ROB 510 Exam-II (Winter 2022)

Prof. Robert Gregg
24 Hour Take-Home Exam
Released: 5pm on Monday, April 25, 2022
Due: 5pm on Tuesday, April 26, 2022

HONOR PLEDGE: Copy (NOW) and SIGN (**after the exam is completed**): I have neither given nor received aid on this exam, nor have I observed a violation of the Engineering Honor Code.

SIGNATURE
(Sign **after** the exam is completed)

LAST NAME (PRINTED), *FIRST NAME*

FILL IN YOUR NAME NOW. COPY THE HONOR CODE NOW.

RULES:

1. **NO COLLABORATION OF ANY KIND**
2. OPEN TEXTBOOK, CLASS NOTES, HOMEWORK
3. GOOGLING SOLUTIONS IS CONSIDERED ACADEMIC DISHONESTY, AND MOST PROBLEMS CANNOT BE EASILY FOUND ON THE WEB ANYWAY
4. CALCULATOR/COMPUTER ALLOWED BUT MUST SHOW CALCULATION STEPS FOR FULL CREDIT
5. SUBMIT QUALITY PHOTOS/SCANS TO GRADESCOPE BY DEADLINE (STRICT)

The maximum possible score is 80. To maximize your own score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly on this booklet.

Problems 1 - 5 (30 points: 5×6)

Instructions. Each part of a question is worth 1.5 points. Submit your answers to questions 1-5 as follows:

1. Download the answer sheet from Canvas (`ROB510 Final Wi2022 TF Answer Sheet.pdf`).
2. Print it, or open it in your favorite PDF viewer app (see Canvas announcement if you need ideas).
3. Clearly mark your answer to each question on the answer sheet.
4. Scan or export your solutions, and upload them to Gradescope.

Do not modify the answer sheet, or attach any extra pages. You do not need to show your work. Answers written directly on the questions below will not be graded.

1. (Robot Kinematics) **Circle True or False as appropriate for the following statements:**

T F (a) A pitch defined as $h = \frac{-\omega^T \hat{\omega} q}{\|w\|^2}$ represents a pure rotation.

T F (b) Given frames A, B, C and body velocities V_{ab}^b, V_{cb}^b, the relationship $V_{ab}^b = -Ad_{g_{bc}} V_{bc}^b$ holds.

T F (c) The configuration space of a 2-DOF revolute-joint robot is a sphere.

T F (d) End-effector manipulability of an n-DOF robot with manipulator Jacobian $J(q) \in \mathbb{R}^{6\times n}$ at a particular configuration $q \in Q$ is associated with the determinant of $J(q)^T J(q)$.

2. (Robot Dynamics) **Circle True or False as appropriate for the following statements:**

T F (a) Given an open-chain manipulator with viscous friction at the joints, the vector $C(q, \dot{q})\dot{q}$ in the equations of motion contains the damping torques.

T F (b) Consider a 4-DOF revolute-joint manipulator with coordinates $q = (q_1, \dots, q_4)^T$ in the usual order. Decompose the inertia/mass matrix $M(q) = \begin{bmatrix} M_a(q) & M_{ab}(q) \\ M_{ab}^T(q) & M_b(q) \end{bmatrix}$, where $M_b(q) \in \mathbb{R}^{2\times 2}$. Then $M_b(q)$ does not depend on q_1, q_2, or q_3.

T F (c) The inertia/mass matrix of any prismatic-joint robot is unbounded.

T F (d) The robot equations of motion are invariant with respect to the location of the fixed spatial frame.

3. (Stability and Passivity) **Circle True or False as appropriate for the following statements:**

Note: For Lyapunov questions, positive definite means $V(x) > 0$ for all $x \neq 0$ and $V(0) = 0$.

T **F** (a) The origin of system

$$\dot{x}_1 = -x_2,$$
$$\dot{x}_2 = x_1 - (1 - x_1^2)x_2$$

is locally exponentially stable.

T **F** (b) Consider a positive-definite Lyapunov function $V(x)$ for a system $\dot{x} = f(x)$ with an asymptotically stable equilibrium point at the origin. The set $E = \{x | V(x) \leq c\}$, for some constant $c > 0$, is a subset of the basin of attraction if $\dot{V}(x) < 0$ for all non-zero $x \in E$.

T **F** (c) Given a positive-definite Lyapunov function $V(x)$, the origin of system $\dot{x} = f(x)$ is unstable if $\dot{V}(x) > 0$ for some x.

T **F** (d) Consider an n-link flexible joint robot with robot joint coordinates $q_1 \in \mathbb{R}^n$, motor shaft coordinates $q_2 \in \mathbb{R}^n$, and motor torques $u \in \mathbb{R}^n$. This robotic system is input-output passive with input u and output $\dot{q}_2$.

4. (Multivariable Robot Control) **Circle True or False as appropriate for the following statements:**

T **F** (a) A flexible-joint robotic arm attached to the International Space Station can stabilize/track a desired set-point using proportional-derivative control with motor shaft feedback.

T **F** (b) Given inertia/mass matrix $M(q)$ and Coriolis matrix $C(q, \dot{q})$ of a robot, the matrix $\dot{M}(q) - 2C(q, \dot{q}) = 0$.

T **F** (c) Consider passivity-based motion control of a fully-actuated n-DOF robot, where we wish to track a constant vector $q^d \in \mathbb{R}^n$. Then the closed-loop dynamics are equivalent to $M(q)(\ddot{q} + \Lambda\dot{q}) + (C(q, \dot{q}) + K)(\dot{q} + \Lambda\tilde{q}) = 0$, where $\tilde{q} = q - q^d$ and K, Λ are diagonal matrices of constant, positive gains.

T **F** (d) Consider energy shaping control of a fully-actuated robot to achieve desired dynamics $\bar{M}(q)\ddot{q} + \bar{C}(q, \dot{q})\dot{q} + \bar{G}(q) = 0$ in place of the original dynamics $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u$. The control law u requires calculation of the inverse of the original inertia/mass matrix $M(q)$.

5. (Force and Underactuated Control) **Circle True or False as appropriate for the following statements:**

T F (a) Consider a nonlinear system $\dot{x} = f(x) + g(x)u$, where $u \in \mathbb{R}^1$, $x \in \mathbb{R}^n$. If the system is (locally) feedback linearizable with transformation $y = T(x)$, then $L_g T_n(x) \neq 0, \forall x \in \mathbb{R}^n$.

T F (b) It is possible to perform hybrid impedance and force control of a robot pushing a block on ice (assume no friction).

T F (c) Consider an n-DOF robot with active coordinates $q_1 \in \mathbb{R}^m$, passive coordinates $q_2 \in \mathbb{R}^{n-m}$, and inertia/mass matrix $M(q) = \begin{bmatrix} M_1(q) & 0 \\ 0 & M_2(q) \end{bmatrix}$. It is possible to perform *collocated* partial feedback linearization.

T F (d) Consider output feedback linearization with output $y = h(q, \dot{q}) \in \mathbb{R}^p$ for a robot with coordinates $q \in \mathbb{R}^n$ and control input $u \in \mathbb{R}^p$, $p < n$. It is possible for this system to have vector relative degree one.

Partial Credit Section of the Exam

For the next problems, partial credit is awarded and you MUST show your work. Unsupported answers, even if correct, receive zero credit. In other words, right answer, wrong reason or no reason could lead to no points. If you come to me and ask whether you have written enough, my answer will be,

"I do not know",

because answering "yes" or "no" would be unfair to everyone else. If you show the steps you followed in deriving your answer, you'll probably be fine. If something was explicitly derived in lecture, handouts or homework, you do not have to re-derive it. You can state it as a known fact and then use it.

6. (20 points) Spring-Loaded Inverted Pendulum (Place your answers in the **boxes** and show your work below.)

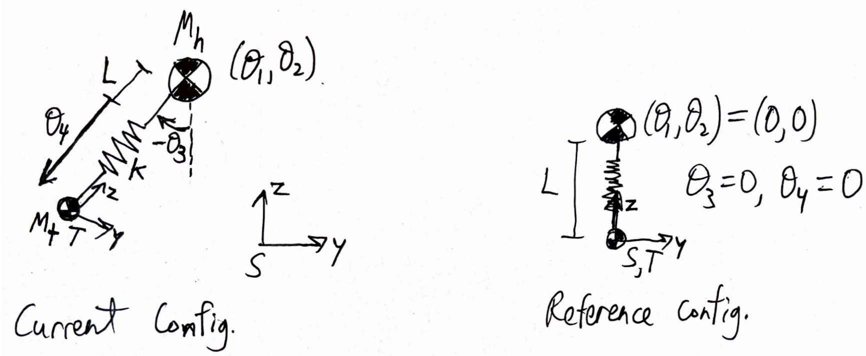

Figure 1: The Spring-Loaded Inverted Pendulum (SLIP) is used to model hopping locomotion. This SLIP has point masses at the hip (M_h) and toe (M_t) resulting in zero link inertia. The Cartesian y-z coordinates of the hip mass are (θ_1, θ_2). The leg orientation is given by θ_3. The springy leg is modeled with a prismatic joint (θ_4) and has leg length $L + \theta_4$, where L corresponds to the resting length of the springy leg (when $\theta_4 = 0$). The leg stiffness is K. The tool frame T is located at the toe, which coincides with the spatial frame S at the reference configuration.

(a) (5 points) Find the twist coordinates ξ_i and $g_{st}(0)$ for the product of exponentials. Use the angle conventions as shown, even if they are not what you would pick.

$\xi_1 = \left[\quad \right], \quad \xi_2 = \left[\quad \right], \quad \xi_3 = \left[\quad \right], \quad \xi_4 = \left[\quad \right], \quad g_{st}(0) = \left[\qquad \right]$

(b) (5 points) Determine the forward kinematic map $g_{st}(\theta)$. You will want to use Mathematica, but be sure to show your steps on the next page (e.g., define each matrix used). Please simplify the solution before writing it here.

$g_{st}(\theta) =$

(c) (6 points) Determine the inertia/mass matrix $M(\theta)$. You will want to use Mathematica, but be sure to show your steps on the next page (e.g., define each matrix used). Please simplify the solution before writing it here.

$M(\theta) =$

(d) (4 points) Find the potential energy $P(\theta)$. You will want to use Mathematica, but be sure to show your steps on the next page (e.g., define each term used). Please simplify the solution before writing it here.

$P(\theta) =$

Show your steps and reasoning below. No reasoning $\Longrightarrow$ no points.

Please show your work for question 6.

7. (15 points) Feedback Linearization of the Reaction Wheel Pendulum (Place your answers in the **boxes** and show your work below.)

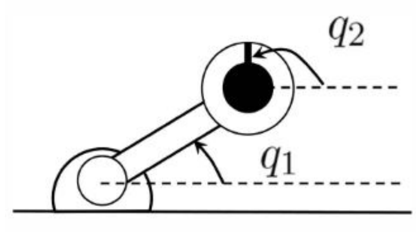

Figure 2: The Reaction Wheel Pendulum is a simple pendulum with a rotating disk at the distal end. Actuating the disk results in a reaction torque to move the pendulum.

The dynamics of the Reaction Wheel Pendulum are given by

$$\begin{aligned} J_1\ddot{q}_1 + mg\ell\cos q_1 &= -u \\ J_2\ddot{q}_2 &= u, \end{aligned}$$

where constants J_1, J_2, m, g, ℓ are all positive. We are going to show that this robot is locally feedback linearizable.

(a) (2 points) Determine the vector fields f and g in the state space dynamics $\dot{x} = f(x) + g(x)u$ for state vector $x = (q_1, q_2, \dot{q}_1, \dot{q}_2)^T$. Do not use a different ordering in the state vector.

$f(x) =$ $\qquad\qquad$ $g(x) =$

(b) (6 points) Determine the vector fields $\mathrm{ad}_f g$, $\mathrm{ad}_f^2 g$, $\mathrm{ad}_f^3 g$.

$\mathrm{ad}_f g =$ $\qquad\qquad$ $\mathrm{ad}_f^2 g =$ $\qquad\qquad$ $\mathrm{ad}_f^3 g =$

(c) (1 points) Determine a region $U \subset \mathbb{R}^4$ for which the vector fields g, $\mathrm{ad}_f g$, $\mathrm{ad}_f^2 g$, and $\mathrm{ad}_f^3 g$ are linearly independent.

$U = \{x| \qquad\qquad \}$

(d) (6 points) Is the distribution $\Lambda = \mathrm{span}\{g, \mathrm{ad}_f g, \mathrm{ad}_f^2 g\}$ involutive? Answer yes or no here but you must prove it below for credit. Hint: The coefficients in the linear combinations are allowed to be smooth functions on $U \subset \mathbb{R}^4$.

Show your steps and reasoning below. No reasoning $\Longrightarrow$ no points.

Please show your work for question 7.

8. (15 points) Robot Control (The following are three short answer questions. You do not need to give a formal proof; only give a few short reasons/calculations why something is TRUE or FALSE. *Part (c) is on the next page.*)

(a) **(5 Points)** Recall that adaptive inverse dynamics achieves the closed-loop error dynamics $\dot{e} = Ae + B\hat{M}^{-1}Y(q,\dot{q},\ddot{q})\tilde{\Theta}$ with adaptation law $\dot{\hat{\Theta}} = \dot{\tilde{\Theta}} = -\Gamma^{-1}Y^T(q,\dot{q},\ddot{q})\hat{M}^{-1}B^TPe$, for tracking error state $e = (\tilde{q}^T, \dot{\tilde{q}}^T)^T \in \mathbb{R}^{2n}$ and parameter error state $\tilde{\Theta} \in \mathbb{R}^{\ell}$, $\ell > n$. Also in the stability analysis with a specific Lyapunov function $V(e, \tilde{\Theta}) > 0$, we found that $\dot{V} = -e^TQe \leq 0$ for $Q > 0$. Then, we can use LaSalle's theorem to show the extended system is asymptotically stable about the origin of extended state vector $(e^T, \tilde{\Theta}^T)^T$.

Circle **T or F**. Give a few short reasons/calculations why this is TRUE or FALSE:

(b) **(5 Points)** We wish to control a planar 2-DOF revolute-joint robot with highly geared motors resulting in high actuator inertia. However, our dynamical model is missing actuator inertia terms in the inertia/mass matrix. We can still use *adaptive inverse dynamics* to achieve asymptotic trajectory tracking ($e \to 0$ as $t \to \infty$). Hint: actuator inertias would be added to the diagonal elements of the inertia/mass matrix associated with the rigid links.

Circle **T or F**. Give a few short reasons/calculations why this is TRUE or FALSE:

(c) **(5 Points)** Force/torque or acceleration feedback is necessary to modify mass/inertia at the robot end-effector during robot-environment interaction (that is, without modifying the environmental force). Hint: Consider a 1-DOF example to justify your answer.

Circle **T or F**. Give a few short reasons/calculations why this is TRUE or FALSE: