Cameras

EECS 442 – Jeong Joon Park

Winter 2024, University of Michigan

https://eecs442.github.io/

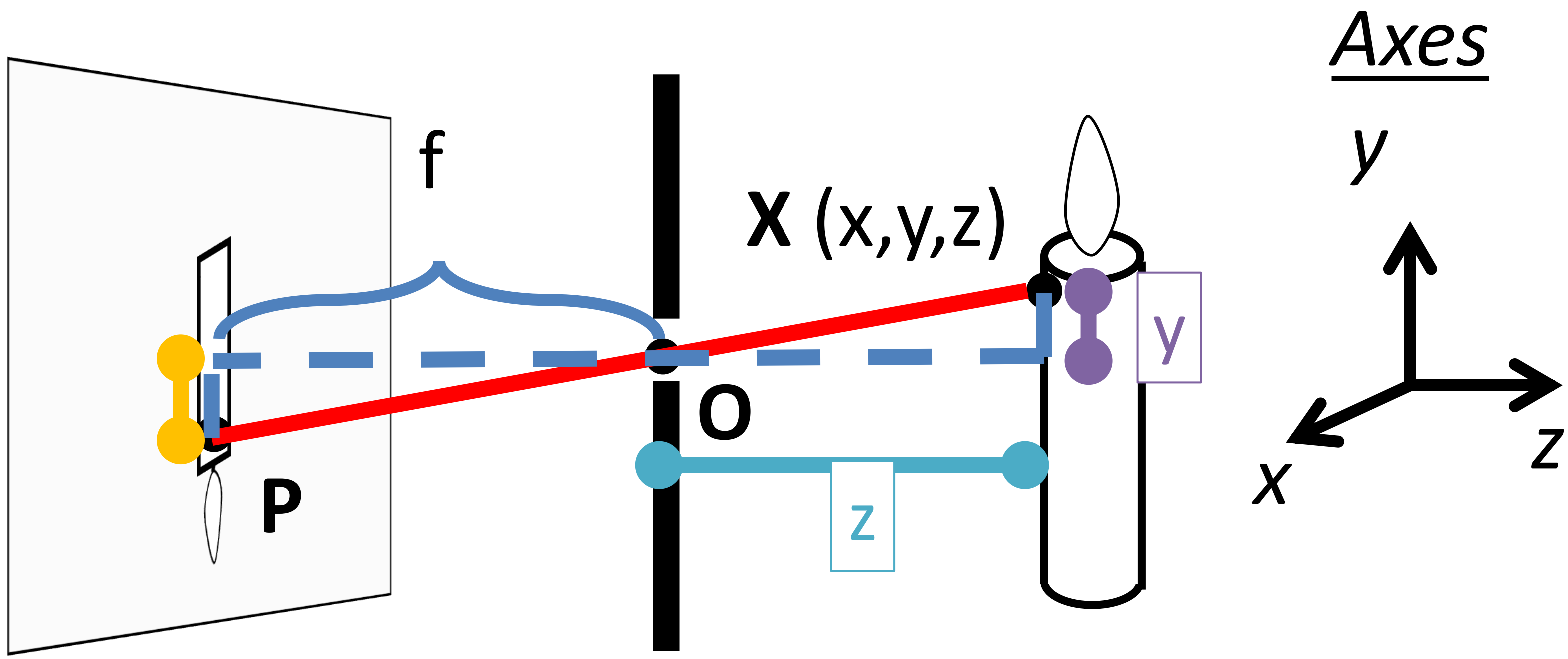
Administrivia

Logistics:

- Discussion sections start today. So please try to attend.

- Office hour schedules can be seen on the class calendar (link found on the course website)
- Lecture note uploaded before the lecture
- If you’re rusty on linear algebra, please look at a linear algebra recap lecture note I’ll post later this week.

Recap: Projection Equations



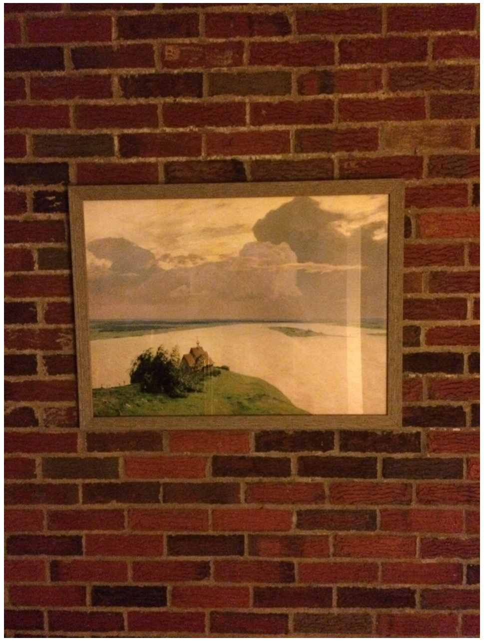
Coordinate system: **O** is origin, XY in image, Z sticks out.

XY is image plane, Z is optical axis.

(x,y,z) projects to (fx/z,fy/z) via similar triangles

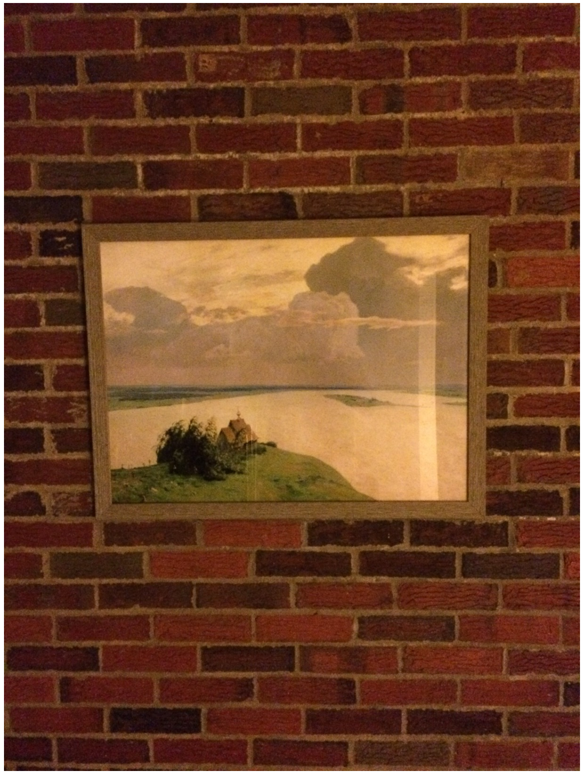
Source: L Lazebnik

Recap: Perspective

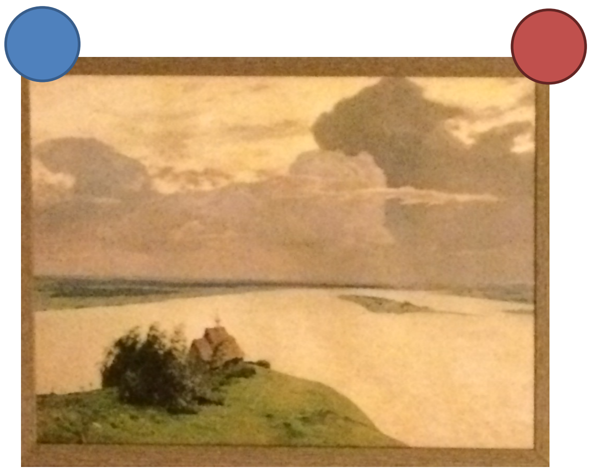
  

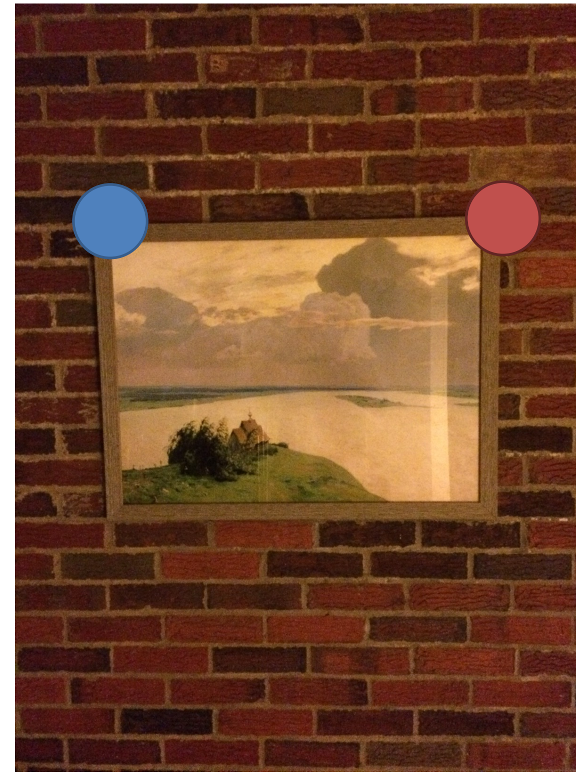
Things looking different when viewed from different angles seems like a nuisance. It's also a cue. **Why?**

Do You Always Get Perspective?



Do You Always Get Perspective?





Y location of blue and red dots in image:

$$\frac{fy}{z_2} \qquad \frac{fy}{z_1} \qquad\qquad \frac{fy}{z} \qquad \frac{fy}{z}$$

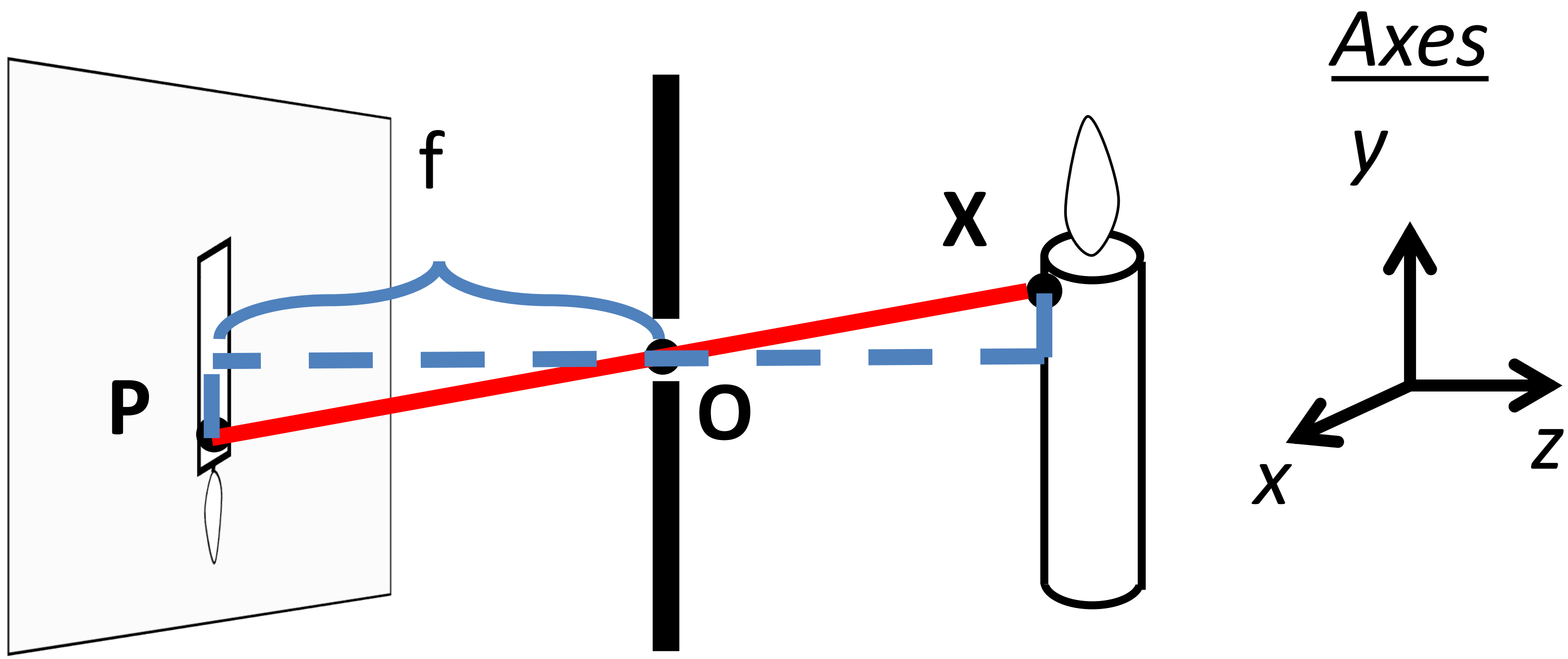
Do You Always Get Perspective?

When plane is fronto-parallel (parallel to camera plane), everything is:

- scaled by f/z
- otherwise is preserved.

Provides us the cue of the view angle!

Projection Equation



$(x,y,z) \rightarrow (fx/z, fy/z)$

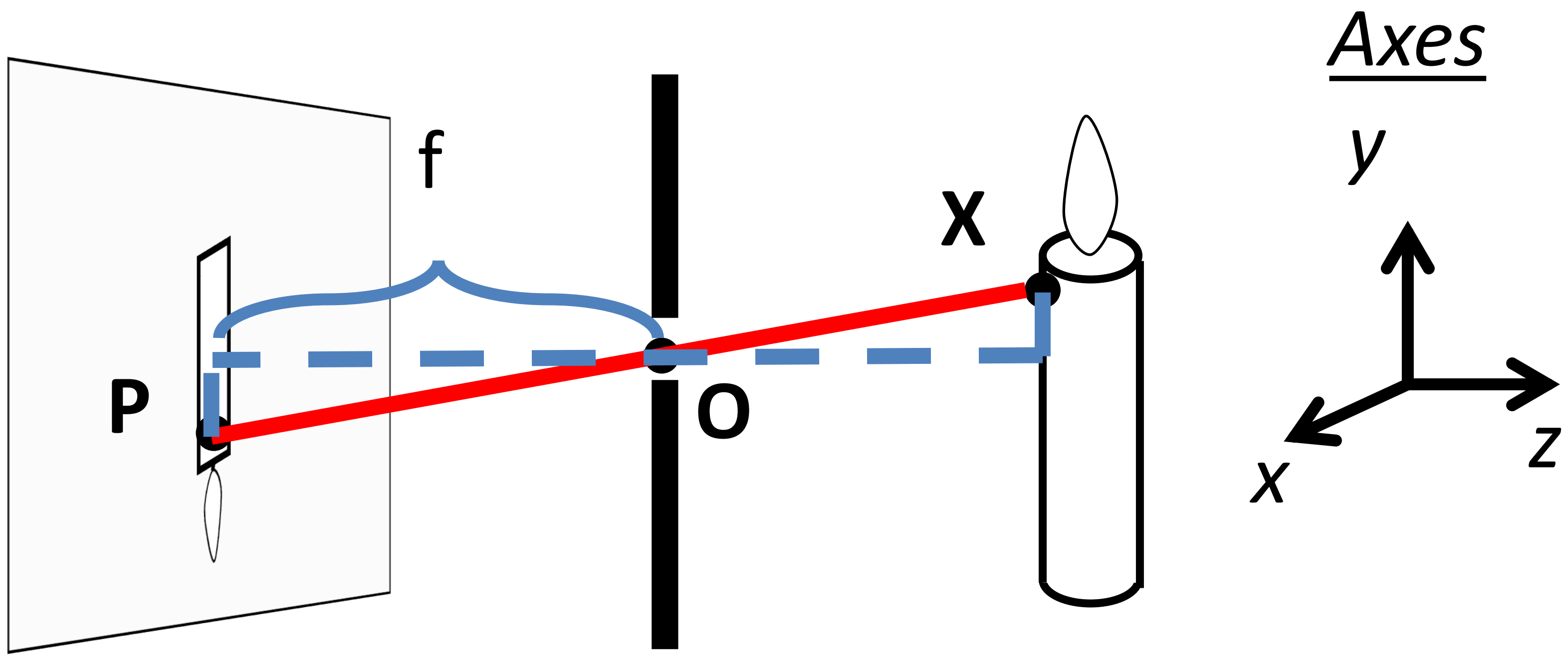
I promised you linear algebra: is this linear?

Nope: division by z is non-linear

(and risks division by 0)

Adapted from S. Seitz slide

Projection Equation



$$(x,y,z) \rightarrow (fx/z, fy/z)$$

No 2x3 matrix satisfy below for any (x,y,z)

$$\begin{bmatrix} fx/z \\ fy/z \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Adapted from S. Seitz slide

Homogeneous Coordinates (2D)

Trick: add a dimension!

This also clears up lots of nasty special cases

Cartesian Point		Homogeneous Point		Cartesian Point
$\begin{bmatrix} x \\ y \end{bmatrix}$	$\longrightarrow$ Concat w=1	$\begin{bmatrix} u \\ v \\ w \end{bmatrix}$	$\longrightarrow$ Divide by w	$\begin{bmatrix} u/w \\ v/w \end{bmatrix}$

What if w = 0?

Infinitely far away point.

Or a direction (not point)

Adapted from M. Hebert slide

Homogeneous Coordinates

Triple / Equivalent

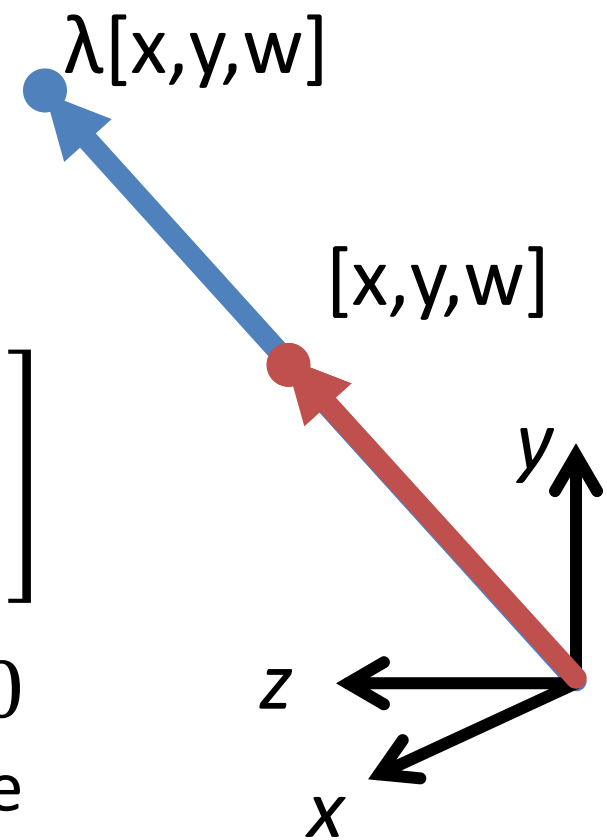
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \boxed{\equiv} \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}$$

$\leftrightarrow$

Double / Equals

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \boxed{=} \lambda \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}$$

$$\lambda \neq 0$$



Two homogeneous coordinates are **equivalent** if they are proportional to each other. **Not = !**

Benefits of Homogeneous Coords

General equation of 2D line:

$$ax + by + c = 0$$

Homogeneous Coordinates

$$\boldsymbol{l}^T \boldsymbol{p} = 0, \qquad \boldsymbol{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \boldsymbol{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

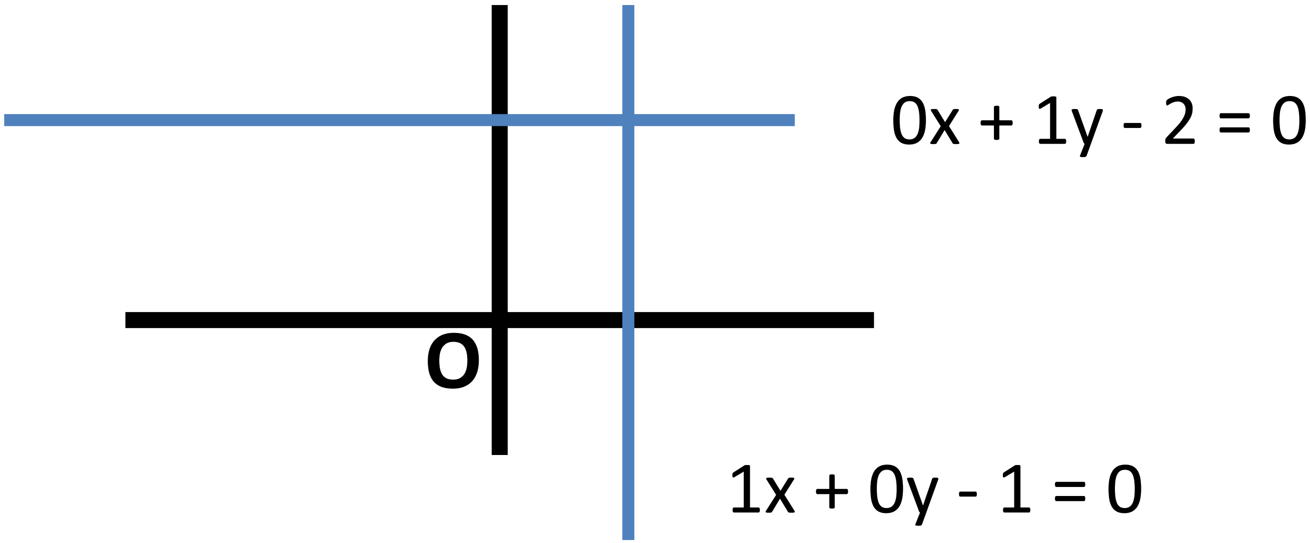
Slide from M. Hebert

Benefits of Homogeneous Coords

- Lines (3D) and points (2D → 3D) are now the same dimension.
- Use the *cross* (x) and *dot product* for:
  - Intersection of lines **l** and **m**: **l** x **m**
  - Line through two points **p** and **q**: **p** x **q**
  - Point **p** on line **l**: $\mathbf{l}^T\mathbf{p}=\mathbf{0}$
- Parallel lines, vertical lines become easy (compared to y=mx+b)

Benefits of Homogeneous Coords

What's the intersection?

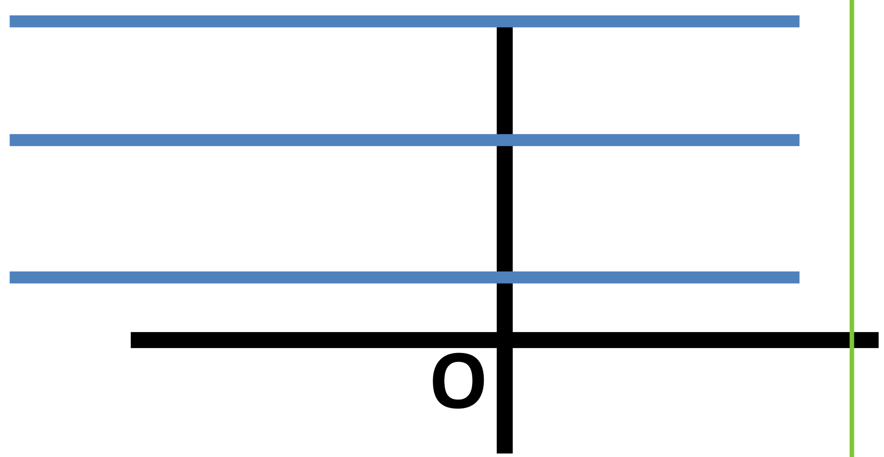


[0,1,-2] x [1,0,-1] = [-1,-2,-1]

Converting back (divide by -1)

(1,2)

Benefits of Homogeneous Coords



$0x + 1y - 3 = 0$

$0x + 1y - 2 = 0$

$0x + 1y - 1 = 0$

Intersection of y=2, y=1

$[0,1,-2] \times [0,1,-1] = [1,0,\boxed{0}]$

means this point is at infinity. it describe a direction rather than a particular point

Do all 3 lines intersect at the same place?

$$l^T p = 0 \quad [0,1,-3]^T[1,0,0] = 0$$

→ Three line intersect at the "same point".

This is a way to show or prove a point is on a line.

可以直接表达出无限远处交会.

Benefits of Homogeneous Coords

这样的linear operation是在Cartesian Coordinate中无法实现的.

Translation is now linear / matrix-multiply

Translate from [u, v] to [u+tx, v+ty]

If w = 1 $$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} u + t_x \\ v + t_y \\ 1 \end{bmatrix}$$

Generically $$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} u + wt_x \\ v + wt_y \\ w \end{bmatrix}$$

Rigid body transforms (rot + trans) now linear

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

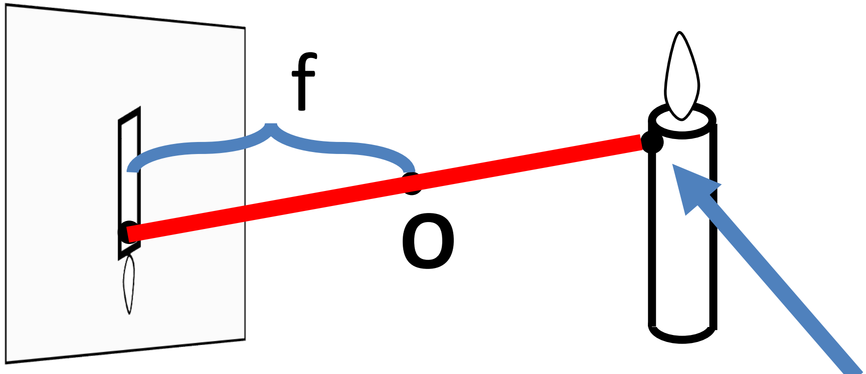
3D Homogeneous Coordinates

Same story: add a coordinate, things are equivalent if they're proportional

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} u \\ v \\ w \\ t \end{bmatrix} \longrightarrow \begin{bmatrix} u/t \\ v/t \\ w/t \end{bmatrix}$$

Projection Matrix

Projection is matrix multiplication

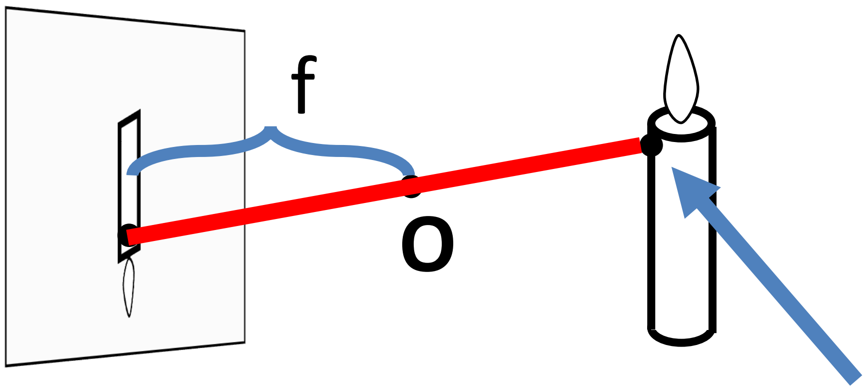


$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide inspired from L. Lazebnik

Projection Matrix

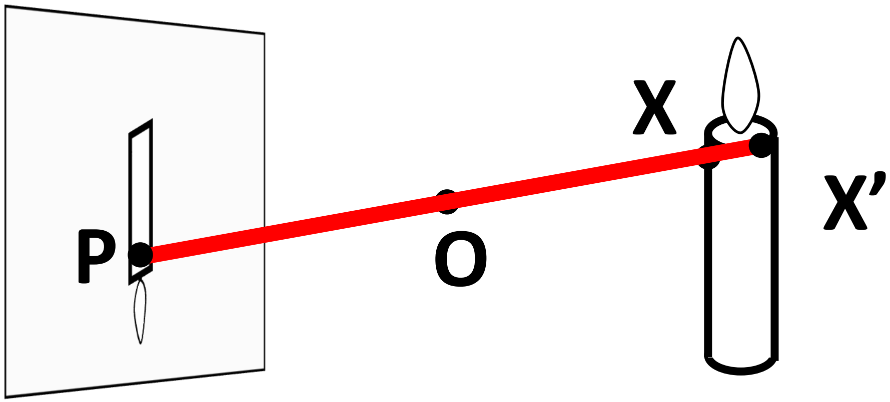
Projection is matrix multiplication



$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \rightarrow \begin{bmatrix} fx/z \\ fy/z \end{bmatrix}$$

Slide inspired from L. Lazebnik

Why ≡ ≠ =



Project X and X' to the image and compare them

YES $\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \equiv \begin{bmatrix} fx' \\ fy' \\ z' \end{bmatrix}$

proportional to each other

NO $\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \begin{bmatrix} fx' \\ fy' \\ z' \end{bmatrix}$

they are not equal to each other

Typical Perspective Model

P: 2D homogeneous point (3D)

X: 3d homogeneous point (4D)

$\boldsymbol{P} \equiv$

$\boldsymbol{X}_{4x1}$

Typical Perspective Model

R: rotation between world system and camera

t: translation between world system and camera

$$\boldsymbol{P} \equiv \qquad [\boldsymbol{R}_{3x3} \quad \boldsymbol{t}_{3x1}] \quad \boldsymbol{X}_{4x1}$$

Typical Perspective Model

adjust the locations of the 2D image plane within the camera systems. Since the image sensor is not always aliged to the center of the pinhole.

f focal length

u0,v0: principal point (image coords of camera origin on retina)

$$\boldsymbol{P} \equiv \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} [\boldsymbol{R}_{3x3} \quad \boldsymbol{t}_{3x1}] \quad \boldsymbol{X}_{4x1}$$

projection matrix

Typical Perspective Model

Intrinsic Matrix K

Extrinsic Matrix [R,t]

$$\boldsymbol{P} \equiv \boxed{\begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}} \; \boxed{\begin{bmatrix} \boldsymbol{R}_{3x3} & \boldsymbol{t}_{3x1} \end{bmatrix}} \; \boldsymbol{X}_{4x1}$$

$$\boldsymbol{P} \equiv \boldsymbol{K}[\boldsymbol{R}, \boldsymbol{t}]\boldsymbol{X} \equiv \boldsymbol{M}_{3x4}\boldsymbol{X}_{4x1}$$

Other Cameras – Orthographic

Orthographic Camera (collapse z dimension)

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{X}_{3x1}$$

Image Credit: Wikipedia

Other Cameras – Orthographic

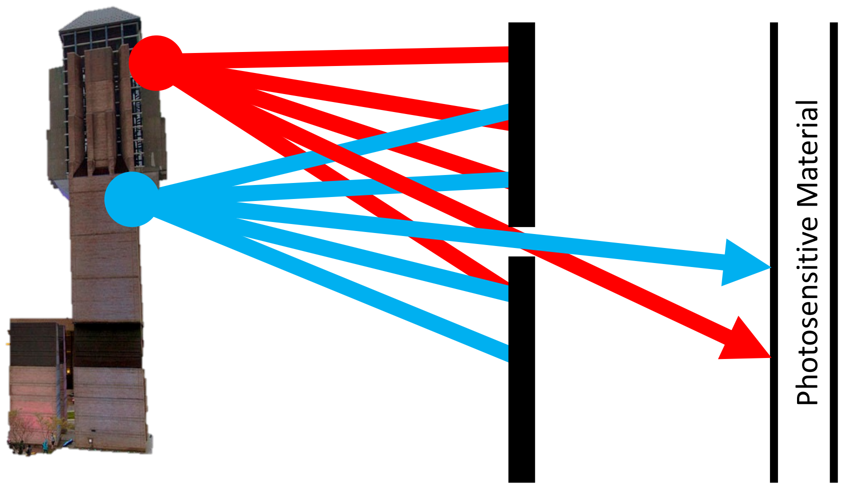
Why does this make things easy and why is this popular in old games?

$$\boldsymbol{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

No Perspective Effects (Less Design)

Less Computation

The Big Issue



Film captures all the rays going through a ***point*** (a p*encil of rays).

How big is a point?

Slide inspired by S. Seitz; image from Michigan Engineering

Math vs. Reality

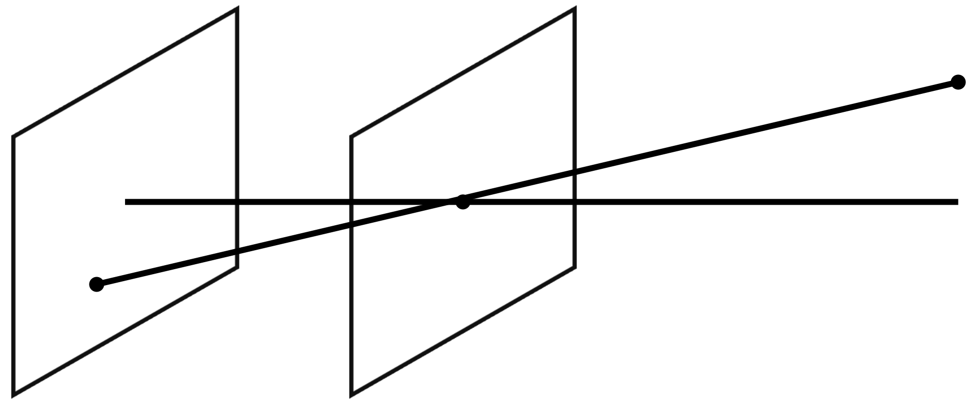
- Math: Any point projects to one point
- Reality:
  - Don’t image points behind the camera / objects
  - Don’t have an infinite sensor resolution
- Other issues
  - Light is limited
  - Spooky stuff happens with infinitely small holes

Limitations of Pinhole Model

<u>Ideal Pinhole</u>

-1 point generates 1 image

-Low-light levels



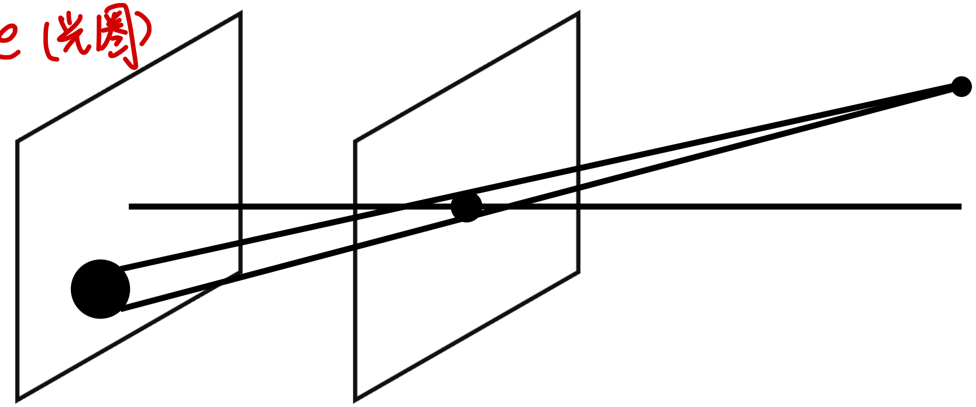
size of pinhole : aperture (光圈)

<u>Finite Pinhole</u>

-1 point generates region

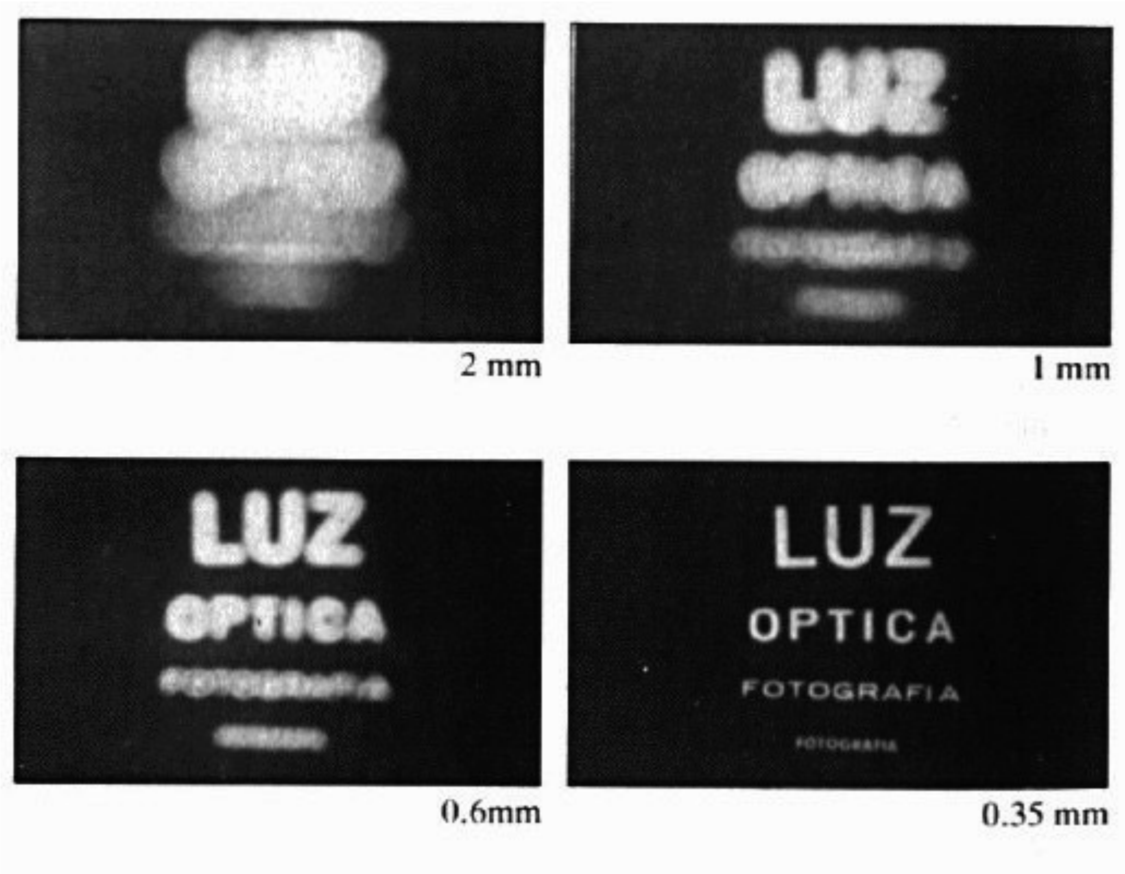
-Blurry.

Why is it blurry?



Slide inspired by M. Hebert

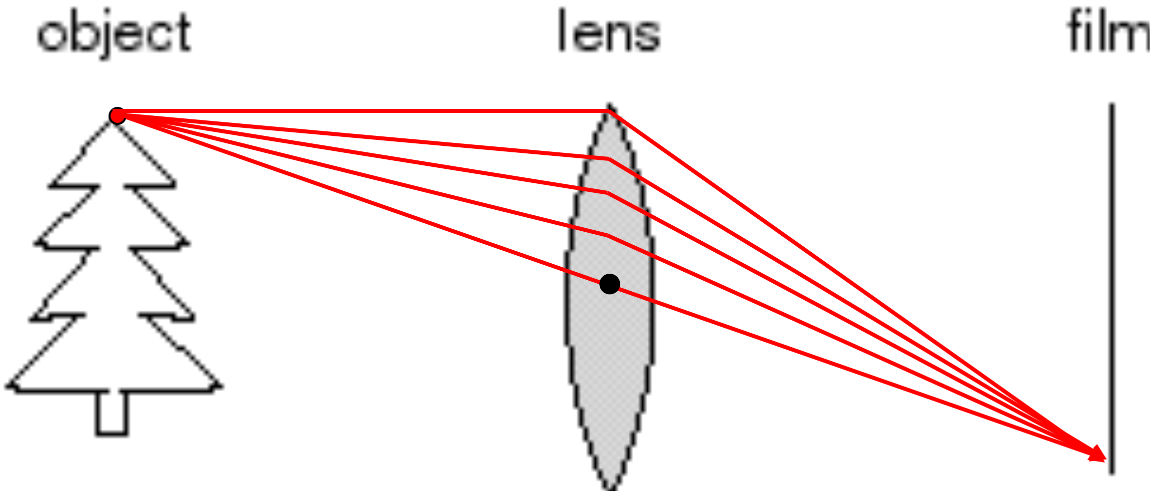
Limitations of Pinhole Model



Low-light & sharpness trade-off

Slide Credit: S. Seitz

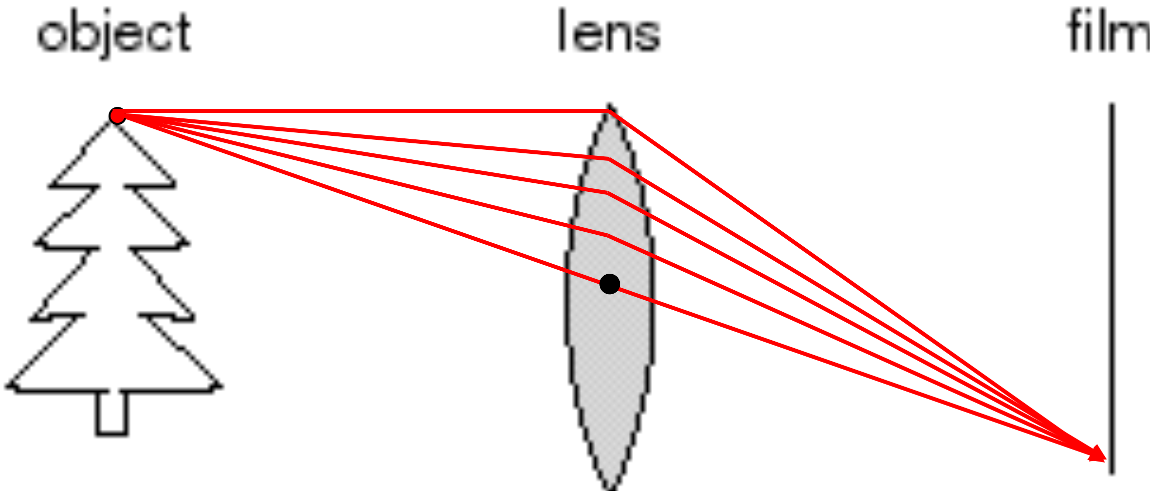
Adding a Lens



- A lens focuses light onto the film
- Thin lens model: rays passing through the center are not deviated (pinhole projection model still holds)
- More light → brighter!

Slide Credit: S. Seitz

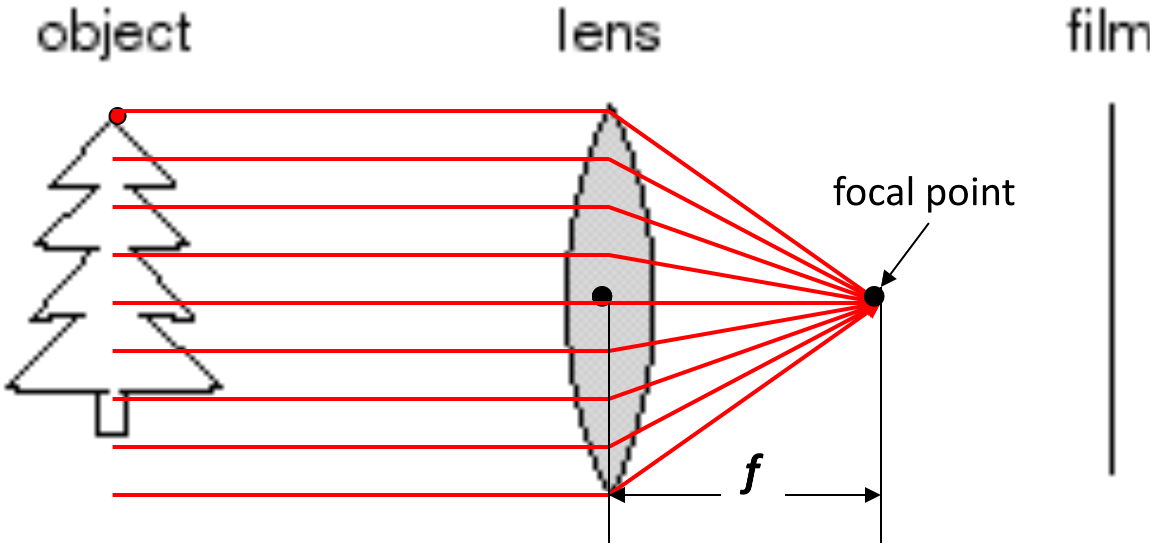
Ideal Lens



- All rays from a single point converge to a single point on the image plane

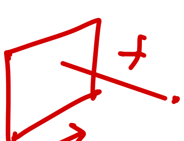
Slide Credit: S. Seitz

Adding a Lens



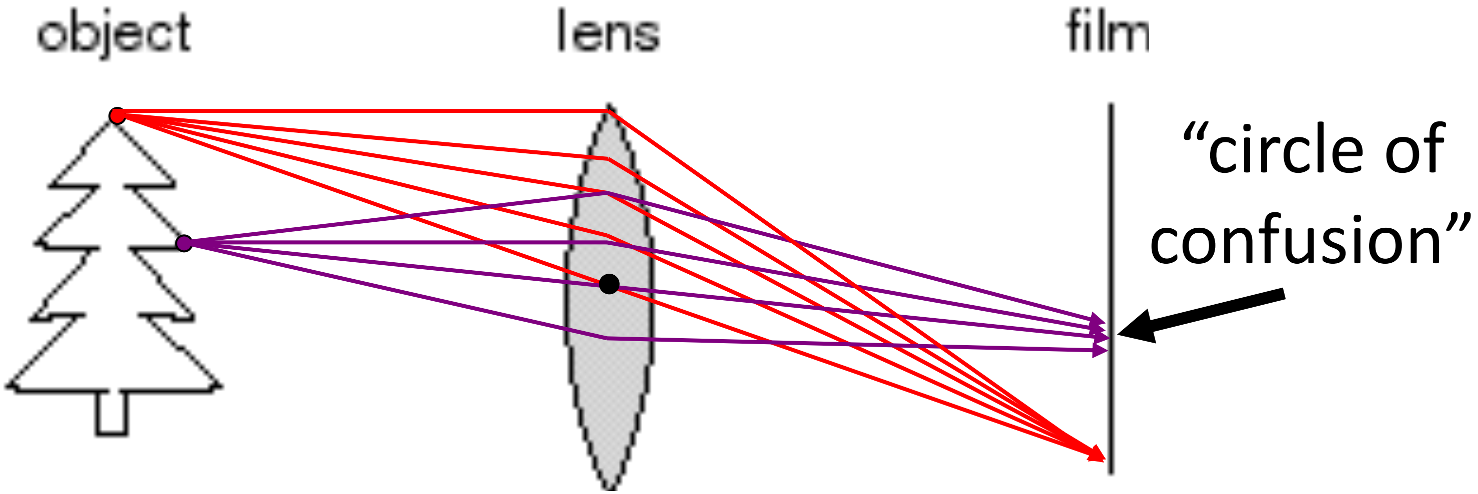
- All rays parallel to the optical axis pass through the *focal point*
- *f* is focal length or focal distance to avoid confusion
- Intrinsic quantity of a lens

we use this



Slide Credit: S. Seitz

What's The Catch?



- Assuming fixed imaging plane and lens
- There's a distance where objects are "in focus"
- Other points project to a "circle of confusion"

Slide Credit: S. Seitz

Thin Lens Formula

We care about images that are in focus.

When is this true?

When two paths from a point hit the same image location.

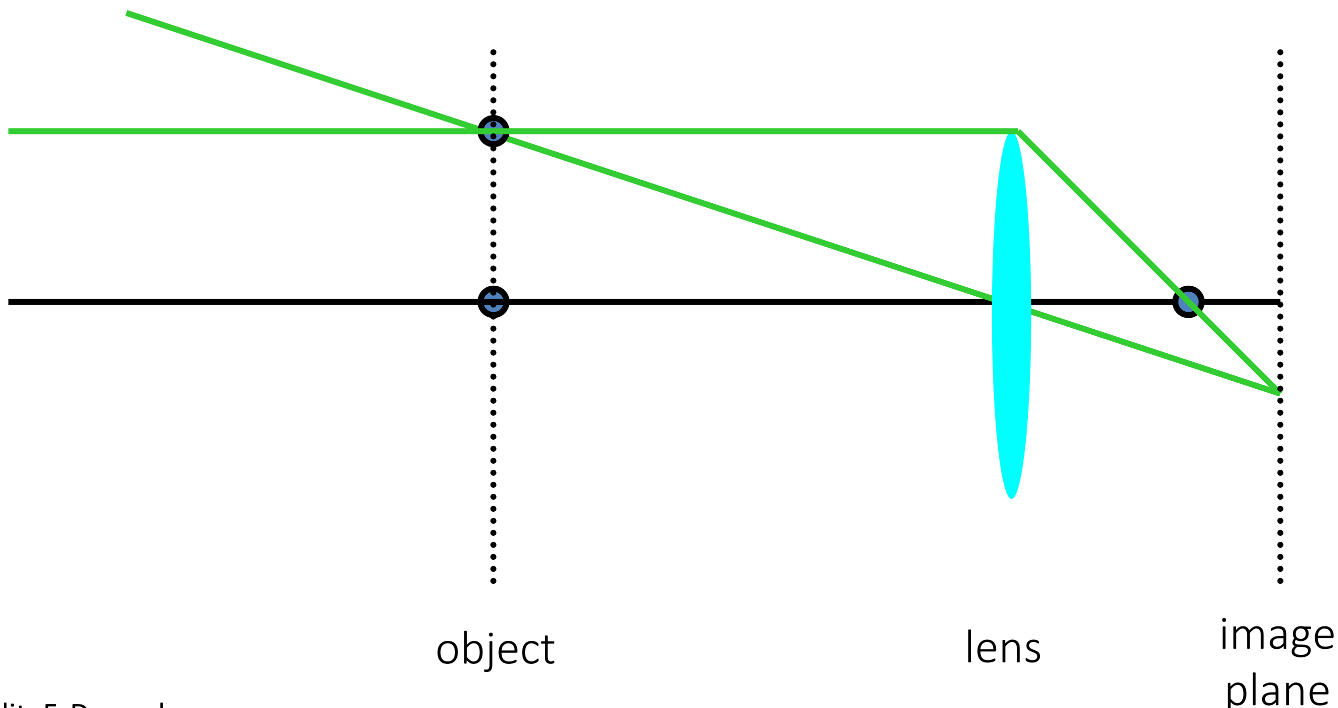


Diagram credit: F. Durand

Thin Lens Formula

When two paths from a point hit the same image location.
D’: “In-focus” image plane distance

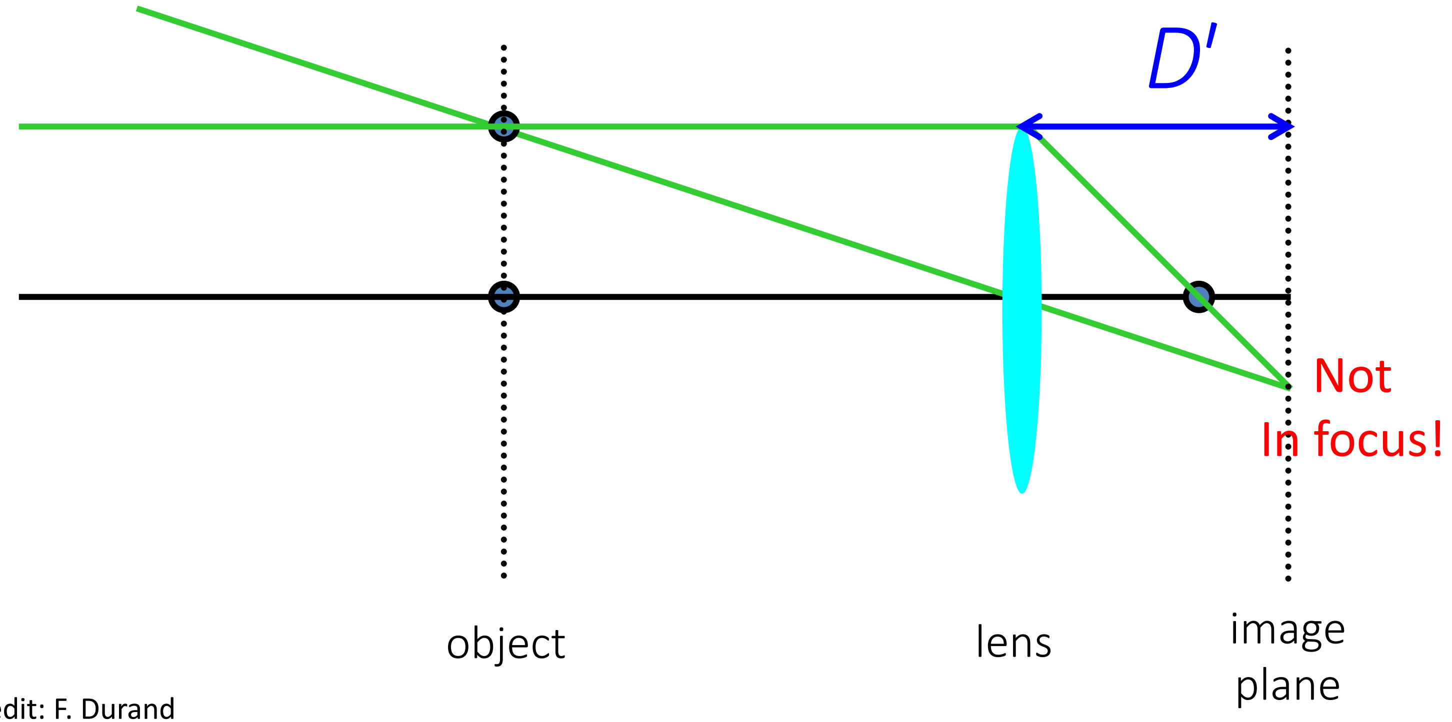


Diagram credit: F. Durand

Thin Lens Formula

If in focus, the Thin Lens model approximates the pinhole model

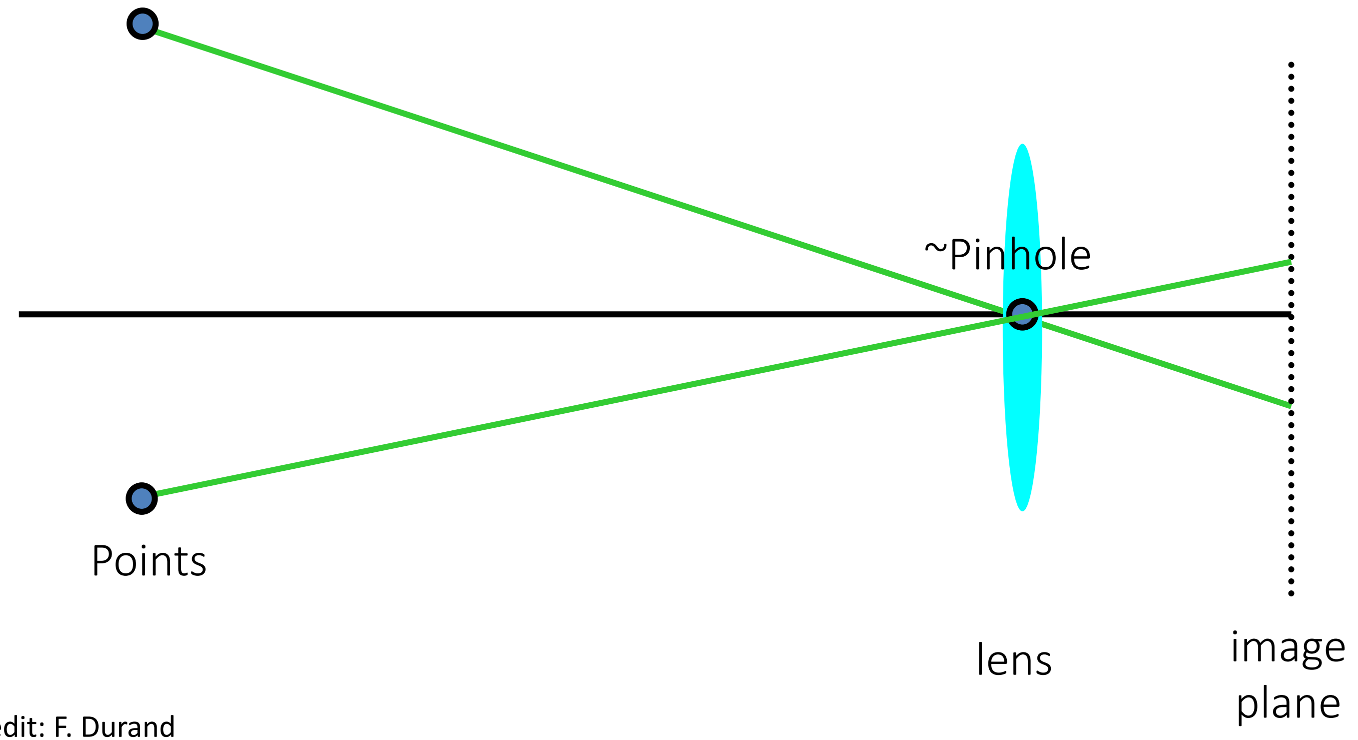


Diagram credit: F. Durand

Thin Lens Formula

Let's derive the relationship between object distance D, in-focus distance D', and focal distance f.

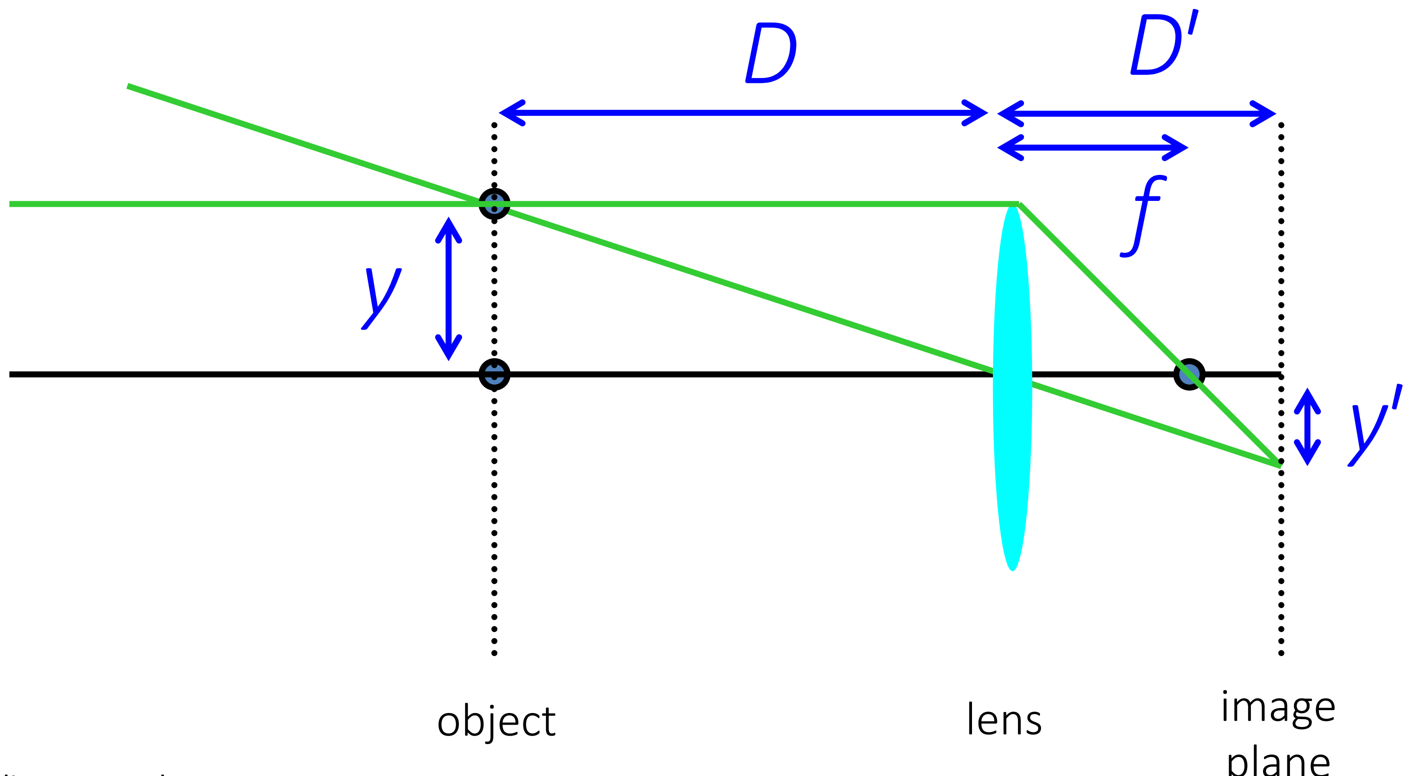


Diagram credit: F. Durand

Thin Lens Formula

One set of similar triangles:

$$\frac{y'}{D' - f} = \frac{y}{f} \longrightarrow \frac{y'}{y} = \frac{D' - f}{f}$$

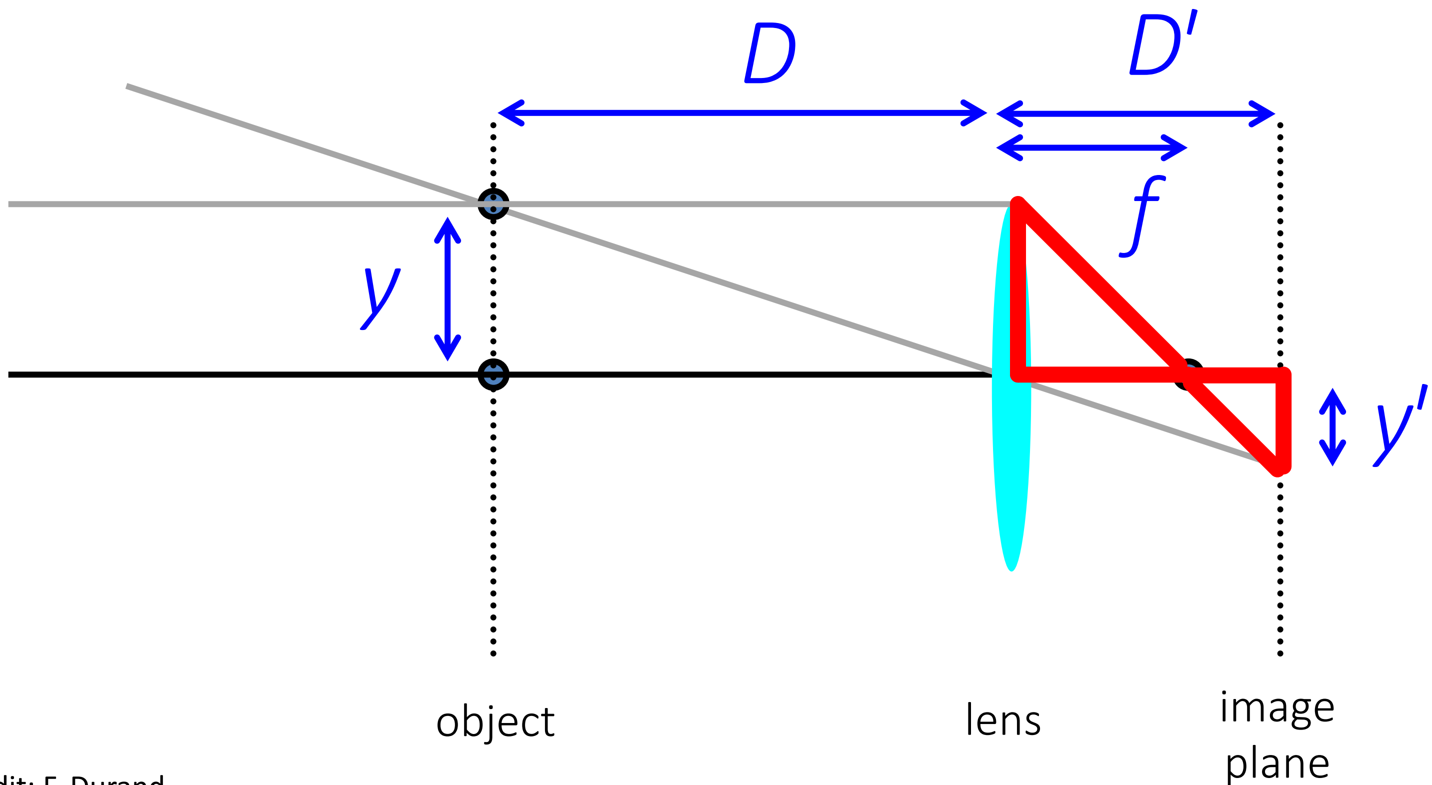


Diagram credit: F. Durand

Thin Lens Formula

Another set of similar triangles:

$$\frac{y'}{D'} = \frac{y}{D} \longrightarrow \frac{y'}{y} = \frac{D'}{D}$$

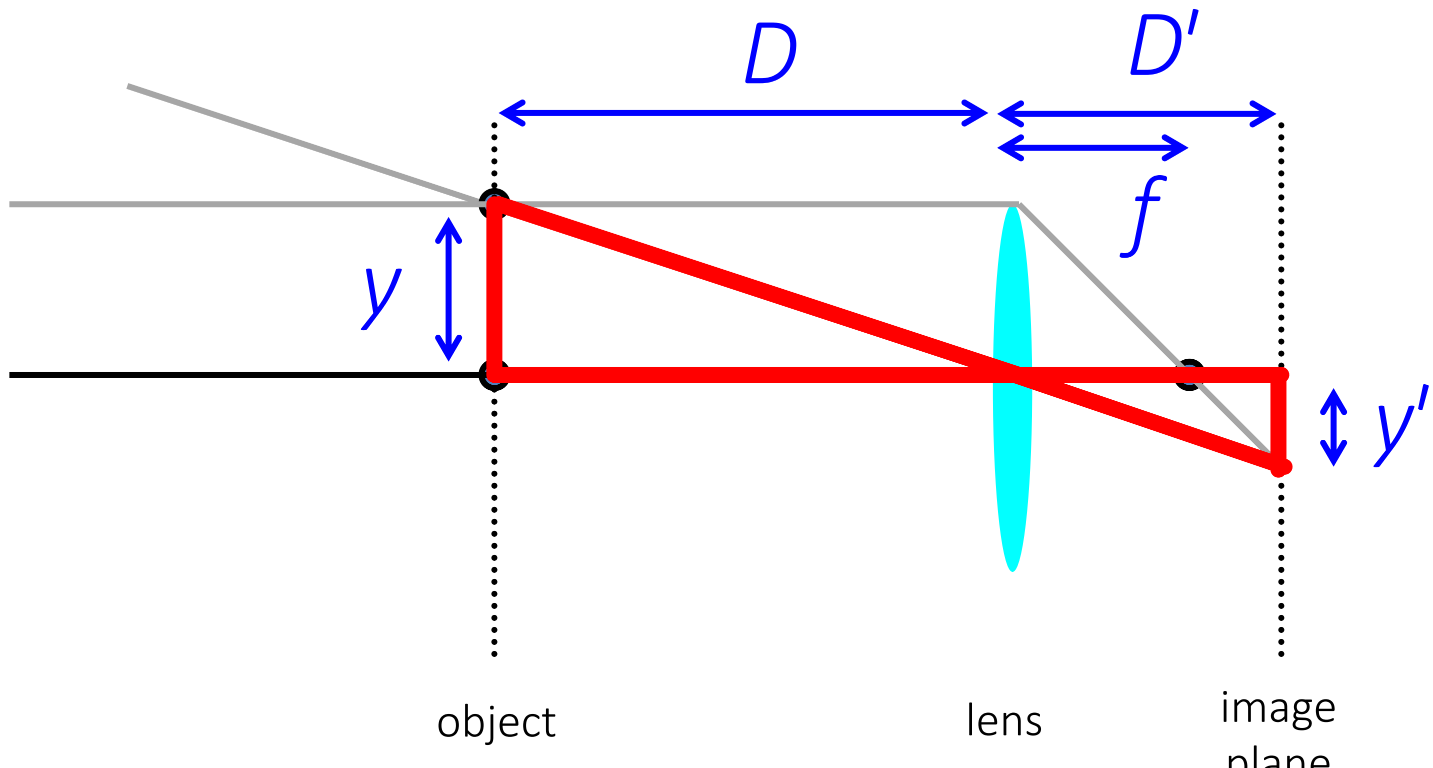


Diagram credit: F. Durand

Thin Lens Formula

Set them equal:

$$\frac{D'}{D} = \frac{D - f}{f} \longrightarrow \frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$

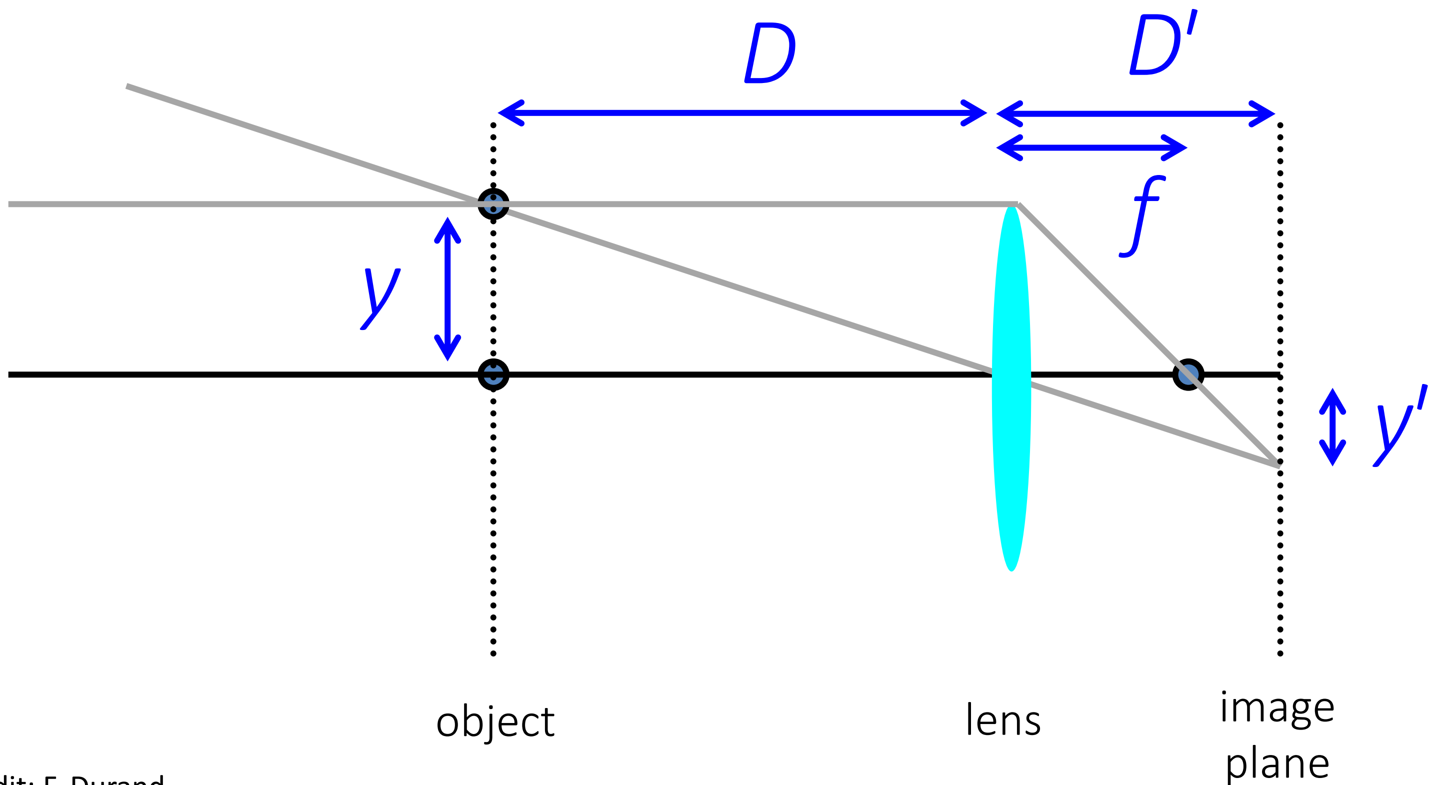


Diagram credit: F. Durand

Thin Lens Formula

How do we take pictures of things at different distances?

$$\frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$

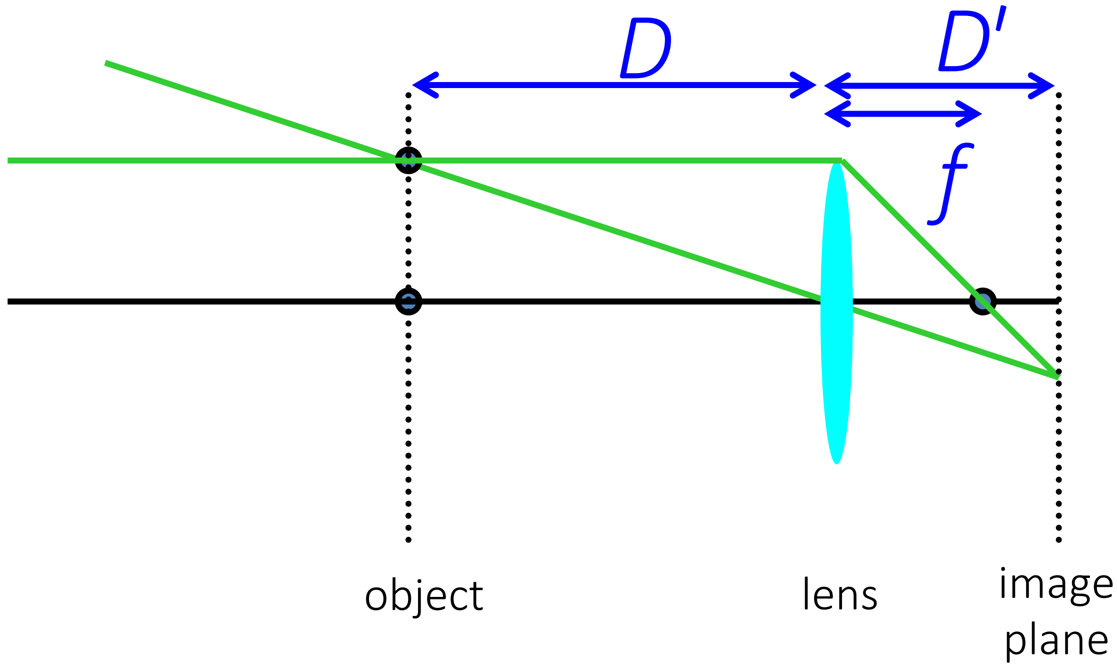


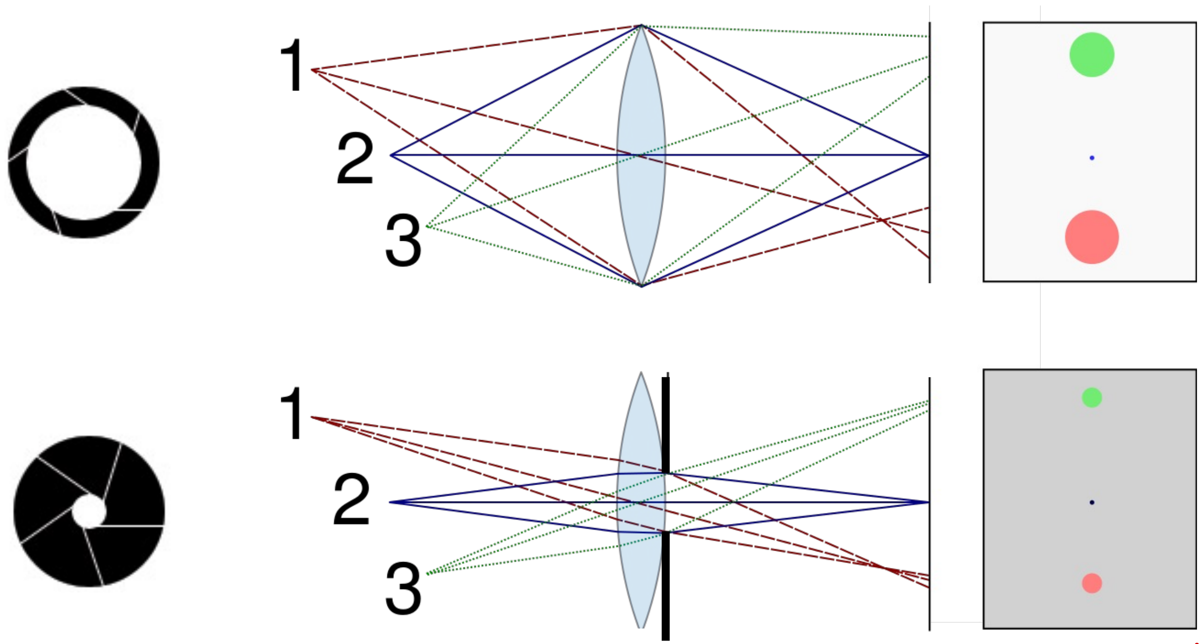
Diagram credit: F. Durand

Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide Credit: A. Efros

Controlling Depth of Field



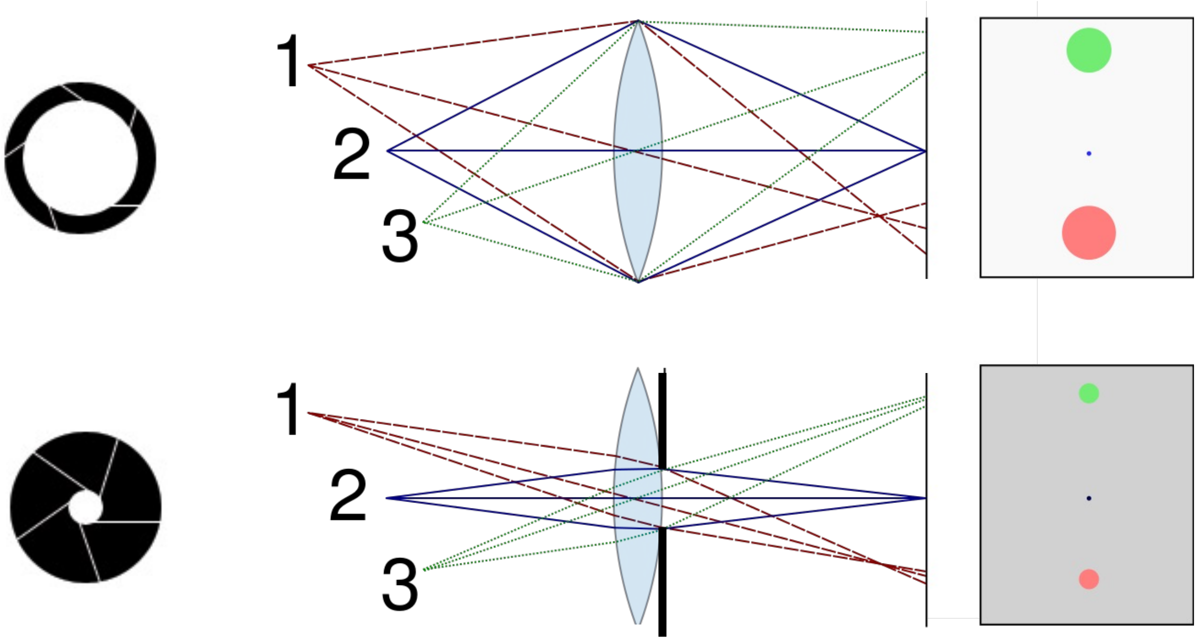
increase the depth of field.

Changing the aperture size affects depth of field

A smaller aperture increases the range in which the object is approximately in focus

Diagram: Wikipedia

Controlling Depth of Field



If a smaller aperture makes everything focused, why don't we just always use it?

Diagram: Wikipedia

Varying the Aperture

Small aperture = large DOF

Large aperture = small DOF

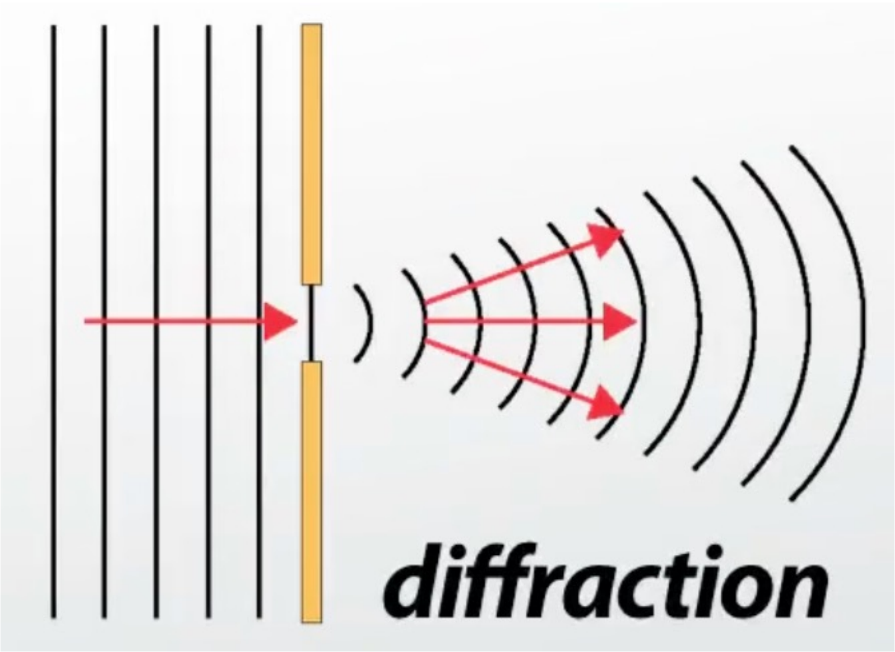
Slide Credit: A. Efros, Photo: Philip Greenspun

Varying the Aperture

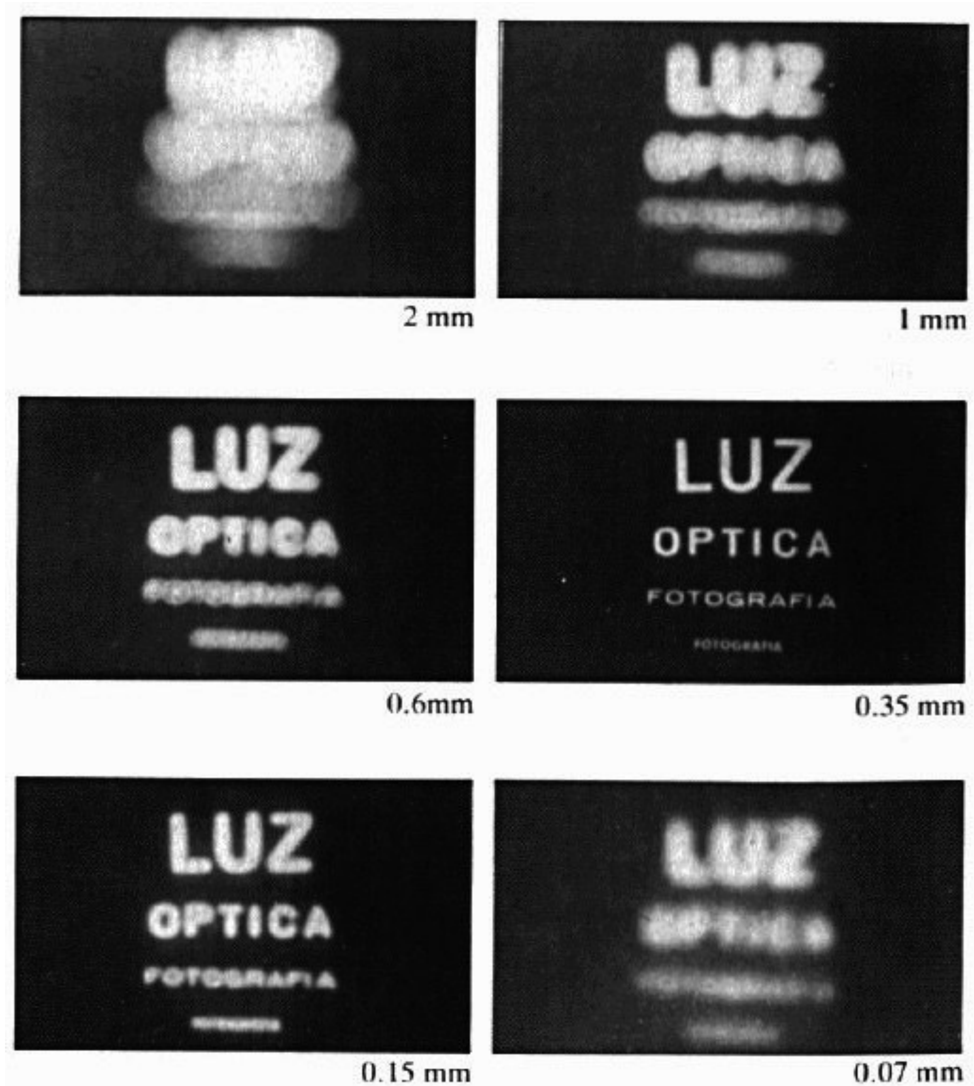
More Light
→ Brighter!

Diffraction 衍射

- Small holes (slits) induce diffraction
- Light doesn't travel like a ray
- See beyond line of sight! (around the corner)



Diffraction



Slide Credit: S. Seitz

Field of View and Focal Length

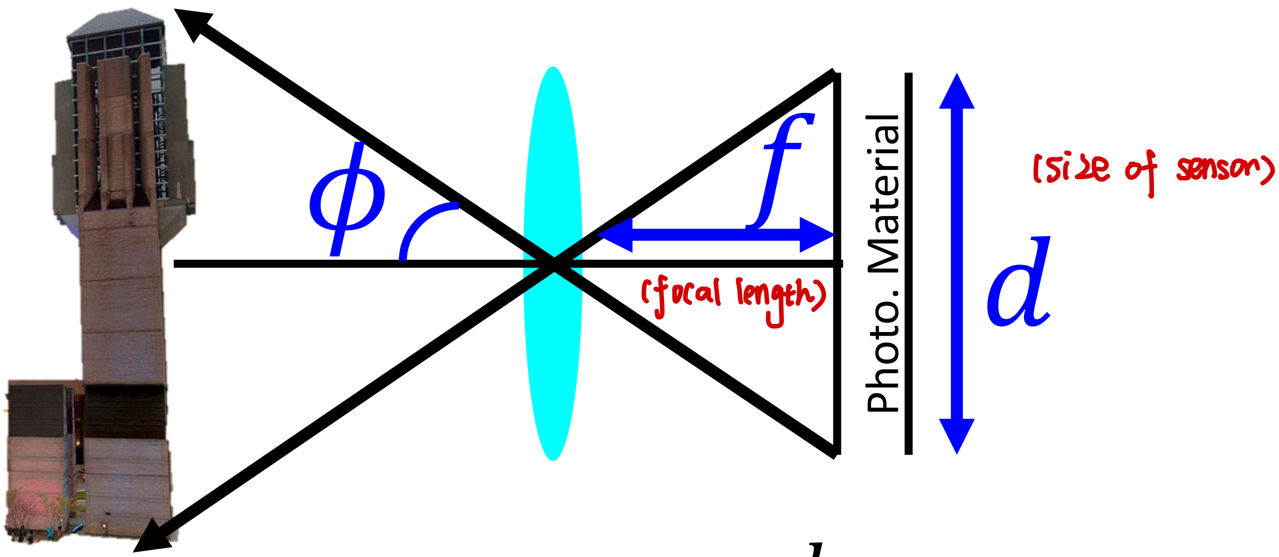
wide-angle

standard

telephoto

Slide Credit: F. Durand

Field of View (FOV)



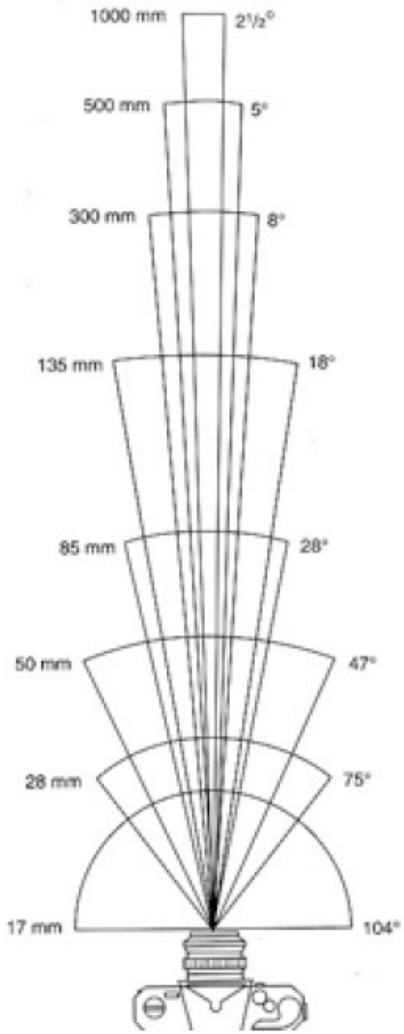
$$\phi = \tan^{-1}\left(\frac{d}{2f}\right)$$

$\tan^{-1}$ is monotonic increasing.

How can I get the FOV bigger?

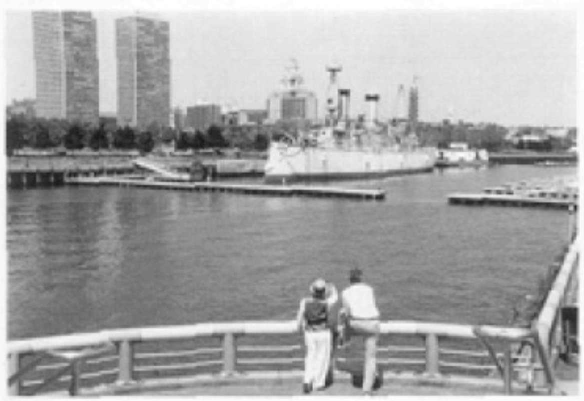
$f\downarrow$ or $d\uparrow$

Field of View




17mm

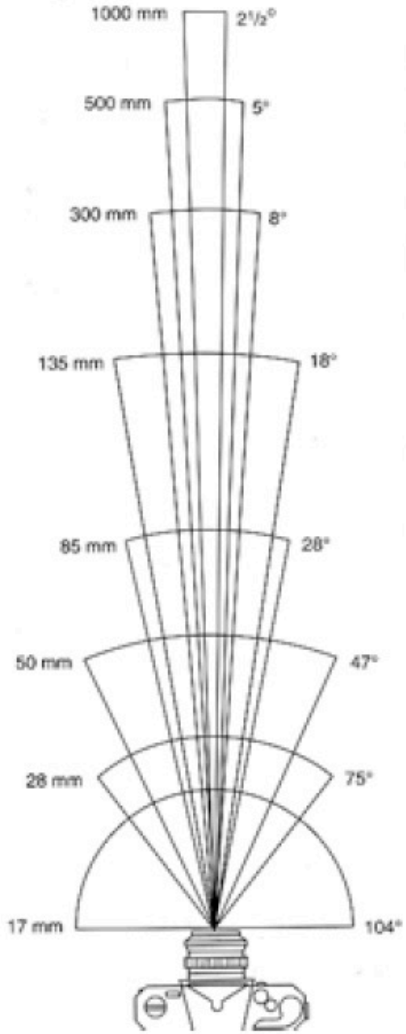

28mm

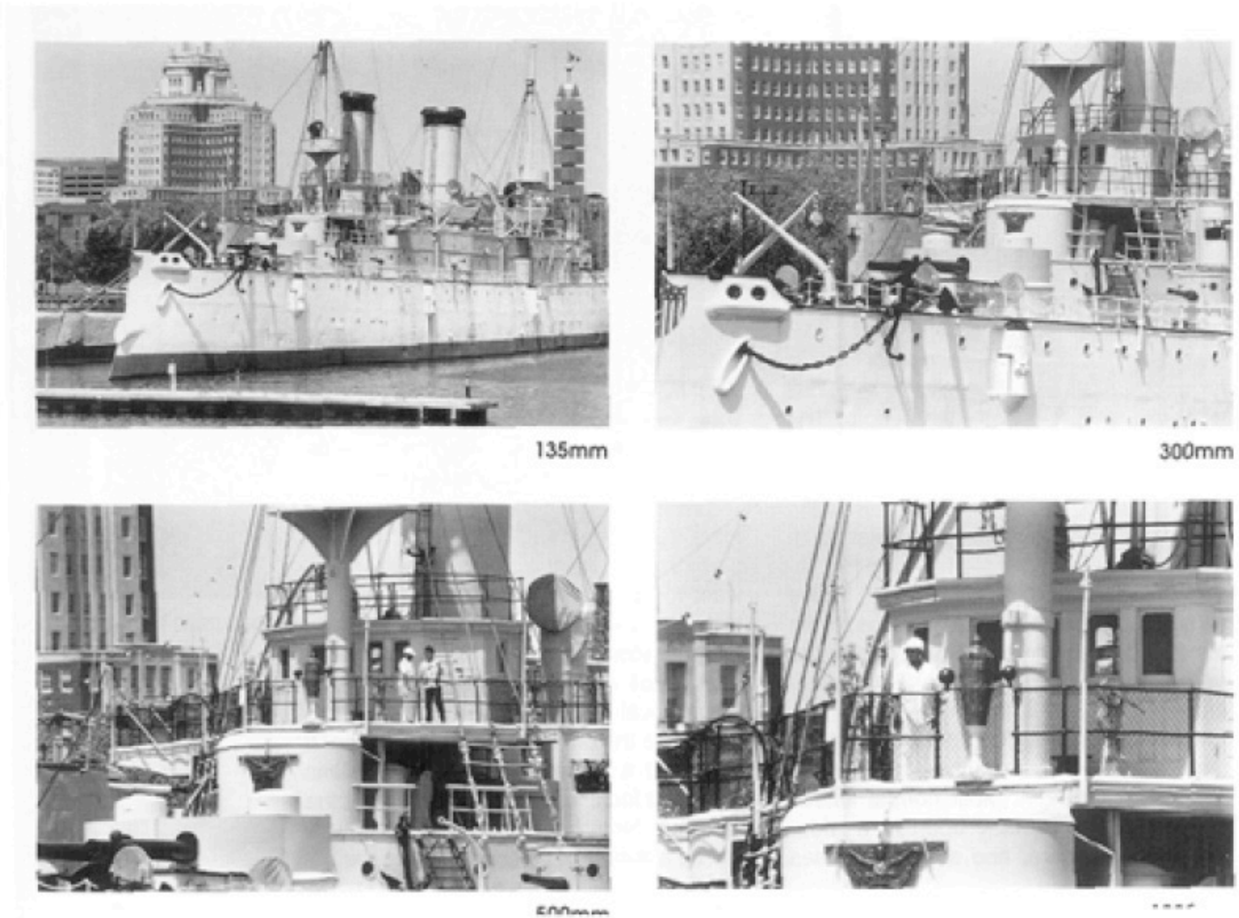

50mm


85mm

Slide Credit: A. Efros

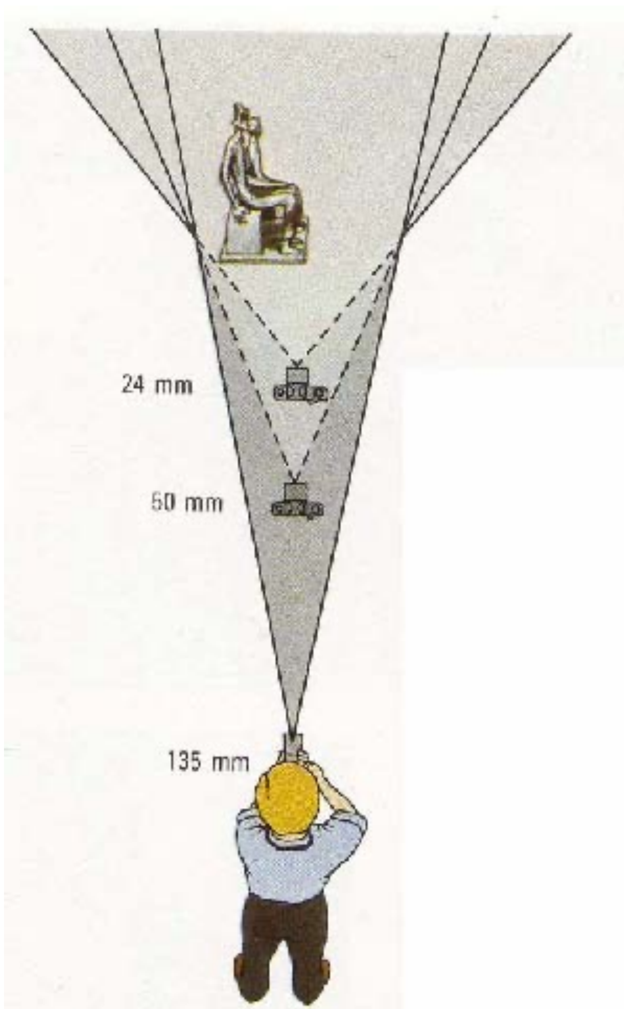
Field of View





Slide Credit: A. Efros

Field of View and Focal Length



Large FOV, small *f*
Camera close to car

Small FOV, large *f*
Camera far from the car

Slide Credit: A. Efros, F. Durand

Dolly Zoom

Change f and distance at the same time

Video Credit: Goodfellas 1990

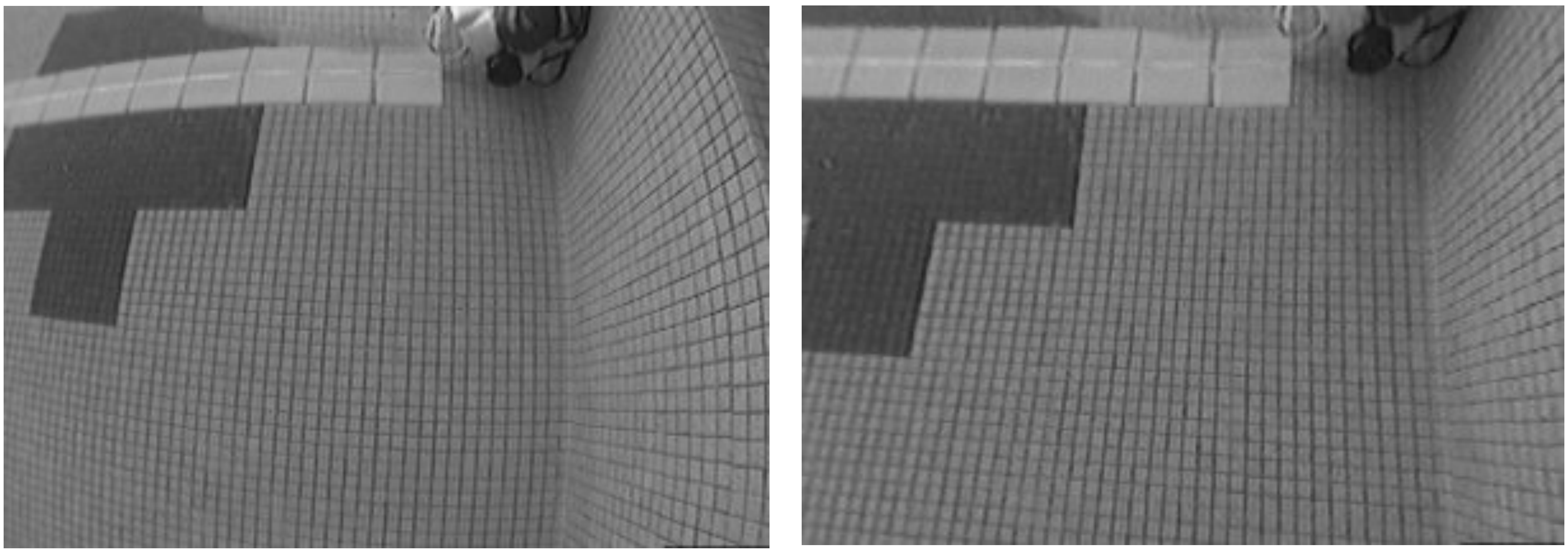
More Bad News!

- First a pinhole…
- Then a thin lens model….

Slide: L. Lazebnik

Lens Flaws: Radial Distortion

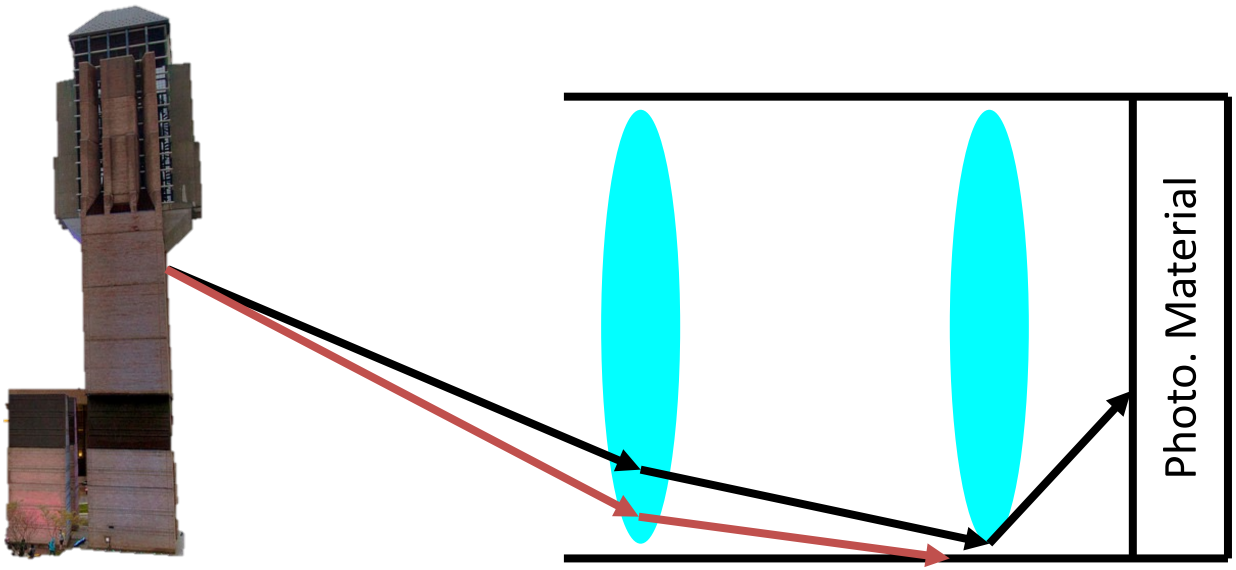
Lens imperfections cause distortions as a function of distance from optical axis



Less common these days in consumer devices

Photo: Mark Fiala, U. Alberta

Vignetting



What happens to the light between the black and red lines?

Edge rays are easier to be blocked!

Slide inspired by L. Lazebnik Slide

Vignetting

Photo credit: Wikipedia (https://en.wikipedia.org/wiki/Vignetting)

Lens Flaws: Chromatic Abberation

Lens refraction index is a function of the wavelength. Colors "fringe" or bleed

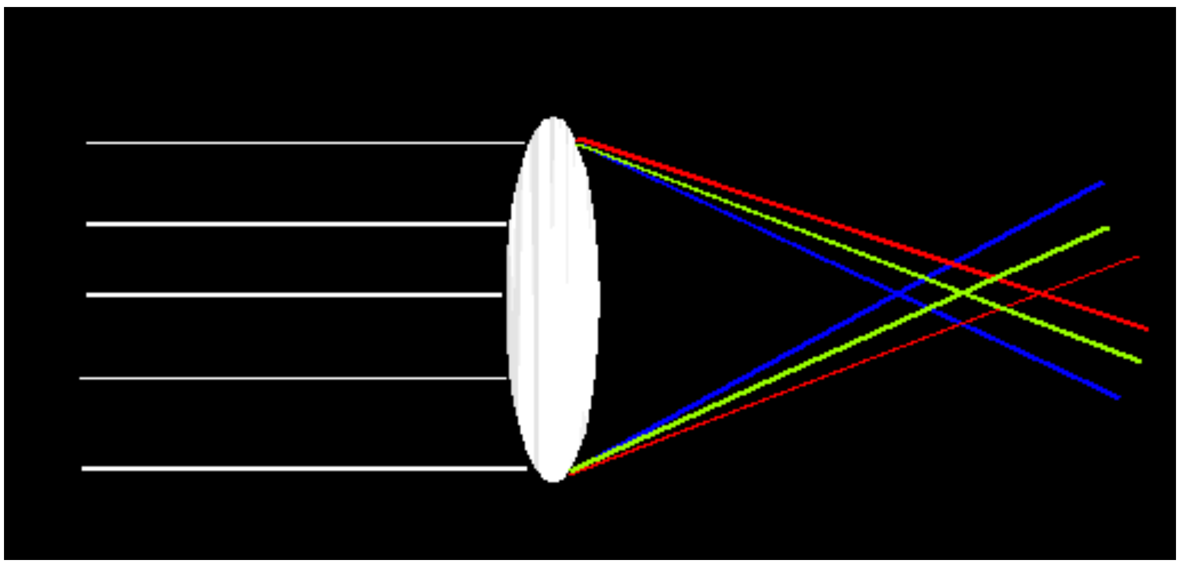
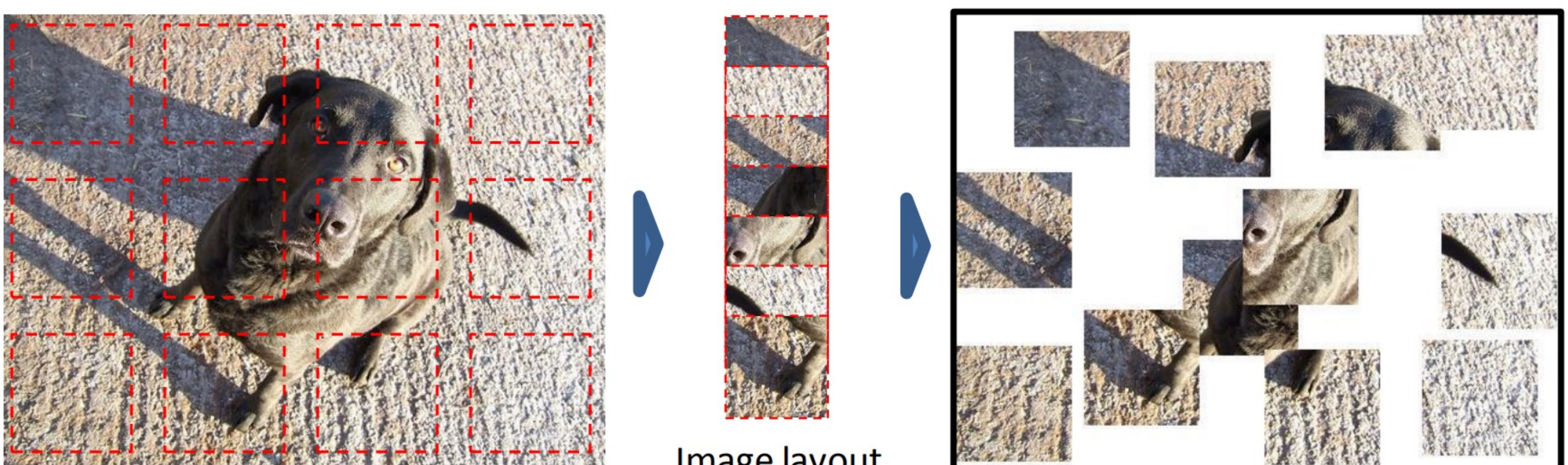


Image credits: L. Lazebnik, Wikipedia

Lens Flaws: Chromatic Abberation

Researchers tried teaching a network about objects by forcing it to assemble jigsaws.



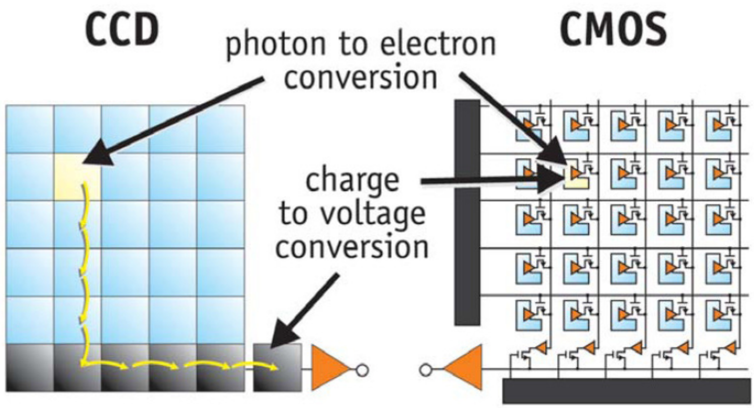
Initial layout, with sampled patches in red

Image layout is discarded

We can recover image layout automatically

Slide Credit: C. Doersch

From Photon to Photo



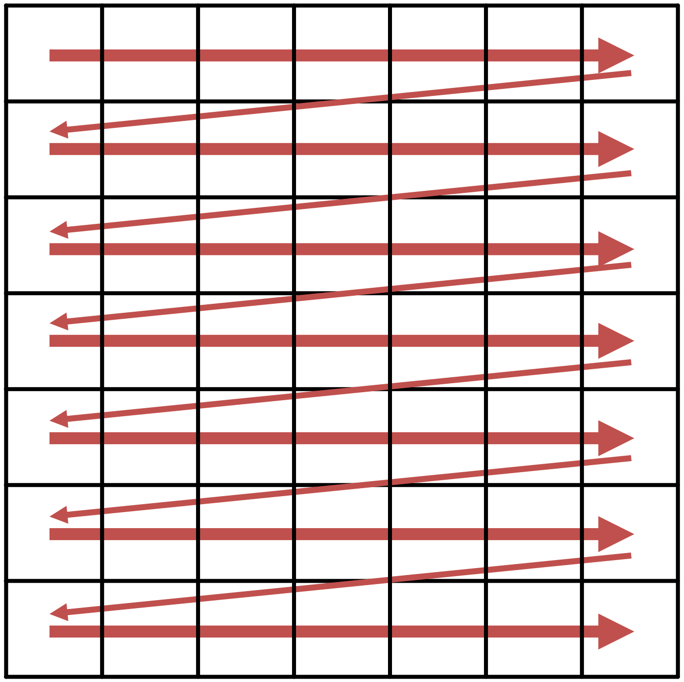
CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

- Each cell in a sensor array is a light-sensitive diode that converts photons to electrons
  - Dominant in the past: **Charge Coupled Device (CCD)**
  - Dominant now: **Complementary Metal Oxide Semiconductor (CMOS)**

Slide Credit: L. Lazebnik, Photo Credit: Wikipedia, Stefano Meroli

From Photon to Photo

Rolling Shutter: pixels read in sequence
Can get global reading, but $$$

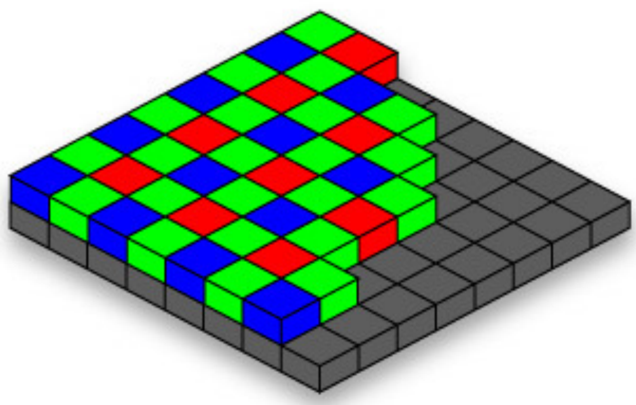


Don’t Panic! In this class:

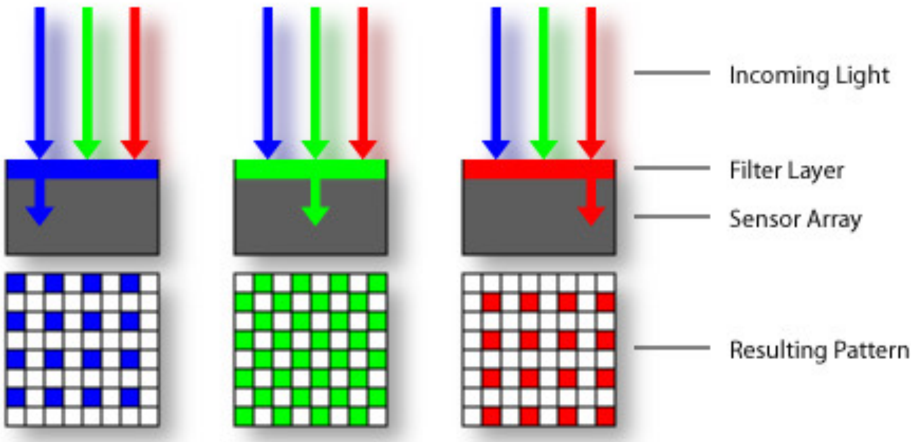
- We mostly assume the images are in focus
- We mostly assume that all complex camera effects are not there
- Assume these effects to be already corrected, e.g., lens distortion or Vignetting
- Ideal Pinhole model is sufficient for many cases

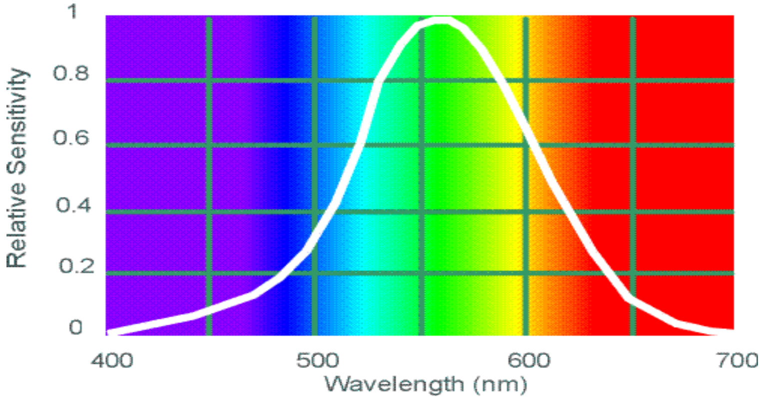
Preview of What's Next

Bayer grid



Demosaicing:
Estimation of missing components from neighboring values





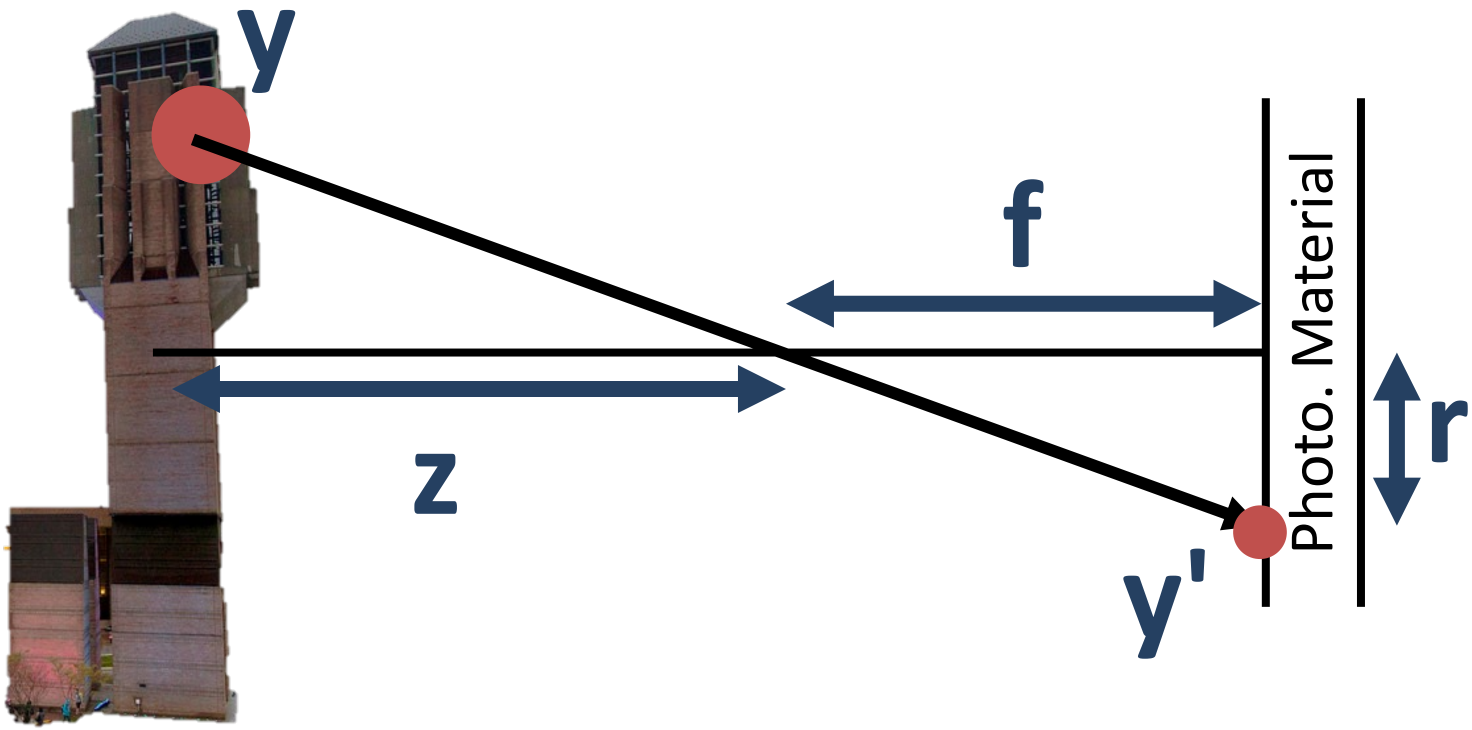
Human Luminance Sensitivity Function

Slide Credit: S. Seitz

For the Curious

- Cut in the interest of time

Radial Distortion Correction



Ideal

$$y' = f\frac{y}{z}$$

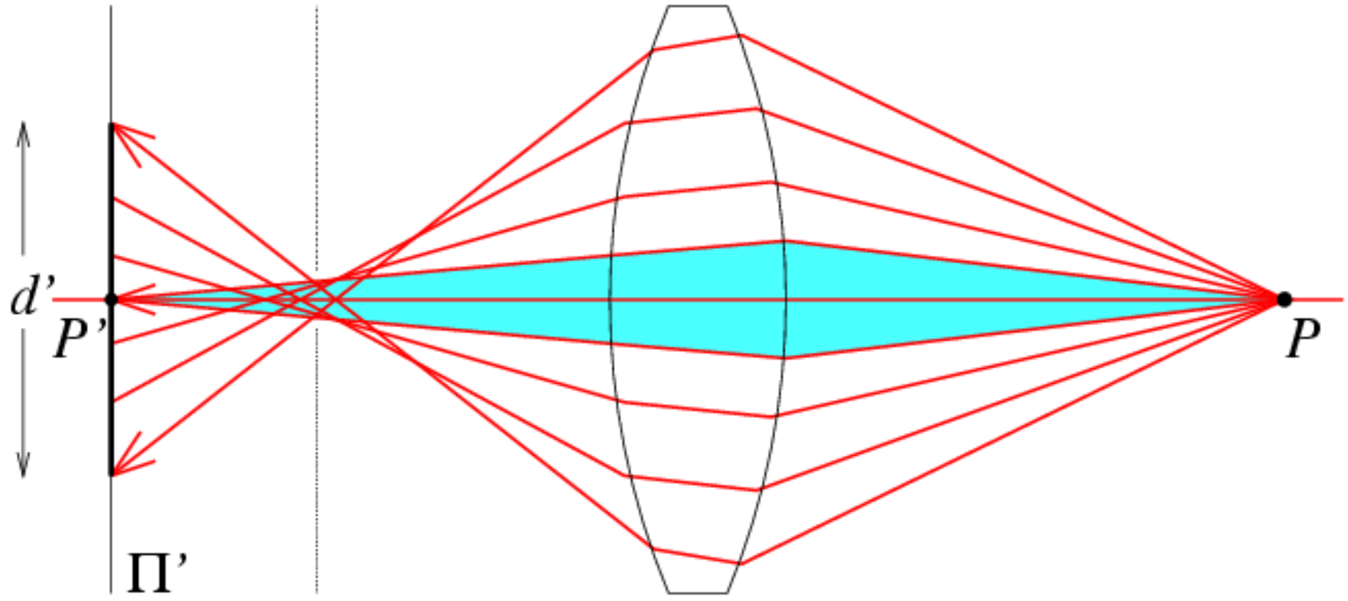
Distorted

$$y' = (1 + k_1 r^2 + \cdots)\frac{y}{z}$$

Lens Flaws: Spherical Abberation

Lenses don't focus light perfectly!

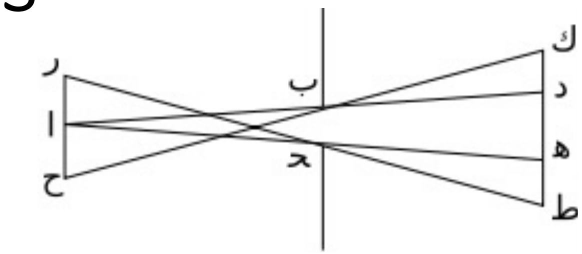
Rays farther from the optical axis focus closer



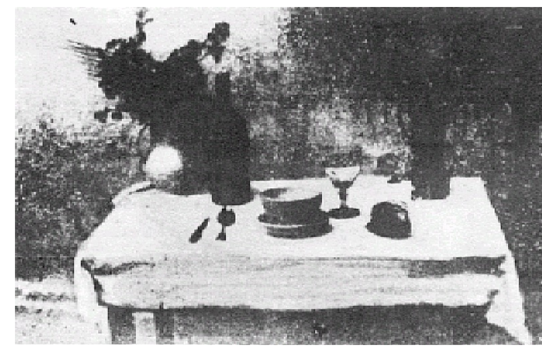
Slide: L. Lazebnik

Historic milestones

- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes


Niepce, "La Table Servie," 1822


Old television camera

Slide Credit: S. Lazebnik

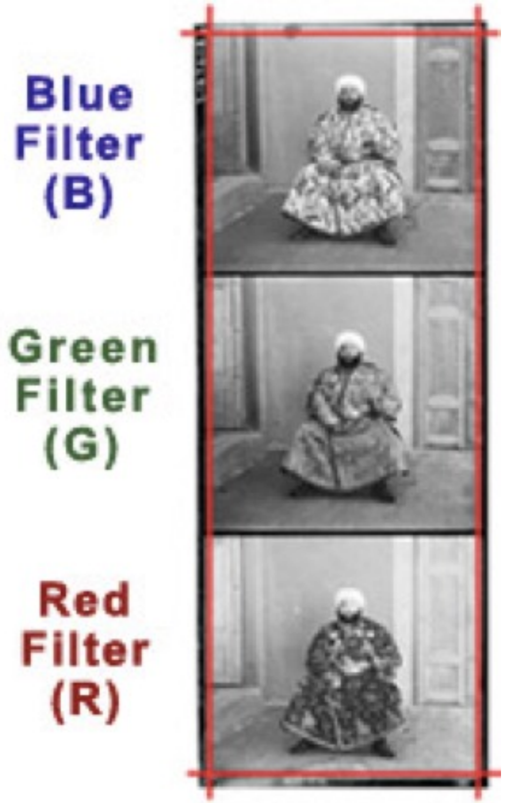
First digitally scanned photograph

- 1957, 176x176 pixels

Slide Credit: http://listverse.com/history/top-10-incredible-early-firsts-in-photography/

Historic Milestone

Sergey Prokudin-Gorskii (1863-1944)
Photographs of the Russian empire (1909-1916)



Slide Credit: S. Maji

Historic Milestone

Slide Credit: S. Maji

Future Milestone

Your job in homework 1:
Make the left look like the right.

Note: it won't quite look like this – this was done by a professional human. But it should look similar