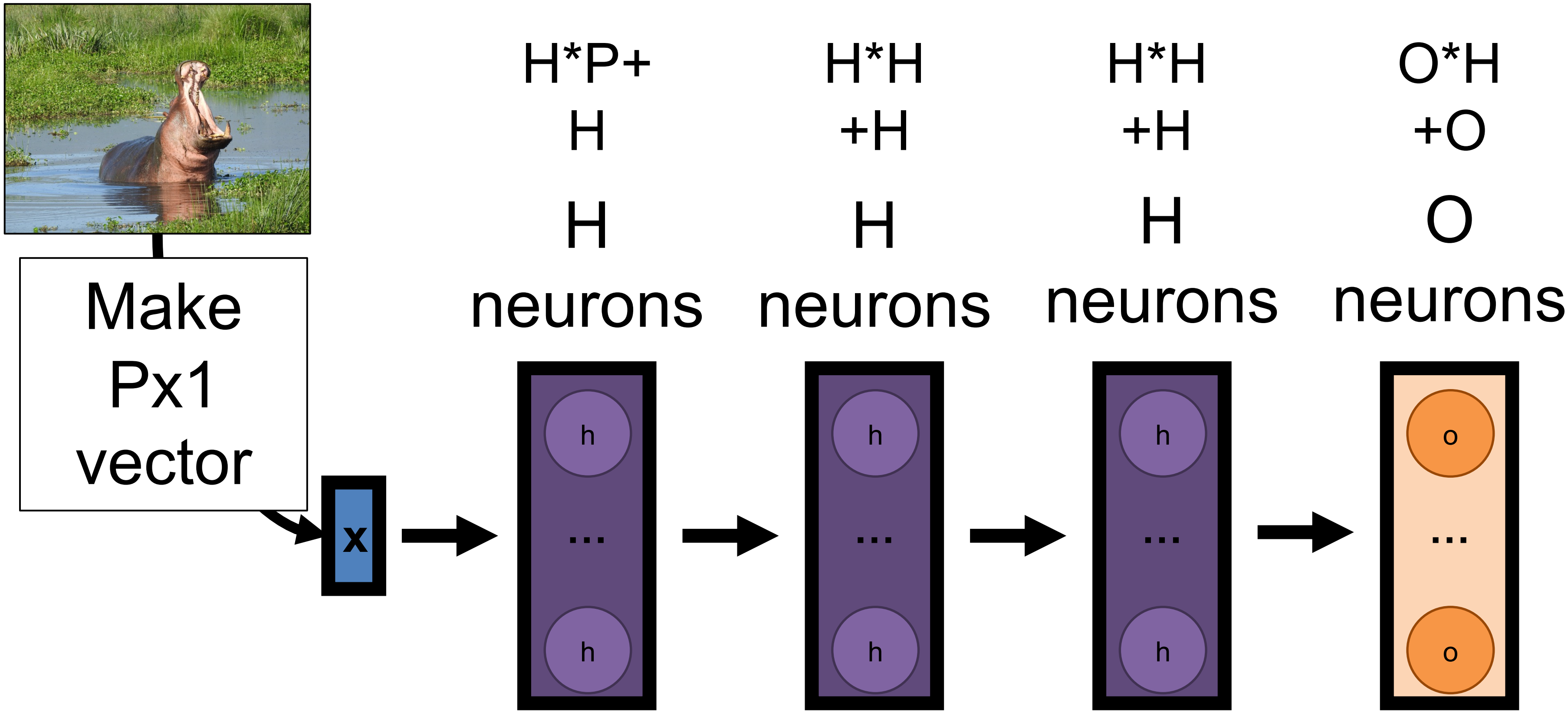
102 million parameters (400MB)

Parameters



- First layer converts **all** visual information into a single H dimensional vector.
- Suppose you want a neuron to represent image gradient at each pixel. **How many neurons do you need?**
- **2P** each pixel has x and y direction gradient.

Parameters



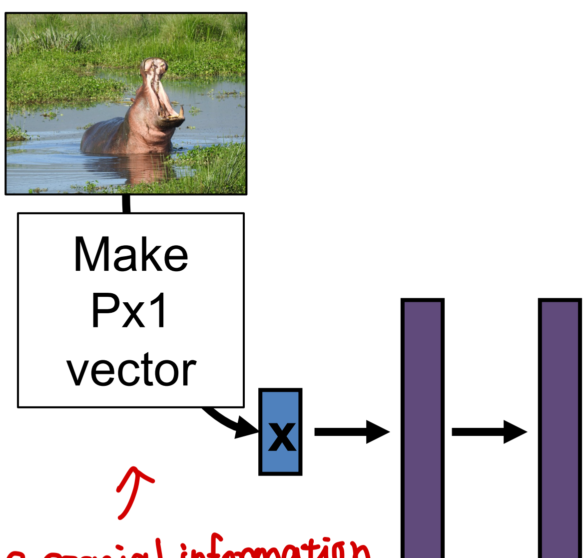
P: 285x350, H: 2P, O: 3

100 billion parameters (400GB)

Convnets

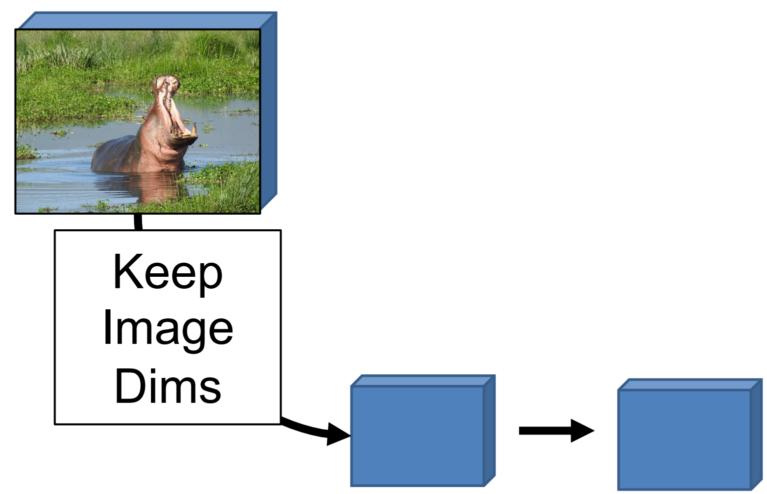
Keep Spatial Resolution Around

FC Neural net:
Data: vector Fx1
Transform: matrix-multiply

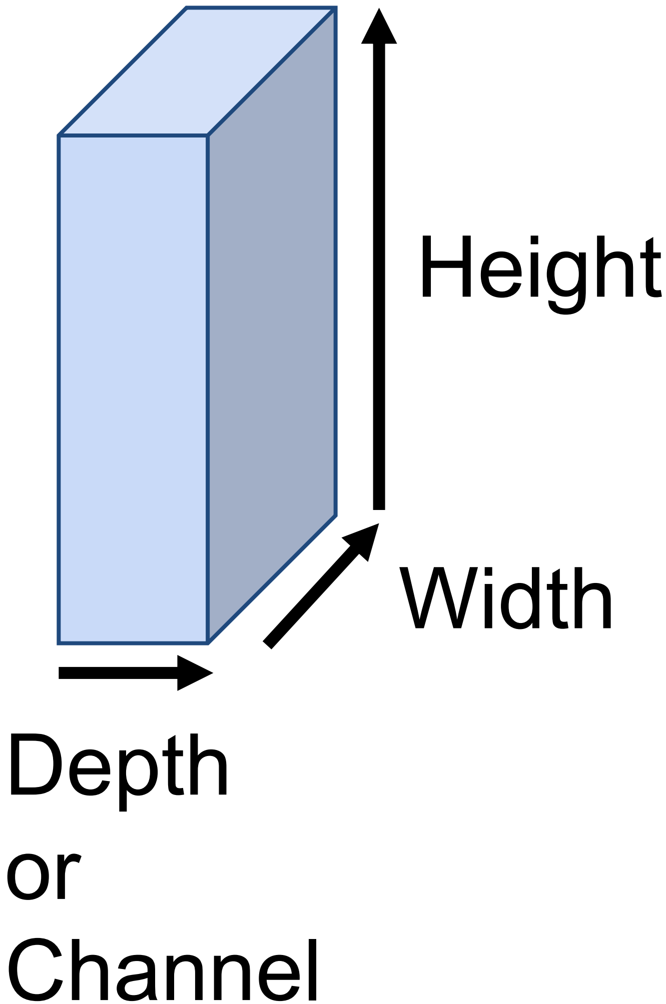


the spacial information is ignored.

Convnet:
Data: image HxWxF
Transform: convolution



Convnet (Feature Volume)

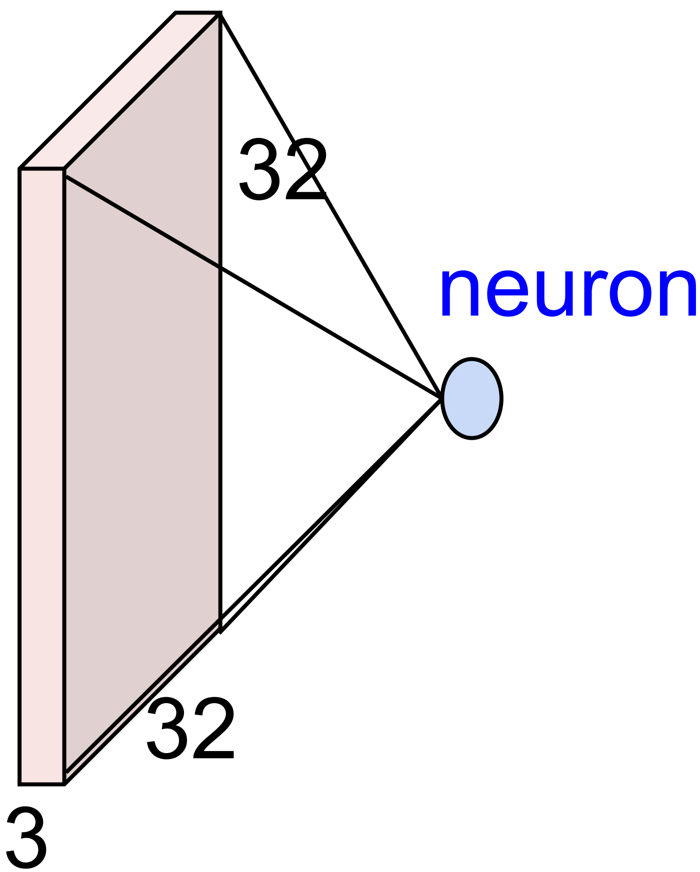


Height: 300
Width: 500
Depth: 3

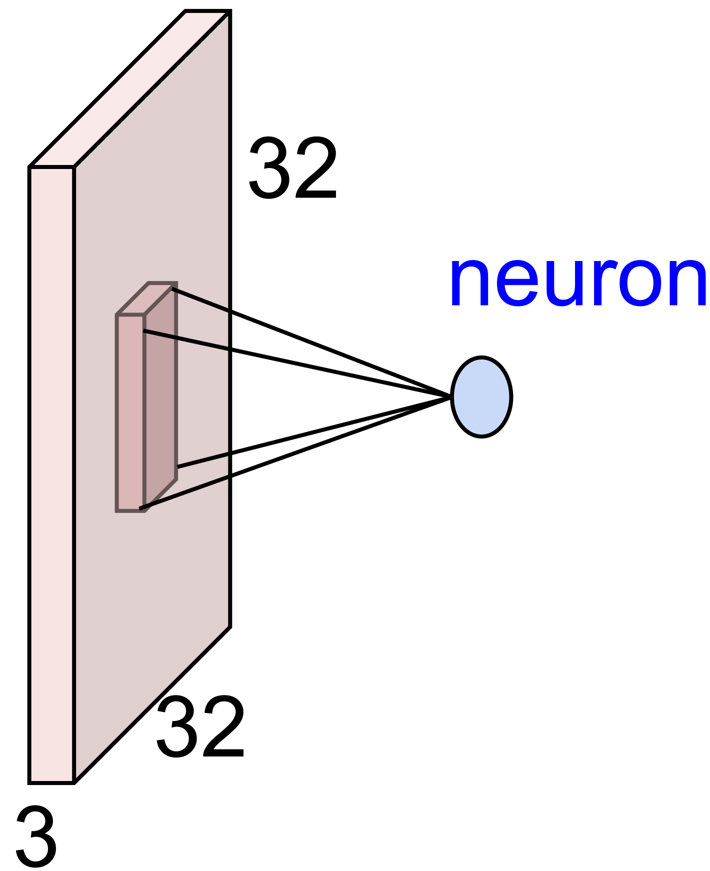
Height: 32
Width: 32
Depth: 3

Convnet

Fully connected:
Connects to everything



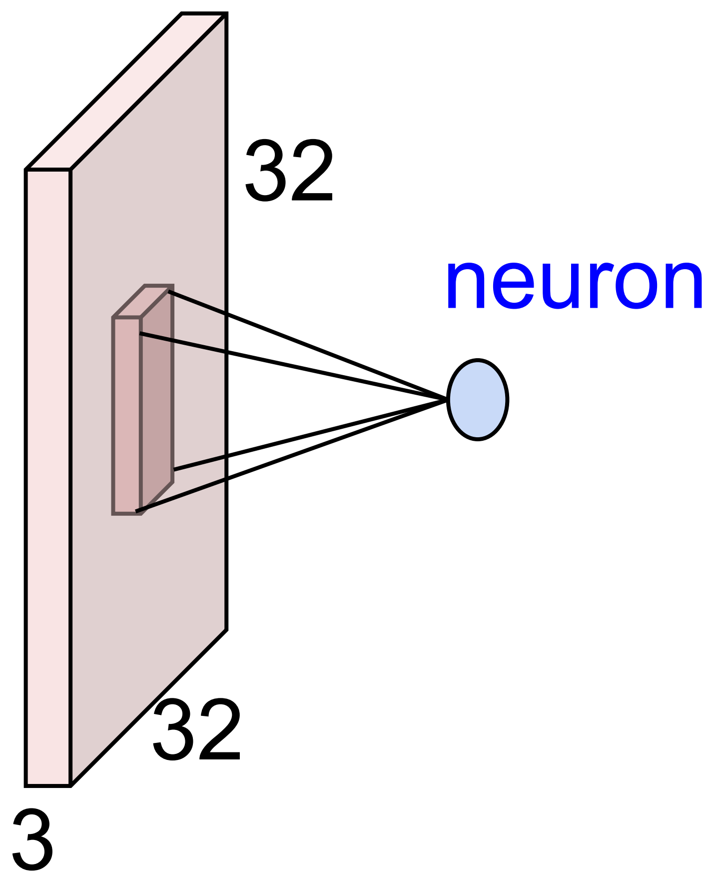
Convnet:
Connects locally
Filter Small & Reused

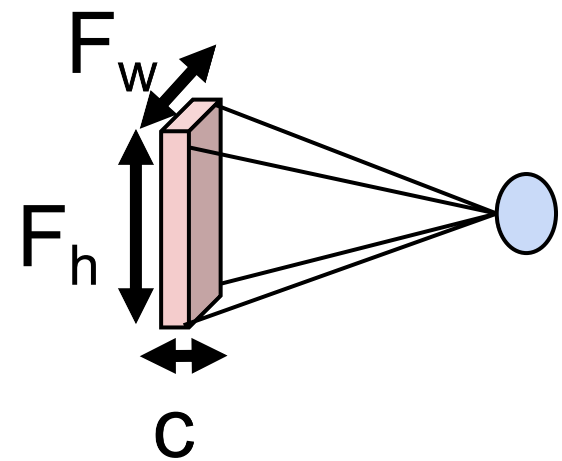


Slide credit: Karpathy and Fei-Fei

Convnet

Neuron is the same: weighted linear average



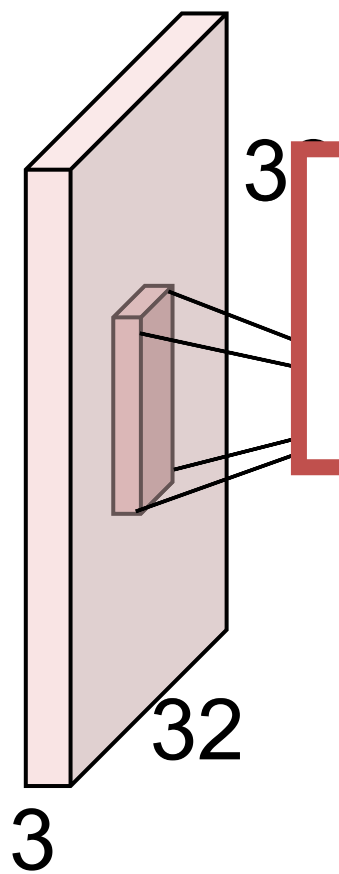


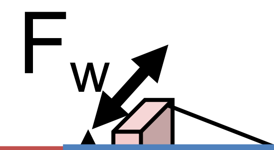
$$\sum_{i=1}^{F_h}\sum_{j=1}^{F_w}\sum_{k=1}^{c} F_{i,j,k} I_{y+i,x+j,k}$$

Slide credit: Karpathy and Fei-Fei

Convnet

Neuron is the same: weighted linear average





Filter is local in space: sum only over F_h x F_w pixels

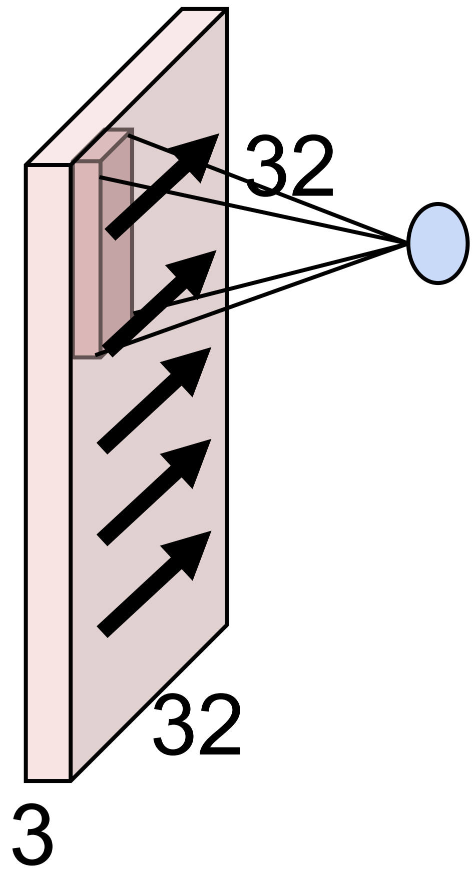
Filter is global over channels/depth: sum over all channels

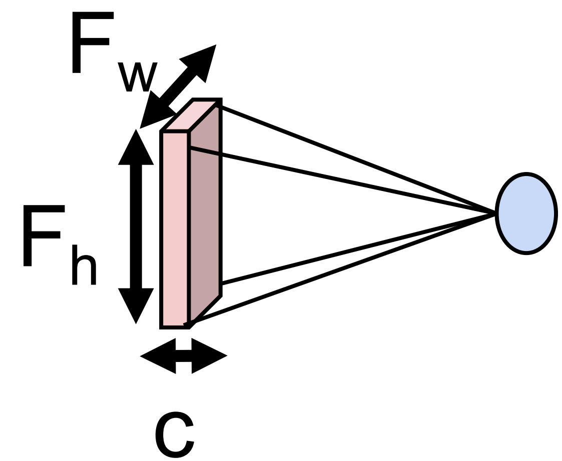
$$\boxed{\sum_{i=1}^{F_h}\sum_{j=1}^{F_w}}\boxed{\sum_{k=1}^{c}} F_{i,j,k} I_{y+i,x+j,k}$$

Slide credit: Karpathy and Fei-Fei

Convnet

Get spatial output by sliding filter over image





$$\sum_{i=1}^{F_h}\sum_{j=1}^{F_w}\sum_{k=1}^{c} F_{i,j,k} I_{y+i,x+j,k}$$

Slide credit: Karpathy and Fei-Fei

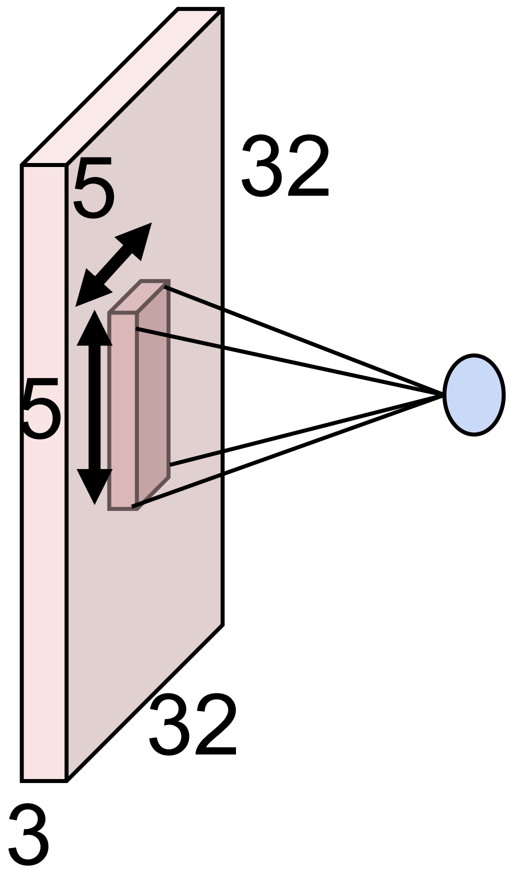
Differences From Earlier Filtering

(a) #input channels can be greater than one
(b) forget you learned the difference between convolution and cross-correlation

$$\text{Output}[1,2] = I[1,2]*F[1,1] + I[1,3]*F[1,2] + \ldots + I[3,4]*F[3,3]$$

Convnet

How big is the output?



Height? 32-5+1=28
Width? 32-5+1=28
Channels? 1

One filter not very useful by itself

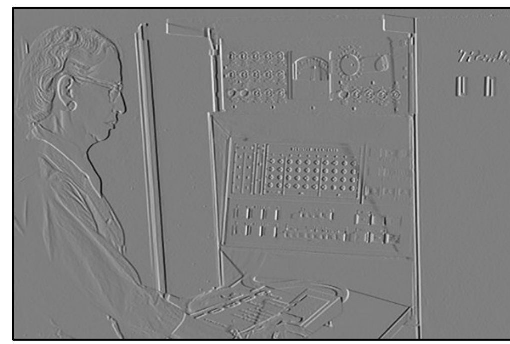
Slide credit: Karpathy and Fei-Fei

Multiple Filters

You’ve already seen this before

Input:
400x600x1

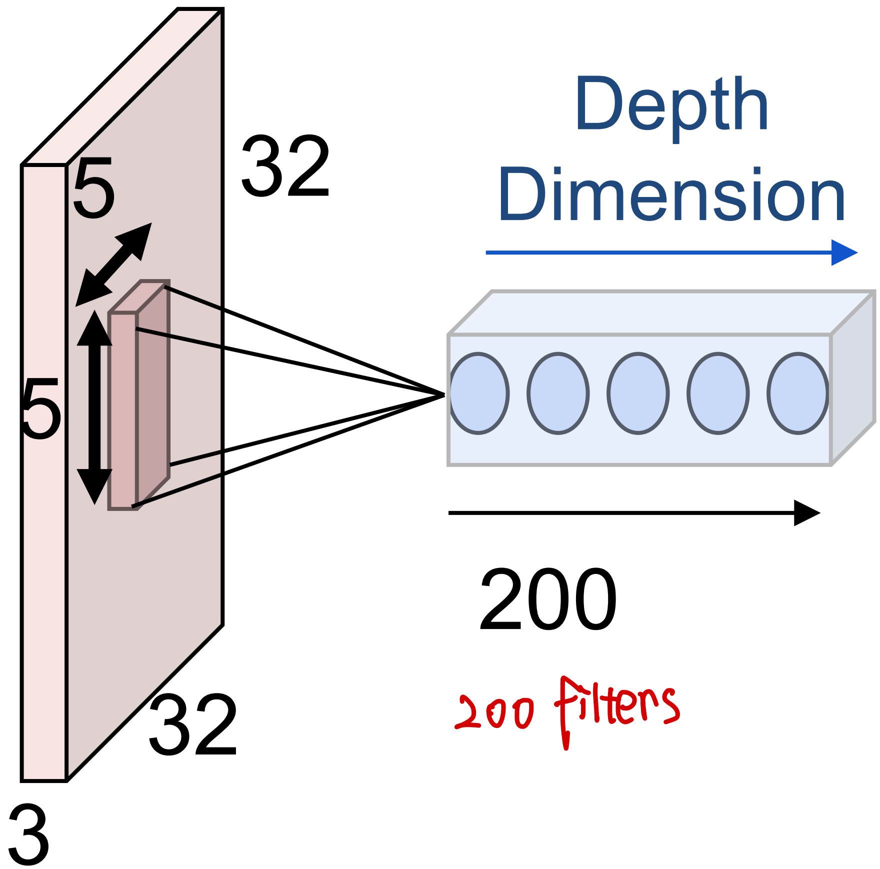
Output:
400x600x2



Convnet

Multiple out channels via multiple filters.

How big is the output?



Height? 32-5+1=28

Width? 32-5+1=28

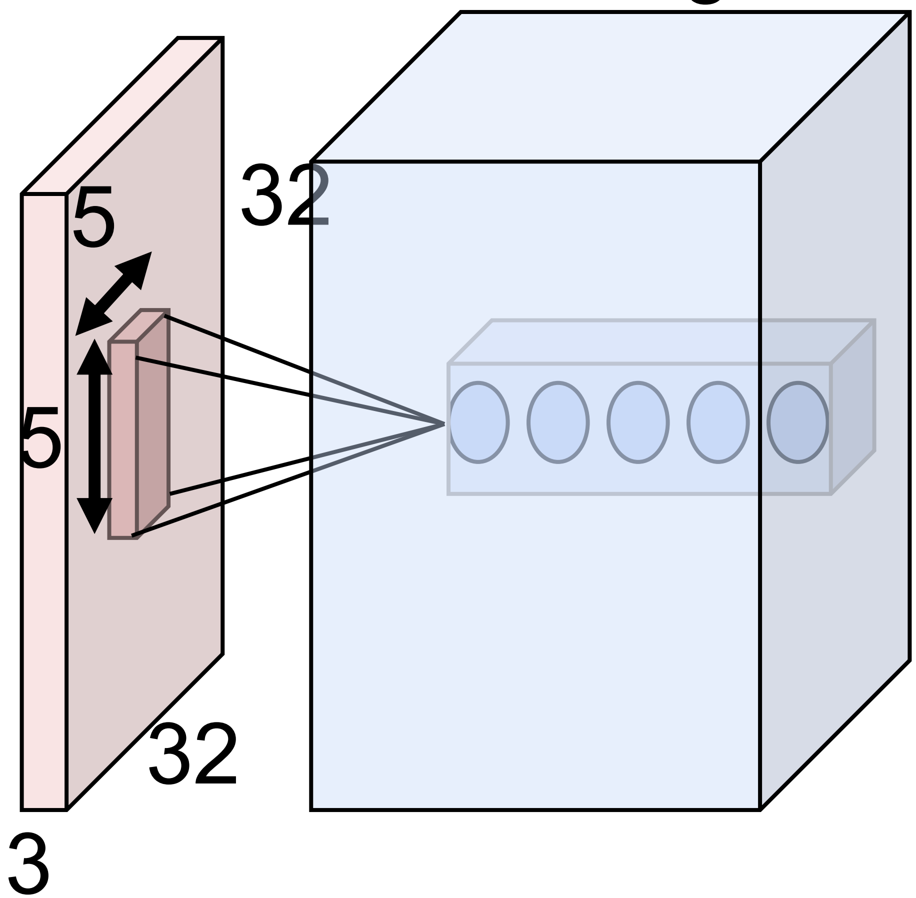
Channels? 200

Slide credit: Karpathy and Fei-Fei

Convnet

Multiple out channels via multiple filters.

How big is the output?



Height? 32-5+1=28

Width? 32-5+1=28

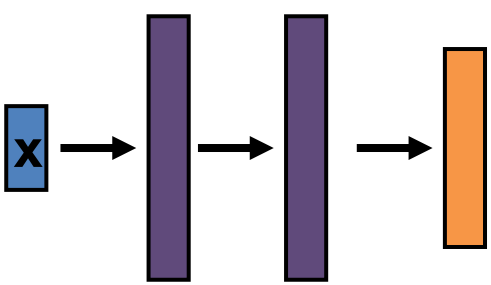
Channels? 200

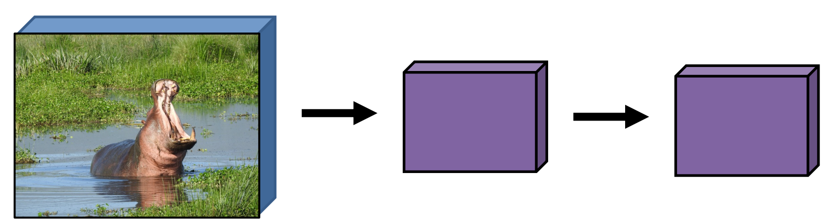
Slide credit: Karpathy and Fei-Fei

Convnet, Summarized

Neural net:
series of matrix-multiplies parameterized by **W**,**b** + nonlinearity/activation
Fit by gradient descent

Convnet:
series of convolutions parameterized by **F,b** + nonlinearity/activation
Fit by gradient descent





One Additional Subtlety – Stride

Warmup: how big is the output spatially?

F11	F12	F13	I14	I15	I16	I17
F21	F22	F23	I24	I25	I26	I27
F31	F32	F33	I34	I35	I36	I37
I41	I42	I43	I44	I45	I46	I47
I51	I52	I53	I54	I55	I56	I57
I61	I62	I63	I64	I65	I66	I67
I71	I72	I73	I74	I75	I76	I77

Normal (Stride 1):
5x5 output

Example credit: Karpathy and Fei-Fei

One Additional Subtlety – Stride

Stride: skip a few (here 2)

F11	F12	F13	I14	I15	I16	I17
F21	F22	F23	I24	I25	I26	I27
F31	F32	F33	I34	I35	I36	I37
I41	I42	I43	I44	I45	I46	I47
I51	I52	I53	I54	I55	I56	I57
I61	I62	I63	I64	I65	I66	I67
I71	I72	I73	I74	I75	I76	I77

Normal (Stride 1):
5x5 output

Example credit: Karpathy and Fei-Fei

One Additional Subtlety – Stride

Stride: skip a few (here 2)

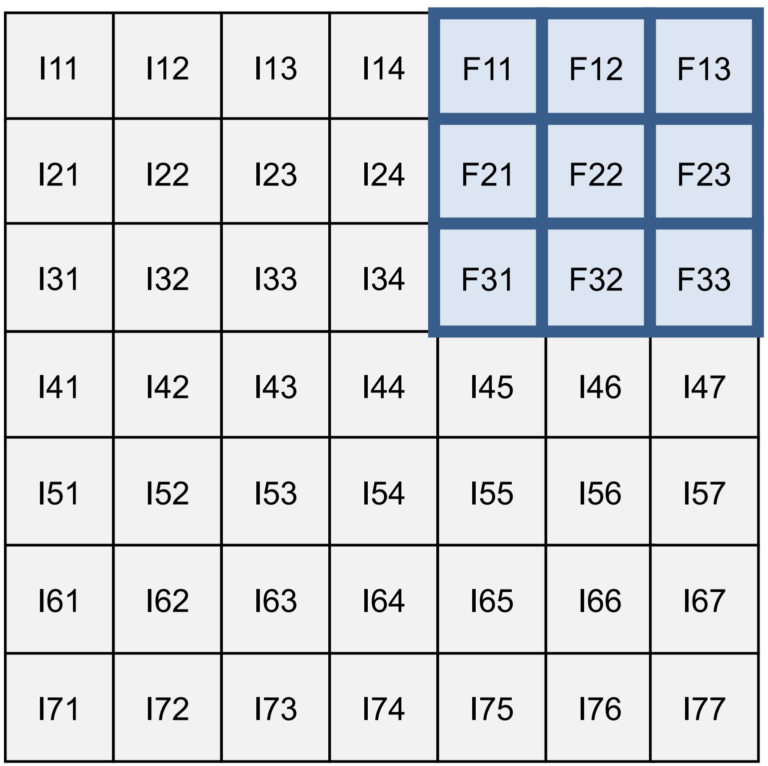
I11	I12	F11	F12	F13	I16	I17
I21	I22	F21	F22	F23	I26	I27
I31	I32	F31	F32	F33	I36	I37
I41	I42	I43	I44	I45	I46	I47
I51	I52	I53	I54	I55	I56	I57
I61	I62	I63	I64	I65	I66	I67
I71	I72	I73	I74	I75	I76	I77

Normal (Stride 1):
5x5 output

Example credit: Karpathy and Fei-Fei

One Additional Subtlety – Stride

Stride: skip a few (here 2)



Normal (Stride 1):
5x5 output

Stride 2 convolution:
3x3 output

Example credit: Karpathy and Fei-Fei

One Additional Subtlety – Stride

What about stride 3?

F11	F12	F13	I14	I15	I16	I17
F21	F22	F23	I24	I25	I26	I27
F31	F32	F33	I34	I35	I36	I37
I41	I42	I43	I44	I45	I46	I47
I51	I52	I53	I54	I55	I56	I57
I61	I62	I63	I64	I65	I66	I67
I71	I72	I73	I74	I75	I76	I77

Normal (Stride 1):
5x5 output

Stride 2 convolution:
3x3 output

Example credit: Karpathy and Fei-Fei

One Additional Subtlety – Stride

What about stride 3?

I11	I12	I13	F11	F12	F13	I17
I21	I22	I23	F21	F22	F23	I27
I31	I32	I33	F31	F32	F33	I37
I41	I42	I43	I44	I45	I46	I47
I51	I52	I53	I54	I55	I56	I57
I61	I62	I63	I64	I65	I66	I67
I71	I72	I73	I74	I75	I76	I77

Normal (Stride 1):
5x5 output

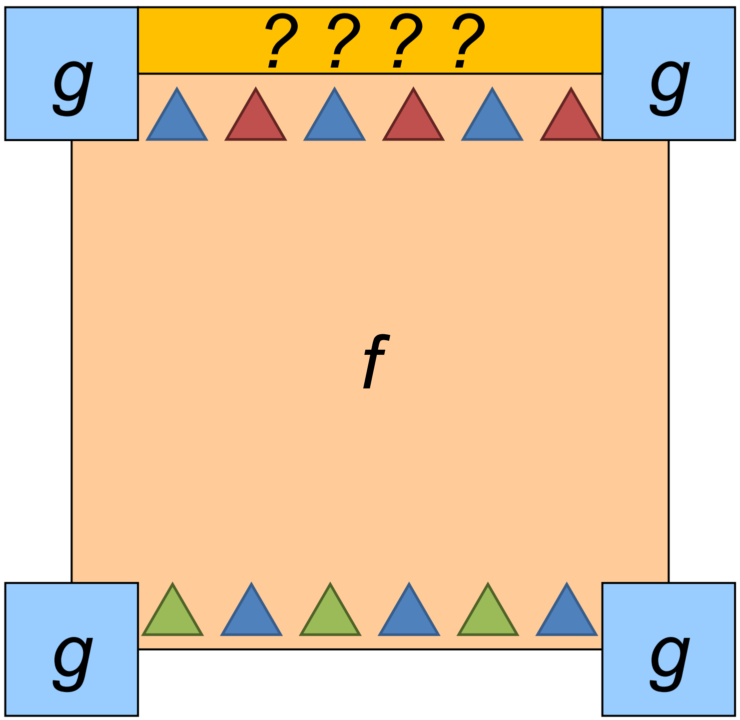
Stride 2 convolution:
3x3 output

Stride 3 convolution:
Not Common!

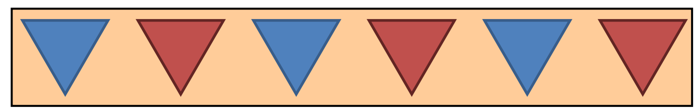
Example credit: Karpathy and Fei-Fei

One Additional Subtlety

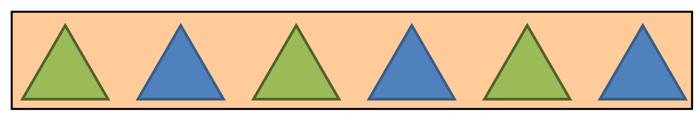
Zero padding is extremely common, although other forms of padding do happen



Symm: fold sides over

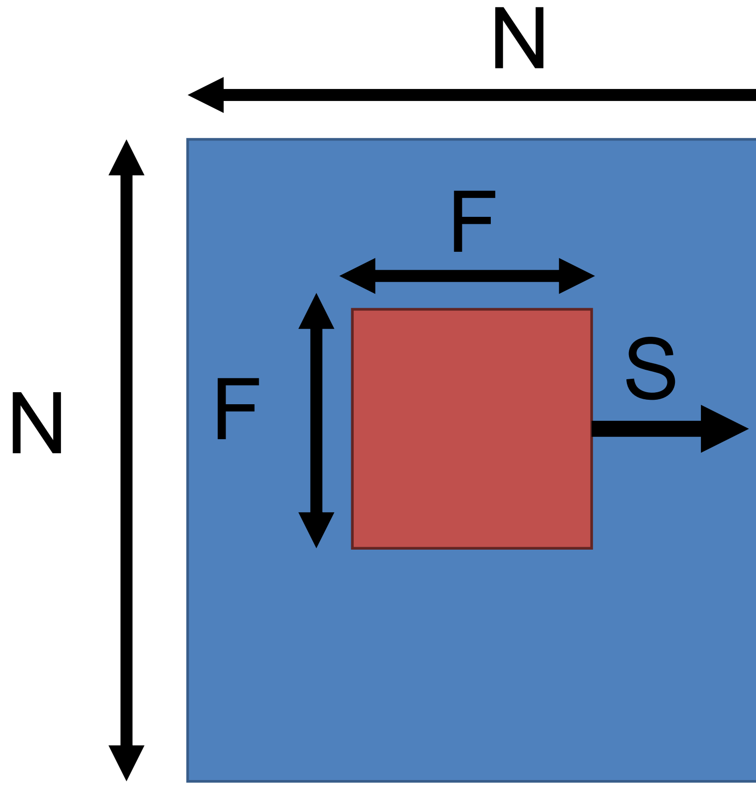


Circular/Wrap: wrap around



pad/fill: add value, often 0

In General

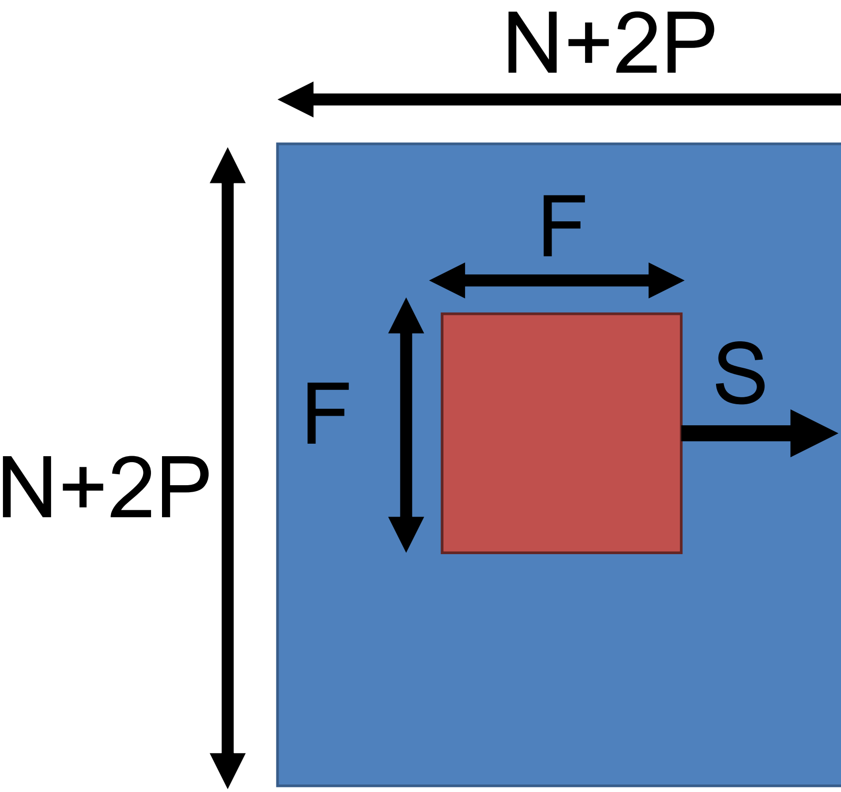


Output Size

$$\left\lfloor \frac{(N-F)}{S} \right\rfloor + 1$$

Slide credit: Karpathy and Fei-Fei

In General

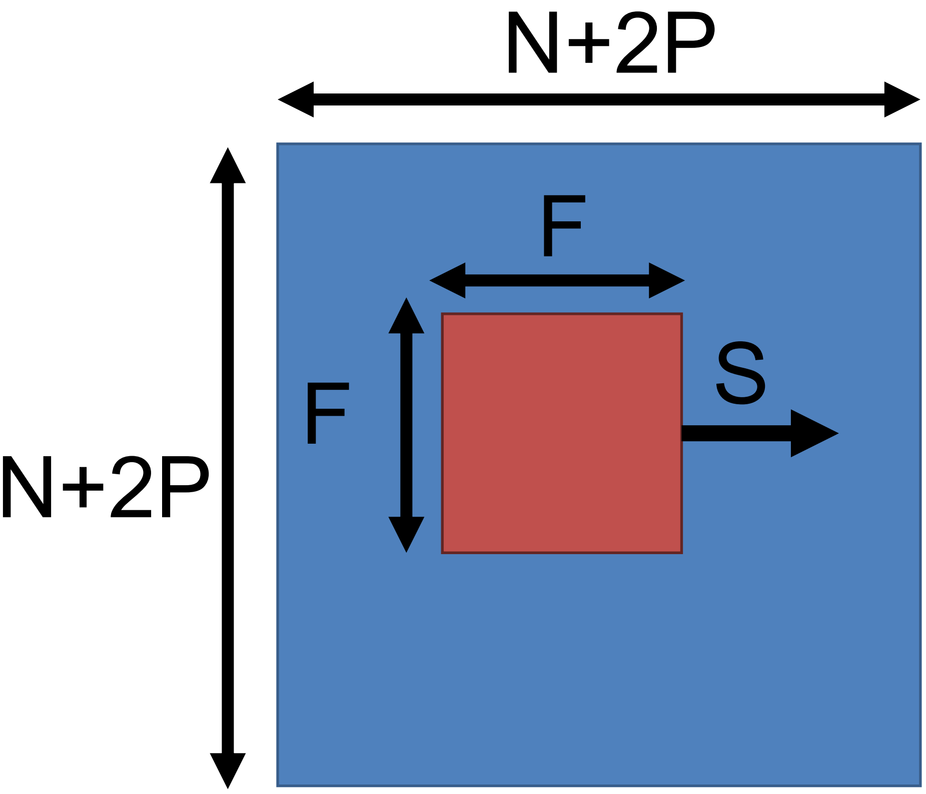


Output Size with Padding

$$\left\lfloor \frac{(N + 2P - F)}{S} \right\rfloor + 1$$

Slide credit: Karpathy and Fei-Fei

In General



With Proper Padding:
Output Size Preserved

$$\left\lfloor \frac{N}{S} \right\rfloor$$

Proper Padding
Size?

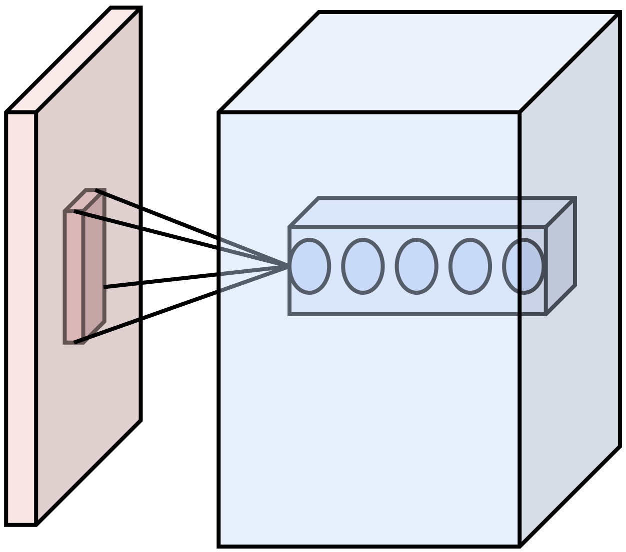
$$\left\lfloor \frac{F}{2} \right\rfloor$$

Slide credit: Karpathy and Fei-Fei

More Examples

Input volume: **32x32x3**
Receptive fields: **5x5**, **stride 1**
Number of neurons: **5**

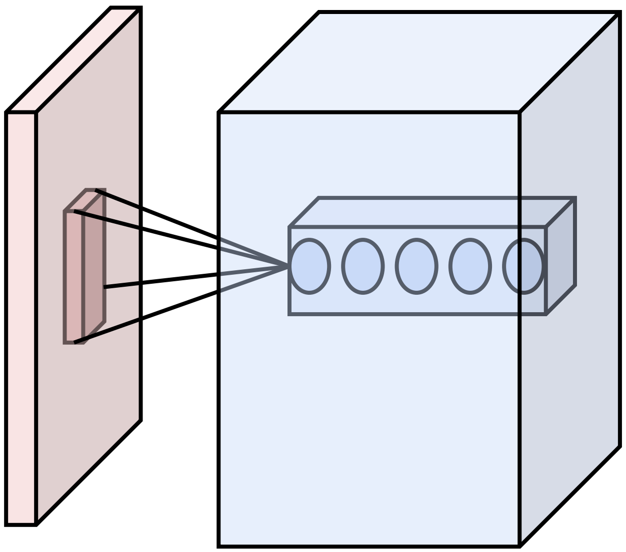
$$\frac{(N - F)}{s} + 1$$



Output volume size?

Slide credit: Karpathy and Fei-Fei

More Examples



Input volume: **32x32x3**

Receptive fields: **5x5**, **stride 1**

Number of neurons: **5**

$$\frac{(N-F)}{s}+1$$

Output volume: (32 - 5) / 1 + 1 = 28, so: **28x28x5**

Number of Parameters?

bias

$\underbrace{5\times5\times3}_{\text{each filter}} \times \underbrace{5}_{\text{we have five filters}} + 5$

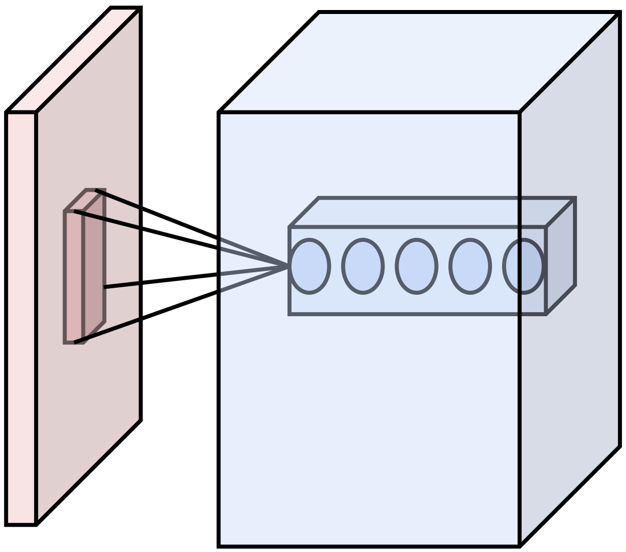
Slide credit: Karpathy and Fei-Fei

More Examples

Input volume: **32x32x3**

Receptive fields: **5x5**, **stride 1**

Number of neurons: **5**



$$\frac{(N-F)}{s}+1$$

Output volume: (32 - 5) / 1 + 1 = 28, so: **28x28x5**

How many parameters? **5x5x3x5 + 5 = 380**

Smaller than FC (32x32x3x5+5)=15365

Slide credit: Karpathy and Fei-Fei

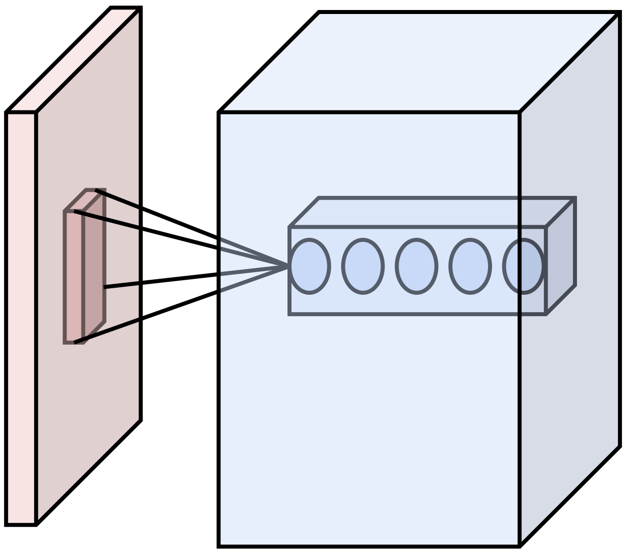
More Examples

Input volume: **32x32x3**

Receptive fields: **5x5**, **stride 3**

Number of neurons: **5**

$$\frac{(N - F)}{S} + 1$$

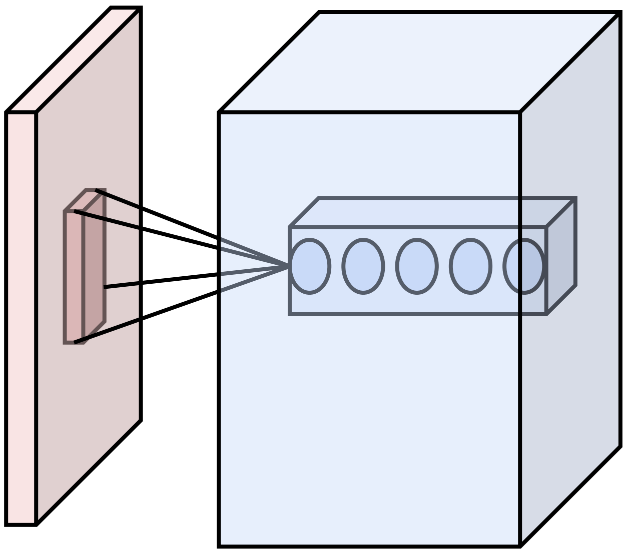


Output volume size?

Slide credit: Karpathy and Fei-Fei

More Examples

Input volume: **32x32x3**
Receptive fields: **5x5**, **stride 3**
Number of neurons: **5**



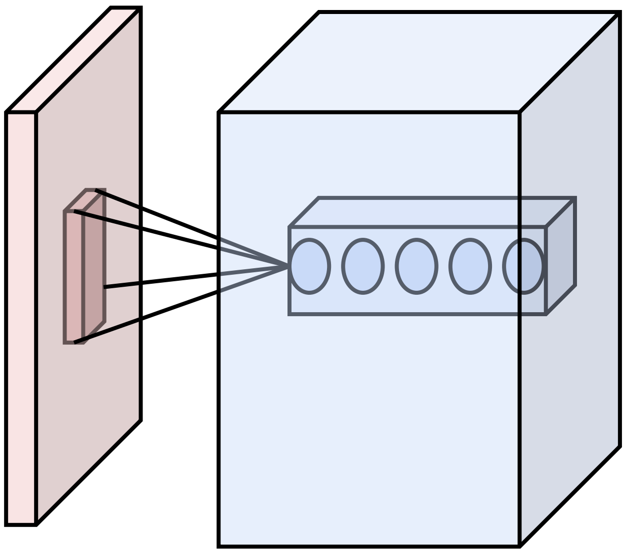
Output volume: (32 - 5) / 3 + 1 = 10, so: **10x10x5**

Number of Parameters?

Slide credit: Karpathy and Fei-Fei

More Examples

Input volume: **32x32x3**
Receptive fields: **5x5**, **stride 3**
Number of neurons: **5**

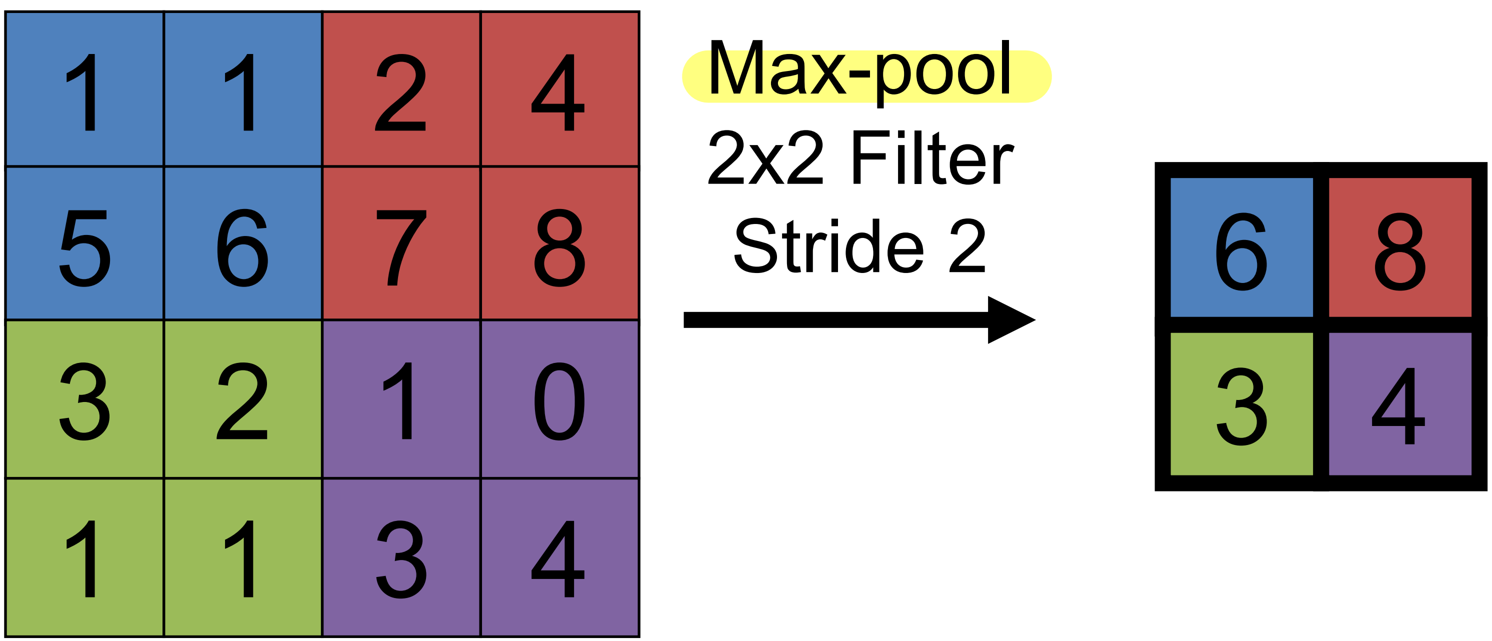


Output volume: (32 - 5) / 3 + 1 = 10, so: **10x10x5**
How many parameters? **5x5x3x5 + 5 = 380. Same!**

Slide credit: Karpathy and Fei-Fei

Other Layers – Pooling

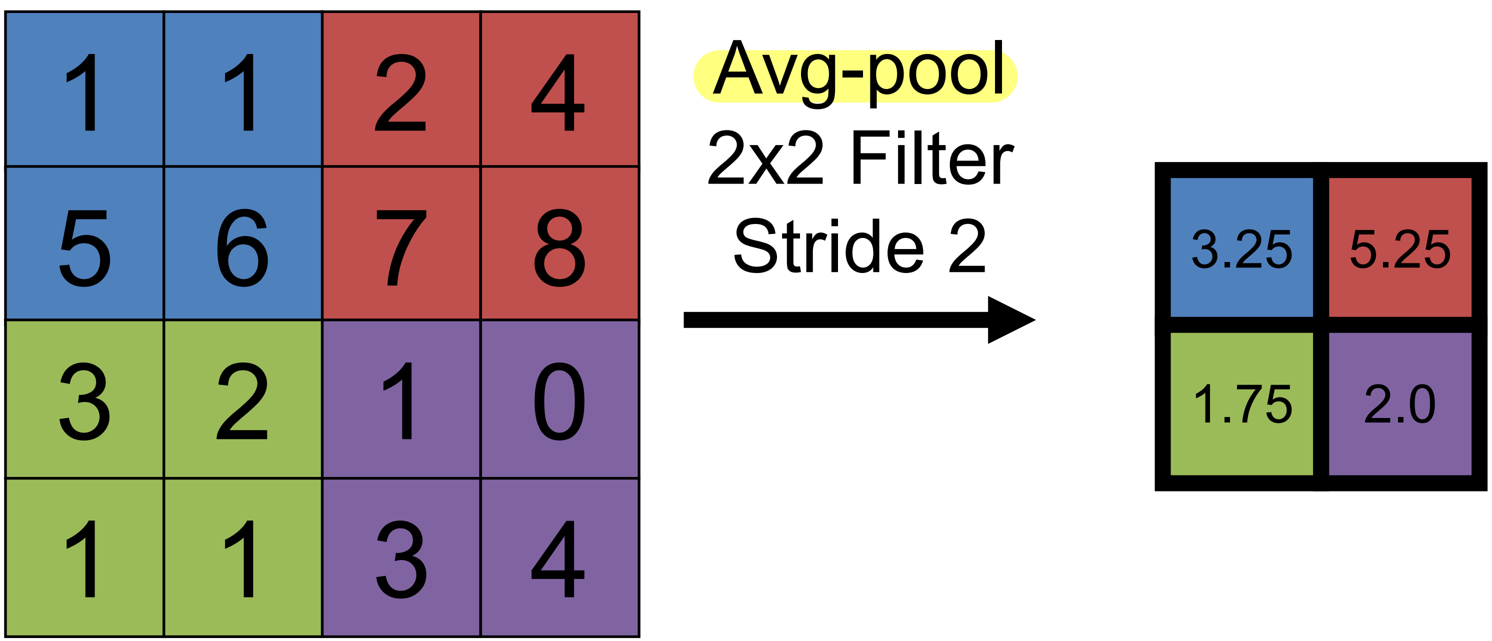
Idea: want spatial resolution of activations / images smaller; applied per-channel



Slide credit: Karpathy and Fei-Fei

Other Layers – Pooling

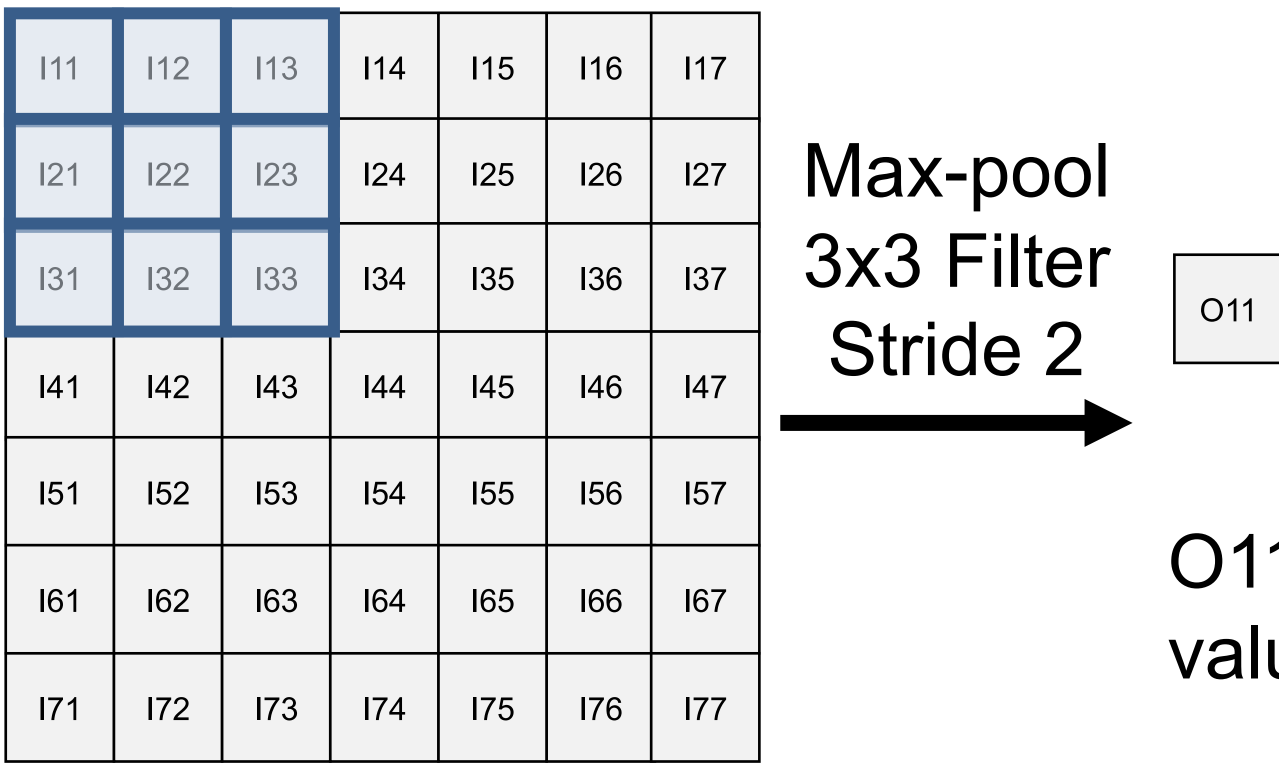
Idea: want spatial resolution of activations / images smaller; applied per-channel



Slide credit: Karpathy and Fei-Fei

Other Layers – Pooling

Idea: want spatial resolution of activations / images smaller; applied per-channel



O11 = maximum value in blue box

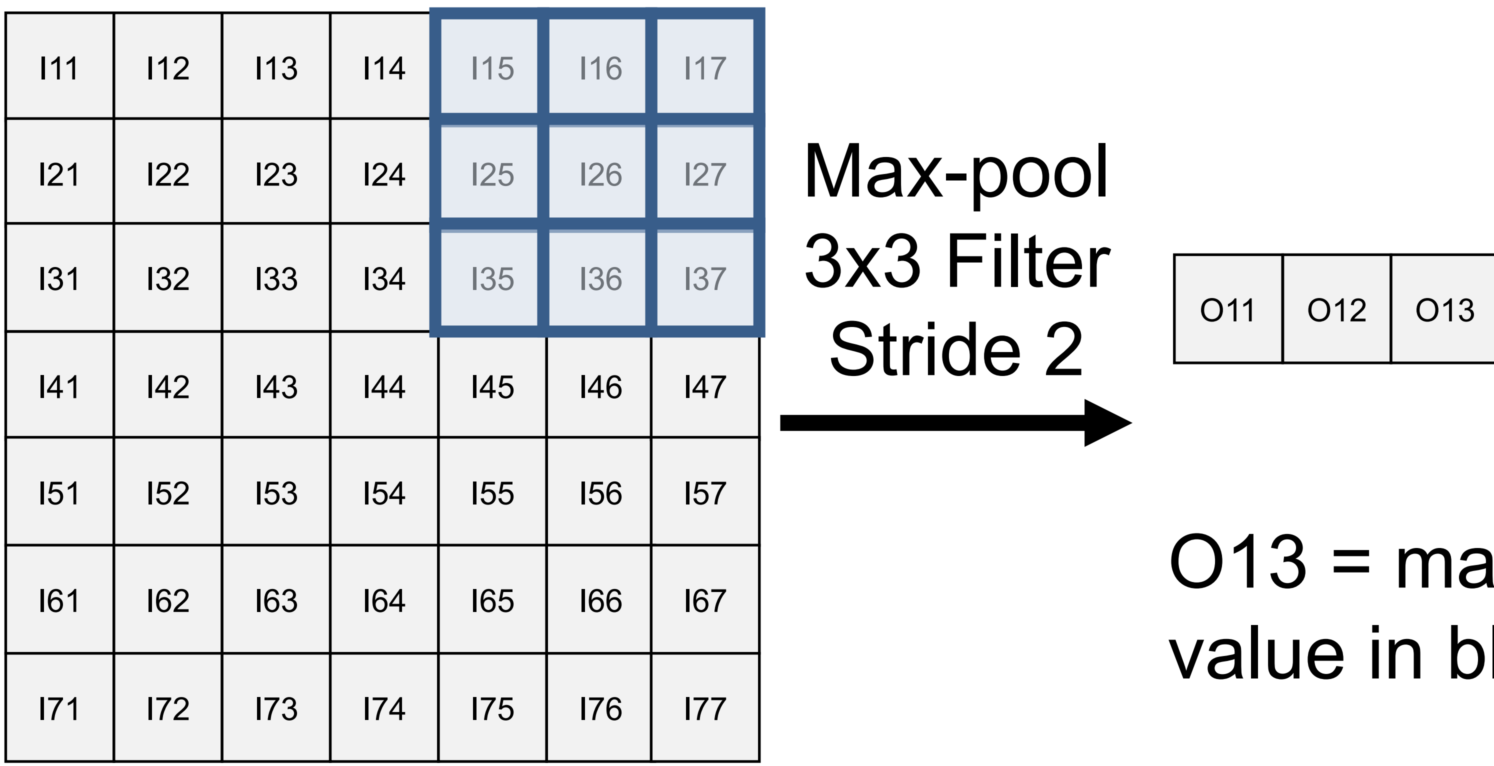
Other Layers – Pooling

Idea: want spatial resolution of activations / images smaller; applied per-channel

O12 = maximum value in blue box

Other Layers – Pooling

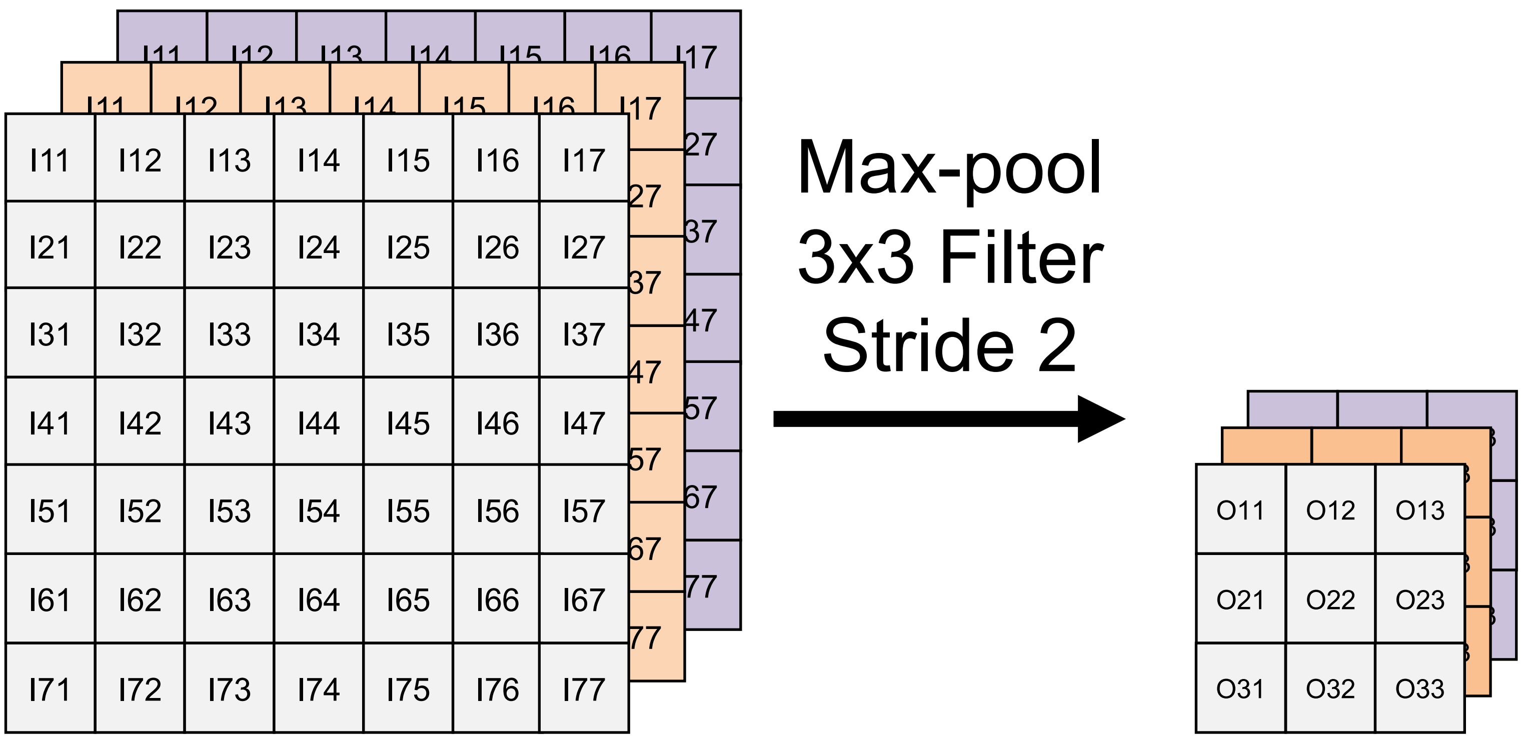
Idea: want spatial resolution of activations / images smaller; applied per-channel



O13 = maximum value in blue box

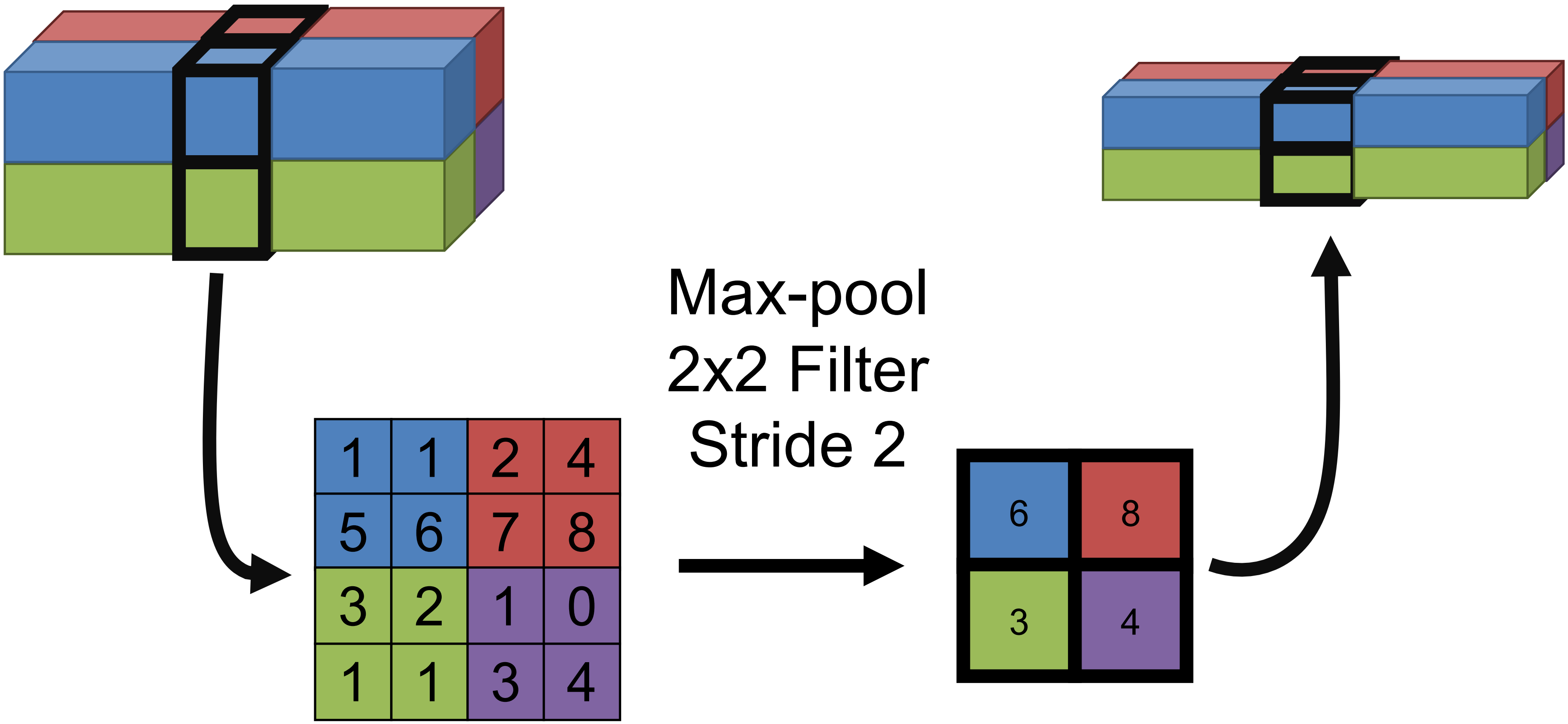
Other Layers – Pooling

Idea: want spatial resolution of activations / images smaller; applied per-channel



Smaller Resolution→Smaller Computation

Squeezing a Loaf of Bread



Example Network

Suppose we want to convert a 32x32x3 image into a 10x1 vector of classification results

Figure Credit: Karpathy and Fei-Fei; see http://cs231n.stanford.edu/

Example Network

input: [32x32x3]
CONV with 10 3x3 filters, stride 1, pad 1:
gives: [32x32x10]
new parameters: (3*3*3)*10 + 10 = 280
RELU
CONV with 10 3x3 filters, stride 1, pad 1:
gives: [32x32x10]
new parameters: (3*3*10)*10 + 10 = 910
RELU
POOL with 2x2 filters, stride 2:
gives: [16x16x10]
parameters: 0

Slide credit: Karpathy and Fei-Fei

Example Network

Previous output: [16x16x10]
CONV with 10 3x3 filters, stride 1:
gives: [16x16x10]
new parameters: (3*3*10)*10 + 10 = 910
RELU
CONV with 10 3x3 filters, stride 1:
gives: [16x16x10]
new parameters: (3*3*10)*10 + 10 = 910
RELU
POOL with 2x2 filters, stride 2:
gives: [8x8x10]
parameters: 0

Slide credit: Karpathy and Fei-Fei

Example Network

Conv, Relu, Conv, Relu, Pool continues until it’s [4x4x10]

Fully-Connected FC layer to 10 neurons
(which are our class scores)
Number of parameters:
$10 * 4 * 4 * 10 + 10 = 1610$

done!

Slide credit: Karpathy and Fei-Fei

An Alternate Conclusion

Conv, Relu, Conv, Relu, Pool continues until it's [4x4x10]

Average POOL 4x4x10 to 10 neurons
Fully-Connected FC layer to 10 neurons
(which are our class scores)
Number of parameters:
10 * 10 + 10 = 110

done!

Slide credit: Karpathy and Fei-Fei

Example Network

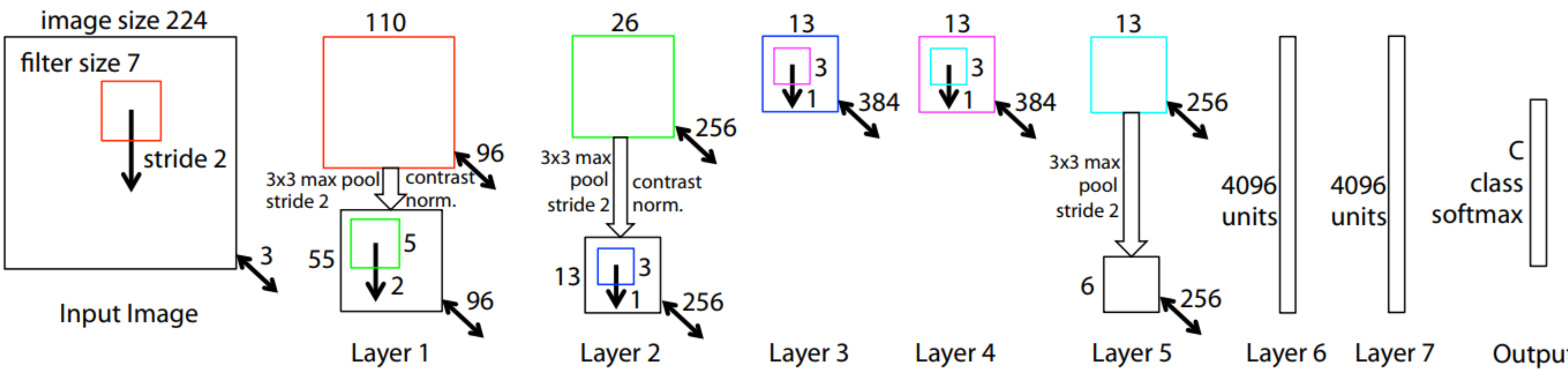
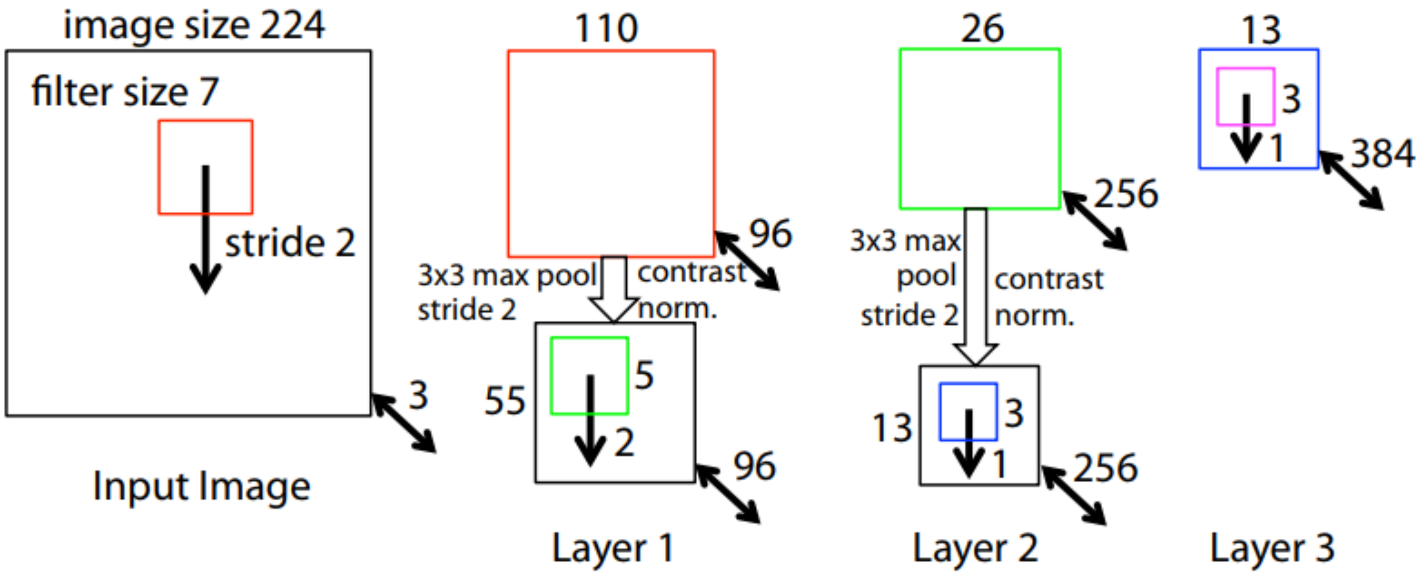


Figure Credit: Zeiler and Fergus, Visualizing and Understanding Convolutional Networks. ECCV 2014

Example Network

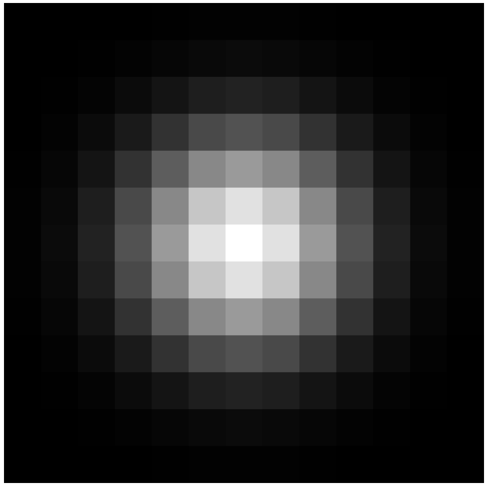


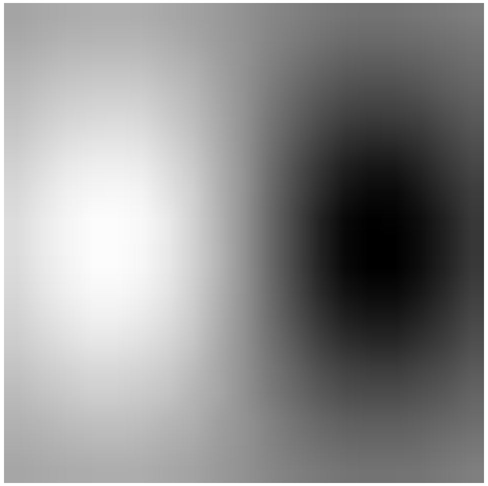
(1) filter image with 96 7x7 filters with stride 2
(2) ReLU
(3) 3x3 max pool with stride 2 (and *contrast normalization – now ignored)*

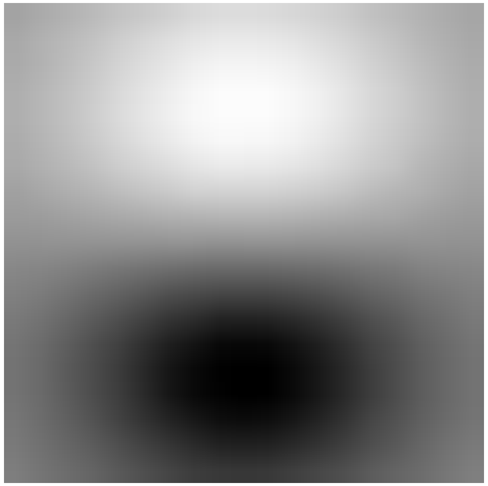
Figure Credit: Zeiler and Fergus, Visualizing and Understanding Convolutional Networks. ECCV 2014

What Do The Filters Represent?

Recall: filters are images and we can look at them







What Do The Filters Represent?

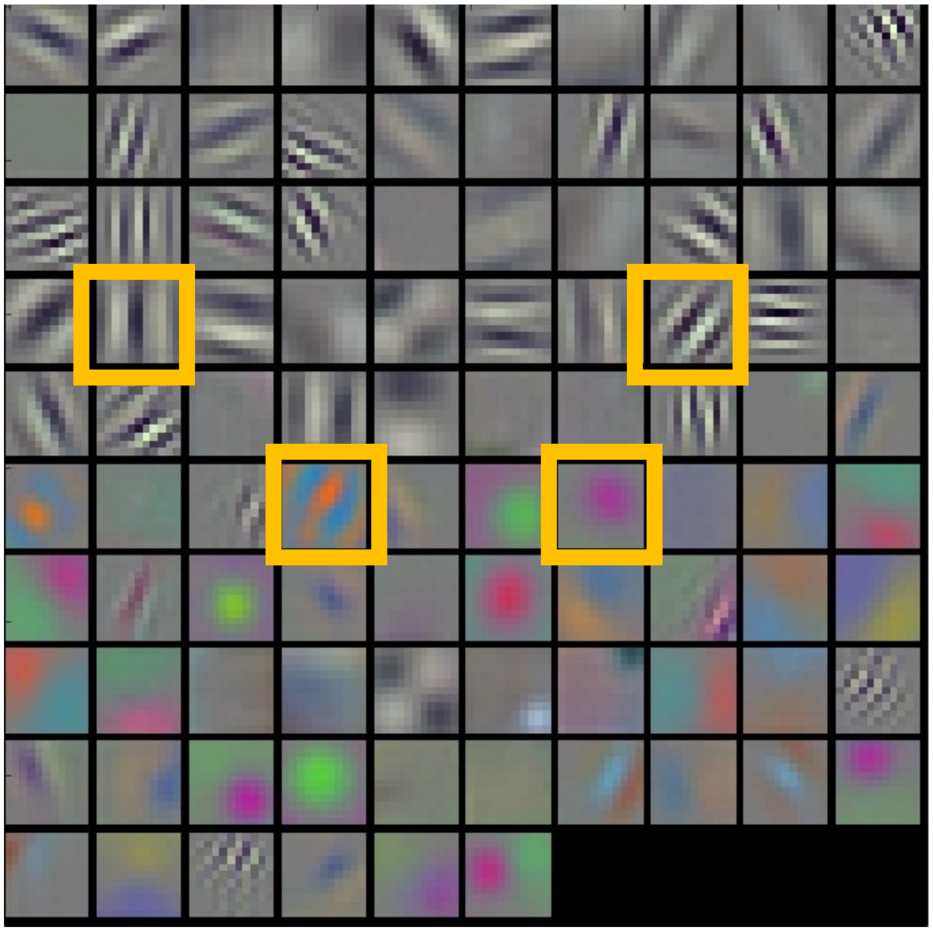


Figure Credit: Karpathy and Fei-Fei

First layer filters (11x11) of a network (AlexNet) trained to distinguish 1000 categories of objects (ImageNet)

Remember these filters go over color.

For the interested: [Gabor filter]

What Do The Filters Do?

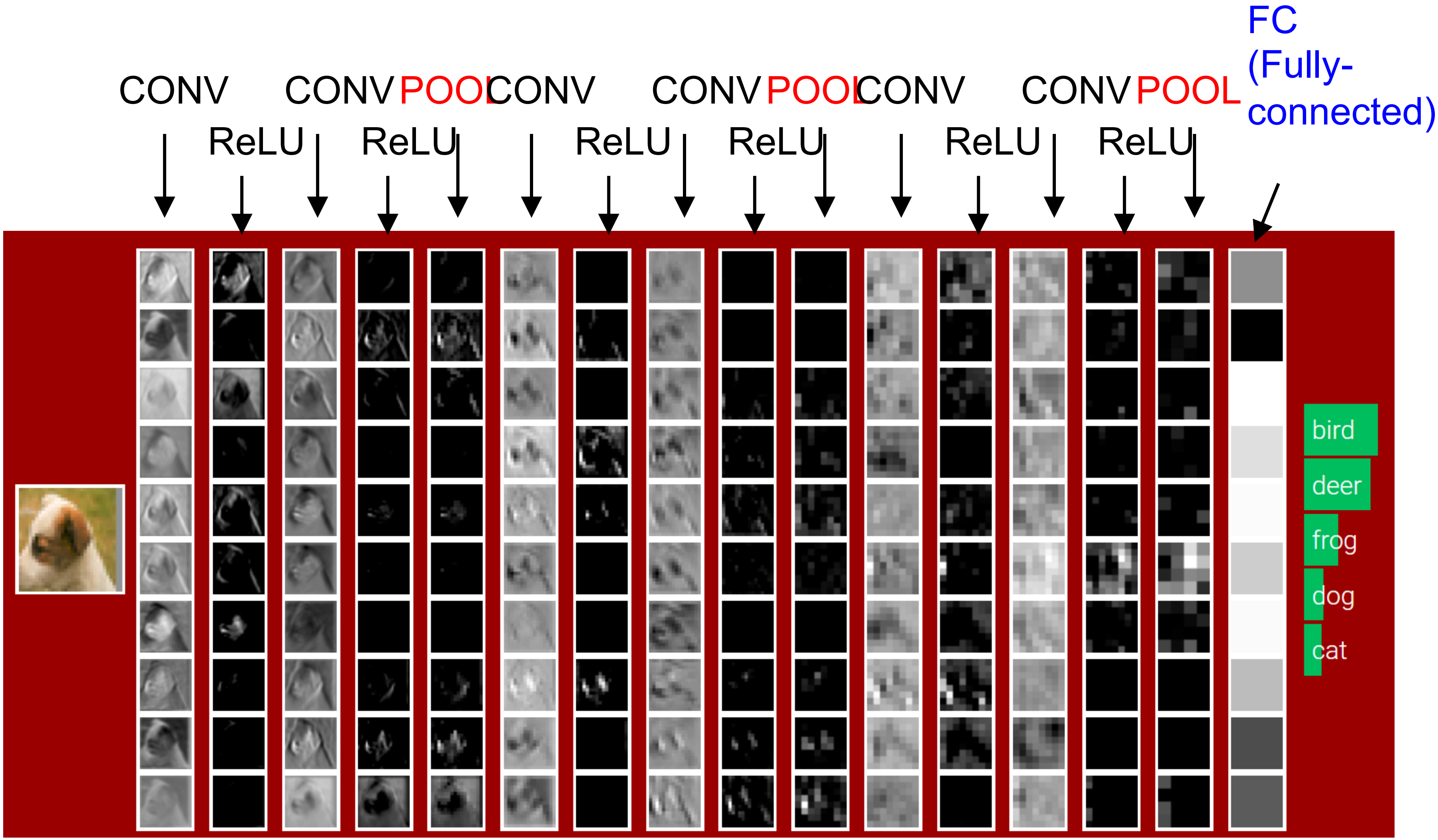


Figure Credit: Karpathy and Fei-Fei; see http://cs231n.stanford.edu/

Next Class: More CNNs