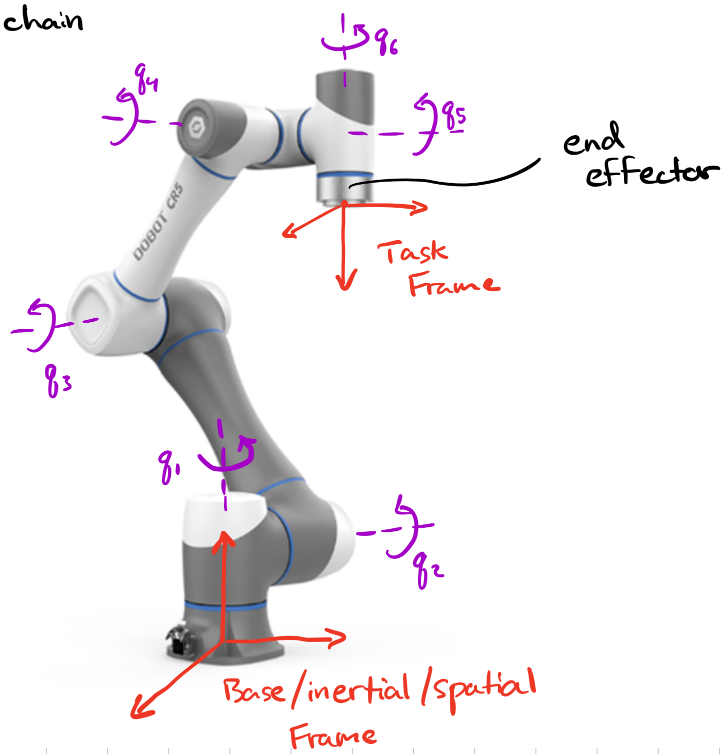
Basic Definitions

Serial kinematic chain



Joints can be revolute or prismatic

- Configuration vector

"degrees of freedom"

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} \quad \text{for } n\text{-DOF robot}$$

unit circle

$$\text{where} \quad q_i \in S^1 \quad \text{for revolute joints}, \quad q_i \in [0, 2\pi)$$

$$q_i \in \mathbb{R}^1 \quad \text{for prismatic joints}$$

- Configuration Space

n-dimensional torus

$$Q = \underbrace{S^1 \times S^1 \times \cdots \times S^1}_{n \text{ times for } n\text{-DOF revolute robot}} = T^n$$

Cartesian product

- Pose of the end effector

$$(p, R)$$

position vector — p

orientation (rotation matrix) — R

- Workspace / Taskspace

$$W = \{\text{All reachable poses}\}$$

- Forward Kinematics

$$F : Q \longrightarrow W$$

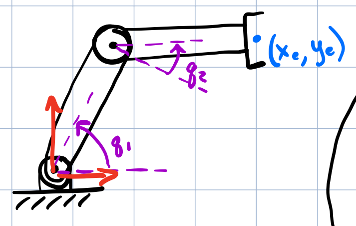
Note: F is onto but it is not one-to-one

- Inverse Kinematics

$$W \longrightarrow Q$$

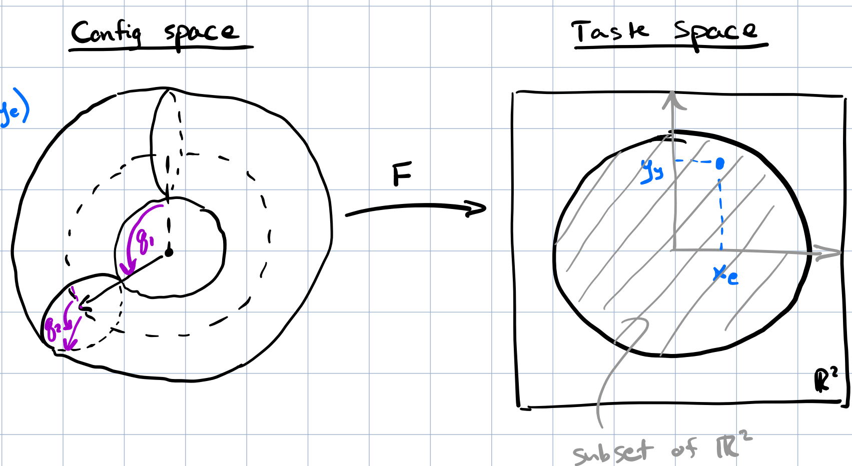
Note: Mapping is not unique

Ex: RR robot



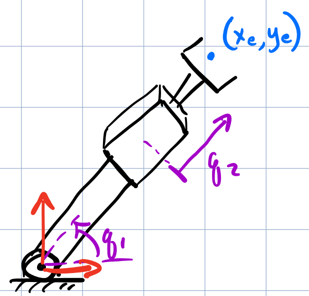
$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$Q = S^1 \times S^1 = T^2$$

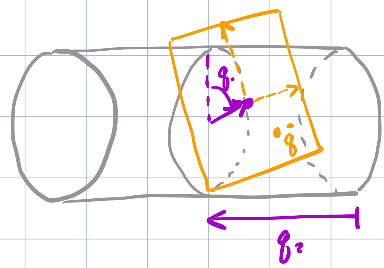


*Assume links 1 & 2 are the same length

Ex: RP robot

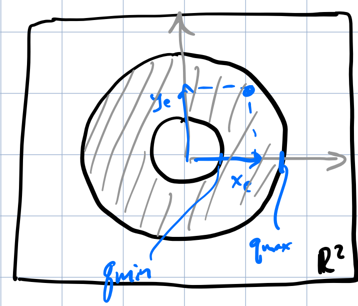


Config Space:



$\xrightarrow{F}$

Task space



$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$Q = S^1 \times \mathbb{R}^1$$

* Assume $q_2 \in [q_{min}, q_{max}]$

- State Vector

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \in \underbrace{\mathcal{X}}_{\text{state space}} = \underbrace{TQ}_{\text{Tangent Bundle}} := \bigcup_{q \in Q} T_q Q$$

Typically we say that $x \in \mathbb{R}^{2n}$ for an n-DOF robot

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \underbrace{f\left(\begin{bmatrix} q \\ \dot{q} \end{bmatrix}\right)}_{\text{drift vector field}} + \underbrace{g\left(\begin{bmatrix} q \\ \dot{q} \end{bmatrix}\right)}_{\text{Matrix of control vector fields}} u \iff \dot{x} = f(x) + g(x)u$$