Lesson 25:

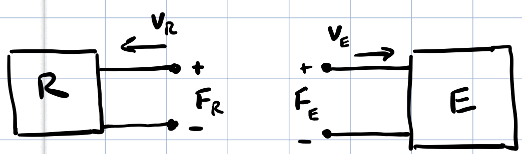
I. Impedence Operators (SHV 10.2.1 / 11.2.1)

II. Impedence Control (SHV 10.3.1 / 11.3.2)

III. Hybrid Impedence Control (SHV 10.3.2 / 11.3.3)

I. Impedence Operators

Robot - Environment are coupled through interaction ports:



Effort	Flow
$F_R = F_E$	$v_R = -v_E$

Mechanical Impedence:

$F(s)$, $V(s)$ are the Laplace Transforms of $F(t)$, $v(t)$
Effort Flow

Impedence $\quad Z(s) := \dfrac{F(s)}{V(s)}$ ← effort over flow

Ex: Mass-spring damper

$$M\dot{v} + Bv + K\int v = F$$

$$\left(Ms + B + K\frac{1}{s}\right)V(s) = F(s) \Rightarrow Z(s) = \frac{F(s)}{V(s)} = Ms + B + K\frac{1}{s}$$

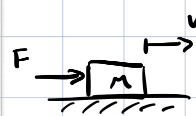
$$= \frac{Ms^2 + Bs + K}{s}$$

Admittance $\quad Y(s) := Z^{-1}(s) = \dfrac{V(s)}{F(s)}$ ← flow over effort

- Classification of Impedence Operators:

  Impedence $Z(s)$ is
  1) Inertial if $|Z(0)| = 0$
  2) Resistive if $|Z(0)| = B < \infty$
  3) Capacitive if $|Z(0)| = \infty$

Ex: Robot pushing mass



- No friction

$$M\dot{v} = F \Rightarrow Z(s) = Ms$$

$$\Rightarrow |Z(0)| = 0 \quad (\text{inertial})$$

- With friction

$$M\dot{v} + Bv = F \Rightarrow Z(s) = Ms + B$$

$$\Rightarrow |Z(0)| = B \quad (\text{resistive})$$

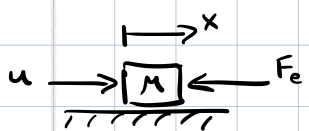
- With spring

$$M\dot{v} + K\int v = F \Rightarrow Z(s) = Ms + \frac{K}{s}$$

$$\Rightarrow |Z(0)| = \infty \quad (\text{capacitive})$$

II. Impedence Control

Ex:



EOM: $M\ddot{x} = u - F_e$

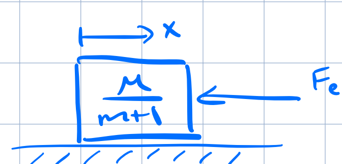
Let $u = -mF_e$

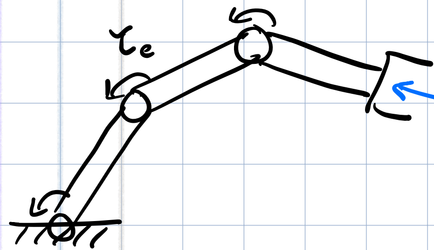
$\Rightarrow$ Closed loop:

$$M\ddot{x} = -mF_e - F_e$$

$$= -F_e(m+1)$$

$$\left(\frac{M}{m+1}\right)\ddot{x} = -F_e$$





F_e - environmental wrench $\Rightarrow \tau_e = -J_b^T F_e$

EOM: $M\ddot{q} + C\dot{q} + G + J_b^T F_e = u$ (1)

- We want a desired impedence at the end effector:
  - Inverse dynamics (inner loop):

$$u = M a_q + C\dot{q} + G + J_b^T a_f$$

outer loop control inputs (TBD)

- Recall from SHV Ch. 9:

  In task space (assume J is square & invertible)

  1.) $a_x = J a_q + \dot{J}\dot{q}$

  $\Rightarrow a_q := J^{-1}(a_x - \dot{J}\dot{q})$

  (first outer loop input)

  2.) $\ddot{x} = J\ddot{q} + \dot{J}\dot{q}$ (from $\dot{x} = J\dot{q}$)

  $\Rightarrow J\ddot{q} = \ddot{x} - \dot{J}\dot{q}$ (we will use later)

- Close inner loop (plug u into (1))

$$M\ddot{q} = M a_q - J^T(F_e - a_f)$$

$$\Rightarrow \ddot{q} = a_q - M^{-1} J^T (F_e - a_f)$$

- Close first outer loop (plug in a_q)

$$\ddot{q} = J^{-1}(a_x - \dot{J}\dot{q}) - M^{-1} J^T (F_e - a_f)$$

$$\Rightarrow J\ddot{q} = a_x - \dot{J}\dot{q} - JM^{-1}J^T(F_e - a_f)$$

$$\Rightarrow \ddot{x} - \cancel{\dot{J}\dot{q}} = a_x - \cancel{\dot{J}\dot{q}} - JM^{-1}J^T(F_e - a_f)$$

$$\Rightarrow \ddot{x} = a_x - JM^{-1}J^T(F_e - a_f)$$

- If F_e is measured, then let $a_f = F_e$,
  (through a 6-axis load cell or nonlinear observer)

$$\Rightarrow \ddot{x} = a_x \longrightarrow$$ can choose a_x to enforce a desired impedance at end effector.

$$a_x = \ddot{x}^d - M_d^{-1}(B_d\dot{\tilde{x}} + K_d\tilde{x} + F_e) \quad \text{where } \tilde{x} = x - x_d$$

$\Rightarrow$ Closed loop:

$$M_d\ddot{\tilde{x}} + B_d\dot{\tilde{x}} + K_d\tilde{x} = -F_e \qquad (*)$$

* Yields way to get inherently "safe" interaction with the environment (e.g. human) with a reasonable choice of M_d, B_d, K_d.

* Impedence control doesn't attempt to regulate motion or force, instead it regulates $\frac{\text{force}}{\text{motion}}$ (impedence)

* Given desired x^d, M_d, B_d, K_d, defined $\tilde{x} = x - x^d$

if $F_e = 0 \Rightarrow \tilde{x} \rightarrow 0$

$F_e \neq 0 \Rightarrow \tilde{x} \not\rightarrow 0$ b.c. disturbance.

III. Hybrid Impedence Control

Want to control robot impedence <u>and</u> motion or force simultaneously.

*Must include environmental dynamics into design:

Z_e: known env. impedence (inertial, resistive, capacative)

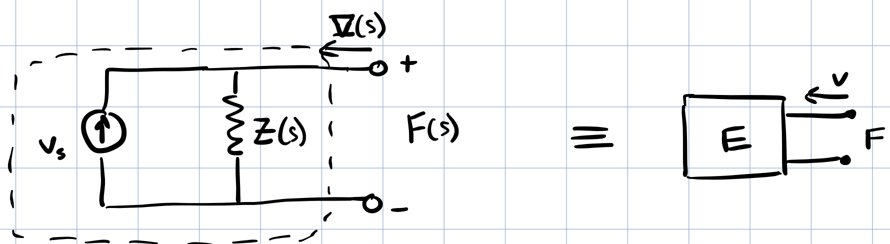
Z_R: desired robot impedence

Assume task-space inverse dynamics: $\ddot{x} = a_x$

<u>Design steps:</u>

1.) If Z_e is capacitive → use Norton network representation

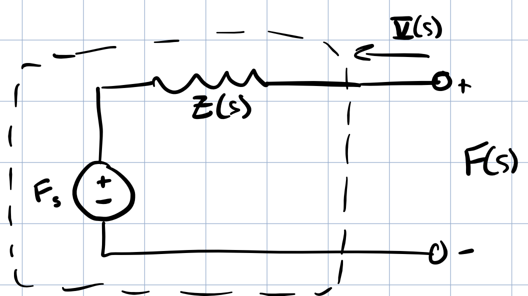
N-network: impedence is in parallel w/ flow source



environment

If Z_e is inertial/resistive → use Thevenin network rep.

T-network: impendence is in series w/ effort source



environment

* <u>Note</u>: Any 1-port network w/passive elements can be expressed as Thevenin or Norton equivalent.

2.) Choose desired Z_R and represent as dual of Z_e

e.g. if Z_e capacitive $\rightarrow$ Z_R Thevenin

if Z_e inertial/resistive $\longrightarrow$ Z_R Norton

3.) Couple R-E ports and design outer-loop control.