ROB 510 Exam-I (Winter 2022)

Prof. Robert Gregg
24 Hour Take-Home Exam
Released: 12pm on Friday, March 11, 2022
Due: 12pm on Saturday, March 12, 2022

HONOR PLEDGE: Copy (NOW) and SIGN (**after the exam is completed**): I have neither given nor received aid on this exam, nor have I observed a violation of the Engineering Honor Code.

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SIGNATURE
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LAST NAME (PRINTED) *FIRST NAME*

FILL IN YOUR NAME NOW. COPY THE HONOR CODE NOW.

RULES:

1. **NO COLLABORATION OF ANY KIND**
2. OPEN TEXTBOOK, CLASS NOTES, HOMEWORK
3. GOOGLING SOLUTIONS IS CONSIDERED ACADEMIC DISHONESTY, AND MOST PROBLEMS CANNOT BE EASILY FOUND ON THE WEB ANYWAY
4. CALCULATOR/COMPUTER ALLOWED BUT MUST SHOW CALCULATION STEPS FOR FULL CREDIT
5. SUBMIT QUALITY PHOTOS/SCANS TO GRADESCOPE BY DEADLINE (STRICT)

The maximum possible score is 80. To maximize your own score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly on this booklet.

Problems 1 - 5 (30 points: 5 $\times$ 6)

Instructions. Each part of a question is worth 1.5 points. Submit your answers to questions 1-5 as follows:

1. Download the answer sheet from Canvas (`ROB510 Midterm Wi2022 TF Answer Sheet.pdf`).
2. Print it, or open it in your favorite PDF viewer app (see Canvas announcement if you need ideas).
3. Clearly mark your answer to each question on the answer sheet.
4. Scan or export your solutions, and upload them to Gradescope.

Do not modify the answer sheet, or attach any extra pages. You do not need to show your work. Answers written directly on the questions below will not be graded.

1. (Rotation Matrices) **Circle True or False as appropriate for the following statements:**

T F (a) Let integer $n \geq 2$ and $R \in SO(n)$, then R can only have non-zero eigenvalues.

T F (b) Given a frame A has orientation R_1 relative to a frame B, a frame C has orientation R_2 relative to frame B, then the orientation of A relative to frame C can be written as $R_2^T R_1$.

T F (c) All orthogonal 3x3 matrices represent rigid rotations.

T F (d) Any orientation $R \in SO(3)$ can be represented by at most two sets of exponential coordinates (ω, θ), where $\omega \in \mathbf{R}^3$ and angle $\theta \in [0, 2\pi)$.

2. (Twists) **Circle True or False as appropriate for the following statements:**

T F (a) $(I - e^{\hat{\omega}\theta}) \cdot q = (I - e^{\hat{\omega}\theta}) \cdot \omega \times v$ for a rotational unit twist $\xi = (v, \omega)$, where $v = -\omega \times q$.

T F (b) Given a twist $\xi = (v, \omega) \in \mathbb{R}^6$ with $\omega \neq 0$, the vector v can always be expressed as $v = -\hat{\omega} q$ for a point q on the rotational axis ω.

T F (c) Given a twist $\xi = (v, \omega) \in \mathbb{R}^6$ $(\omega \neq 0)$ that generates a screw motion about some axis in space, the screw axis must go through the point $q = \frac{w \times v}{\|w\|^2}$.

T F (d) The 2-norm of a unit twist $\xi = (v, \omega) \in \mathbb{R}^6$ is equal to 1 for both pure rotation and pure translation.

3. (Forward Kinematics) **Circle True or False as appropriate for the following statements:**

T **F** (a) Let p be a point attached to the tool frame T at the robot's reference configuration (defined with respect to spatial frame S). The operation $g_{st}(\theta)\bar{p}$ gives the point at the robot's current configuration associated with configuration vector θ.

T **F** (b) For a 3-DOF robot, $g_{st}(\theta) = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}e^{\hat{\xi}_3\theta_3}g_{st}(0) = e^{\hat{\xi}'_3\theta_3}e^{\hat{\xi}'_2\theta_2}e^{\hat{\xi}_1\theta_1}g_{st}(0)$, where $\xi'_3 = \mathrm{Ad}_{(e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2})}\xi_3$ and $\xi'_2 = \mathrm{Ad}_{(e^{\hat{\xi}_1\theta_1})}\xi_2$.

T **F** (c) Let L_1 and L_2 be adjacent link frames, defined at a robot's 1st and 2nd joint, respectively. Then, $g_{\ell_1,\ell_2}(\theta) = e^{\hat{\xi}_2\theta_2}g_{\ell_1,\ell_2}(0)$ where $\hat{\xi}_2$ is the standard twist for the 2nd joint defined at the reference configuration with respect to the spatial frame.

T **F** (d) The product of exponentials can also be written as $g_{st}(\theta) = g_{st}(0)e^{\hat{\xi}'_1\theta_1}e^{\hat{\xi}'_2\theta_2}...e^{\hat{\xi}'_n\theta_n}$, where $\xi'_i = \mathrm{Ad}^{-1}_{g_{st}(0)}\xi_i$ for $i = 1, 2, ..., n$.

4. (Inverse Kinematics) **Circle True or False as appropriate for the following statements:**

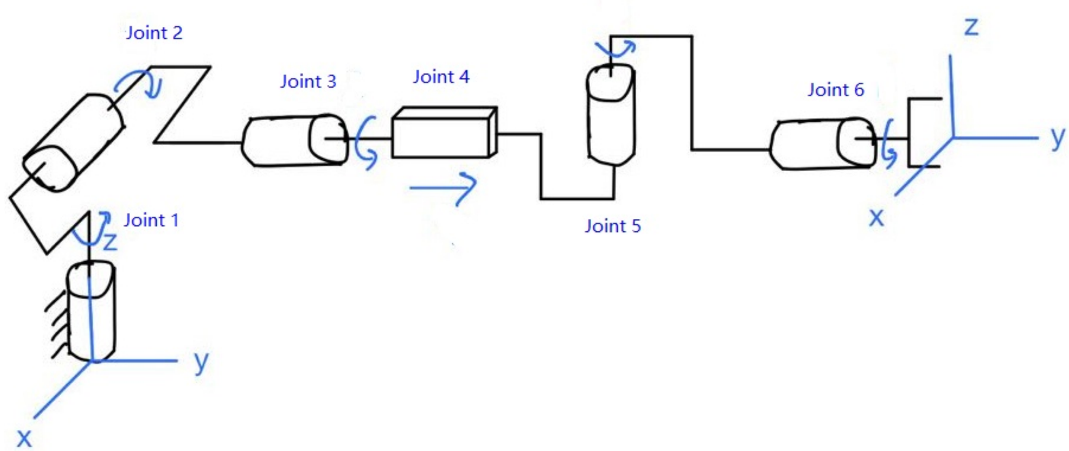

Figure 1: 6-DOF robot manipulator in its reference configuration. Follow the right-hand rule if you need to define the direction of a joint angle. Assume the desired end-effector configuration g_d is known. Each ξ_i corresponds to the i_{th} joint.

T **F** (a) Paden–Kahan Subproblem 2 can be used immediately to find θ_1 and θ_2 (before solving other joints).

T **F** (b) Suppose the sequence of twist 3 and twist 4 were reversed, i.e., joint 3 becomes prismatic and joint 4 becomes revolute (still about y-axis). The solution(s) for the inverse kinematics will be the same.

T **F** (c) The solution of θ_3 is not unique (i.e., number of solutions > 1) if the value of the other joints are given.

T **F** (d) Given a robot manipulator and its desired end-effector position g_d, the maximum number of solutions is independent of the way the inverse kinematics are analytically solved.

5. (Rigid Body Velocities, Wrenches, Jacobians, and Singularities) **Circle True or False as appropriate for the following statements:**

T **F** (a) The dot product of velocity twist coordinates and a wrench is independent of the coordinate frame of the twist and wrench.

T **F** (b) Given $e^{\hat{\xi}_1\theta} = \begin{bmatrix} R_1 & p_1 \\ 0 & 1 \end{bmatrix}$ and $e^{\hat{\xi}_2\theta} = \begin{bmatrix} R_2 & p_2 \\ 0 & 1 \end{bmatrix}$, $\mathrm{Ad}_{e^{\hat{\xi}_1\theta}e^{\hat{\xi}_2\theta}} = \begin{bmatrix} R_1R_2 & R_1\hat{p}_2R_2 + \hat{p}_1R_1R_2 \\ 0 & R_1R_2 \end{bmatrix}$.

T **F** (c) If n non-zero wrenches F_{c_i} are applied to a rigid body at the corresponding reference frames C_i for $i = 1, \ldots, n$, the net wrench F_p with respect to a reference frame P is invariant to the location of P on the rigid body.

T **F** (d) Let $J^b_{st}(\theta) \in \mathbb{R}^{6\times n}$ be the body manipulator Jacobian of a n-DOF robot, $n < 6$. If it is possible to map a joint velocity vector $\dot{\theta}$ to a unique tool velocity V^b_{st}, then it is also possible to map a joint torque vector τ to a unique tool wrench F^b.

Partial Credit Section of the Exam

For the next problems, partial credit is awarded and you MUST show your work. Unsupported answers, even if correct, receive zero credit. In other words, right answer, wrong reason or no reason could lead to no points. If you come to me and ask whether you have written enough, my answer will be,

"I do not know",

because answering "yes" or "no" would be unfair to everyone else. If you show the steps you followed in deriving your answer, you'll probably be fine. If something was explicitly derived in lecture, handouts or homework, you do not have to re-derive it. You can state it as a known fact and then use it.

6. (20 points) Forward Kinematics (Place your answers in the **boxes** and show your work below.)

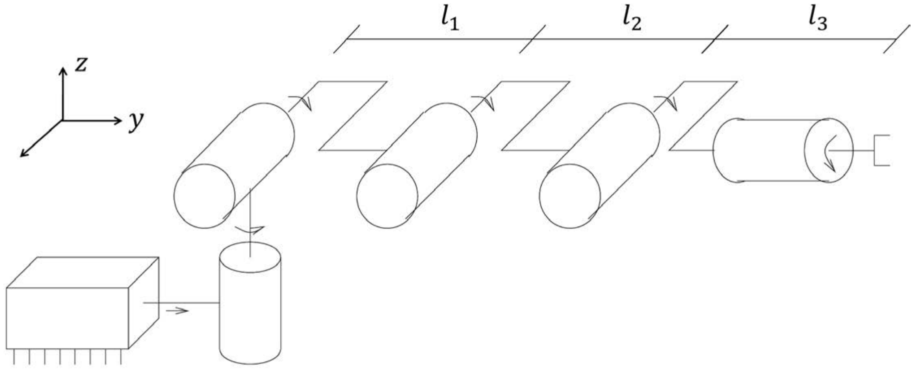

Figure 2: 6-DOF robot manipulator in its reference configuration. Assume the reference displacement of the prismatic joint is zero (so the spatial frame is located at the center of the third joint in the reference configuration), the link length between joints 2 and 3 is zero, and the tool frame has the same reference orientation as the spatial frame.

(a) (9 points) For the manipulator shown, find the twist coordinates ξ_i and $g_{st}(0)$ for the product of exponentials. Use the angle conventions as shown, even if they are not what you would pick.

(a) $\xi_1 = \left[\quad \right]$, $\xi_2 = \left[\quad \right]$, $\xi_3 = \left[\quad \right]$, $\xi_4 = \left[\quad \right]$, $\xi_5 = \left[\quad \right]$, $\xi_6 = \left[\quad \right]$, $g_{st}(0) = \left[\quad \right]$

(b) (2 points) If we replace the sixth joint with a helical joint (e.g., for drilling) that travels 0.01 m per revolution about the same axis, what would the twist coordinates ξ_6 become? Assume SI units for all terms.

(c) $\xi_6 = \left[\quad \right]$

(c) (9 points) Extract the (unit) twist coordinates associated with $g_{s\ell_2}(\theta)$, assuming L_2 is the second link's frame defined at the second joint center. This represents the equivalent twist axis taking frame L_2 to frame S.
Hint: Look at the assumptions below Fig. 2. Use Mathematica to simplify trigonometry. The answer will depend on θ.

(b) $\xi_{g_{s\ell_2}(\theta)} = \left[\quad \right]$

Show your steps and reasoning below. No reasoning $\Longrightarrow$ no points.

Please show your work for question 6.

7. (15 points) Manipulator Jacobian (Place your answers in the **boxes** and show your work below.)

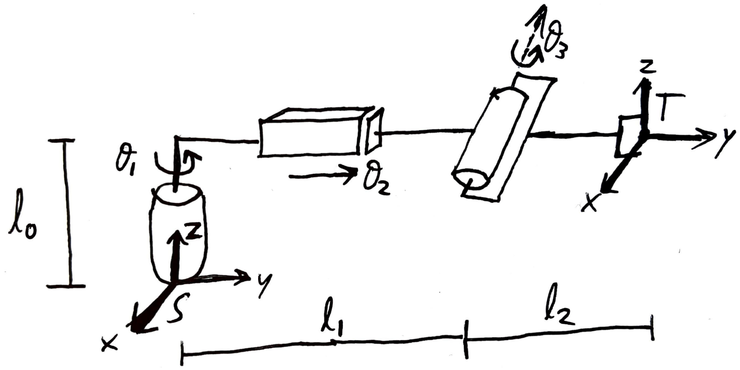

Figure 3: 3-DOF robot manipulator in its reference configuration. Length l_1 is defined when prismatic joint $\theta_2 = 0$. Assume the third axis is parallel to the x-axis.

(a) (12 points) Find the *body* manipulator Jacobian $J^b_{st}(\theta)$ for the 3-DOF robot shown above.

(a) $J^b_{st}(\theta) = \begin{bmatrix} \; \end{bmatrix}$

(b) (3 points) Does this manipulator have singular configuration(s) at which the rank of $J^b_{st}(\theta)$ drops? If yes, give an example singular configuration θ. If no, say why.

(b)

Show your steps and reasoning below. No reasoning $\Longrightarrow$ no points.

Please show your work for question 7.

8. (15 points) Rotation Matrices (Place your answers in the **boxes** and show your work below.)

(a) (5 points) Find the rotation matrix associated with a 180 deg rotation about an equivalent axis of $(0, 1/\sqrt{2}, 1/\sqrt{2})^T$.

(a) $R = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

(b) (10 points) Consider a sequence of Y-Z-X Euler angles with corresponding angles α, β, and γ. Find a set of Euler angles that produces the same orientation as part (a).

(b) $\alpha =$	$\beta =$	$\gamma =$

Show your steps and reasoning below. No reasoning $\implies$ no points.

Extra space for question 8.