Lesson 23:

I. Adaptive IDC (cont'd) (SHV 9.3.4 / 7.3.2)

II. Passivity - Based Control

   A. Exact Model Case

   B. Inexact Model Case

I. Adaptive IDC (cont'd)

Given model estimate

$$\hat{M}\ddot{q} + \hat{C}\dot{q} + \hat{G} = Y(q,\dot{q},\ddot{q})\hat{\Theta}$$

We will "adapt" $\hat{\Theta}$ over time to acheive perfect trajectory tracking.

- Def. inner-loop control input

$$u := \hat{M} a_q + \hat{C}\dot{q} + \hat{G}$$

- Def. outer-loop control input

$$a_q := \ddot{q}^d(t) - K_d\dot{\tilde{q}} - K_p\tilde{q} \quad , \text{ where } \tilde{q} = q - q^d$$

$\Rightarrow$ Closed loop dynamics

$$\ddot{\tilde{q}} + K_d\dot{\tilde{q}} + K_p\tilde{q} = \hat{M}^{-1} Y(q,\dot{q},\ddot{q})\tilde{\Theta}$$

- Let $e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix}$ be error state vector

$$A = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$\Rightarrow \dot{e} = Ae + B\Phi\tilde{\Theta} \quad , \text{ where } \Phi := \hat{M}^{-1} Y(q,\dot{q},\ddot{q})$$

- Treat $\hat{\Theta}$ as an extended system state

Propose: $\dot{\hat{\Theta}} = -\Gamma^{-1}\Phi^T B^T P e$ ——— (1)

for const. $\Gamma > 0$ (adaptation gains)

matrix $P > 0$ from Lyapunov eq. $A^T P + PA = -Q$

- Lyapunov analysis of $e = 0$

Let $V = e^T P e + \tilde{\Theta}^T \Gamma \tilde{\Theta} > 0 \quad \forall (e, \tilde{\Theta}) \neq 0$

$$\Rightarrow \dot{V} = 2e^T P \dot{e} + 2\tilde{\Theta}^T \Gamma \dot{\tilde{\Theta}}, \quad \text{where } \dot{\tilde{\Theta}} = \dot{\hat{\Theta}}$$

$$= 2e^T P \left[Ae + B\Phi\tilde{\Theta} \right] + 2\tilde{\Theta}^T \Gamma \dot{\tilde{\Theta}}$$

(Note: $-e^T Q e = e^T(A^T P + PA)e = e^T A^T P e + e^T P A e$

$= (e^T A^T P e)^T + e^T P A e = 2e^T P A e$)

$$= -e^T Q e + 2\tilde{\Theta}^T \left[\underbrace{\Phi^T B^T P e + \Gamma \dot{\tilde{\Theta}}}_{=0\text{, after pluggin in (1)}} \right]$$

$= -e^T Q e \leq 0$, because $\tilde{\Theta}$ (or $\hat{\Theta}$) can be any thing when $\dot{V} = 0$

$\Rightarrow$ Stability in the sense of Lyapunov (S.I.S.L.)

↳ Tracking error e and parameter error $\tilde{\Theta}$ are bounded.

- Can also prove that $e \xrightarrow[t \to \infty]{} 0$ by <u>Barbalat's Thm</u> (from boundedness prop. of SISL)

(see SHV Appendix C)

$\Rightarrow$ Perfect tracking of $q^d(t)$, but not necessarily perfect parameter estimation (only bounded parametric error)

* Need to measure accelerations $\ddot{q}$ in $\Phi(q,\dot{q},\ddot{q})$ which are noisy signals!

$\rightarrow$ Can get around this using Passivity-Based Control, which exploit structure of robot dynamics.

II. Passivity-Based Motion Control

- Inverse Dynamics (i.e "computed torque") relies on cancelling all non-linearities in dynamics to obtain linear dynamics.
  - $\rightarrow$ May be too aggressive, may be possible to acheive desired result with some non-linearities.

Define quantities

$$v := \dot{q}^d - \Lambda \tilde{q} \quad , \quad \text{where } \tilde{q} = q - q^d \ , \quad \text{diag. } \Lambda > 0$$

$$a := \dot{v} = \ddot{q}^d - \Lambda \dot{\tilde{q}}$$

$$r := \dot{q} - v = \dot{\tilde{q}} + \Lambda \tilde{q}$$

A. Exact Model Case

Let $u = Ma + Cv + G - Kr$, where diag. $K > 0$

$\Rightarrow$ Closed-loop

$$M(q)\dot{r} + C(q,\dot{q})r + Kr = 0 \ , \quad \text{where } \dot{r} = \ddot{q} - \dot{v}$$

$$= \ddot{\tilde{q}} + \Lambda \dot{\tilde{q}}$$

Still a coupled system of NL equations with original inertia/mass and Coriolis matrices.

- Let $e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix}$

- Lyapunov analysis of $e = 0$

$$V = \frac{1}{2} r^T M(q) r + \tilde{q}^T \Lambda K \tilde{q} > 0 \qquad \forall e \neq 0$$

and radially unbounded

$$\Rightarrow \dot{V} = r^T M \dot{r} + \frac{1}{2} r^T \dot{M} r + 2 \tilde{q}^T \Lambda K \dot{\tilde{q}}$$

$$= -r^T [Cr + Kr] + \frac{1}{2} r^T \dot{M} r + 2 \tilde{q}^T \Lambda K \dot{\tilde{q}}$$

$$= \frac{1}{2} r^T \cancel{(\dot{M} - 2C)}^{\,0} r - r^T K r + 2 \tilde{q}^T \Lambda K \dot{\tilde{q}}$$

$$= -(\dot{\tilde{q}} + \Lambda \tilde{q})^T K (\dot{\tilde{q}} + \Lambda \tilde{q}) + 2 \tilde{q}^T \Lambda K \dot{\tilde{q}}$$

$$= -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \cancel{2 \tilde{q}^T \Lambda K \dot{\tilde{q}}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \cancel{2 \tilde{q}^T \Lambda K \dot{\tilde{q}}} \qquad \left(\text{Note: } (\Lambda K)^T = \Lambda K\right)$$

$$= -e^T Q e, \quad \text{where} \quad Q = \begin{bmatrix} \Lambda K \Lambda & 0 \\ 0 & K \end{bmatrix} > 0$$

$$< 0 \quad \forall e \neq 0$$

$\Rightarrow$ G.A.S

B. Inexact Model Case

Next time!