Project 3: Potential and Efficient Solution Techniques

Due Oct 28

- Solve the following boundary value problem, with domain $\varOmega = (0, L)$, analytically:

$$\boxed{\begin{array}{l} \frac{d}{dx}\left(E(x)\frac{du}{dx}\right) = xk^3 cos(\frac{2\pi kx}{L}) \\ \\ E(x) = 10\ DIFFERENT\ SEGMENTS\ (SEE\ BELOW) \\ \\ k = 12,\ L = 1,\ u(0) = -0.3,\ u(L) = 0.7 \end{array}} \quad (0.1)$$

  For E

$$\boxed{\begin{array}{l} FOR\ 0.0 < x < 0.1\ E = 2.5 \\ FOR\ 0.1 < x < 0.2\ E = 1.0 \\ FOR\ 0.2 < x < 0.3\ E = 1.75 \\ FOR\ 0.3 < x < 0.4\ E = 1.25 \\ FOR\ 0.4 < x < 0.5\ E = 2.75 \\ FOR\ 0.5 < x < 0.6\ E = 3.75 \\ FOR\ 0.6 < x < 0.7\ E = 2.25 \\ FOR\ 0.7 < x < 0.8\ E = 0.75 \\ FOR\ 0.8 < x < 0.9\ E = 2.0 \\ FOR\ 0.9 < x < 1.0\ E = 1.0 \end{array}} \quad (0.2)$$

- Solve this with the finite element method using linear equal-sized elements. Use 100, 1000 and 10000 elements. You are to write a Preconditioned Conjugate-Gradient solver. Use the diagonal preconditioning given in the notes. The data storage is to be element by element (symmetric) and the matrix vector multiplication is to be done element by element.
- You are to plot the solution (nodal values) for each N.
- You are to plot

$$\boxed{\begin{array}{l} e^N \stackrel{\text{def}}{=} \frac{||u-u^N||_{E(\varOmega)}}{||u||_{E(\varOmega)}}, \\ \\ ||u||_{E(\varOmega)} \stackrel{\text{def}}{=} \sqrt{\int_{\varOmega} \frac{du}{dx} E \frac{du}{dx}\, dx}, \end{array}} \quad (0.3)$$

  for each N.

- You are to plot

$$\boxed{POTENTIAL\ ENERGY = \mathcal{J}(u^N)} \tag{0.4}$$

  for each N.
- You are to plot the number of PCG-solver iterations for each N for a stopping tolerance of 0.000001.
- Use a Gauss integration rule of level 5.
- Check your Conjugate Gradient generated results against a regular Gaussian solver, for example the one available in Matlab.