Detectors and Descriptors

EECS 442 – Jeong Joon Park
Winter 2024, University of Michigan
https://eecs442.github.io/

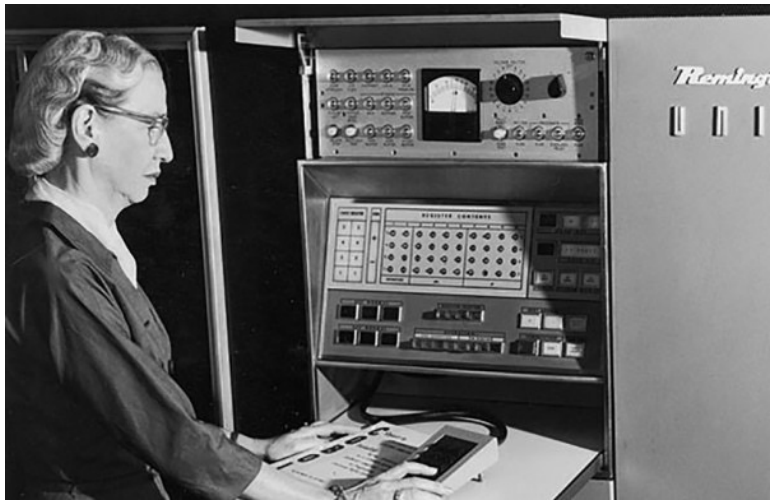
Goal

Extract Characteristics from this image

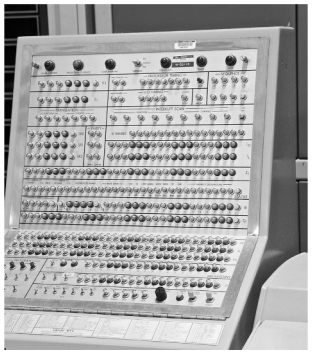
How big is this image as a vector?

389x600 = 233,400 dimensions **(big)**

Applications To Have In Mind



Part of the same photo?



Same computer from another angle?

Applications To Have In Mind

Building a 3D Reconstruction Out Of Images

Slide Credit: N. Seitz

Applications To Have In Mind

(panorama)

Stitching photos taken at different angles

One Example

Given two images: how do you align them?

One Solution

```
for y in range(-ySearch,ySearch+1):
    for x in range(-xSearch,xSearch+1):
        #Touches all HxW pixels!
        check_alignment_with_images()
```

One Motivating Example

Given these images: how do you align them?

These aren't off by a small 2D translation but instead by a 3D rotation + translation of the camera.

Photo credit: M. Brown, D. Lowe

One Solution

```
for y in yRange:
    for x in xRange:
        for z in zRange:
            for xRot in xRotVals:
                for yRot in yRotVals:
                    for zRot in zRotVals:
                        #touches all HxW pixels!
                        check_alignment_with_images()
```

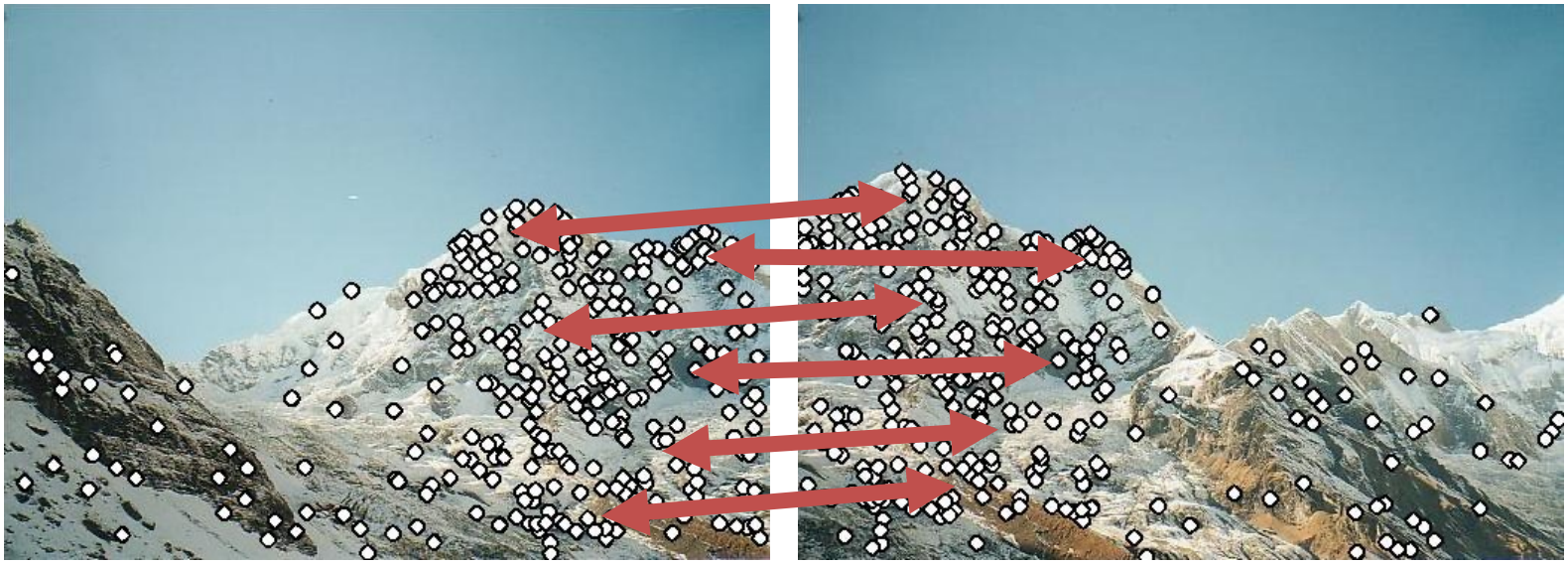
This code should make you really unhappy

An Alternate Approach

Given these images: how would **you** align them?

An Alternate Approach

Finding and Matching



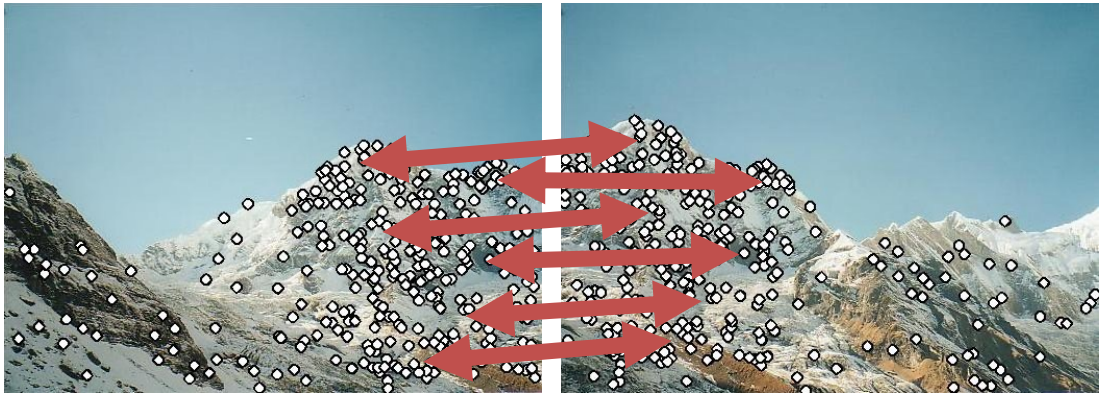
1: find corners+features

2: match based on local image data

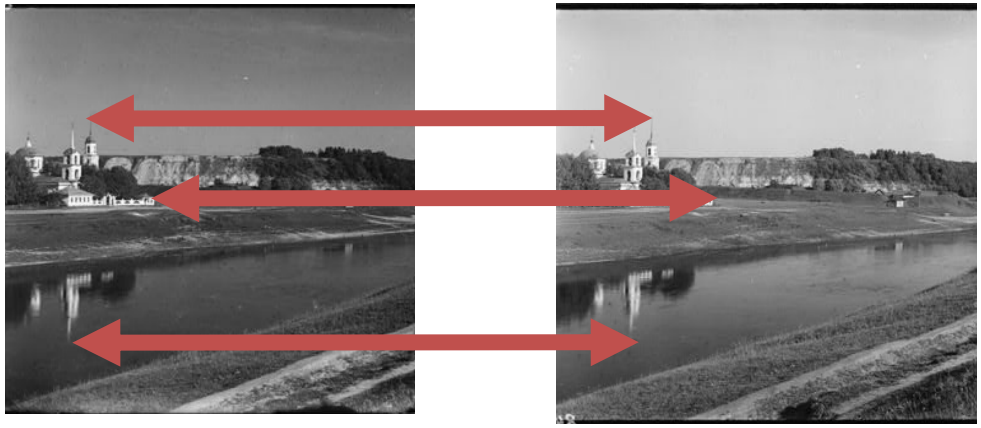
Slide Credit: S. Lazebnik, original figure: M. Brown, D. Lowe

What Now?

Given pairs **p1**,**p2** of correspondence, **how do I align?**

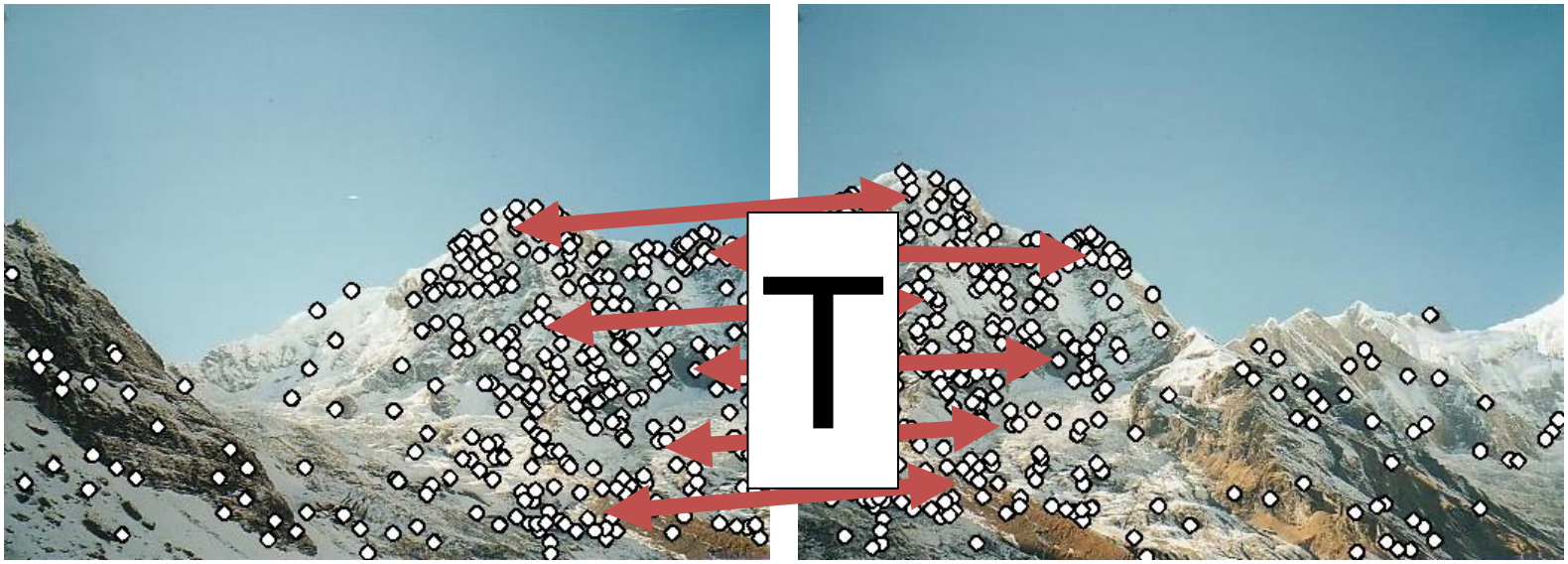


Consider translation-only case.



An Alternate Approach

Solving for a Transformation



3: Solve for transformation T (e.g. such that **p1** ≡ **T p2**) that fits the matches well

Note the homogeneous coordinates, you'll see them again.

Slide Credit: S. Lazebnik, original figure: M. Brown, D. Lowe

An Alternate Approach

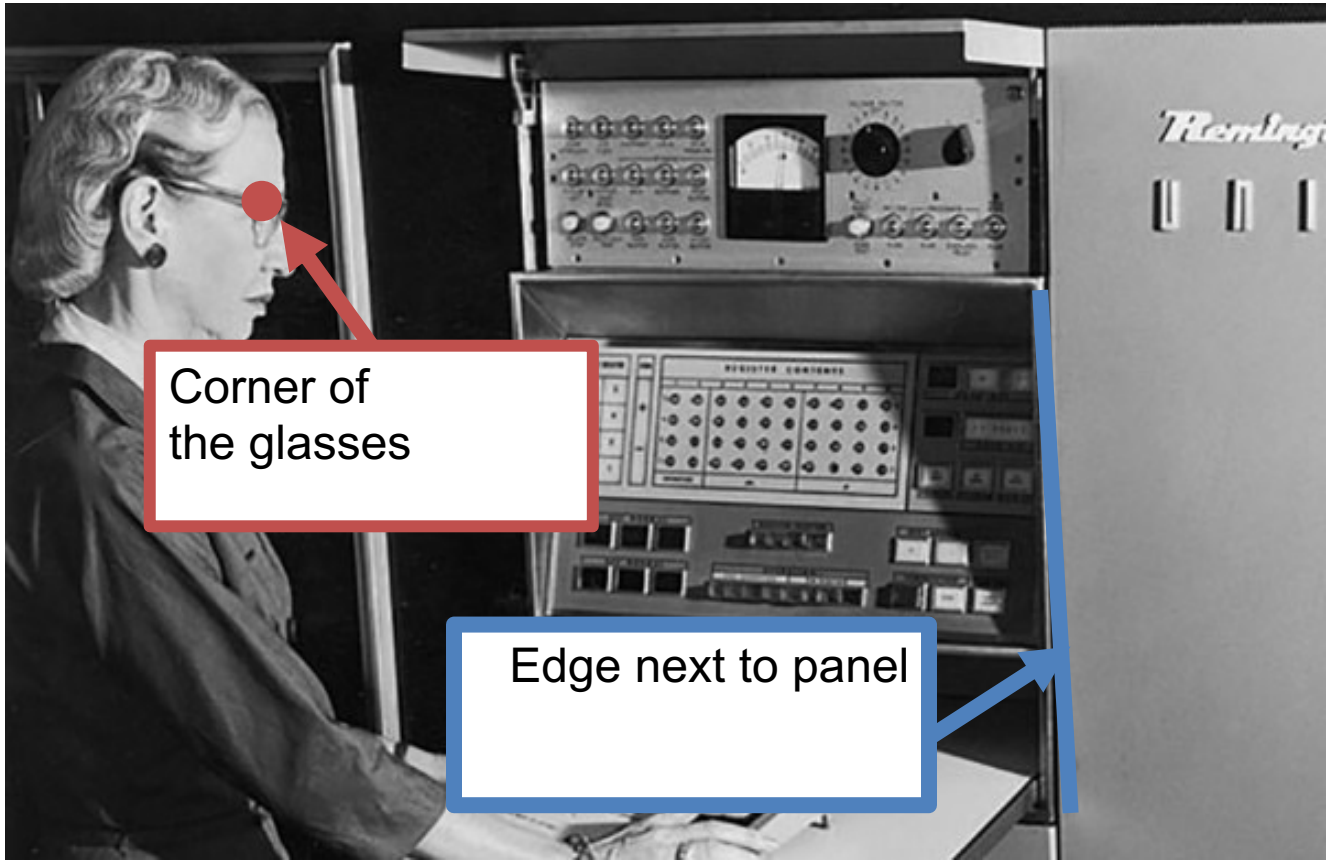
Blend Them Together

Key insight: we don’t work with full image. We work with only parts of the image (features).

Photo Credit: M. Brown, D. Lowe

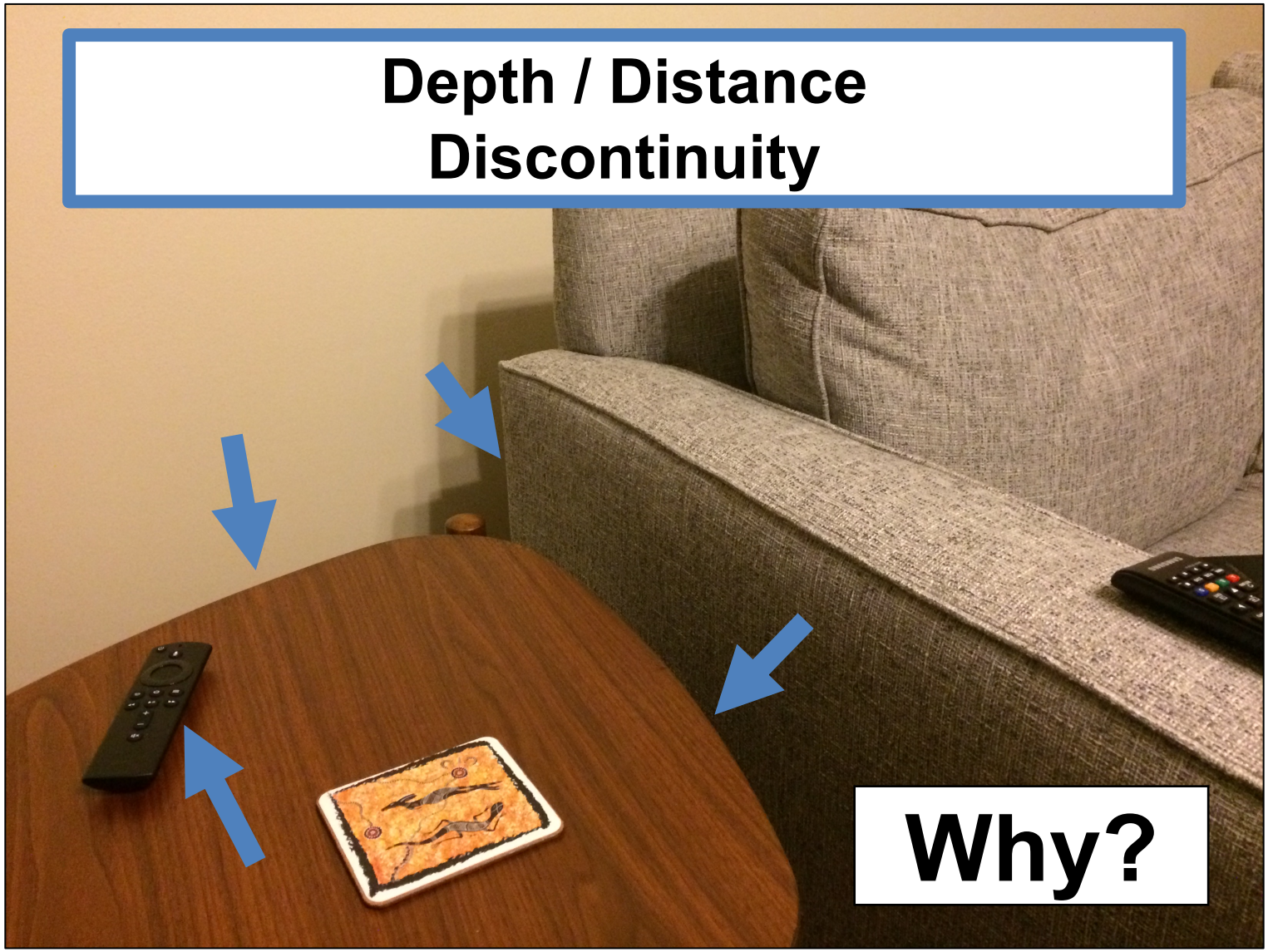
Today

Finding edges (part 1) and corners (part 2) in images.

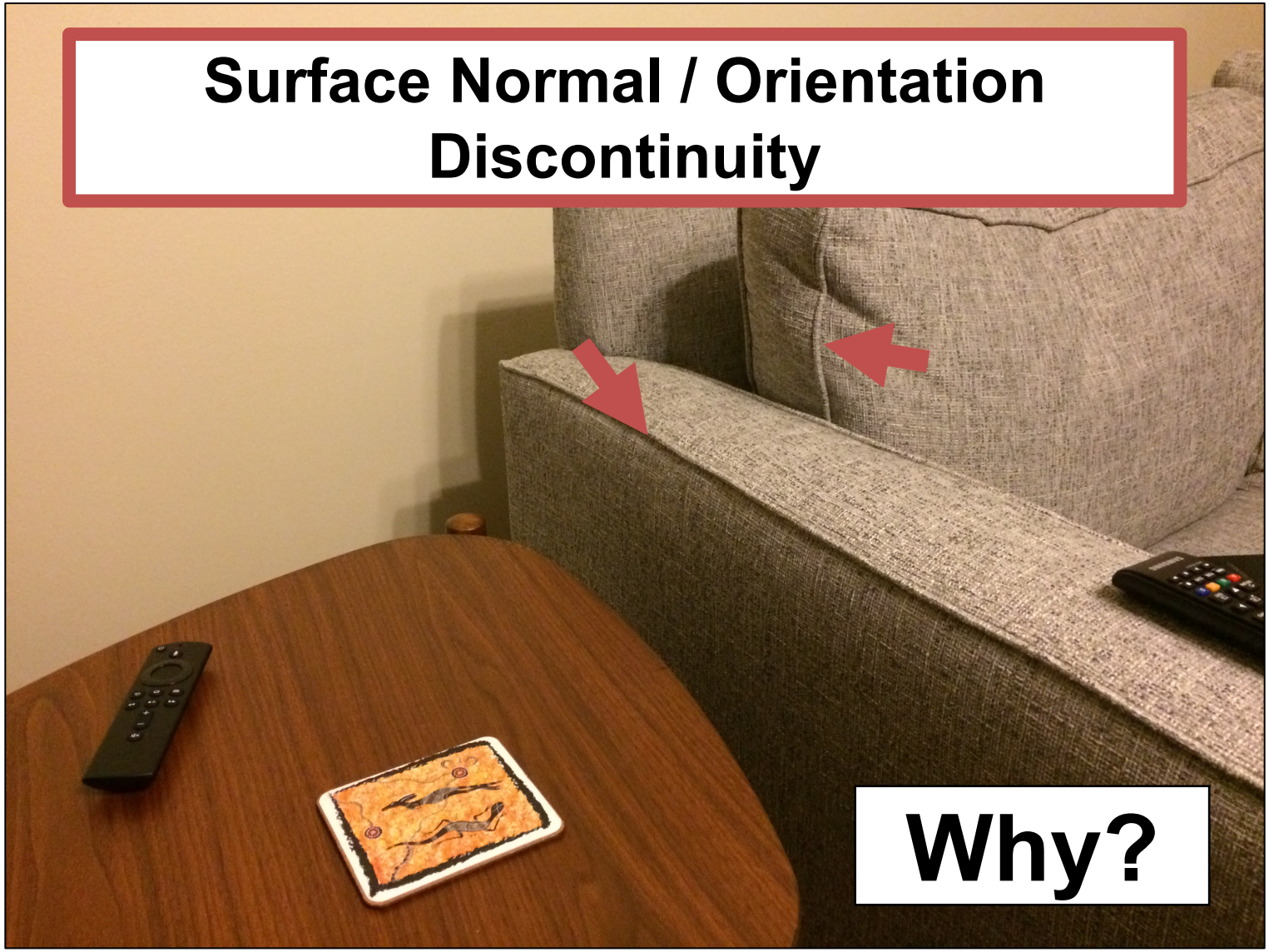


Where do Edges Come From?

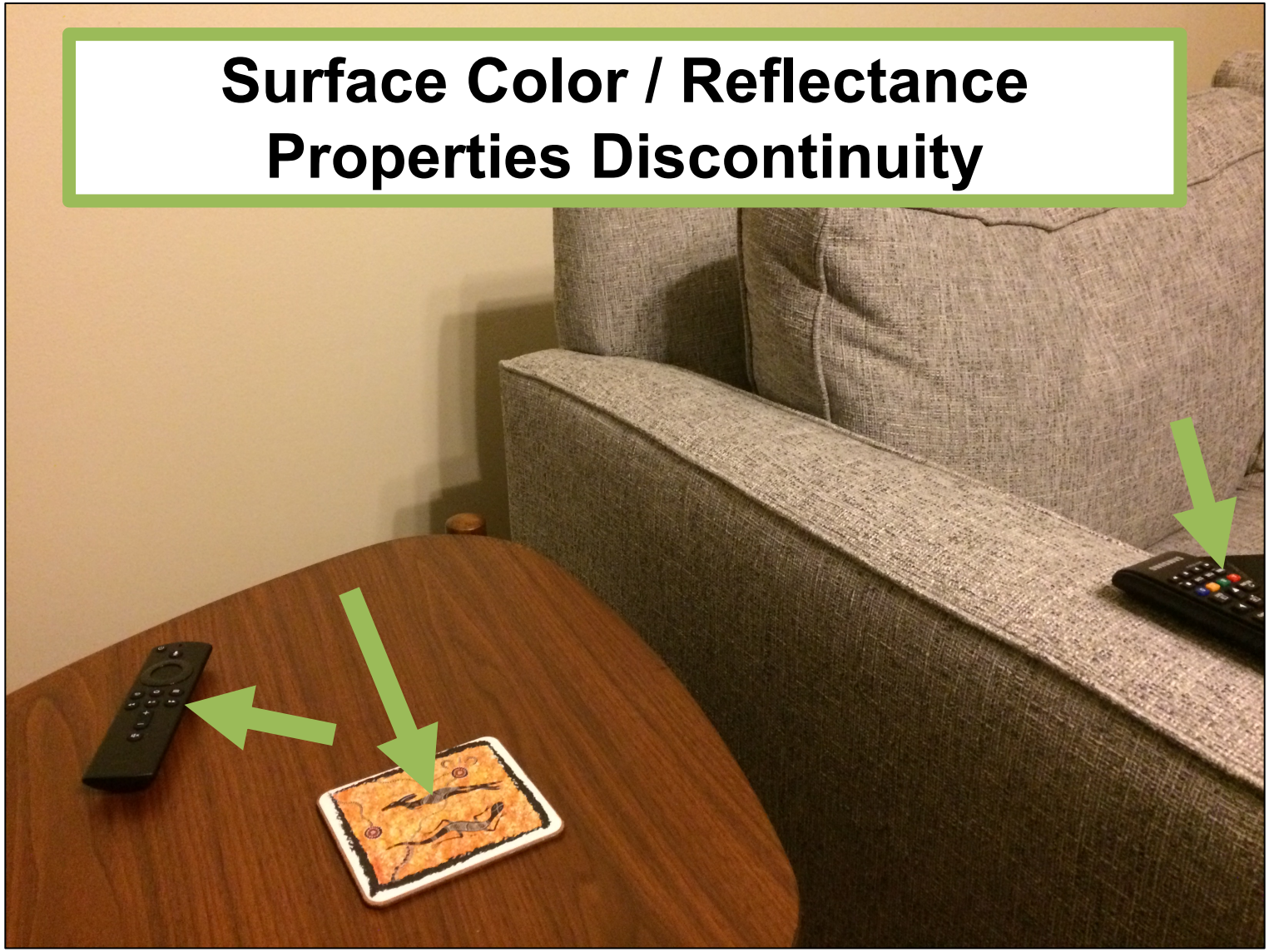
Where do Edges Come From?



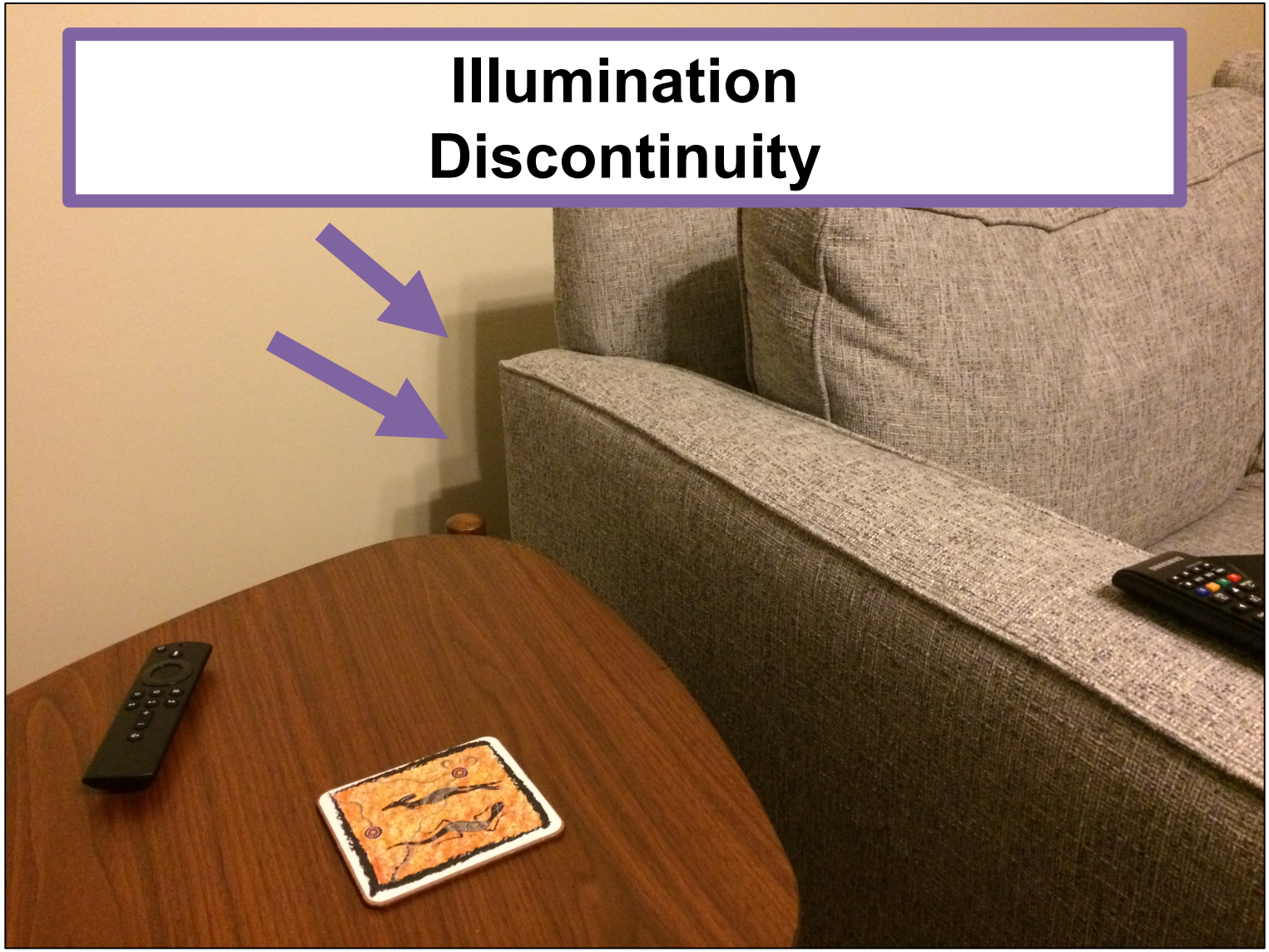
Where do Edges Come From?



Where do Edges Come From?



Where do Edges Come From?



Last Time

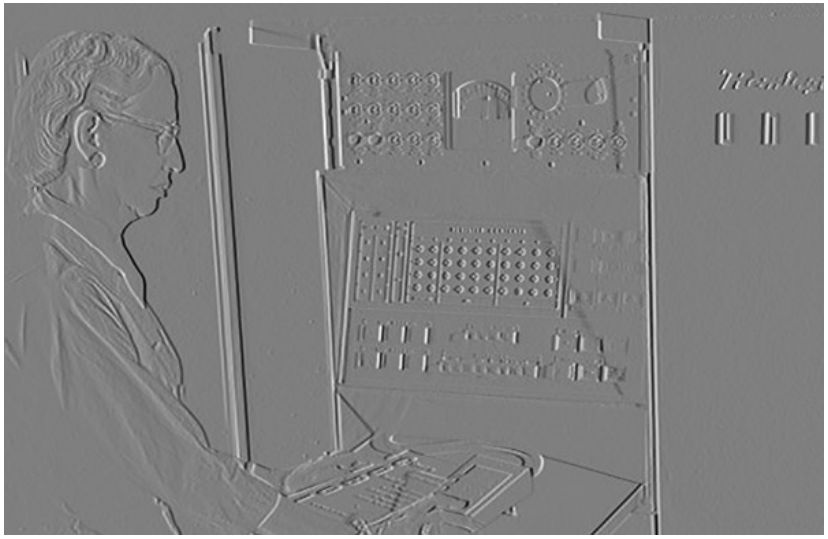
-1	0	1

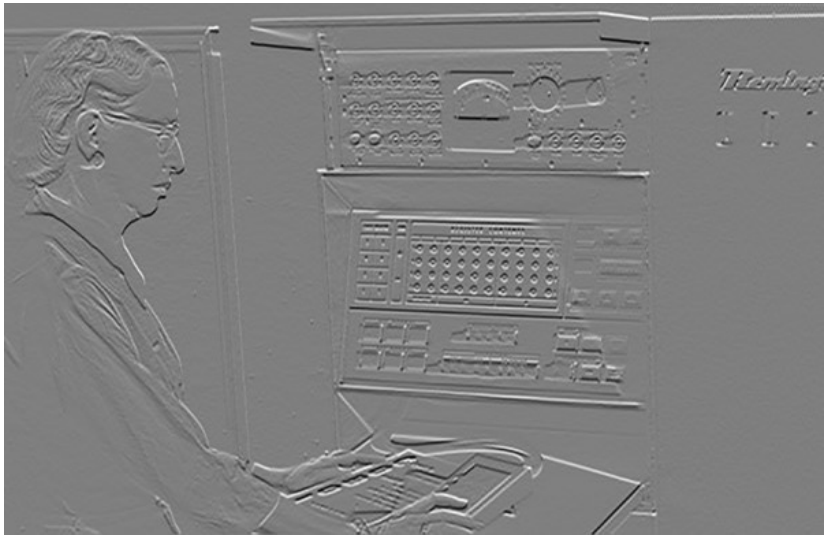
I_x

-1	0	1

^{T}

I_y





Derivatives

Remember derivatives?

Derivative: rate at which a function f(x) changes at a point as well as the direction that increases the function

Gradient: all of the partial derivatives (derivatives in only one direction) stacked together.

What Should I Know?

- Gradients are simply partial derivatives per-dimension: if $\boldsymbol{x}$ in $f(\boldsymbol{x})$ has n dimensions, $\nabla_f(x)$ has n dimensions
- Gradients point in direction of ascent and tell the rate of ascent
- If a is minimum of $f(\boldsymbol{x}) \rightarrow \nabla_{\mathrm{f}}(\mathrm{a}) = \mathbf{0}$
- Reverse is not true, especially in high-dimensional spaces

Last Time

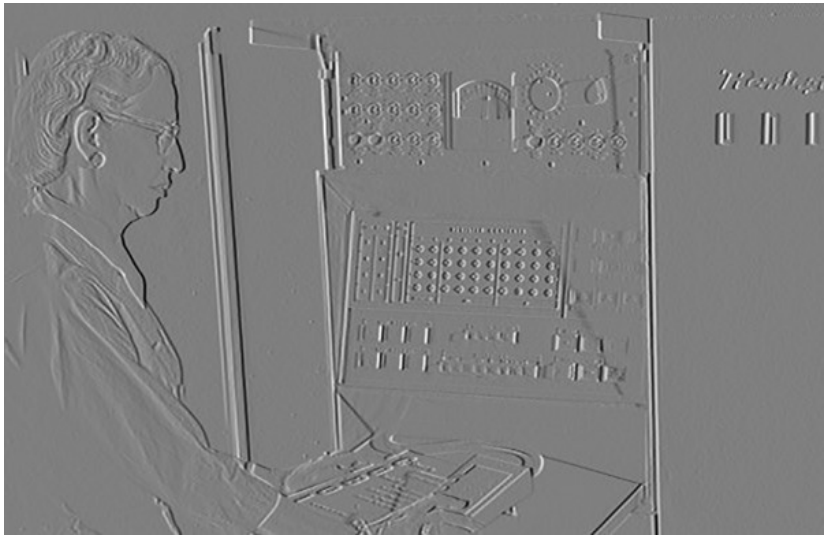
-1	0	1

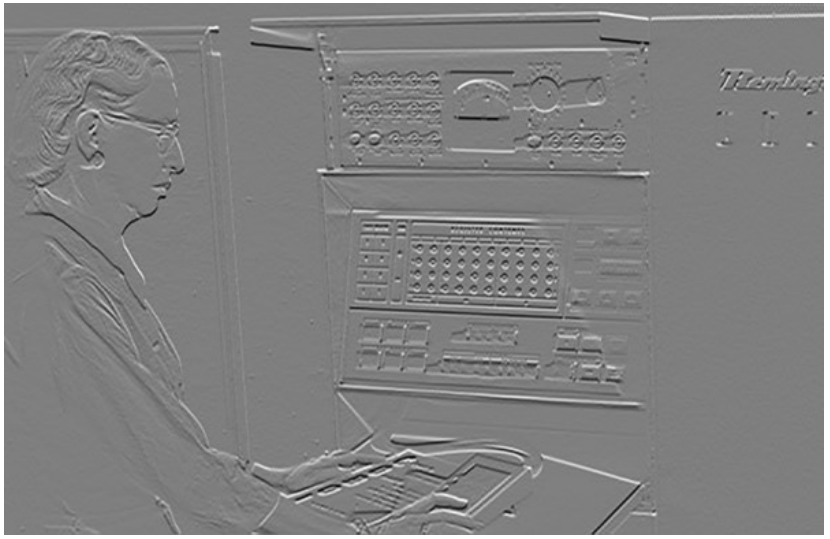
I_x

-1	0	1

^{T}

I_y





Why Does This Work?

Image is function f(x,y)

Remember:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x,y)}{\epsilon}$$

Approximate:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1,y) - f(x,y)}{1}$$

-1	1

Another one:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1,y) - f(x-1,y)}{2}$$

-1	0	1

The reason why use the 3*3 metric instead of 1*3 vector is that we can rely on more pixels and thus we can more reliably estimate the gradient

Other Differentiation Operations

	Horizontal	Vertical
Prewitt	$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$
Sobel	$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Why might people use these compared to [-1,0,1]?

The function f(x,y), input the x and y coordinates and output the intensity of the image at that given x and y position.

Images as Functions

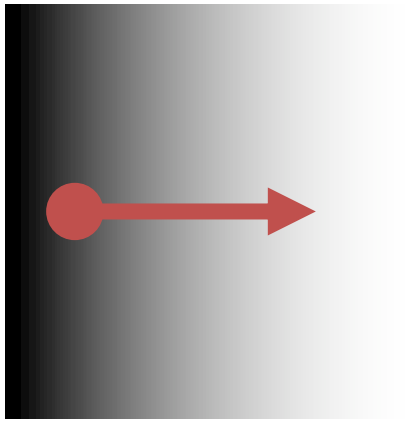
Key idea: can treat image a function of x,y.

$$\nabla I(x,y) = \begin{bmatrix} \frac{\partial I}{\partial x}(x,y) \\ \\ \frac{\partial I}{\partial y}(x,y) \end{bmatrix}$$

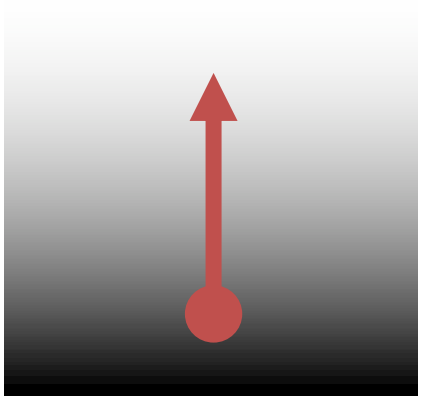
How much the intensity of the image changes as you go horizontally at (x,y)
(Let's call it "I_x")

Image Gradient Direction

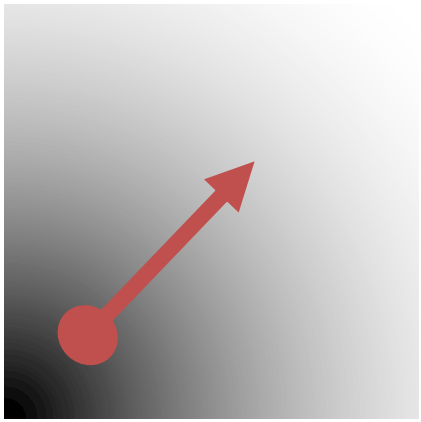
Some gradients



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



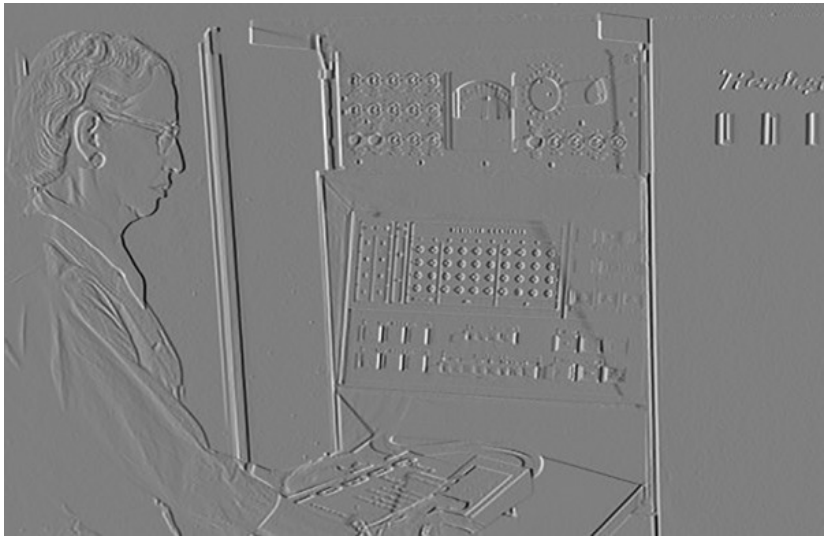
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

Figure Credit: S. Seitz

Image Gradient

Gradient: direction of maximum change.
What's the relationship to edge direction?

Ix



Iy

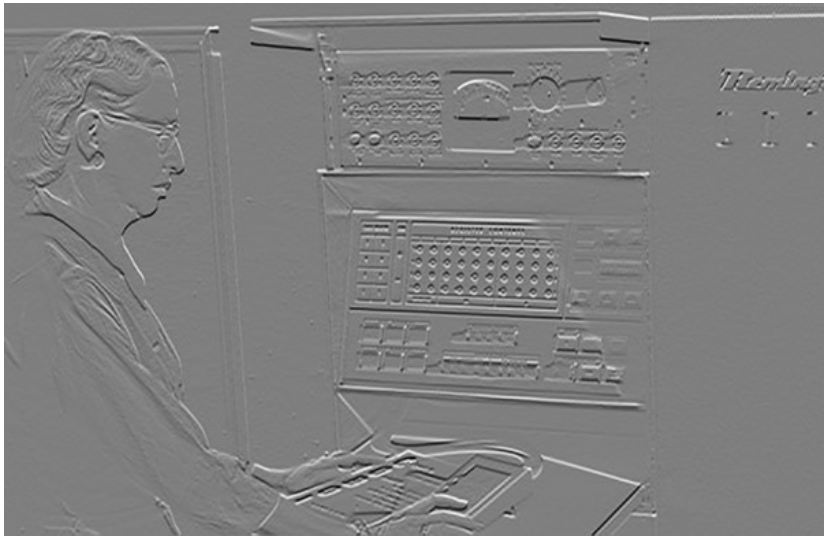
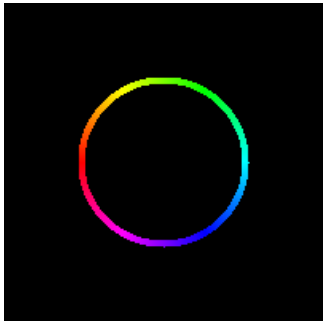


Image Gradient

$(Ix^2 + Iy^2)^{1/2}$: magnitude

Image Gradient

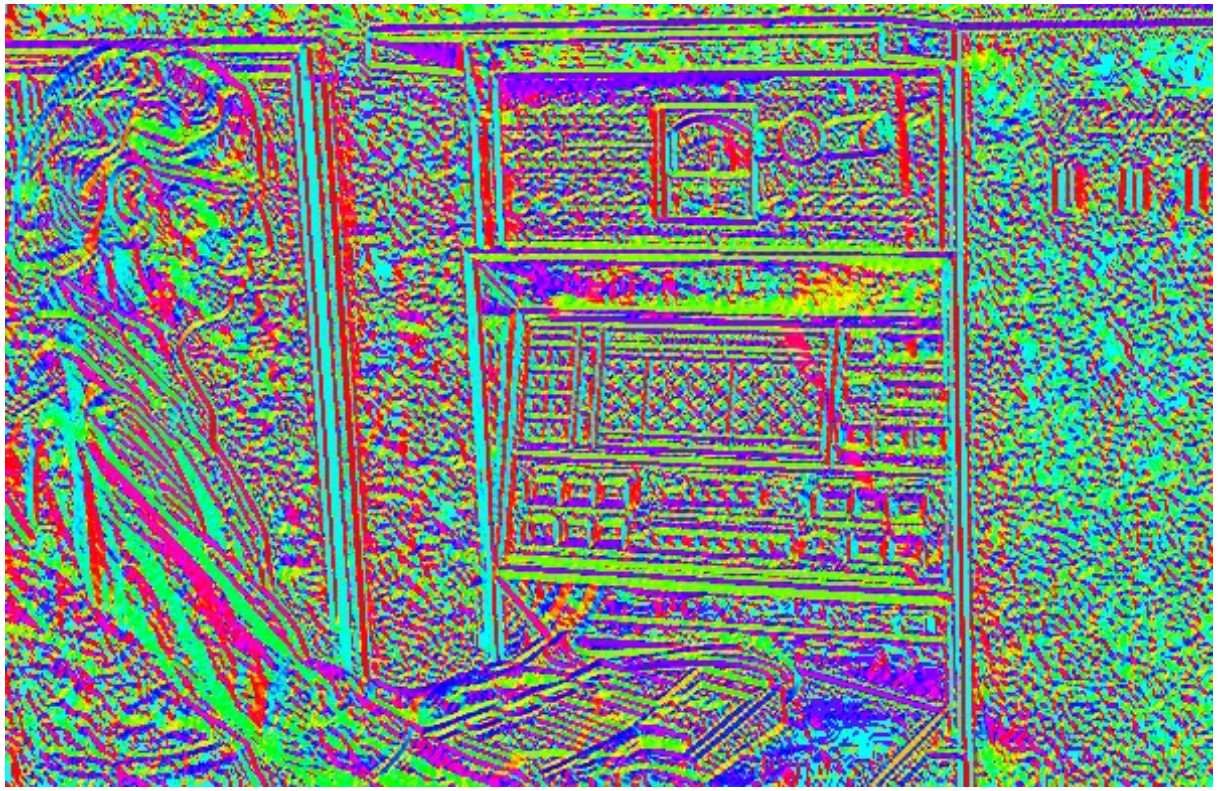
atan2(Iy,Ix): orientation

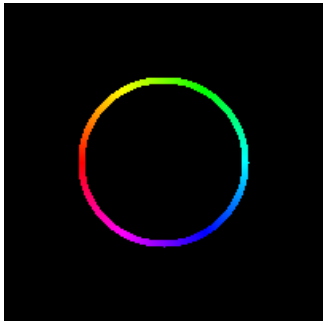


I’m making the lightness equal to gradient magnitude

Image Gradient

atan2(Iy,Ix): orientation



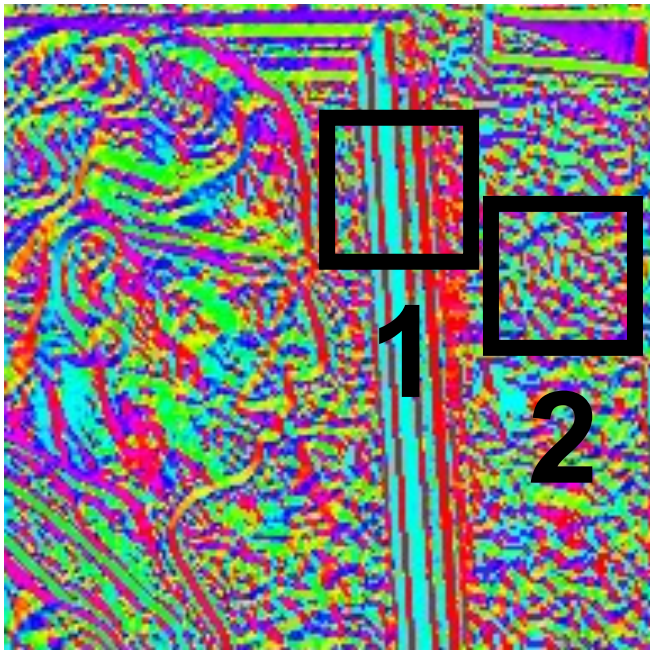


Now I'm showing *all* the gradients

Image Gradient

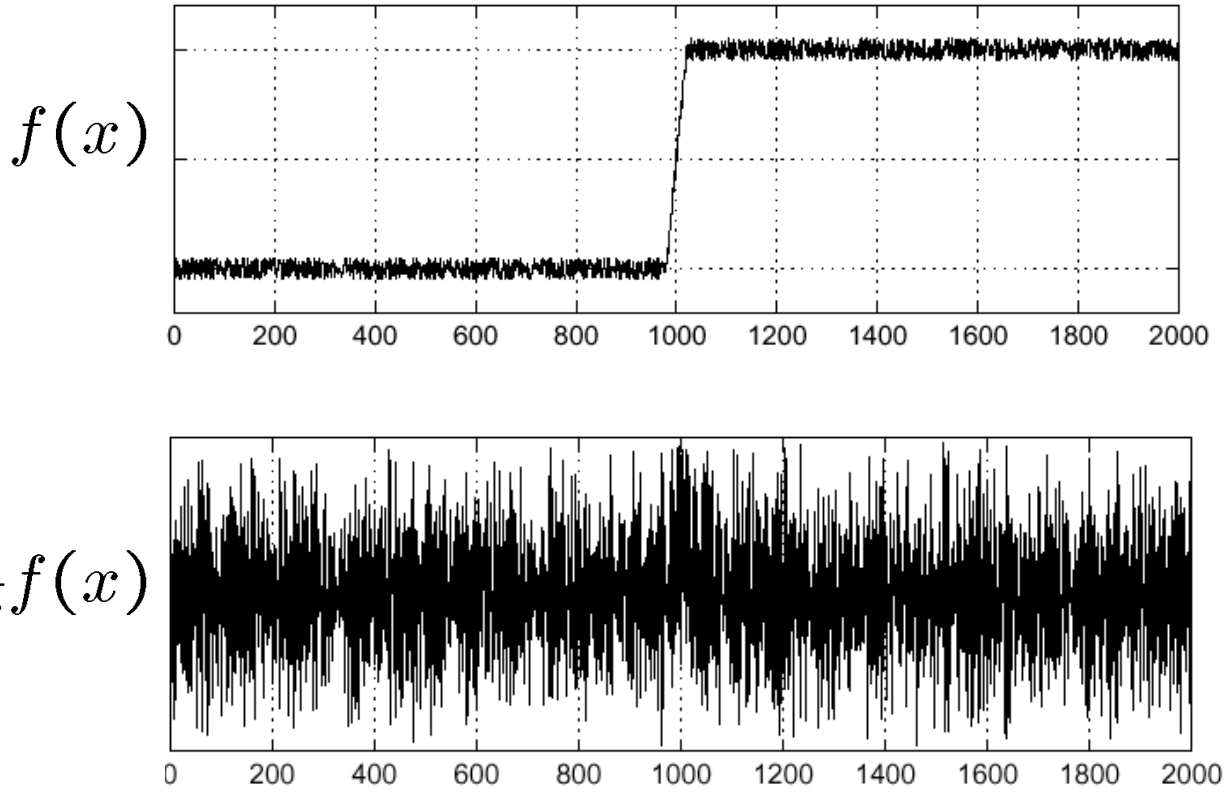
atan2(Iy,Ix): orientation

Why is there structure at 1 and not at 2?



Noise

Consider a row of f(x,y) (i.e., fix y)



Slide Credit: S. Seitz

Noise

Conv. image + per-pixel noise with

-1	0	1

$I_{i,j} =$ True image $\quad \epsilon_{i,j} \sim N(0, \sigma^2)$

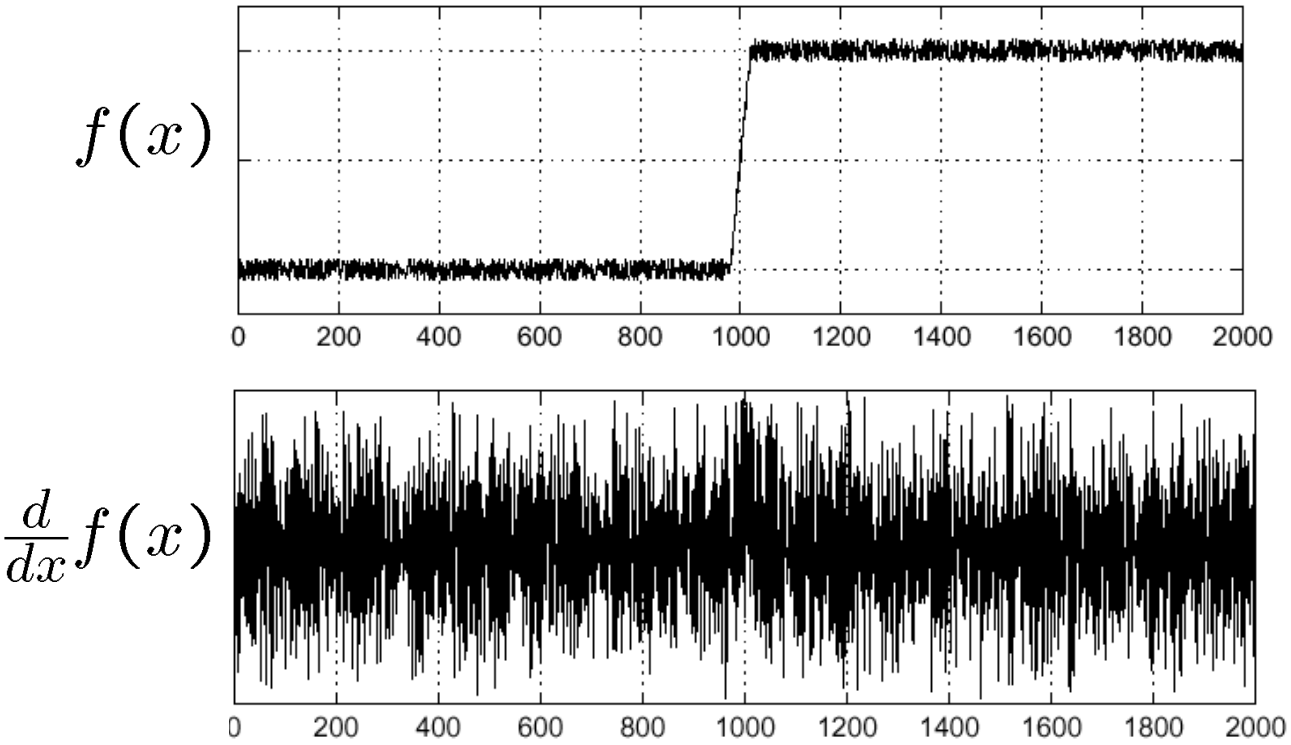
$$D_{i,j} = (I_{i,j+1} + \epsilon_{i,j+1}) - (I_{i,j-1} + \epsilon_{i,j-1})$$

$$D_{i,j} = \underbrace{(I_{i,j+1} - I_{i,j-1})}_{\text{True difference}} + \underbrace{\epsilon_{i,j+1} - \epsilon_{i,j-1}}_{\text{Sum of 2 Gaussians}}$$

$\epsilon_{i,j} - \epsilon_{k,l} \sim N(0, 2\sigma^2) \rightarrow$ Variance doubles!

Noise

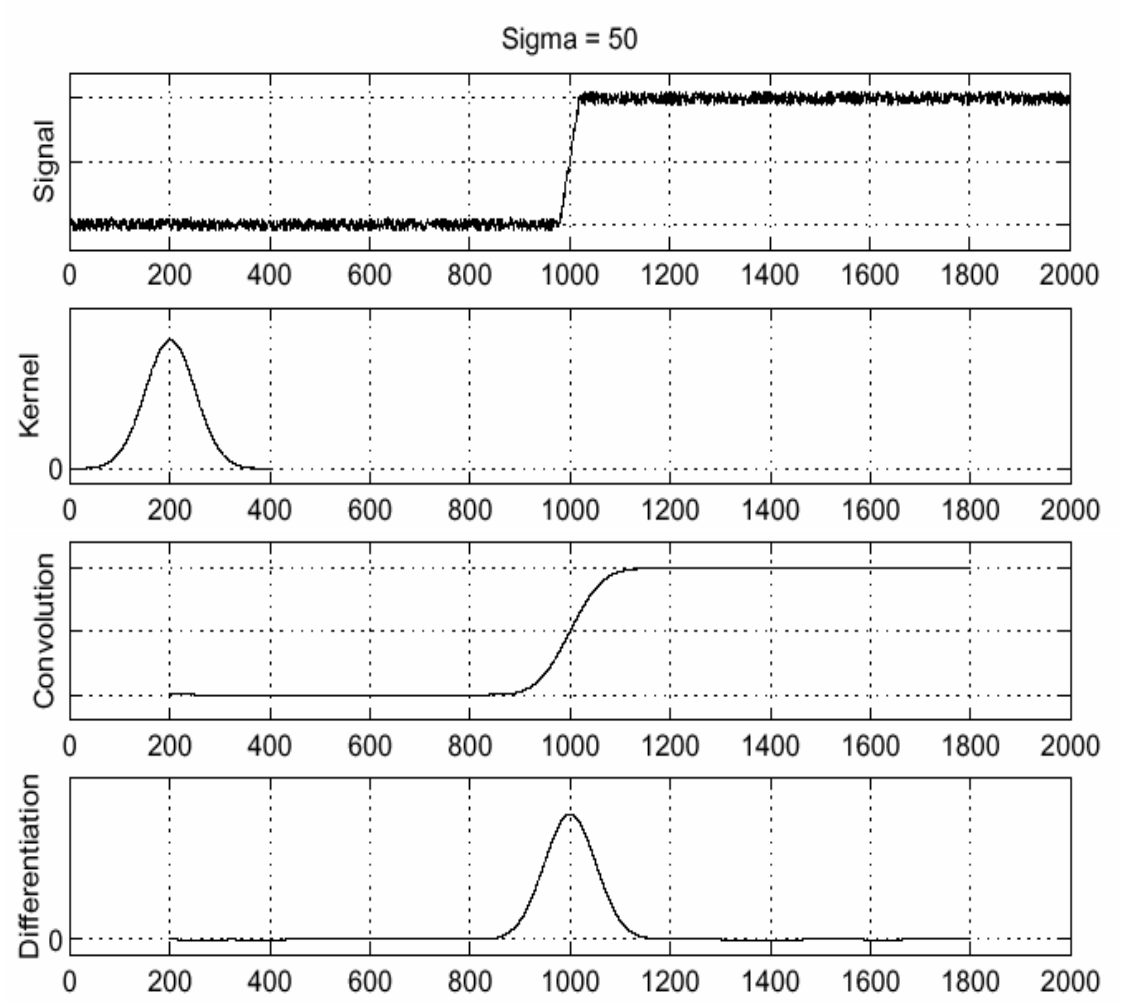
Consider a row of f(x,y) (i.e., make y constant)



How can we use the last class to fix this?

Slide Credit: S. Seitz

Handling Noise



f

g

$f * g$

$\frac{d}{dx}(f * g)$

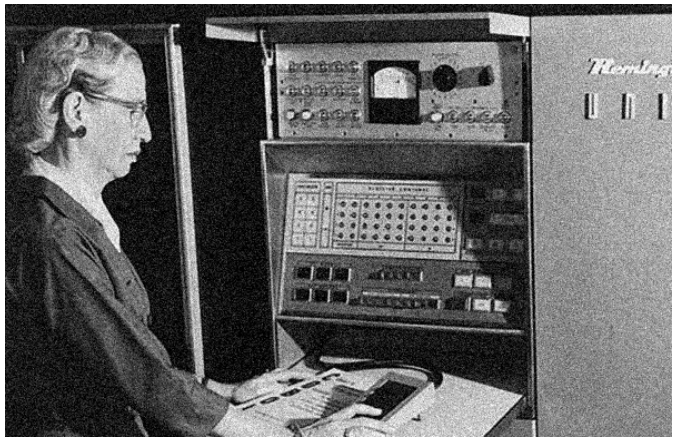
Slide Credit: S. Seitz

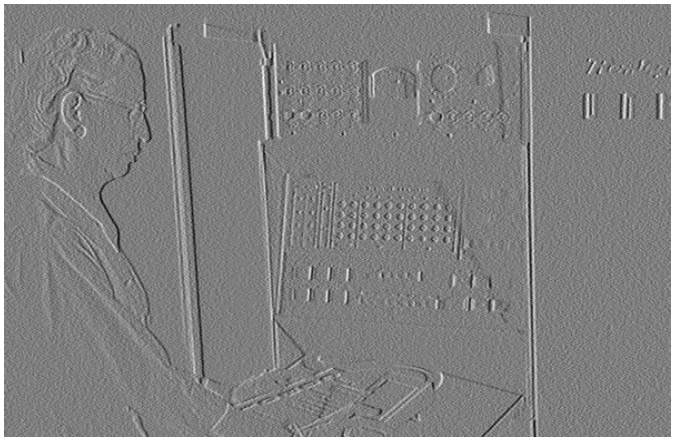
Noise in 2D

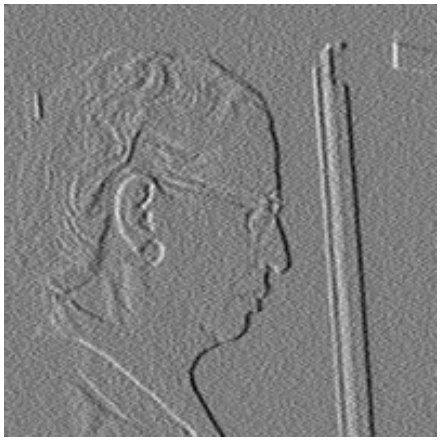
Noisy Input

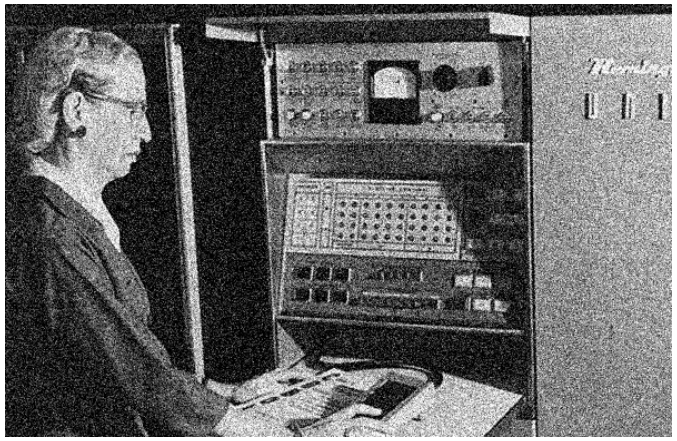
Ix via [-1,01]

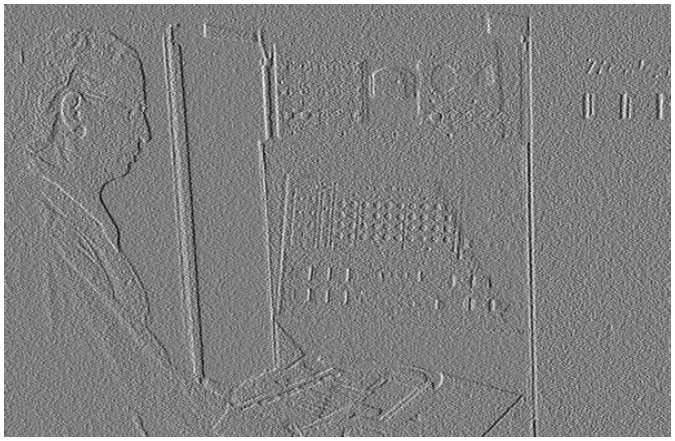
Zoom









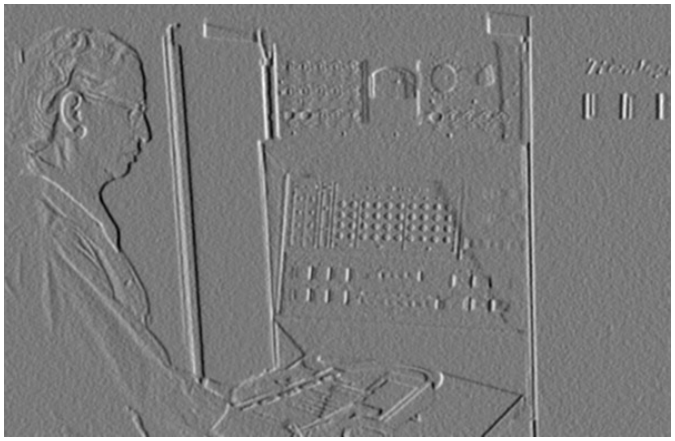


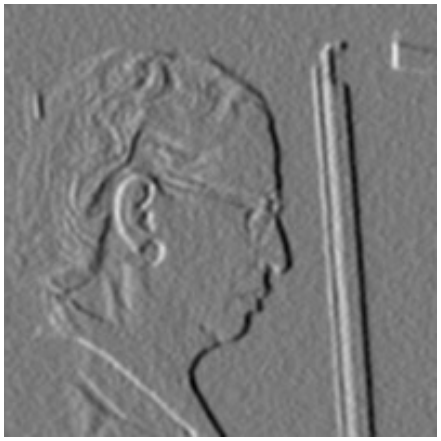
Noise + Smoothing

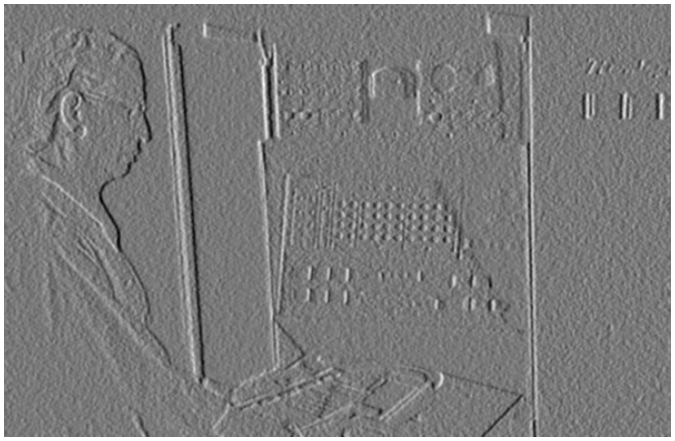
Smoothed Input

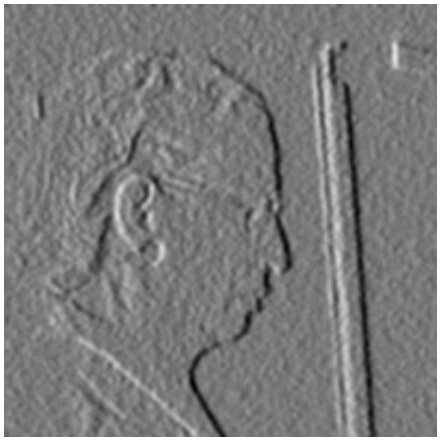
Ix via [-1,01]

Zoom









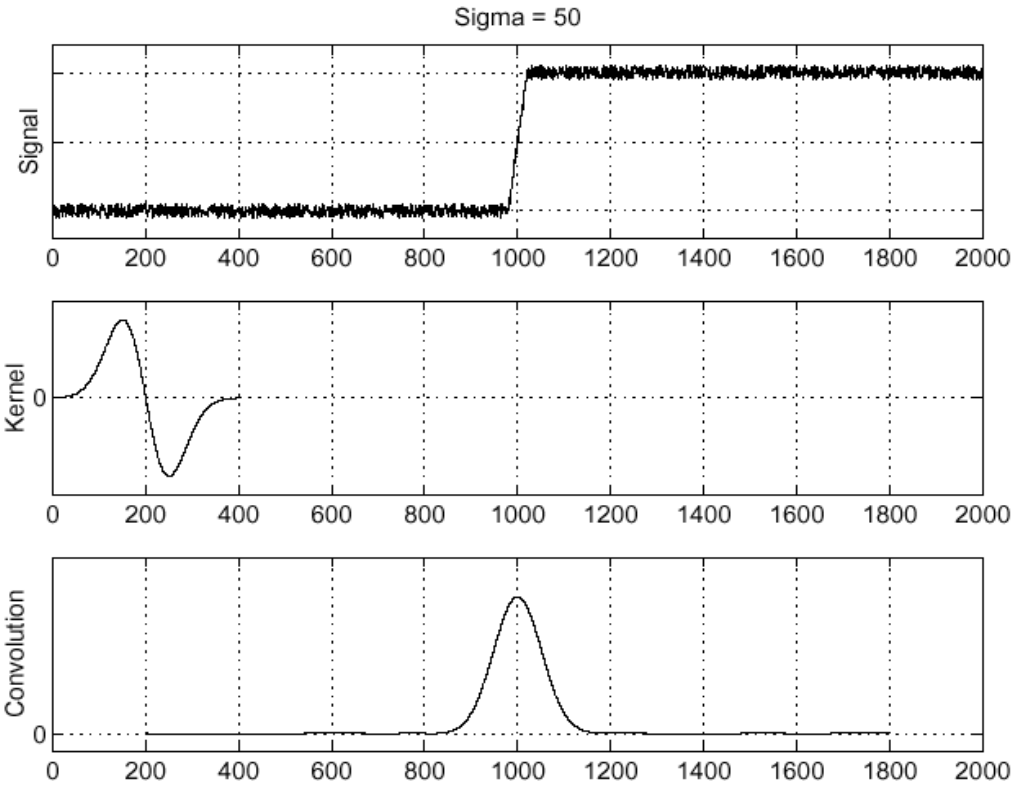
Let's Make It One Pass (1D)

$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

f

$\frac{d}{dx} g$

$f * \frac{d}{dx} g$



Slide Credit: S. Seitz

Proof of $\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$

Definition of Convolution

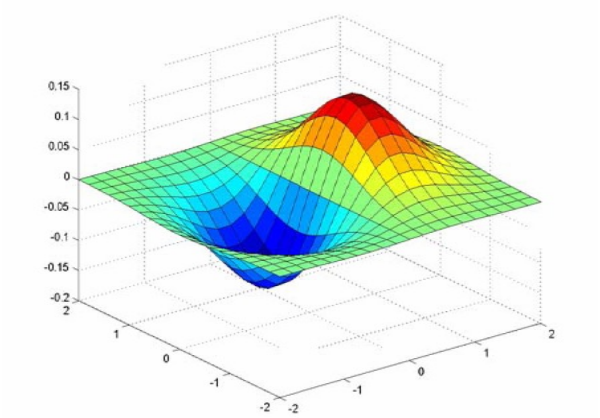
$$(f * g)(x) = \int_{-\infty}^{+\infty} f(t) \cdot g(x - t)\, dt$$

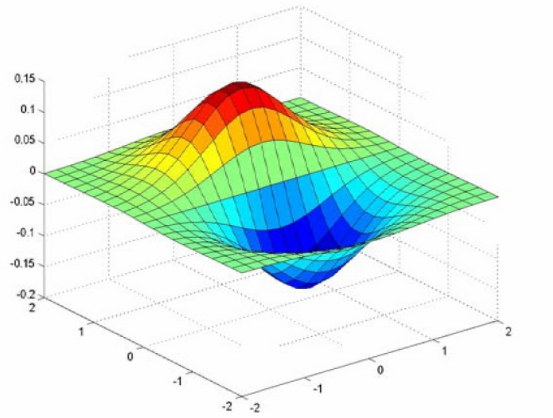
Using Linearity of Differential Operator

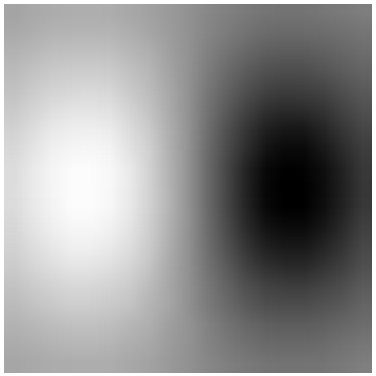
$$\frac{d}{dx}(f * g)(x) = \frac{d}{dx} \int_{-\infty}^{+\infty} f(t) \cdot g(x - t)\, dt = \int_{-\infty}^{+\infty} f(t) \cdot \frac{d}{dx} g(x - t)\, dt$$

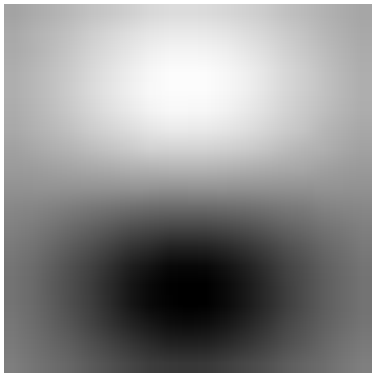
Let's Make It One Pass (2D)

Gaussian Derivative Filter







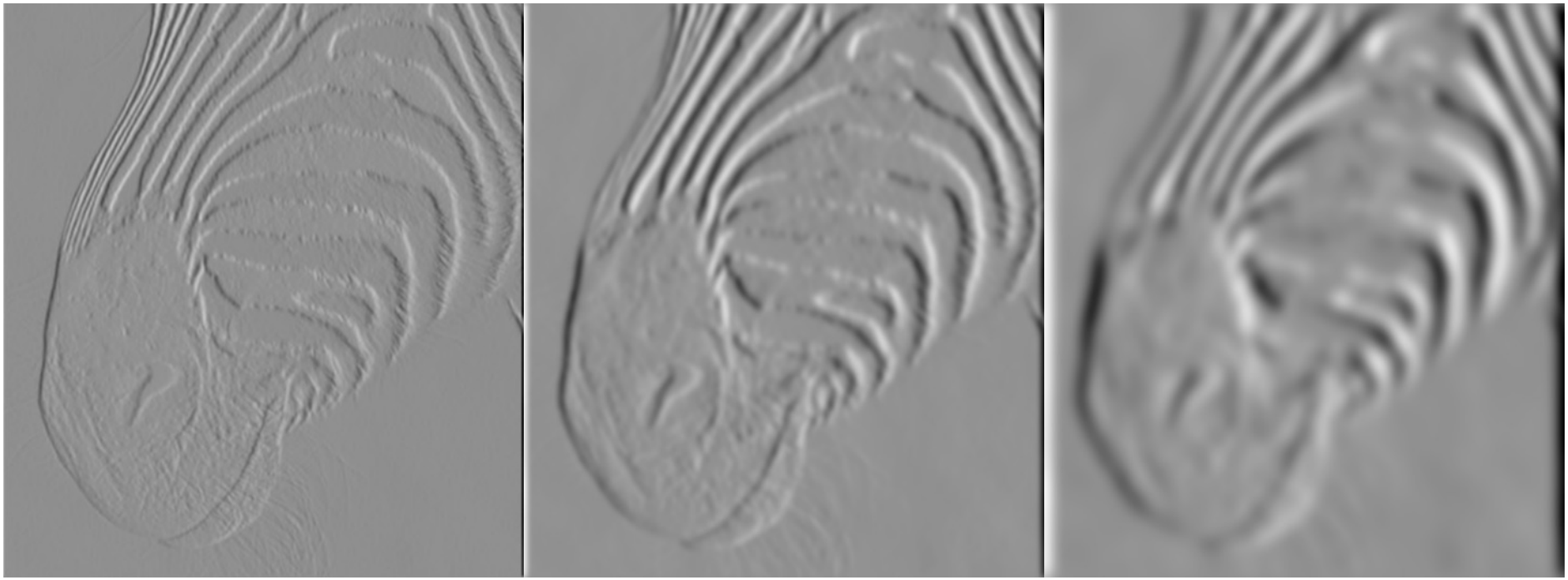


Which one finds the X direction?

Slide Credit: L. Lazebnik

Applying the Gaussian Derivative

SD: 1 pixel | 3 pixels | 7 pixels

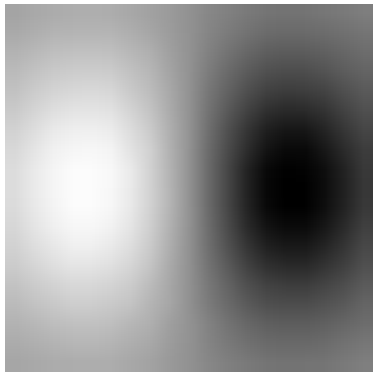


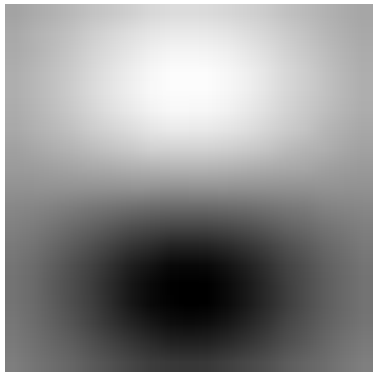
Removes noise, but blurs edge

Slide Credit: D. Forsyth

Compared with the Past

Gaussian Derivative





Sobel Filter

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

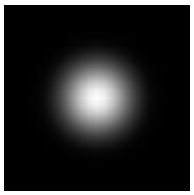
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

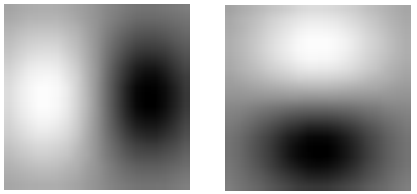
Why would anybody use the bottom filter?

1. The Gaussian derivative is expensive to compute.
2. only the liner filter we can do the derivative of the filter first

Filters We've Seen

	Smoothing	Derivative
Example	Gaussian	Deriv. of gauss
Goal	Remove noise	Find edges
Only +?	**Yes**	**No**
Sums to	1	0





Why sum to 1 or 0, intuitively?

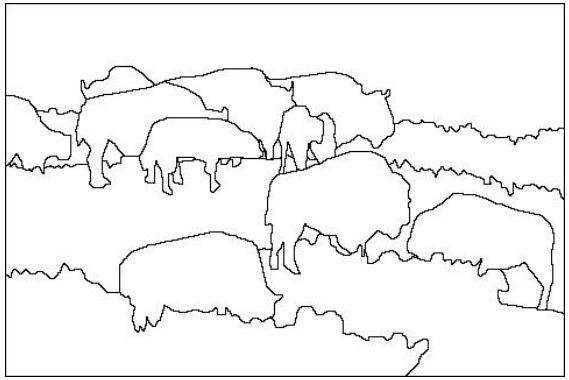
Don't want to change the intense of the picture.

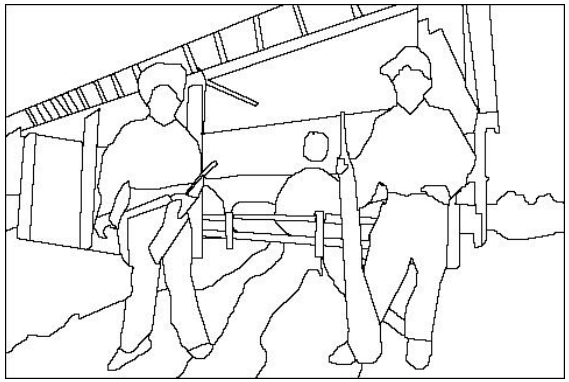
We want to left and right to be symmetric when we do the compare.

Slide Credit: J. Deng

Problems

Image | human segmentation | gradient magnitude




Still an unsolved problem

Localizing Reliably

- Suppose you need to meet someone but you can't use your cell phone to coordinate
- Where do you agree to meet?

A: Along the Huron river

B: Along State Street

C: At Liberty and State Street

D: On North Campus

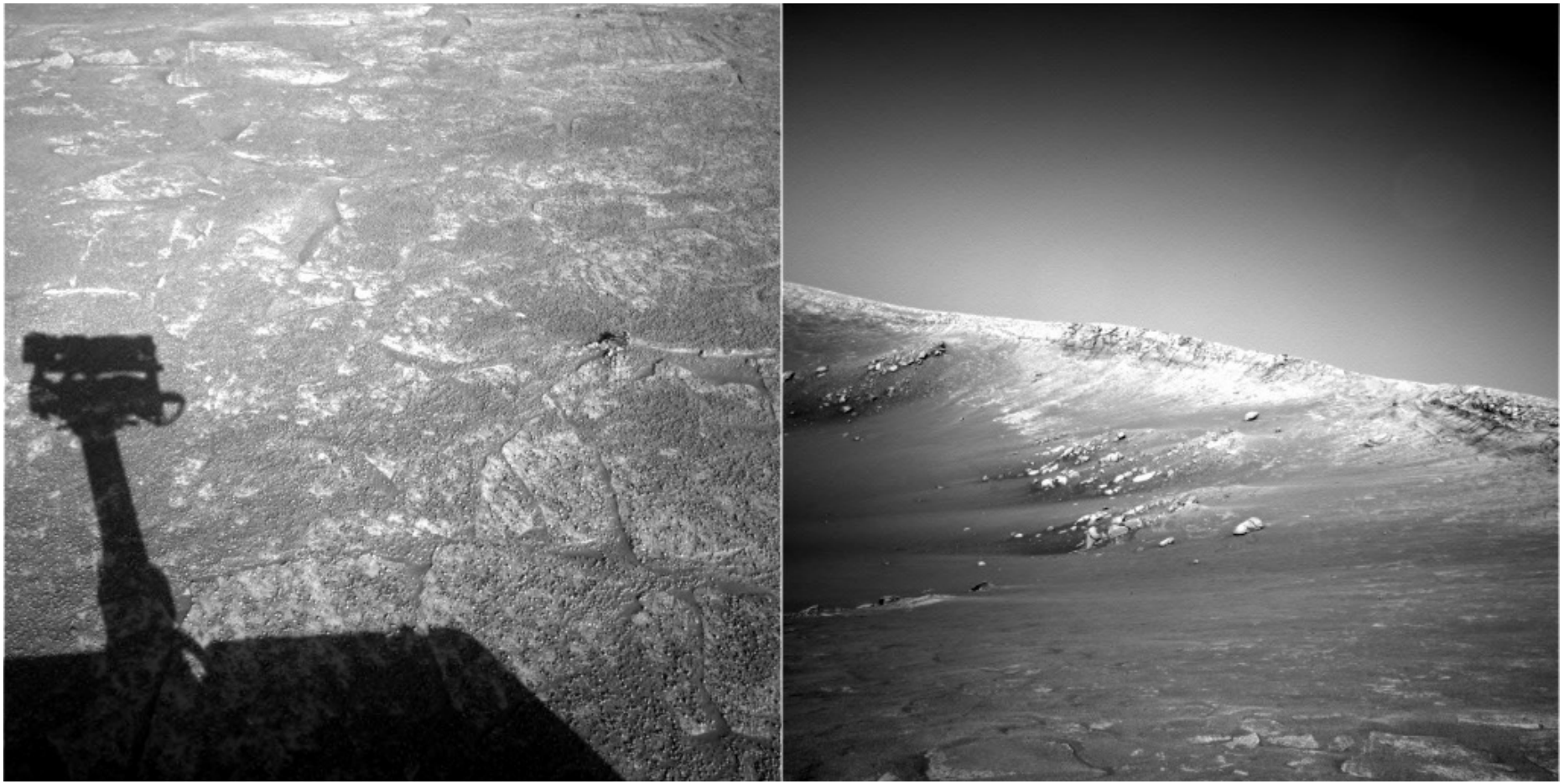
Corners over a line!

Desirables

- Repeatable: should find same things even with distortion
- Saliency: each feature should be distinctive
- Compactness: shouldn’t just be all the pixels
- Locality: should only depend on local image data

Property list: S. Lazebnik

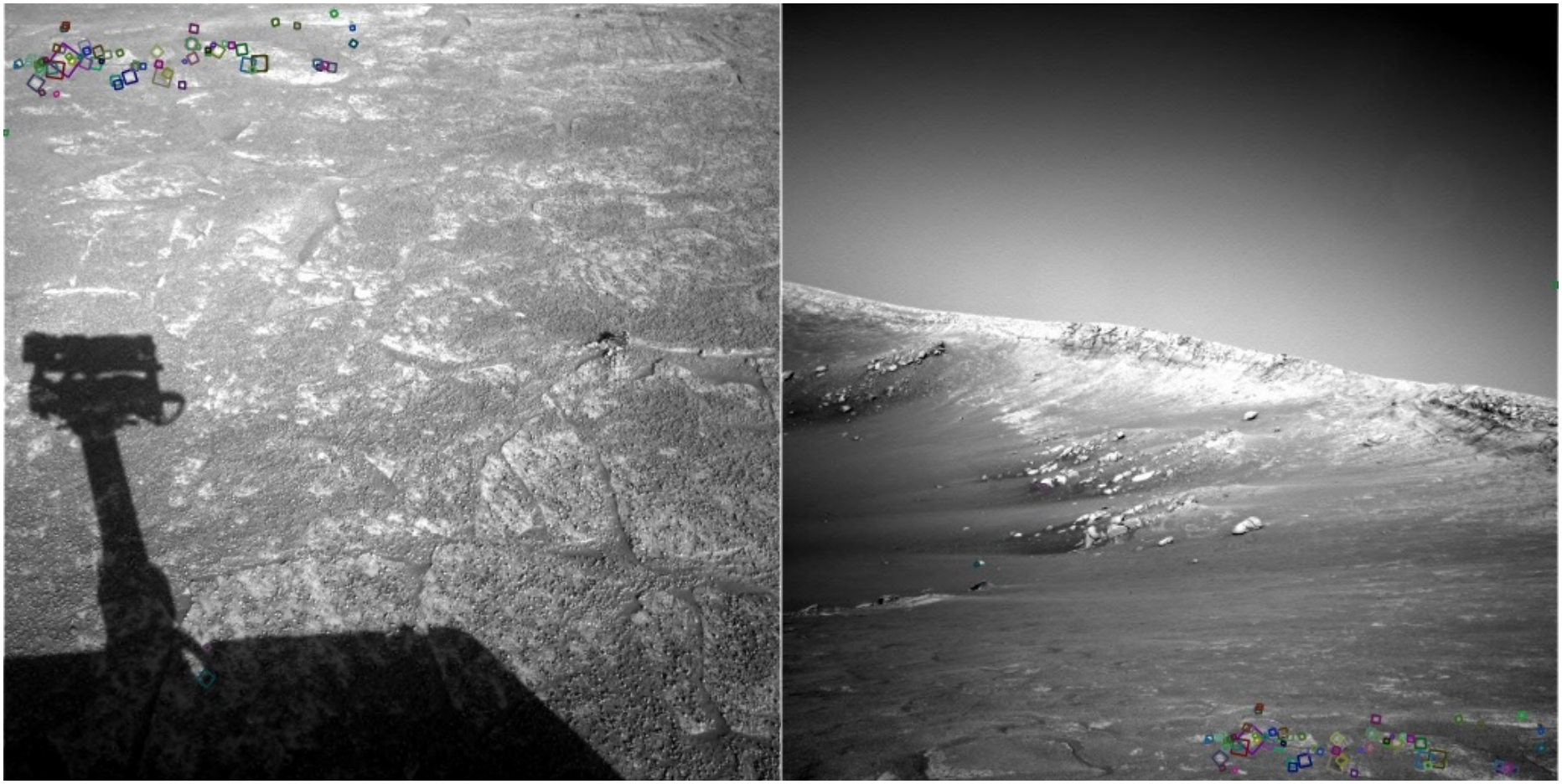
Example



Can you find the correspondences?

Slide credit: N. Snavely

Example Matches

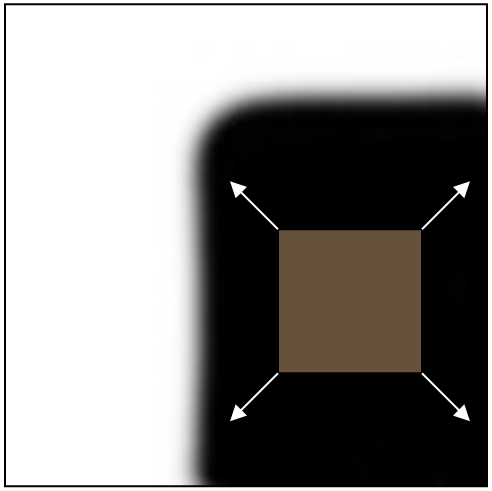


Look for the colored squares

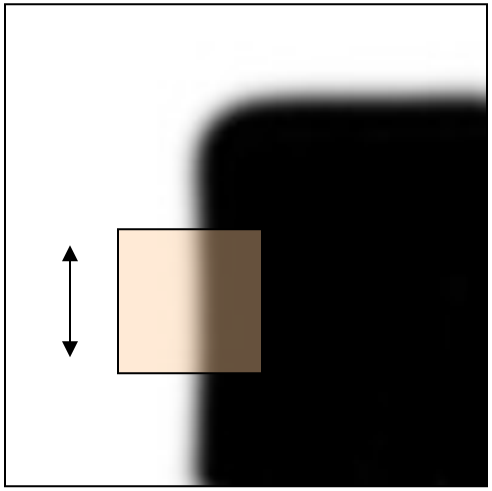
Slide credit: N. Snavely

Basic Idea

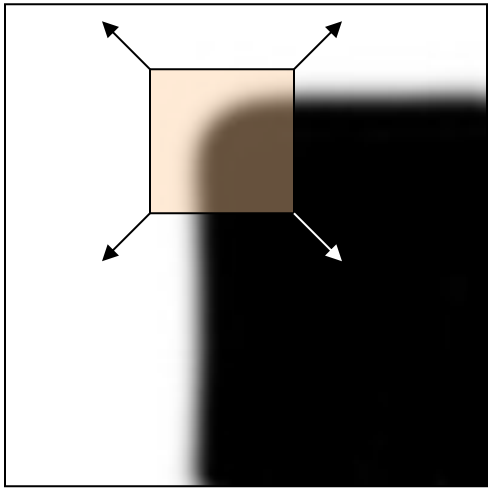
Should see where we are based on small window, or any shift → big intensity change.



"flat" region:
no change in all directions



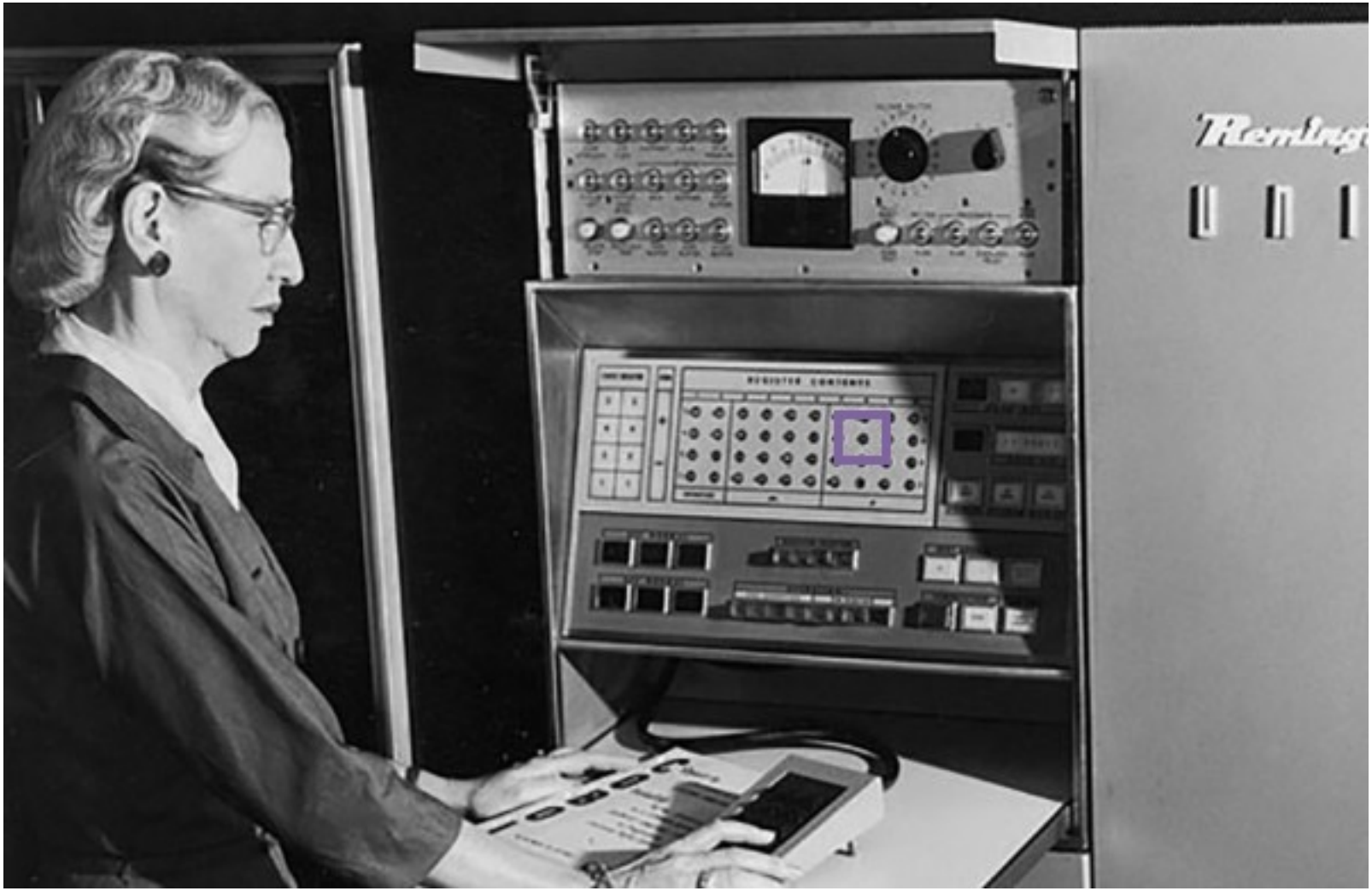
"edge":
no change along the edge direction



"corner":
significant change in all directions

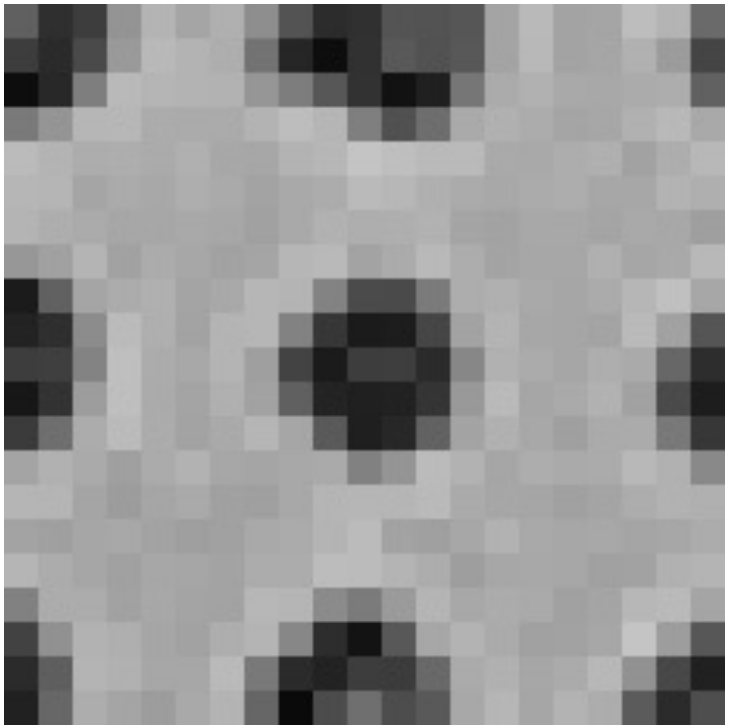
Slide Credit: S. Lazebnik

Formalizing Corner Detection

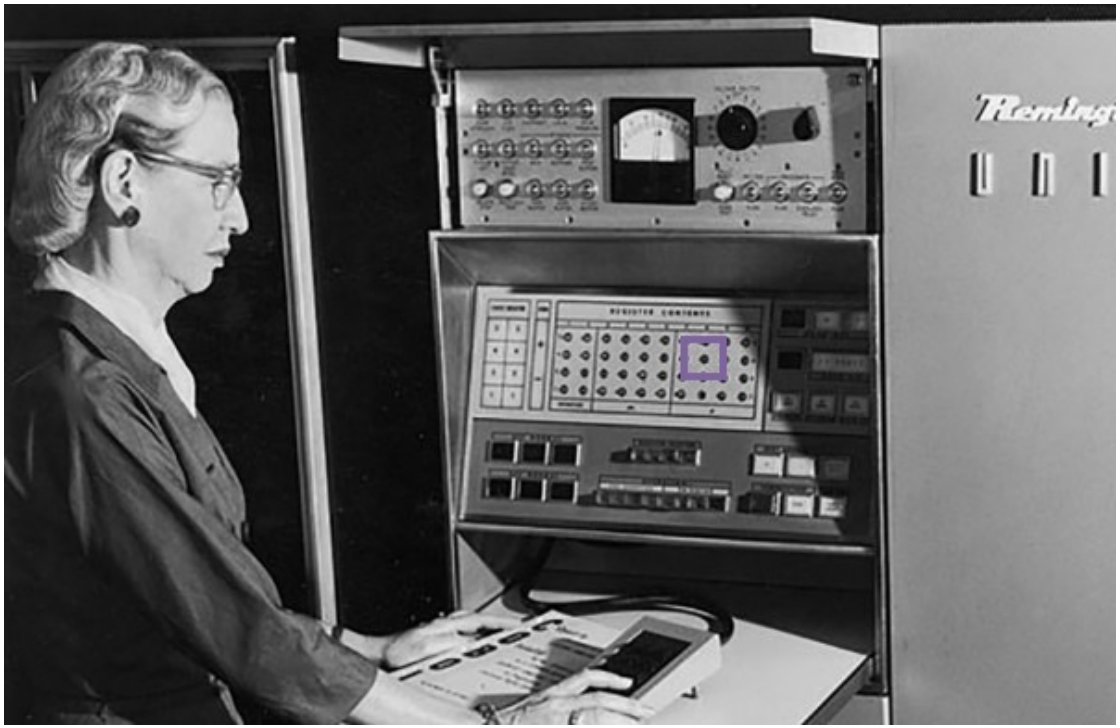


Formalizing Corner Detection

Zoom-In at x,y

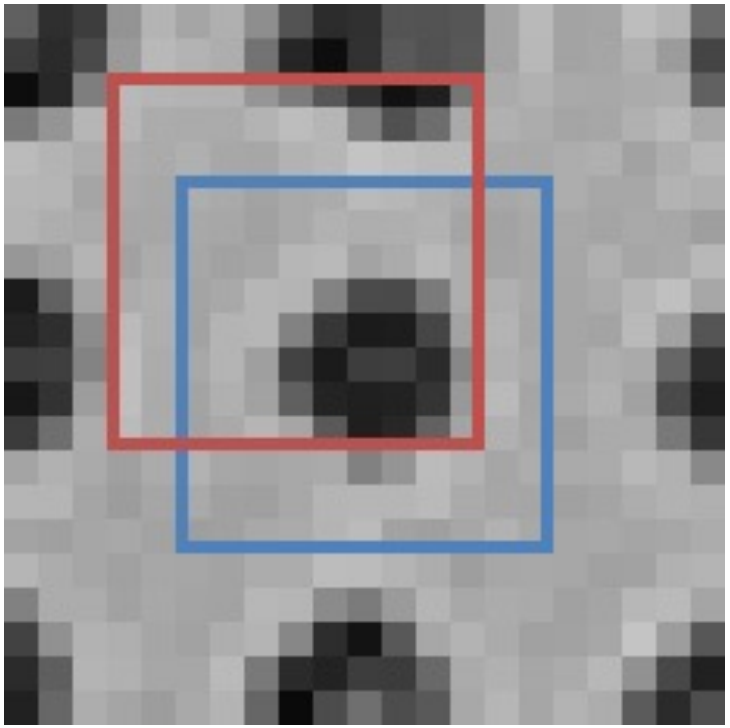


Original Image



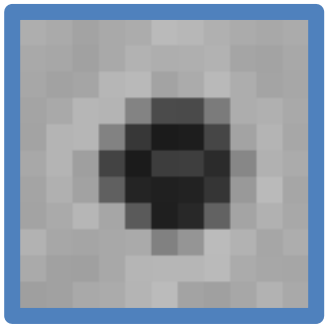
Formalizing Corner Detection

Zoom-In at x,y



Window without and with Offset

“Window”
At x+u, y+v
Here: u=-2,v=-3

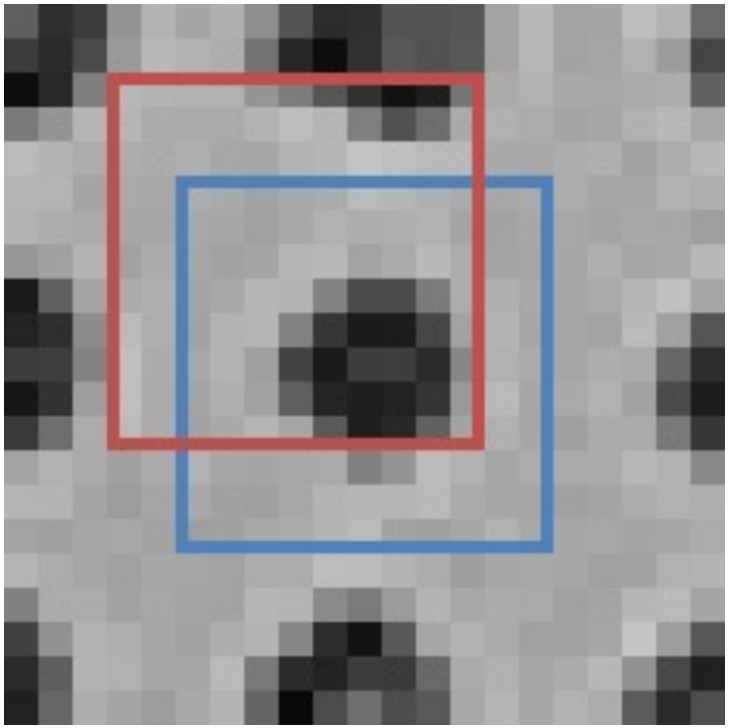


“Window”
At x, y

How might we measure similarity?

Formalizing Corner Detection

Zoom-In at x,y



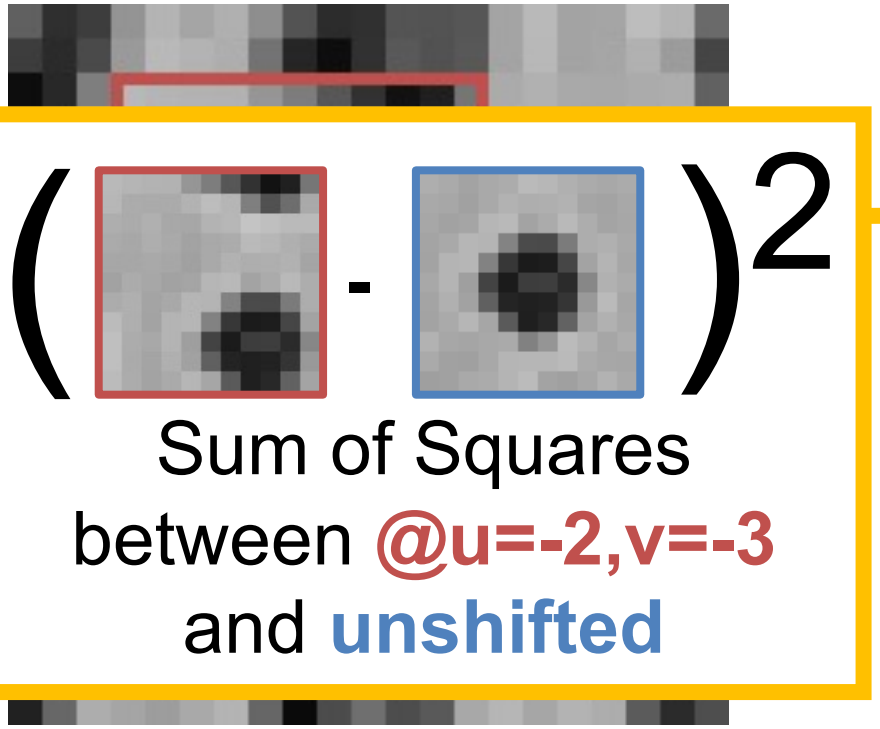
Error (Sum Sqs) for u,v offset

$$E(u,v) = \sum_{(x,y)\in W} (I[x+u, y+v] - I[x,y])^2$$

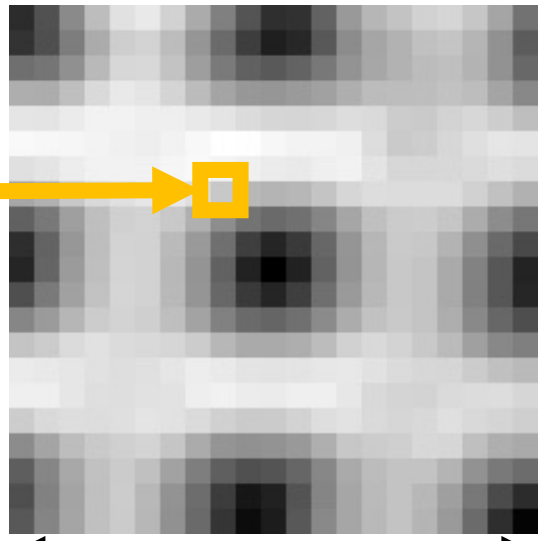
$$\left(\quad - \quad \right)^2$$

Formalizing Corner Detection

Zoom-In at x,y



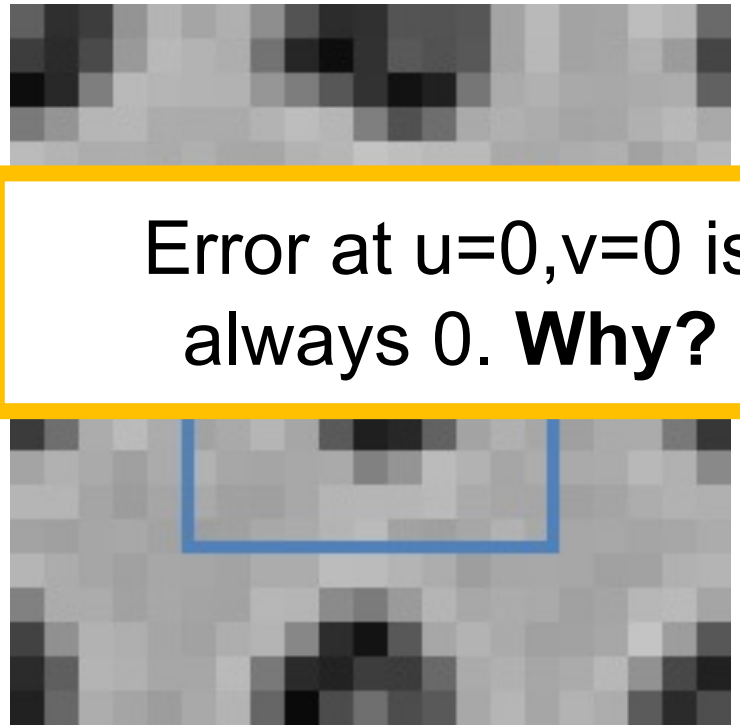
Error (Sum Sqs) for u,v offset



Sum of Squares between **@u=-2,v=-3** and **unshifted**

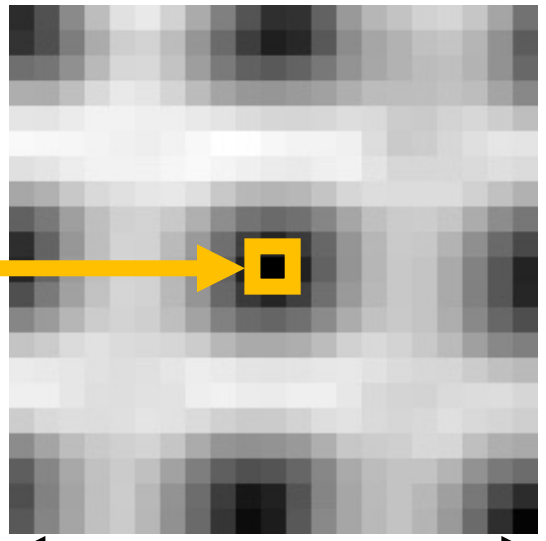
Formalizing Corner Detection

Zoom-In at x,y



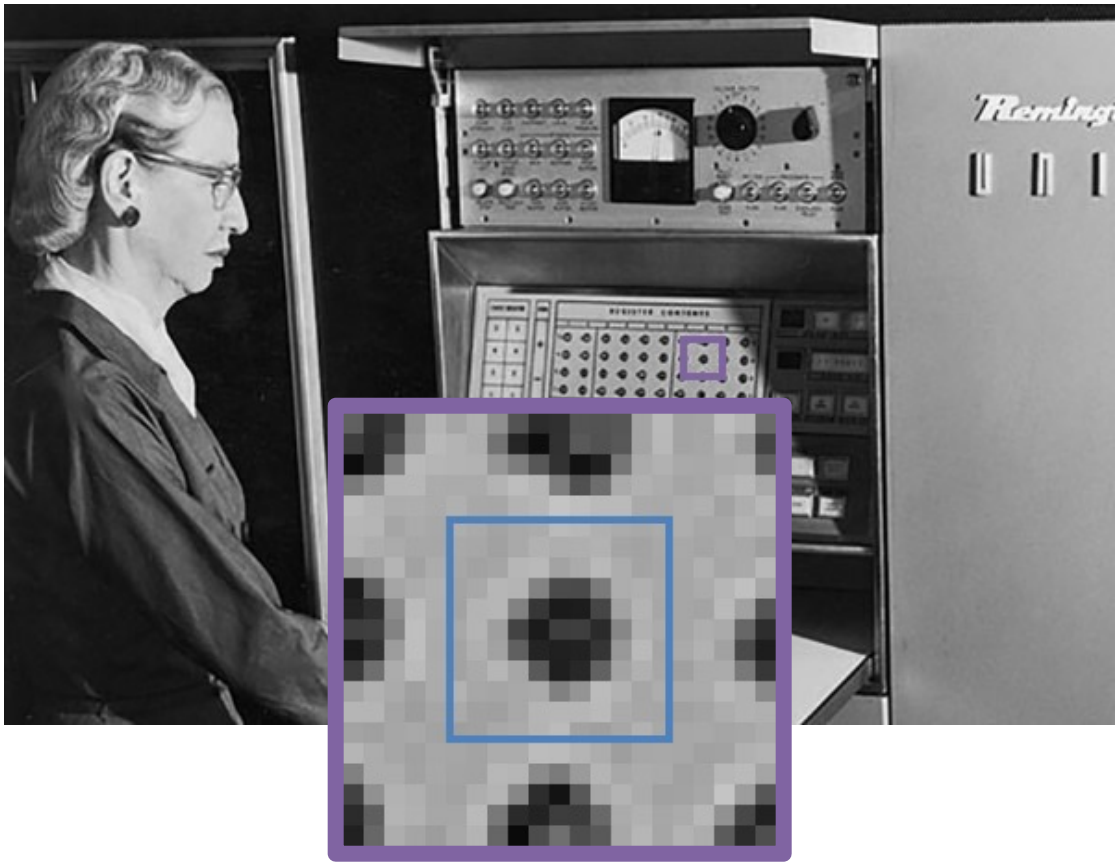
Error at u=0,v=0 is always 0. **Why?**

Error (Sum Sqs) for u,v offset



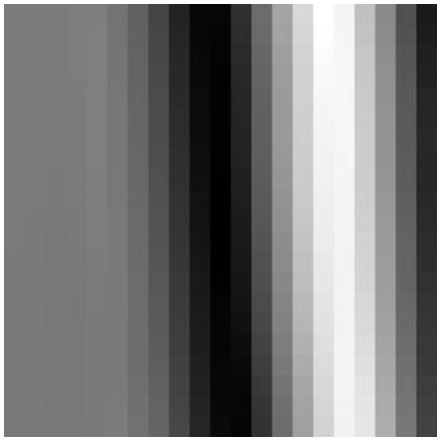
Match The Location and Plot

Original Image and Zoom-In

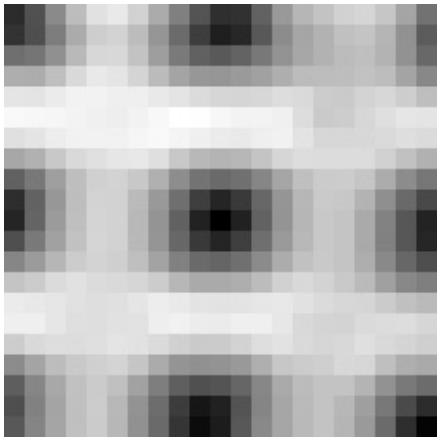


Error Options

A

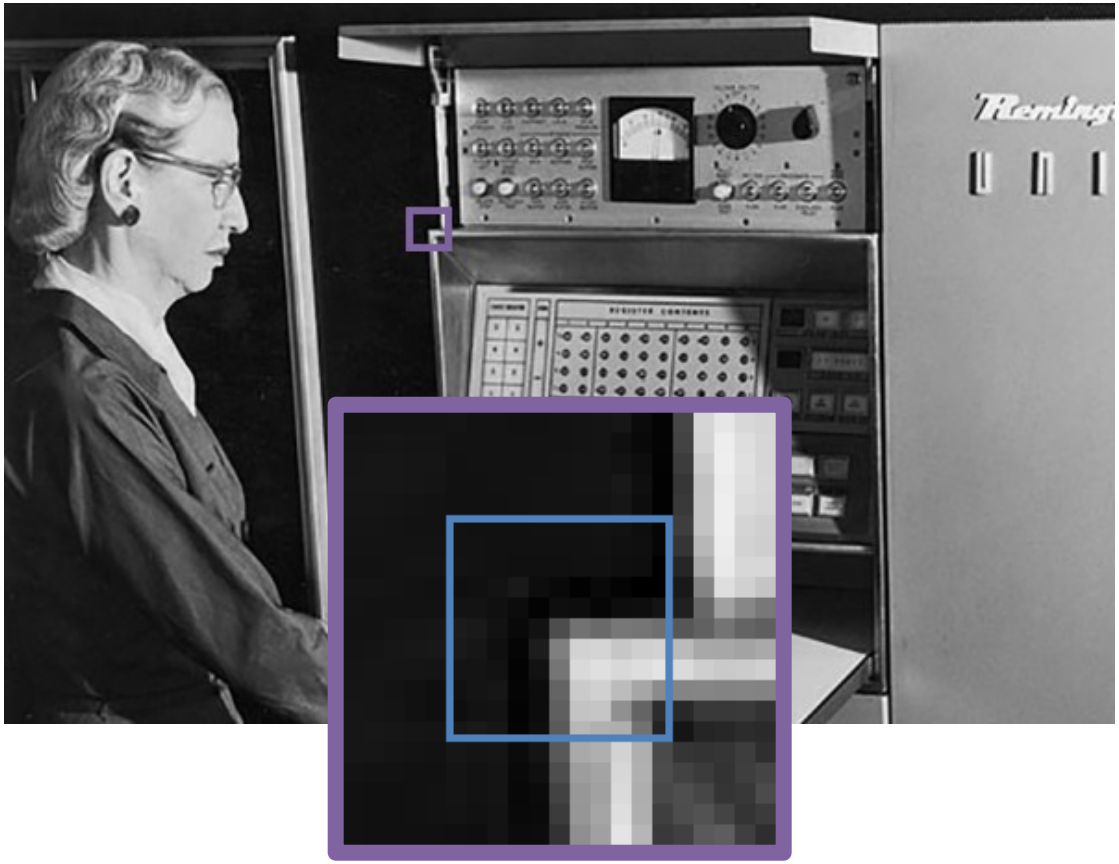


B



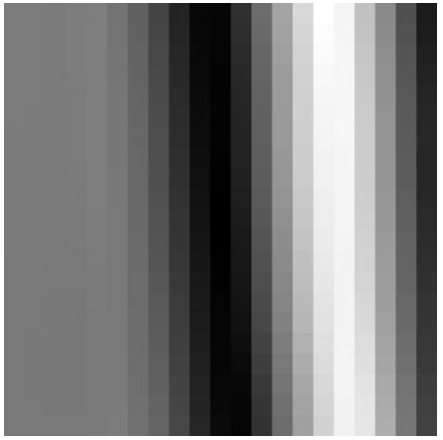
Match The Location and Plot

Original Image and Zoom-In

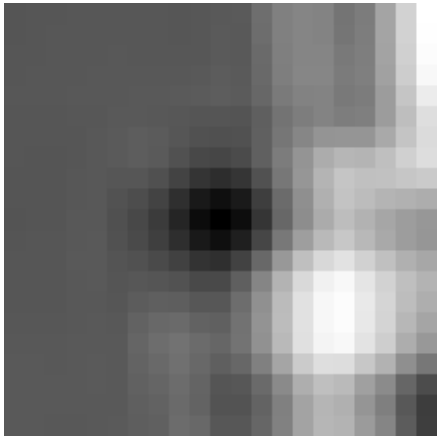


Error Options

A

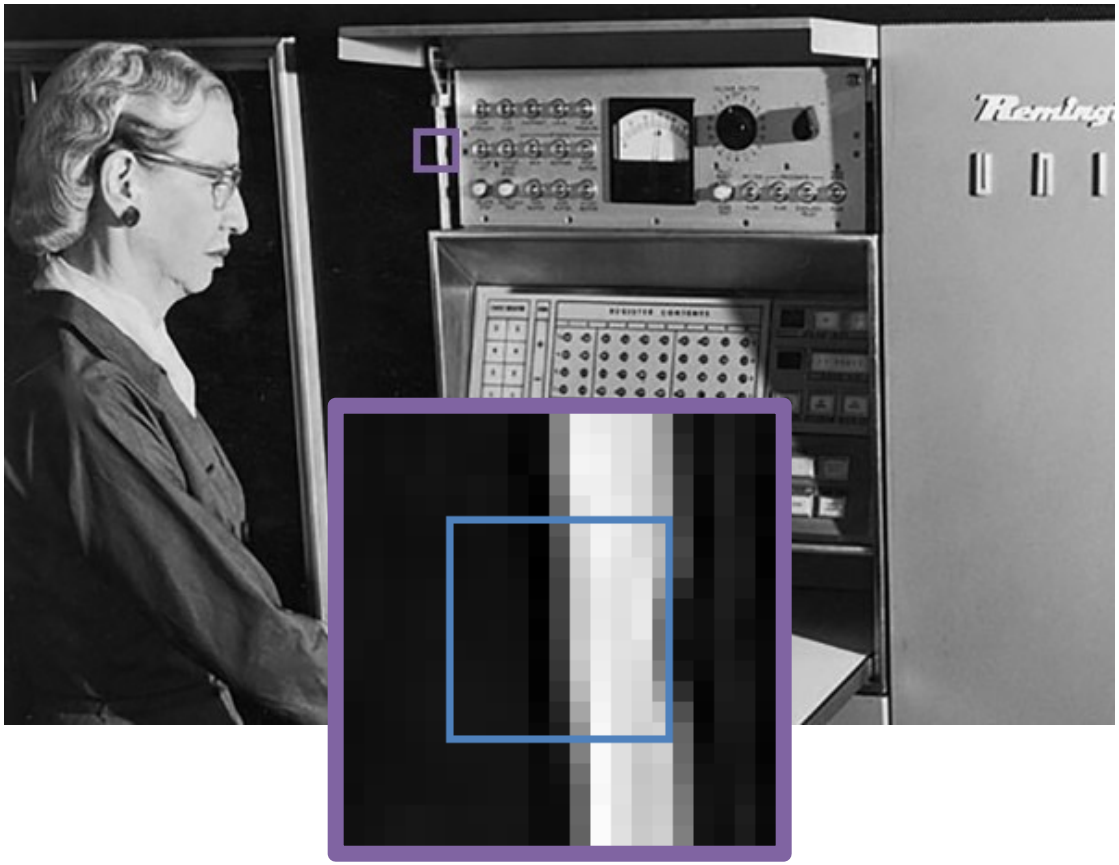


B



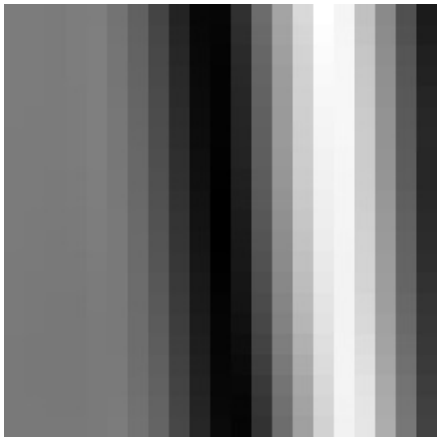
Match The Location and Plot

Original Image and Zoom-In

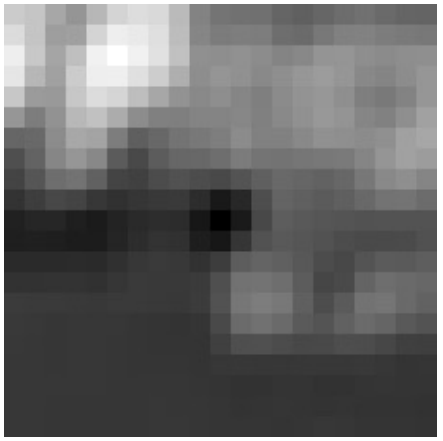


Error Options

A

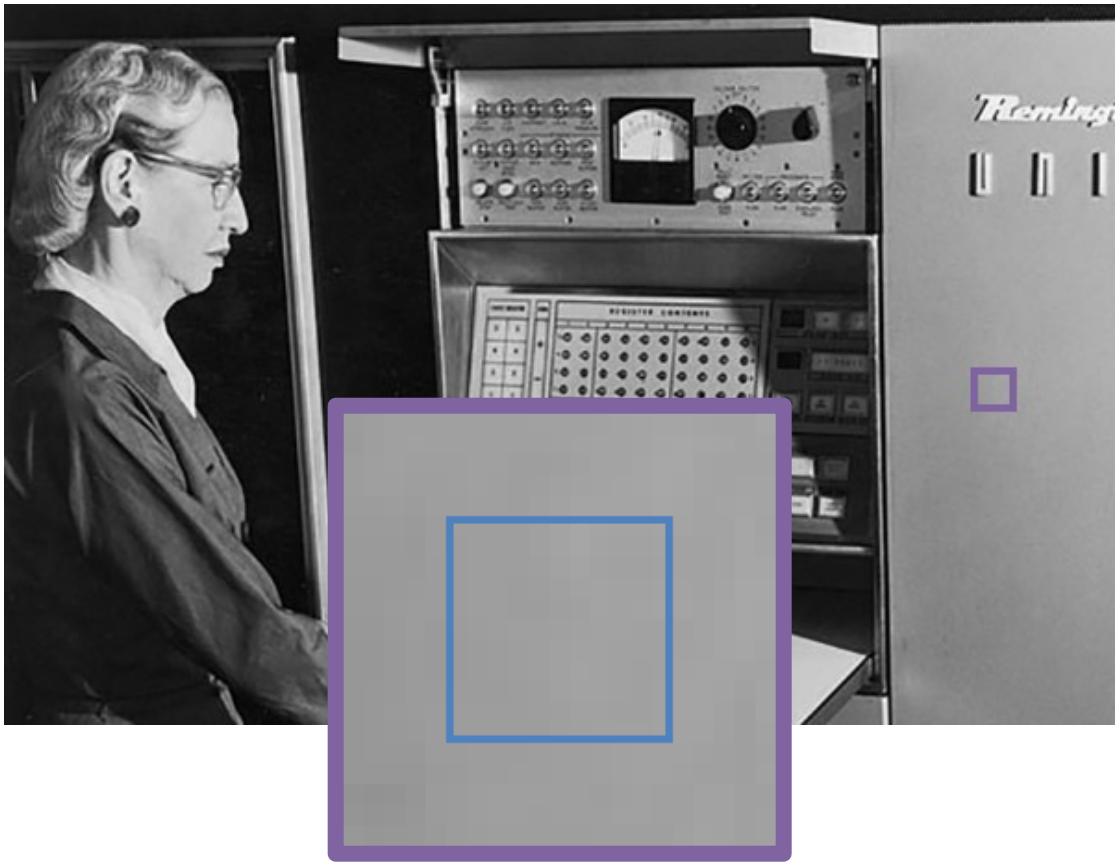


B



Match The Location and Plot

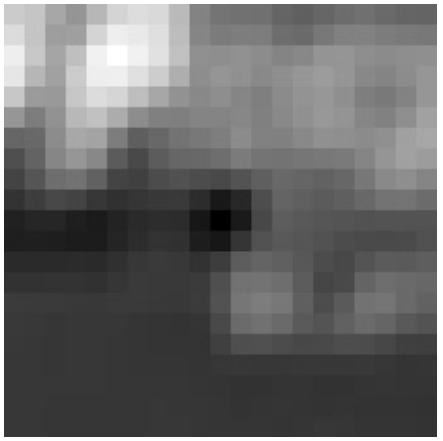
Original Image and Zoom-In



Error Options

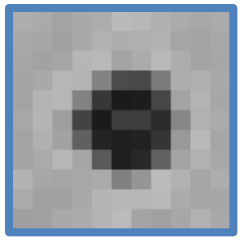
A

B



Ok But Back To Math

$$E(u,v) = \sum_{(x,y)\in W} (I[x+u, y+v] - I[x,y])^2$$

$$\left(\text{  } - \text{  } \right)^2$$

Shifting windows around is expensive!
We'll find a trick to approximate this.

Note: only need to get the gist

Aside: Taylor Series for Images

Recall Taylor Series – way of *linearizing* a function:

$$f(x+d) \approx f(x) + \frac{\partial f(x)}{\partial x} d$$

$$f(x+u, y+v) \approx f(x,y) + \frac{\partial f(x,y)}{\partial x} u + \frac{\partial f(x,y)}{\partial y} v$$

Do the same with images, treating them as function of x, y

$$I(x+u, y+v) \approx I(x,y) + I_x u + I_y v$$

For brevity: Ix = Ix(x,y) at point (x,y), Iy = Iy(x,y) at point (x,y)

Formalizing Corner Detection

$$E(u,v) = \sum_{(x,y)\in W} (I[x+u, y+v] - I[x,y])^2$$

Taylor series expansion for I at every single point in window

$$\approx \sum_{(x,y)\in W} \left(I[x,y] + I_x u + I_y v - I[x,y]\right)^2$$

Cancel

$$= \sum_{(x,y)\in W} \left(I_x u + I_y v\right)^2$$

Expand

$$= \sum_{(x,y)\in W} I_x^2 u^2 + 2 I_x I_y uv + I_y^2 v^2$$

For brevity: Ix = Ix at point (x,y), Iy = Iy at point (x,y)

Formalizing Corner Detection

By linearizing image, we can approximate E(u,v) with quadratic function of u and v

$$E(u,v) \approx \sum_{(x,y)\in W} \left(I_x^2 u^2 + 2I_x I_y uv + I_y^2 v^2\right)$$

$$= [u,v]\boldsymbol{M}[u,v]^T$$

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y\in W} I_x^2 & \sum_{x,y\in W} I_x I_y \\ \sum_{x,y\in W} I_x I_y & \sum_{x,y\in W} I_y^2 \end{bmatrix}$$

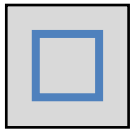
M is called the second moment matrix
Simple algebra: Try at home!

Intuitively what is M?

Pretend gradients are *either* vertical or horizontal at a pixel (so Ix Iy = 0)

Obviously Wrong!

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} \approx \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

a,b both small: flat  $\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

One big, other small: edge  $\begin{bmatrix} 50 & 0 \\ 0 & 0.1 \end{bmatrix}$ or $\begin{bmatrix} 0.1 & 0 \\ 0 & 50 \end{bmatrix}$

a,b both big: corner  $\begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$

Intuitively what is M?

General case: so Ix Iy ≠ 0

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} \approx ? \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

a,b both small: flat

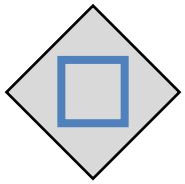


Image might be rotated by rotation θ!

One big, other small: edge

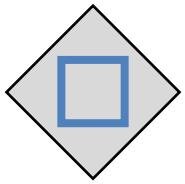
a,b both big: corner

Intuitively what is M?

General case: so Ix Iy ≠ 0

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

a,b both small: flat



If image rotated by rotation θ / matrix **V**

One big, other small: edge

M will look like

$$\boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

a,b both big: corner

So What Now?

Can calculate M at pixel, by summing nearby gradients, but need to find a and b.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

Given **M**, can decompose it into eigenvectors **V** and eigenvalues λ_1, λ_2 with $\mathbf{M} = \boldsymbol{V}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$.

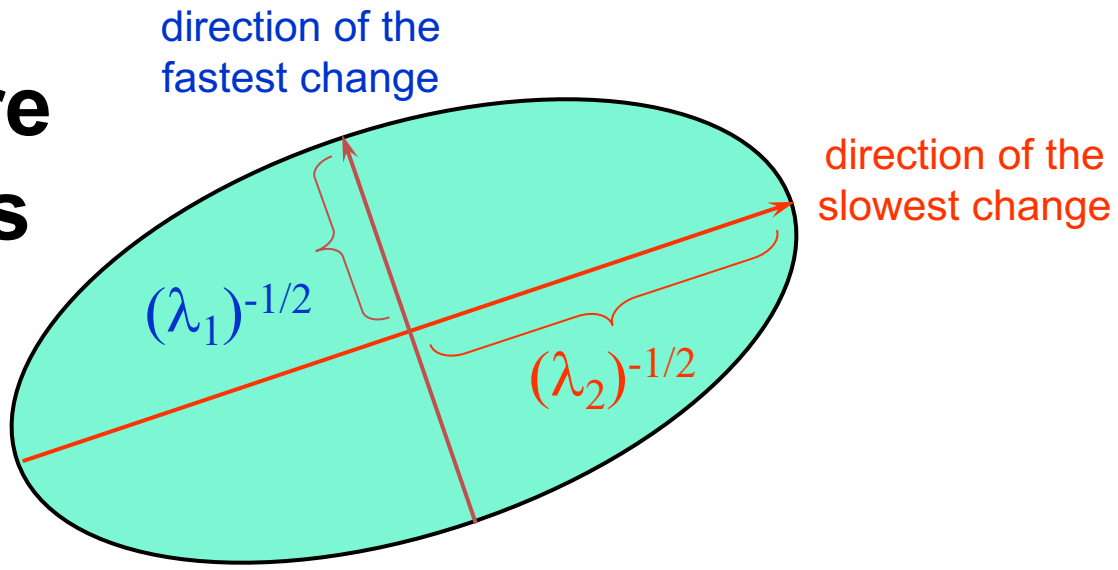
Eigenvalue Decomposition

Review: Quadratic Forms

We can look at the shape of this ellipse by decomposing a symmetric matrix M into a rotation + scaling

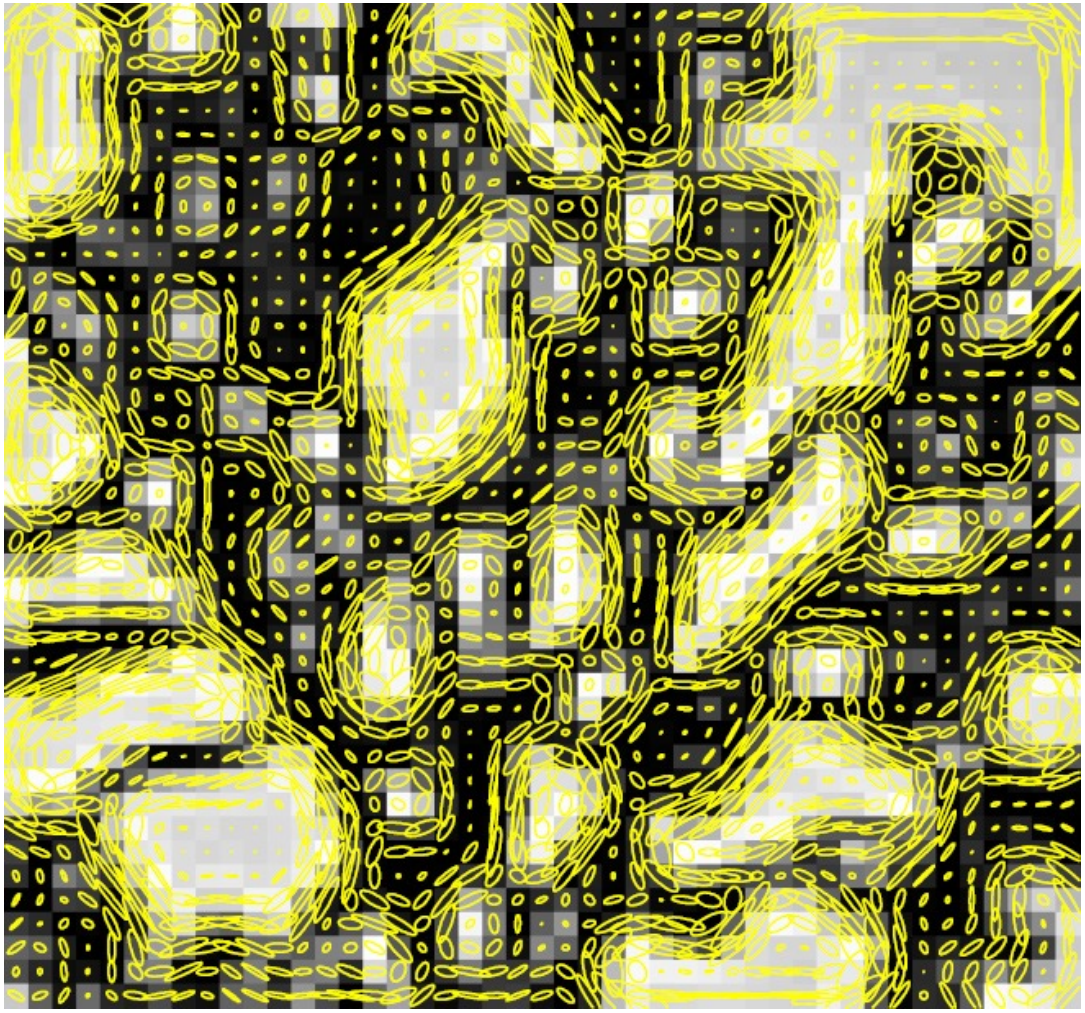
$$\boldsymbol{M} = \boldsymbol{V}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$$

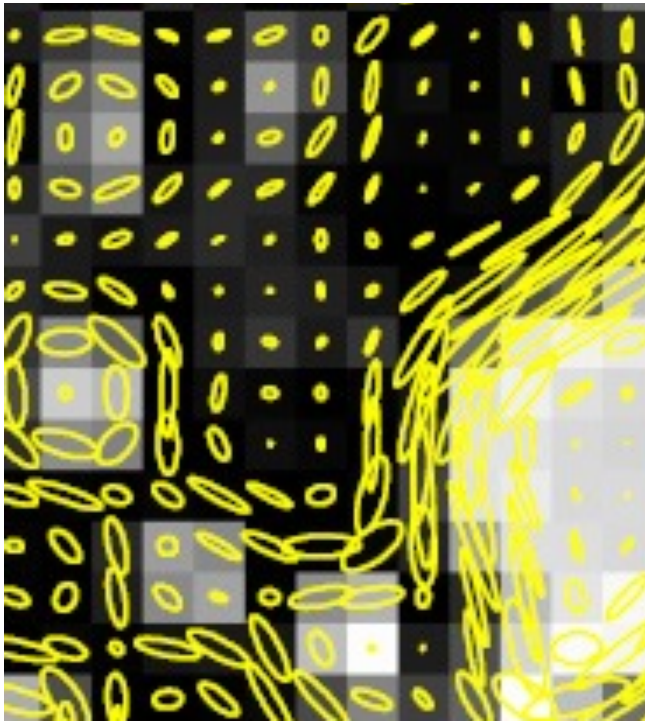
λ_1 and λ_2 are eigenvalues



Slide credit: S. Lazebnik

Visualizing M





Technical note: M is often best *visualized* by first taking inverse, so long edge of ellipse goes along edge

Slide credit: S. Lazebnik

So What Now?

Can calculate M at pixel, by summing nearby gradients, but need to find a and b.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

Given **M**, can decompose it into eigenvectors **V** and eigenvalues λ_1, λ_2 with $\mathbf{M} = \boldsymbol{V}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$.

Rotation direction factored our by V

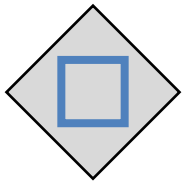
λ_1, λ_2 **only describe magnitude!**

Intuitively what is M?

Can calculate M at pixel, by summing nearby gradients, but need to find λ_1 and λ_2.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$$

λ_1, λ_2 both small: flat



One big, other small: edge

λ_1, λ_2 both big: corner

So What Now?

Can calculate M at pixel, by summing nearby gradients, but need to find λ_1 and λ_2.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$$

Given **M**, can decompose it into eigenvectors **V** and eigenvalues λ_1, λ_2 with $\mathbf{M} = \boldsymbol{V}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$.

Slow. Why?

So What Now?

Can calculate M at pixel, by summing nearby gradients, but need to find a and b.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V^{-1}} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \boldsymbol{V}$$

Instead: compute quantity R from **M**

$$R = \det(\boldsymbol{M}) - \alpha \operatorname{trace}(\boldsymbol{M})^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

Easy fast formula for 2x2

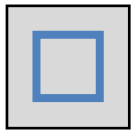
Fast – sum the diagonal

Empirical value, usually 0.04-0.06

So What Now?

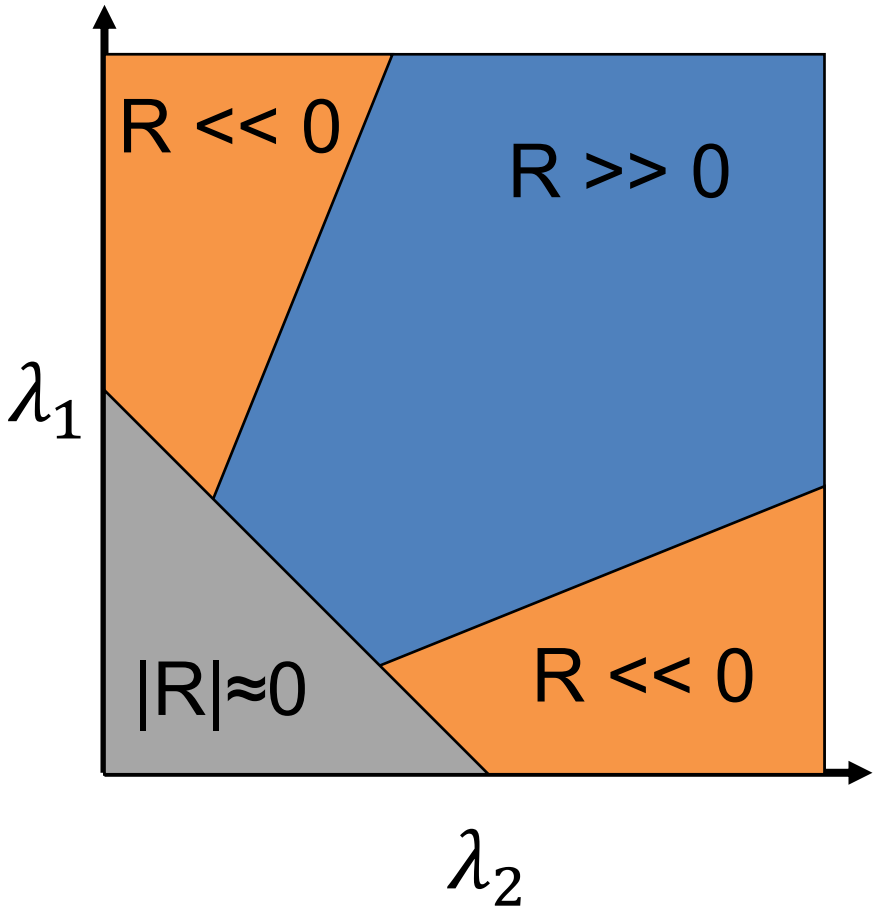
R tells us whether we're at a corner, edge, or flat

$$R = \det(\boldsymbol{M}) - \alpha \operatorname{trace}(\boldsymbol{M})^2 = \lambda_1\lambda_2 - \alpha\,(\lambda_1 + \lambda_2)^2$$

flat  $\lambda_1, \lambda_2 \approx 0$

edge  $\lambda_1 \gg \lambda_2 \gg 0$
$\lambda_2 \gg \lambda_1 \gg 0$

corner  $\lambda_1 \approx \lambda_2 \gg 0$



Remake of standard diagram from S. Lazebnik from original Harris paper.

What Do I Need To Know?

- Need to be able to take derivatives of image
- Need to be able to compute the entries of **M** at every pixel.
- Should know that some properties of **M** indicate whether a pixel is a corner or not.

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix}$$

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} w(x,y) I_x^2 & \sum_{x,y \in W} w(x,y) I_x I_y \\ \sum_{x,y \in W} w(x,y) I_x I_y & \sum_{x,y \in W} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Slide credit: S. Lazebnik

In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R

$$R = \det(\boldsymbol{M}) - \alpha\, trace(\boldsymbol{M})^2$$
$$= \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

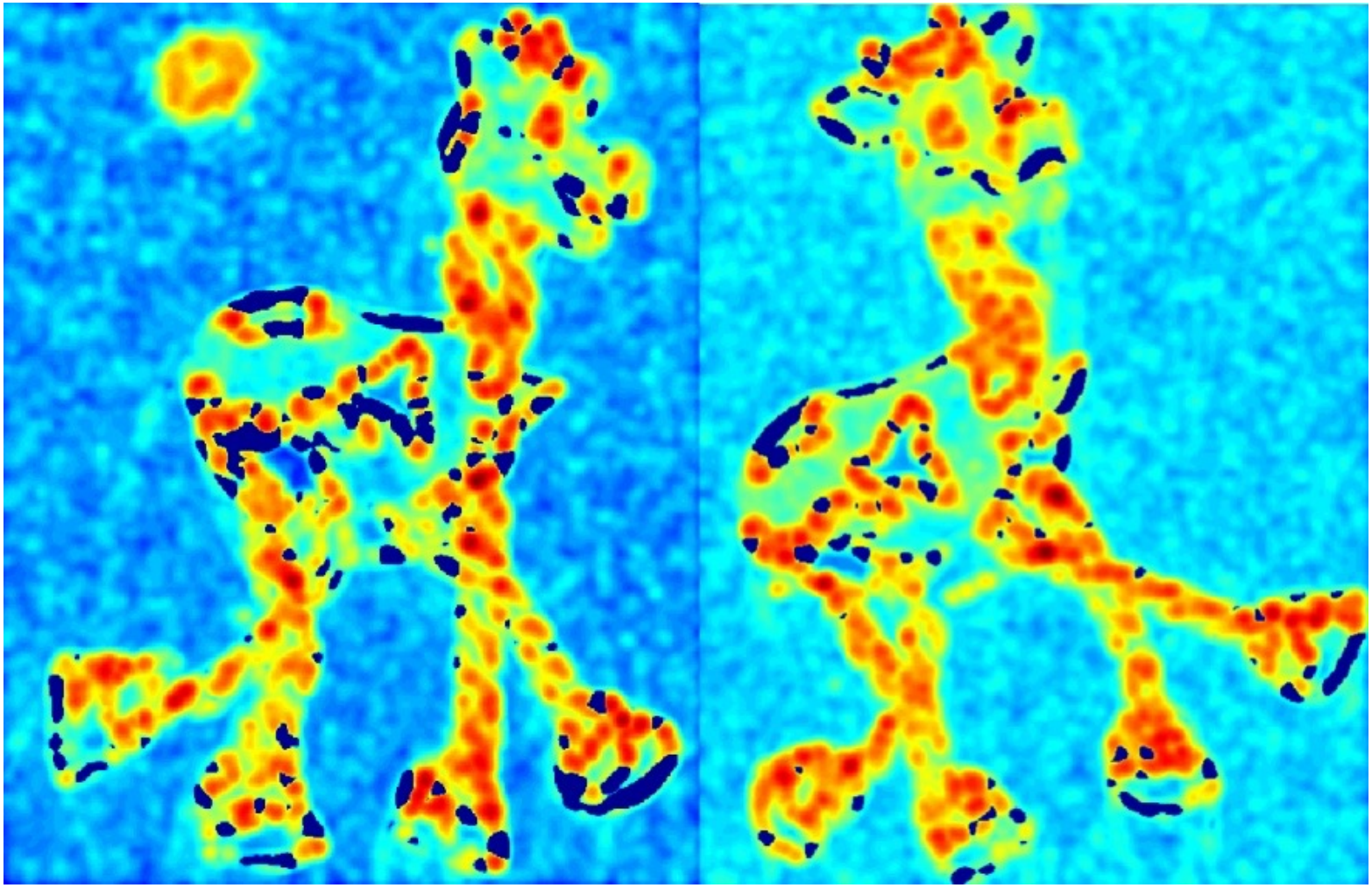
C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Slide credit: S. Lazebnik

Computing R

Slide credit: S. Lazebnik

Computing R



Slide credit: S. Lazebnik

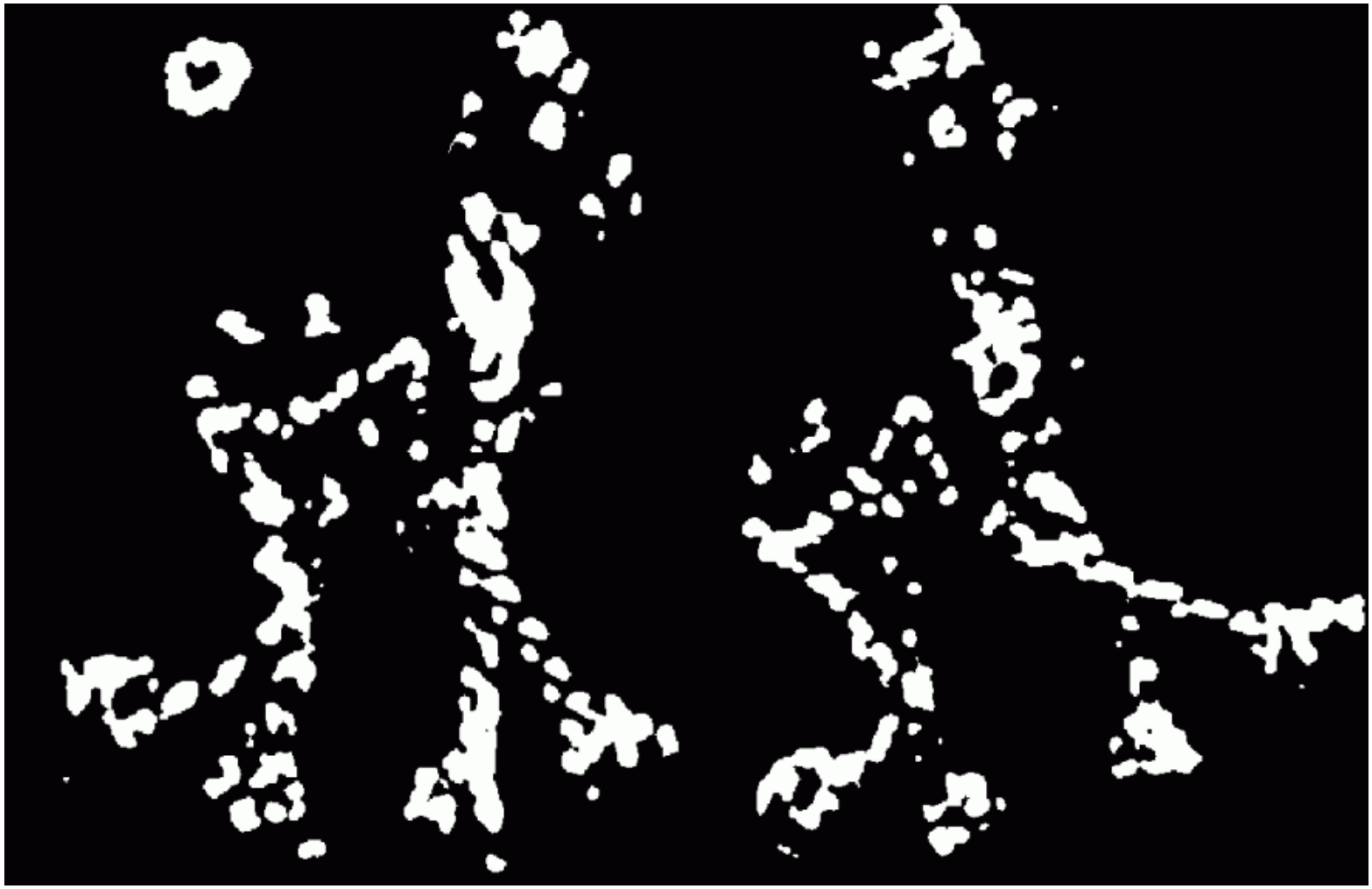
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded R



Slide credit: S. Lazebnik

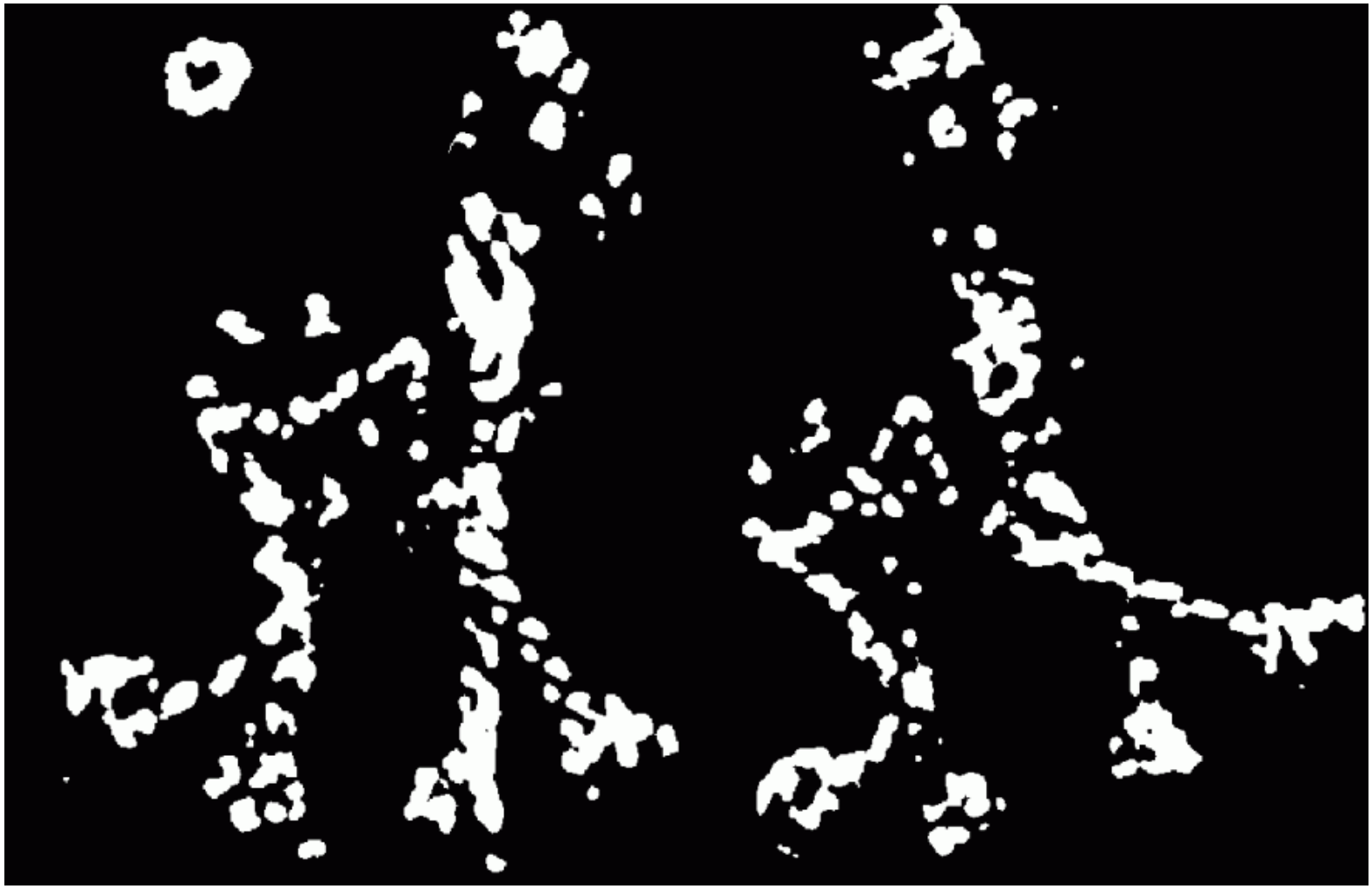
In Practice

1. Compute partial derivatives Ix, Iy per pixel
2. Compute **M** at each pixel, using Gaussian weighting w
3. Compute response function R
4. Threshold R
5. Take only local maxima (called non-maxima suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

Slide credit: S. Lazebnik

Thresholded, NMS R



Slide credit: S. Lazebnik

Final Results

Slide credit: S. Lazebnik

Desirable Properties

If our detectors are repeatable, they should be:

- **Invariant** to some things: image is transformed and corners remain the same
- **Covariant/equivariant** with some things: image is transformed and corners transform with it.

Slide credit: S. Lazebnik

Recall Motivating Problem

Images may be different in lighting and geometry

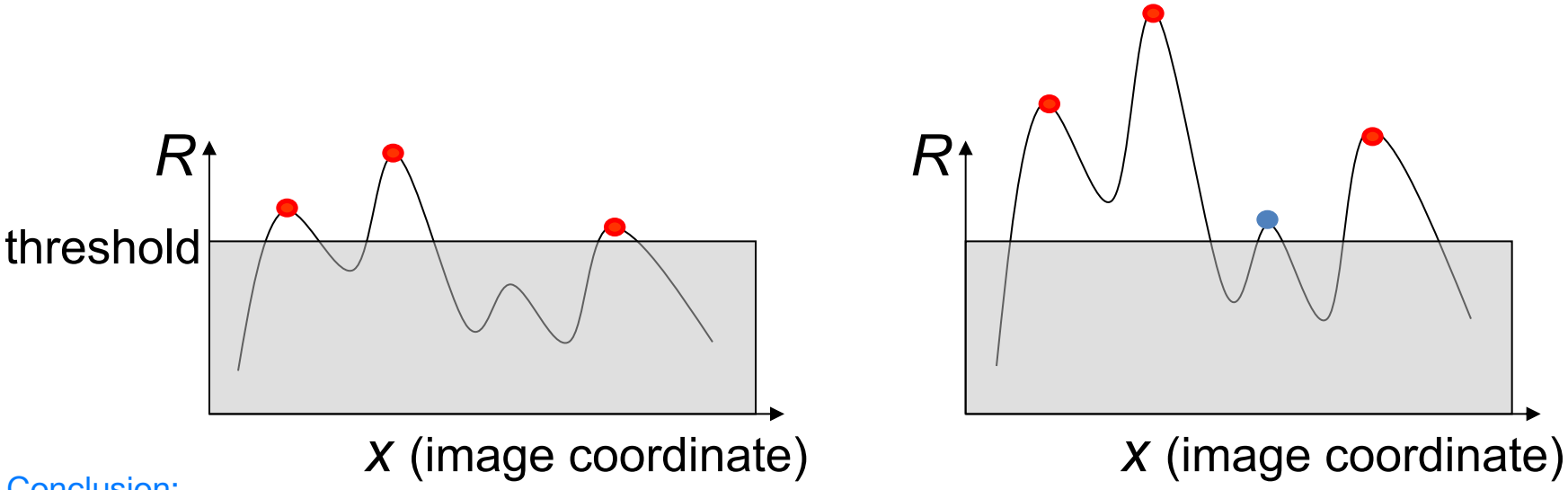
These M matrix is resilient to intensity changes, mostly because these peaks are preserved, but the constant a might pull out some of the peaks above the threshold, and you will have the new corner detected.

Affine Intensity Change

$$I_{new} = aI_{old} + b$$

M only depends on derivatives, so b is irrelevant

But a scales derivatives and there's a threshold

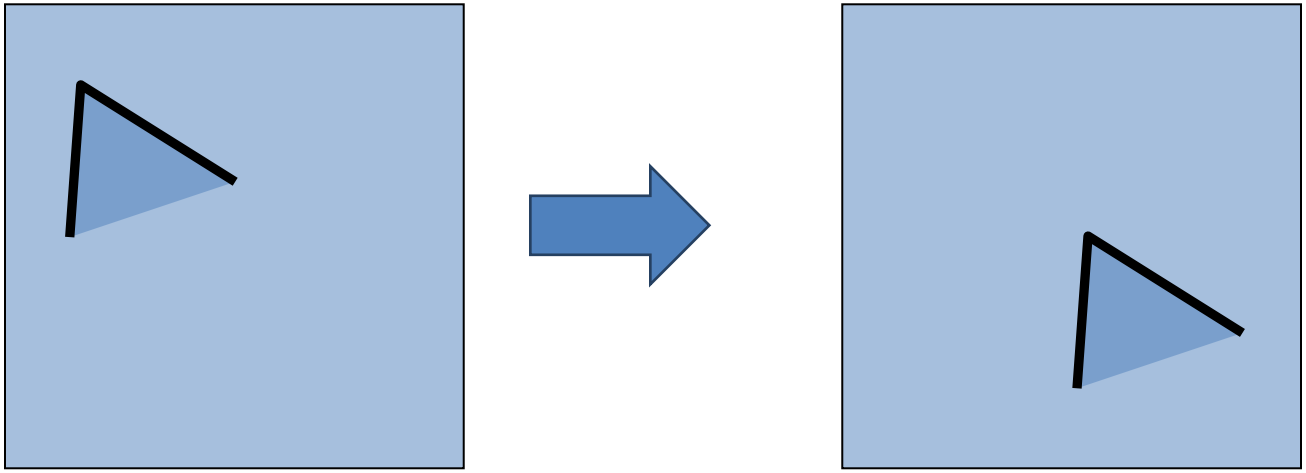


Conclusion:

Partially invariant to affine intensity changes

Slide credit: S. Lazebnik

Image Translation



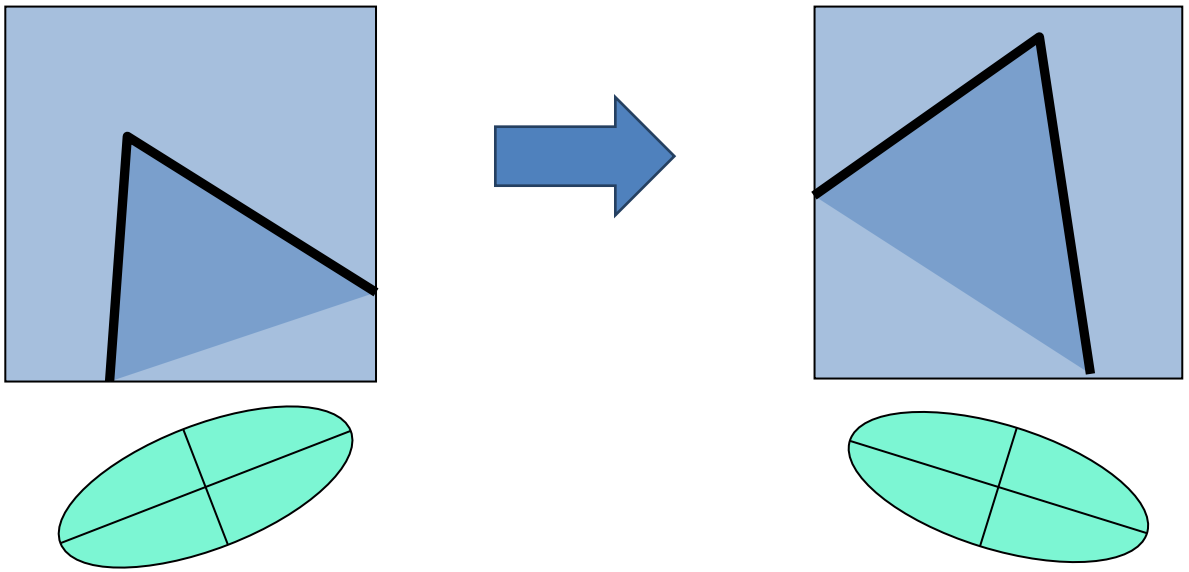
All done with convolution. Convolution is translation invariant.

Equivariant with translation

Slide credit: S. Lazebnik

It will change the eigenvector of M but will not change the eigenvalue of the M. So if you just pick the pixel with the high eight value then you still pick the same corner.

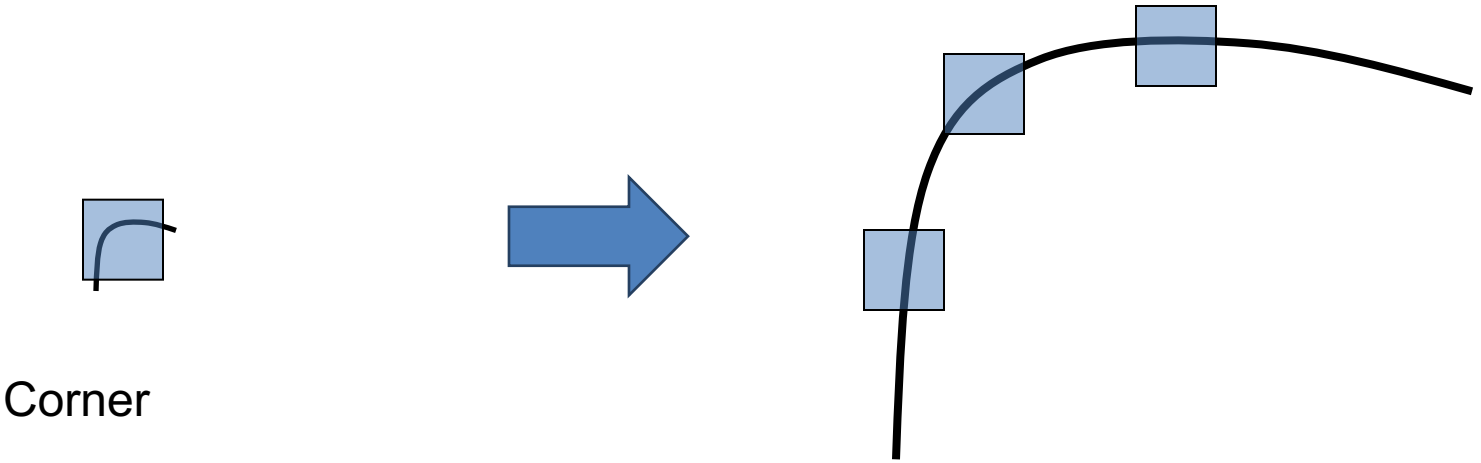
Image Rotation



Rotations just cause the corner rotation to change.
Eigenvalues remain the same.

Equivariant with rotation

Image Scaling



One pixel can become many pixels and vice-versa.

Not equivariant with scaling
Fix: Next Class

For the Curious

Review: Quadratic Forms

Suppose have symmetric matrix **M,** scalar a, vector [u,v]:

$$E([u,v]) = [u,v]\boldsymbol{M}[u,v]^T$$

Then the isocontour / slice-through of E, i.e.

$$E([u,v]) = a$$

is an ellipse.

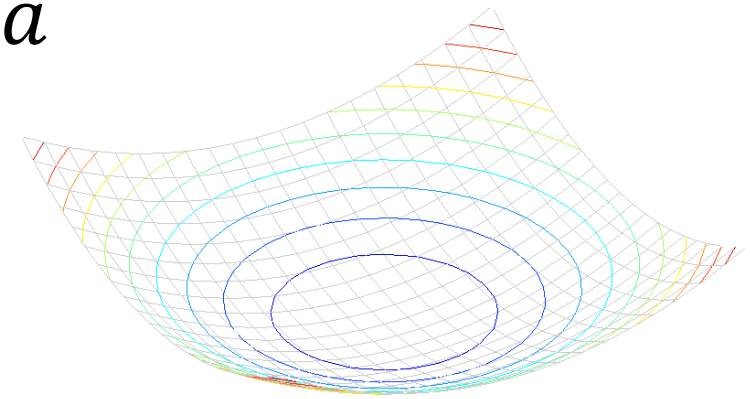


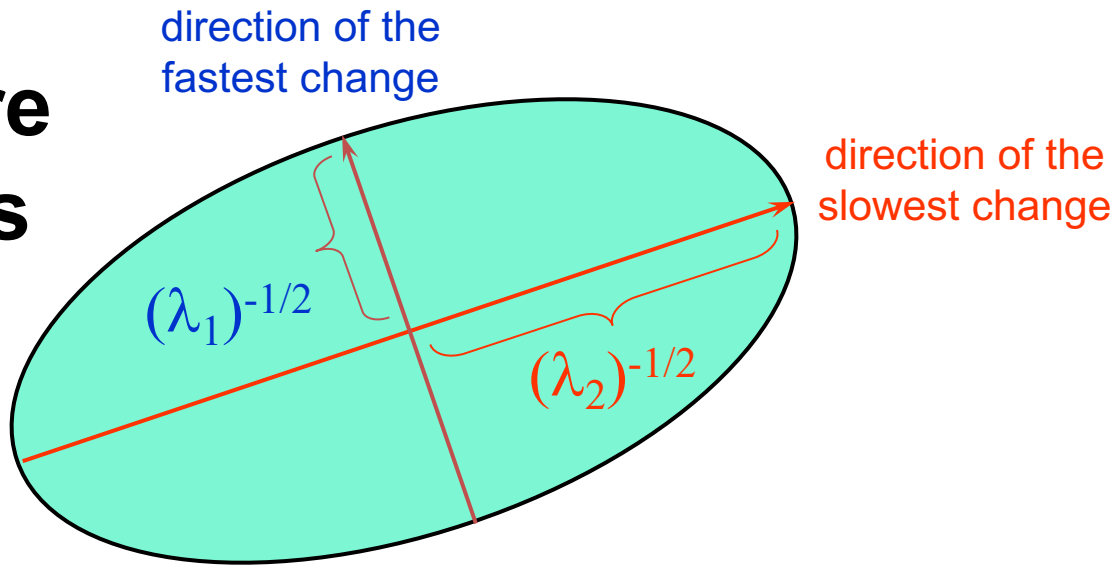
Diagram credit: S. Lazebnik

Review: Quadratic Forms

We can look at the shape of this ellipse by decomposing M into a rotation + scaling

$$\boldsymbol{M} = \boldsymbol{V}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$$

λ_1 and λ_2 are eigenvalues



Slide credit: S. Lazebnik

Interpreting The Matrix M

The second moment matrix tells us how quickly the image changes and in which directions.

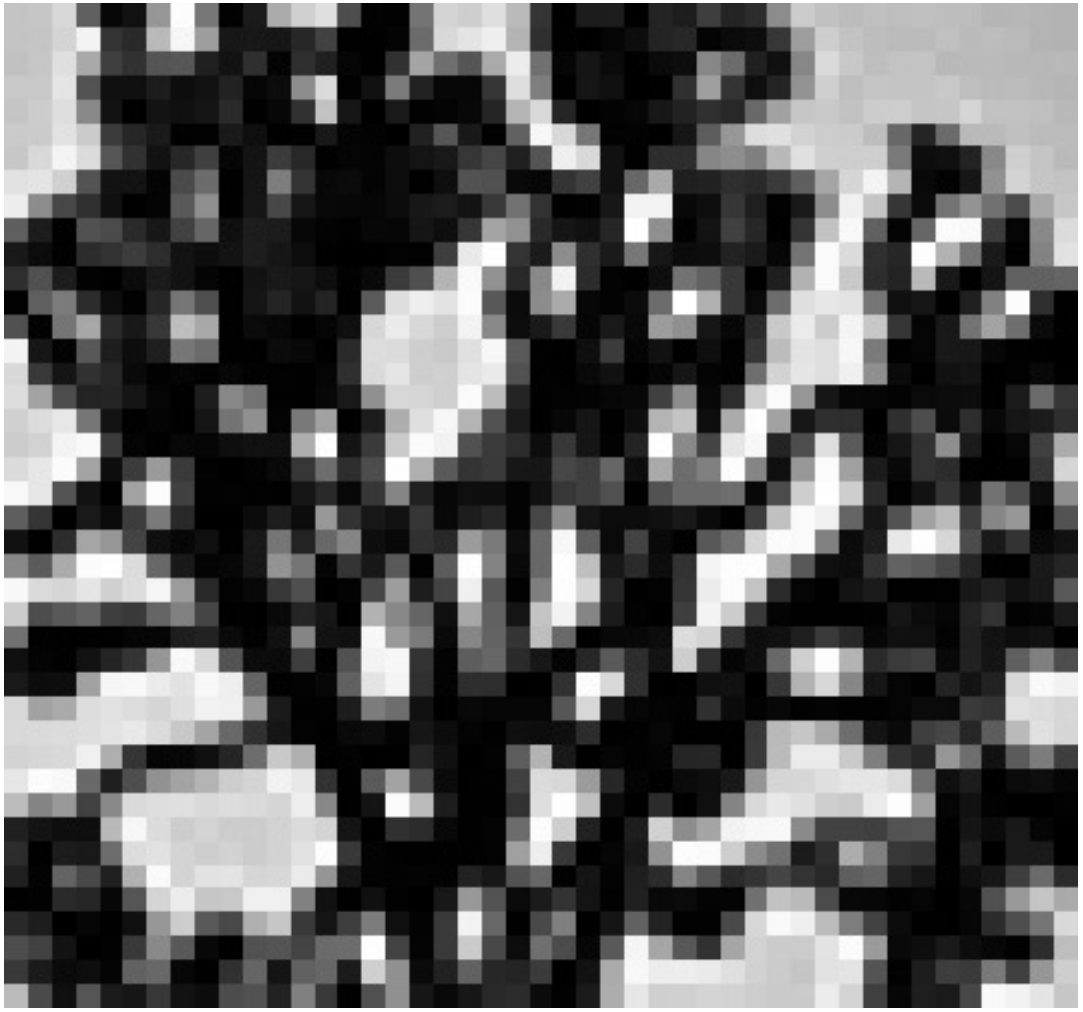
Can compute at each pixel

$$\boldsymbol{M} = \begin{bmatrix} \sum_{x,y \in W} I_x^2 & \sum_{x,y \in W} I_x I_y \\ \sum_{x,y \in W} I_x I_y & \sum_{x,y \in W} I_y^2 \end{bmatrix} = \boldsymbol{V}^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \boldsymbol{V}$$

Directions

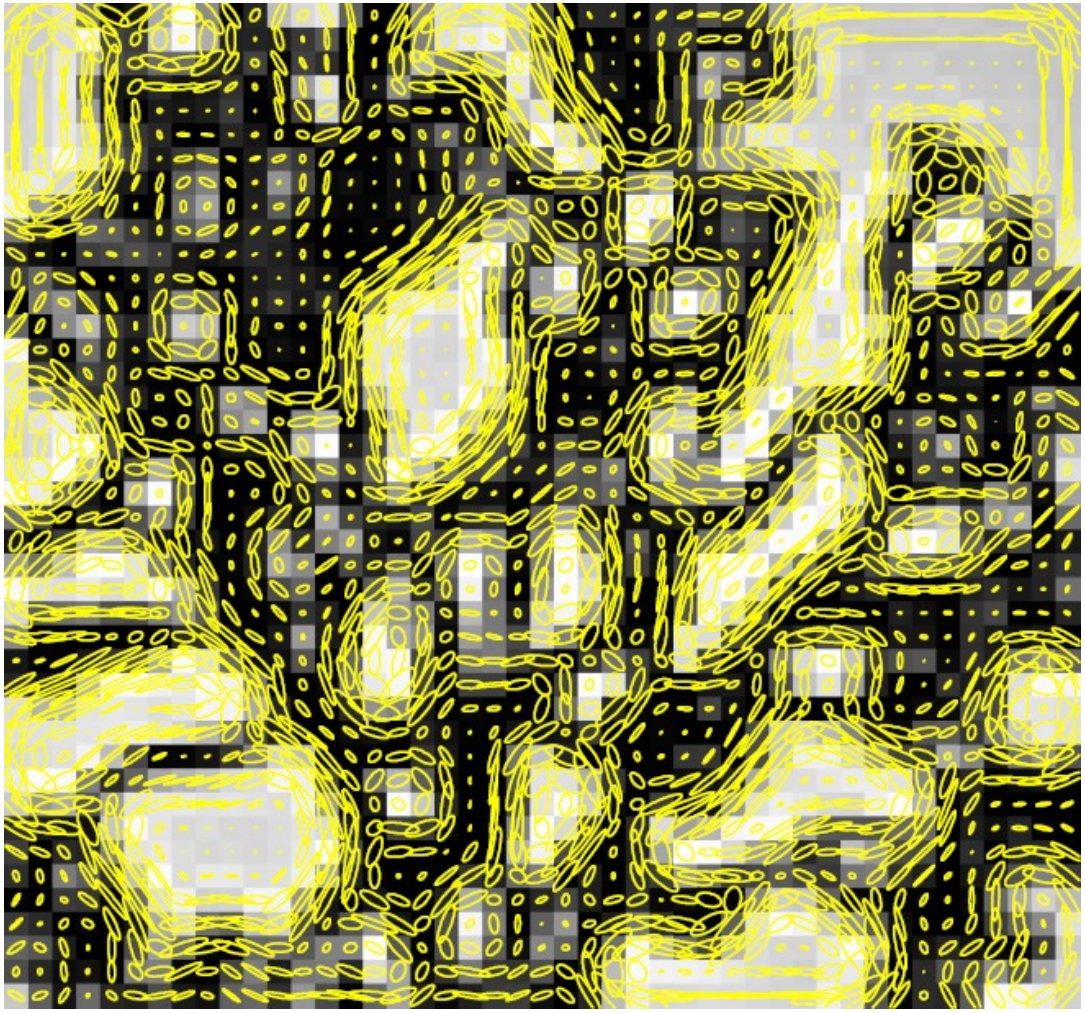
Amounts

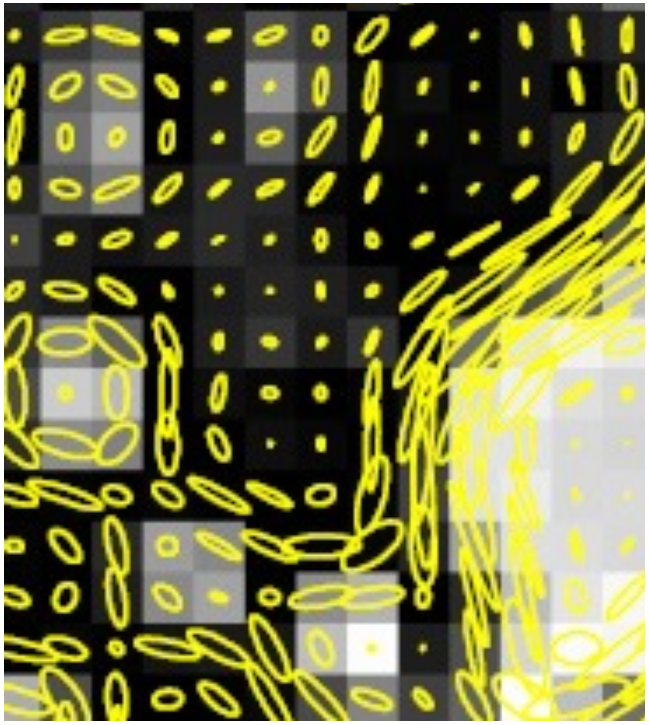
Visualizing M



Slide credit: S. Lazebnik

Visualizing M

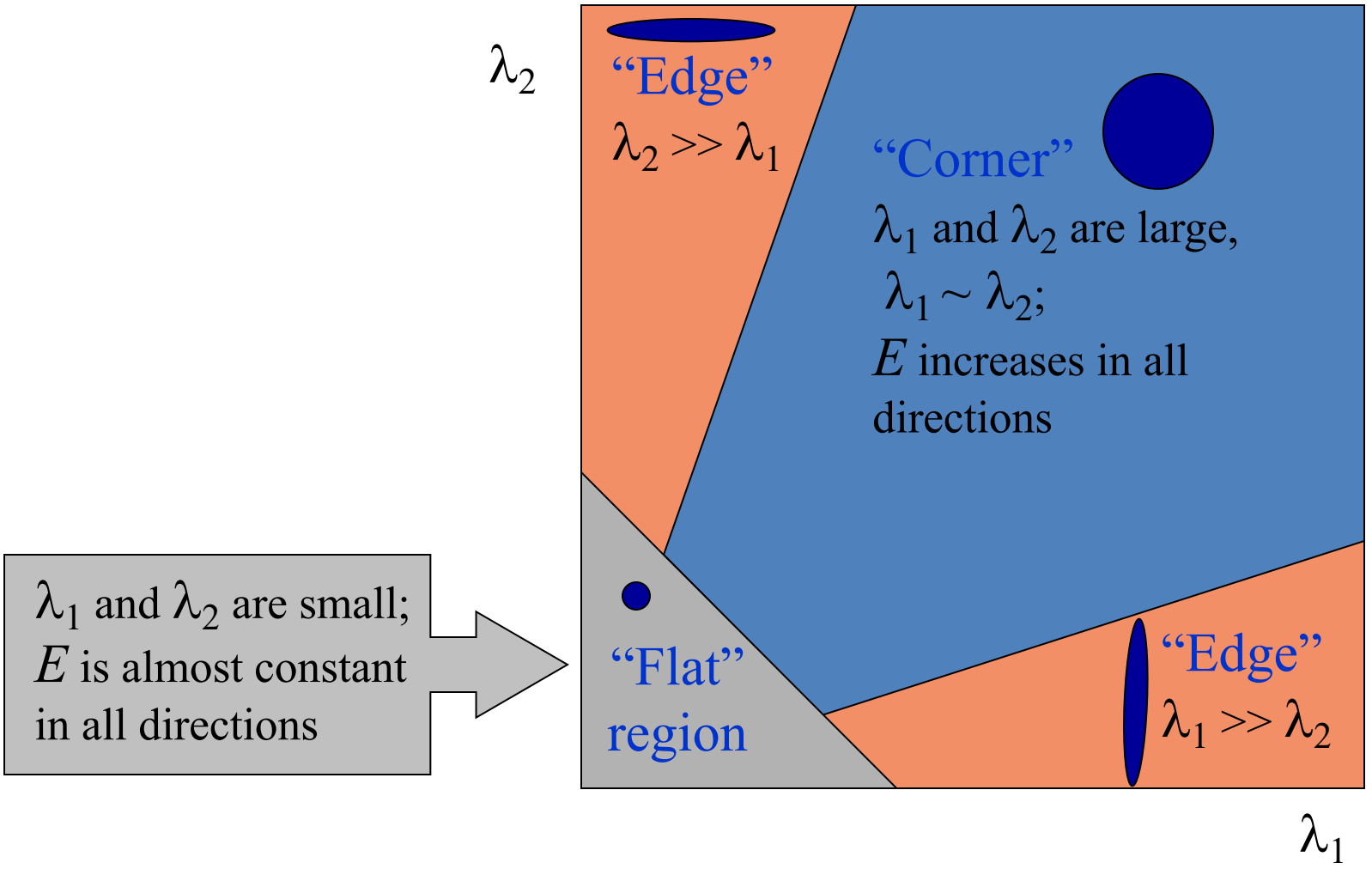




Technical note: M is often best *visualized* by first taking inverse, so long edge of ellipse goes along edge

Slide credit: S. Lazebnik

Interpreting Eigenvalues of M

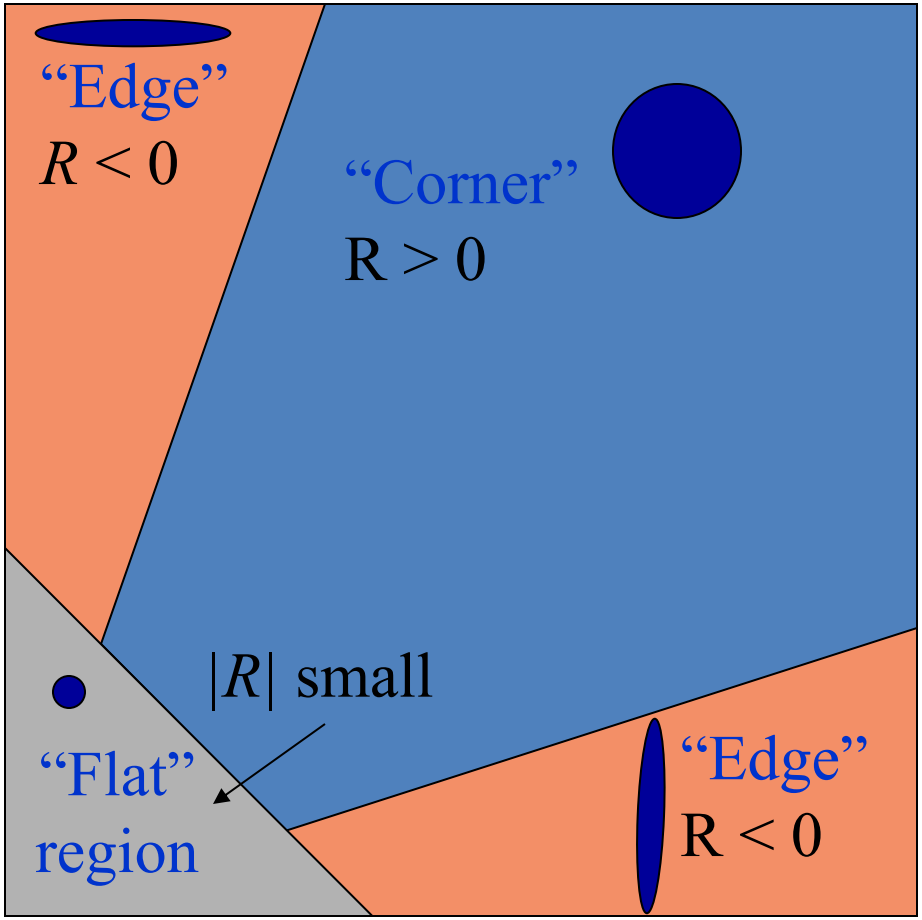


Slide credit: S. Lazebnik; Note: this refers to previous ellipses, not original M ellipse. Other slides on the internet may vary

Putting Together The Eigenvalues

$$R = \det(\boldsymbol{M}) - \alpha\, trace(\boldsymbol{M})^2$$
$$= \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Slide credit: S. Lazebnik; Note: this refers to previous ellipses, not original M ellipse. Other slides on the internet may vary

Corners

Slide Credit: S. Lazebnik

Derivatives Review

Given quadratic function f(x)

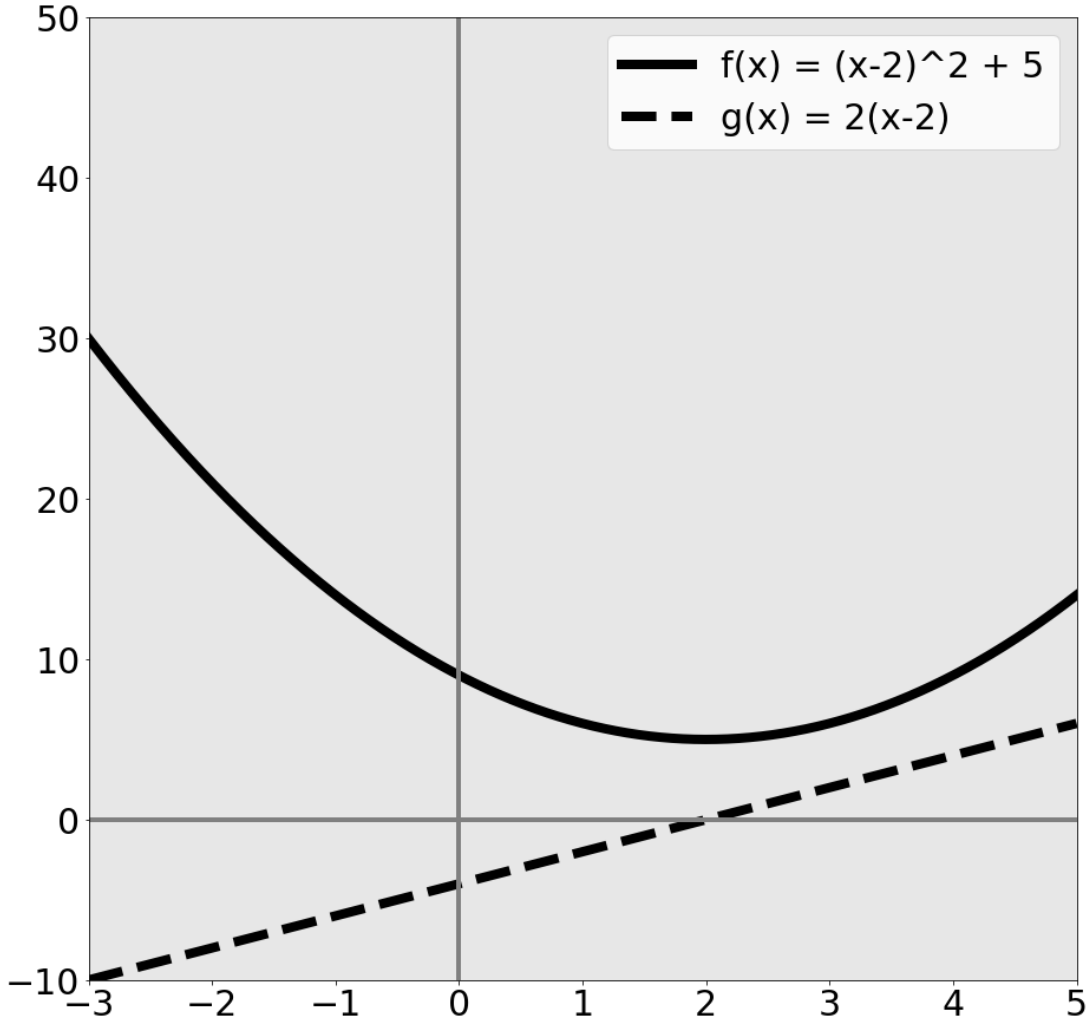
$$f(x) = (x-2)^2 + 5$$

$f(x)$ is function

$$g(x) = f'(x)$$

aka

$$g(x) = \frac{d}{dx} f(x)$$



Given quadratic function f(x)

$$f(x) = (x-2)^2 + 5$$

What's special about x=2?

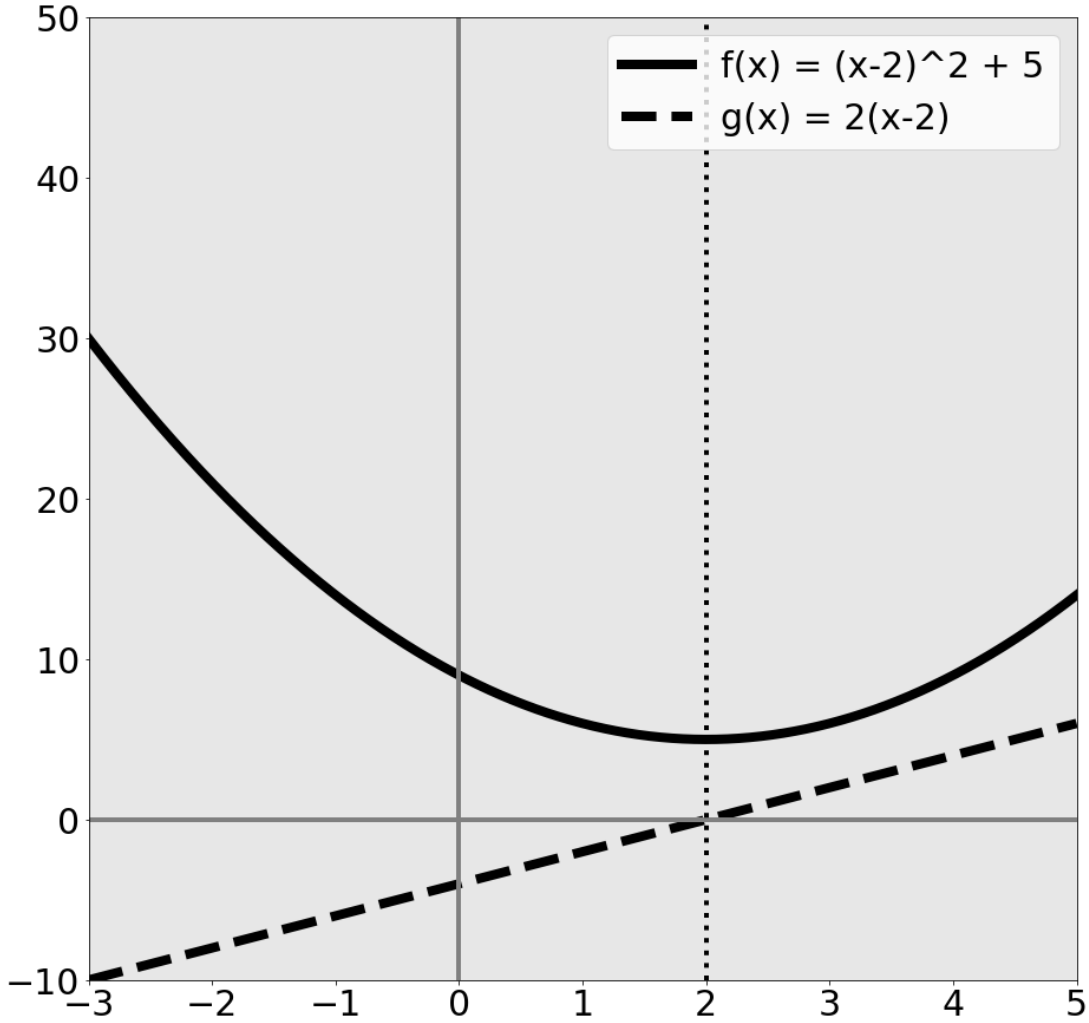
$f(x)$ minim. at 2

$g(x) = 0$ at 2

a = minimum of f $\rightarrow$

$g(a) = 0$

Reverse is ***not true***



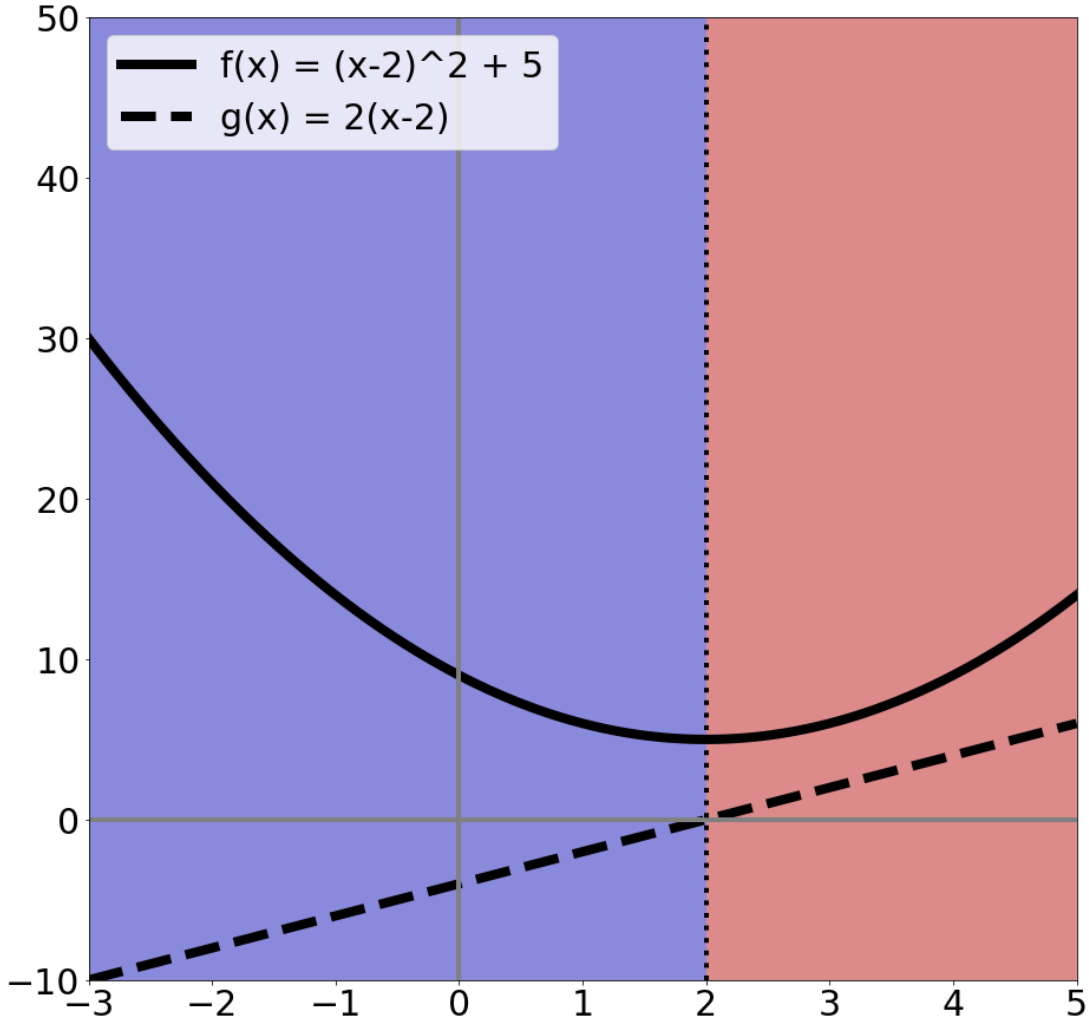
Rates of change

$$f(x) = (x-2)^2 + 5$$

Suppose I want to increase f(x) by changing x:

Blue area: move left
Red area: move right

Derivative tells you direction of ascent and rate



What Calculus Should I Know

- Really need intuition
- Need chain rule
- Rest you should look up / use a computer algebra system / use a cookbook
- Partial derivatives (and that's it from multivariable calculus)

Partial Derivatives

- Pretend other variables are constant, take a derivative. That's it.
- Make our function a function of two variables

$$f(x) = (x-2)^2 + 5$$

$$\frac{\partial}{\partial x} f(x) = 2(x-2) * 1 = 2(x-2)$$

$$f_2(x,y) = (x-2)^2 + 5 + (y+1)^2$$

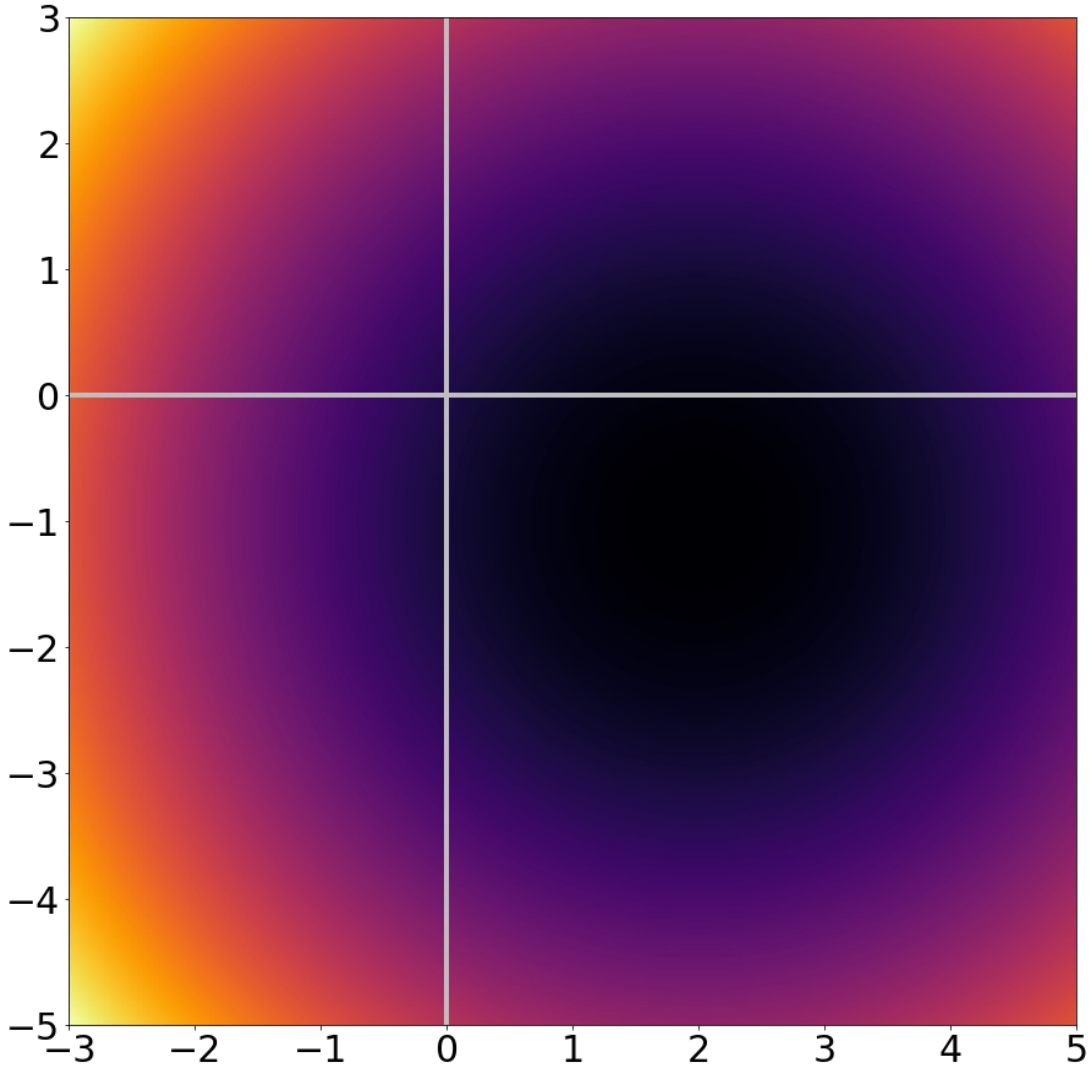
Pretend it's constant → derivative = 0

$$\frac{\partial}{\partial x} f_2(x) = 2(x-2)$$

Zooming Out

$$f_2(x, y) = (x - 2)^2 + 5 + (y + 1)^2$$

Dark = f(x,y) low
Bright = f(x,y) high



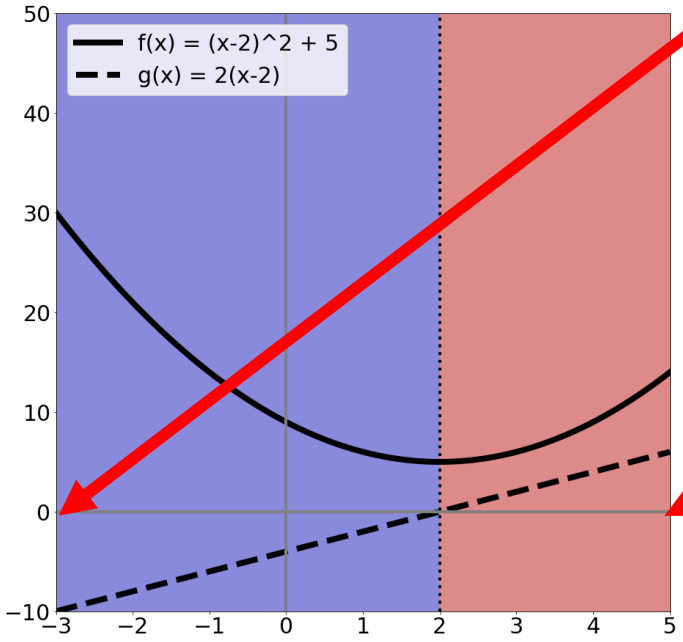
Taking a slice of

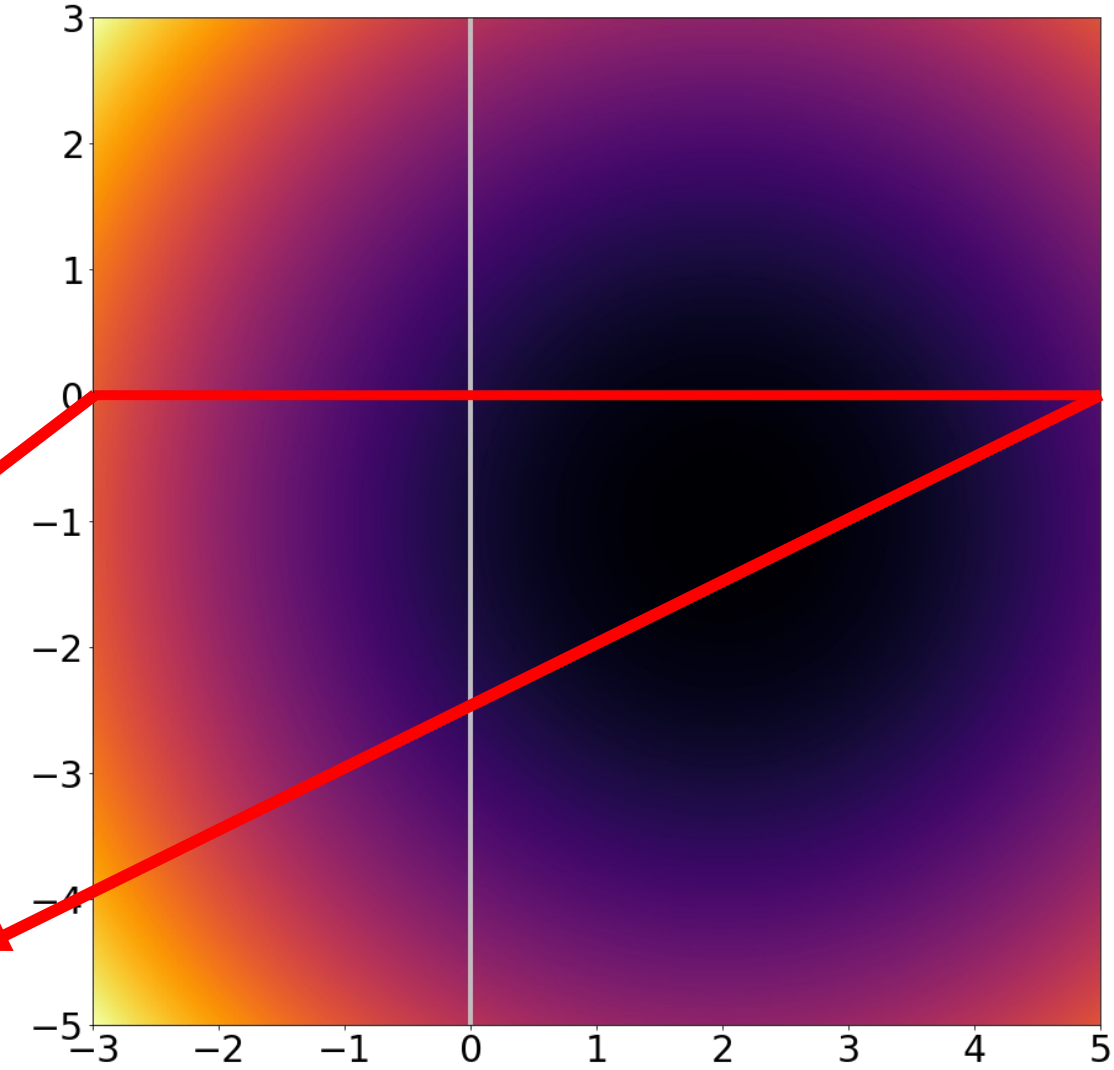
$$f_2(x, y) = (x-2)^2 + 5 + (y+1)^2$$

Slice of y=0 is the function from before:

$$f(x) = (x-2)^2 + 5$$

$$f'(x) = 2(x-2)$$

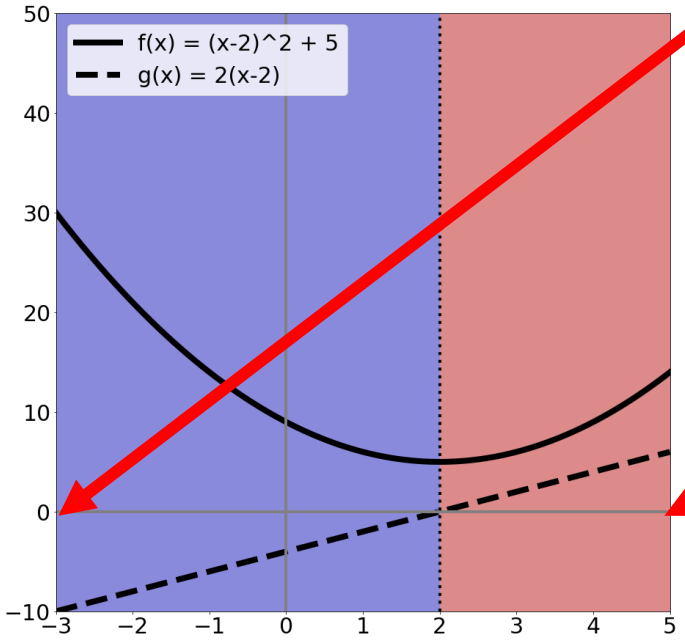


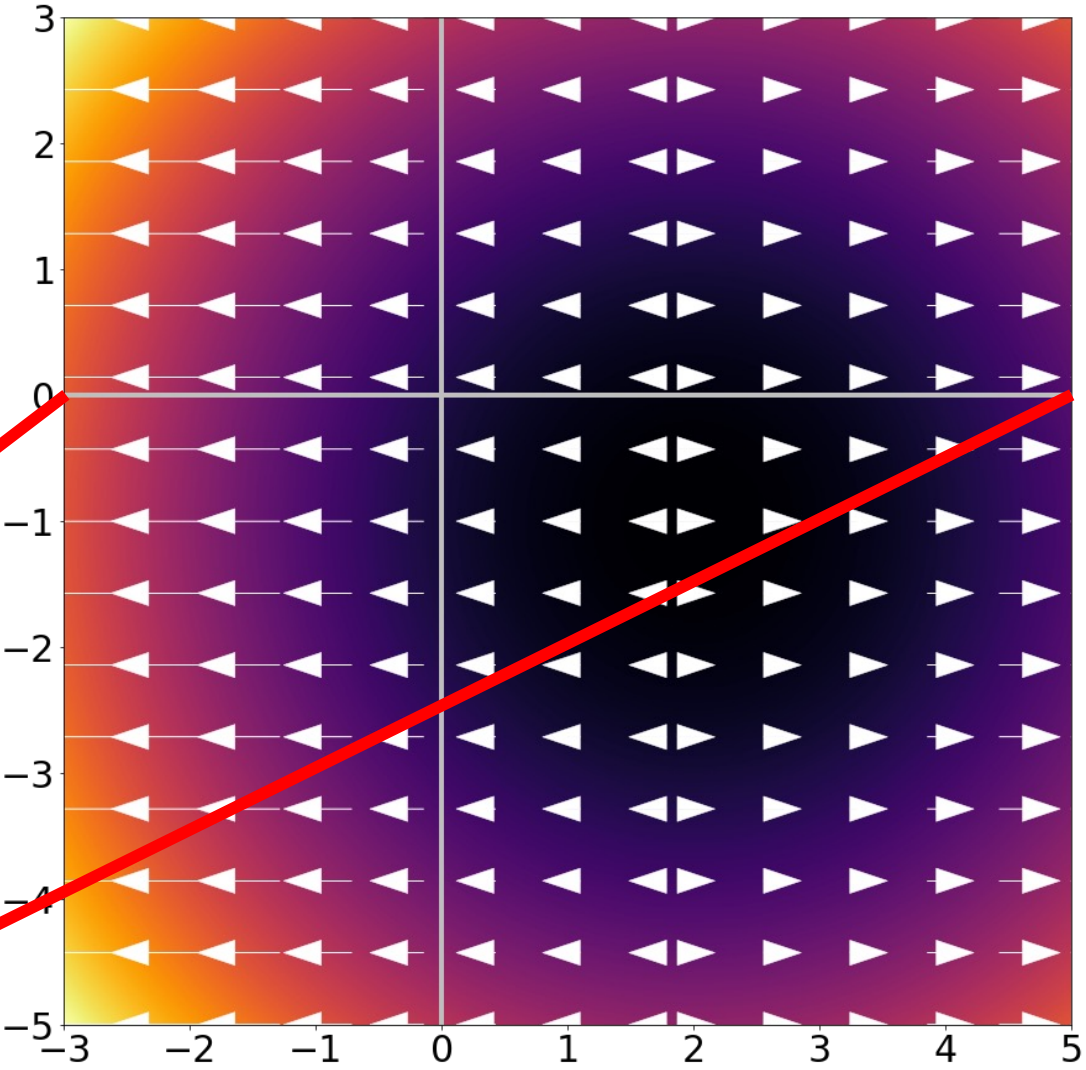


Taking a slice of

$$f_2(x, y) = (x - 2)^2 + 5 + (y + 1)^2$$

$\frac{\partial}{\partial x} f_2(x, y)$ is rate of change & direction in x dimension





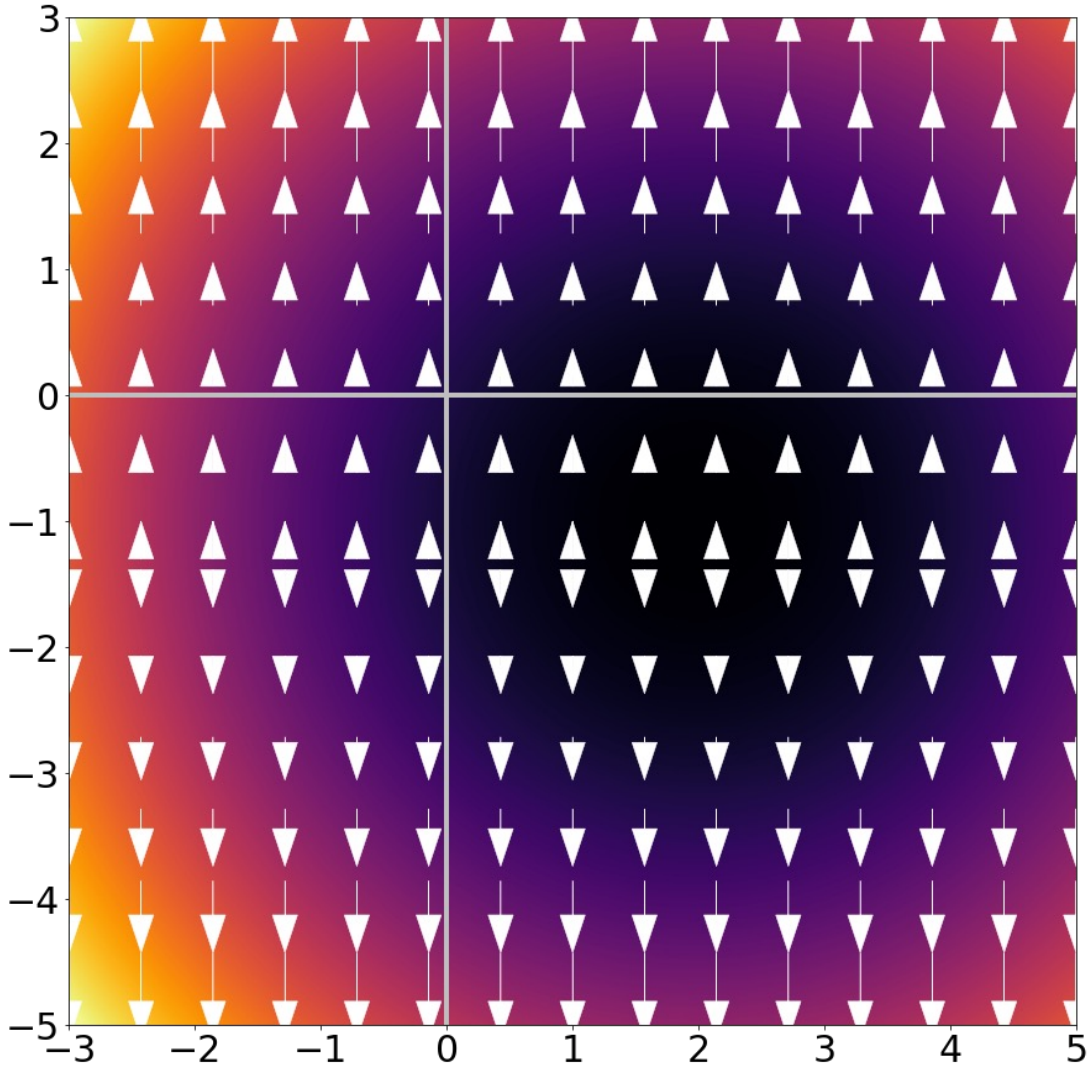
Zooming Out

$$f_2(x, y) = (x - 2)^2 + 5 + (y + 1)^2$$

$\frac{\partial}{\partial y} f_2(x, y)$ is

$2(y + 1)$

and is the rate of change & direction in y dimension



Zooming Out

$$f_2(x, y) = (x-2)^2 + 5 + (y+1)^2$$

Gradient/Jacobian: Making a vector of

$$\nabla_f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

gives rate and direction of change.

Arrows point OUT of minimum / basin.