Lesson 14:

I. Manipulator Jacobians (cont'd) (MLS 3.4)

II. Singularities (MLS 3.4.3)

III. Manipulability (MLS 3.4.4)

IV. Wrenches & Joint torques (MLS 3.4.2)

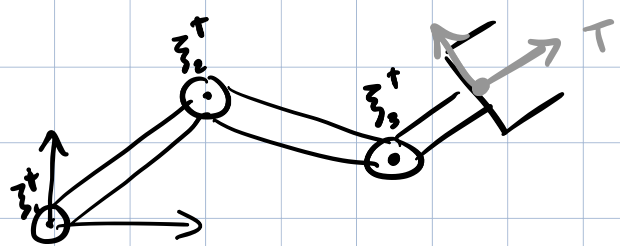
I. Manipulator Jacobians (cont'd)

Recall: Body manipulator Jacobian J_{st}^b satisfies

$$V_{st}^b = J_{st}^b(\theta)\,\dot{\theta}, \quad \text{where} \quad J_{st}^b(\theta) = \left[\xi_1^\dagger, \xi_2^\dagger, \ldots, \xi_n^\dagger\right]$$

$$\xi_i^\dagger = Ad^{-1}_{(e^{i} \cdots e^{n} g_{st}(0))}\,\xi_i$$

- Columns $\xi_i^\dagger$ correspond to joint twist coords w.r.t. current config of the tool frame.



- Derived in similar manner to the spatial case from

$$\hat{V}_{st}^b = g_{st}^{-1}(\theta)\,\dot{g}_{st}(\theta)$$

- Can relate body & spatial Jacobians by

$$J_{st}^s(\theta) = Ad_{g_{st}(\theta)}\,J_{st}^b(\theta)$$

- Can use the body or spatial Jacobian to compute the inst. velocity of a point, specifically attached to end effector

$$\bar{v}_q^b = \hat{V}_{st}^b \bar{q}_b = \left(J_{st}^b(\theta)\dot{\theta}\right)^\wedge \bar{q}_b \qquad \text{(in body/tool coords)}$$

$$\bar{v}_q^s = \hat{V}_{st}^s \bar{q}_s = \left(J_{st}^s(\theta)\dot{\theta}\right)^\wedge \bar{q}_s \qquad \text{(in spatial coords)}$$

<u>Control implications:</u>

- Can use the manipulator Jacobian to move a robot from one end-effector config. to another <u>without</u> calculating inv. kinematics explicitly.

- If J_{st} is square (e.g. 6-DOF in $W \subset SE(3)$) and invertible, then

$$\dot{\theta}(t) = \left(J_{st}^s(\theta(t))\right)^{-1} V_{st}^s(t)$$

desired velocity profile

$$= \left(J_{st}^b(\theta(t))\right)^{-1} V_{st}^b(t)$$

note: or body coords by reverse order

- Calculate $\hat{V}_{st}^s(t) = \dot{g}_{st}(t)\, g_{st}^{-1}(t)$ from a desired workspace path $g_{st}(t)$ from $g_{st}(0)$ to $g_{st}(T)$
- Robot controls desired $\dot{\theta}(t)$

II. Singularities

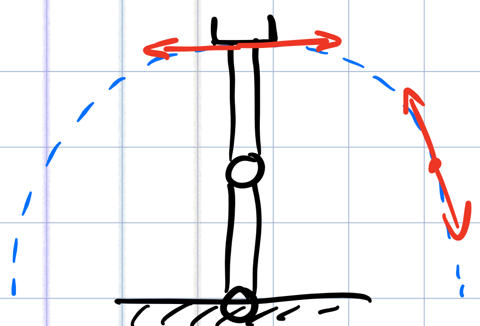
<u>Def</u>: A singular config. is a robot config. at which the manipulater Jacobian J_{st} drops rank.

- For square J_{st}, $\det(J_{st}) = 0$ at rank drop.

• Singularity means the robot is unable to acheive inst. motion in at least one direction of the workspace.

- When approaching singular config. increasingly faster joint velocities are required for increasingly smaller end effector velocities.
- Avoid singularities when conttolling via end effector velocity.

<u>Ex</u>: For planar 2-DOF arm where $W \subset \mathbb{R}^2$



the boundary of the workspace is a set of singular configs.

<u>Ex</u>: Honda ASIMO

What happens when J_{st} is not square?

For n-DOF robot, $J_{st} \in \mathbb{R}^{\dim W \times n}$, where $\dim W \leq 6$,

if $n \neq \dim W$ then J_{st} is not square!

<u>Case 1:</u> $(n < \dim W)$

The end effector starts with fewer DOFs than workspace, and loses workspace DOFs at singular configurations.

$\Rightarrow$ Cannot invert Jacobian for full workspace (can work w/ a lower-dimensional workspace)

<u>Case 2:</u> $(n > \dim W)$

This is a <u>redundant</u> manipulator.

Has "internal" DOFs to avoid $\operatorname{rank}(J_{st})$ dropping.

How do you "invert" J_{st} in this case?

Use pseudo-inverse to calculate (non-unique) joint velocities for a desired V_{st}:

$$\dot{\Theta} = J_{st}^T \underbrace{\left(J_{st} J_{st}^T \right)}_{\dim W \times \dim W}^{-1} V_{st}$$

$\dim W \times \dim W \longrightarrow$ Invertibility requires $\operatorname{rank}(J_{st}) = \dim W$

or

$$\dot{\Theta} = A J_{st}^T \left(J_{st} A J_{st}^T \right)^{-1} V_{st}$$

, for weighting matrix $A \in \mathbb{R}^{n \times n}$, $A > 0$

III. Manipulability

Recall workspace $W = \{ g_{st}(\theta) \mid \theta \in Q \} \subset SE(3)$

note: "reachable workspace" and "dexterous workspace"
(just positions) (all positions where you can freely orient the end effector)

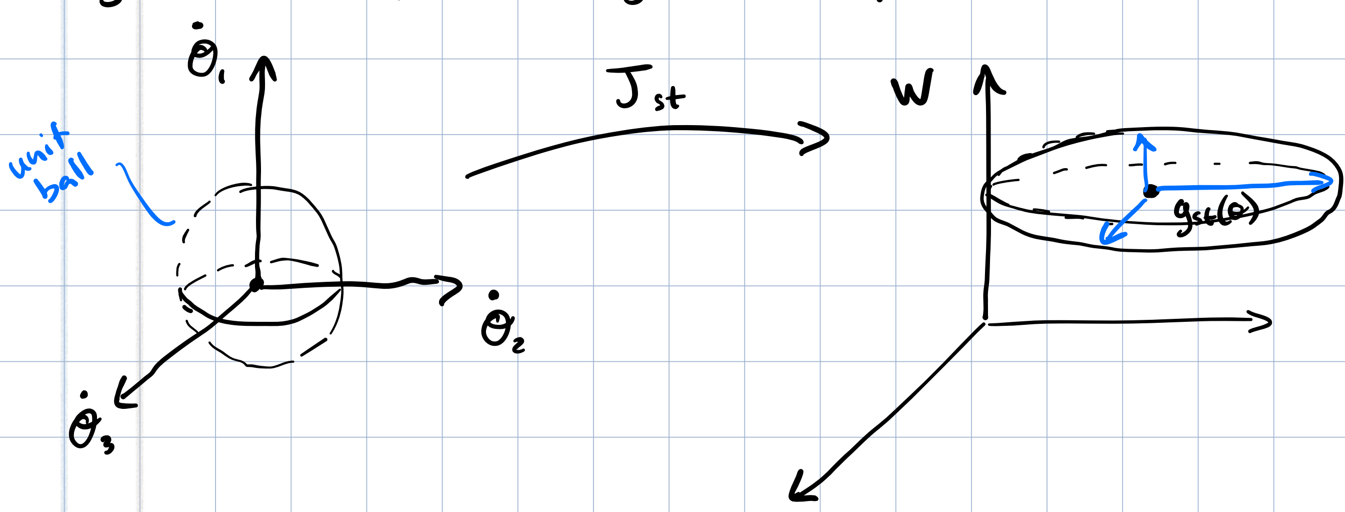
Manipulability describes robot's ability to move freely in all directions of the workspace.

- Local property characterized by the manipulator Jacobian, relating inf. joint motions to inf. workspace motions.

- Manipulability determined by singular values of J_{st}
  - For square J_{st}, can define as

$$\mu(\theta) := \det(J_{st}(\theta)) = \overbrace{\prod_{i=1}^{n}}^{\text{product of}} \lambda_i(J_{st}(\theta))$$

- Measures the volume of velocity ellipsoid in workspace generated by unit joint velocity vectors:



- Principal axes of ellipsoid correspond to eigenvectors of J_{st}
- Magnitude of principal axes correspond to eigenvalues of J_{st}