Project 4: Error estimation and adaptive meshing-using the exact solution as a test

Due Nov 10

- Consider the boundary value problem $\frac{d}{dx}\left(E\frac{du}{dx}\right) = f(x)$, $E = 1$, with domain $\Omega = (0, L)$, $L = 1$, and solution $u(x) = \cos(10\pi x^5)$.
- Compute the finite element solution u^N to this problem using linear equal-sized elements. Determine how many elements are needed in order to achieve

$$e^N \stackrel{\text{def}}{=} \frac{||u - u^N||_{E(\Omega)}}{||u||_{E(\Omega)}} \leq TOL = 0.05\,,$$

$$||u||_{E(\Omega)} \stackrel{\text{def}}{=} \sqrt{\int_\Omega \frac{du}{dx} E \frac{du}{dx} dx}$$

- Plot I versus A_I, where

$$A_I^2 \stackrel{\text{def}}{=} \frac{\frac{1}{h_I}||u - u^N||^2_{E(\Omega_I)}}{\frac{1}{L}||u||^2_{E(\Omega)}}\,.$$

  Here I is the element index, h_I is the length of element I, and

$$||u||^2_{E(\Omega_I)} \stackrel{\text{def}}{=} \int_{\Omega_I} \frac{du}{dx} E \frac{du}{dx} dx\,.$$

- Modify your code from HW 1 so that it can automatically refine the mesh the following criterion:
- Refine the mesh (by dividing elements into two) until $A_I < TOL_E$ for all I. Use this criterion to refine your mesh, starting with $N = 20$ equal-sized elements:
  - Determine how many elements are needed to achieve $A_I < TOL_E = 0.05$ for all I.
  - Plot the final solution, together with the exact solution.
  - Tabulate the final number of elements that fall into each of the initial 20 elements.
  - Plot X_I versus A_I for the final solution (X_I = position of node I).