Optimal Dispatch for Drone Delivery Services: Static/Dynamic Models, Properties, Cutting Planes and City-Scale Simulation

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***Abstract*—This paper investigates the optimal delivery problem for drones in the low-altitude economy, aiming to optimize the assignment and sequencing of orders across multiple drone stations to minimize total service time. The problem is applicable across various on-demand delivery schemes, including food delivery, medical transport, parcel handling, and document delivery. We formulate the static version of the problem as a Mixed-Integer Programming (MIP) model, analyze its optimality properties, and provide formal proofs. Based on these properties, we propose several effective (optimality, dominance, and lower bound) cutting planes and develop an exact branch-and-cut algorithm. To validate the effectiveness of the proposed approach at practical scales, we develop a city-scale simulation system and extend the static model to its dynamic version to handle system changes over time. Additionally, we propose an approximate model based on demand aggregation to optimize locker locations for more realistic city-scale simulations. Extensive numerical experiments demonstrate that the proposed algorithm significantly outperforms the original MIP model in terms of solution speed, solvable problem sizes, and convergence rate. City-scale simulations reveal that drone packing, weighing, and launch times significantly impact system throughput. This study also offers insights for optimizing various urban drone delivery operations.**

***Index Terms*—Low-altitude economy, drone delivery service, order assignment and sequencing, dynamic order dispatch, cutting planes, mixed integer programming, branch-and-cut**

I. Introduction

In recent years, urban areas worldwide have witnessed a rapid increase in the deployment of drone-based delivery services, driven by the growing demand for fast and flexible last-mile logistics solutions [1]. Compared to traditional ground transportation, unmanned aerial vehicles (UAVs) offer distinct advantages such as direct routing, reduced congestion, lower emissions, and potentially lower operational costs [1]–[4] . These benefits make drones an attractive alternative for various applications, including parcel and food delivery, medical transport, document delivery, and other time-sensitive on-demand services [5]–[9].

Despite its promising potential, the implementation of drone delivery systems in urban environments involves a range of challenges, including real-time operational control, airspace coordination, infrastructure siting, and regulatory compliance, weather uncertainty, among others [10]–[12]. Among these, order dispatch—which encompasses both the assignment of orders to takeoff stations and the sequencing of departures—constitutes a particularly critical and computationally intensive aspect of real-time operations, and forms the core focus of this study [13]. However, to the best of our knowledge, this problem has not yet been formally studied in prior research.

In on-demand drone delivery systems, order dispatch optimization typically requires assigning orders to stations and sequencing their launches while considering packaging time. This joint assignment-sequencing problem is highly complex to solve and becomes even more challenging in dynamic settings with continuous order arrivals and system changes. Consequently, there is a urgent need for developing efficient models dispatch algorithms to enhance system performance and support the economic scalability of drone delivery services.

To this end, this paper develops an efficient optimization model for the order dispatch in drone delivery system, analyzes key structural properties of the problem, introduces effective cutting planes and embeds them into an exact branch-and-cut framework. In addition, we develop a dynamic city-scale simulation platform to evaluate the proposed approach under realistic operational conditions and conduct a case study in a major city in China.

The contributions of this paper are summarized as follows:

1) We introduce the joint assignment-sequencing problem in the drone delivery systems, and propose its static and dynamic mixed integer programming formulations.
2) We derive several important properties of the studied problem and propose the corresponding effective cutting planes that can significantly boost the solution.
3) We design a tailored, high-performance exact branch-and-cut algorithm to solve the model efficiently.
4) We develop a dynamic, city-level multifunctional simulation platform capable of supporting real-time dynamic order dispatching and locker location optimization, etc.

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