Homework 5

Problem 1

In this problem, use Mathematica with the screw theory package. Copy and paste your code in the homework.
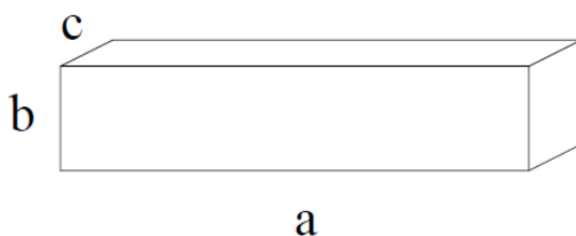

Figure 2: Block

Consider a uniform rectangular solid of mass m and dimensions $a \times b \times c$ as shown in Figure 2. The inertia tensor with respect to a coordinate frame located at the geometric center of the solid can be computed in terms of a, b, c and m.

For each of the two-link planar robots shown on the next page, assume that the links are uniform rectangular solids as above. Let $a = 1$, $b = 0.2$, and $m = 2$ in each case. The value of c does not matter since the motion is planar. You may take $c = 0$ for simplicity. Assume that the gravity vector is in the direction of the negative y–axis. **Define the positive x-axis pointing towards right, positive y-axis pointing upwards, and positive z-axis pointing out of the page.** Robots (i), (iii), and (iv) are shown with angles to help understand where the links are. For the zero configuration of each of these robots assume that the links that are shown at 45 degrees are only along the x-direction. **This means that the two links for (i) and (iii) are both along x only**.

(a) Compute the inertia tensor and center of mass vector for each link.

(b) Compute the Euler-Lagrange dynamic equations using the Lagrangian function L.

(c) Output the inertia matrix $D(q)$, the Coriolis/centrifugal matrix $C(q, \dot{q})$, and the gravitational torques/forces vector $g(q)$ for each robot.

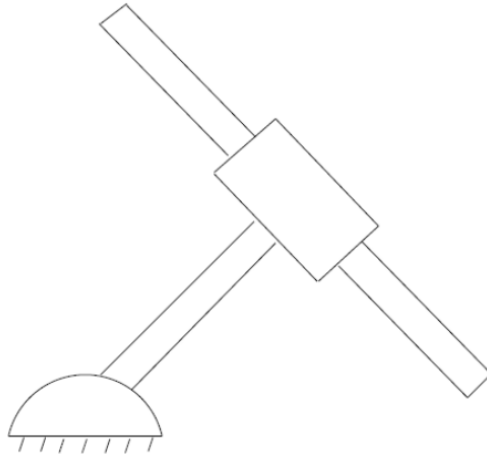(d) Compute the expression $\dot{D}(q) - 2C(q, \dot{q})$ for each robot and verify that it is a skew symmetric matrix.
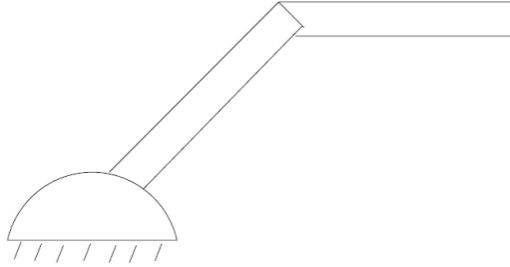
(i) A two-link RR manipulator
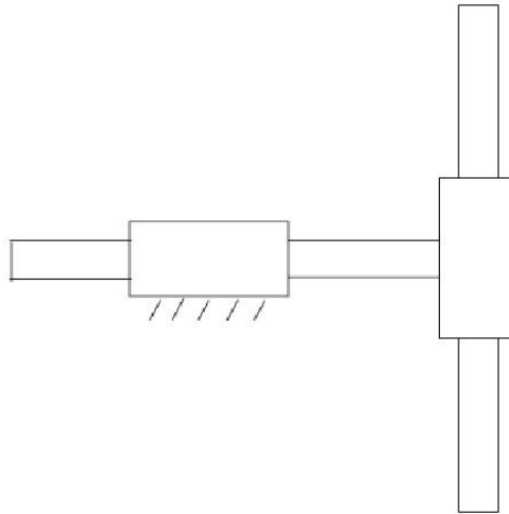
(ii) A two-link PP manipulator.
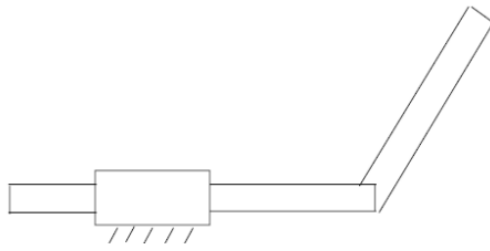
(iii) A two-link PR manipulator.

(iv) A two-link RP manipulator.

Problem 2

Consider the nonlinear system:

$$\dot{x}_1 = x_1 - x_1 x_2$$
$$\dot{x}_2 = 2x_1^2 - 2x_2$$

Find the equilibrium points and investigate local stability around each equilibrium point.

Problem 3

Consider the nonlinear system:

$$\dot{x}_1 = -x_1 - x_1 x_2^2$$
$$\dot{x}_2 = -x_2 - x_2 x_1^2$$

Show that (0,0) is the unique equilibrium point and investigate local stability. Investigate global stability using the Lyapunov function candidate.

$$V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$$