Project 2: Higher Order Elements

- Consider the following boundary value problem, with domain $\Omega = (0, L)$:

$$\boxed{\begin{array}{l} \frac{d}{dx}\left(E\frac{du}{dx}\right) = xk^3cos(\frac{2\pi kx}{L}) \\ E = 0.2 \\ k = 12 \\ L = 1 \\ u(0) = \Delta_1 = given\ constant = 3 \\ u(L) = \Delta_2 = given\ constant = -1 \end{array}} \tag{0.1}$$

- Solve this with the finite element method using order p equal-sized elements. In order to achieve

$$\boxed{\begin{array}{l} e^N \stackrel{\text{def}}{=} \frac{||u-u^N||_{E(\Omega)}}{||u||_{E(\Omega)}} \leq TOL = 0.04, \\ ||u||_{E(\Omega)} \stackrel{\text{def}}{=} \sqrt{\int_\Omega \frac{du}{dx} E \frac{du}{dx}\, dx} \end{array}} \tag{0.2}$$

  how many finite elements (N) are needed for

$$\boxed{\begin{array}{l} p = 1 \Rightarrow N =? \\ p = 2 \Rightarrow N =? \\ p = 3 \Rightarrow N =? \end{array}} \tag{0.3}$$

- Plot the numerical solutions for several values of N, for each p, along with the exact solution
- Plot e^N as a function of the element size h for each p
- Plot e^N as a function of the number of degrees of freedom for each p
- Determine the relationship between the error and the element size for each p
- Note: Please be careful with the quadrature order...you will need higher order Gauss rules for quadratic and cubic elements.