UM EECS 270 F22
Introduction to Logic Design

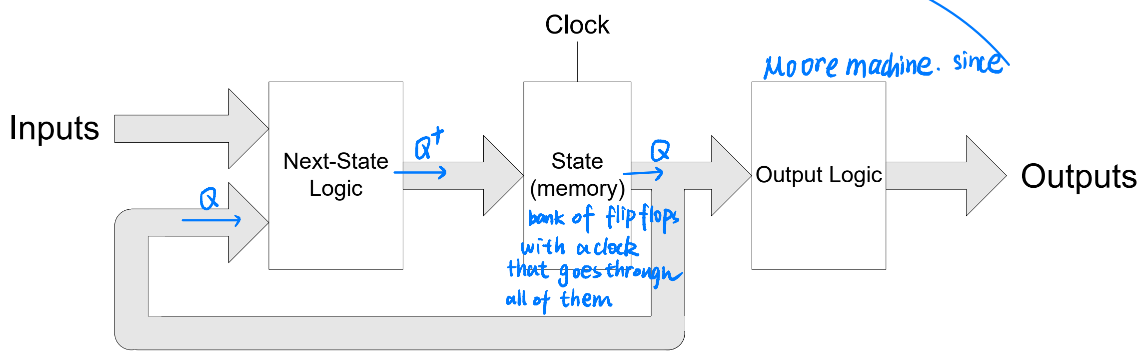
10. Analysis of Sequential Circuits

Sequential Circuits

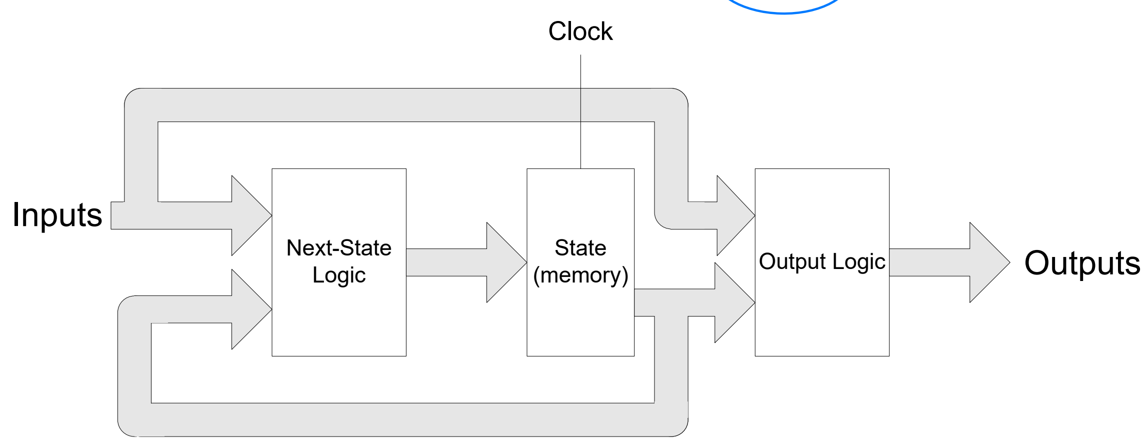
- Modeled as **Finite** State Machines
- Timing behavior:
  - Synchronous (clocked)
  - Asynchronous

Our Focus: **Synchronous** Sequential Circuits

- Sequential Circuit Components:
  - **Next state logic** *(combinational)*: next state = f(current state, inputs)
  - **Memory** *(sequential)*: stores state in terms of state variables
  - **Output logic** *(combinational)*:
    - **<u>Moore Output</u>**: output = g(only current state)



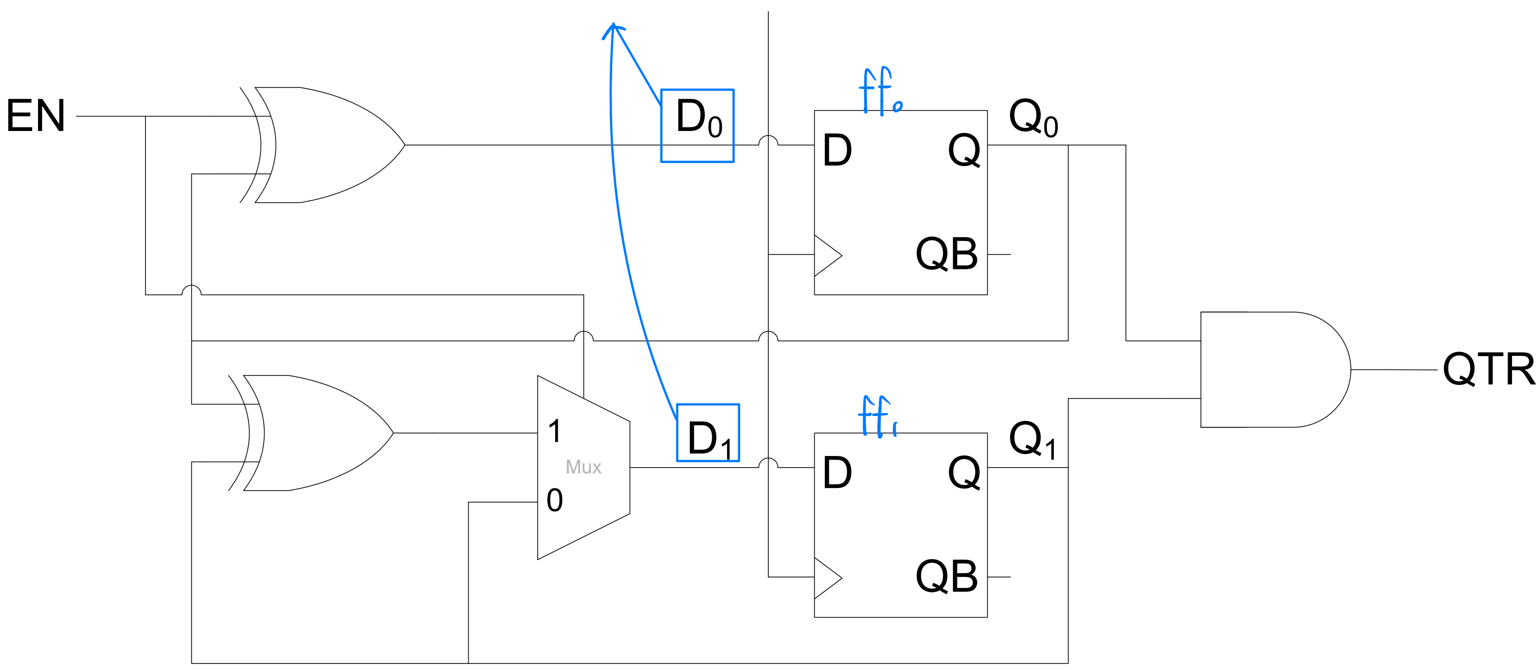
    - **<u>Mealy Output</u>**: output= g(current state, also inputs)



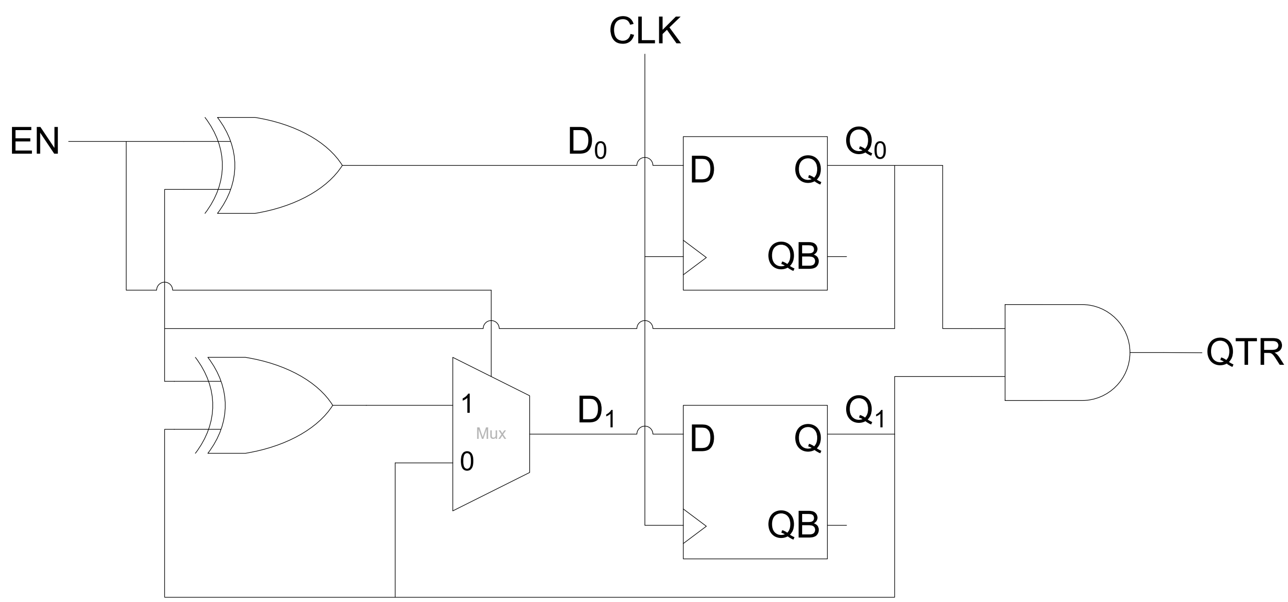
Sequential Circuit Analysis

- Goal: Given a sequential circuit, describe the circuit's behavior

functions of the primary input "EN"



- **Excitation equations** describe memory (FF or latch) input signals as a function of inputs and current state (i.e., state variables)

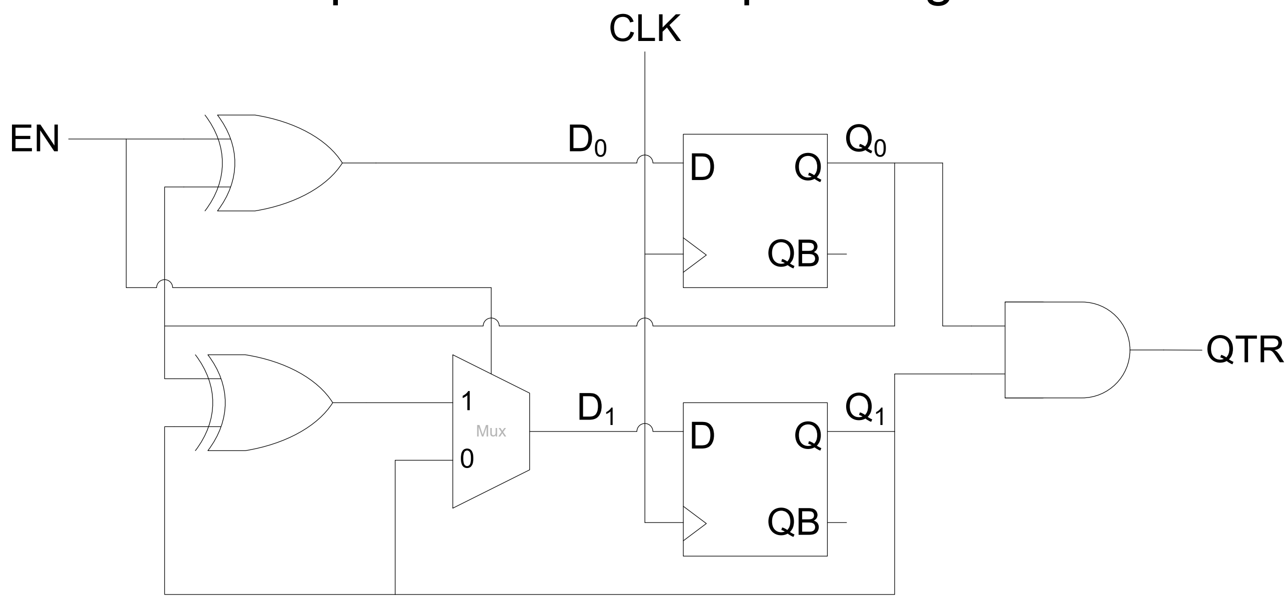


Excitation Equations:

$$D_0 = EN \oplus Q_0$$

$$D_1 = EN \cdot (Q_0 \oplus Q_1) + \overline{EN} \cdot Q_1$$

- **Transition equations** describe the next state as a function of inputs and current state
  - Generated by substituting the excitation equations into the characteristic equation for the sequential gates



D FF Characteristic Eqn:

$$Q^+ = D$$

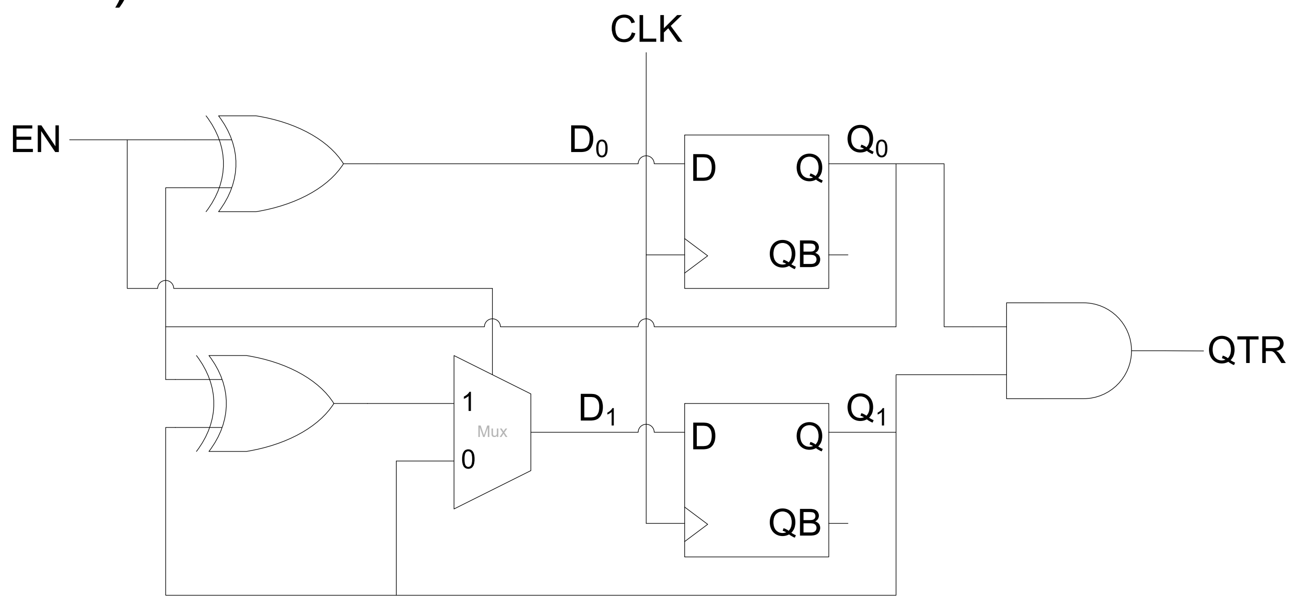
Transition Equations:

$$Q_0^+ = D_0 = EN \oplus Q_0$$

$$Q_1^+ = D_1 = EN \cdot (Q_0 \oplus Q_1) + \overline{EN} \cdot Q_1$$

This step is trivial when using D FFs!

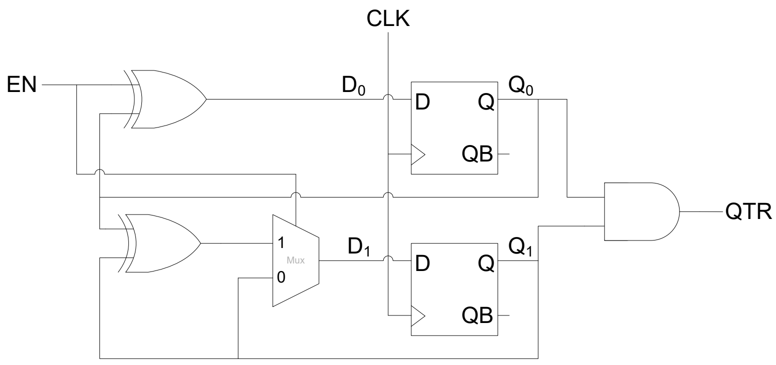
- **Output equations** describe the output signals as a function of the current state (for a Moore machine) or as a function of the current state and inputs (for a Mealy machine)



Output Equation:

$$QTR = Q_0 \cdot Q_1$$

- The **transition/output table** shows the next state and output for every current state/input combination
  - Entries of the table are obtained from the transition equations and the output equations



Transition Equations:

$$Q_0^+ = D_0 = EN \oplus Q_0$$

$$Q_1^+ = D_1 = EN \cdot (Q_0 \oplus Q_1) + \overline{EN} \cdot Q_1$$

Output Equation:

$$QTR = Q_0 \cdot Q_1$$

Transition/Output Table:

current state		input EN		output
Q_1	Q_0	0	1	QTR
0	0	00	01	0
0	1	01	10	0
1	0	10	11	0
1	1	11	00	1
		$Q_1^+ Q_0^+$ next state		

- **State labels** are a one-to-one mapping from state encodings to state names

Q_1	Q_0	State name
0	0	A
0	1	B
1	0	C
1	1	D

- The **state/output table** has the same format as the transition table, but state names are substituted in for state encodings

Transition/Output Table:

Q_1	Q_0	EN = 0 ($Q_1^+ Q_0^+$)	EN = 1 ($Q_1^+ Q_0^+$)	QTR
0	0	00	01	0
0	1	01	10	0
1	0	10	11	0
1	1	11	00	1

State Assignments

Q_1	Q_0	State name
0	0	A
0	1	B
1	0	C
1	1	D

State/Output Table:

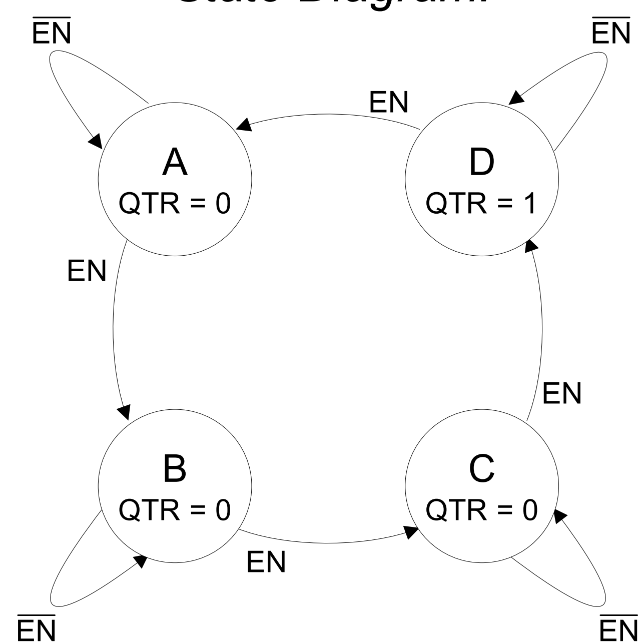
S	EN = 0 (S^+)	EN = 1 (S^+)	QTR
A	A	B	0
B	B	C	0
C	C	D	0
D	D	A	1

- A **state diagram** is a graphical representation of the information in the state/output table
- **Nodes** (or vertices) represent states
  - Moore machines: output values are written in state node
- **Arcs** (or edges) represent state transitions
  - Labeled with a **transition expression**
    - when an arc's transition expression evaluates to 1 for a given input combination, that arc is followed to the next state
  - Mealy machines: output values (or expressions) are written on arcs

State/Output Table:

	EN		
S	0	1	QTR
A	A	B	0
B	B	C	0
C	C	D	0
D	D	A	1
	S^+		

State Diagram:



Sequential Circuit Analysis Recap

1) Find the circuit’s **excitation equations**
2) Using the excitation and characteristic equations, write the circuit’s **transition equations**
3) Write the circuit’s **output equations**
4) From the transition and output equations, create the circuit’s **transition/output table**
5) Create **state labels**
6) Using the transition table and state labels, create the **state table**
7) (optional) Draw the circuit’s **state diagram**