CS 514/ Math 514 Numerical Analysis
Spring 2018
Homework 6
Given: April 5, 2018; Due: **April 12, 2018** 11:45 A.M. (15 minutes before class).
Note that you have only one week to do this HW.

You are permitted to discuss these problems with **one** other student, and you should indicate the name of your collaborator in your answer to the first problem in this set. If you worked on your own, please indicate this on your answer. You should write up your own solution, and you are not permitted to either share a written copy of your solution or copy some one else's written solution. Similarly with programs, you are not permitted to share your code with another student. If you need help with debugging your code, you should describe what the problem is without sharing or showing your code to another student. If you are stuck on a problem or on a code, you could ask the TAs for help on Piazza. Please do not discuss solutions to the HW problems on Piazza before they are graded.

If you copy someone else's work, or let your work be copied, you will get zero points for the entire HW, a loss of one letter grade in the course, and you will be reported to the Dean of Students. I follow the course policies described by Professor Gene Spafford (of Purdue Computer Science department) at the URL `http://spaf.cerias.purdue.edu/~spaf/cpolicy.html` If you are not familiar with this policy, please read it! Please submit your assignments using Blackboard, preferably using either a Tex-generated or Word document.

0. If you collaborated with a student in answering these questions, please write down your collaborator's name. If you did not collaborate with anyone, please indicate that too.

1. **10 points**
   Given the values

$$f(-1) = 4, f(0) = 1, f(1) = 4,$$
$$f'(-1) = -8, f'(0) = 0, f'(1) = 8,$$

   find the piecewise cubic Hermite interpolating polynomial through the points -1, 0 and 1.

2. **15 points** This problem is quite similar to the problem we worked out in class for best approximation.

   (a) Approximate the function $\exp(-x)$ on the interval $[-1\ 1]$ (in the continuous least squares sense) with a cubic polynomial $v^*(x)$ that minimizes the integral of $(\exp(-x) - v(x))^2$ in the interval. Note that the weight function is one, and you can use the Legendre polynomials here. The first three Legendre polynomials are listed on page 371 of the textbook.

(b) Next, plot the function $\exp(-x)$ and the approximating quadratic polynomial $v^*(x)$ on the interval $-1\ 1]$ using Matlab or another program. If your computations are correct, then the cubic approximation should be a reasonably good fit of the exponential function. Submit your plot. (This is a way for you to check your computations and make sure that they are correct!)

Evaluate the function and the cubic polynomial at the points `-1:0.01:1` and report the maximum difference of their values, and the x-value where it is attained. You need to submit only the maximum difference, and where it is attained.

3. **15 points** This problem is a modified version of Problem 2 from Section 12.5.

(a) Approximate the function $\cos(\pi t)$ on the interval $[0\ 1]$ (in the continuous least squares sense) with a second degree polynomial $v^*(t)$ that minimizes the integral of $(\cos(\pi t) - v(t))^2$ in the interval. Again, the weight function is one, and you can use the Legendre polynomials here, but you have to transform the variable x of the Legendre polynomials to the variable t defined in the interval $[0\ 1]$. (See Example 12.2 in the textbook.)

(b) Next, plot the function $\cos(\pi t)$ and the approximating quadratic polynomial $v^*(t)$ on the interval $[0\ 1]$ using Matlab or another program. If your computations are correct, then the quadratic should be a reasonably good fit of the cosine function. Submit your plot. (This is a way for you to check your computations!)

Evaluate the function and the quadratic polynomial at the points `0:0.001:1` and report the maximum difference of their values, and the x-value where it is attained. You need to submit only the maximum difference, and where it is attained.