Course: MECH 567: Robot Kinematics & Dynamics Assigned Date: 02/15/2024

Instructor: Daniel Bruder, PhD

Due: 11:59 PM on Thursday, 03/07/2024

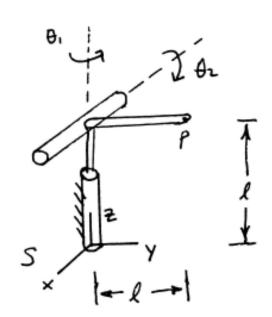
Submit by uploading on canvas

# Homework 3

## **Problem 1**

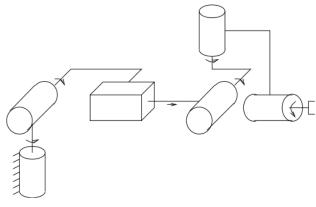
Use Paden-Kahan subproblems 1 & 2 to find  $\theta_1$  and  $\theta_2$  necessary to rotate an initial point  $\mathbf{p} = \begin{bmatrix} 0 \\ l \\ l \end{bmatrix}$ 

to a final position q = 
$$\begin{bmatrix} -l/\sqrt{2} \\ 0 \\ l+l/\sqrt{2} \end{bmatrix}$$
. See MLS Pgs. 99-103.



#### **Problem 2**

Show how you would solve for the inverse kinematics of the manipulator shown in Figure 3.24 (iii) of MLS book, given a desired  $g_d$ . Show which subproblems you would solve and which points you would use to solve them (or how you would use geometric reasoning instead of one or more of the subproblems). How many solutions are possible?



(iii) Stanford manipulator

#### **Problem 3**

- (a) A body B is initially located with origin (1,2,0) relative to a fixed frame A, and coordinate axes parallel to A. What is the spatial velocity  $V_{ab}^s$  if the body rotates at  $\frac{\pi}{2}$  rad/s about its z axis?
- (b) What is the initial velocity, relative to the A frame (i.e.,  $v_{q_a}$ ), of a point with coordinates  $q_b = (0, 1, 0)$  in the body frame? Show your math.

#### **Problem 4**

- (a) Continued with Problem 3, what is the location of the point q after 1 second, in A coordinates? Show your math.
- (b) Suppose that B is translating with a velocity of  $2\pi$  units per second along the z axis, in addition to the rotation described in Problem 3(a). Now what is  $V_{ab}^s$ ? What would the location of the point q be after 1 second, in A coordinates? Again, show your math.

### **Problem 5**

Figure 2.17 shows a two degree of freedom manipulator. Let  $l_0$ ,  $l_1$ ,  $l_2$  be the link length parameters and  $\theta_1$ ,  $\theta_2$  the joint angle variables of link 1 and link 2, respectively. The coordinate axes for this problem have z pointing up, y pointing to the right, and x pointing out of the page. Use the Right-Hand-Rule (RHR) to determine the sign of the rotation. For simplicity, assume  $C_1$ ,  $C_2$ , & C3 are on the same plane. **Use Mathematica to solve the following problems:** 

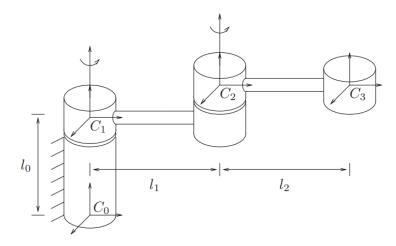


Figure 2.17: A two degree of freedom manipulator.

- (a) Express the position and orientation of frame C3 relative to frame C0 in terms of the joint angle variables and the link parameters.
- (b) Compute the spatial velocity of C3 relative to C0 as functions of the joint angles and the joint rates.
- (c) Compute the body velocity of C3 relative to C0 as functions of the joint angles and the joint rates.