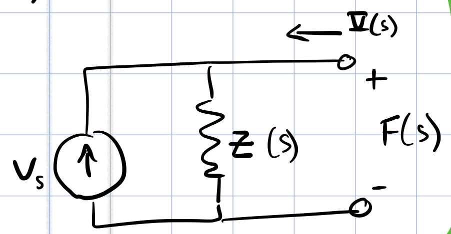
Warm-up Problem: https://join.iclicker.com/MMAW

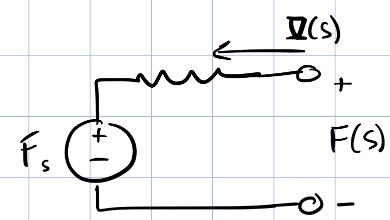
You want to design a hybrid impedence controller that yields Z_R, a desired robot impedence. The environment impedence is known, $\boxed{Z_e(s) = \frac{1}{s}}$

Which equivalent circuit diagram should you use to represent the environment?

(a.) N-network



(b.) T-network



(c.) None of the above

Z_e is capacative, since $Z_e(0) = \infty$

Lesson 26:

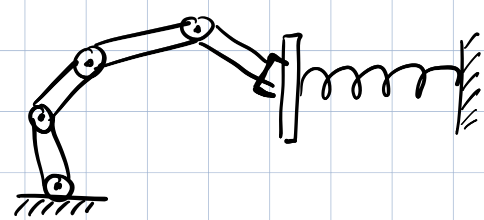
I. Hybrid Impedence Control (SHV 10.3.2 / 11.3.3)

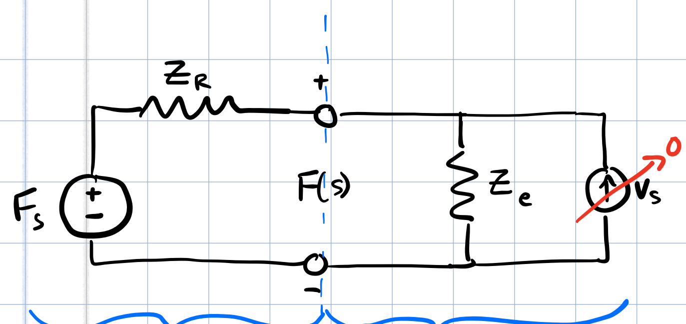
A. Capacative Environment Example.

B. Inertial Environment Example.

I. Hybrid Impedence Control

A. Example: Capacative Environment / Force Control





Assume env. passive: $V_s = 0$

Robot T-network w/ desired Z_R, where $Z_R(0) \neq \infty$ (not capacative)

environment N-network

$$\Rightarrow \frac{F}{F_s} = \frac{Z_e(s)}{Z_e(s) + Z_R(s)} \quad \text{(by Kirchoff's laws)}$$

can control force: $F_s = \frac{F_d}{s}$ (step input)

Error $E(s) = F - F_s = \dfrac{-Z_R(s)}{Z_e(s) + Z_R(s)} \cdot \dfrac{F_d}{s}$

Final Value Theorem: $e_{ss} := \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\, E(s)$

$\Rightarrow e_{ss} = \dfrac{-Z_R(0)}{Z_e(0) + Z_R(0)} \cdot F_d$, where $Z_e(0) = \infty$ (capacitive)

$= 0$ $\qquad Z_R(0) \neq \infty$ (not capacitive)

$\Rightarrow$ Means we can control both impedence and force.

<u>Next Step</u>: Determine control law to acheive Z_R and F_d

Assume $\ddot{x} = a_x$ (from inverse dynamics control)

Design a_x:

Assume $Z_R = \underbrace{M_c s}_{\text{inertial term}} + \underbrace{Z_{REM}}_{\text{damping term}}$

Assume task-space inv. dynamics:

$$\ddot{x} = a_x := -\frac{1}{M_c} Z_{REM}\, \dot{x} + \frac{1}{M_c}(F_s - F)$$

<u>note</u>: needs vel. and force feedback.

⇒ Closed-loop dynamics (Laplace)

$$s^2 X(s) = -\frac{1}{M_c} Z_{REM}\, s X(s) + \frac{1}{M_c}(F_s - F)$$

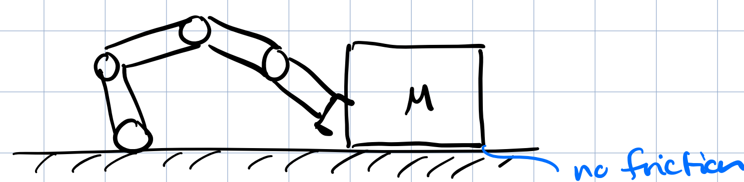
$$s\underbrace{(M_c s + Z_{REM})}_{Z_R} X(s) = F_s - F$$

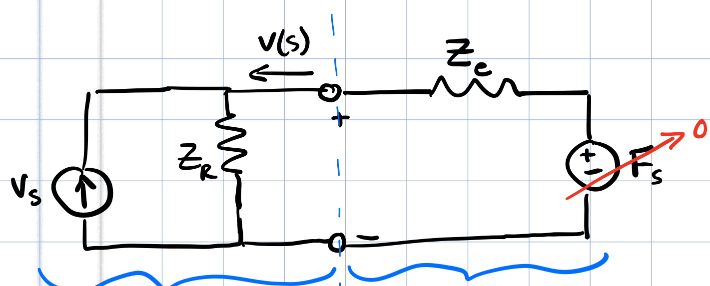
⇒ Closed-loop dynamics (time-domain)

$$\boxed{Z_R \dot{x} = F_s - F} \longrightarrow 0$$

* Impedence + force control!

B. <u>Example</u>: Inertial Environment / Velocity Control





Robot w/desired Z_R, where $Z_R(0) \neq 0$ (not inertial)

Env, $Z_e(0) = 0$

Assume env. passive: $F_s = 0$

$$\Rightarrow \frac{\underline{V}}{\underline{V}_s} = \frac{Z_R(s)}{Z_e(s) + Z_R(s)} \quad \text{(Kirchoff's Laws)}$$

Can control velocity:

$$\underline{V}_s = \frac{V_d}{s} \quad \text{(step velocity with set point } v_d\text{)}$$

Define velocity error:

$$E(s) = \underline{V}(s) - \underline{V}_s(s) = \frac{-Z_e(s)}{Z_R(s) + Z_e(s)} \cdot \frac{V_d}{s}$$

Apply F.V.T.

$$e_{ss} = \lim_{s \to 0} \frac{-Z_e(s)}{Z_R(s) + Z_e(s)} V_d = 0 \checkmark$$

<u>Next step</u>: Determine control law to acheive Z_R, V_d

Let $Z_R = M_c s + \underbrace{Z_{REM}}_{\text{non-zero (damping + stiffness)}}$

Assume task-space inv. dynamics

$$\ddot{x} = a_x := \ddot{x}^d + \frac{1}{M_c} Z_{REM}(\dot{x}^d - \dot{x}) + \frac{1}{M_c} F$$

Closed-loop dynamics:

$$\mathcal{L}\left[(\ddot{x} - \ddot{x}^d) M_c = Z_{REM}(\dot{x}^d - \dot{x}) + F \right]$$

$$s^2 \left(\underline{X}(s) - \underline{X}^d(s) \right) M_c = \left(Z_R - M_c s \right) s \left(\underline{X}^d(s) - \underline{X}(s) \right) + F(s)$$

$$s^2\cancel{\left(\mathbb{X}(s) - \mathbb{X}^d(s)\right)M_c} = Z_R\, s\left(\mathbb{X}^d - \mathbb{X}\right) + \cancel{s^2 M_c\left(\mathbb{X} - \mathbb{X}^d\right)} + F(s)$$

$$\mathcal{L}^{-1}\left[\; Z_R\, s\left(\mathbb{X} - \mathbb{X}^d\right) = F(s) \;\right]$$

$$\boxed{Z_R \underbrace{\left(\dot{x} - \dot{x}^d\right)}_{\to 0} = F}$$

* Impedance & velocity control