Table of Contents

1(a)

```
x0=[1;-1];
N=50;
A=[0.77,-0.35;0.49,0.91];
B=[0.04;0.15];
Q=[500,0;0,100];
R=1;
P=[1500,0;0,100];
[K,P] = lqrBatch(A,B,Q,R,P,N);
U0opt=K*x0
J0opt=x0'*P*x0
```

```
U0opt =

    4.0818
   -3.8039
   -3.0017
   -1.6477
   -0.8106
   -0.3813
   -0.1757
   -0.0801
   -0.0364
   -0.0165
   -0.0075
   -0.0034
   -0.0015
   -0.0007
   -0.0003
   -0.0001
   -0.0001
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
```

```
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000


J0opt =

   1.8723e+03
```

1(b)

```
x0=[1;-1];
N=50;
A=[0.77,-0.35;0.49,0.91];
B=[0.04;0.15];
Q=[500,0;0,100];
R=1;
P=[1500,0;0,100];
nx = size(A,1);
nu = size(B,2);
Sx = zeros(nx*(N+1),nx);
Su = zeros(nx*(N+1),nu*N);
Sx(1:nx,:) = eye(nx);

for i=1:N
    Sx(nx*i+1:nx*(i+1),:) = A*Sx(nx*(i-1)+1:nx*i,:);
    Su(nx*i+1:nx*(i+1),1:i*nu) = [A*Su(nx*(i-1)+1:nx*i,1:(i-1)*nu) B];
end
Qbar  = blkdiag(kron(eye(N),Q),P);
Rbar = kron(eye(N),R);
H=Su'*Qbar*Su+Rbar;
```

```
F=Sx'*Qbar*Su;
u=sdpvar(N,1);
x=sdpvar(2,1);
C=x==x0;
obj=u'*H*u+2*x'*F*u+x'*Sx'*Qbar*Sx*x;
Options = sdpsettings('solver','quadprog');
out=optimize(C,obj,Options);
double(u)
double(obj)
```

```
Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-
decreasing in
feasible directions, to within the default value of the optimality
 tolerance,
and constraints are satisfied to within the selected value of the
 constraint tolerance.
```

```
ans =

    4.0818
   -3.8039
   -3.0017
   -1.6477
   -0.8106
   -0.3813
   -0.1757
   -0.0801
   -0.0364
   -0.0165
   -0.0075
   -0.0034
   -0.0015
   -0.0007
   -0.0003
   -0.0001
   -0.0001
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
```

```
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
   -0.0000
    0.0000
   -0.0000
    0.0000


ans =

   1.8723e+03
```

1(c)

```
x0=[1;-1];
N=50;
A=[0.77,-0.35;0.49,0.91];
B=[0.04;0.15];
Q=[500,0;0,100];
R=1;
nx = size(A,1);
nu = size(B,2);
P = zeros(nx,nx,N+1);
PN=[1500,0;0,100];
P(:,:,N+1) = PN;
F=zeros(1,2,N);
for i=N:-1:1
    F(:,:,i)= -inv(R+B'*P(:,:,i+1)*B)*B'*P(:,:,i+1)*A;

    P(:,:,i) = Q + A'*P(:,:,i+1)*A - A'*P(:,:,i+1)*B*inv(R+B'*P(:,:,i
+1)*B)*B'*P(:,:,i+1)*A;

end
Jopt=x0'*P(:,:,1)*x0;
```

1(d)

```
%%Batch
D=[0.1;0.1];
N=50;
w=(10^0.5)*randn(1,51);
w(1)=0;
D=[0;0];
for i=1:N
    D=[D;0.1*w(i+1);0.1*w(i+1)];
end


x0=[1;-1];

A=[0.77,-0.35;0.49,0.91];
B=[0.04;0.15];
Q=[500,0;0,100];
R=1;
P=[1500,0;0,100];
nx = size(A,1);
nu = size(B,2);
Sx = zeros(nx*(N+1),nx);
Su = zeros(nx*(N+1),nu*N);
Sx(1:nx,:) = eye(nx);

for i=1:N
    Sx(nx*i+1:nx*(i+1),:) = A*Sx(nx*(i-1)+1:nx*i,:);
    Su(nx*i+1:nx*(i+1),1:i*nu) = [A*Su(nx*(i-1)+1:nx*i,1:(i-1)*nu) B];
end
Qbar  = blkdiag(kron(eye(N),Q),P);
Rbar = kron(eye(N),R);
H=Su'*Qbar*Su+Rbar;
F=Sx'*Qbar*Su;
u=sdpvar(N,1);
x=sdpvar(2,1);
C=x==x0;
obj=u'*H*u+2*(x'*F+D'*Qbar*Su)*u+x'*Sx'*Qbar*Sx*x;
Options = sdpsettings('solver','quadprog');
out=optimize(C,obj,Options);
double(u)

%%recursive
x0=[1;-1];
N=50;
A=[0.77,-0.35;0.49,0.91];
B=[0.04;0.15];
Q=[500,0;0,100];
R=1;
P = zeros(nx,nx,N+1);
PN=[1500,0;0,100];
P(:,:,N+1) = PN;
F=zeros(1,2,N);
```

```
for i=N:-1:1
    F(:,:,i)= -inv(R+B'*P(:,:,i+1)*B)*(B'*P(:,:,i+1)*A);

    P(:,:,i) = Q + A'*P(:,:,i+1)*A - A'*P(:,:,i+1)*B*inv(R+B'*P(:,:,i
+1)*B)*B'*P(:,:,i+1)*A;

end
U=[F(:,:,1)*x0];
for i=2:N
    x=A*x0+B*U(i-1)+D(2*i-1:2*i);
    U=[U;F(:,:,1)*x+D(2*i+1:2*i+2)'*P(:,:,i+1)*B];
end
double(U)
```

Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the selected value of the constraint tolerance.

ans =

```
    5.0217
   -2.2012
   -4.2050
   -4.9855
    3.8564
   -1.7814
    0.8768
   -2.8319
    4.8868
   -0.6495
   -0.6134
   -1.5402
   -2.3953
    1.7083
    3.5958
   -0.2112
    1.4546
   -7.0296
    5.2024
   -2.2656
    0.9186
   -3.3881
    2.1216
    1.8329
    0.9868
```

```
   -3.3443
    1.7480
    1.2106
   -2.8682
    2.3505
    0.7650
   -2.9899
    4.8137
   -2.5946
   -1.3827
    1.3349
   -1.3787
    3.0186
    1.6216
   -1.1815
   -1.2264
   -5.0153
    5.2126
   -2.2405
   -1.0349
    3.2183
   -2.7784
    2.6011
   -5.1588
    4.4457


ans =

    4.0818
  -15.3959
   18.4813
    2.0745
  -15.9378
   23.1386
  -26.8243
   42.4982
  -59.5547
   63.1434
  -55.7008
   69.1714
  -51.3768
   48.8567
  -68.5786
   62.9666
  -74.8736
  109.0700
 -123.1186
  135.0140
 -139.3490
  152.0262
 -164.3145
  148.1913
 -163.2524
```

```
 173.8362
-177.3402
 175.6686
-156.2836
 156.9141
-154.9294
 175.8619
-190.5057
 202.2775
-192.9556
 188.0524
-184.4796
 168.9701
-181.0333
 185.8552
-176.4796
 209.2783
-224.7155
 229.4296
-230.9580
 206.0846
-206.8405
 185.0673
-171.1184
 148.3418
```

2(a)

```
tic
x0=[-1;-1];
N=3;
A=[1,1;0,1];
B=[0;1];
Q=eye(2);
R=0.1;
P=eye(2);
nx = size(A,1);
nu = size(B,2);
Sx = zeros(nx*(N+1),nx);
Su = zeros(nx*(N+1),nu*N);
Sx(1:nx,:) = eye(nx);

u=sdpvar(N,1);
x=sdpvar(2,N+1);
C=[x(:,1)==x0,abs(u)<=1];
for i=1:N
    Sx(nx*i+1:nx*(i+1),:) = A*Sx(nx*(i-1)+1:nx*i,:);
    Su(nx*i+1:nx*(i+1),1:i*nu) = [A*Su(nx*(i-1)+1:nx*i,1:(i-1)*nu) B];
    C=[C,abs(x(:,i+1))<=15];
end
Qbar  = blkdiag(kron(eye(N),Q),P);
Rbar = kron(eye(N),R);
```

```
H=Su'*Qbar*Su+Rbar;
F=Sx'*Qbar*Su;


obj=u'*H*u+2*x(:,1)'*F*u+x(:,1)'*Sx'*Qbar*Sx*x(:,1);
Options = sdpsettings('solver','quadprog');
out=optimize(C,obj,Options);
double(u)
double(obj)
toc
```

```
Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-
decreasing in
feasible directions, to within the default value of the optimality
 tolerance,
and constraints are satisfied to within the selected value of the
 constraint tolerance.



ans =

    1.0000
    0.9129
   -0.8299


ans =

   12.2743

Elapsed time is 0.420515 seconds.
```

2(b)

```
tic
x0=[-1;-1];
N=3;
A=[1,1;0,1];
B=[0;1];
Q=eye(2);
R=0.1;
P=eye(2);
nx = size(A,1);
nu = size(B,2);
Sx = zeros(nx*(N+1),nx);
Su = zeros(nx*(N+1),nu*N);
Sx(1:nx,:) = eye(nx);
```

```
u=sdpvar(N,1);
x=sdpvar(2,N+1);
C=[x(:,1)==x0,abs(u)<=1];
for i=1:N
    Sx(nx*i+1:nx*(i+1),:) = A*Sx(nx*(i-1)+1:nx*i,:);
    Su(nx*i+1:nx*(i+1),1:i*nu) = [A*Su(nx*(i-1)+1:nx*i,1:(i-1)*nu) B];
    C=[C,x(:,i+1)==A*x(:,i)+B*u(i),abs(x(:,i+1))<=15];
end
Qbar  = blkdiag(kron(eye(N),Q),P);
Rbar = kron(eye(N),R);
H=Su'*Qbar*Su+Rbar;
F=Sx'*Qbar*Su;


obj=u'*H*u+2*x(:,1)'*F*u+x(:,1)'*Sx'*Qbar*Sx*x(:,1);
Options = sdpsettings('solver','quadprog');
out=optimize(C,obj,Options);
double(u)
double(obj)
toc
```

```
Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-
decreasing in
feasible directions, to within the default value of the optimality
 tolerance,
and constraints are satisfied to within the selected value of the
 constraint tolerance.



ans =

    1.0000
    0.9129
   -0.8299


ans =

   12.2743

Elapsed time is 0.287770 seconds.
```

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