Warm-up Problem: https://join.iclicker.com/MMAW

In last class, we showed that in the exact model case, PBC yields the following closed-loop dynamics

$$M(q)\dot{r} + C(q,\dot{q})r + Kr = 0, \quad \text{where} \quad r = \dot{\tilde{q}} + \Lambda\tilde{q}$$

Consider the Lyapunov function candidate,

$$V = \frac{1}{2} r^T M(q) r$$

What can you say about the equilibrium point

$e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix} = 0$ based on this Lyapunov function?

(a.) S.I.S.L (c.) Unstable

(b.) G.A.S. (d.) None of the above

Compute $\dot{V}$

$$\dot{V} = r^T \underline{M(q)\dot{r}} + \frac{1}{2} r^T \dot{M}(q) r$$

$$= r^T(-Cr - Kr) + \frac{1}{2} r^T \dot{M} r$$

$$= \cancel{\frac{1}{2} r^T(\dot{M} - 2C) r} - r^T K r$$

$$= -r^T K r \leq 0,$$ since it equals 0 for any $\dot{\tilde{q}} = -\Lambda\tilde{q}$, not only at $e = 0$.

Lesson 24:

I. Passivity-Based Control (SHV 9.4 / 7.4)

  A. Inexact Model Case

II. Force Control (SHV 10/11)

  A. Rigid Interaction / Constraints (SHV 10.1 / 11.1)

  B. Network Models & Impedence (SHV 10.2 / 11.2)

I. Passivity-Based Control

A. Inexact Model Case

- PB Robust Control (SHV 9.4.1 / 7.4.1)

  Advantages over robust control is that it eliminates assumption of closeness of $\hat{M}$ to M, and simplifies computation of uncertainty bounds.

- PB Adaptive Control (SHV 9.4.2 / 7.4.2)

  Advantage over adaptive control is eliminates need for acceleration feedback.

PB Adaptive Control:

- Inexact $\hat{M}, \hat{C}, \hat{G}$ based on parametric uncertainty with parameter estimates $\hat{\Theta}$

- Use PB control law

$$u = \hat{M}a + \hat{C}v + \hat{G} - Kr$$

$$= Y(q, \dot{q}, a, v)\hat{\Theta} - Kr$$

Recall:

$v = \dot{q}^d - \Lambda\tilde{q}$

$a = \ddot{q}^d - \Lambda\dot{\tilde{q}}$

$r = \dot{\tilde{q}} + \Lambda\tilde{q}$

$\Rightarrow$ Closed-loop

$$M\dot{r} + Cr + Kr = \underbrace{Y(q, \dot{q}, a, v)\tilde{\Theta}}_{\text{residual}}$$

, where $\tilde{\Theta}$ is parameter error vector $= \hat{\Theta} - \Theta$

• Extend state vector with $\hat{\Theta}$, and choose gradient-based update law:

$$\dot{\hat{\Theta}} = \dot{\tilde{\Theta}} = -\Gamma^{-1} Y^T(q, \dot{q}, a, v)\, r \quad , \quad \text{for } \Gamma > 0$$

(adaptation gains)

• Lyapunov analysis w/ extended state vector $(\tilde{q}, \dot{\tilde{q}}, \tilde{\Theta})$:

$$V = \frac{1}{2} r^T M(q) r + \tilde{q}^T \Lambda K \tilde{q} + \frac{1}{2} \tilde{\Theta}^T \Gamma \tilde{\Theta} > 0$$

$$\Rightarrow \dot{V} = -\tilde{q}^T \Lambda^T K \Lambda \tilde{q} - \dot{\tilde{q}}^T K \dot{\tilde{q}} + \tilde{\Theta}^T \cancelto{0}{\left(\Gamma \dot{\hat{\Theta}} + Y^T r\right)}$$

plug in adaptation law

$$= -e^T Q e \le 0 \Rightarrow \text{S.I.S.L.}$$

where $e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix}$ and $Q = \begin{bmatrix} \Lambda^T K \Lambda & 0 \\ 0 & K \end{bmatrix} > 0$

* Use Barbalat's Lemma to $\tilde{q} \underset{t \to \infty}{\to} 0$, from the boundedness property of S.I.S.L. (SHV Appendix C)

II. Force Control

Interaction between robot (R) and environment (E)

- Rigid and Compliant
- Control design (Impedence control)

A. Rigid Interaction / Constraints

Assume ideal conditions: rigid bodies/environment w/o friction.

Duality between motion and force:

1.) Motion space $\mathcal{M}$, $\dim(\mathcal{M}) = 6$

- Twist $\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} \in \mathcal{M}$ representing inst. velocities of end effector.

2.) Force space $\mathcal{F}$, $\dim(\mathcal{F}) = 6$

- Wrench $F = \begin{bmatrix} f \\ \tau \end{bmatrix} \in \mathcal{F}$ representing inst. force.

Def: $\xi \in \mathcal{M}$ and $F \in \mathcal{F}$ are reciprocal if

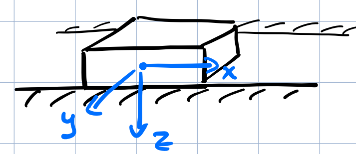
$$\xi^T F = v^T f + \omega^T \tau = 0$$

i.e. zero inst. work (power)

- Natural constraints from environment
- Artificial contraints from specified task

} characterize rigid interaction.

Ex: Erasing a white board $\left(\begin{matrix} \text{w/o friction} \\ \text{\& gravity} \end{matrix} \right)$



- Natural constraints

$$v_z = 0, \quad \omega_y = 0, \quad \omega_x = 0$$

$$f_y = 0, \quad f_x = 0, \quad \tau_z = 0$$

- Artificial constraints

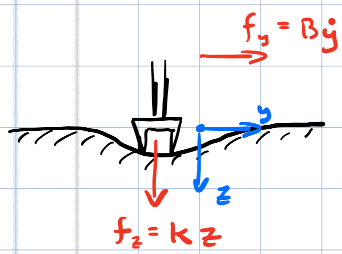
$$f_z = f^d, \quad v_y = v_y^d, \quad v_x = v_x^d$$

$\Rightarrow \xi^T F = v^T f + \omega^T \tau = 0 \Rightarrow$ Forces of constraints do no work in directions of motion.

* Not true w/ compliance or friction

↳ motion is coupled w/ force

Model environmental reaction to motion (friction) & deflection.



$$\Rightarrow \xi^T F \neq 0$$

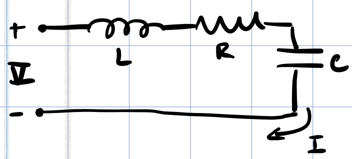
In z-direction

$$\int_0^t v_z(\xi)\, f_z(\xi)\, d\xi = \frac{1}{2}k\left(z^2(t) - z^2(0)\right)$$

B. Network Models & Impedence

Network theory can be used to model R-E interaction

Electrical circuit

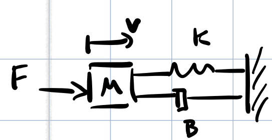


I = current (flow of charge)

V = voltage (effort)

V I = power

Mechanical system



F = force (effort)

v = velocity (flow of position)

<u>Flow</u> variables $\quad I \sim v$

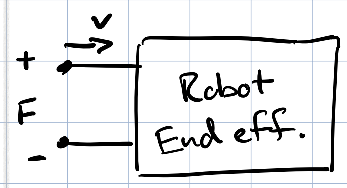
<u>Effort</u> variables $\quad V \sim F$

$L \sim M$ "inertia"/resistance to change in flow

$R \sim B$ dissipate energy

$C \sim K$ store storage

1-port network model for end effector



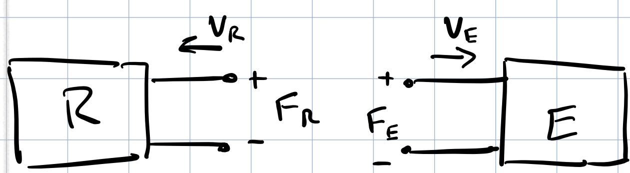
modeled as forced mass-spring damper

F, v are "port variables"

$$M\ddot{x} + B\dot{x} + Kx = F$$

$$M\dot{v} + Bv + K\int v = F$$

* R-E are coupled through interaction ports:



$$F_R = F_E$$

$$v_R = -v_E$$