Using FMINCON to solve dynamic optimization problems

Two approaches are presented, both reformulating the problem to a general nonlinear program. In one case, "substitution", the decision variable consists solely of the input sequence (as the resulting state sequence is an explicit function of the intial condition and the input sequence). In the other case, both the input **and** state sequences are decision variables, and the optimization problem includes equality constraints which constrain the state and input sequences to satisfy the system's governing equation.

Contents

Some parameters

```
TS = 0.05;
N = 120;
TFinal = TS*N;
```

Continuous dynamics

$$\dot{x}_1 = \sin(x_1) + \frac{\gamma}{1+x_1^2}\tan^{-1}(x_2)$$

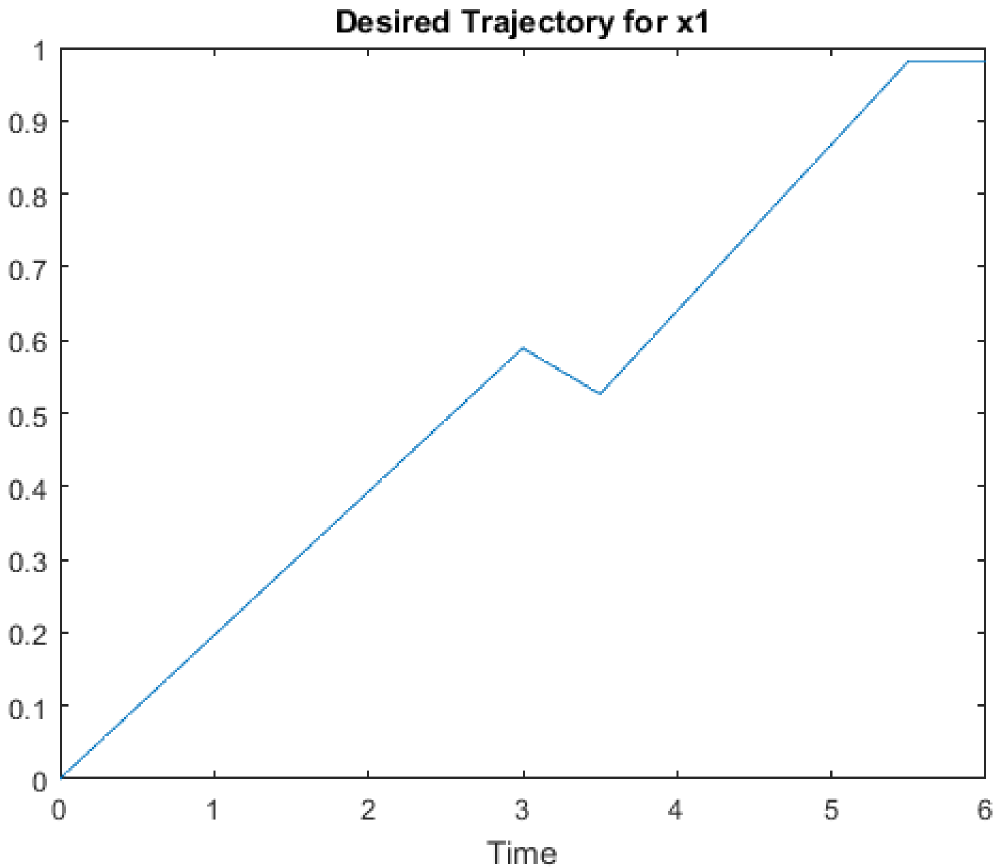
$$\dot{x}_2 = -\frac{1}{\tau}(x_2 - u)$$

```
tau = 0.2;
gamma = 10;
fDyn = @(x,u) [-sin(x(1))+gamma/(1+x(1)^2)*atan(x(2)); -1/tau*(-u+x(2))];
nx = 2;
nu = 1;
```

Desired State trajectory

Create specific piecewise-linear desired trajectory for x1.

```
tValues = [0 3 3.5 5.5 6];
xDesValues = [0 0.75*pi/4 0.67*pi/4 1.25*pi/4 1.25*pi/4];
tGrid = 0:TS:TFinal;
xDesGrid = interp1(tValues, xDesValues, tGrid);
plot(TS*(0:N), xDesGrid)
title('Desired Trajectory for x1')
xlabel('Time')
```



Create anonymous function representing objective function

Write a function which has two input arguments: a state sequence and an input sequence, and returns the value of the objective.

```
objJ = @(xSeq, uSeq) TS*norm(xSeq(1,:)-xDesGrid)^2;
```

Constraint Functions

The inequality constraint function accepts two arguments - a state and input sequence, and returns a vector of values. `fmincon` interprets this as values that are constrained to all be <= 0. In our example, we have constraints on the final value of `x1` (should be close to final value of `xDesGrid`), as well as constraints on the magnitude of $\dot{x}_2$. Here we put the rate-bound on x_2 as 0.07.

```
RBValue = 0.07;
```

```
cIE = @(xSeq, uSeq) [0.975*xDesGrid(end)-xSeq(1,end);...
     xSeq(1,end)-1.025*xDesGrid(end);...
    (-1/tau*(-uSeq+xSeq(2,1:end-1))')-RBValue;...
    -(-1/tau*(-uSeq+xSeq(2,1:end-1))')-RBValue];
```

This control problem has no explicit equality constraints.

```
cE = @(xSeq, uSeq) [];
```

Create `parm` object, which has details of the problem

Look in `fminconDynamicSystemTemplate` to see what variables should be specified as fields of `parm`. They are listed below with short comments to remind you of their meanings.

```
parm.fh = fDyn;         % handle to continuous-time dynamics
parm.ts = TS;           % discretization time
parm.nu = nu;           % number of control inputs
parm.nx = nx;           % number of states
parm.x0 = zeros(nx,1); % initial condition for state
parm.Jh = objJ;         % handle to objective function (acts on state and input sequence)
parm.cIEh = cIE;        % handle to inequality constraint function
parm.cEh = cE;          % handle to equality constraint function (do not include state equatio
ns)
```

Case 1: only the input is treated as a decision variable

By setting the messages to `fminconDynamicSystemTemplate` to `'obj'` and `'constraint'`, it is assumed that the only decision variables are the input sequence, and the state sequence will automatically be solved for, given the initial condition, dynamic equations, and input sequence. Let `nDV` denote the number of decision variables. Since the input dimension is known (`parm.nu`) and the horizon (`N`) is known, the number of decision variables is easy to deermine.

```
nDV = parm.nu*N;
dvInit = randn(nDV,1);  % give FMINCON a random initial value for decision variable
fmcOBJ = @(dv) fminconDynamicSystemTemplate(dv, parm, 'obj');
fmcCON = @(dv) fminconDynamicSystemTemplate(dv, parm, 'constraint');
```

Call FMINCON

```
options = optimoptions('fmincon','MaxFunctionEvaluations',20000,'display','iter');
[uDV,FVAL,EXITFLAG,OUTPUT,LAMBDA,GRAD] = fmincon(fmcOBJ,dvInit,...
    [],[],[],[],[],[],fmcCON,options);
% The returned variable |uDV| should be a column vector, of dimension
% |nDV|.  It represents the optimal input trajectory, as determined by the
% optimization.
```

```
                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0     121    3.069091e+00    1.528e+01    8.947e-01
```

1	242	1.943241e+00	7.973e+00	9.626e-01	4.906e+00
2	363	4.963573e-01	4.323e+00	3.969e-01	2.649e+00
3	484	3.587988e-01	1.254e+00	3.929e-01	2.207e+00
4	605	3.956191e-02	3.782e-01	2.289e-01	5.148e-01
5	726	2.607185e-02	9.003e-02	2.087e-01	2.279e-01
6	848	2.337256e-02	2.209e-02	1.156e-01	3.580e-02
7	969	2.216128e-02	0.000e+00	4.963e-02	3.319e-02
8	1090	2.081750e-02	0.000e+00	2.406e-02	1.067e-02
9	1211	1.842743e-02	0.000e+00	2.174e-03	1.027e-02
10	1332	7.248300e-03	0.000e+00	3.763e-03	7.885e-02
11	1453	4.499085e-03	0.000e+00	3.411e-03	3.078e-02
12	1574	3.804180e-03	0.000e+00	1.327e-03	1.181e-02
13	1695	3.753184e-03	0.000e+00	2.749e-04	2.163e-03
14	1816	3.752684e-03	0.000e+00	2.000e-04	6.386e-04
15	1937	2.419953e-03	0.000e+00	1.512e-03	2.711e-02
16	2058	1.770221e-03	0.000e+00	9.268e-04	1.961e-02
17	2179	1.653241e-03	0.000e+00	4.624e-04	4.630e-03
18	2300	1.633029e-03	0.000e+00	2.825e-04	2.086e-03
19	2421	1.632193e-03	0.000e+00	1.312e-04	2.519e-03
20	2542	1.634261e-03	0.000e+00	1.022e-04	1.099e-03
21	2663	1.633999e-03	0.000e+00	6.117e-05	8.916e-04
22	2784	1.632492e-03	0.000e+00	4.000e-05	3.385e-04
23	2905	1.174852e-03	0.000e+00	1.043e-03	2.214e-02
24	3026	9.677132e-04	0.000e+00	5.324e-04	2.099e-02
25	3147	9.506039e-04	0.000e+00	3.215e-04	2.105e-03
26	3268	9.435168e-04	0.000e+00	1.716e-04	2.756e-03
27	3389	9.445846e-04	0.000e+00	1.059e-04	1.634e-03
28	3510	9.457759e-04	0.000e+00	6.567e-05	1.113e-03
29	3631	9.455788e-04	0.000e+00	4.491e-05	5.756e-04
30	3752	9.449102e-04	0.000e+00	3.196e-05	4.762e-04

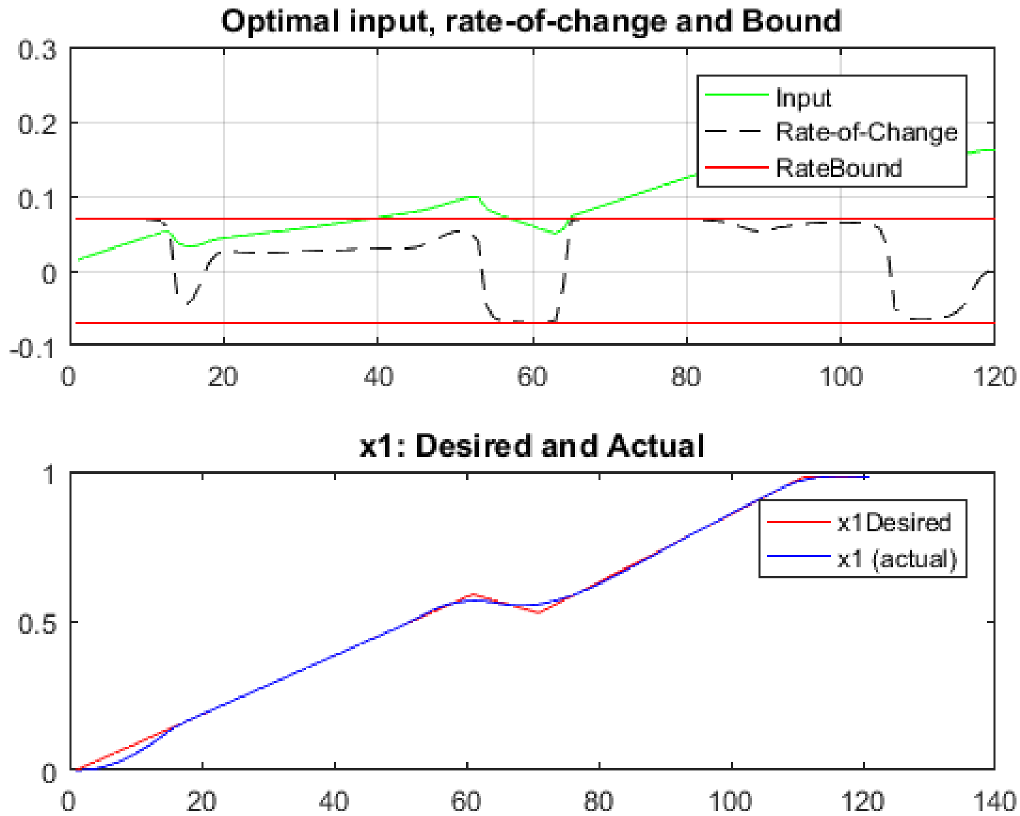
Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	3873	9.445771e-04	0.000e+00	2.064e-05	3.211e-04
32	3994	9.446149e-04	0.000e+00	1.527e-05	1.737e-04
33	4115	9.447201e-04	0.000e+00	9.055e-06	1.030e-04
34	4236	9.447612e-04	0.000e+00	8.000e-06	6.936e-05
35	4357	8.165063e-04	0.000e+00	8.542e-04	1.609e-02
36	4478	7.614662e-04	0.000e+00	6.426e-04	1.124e-02
37	4599	7.568039e-04	0.000e+00	2.290e-04	1.751e-03
38	4720	7.559286e-04	0.000e+00	1.490e-04	1.555e-03
39	4841	7.554135e-04	0.000e+00	1.087e-04	1.619e-03
40	4962	7.550755e-04	0.000e+00	6.922e-05	1.330e-03
41	5083	7.548207e-04	0.000e+00	5.768e-05	1.120e-03
42	5204	7.546467e-04	0.000e+00	5.194e-05	6.545e-04
43	5325	7.544040e-04	0.000e+00	5.088e-05	7.845e-04
44	5446	7.542283e-04	0.000e+00	3.718e-05	7.143e-04
45	5567	7.542270e-04	0.000e+00	2.071e-05	4.331e-04
46	5688	7.542910e-04	0.000e+00	2.679e-05	2.710e-04
47	5809	7.543609e-04	0.000e+00	3.106e-05	3.076e-04
48	5930	7.543806e-04	0.000e+00	2.712e-05	3.124e-04
49	6051	7.543394e-04	0.000e+00	1.563e-05	2.557e-04
50	6172	7.542714e-04	0.000e+00	1.062e-05	1.771e-04
51	6293	7.542202e-04	0.000e+00	8.271e-06	1.112e-04
52	6414	7.541945e-04	0.000e+00	8.039e-06	7.870e-05
53	6535	7.541887e-04	0.000e+00	5.009e-06	6.456e-05
54	6656	7.541966e-04	0.000e+00	4.858e-06	5.132e-05

```
    55     6777    7.542077e-04    0.000e+00    3.514e-06    4.052e-05
    56     6898    7.542162e-04    0.000e+00    3.249e-06    3.360e-05
    57     7019    7.542201e-04    0.000e+00    2.477e-06    3.141e-05
    58     7140    7.542194e-04    0.000e+00    2.751e-06    2.889e-05
    59     7261    7.542165e-04    0.000e+00    2.126e-06    2.452e-05
    60     7382    7.542129e-04    0.000e+00    1.955e-06    2.242e-05

                                                First-order      Norm of
  Iter F-count            f(x)  Feasibility   optimality         step
    61     7503    7.542097e-04    0.000e+00    1.600e-06    1.874e-05
    62     7624    7.138124e-04    0.000e+00    2.194e-04    1.143e-02
    63     7745    7.037103e-04    0.000e+00    1.857e-04    7.272e-03
    64     7866    7.026355e-04    0.000e+00    1.252e-04    1.461e-03
    65     7987    7.022736e-04    0.000e+00    7.293e-05    8.004e-04
    66     8108    7.022133e-04    0.000e+00    6.910e-05    6.885e-04
    67     8229    7.021464e-04    0.000e+00    4.998e-05    5.497e-04
    68     8350    7.020886e-04    0.000e+00    2.094e-05    3.636e-04
    69     8471    7.020414e-04    0.000e+00    2.333e-05    3.903e-04
    70     8592    7.020128e-04    0.000e+00    1.754e-05    3.258e-04
    71     8713    7.019946e-04    0.000e+00    1.601e-05    2.688e-04
    72     8834    7.019833e-04    0.000e+00    1.799e-05    3.264e-04
    73     8955    7.019757e-04    0.000e+00    1.722e-05    4.387e-04
    74     9076    7.019697e-04    0.000e+00    1.223e-05    4.245e-04
    75     9197    7.019618e-04    0.000e+00    9.976e-06    3.435e-04
    76     9318    7.019511e-04    0.000e+00    1.104e-05    2.588e-04
    77     9439    7.019426e-04    0.000e+00    1.144e-05    1.676e-04
    78     9560    7.019358e-04    0.000e+00    9.531e-06    1.369e-04
    79     9681    7.019313e-04    0.000e+00    5.802e-06    1.478e-04
    80     9802    7.019310e-04    0.000e+00    5.699e-06    1.516e-04
    81     9923    7.019340e-04    0.000e+00    6.609e-06    1.754e-04
    82    10044    7.019378e-04    0.000e+00    6.741e-06    1.853e-04
    83    10165    7.019390e-04    0.000e+00    5.359e-06    1.421e-04
    84    10286    7.019385e-04    0.000e+00    4.077e-06    1.290e-04
    85    10407    7.019358e-04    0.000e+00    3.434e-06    9.512e-05
    86    10528    7.019320e-04    0.000e+00    4.066e-06    6.577e-05
    87    10649    7.019281e-04    0.000e+00    4.712e-06    7.350e-05
    88    10770    7.019256e-04    0.000e+00    3.595e-06    7.874e-05
    89    10891    7.019250e-04    0.000e+00    1.980e-06    6.536e-05
    90    11012    7.019253e-04    0.000e+00    2.152e-06    5.642e-05

                                                First-order      Norm of
  Iter F-count            f(x)  Feasibility   optimality         step
    91    11133    7.019255e-04    0.000e+00    2.811e-06    4.993e-05
    92    11254    7.019253e-04    0.000e+00    2.689e-06    5.353e-05
    93    11375    7.019242e-04    0.000e+00    1.395e-06    5.910e-05
    94    11496    7.019229e-04    0.000e+00    1.868e-06    4.918e-05
    95    11617    7.019221e-04    0.000e+00    1.970e-06    3.799e-05
    96    11738    7.019217e-04    0.000e+00    1.483e-06    4.231e-05
    97    11859    7.019218e-04    0.000e+00    9.874e-07    3.080e-05

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in 
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.
```

Recover input/state trajectories and plot

Reshape the decision variable into a sequencte of input values.

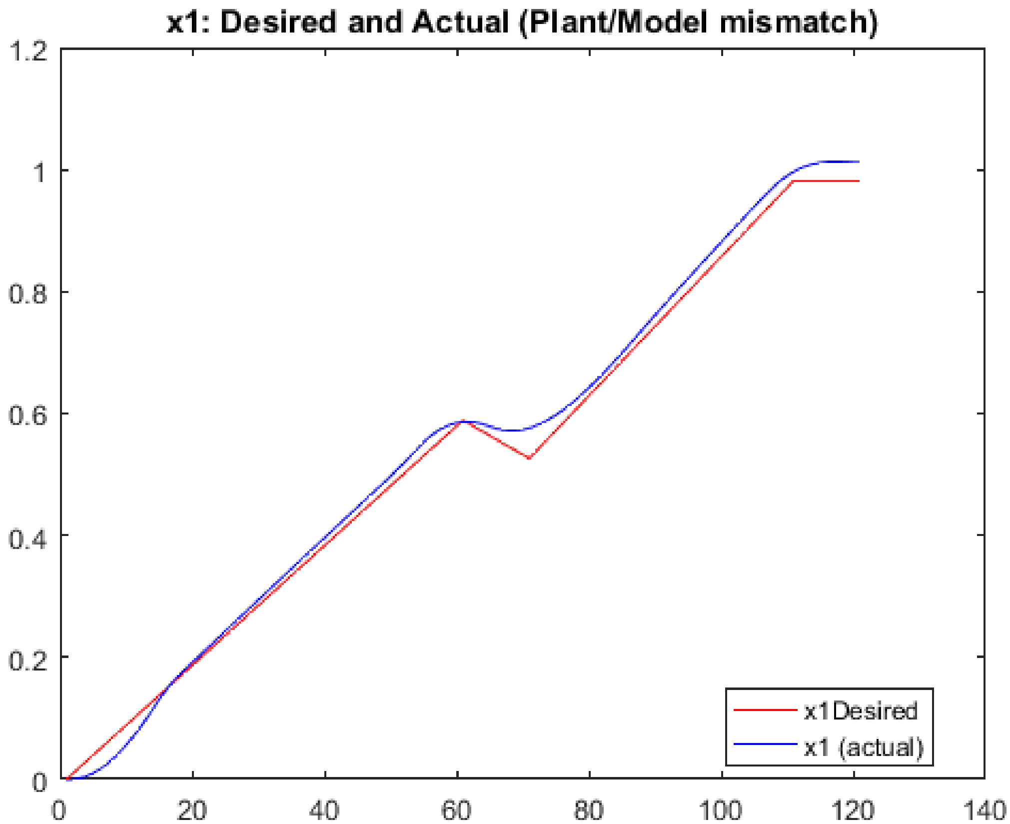
```
uSeq = reshape(uDV,[nu,N]);
% Simulate the discretized system, starting from the specified initial
% condition, using the optimal input.
xSeq = zeros(nx,N+1);
xSeq(:,1) = parm.x0;
for i=1:N
    xSeq(:,i+1) = xSeq(:,i) + TS*fDyn(xSeq(:,i), uSeq(:,i));
end
% Reconstruct the signal ($\dot x}_2$) that should be bounded by |RBvalue|
x2Dot = -1/tau*(-uSeq+xSeq(2,1:end-1));
clf
subplot(2,1,1)
plot(1:N,uSeq,'g',1:N,x2Dot(1:N),'k--',1:N,...
   repmat(RBValue,[1 N]),'r',1:N,repmat(-RBValue,[1 N]),'r')
legend('Input','Rate-of-Change','RateBound')
title('Optimal input, rate-of-change and Bound')
grid on
subplot(2,1,2)
plot(1:N+1, xDesGrid, 'r', 1:N+1,xSeq(1,:),'b')
legend('x1Desired', 'x1 (actual)')
title('x1: Desired and Actual')
```



Effect of uncertainties on system response

Create new dynamics, with slightly different parameters. Use the optimized control signal (optimized on original plant model) on this perturbed plant model. See the degradation in the tracking performance.

```
tauPerturbed = 0.22;
gammaPerturbed = 10.5;
fDynPerturbed = @(x,u) [-sin(x(1))+gammaPerturbed/(1+x(1)^2)*atan(x(2)); ...
   -1/tauPerturbed*(-u+x(2))];
xSeq = zeros(2,N+1);
uSeq = reshape(uDV,[parm.nu,N]);
xSeq(:,1) = parm.x0;
for i=1:N
    xSeq(:,i+1) = xSeq(:,i) + TS*fDynPerturbed(xSeq(:,i), uSeq(:,i));
end
clf
plot(1:N+1, xDesGrid, 'r', 1:N+1,xSeq(1,:),'b')
legend('x1Desired', 'x1 (actual)', 'location','best')
title('x1: Desired and Actual (Plant/Model mismatch)')
```



This degradation points to the advantage of feedback - instead of computing 120 control actions (here), and just applying them, it is advantageous to apply some portion of the optimal input sequence, measure the actual response, and then reoptimize the input over some time-interval of the future (perhaps another 120 steps). Even if there is a mismatch between the true plant behavior and the plant model (used in optimization), the continuous act of reassessing (based on measurement) and reoptimization provides feedback which leads to corrective actions. We will revisit this extensively in the MPC/receding horizon control section.

Solve problem with equality constraints to represent dynamics

There are still no additional equality constraints. However, by setting the messages to `fminconDynamicSystemTemplate` to have the suffix `StateEq`, the optimization problem is formulated with decision variables for both input **and** state, and the state-equations are imposed as equality constraints (see the variable `StateEvolveMismatch` in the code for `fminconDynamicSystemTemplate`).

```
nDV = (parm.nu + parm.nx)*N;
dvInit = randn(nDV,1);
fmcOBJ = @(dv) fminconDynamicSystemTemplate(dv, parm, 'objStateEQ');
fmcCON = @(dv) fminconDynamicSystemTemplate(dv, parm, 'constraintStateEQ');
```

Call FMINCON

```
options = optimoptions('fmincon','MaxFunctionEvaluations',80000,'display','iter');
[uxDV,FVAL,EXITFLAG,OUTPUT,LAMBDA,GRAD] = fmincon(fmcOBJ,randn(nDV,1),...
    [],[],[],[],[],[],fmcCON,options);
```

```
                                            First-order      Norm of
```

Iter	F-count	f(x)	Feasibility	optimality	step
0	361	9.483837e+00	2.150e+01	1.804e+00	
1	722	7.232506e+00	1.797e+01	1.809e+00	3.393e+00
2	1083	2.585848e+00	7.081e+00	1.792e+00	1.051e+01
3	1444	1.365645e+00	2.743e+00	1.181e+00	4.228e+00
4	1805	9.337296e-01	1.241e+00	5.968e-01	1.682e+00
5	2166	4.835873e-01	6.200e-01	3.627e-01	1.617e+00
6	2528	2.873504e-01	5.395e-01	3.676e-01	8.551e-01
7	2889	5.744935e-02	2.107e-01	2.124e-01	2.009e+00
8	3251	4.138454e-02	9.623e-02	1.375e-01	4.742e-01
9	3612	3.463719e-02	1.347e-03	1.001e-01	7.712e-01
10	3973	2.942671e-02	3.120e-06	2.427e-02	7.746e-02
11	4334	2.552461e-02	5.262e-06	3.051e-03	6.760e-02
12	4695	8.771866e-03	1.571e-04	3.421e-03	4.067e-01
13	5056	5.544621e-03	4.229e-05	4.581e-03	1.627e-01
14	5417	4.748609e-03	2.681e-06	4.295e-03	3.696e-02
15	5778	3.984464e-03	2.251e-06	9.664e-04	3.418e-02
16	6139	3.743529e-03	1.450e-06	6.483e-04	2.868e-02
17	6500	3.772539e-03	2.867e-07	3.997e-04	1.287e-02
18	6861	3.778303e-03	8.606e-08	2.641e-04	4.173e-03
19	7222	3.764253e-03	1.191e-07	2.000e-04	4.746e-03
20	7583	2.461776e-03	9.544e-06	7.255e-04	7.798e-02
21	7944	1.804635e-03	5.034e-06	9.070e-04	5.227e-02
22	8305	1.670882e-03	3.389e-07	5.534e-04	1.310e-02
23	8666	1.637171e-03	7.020e-08	3.488e-04	4.574e-03
24	9027	1.628547e-03	9.478e-08	1.682e-04	6.062e-03
25	9388	1.635369e-03	1.804e-08	1.292e-04	2.505e-03
26	9749	1.636717e-03	6.054e-09	7.368e-05	1.511e-03
27	10110	1.634652e-03	2.322e-09	4.000e-05	9.636e-04
28	10471	1.205411e-03	4.504e-06	7.009e-04	4.754e-02
29	10832	9.939367e-04	2.089e-06	5.525e-04	3.327e-02
30	11193	9.716199e-04	7.838e-08	2.980e-04	6.262e-03

				First-order	Norm of
Iter	F-count	f(x)	Feasibility	optimality	step
31	11554	9.594558e-04	6.153e-08	1.763e-04	6.031e-03
32	11915	9.538354e-04	5.210e-08	1.631e-04	4.795e-03
33	12276	9.491614e-04	1.211e-07	1.385e-04	6.097e-03
34	12637	9.472295e-04	4.237e-08	1.182e-04	3.531e-03
35	12998	9.461301e-04	6.568e-09	8.320e-05	1.529e-03
36	13359	9.454038e-04	6.534e-10	6.981e-05	7.343e-04
37	13720	9.447820e-04	1.150e-09	2.827e-05	7.272e-04
38	14081	9.445654e-04	1.252e-09	2.514e-05	5.706e-04
39	14442	9.446458e-04	1.743e-09	2.514e-05	6.043e-04
40	14803	9.447668e-04	1.272e-09	1.981e-05	4.673e-04
41	15164	9.448300e-04	1.102e-09	1.482e-05	4.268e-04
42	15525	9.448310e-04	5.515e-10	1.138e-05	3.549e-04
43	15886	9.448055e-04	1.199e-10	9.071e-06	2.721e-04
44	16247	9.447817e-04	4.441e-11	8.000e-06	1.710e-04
45	16608	8.137413e-04	1.870e-06	3.730e-04	2.955e-02
46	16969	7.622989e-04	1.213e-06	5.367e-04	1.680e-02
47	17330	7.581975e-04	5.950e-08	2.841e-04	3.335e-03
48	17691	7.563655e-04	1.448e-08	1.331e-04	2.729e-03
49	18052	7.561880e-04	3.941e-09	1.135e-04	1.325e-03
50	18413	7.556597e-04	1.645e-08	6.628e-05	2.126e-03
51	18774	7.552056e-04	2.606e-09	5.271e-05	1.401e-03
52	19135	7.547972e-04	1.708e-09	5.271e-05	9.891e-04

53	19496	7.545673e-04	4.171e-10	4.958e-05	5.919e-04
54	19857	7.542772e-04	1.067e-09	3.522e-05	1.002e-03
55	20218	7.541694e-04	8.798e-10	2.687e-05	9.097e-04
56	20579	7.542406e-04	1.051e-09	2.687e-05	7.433e-04
57	20940	7.543021e-04	1.744e-10	1.842e-05	3.459e-04
58	21301	7.543179e-04	1.826e-10	1.361e-05	2.754e-04
59	21662	7.542905e-04	9.209e-11	1.106e-05	2.179e-04
60	22023	7.542514e-04	3.133e-11	8.701e-06	1.740e-04

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
61	22384	7.542223e-04	2.478e-11	6.329e-06	1.391e-04
62	22745	7.542103e-04	2.605e-11	4.661e-06	1.263e-04
63	23106	7.542121e-04	2.443e-11	4.661e-06	1.163e-04
64	23467	7.542140e-04	2.926e-11	4.661e-06	9.320e-05
65	23828	7.542124e-04	1.614e-11	3.753e-06	7.438e-05
66	24189	7.542093e-04	6.902e-12	3.186e-06	6.340e-05
67	24550	7.542080e-04	1.768e-12	2.672e-06	4.635e-05
68	24911	7.542097e-04	1.793e-12	1.907e-06	3.394e-05
69	25272	7.542124e-04	1.415e-12	1.600e-06	3.071e-05
70	25633	7.149915e-04	6.674e-07	1.845e-04	1.560e-02
71	25994	7.039657e-04	1.915e-07	1.857e-04	8.266e-03
72	26355	7.033531e-04	5.627e-09	1.730e-04	1.614e-03
73	26716	7.025979e-04	8.741e-09	8.024e-05	1.459e-03
74	27077	7.024364e-04	5.327e-09	7.396e-05	1.221e-03
75	27438	7.023183e-04	2.772e-09	6.161e-05	1.066e-03
76	27799	7.021888e-04	2.061e-09	3.320e-05	8.775e-04
77	28160	7.021146e-04	8.197e-10	2.931e-05	4.945e-04
78	28521	7.020589e-04	6.004e-10	1.389e-05	4.399e-04
79	28882	7.020159e-04	6.863e-10	1.260e-05	4.521e-04
80	29243	7.019873e-04	3.550e-10	1.182e-05	4.243e-04
81	29604	7.019720e-04	1.192e-10	1.114e-05	3.194e-04
82	29965	7.019616e-04	1.294e-10	9.959e-06	2.687e-04
83	30326	7.019541e-04	7.075e-11	8.711e-06	2.158e-04
84	30687	7.019470e-04	5.216e-11	8.711e-06	2.174e-04
85	31048	7.019412e-04	3.774e-11	5.795e-06	2.109e-04
86	31409	7.019367e-04	1.231e-10	5.721e-06	2.086e-04
87	31770	7.019334e-04	1.448e-10	5.721e-06	1.994e-04
88	32131	7.019312e-04	2.960e-11	2.938e-06	1.181e-04
89	32492	7.019294e-04	1.431e-11	2.938e-06	8.588e-05
90	32853	7.019278e-04	1.475e-11	2.938e-06	1.104e-04

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
91	33214	7.019273e-04	1.285e-11	2.307e-06	9.248e-05
92	33575	7.019278e-04	4.304e-12	2.248e-06	6.302e-05
93	33936	7.019288e-04	7.772e-12	2.248e-06	6.340e-05
94	34297	7.019294e-04	7.007e-12	2.248e-06	6.589e-05
95	34658	7.019294e-04	1.494e-11	2.036e-06	7.953e-05
96	35019	7.019280e-04	2.393e-11	1.919e-06	9.146e-05
97	35380	7.019261e-04	9.067e-12	1.811e-06	7.318e-05
98	35741	7.019255e-04	2.398e-12	1.730e-06	4.927e-05
99	36102	7.019259e-04	3.357e-12	1.628e-06	5.758e-05
100	36463	7.019267e-04	3.103e-12	1.516e-06	6.048e-05
101	36824	7.019268e-04	3.320e-12	1.402e-06	5.942e-05
102	37185	7.019255e-04	1.540e-12	1.294e-06	5.588e-05
103	37546	7.019234e-04	1.806e-12	1.212e-06	5.159e-05

```
  104   37907    7.019218e-04    2.264e-12    1.169e-06    4.660e-05
  105   38268    7.019212e-04    1.610e-12    1.110e-06    4.270e-05
  106   38629    7.019216e-04    8.221e-13    9.437e-07    3.976e-05

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the default value of the optimality tolerance,
and constraints are satisfied to within the default value of the constraint tolerance.
```

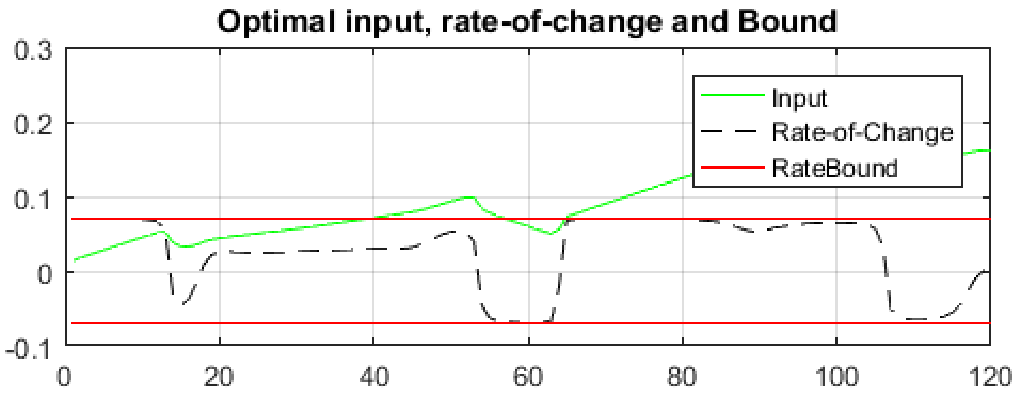
Check size of decision variable, and verify that this makes sense to you

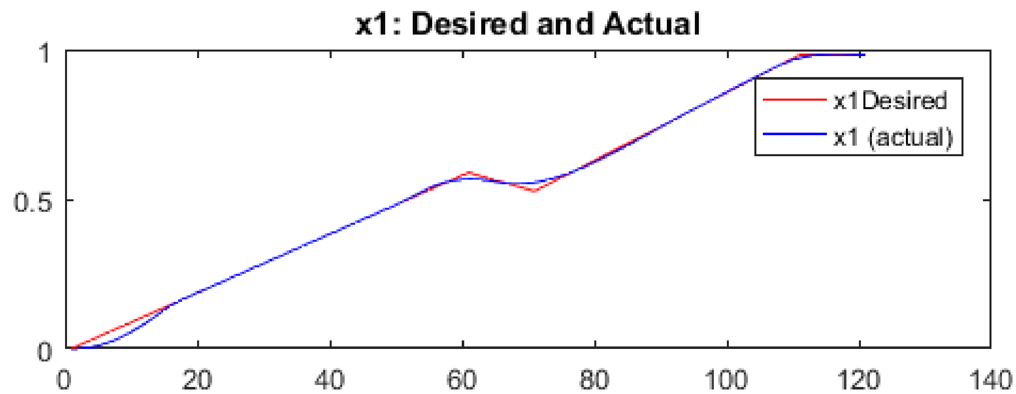
```
numel(uxDV)
```

```
ans =

   360
```

Recover input/state trajectories and plot

```
uSeq = reshape(uxDV(1:N*parm.nu),[parm.nu,N]);
xSeq = [parm.x0 reshape(uxDV(N*parm.nu+1:end),[parm.nx,N])];
x2Dot = -1/tau*(-uSeq+xSeq(2,1:end-1));
clf
subplot(2,1,1)
plot(1:N,uSeq,'g',1:N,x2Dot(1:N),'k--',1:N,...
   repmat(RBValue,[1 N]),'r',1:N,repmat(-RBValue,[1 N]),'r')
legend('Input','Rate-of-Change','RateBound')
title('Optimal input, rate-of-change and Bound')
grid on
subplot(2,1,2)
plot(1:N+1, xDesGrid, 'r', 1:N+1,xSeq(1,:),'b')
legend('x1Desired', 'x1 (actual)')
title('x1: Desired and Actual')
```





Attribution

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