ME567/EECS567/ROB510 Final Exam (Winter 2024)

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24 Hour Take-Home Exam

Released: 12pm on Tuesday, April 30, 2024

Due: 12pm on Wednesday, May 1, 2024

HONOR PLEDGE: Copy (NOW) and SIGN (**after the exam is completed**): I have neither given nor received aid on this exam, nor have I observed a violation of the Engineering Honor Code.

SIGNATURE
(Sign **after** the exam is completed)

LAST NAME (PRINTED), *FIRST NAME*

FILL IN YOUR NAME NOW. COPY THE HONOR CODE NOW.

RULES:

1. **NO COLLABORATION OF ANY KIND**
2. OPEN TEXTBOOK, CLASS NOTES, HOMEWORK
3. SEARCHING THE INTERNET FOR SOLUTIONS (E.G., GOOGLE, CHATGPT) IS CONSIDERED ACADEMIC DISHONESTY, AND MOST PROBLEMS CANNOT BE EASILY FOUND ON THE WEB ANYWAY
4. CALCULATOR/COMPUTER ALLOWED BUT MUST SHOW CALCULATION STEPS FOR FULL CREDIT
5. SUBMIT QUALITY PHOTOS/SCANS TO CANVAS BY DEADLINE (STRICT)

The maximum possible score is 50. To maximize your own score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly on this booklet.

Problems 1 - 5 (20 points: 5 × 4)

Instructions. Each part of a question is worth 1 point. You do not need to show your work.

1. (Robot Kinematics) **Circle True or False as appropriate for the following statements:**

T **F** (a) A planar rigid body transformation $g = (p, R) \in SE(2)$ comprises a translation $p \in \mathbb{R}^2$ and 2x2 rotation matrix $R \in SO(2)$. The associated twist $\hat{\xi} \in se(2)$ is given by $\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{3\times 3}$ where $\hat{\omega} \in \mathbb{R}^{2\times 2}$, $v \in \mathbb{R}^2$. Moreover, $\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} \in \mathbb{R}^3$ with $\omega \in \mathbb{R}^1$.

T **F** (b) Elements of SO(2) commute, i.e., $R_1 \cdot R_2 = R_2 \cdot R_1$ for any $R_1, R_2 \in \mathrm{SO}(2)$.

T **F** (c) For inverse kinematics, subproblem 1 can be used to find θ such that $e^{\hat{\xi}\theta}\bar{p} = \bar{q}$ where homogeneous point $\bar{p}$ is known but $\bar{q}$ is unknown.

T **F** (d) For a n-DoF manipulator, let V_{st}^b be the end-effector's body velocity and $\dot{\theta}$ be the joint velocity vector, then the i-th column of $\frac{\partial V_{st}^b}{\partial \dot{\theta}}$ depends on $\theta_1, \cdots, \theta_{i-1}$.

2. (Robot Dynamics) **Circle True or False as appropriate for the following statements:**

T **F** (a) Let $M(q)$ be the mass/inertia matrix of a n-DoF manipulator with revolute joints. There exists a constant $\sigma \geq \lambda_M(q)$ such that $\sigma I - M(q)$ is positive-definite for all $q \in Q$, where $\lambda_M(q)$ is the maximum eigenvalue of $M(q)$.

T **F** (b) Consider a n-DoF revolute-joint robot with parallel elasticity at the joints, i.e., each joint has a parallel elastic element that produces a torque proportional to the joint displacement, $\tau_{\mathrm{ela},i} = -K_i q_i$ for $K_i > 0$, $i = 1, \ldots, n$. Let $L(q, \dot{q})$ be the Lagrangian accounting for kinetic and potential energy, then the dynamics are described using the E-L equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau_{\mathrm{ela}}$ (assuming no control inputs for simplicity).

T **F** (c) The products of inertia (off-diagonal elements of the inertia tensor $\mathcal{I}$) are always equal to zero.

T **F** (d) Let $L(q, \dot{q})$ be the Lagrangian, then $\dot{q}^T(\frac{\partial L}{\partial \dot{q}} - 2C\dot{q}) = 0$, where C is the Coriolis matrix.

3. (Stability and Passivity) **Circle True or False as appropriate for the following statements:**

T **F** (a) A 2-dimensional nonlinear system $\dot{x} = f(x)$ can have zero, finite, or infinite equilibrium points.

T **F** (b) For scalar $\dot{x} = \alpha x^5$, the equilibrium point $x = 0$ is exponentially stable for any real $\alpha < 0$.

T **F** (c) Given a linear system $\dot{x} = Ax$, the system is stable if for some symmetric positive-definite Q, the P solving the Lyapunov equation $A^T P + PA + Q = 0$ is positive definite.

T **F** (d) Consider the first-order nonlinear system $\dot{x} = f(x)$ with $x \in \mathbb{R}$, $f(0) = 0$. Suppose there is a Lyapunov function $V(x) > 0$ (positive definite) such that $\frac{\partial V}{\partial x} f(x) < 0$ (negative definite), then $\bar{x} = 0$ is also a (locally) asymptotically stable equilibrium point for the system $\dot{x} = f(x) \cdot (x - 2)$.

4. (Multivariable Robot Control) **Circle True or False as appropriate for the following statements:**

T **F** (a) Consider a robot with non-zero gravity term $G(q)$. Given finite gain matrices $K_p, K_d > 0$, there may exist one or more specific set-points for which PD control can achieve perfect tracking.

T **F** (b) Consider adaptive inverse dynamics for a n-DOF robot with trajectory tracking error state $e \in \mathbb{R}^{2n}$ and parametric error state $\tilde{\Theta} \in \mathbb{R}^p$. Then $V(e, \tilde{\Theta}) = e^T Pe$ is a positive-definite function if $P > 0$.

T **F** (c) Potential energy shaping can be used to virtually rotate Earth's gravity vector in the closed-loop dynamics of a fully actuated robot.

T **F** (d) Task-space inverse dynamics requires inversion of the joint-space dynamics (i.e., joint acceleration).

5. (Force/Impedence Control) **Circle True or False as appropriate for the following statements:**

T **F** (a) The impedence $Z(s) = 2s + 1$ is said to be inertial.

T **F** (b) For a nonzero environmental force $F \neq 0$, impedance control can achieve desired impedance properties of the end effector, but can not necessarily achieve tracking of a reference trajectory $x^d(t)$.

T **F** (c) It is not possible for a robot end effector to achieve hybrid impedance and velocity control with an elastic environment.

T **F** (d) Using hybrid impedance control, it is possible to simultaneously control the velocity and force at the end effector.

Partial Credit Section of the Exam

For the next problems, partial credit is awarded and you MUST show your work. Unsupported answers, even if correct, receive zero credit. In other words, right answer, wrong reason or no reason could lead to no points. If you come to me and ask whether you have written enough, my answer will be,

"I do not know",

because answering "yes" or "no" would be unfair to everyone else. If you show the steps you followed in deriving your answer, you'll probably be fine. If something was explicitly derived in lecture, handouts or homework, you do not have to re-derive it. You can state it as a known fact and then use it.

6. (20 points) 2 DOF Robot

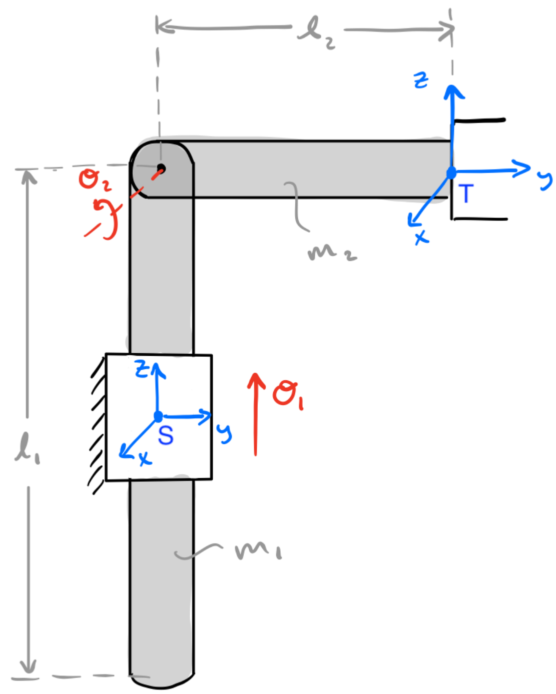

Figure 1: This 2 DOF robot consists of two links with masses m_1 and m_2. They can be considered to be thin rigid rods of uniform density. The first joint is prismatic and the second joint is revolute. In its reference configuration (shown), the midpoint of link 1 is coincident with the origin of the spatial frame, and link 2 is parallel to the y-axis of the spatial frame. Assume gravity is pointing downward, in the direction of the negative z-axis of the spatial frame.

(a) (4 points) Derive the body Jacobian $J^b_{st}(\theta)$ for this robot.

(b) (6 points) Derive the intertia/mass matrix $M(\theta) \in \mathbb{R}^{2\times 2}$.

(c) (4 points) Derive the gravitational forces/torques vector $G(\theta) \in \mathbb{R}^2$.

(d) (6 points) For the remainder of this question, assume that

$$q = \begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix}, \quad M(\theta) = \begin{bmatrix}2 & \cos\theta_2 \\ \cos\theta_2 & \frac{4}{3}\end{bmatrix}, \quad C(\theta,\dot{\theta}) = \begin{bmatrix}0 & -4\dot{\theta}_2\sin\theta_2 \\ 0 & 0\end{bmatrix}, \quad G(\theta) = \begin{bmatrix}20 \\ 10\cos\theta_2\end{bmatrix}.$$

Is the following control law locally asymptotically stable about the desired configuration $q^d = [0,0]^\top$?

$$u = -\begin{bmatrix}2 & 0 \\ 0 & 2\end{bmatrix} q - \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\dot{q} - \begin{bmatrix}-20 \\ 2\sin\theta_2 - 10\cos\theta_2\end{bmatrix}$$

Justify your answer.

Show your steps and reasoning below. No reasoning $\implies$ no points. Box your final answers.

Please show your work for question 6.

Additional page for question 6. Please insert additional pages to show your work if needed.

7. (10 points) Short Answer Questions (The following are three short answer questions. You do not need to give a formal proof; only give reasons/calculations why something is TRUE or FALSE.)

(a) **(5 Points)** There exist manipulators (with at least 2 DOFs) for which the order of the product of exponentials does not matter, i.e., all matrix exponentials commute. If True, give an example. If False, provide justification.

Circle **T or F**. Give reasons/calculations why this is TRUE or FALSE:

(b) **(5 Points)** Consider the n-link flexible joint robot with state $(q_1, q_2, \dot{q}_1, \dot{q}_2) \in \mathbb{R}^{4n}$ and gravity-free dynamics

$$\begin{aligned} M(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + K(q_1 - q_2) &= 0 \\ J\ddot{q}_2 + K(q_2 - q_1) &= u, \end{aligned}$$

where $u := -K_p\tilde{q}_2 - K_d\dot{q}_2$ for diagonal $K_p, K_d > 0$ and $\tilde{q}_2 := q_2 - q_d$ for constant vector $q_d \in \mathbb{R}^n$. Then, the Lyapunov function $V = \frac{1}{2}\dot{q}_1^T M(q_1)\dot{q}_1 + \frac{1}{2}\dot{q}_2^T J\dot{q}_2 + \frac{1}{2}(q_1 - q_2)^T K(q_1 - q_2)$ can be used to prove global asymptotic stability (with the help of LaSalle).

Circle **T or F**. Give reasons/calculations why this is TRUE or FALSE: