EECS 270 Fall 2021
Homework 3

Due Friday, September 24 @ **5:00 PM** on Gradescope

This is an individual assignment, all of the work should be your own.
Write neatly or type and show all your work for full credit.
Have your name and unique name on the front page of your submission.

Submit on Gradescope.
Total Points: 80

1. **[20 points]** *Algebraic Manipulation:* Using only algebraic manipulation, in the style covered in lecture, verify each of the following equalities, stating all postulates and theorems (with their number and name if they have one) you use in your verification:

   a. **[5]** $(x + xy)x' = 0$

   b. **[5]** $x + yz + x'yz = (x + y)(x + z(z + y))$

   c. **[5]** $x + xy + x'y + x'z = x + y + zzzzz + www'$

   d. **[5]** $bc + ab' + ((ab' + ab)' + c')' = bc + ab'$

a) $(x+x)(x+y)x' = 0$ distribution

$(xx')(x+y) = 0$ Idempotency

$0 = 0$ Complements

b) $x+yz+x'yz = (x+y)(x+z(z+y))$

$x+yz+x'yz = (x+y)(x+zz+zy)$ distributivity

$x+yz+x'yz = (x+y)(x+z+zy)$ Idempotency

$x+yz+x'yz = xx+xy+xz+yz+xzy+zyy$ distributivity

2. **[10 points]** *Truth Tables:* For the following questions assume $f = c(a \oplus b') + a'c + bc'$.

   a. **[5]** Create a truth table for f.

   b. **[5]** Simulate f in mMdelSim for all 8 combinations of a, b, and c with a 20 ns between each test case. Be sure to show the output of f.

3. **[20 points]** *SOP, POS, Minterms, and Maxterms:* Please use Table 1 for the following questions.

   a. **[5]** Write the canonical sum-of-products for g in Table 1.

   b. **[5]** Write g's minterms using the $\sum$ notation.

   c. **[5]** Write the canonical product-of-sums for g in Table 1.

   d. **[5]** Write g's maxterms using the $\prod$ notation.

xyz	g
000	1
001	0
010	1
011	0
100	1
101	1
110	0
111	1

Table 1: Truth table for Question 3.

4. **[10 points]** Shannon's Expansion Theorem: For the following questions let $f = x'y'z + xyz' + xy'z' + xyz + x'yz$.

   a. **[5]** Decompose $f(x, y, z)$ around x using Shannon's Expansion Theorem.

   b. **[5]** Decompose $f(x, y, z)$ around y, z using Shannon's Expansion Theorem.

b) $x+yz+x'yz=(x+y)(x+z(z+y))$

$x+yz+x'yz=(x+y)(x+zz+zy)$ distributivity

$x+yz+x'yz=(x+y)(x+z+zy)$ Idempotenoy

$x+yz+x'yz=x(x+y)+z(x+y)+zy(x+y)$ distribution

$x+yz = x + z(x+y)+zy(x+y)$ absorption (twice)

$x+yz = x+zx+zy+zyx+zyy$ distribution (twice)

$x+yz = x+zx+zy+zyx+zy$ idempotency

$x+yz = x+zx+zy+zyx$ absorption.

$x+yz = x+zx+zy$ absorption

$x+yz = x+zy$ absorption.

$x+zy = x+zy$ Commutativity.

c)

[5] $x + xy + x'y + x'z = x + y + zzzzz + www'$

$x+xy+x'y+x'z = x+y+z+ww'$ idempotency (twice)

$x+x'y+x'z = x+y+z+0$ absorption / complement

$x+y+x'z = x+y+z$ no name

$y+x+x'z = x+y+z$ Commutativity

$y+x+z = x+y+z$ no name

$x+y+z = x+y+z$ Commutativity

d. **[5]** $bc + ab' + ((ab' + ab)' + c')' = bc + ab'$

$bc+ab'+((ab'+ab)')'c = bc+ab'$ De Morgan's

$bc+ab'+(ab'+ab)c = bc+ab'$ Involution.

$bc+ab'+ab'c+abc = bc+ab'$ distributivity

$bc+ab'+abc = bc+ab'$ absorption.

$bc+ab' = bc+ab'$ absorption.

$$f = c(a \oplus b') + a'c + bc'.$$

$ab+a'b'$

a) $= c(ab+a'b')+a'c+bc'$

$= abc+a'b'c+a'(b+b')c+(a+a')bc'$

$= abc+a'b'c+a'bc+a'b'c+abc'+a'bc'$

abc	$a'b'c$	$a'bc$	$a'b'c$	abc'	$a'bc'$
111	001	011	001	110	010
m_7	m_1	m_3	m_1	m_6	m_2

$m_1+m_2+m_3+m_7$.

	abc	f
0	000	0
1	001	1
2	010	1
3	011	1
4	100	0
5	101	0
6	110	0 ✗ 1
7	111	1 ✗ 0

[20 points] *SOP, POS, Minterms, and Maxterms:* Please use Table 1 for the following questions.

a. **[5]** Write the canonical sum-of-products for g in Table 1.

b. **[5]** Write g's minterms using the $\sum$ notation.

c. **[5]** Write the canonical product-of-sums for g in Table 1.

d. **[5]** Write g's maxterms using the $\prod$ notation.

	xyz	g
0	000	1
1	001	0
2	010	1
3	011	0
4	100	1
5	101	1
6	110	0
7	111	1

a) $g = m_0 + m_2 + m_4 + m_5 + m_7$

$= x'y'z' + x'yz' + xy'z' + xy'z + xyz$

b) $\Sigma_{xyz}(0.2.4.5.7)$

c) $g = M_1 + M_3 + M_6$

$= xyz' + xy'z' + x'y'z$

d $\prod_{xyz}(1.3.6)$

4. **[10 points]** Shannon's Expansion Theorem: For the following questions let $f = x'y'z + xyz' + xy'z' + xyz + x'yz$.

a. **[5]** Decompose $f(x, y, z)$ around x using Shannon's Expansion Theorem.

b. **[5]** Decompose $f(x, y, z)$ around y, z using Shannon's Expansion Theorem.

a) $f(x.y.z) = x'\overset{f_{x'}}{f(0,y.z)} + x\,\overset{f_x}{f(1.y.z)}$

b) $f(x.y.z) = y'f(x.0.z) + y\,f(x.1.z)$

$x'(y'z+yz) + x(yz'+y'z+yz)$

$y'(x'z+xz') + y(xz'+xz+x'z)$

5. **[10 points]** *Complement and Dual:* Let $f'$and f^d denote the complement and dual, respectively, of function f. Let $f = xy'z + yz + xz'$

   a. **[4]** What is f'?

   b. **[4]** What is f^d?

   c. **[2]** A function is self-dual if $f = f^d$. Is function f self-dual?

6. **[10 points]** *Functionally Complete:* Prove that the following are functionally complete.

   a. **[5]** NOR

   b. **[5]** 2to1 MUX

5. a) $f' = (x'+y+z)(y'+z')(x'+z)$

b $f^d = (x+y'+z)(y+z)(x+z')$.

c) $f^d = (x+y'+z)(y+z+xx')(x+z'+yy')$

$= (x+y'+z)(y+z+x)(y+z+x')(x+z'+y)(x+z'y')$

$= 010 \quad 000 \quad 001 \quad 010 \quad 011$

$M_2 \quad M_0 \quad M_1 \quad M_2 \quad M_3$

$\Pi_{xyz}(0.1.2.3)$.

$f = xy'z + (x'+x)yz + x(y'+y)z'$

$= xy'z + x'yz + xyz + xy'z' + xyz'$

$101 \quad 011 \quad 111 \quad 100 \quad 110$

$m_5 \quad m_3 \quad m_7 \quad m_4 \quad m_6$

$\Sigma_{xyz} = (3.4.5.6.7)$.

$f = \Sigma_{xyz}(3.4.5.6.7) = \Pi_{xyz}(0.1.2)$.

$f^d = \Pi_{xyz} = (0.1.2.3)$ $\neq$.

6. x NOR y : $x'y'$

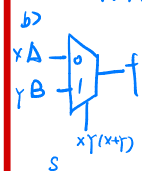
$\underline{x'y'}$ NOR $\underline{x'y'}$: $(x'y')'(x'y')' = (x+y)(x+y)$

(xNORy) (xNORy)

$= (x+y). \rightarrow$ "+"

x NOR x : $x'x' = x'$ "implement"

b)



$f = SB + S'A$

$f = xY + x'x$

$= xY \rightarrow$ and.

$f = x'Y + xx$

$= x'Y + x$

$= x+Y. \rightarrow$ or.