ME 567/EECS 567/ROB 510 Midterm Exam (Winter 2024)

Prof. Daniel Bruder
24 Hour Take-Home Exam
Released: 10am on Wednesday, March 20, 2024
Due: 10am on Thursday, March 21, 2024

HONOR PLEDGE: Copy (NOW) and SIGN (**after the exam is completed**): I have neither given nor received aid on this exam, nor have I observed a violation of the Engineering Honor Code.

SIGNATURE
(Sign **after** the exam is completed)

LAST NAME (PRINTED), *FIRST NAME*

FILL IN YOUR NAME NOW. COPY THE HONOR CODE NOW.

RULES:

1. **NO COLLABORATION OF ANY KIND**
2. OPEN TEXTBOOK, CLASS NOTES, HOMEWORK
3. SEARCHING THE INTERNET FOR SOLUTIONS (E.G., GOOGLE, CHATGPT) IS CONSIDERED ACADEMIC DISHONESTY, AND MOST PROBLEMS CANNOT BE EASILY FOUND ON THE WEB ANYWAY
4. CALCULATOR/COMPUTER ALLOWED BUT MUST SHOW CALCULATION STEPS FOR FULL CREDIT
5. SUBMIT QUALITY PHOTOS/SCANS TO CANVAS BY DEADLINE (STRICT)

The maximum possible score is 50. To maximize your own score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly on this booklet.

Problems 1 - 4 (16 points: 4×4)

Instructions. Each part of a question is worth 1 point. You do not need to show your work.

1. (Rotation Matrices) **Circle True or False as appropriate for the following statements:**

T **F** (a) A given rotation matrix $R \in SO(3)$ is parameterized by a unique set of zyz Euler angles α, β, γ.

T **F** (b) A given set of zyz Euler angles α, β, γ specifies a unique rotation matrix $R \in SO(3)$.

T **F** (c) Every rotation matrix $R \in SO(3)$ is skew symmetric.

T **F** (d) Given $R \in SO(3)$, there exists a unique $\omega \in \mathbb{R}^3$, $\|\omega\| = 1$ and unique $\theta \in \mathbb{R}$ such that $R = e^{\hat{\omega}\theta}$.

2. (General Rigid Body Transformations) **Circle True or False as appropriate for the following statements:**

T **F** (a) Any rigid body motion can be realized by a screw motion (rotation about an axis combined with translation parallel to that axis).

T **F** (b) Given twist coordinates $\xi = (v, \omega) \in \mathbb{R}^6$ with $\omega \neq 0$ and $\|\omega\| = 1$, the vector v can always be expressed as $v = -\hat{\omega} q + h\omega$ for a point q on the rotational axis ω and some $h \in \mathbb{R}$.

T **F** (c) For a given rigid body transformation $g \in SE(3)$, there is a unique $\hat{\xi} \in se(3)$ and $\theta \in \mathbb{R}$ such that $\bar{g} = e^{\hat{\xi}\theta}$.

T **F** (d) $\|\bar{g}\bar{p} - \bar{g}\bar{q}\| = \|\bar{p} - \bar{q}\|$ for $g \in SE(3)$ and points $p, q \in \mathbb{R}^3$.

3. (Kinematics) **Circle True or False as appropriate for the following statements:**

T **F** (a) $e^{\hat{\xi}_n\theta_n}\cdots e^{\hat{\xi}_1\theta_1}g_{st}(0) = e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_n\theta_n}g_{st}(0)$

T **F** (b) The forward kinematic map $g_{st}(\theta)$ derived using the product of exponentials is equivalent to the forward kinematic map dervied using D-H parameters.

T **F** (c) $\exp(\hat{\xi}_1)\exp(\hat{\xi}_2)\exp(\hat{\xi}_3) = \exp(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3)$ for $\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3 \in se(3)$.

T **F** (d) Multiple solutions for inverse kinematics can exist even when the robot has fewer joints than the dimension of workspace.

4. (Rigid Body Velocities) **Circle True or False as appropriate for the following statements:**

T **F** (a) The axes of the spatial frame A and a body frame B are parallel (i.e., $R_{ab} = I$), and the origin of body frame B is moving in the direction of it's own y-axis at 1m/s. In this case, $V^s_{ab} = V^b_{ab}$.

T **F** (b) The origin of a body frame B is stationary relative to the origin of the spatial frame A. In this case, $v^s_{ab} = v^b_{ab}$.

T **F** (c) Let A, B be inertial frames and let C be a body frame. It is always true that $V^b_{ac} = V^b_{bc}$.

T **F** (d) Spatial and body velocity twists satisfy the following relationship $\hat{V}^s_{ab} = Ad_{g_{ab}}\hat{V}^b_{ab}$, where A is the spatial frame, B is the body frame, and $\hat{V}^s_{ab}, \hat{V}^b_{ab} \in SE(3)$.

Partial Credit Section of the Exam

For the next problems, partial credit is awarded and you MUST show your work. Unsupported answers, even if correct, receive zero credit. In other words, right answer, wrong reason or no reason could lead to no points. If you come to me and ask whether you have written enough, my answer will be,

"I do not know",

because answering "yes" or "no" would be unfair to everyone else. If you show the steps you followed in deriving your answer, you'll probably be fine. <u>If something was explicitly derived in lecture, handouts or homework, you do not have to re-derive it. You can state it as a known fact and then use it.</u>

If you need more space to show your work than what is provided, feel free to insert extra pages after each question.

5. (16 points) Forward Kinematics

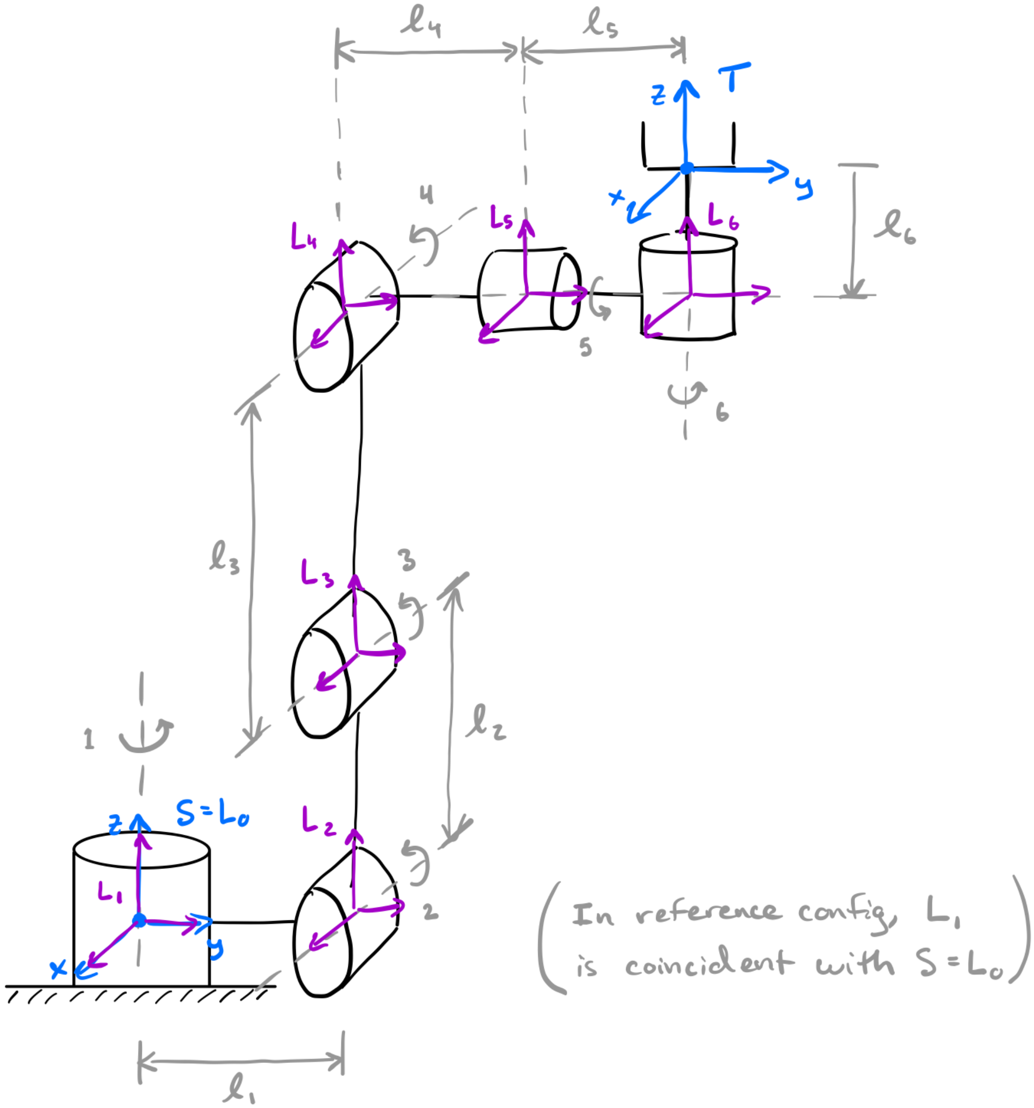

Figure 1: UR5 robot arm in its reference configuration. All joint angles follow the right-hand rule; use these angle conventions. Assume the given lengths l_i are non-zero.

(a) (2 points) Find $g_{st}(0)$ for the product of exponentials.

(b) (6 points) Find the twist coordinates ξ_i, for $i = 1, \ldots, 6$, for the product of exponentials.

(c) (4 points) Find the D-H twist coordinates $\xi_{i-1,i}$, for $i = 1, \ldots, 6$, representing the i-th joint twist in the coordinates of link frame L_{i-1}. Link frames are defined at the proximal joint of the link like usual; assume link frames have the same orientation as the spatial frame at the reference configuration.

(d) (4 points) Derive the relative transformation $g_{\ell_3,\ell_4}(\theta_4)$ between link frames L_3 and L_4 using $\xi_{3,4}$ from part (c). Write out the 4x4 matrix.

Show your steps and reasoning below. No reasoning $\Longrightarrow$ no points. Box your final answers.

Please show your work for question 5.

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6. (18 points) Manipulator Jacobian and Wrench

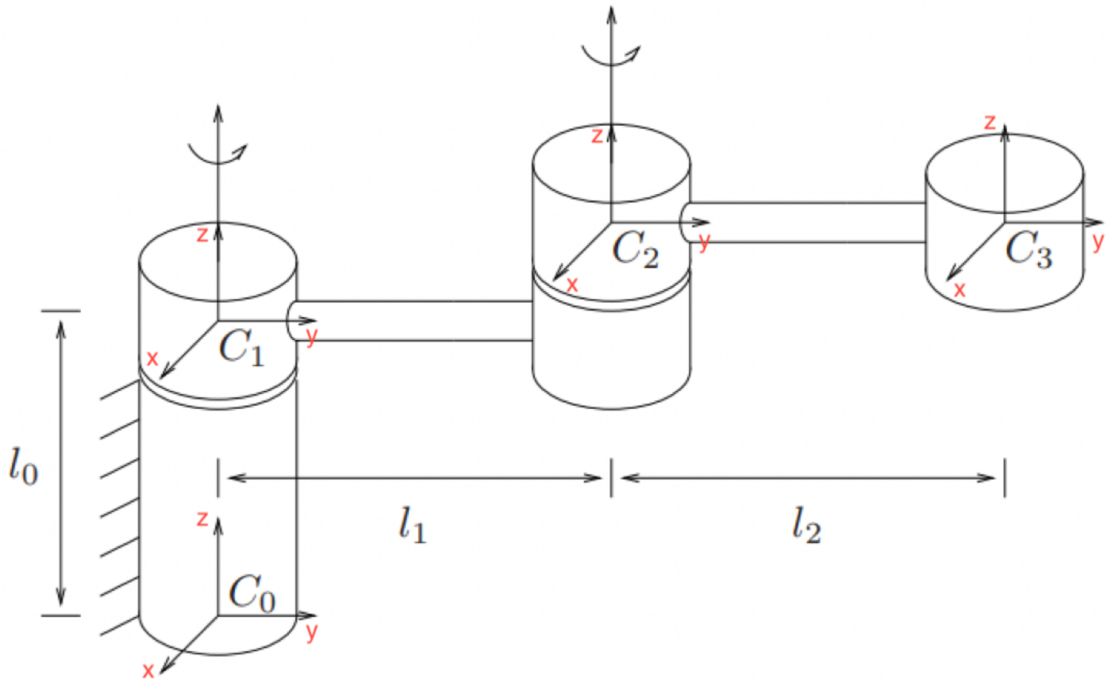

Figure 2: 2-DoF robot manipulator in its reference configuration.

(a) (6 points) Find the *spatial* manipulator Jacobian $J^s_{st}(\theta)$ for the 2-DoF robot shown above. Do this by hand and show your work (you may use Mathematica or the like to check your work, but you must show how you got to your answer).

(b) (4 points) Say a body wrench $F_{C_2} = [1, 1, 0, 0, 0, -1]^\top$ is applied to the origin of C_2. Find the an equivalent body wrench F_{C_3} applied at the origin of C_3.

(c) (6 points) Let C_0 be the spatial frame, and let q^s represent a point attached to the origin of the C_3 frame, expressed in spatial coordinates. Find the homogeneous velocity of the point expressed in the spatial frame, $\bar{v}^s_q$, given $\theta_1 = 0, \theta_2 = \frac{\pi}{2}$, for any $\dot{\theta}_1, \dot{\theta}_2 \in \mathbb{R}$.

(d) (2 points) Does the robot have any singular configurations? What are they?

Show your steps and reasoning below. No reasoning $\implies$ no points. Box your final answers.

Please show your work for question 6.

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Please show your work for question 6.