Project 1: The Basics of FEM

- Solve the followng boundary value problem, with domain $\Omega = (0, L)$, analytically:

$$\boxed{\begin{array}{l} \frac{d}{dx}\left(E\frac{du}{dx}\right) = k^2 sin(\frac{2\pi kx}{L}) \\ E = given\ constant = 0.1 \\ k = given\ constant \\ L = 1 \\ u(0) = \Delta_1 = given\ constant = 0 \\ u(L) = \Delta_2 = given\ constant = 1 \end{array}} \tag{0.1}$$

- Now solve this with the finite element method using linear equal-sized elements. In order to achieve,

$$\boxed{\begin{array}{l} e^N \overset{\text{def}}{=} \frac{||u-u^N||_{E(\Omega)}}{||u||_{E(\Omega)}} \leq TOL = 0.05, \\ \\ ||u||_{E(\Omega)} \overset{\text{def}}{=} \sqrt{\int_\Omega \frac{du}{dx} E \frac{du}{dx}\, dx} \end{array}} \tag{0.2}$$

  How many finite elements (N) are needed for

$$\boxed{\begin{array}{l} k = 1 \Rightarrow N = ? \\ k = 2 \Rightarrow N = ? \\ k = 4 \Rightarrow N = ? \\ k = 8 \Rightarrow N = ? \\ k = 16 \Rightarrow N = ? \\ k = 32 \Rightarrow N = ? \end{array}} \tag{0.3}$$

  You should set up a general matrix equation and solve it using Gaussian elimination. Later we will use other types of more efficient solvers. Plot the numerical solutions for $N = 2, 4, 8, 16, ...$, for each k, along with the exact solution. Also make a plot of the e^N for each k.

Remarks: You should write a general one-dimensional code where you specify the number of elements. Your code should partition the domain automatically. However, if you want to make the code more general (for future assignments), you should put in the following features:

- element endpoint locations (different sized elements)
- the possibility for different material values for each element ($E(x)$)

Assignment Format

- All assignments must be typed-nothing handwritten!
- Be concise, shorter is better-provided you do not delete essential information!
- You are encouraged to talk and work with one another. Please see me for any problems, theoretical, coding etc...
- Warning: Make sure the code runs, because we may sit together, login, and test it...

Assignment Write-up General Outline

1. Introduction to the problem: explain it to a layman...
2. Objectives: what are the goals...
3. Your procedure: Brief explanation, flow-charts, difficulties, assumptions, etc...
4. Findings: figures, plots and tables... (Make sure they are readable!)
5. Observations and discussion: some interpretation and insight into the results ...
6. Appendix: the messy stuff like your code or raw data...