



UM EECS 270 W22

Introduction to Logic Design

20. Sequential Design with JK Flip-Flops



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20. Sequential Design with JK Flip-Flops

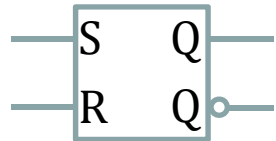
Latch and FF Review



- Basic cross-coupled SR latch
 - Set, Reset, Hold, and **Invalid**
- D latch
 - Set and Reset
- Clocked *level-sensitive* D latch
 - Set, Reset, and Hold *we need another input*
- Clocked *edge-triggered* D flip-flop
 - Set, Reset, and Hold
- Synchronous (clocked) latches and flip-flops enable separation of functional and timing considerations
 - Functional: ignore timing and focus on functional correctness
 - Timing: ignore function and ensure setup and hold constraints are satisfied
- But are there other types of latches besides SR and D?

Functional Classification of Memory Elements

SR Device

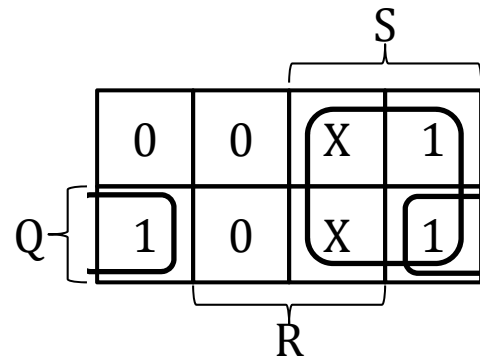


Function
(next-state)
Table

S	R	Q^+	Function
0	0	Q	Hold
0	1	0	Reset
1	0	1	Set
1	1	—	Invalid

Characteristic
(next-state)
Equation

$$Q^+ = S + R'Q$$

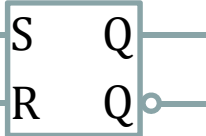
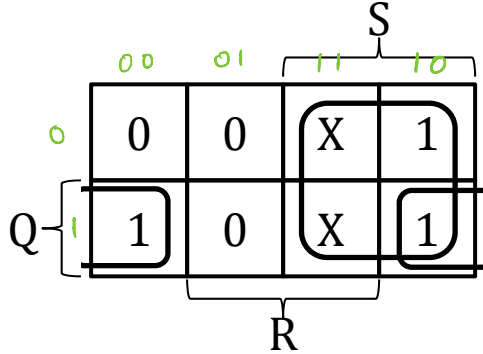


Excitation
(required-input)
Table

Q	Q^+	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

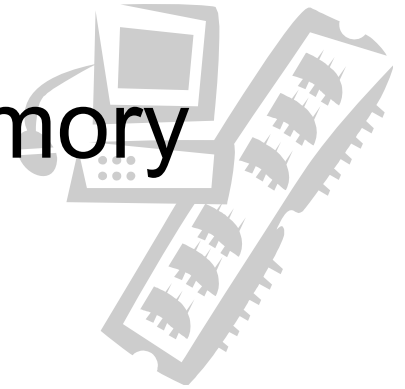
Functional Classification of Memory Elements

SR Device

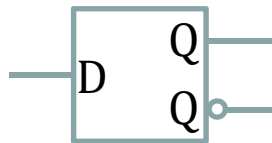
Symbol	Function (next-state) Table	Characteristic (next-state) Equation	Excitation (required-input) Table																																																													
	<p>3 input truth table.</p> <table><tr><th>S</th><th>R</th><th>Q</th><th>Q⁺</th><th>Function</th></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td><td rowspan="2">Hold</td></tr><tr><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td><td>0</td><td rowspan="2">Reset</td></tr><tr><td>0</td><td>1</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td><td rowspan="2">Set</td></tr><tr><td>1</td><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>X</td><td rowspan="2">Invalid</td></tr><tr><td>1</td><td>1</td><td>1</td><td>X</td></tr></table>	S	R	Q	Q ⁺	Function	0	0	0	0	Hold	0	0	1	1	0	1	0	0	Reset	0	1	1	0	1	0	0	1	Set	1	0	1	1	1	1	0	X	Invalid	1	1	1	X	<p>either by holding or resetting.</p> $Q^+ = S + R'Q$ 	<table><tr><th>Q</th><th>Q⁺</th><th>S</th><th>R</th></tr><tr><td>0</td><td>0</td><td>0</td><td>X</td></tr><tr><td>0</td><td>1</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>X</td><td>0</td></tr></table> <p>eg. If I want to go from 0 to 0, how do I apply my input?</p> <p>here we want to make a transition (all possible combination) what input do I apply to achieve this.</p>	Q	Q ⁺	S	R	0	0	0	X	0	1	1	0	1	0	0	1	1	1	X	0
S	R	Q	Q ⁺	Function																																																												
0	0	0	0	Hold																																																												
0	0	1	1																																																													
0	1	0	0	Reset																																																												
0	1	1	0																																																													
1	0	0	1	Set																																																												
1	0	1	1																																																													
1	1	0	X	Invalid																																																												
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Q	Q ⁺	S	R																																																													
0	0	0	X																																																													
0	1	1	0																																																													
1	0	0	1																																																													
1	1	X	0																																																													

Functional Classification of Memory Elements

D Device



Symbol



Function
(next-state)
Table

D	Q^+	Function
0	0	Reset
1	1	Set

Characteristic
(next-state)
Equation

$$Q^+ = D$$

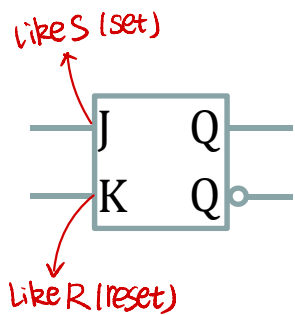
Excitation
(required-input)
Table

Q	Q^+	D
0	0	0
0	1	1
1	0	0
1	1	1

$$D = Q^+$$

Functional Classification of Memory Elements

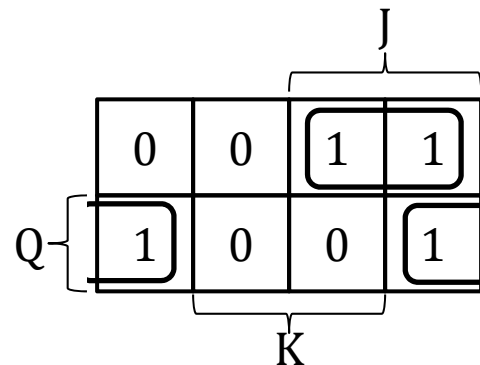
JK Device



J	K	Q^+	Function
0	0	Q	Hold
0	1	0	Reset
1	0	1	Set
1	1	Q'	Toggle

Characteristic
(next-state)
Equation

$$Q^+ = JQ' + K'Q$$

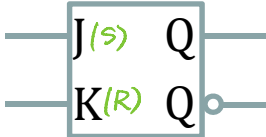
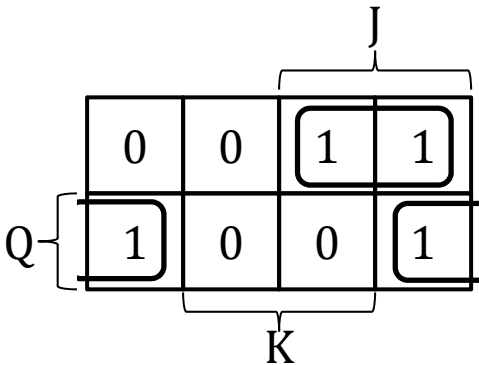


Excitation
(required-input)
Table

Q	Q^+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Functional Classification of Memory Elements

JK Device

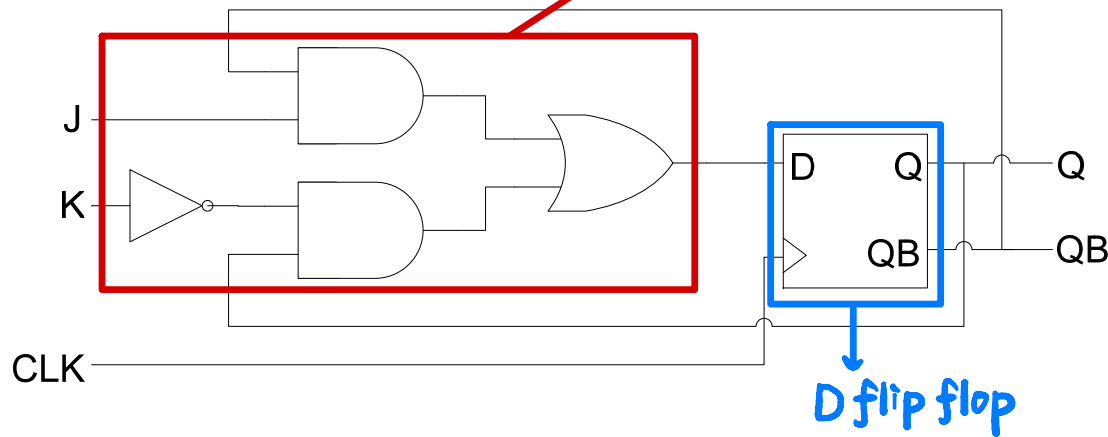
Symbol	Function (next-state) Table	Characteristic (next-state) Equation	Excitation (required-input) Table																																																													
	<table><tr><th>J</th><th>K</th><th>Q</th><th>Q⁺</th><th>Function</th></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td><td rowspan="2">Hold</td></tr><tr><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td><td>0</td><td rowspan="2">Reset</td></tr><tr><td>0</td><td>1</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td><td rowspan="2">Set</td></tr><tr><td>1</td><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td><td>1</td><td rowspan="2">Toggle</td></tr><tr><td>1</td><td>1</td><td>1</td><td>0</td></tr></table> <p>Handwritten notes on the table: - Red boxes around (0,0) to (0,1) with "k don't care" - Green boxes around (1,0) to (1,1) with "J don't care" - Orange boxes around (0,1) to (1,1) with "J don't care" - Purple boxes around (1,0) to (1,1) with "k don't care"</p>	J	K	Q	Q ⁺	Function	0	0	0	0	Hold	0	0	1	1	0	1	0	0	Reset	0	1	1	0	1	0	0	1	Set	1	0	1	1	1	1	0	1	Toggle	1	1	1	0	$Q^+ = JQ' + K'Q$ 	<table><tr><th>Q</th><th>Q⁺</th><th>J</th><th>K</th></tr><tr><td>0</td><td>0</td><td>0</td><td>X</td></tr><tr><td>0</td><td>1</td><td>1</td><td>X</td></tr><tr><td>1</td><td>0</td><td>X</td><td>1</td></tr><tr><td>1</td><td>1</td><td>X</td><td>0</td></tr></table>	Q	Q ⁺	J	K	0	0	0	X	0	1	1	X	1	0	X	1	1	1	X	0
J	K	Q	Q ⁺	Function																																																												
0	0	0	0	Hold																																																												
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0	1	0	0	Reset																																																												
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1	0	0	1	Set																																																												
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1	1	0	1	Toggle																																																												
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Q	Q ⁺	J	K																																																													
0	0	0	X																																																													
0	1	1	X																																																													
1	0	X	1																																																													
1	1	X	0																																																													

use the D flip flop to impliment a Jk flip flop

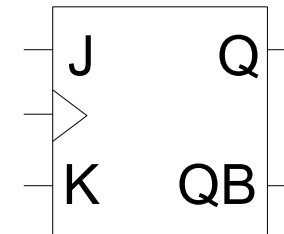
Implementing a JK Flip-Flop



$$Q^+ = JQ' + K'Q$$

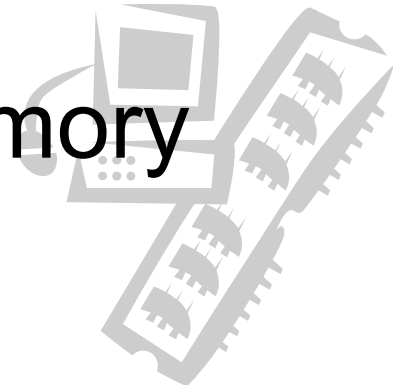


$$D = Q^+ = JQ' + K'Q$$

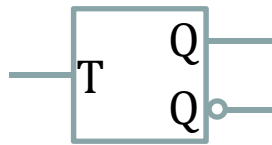


Functional Classification of Memory Elements

T Device



Symbol



Function
(next-state)
Table

T	Q^+	Function
0	Q	Hold
1	Q'	Toggle

Characteristic
(next-state)
Equation

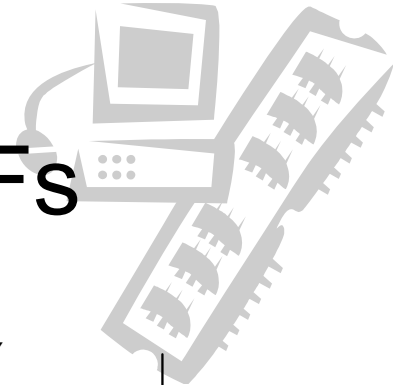
$$Q^+ = T \oplus Q$$

Excitation
(required-input)
Table

Q	Q^+	T
0	0	0
0	1	1
1	0	1
1	1	0

$$T = Q \oplus Q^+$$

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

$\overline{S^+}$

S	Q ₁ Q ₀
A	0 0
B	0 1
C	1 1
D	1 0

Q ₁ Q ₀	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$\overline{Q_1^+ Q_0^+}$

Q ₁ Q ₀	X Y				X Y				X Y			
	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00								
01	00	01	00	11								
11	00	10	00	00								
10	00	01	00	11								

$\overline{Q_1^+ Q_0^+}$

X Y			
00	01	11	10

$J_1 K_1$

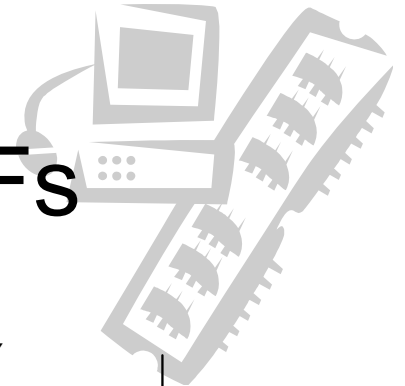
X Y			
00	01	11	10

$J_0 K_0$

QQ ⁺	JK
00	0X
01	1X
10	X1
11	X0

Excitation
(required-input)
Table

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

S^+

S	Q ₁	Q ₀
A	0	0
B	0	1
C	1	1
D	1	0

Q ₁ Q ₀	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$Q_1^+ Q_0^+$

XY					XY					XY				
Q ₁ Q ₀	00	01	11	10	Q ₁ Q ₀	00	01	11	10	Q ₁ Q ₀	00	01	11	10
00	00	01	00	00	0X (0→0)	0X	0X	0X	0X					
01	00	01	00	11	0X (0→0)	0X	0X	1X	1X					
11	00	10	00	00	X1 (1→0)	X0	X1	X1	X1					
10	00	01	00	11	X1 (1→0)	X1	X1	X0	X0					

QQ ⁺	JK
00	0X
01	1X
10	X1
11	X0

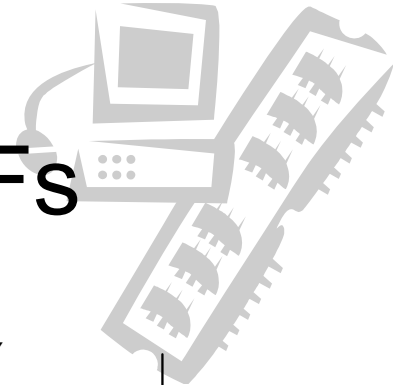
Excitation
(required-input)
Table

If D flip-flop: $D_1 = Q_1^+$
 $D_0 = Q_0^+$ } use the k map to find the expression

$J_1 K_1$
↑
k map

$J_0 K_0$
↑
k map

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

S^+

S	Q_1	Q_0
A	0	0
B	0	1
C	1	1
D	1	0

$Q_1 Q_0$	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$Q_1^+ Q_0^+$

$Q_1 Q_0$	X Y				$Q_1 Q_0$	X Y				$Q_1 Q_0$	X Y			
	00	01	11	10		00	01	11	10		00	01	11	10
00	00	01	00	00	0X									
01	00	01	00	11										
11	00	10	00	00										
10	00	01	00	11										

$Q_1^+ Q_0^+$

X Y			
00	01	11	10

$J_1 K_1$

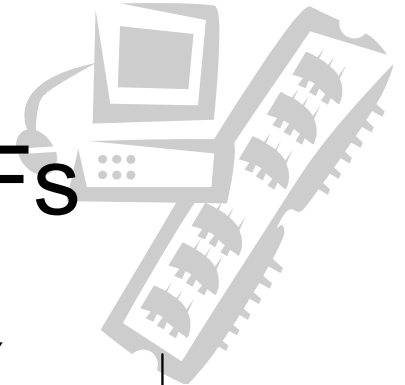
X Y			
00	01	11	10

$J_0 K_0$

$Q Q^+$	JK
00	0X
01	1X
10	X1
11	X0

Excitation
(required-input)
Table

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

$\overline{S^+}$

S	Q_1	Q_0
A	0	0
B	0	1
C	1	1
D	1	0

$Q_1 Q_0$	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$\overline{Q_1^+ Q_0^+}$

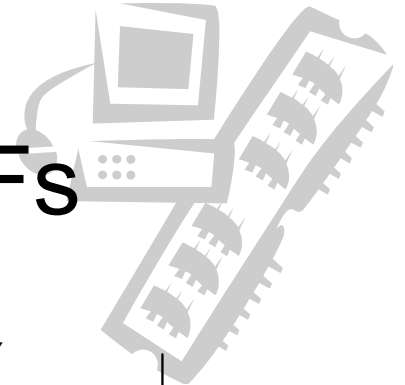
$Q_1 Q_0$	X Y				X Y				X Y			
	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X							
01	00	01	00	11	0X							
11	00	10	00	00								
10	00	01	00	11								

$\overline{Q_1^+ Q_0^+}$ $J_1 K_1$ $J_0 K_0$

$Q Q^+$	JK
00	0X
01	1X
10	X1
11	X0

Excitation
(required-input)
Table

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

$\overline{S^+}$

S	Q_1	Q_0
A	0	0
B	0	1
C	1	1
D	1	0

$Q_1 Q_0$	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$\overline{Q_1^+ Q_0^+}$

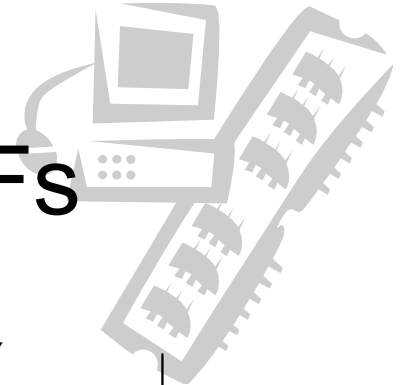
$Q_1 Q_0$	X Y					X Y					X Y				
	00	01	11	10		00	01	11	10		00	01	11	10	
00	00	01	00	00	0X										
01	00	01	00	11	0X										
11	00	10	00	00	X1										
10	00	01	00	11											

$\overline{Q_1^+ Q_0^+}$ $\overline{J_1 K_1}$ $\overline{J_0 K_0}$

$Q Q^+$	JK
00	0X
01	1X
10	X1
11	X0

Excitation
(required-input)
Table

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

$\overline{S^+}$

S	Q_1	Q_0
A	0	0
B	0	1
C	1	1
D	1	0

$Q_1 Q_0$	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$\overline{Q_1^+ Q_0^+}$

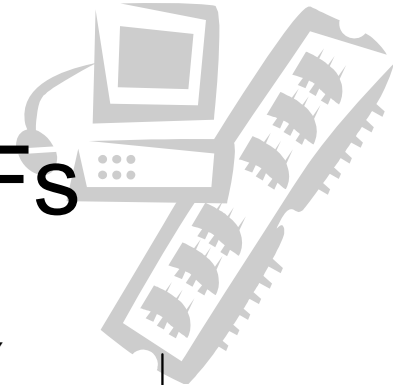
$Q_1 Q_0$	X Y					X Y					X Y				
	00	01	11	10		00	01	11	10		00	01	11	10	
00	00	01	00	00	0X										
01	00	01	00	11	0X										
11	00	10	00	00	X1										
10	00	01	00	11	X1										

$\overline{Q_1^+ Q_0^+}$ $J_1 K_1$ $J_0 K_0$

$Q Q^+$	JK
00	0X
01	1X
10	X1
11	X0

Excitation
(required-input)
Table

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

$\overline{S^+}$

S	Q_1	Q_0
A	0	0
B	0	1
C	1	1
D	1	0

$Q_1 Q_0$	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$\overline{Q_1^+ Q_0^+}$

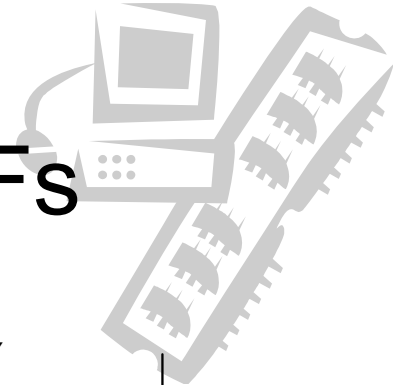
$Q_1 Q_0$	X Y					X Y					X Y				
	00	01	11	10		00	01	11	10		00	01	11	10	
00	00	01	00	00	0X										
01	00	01	00	11	0X										
11	00	10	00	00	X1										
10	00	01	00	11	X1										

$\overline{Q_1^+ Q_0^+}$ $J_1 K_1$ $J_0 K_0$

$Q Q^+$	JK
00	0X
01	1X
10	X1
11	X0

Excitation
(required-input)
Table

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

$\overline{S^+}$

S	Q_1	Q_0
A	0	0
B	0	1
C	1	1
D	1	0

$Q_1 Q_0$	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$\overline{Q_1^+ Q_0^+}$

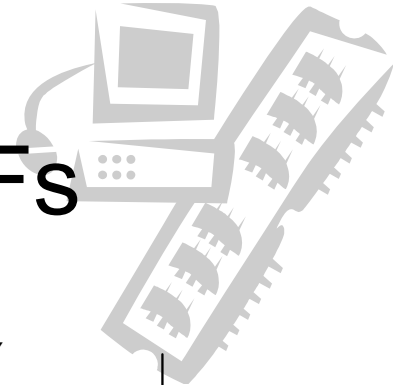
$Q_1 Q_0$	X Y				X Y				X Y			
	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X	0X						
01	00	01	00	11	0X	0X						
11	00	10	00	00	X1	X0						
10	00	01	00	11	X1	X1						

$\overline{Q_1^+ Q_0^+}$ $J_1 K_1$ $J_0 K_0$

$Q Q^+$	JK
00	0X
01	1X
10	X1
11	X0

Excitation
(required-input)
Table

Combination Lock with JK FFs



S	X Y				UNLOCK
	00	01	11	10	
A	A	B	A	A	0
B	A	B	A	C	0
C	A	D	A	A	0
D	A	B	A	C	1

S^+

S	Q ₁	Q ₀
A	0	0
B	0	1
C	1	1
D	1	0

Q ₁ Q ₀	X Y				UNLOCK
	00	01	11	10	
00	00	01	00	00	0
01	00	01	00	11	0
11	00	10	00	00	0
10	00	01	00	11	1

$Q_1^+ Q_0^+$

XY					XY				XY			
Q ₁ Q ₀	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X	0X	0X	0X	0X	1X	0X	0X
01	00	01	00	11	0X	0X	0X	1X	X1	X0	X1	X0
11	00	10	00	00	X1	X0	X1	X1	X1	X1	X1	X1
10	00	01	00	11	X1	X1	X1	X0	0X	1X	0X	1X

$Q_1^+ Q_0^+$ $J_1 K_1$ $J_0 K_0$

two k-map
 • one for J₁
 • one for K₁

two k-map
 • one for J₀
 • one for K₀

Excitation
 (required-input)
 Table

Combination Lock with JK FFs



	XY				XY				XY			
Q_1Q_0	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X	0X	0X	0X	0X	1X	0X	0X
01	00	01	00	11	0X	0X	0X	1X	X1	X0	X1	X0
11	00	10	00	00	X1	X0	X1	X1	X1	X1	X1	X1
10	00	01	00	11	X1	X1	X1	X0	0X	1X	0X	1X
	$Q_1^+Q_0^+$				J_1K_1				J_0K_0			

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
J_1				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
K_1				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
J_0				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
K_0				

Combination Lock with JK FFs



	XY				XY				XY			
Q_1Q_0	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X	0X	0X	0X	0X	1X	0X	0X
01	00	01	00	11	0X	0X	0X	1X	X1	X0	X1	X0
11	00	10	00	00	X1	X0	X1	X1	X1	X1	X1	X1
10	00	01	00	11	X1	X1	X1	X0	0X	1X	0X	1X
	$Q_1^+Q_0^+$				J_1K_1				J_0K_0			

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
J_1				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
K_1				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
J_0				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
	11			
	10			
K_0				

Combination Lock with JK FFs



Q_1Q_0	XY				XY				XY			
	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X	0X	0X	0X	0X	1X	0X	0X
01	00	01	00	11	0X	0X	0X	1X	X1	X0	X1	X0
11	00	10	00	00	X1	X0	X1	X1	X1	X1	X1	X1
10	00	01	00	11	X1	X1	X1	X0	0X	1X	0X	1X
	$Q_1^+Q_0^+$				J_1K_1				J_0K_0			

		XY			
		00	01	11	10
Q_1	00				
	01				1
	11	X	X	X	X
	10	X	X	X	X
		J_1			

		XY			
		00	01	11	10
Q_1	00				
	01				
	11				
	10				
		K_1			

		XY			
		00	01	11	10
Q_1	00				
	01				
	11				
	10				
		J_0			

		XY			
		00	01	11	10
Q_1	00				
	01				
	11				
	10				
		K_0			

Combination Lock with JK FFs



	XY				XY				XY			
Q_1Q_0	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0 X	0 X	0 X	0 X	0X	1X	0X	0X
01	00	01	00	11	0 X	0 X	0 X	1 X	X1	X0	X1	X0
11	00	10	00	00	X 1	X 0	X 1	X 1	X1	X1	X1	X1
10	00	01	00	11	X 1	X 1	X 1	X 0	0X	1X	0X	1X
	$Q_1^+Q_0^+$				J_1K_1				J_0K_0			

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			1
Q_1	11	X	X	X
	10	X	X	X
J_1				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
Q_1	11			
	10			
K_1				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
Q_1	11			
	10			
J_0				

	XY			
	00	01	11	10
Q_1Q_0	00			
	01			
Q_1	11			
	10			
K_0				

Combination Lock with JK FFs



Q_1Q_0	XY				XY				XY			
	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X	0X	0X	0X	0X	1X	0X	0X
01	00	01	00	11	0X	0X	0X	1X	X1	X0	X1	X0
11	00	10	00	00	X1	X0	X1	X1	X1	X1	X1	X1
10	00	01	00	11	X1	X1	X1	X0	0X	1X	0X	1X
	$Q_1^+Q_0^+$				J_1K_1				J_0K_0			

		XY			
		00	01	11	10
Q_1	00				
	01				1
	11	X	X	X	X
	10	X	X	X	X
		J_1			

		XY			
		00	01	11	10
Q_1	00	X	X	X	X
	01	X	X	X	X
	11	1		1	1
	10	1	1	1	
		K_1			

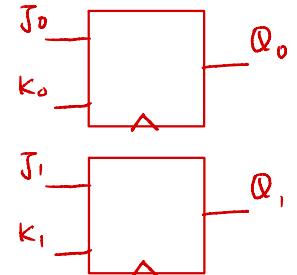
		XY			
		00	01	11	10
Q_1	00				
	01				
	11				
	10				
		J_0			

		XY			
		00	01	11	10
Q_1	00				
	01				
	11				
	10				
		K_0			

Combination Lock with JK FFs



	XY				XY				XY			
Q_1Q_0	00	01	11	10	00	01	11	10	00	01	11	10
00	00	01	00	00	0X	0X	0X	0X	0X	1X	0X	0X
01	00	01	00	11	0X	0X	0X	1X	X1	X0	X1	X0
11	00	10	00	00	X1	X0	X1	X1	X1	X1	X1	X1
10	00	01	00	11	X1	X1	X1	X0	0X	1X	0X	1X
	$Q_1^+Q_0^+$				J_1K_1				J_0K_0			



	XY			
	00	01	11	10
Q_1Q_0 00				
01				1
11	X	X	X	X
10	X	X	X	X
	J_1			

	XY			
	00	01	11	10
Q_1Q_0 00	X	X	X	X
01	X	X	X	X
11	1		1	1
10	1	1	1	
	K_1			

	XY			
	00	01	11	10
Q_1Q_0 00		1		
01	X	X	X	X
11	X	X	X	X
10		1		1
	J_0			

	XY			
	00	01	11	10
Q_1Q_0 00	X	X	X	X
01	1		1	
11	1	1	1	1
10	X	X	X	X
	K_0			



Lab 7 Hint



