Newton's Method

Announcements

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- All info and materials will be posted to cs61a.org/extra.html

Lambda Expressions

(Demo)

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```
>>> square(4)
16
```

Newton's Method

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`Quickly finds accurate approximations to zeroes of differentiable functions!`

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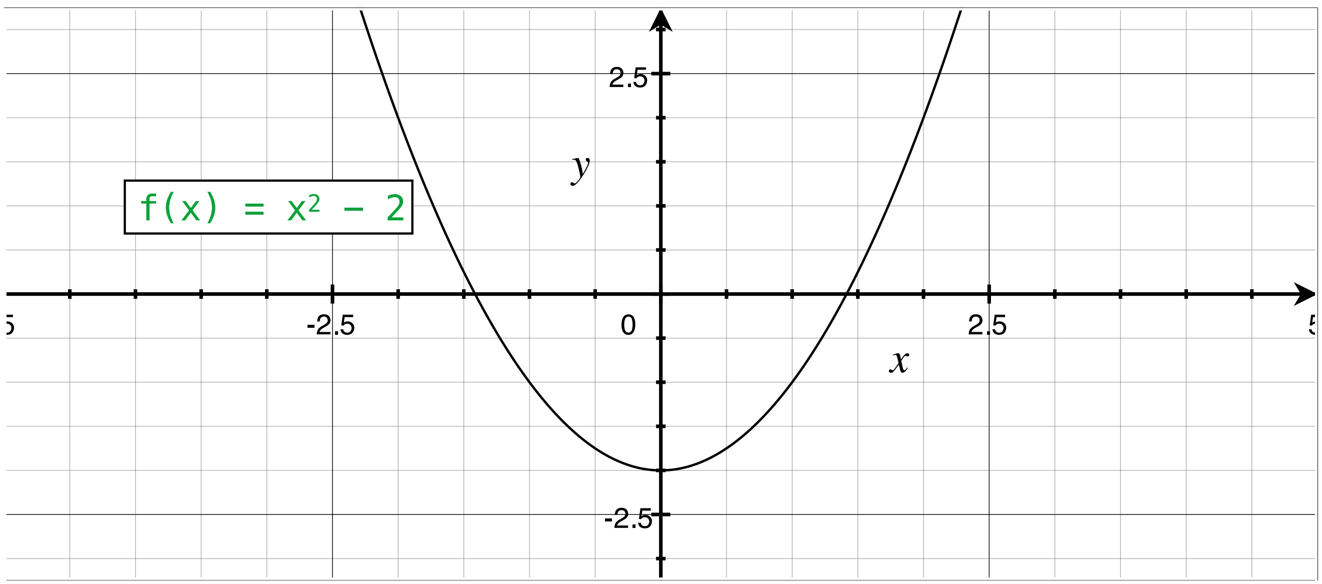

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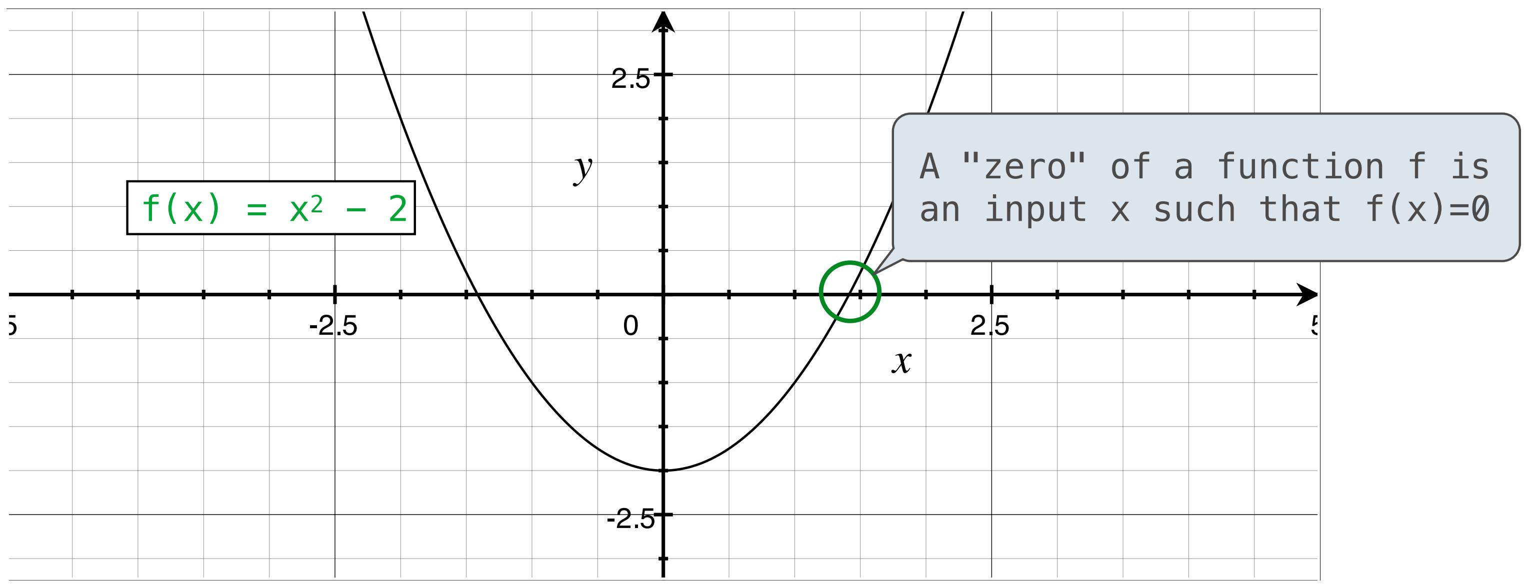

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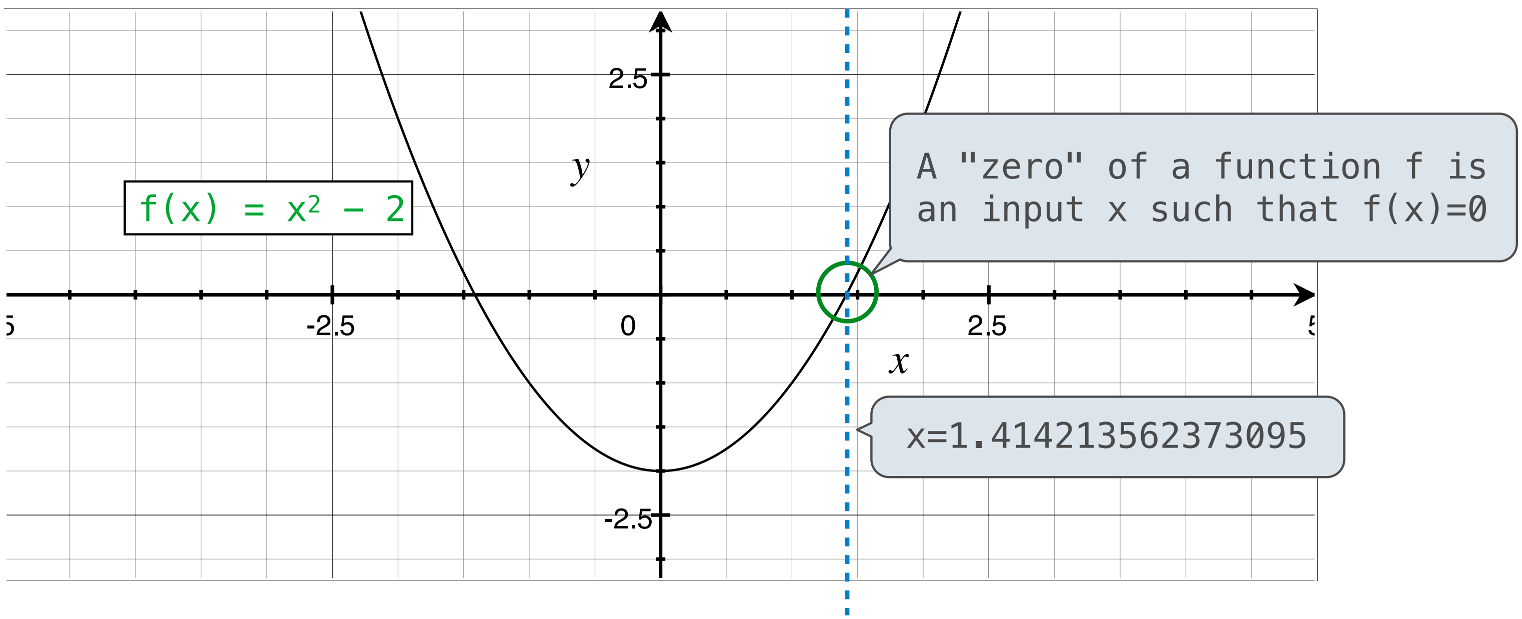

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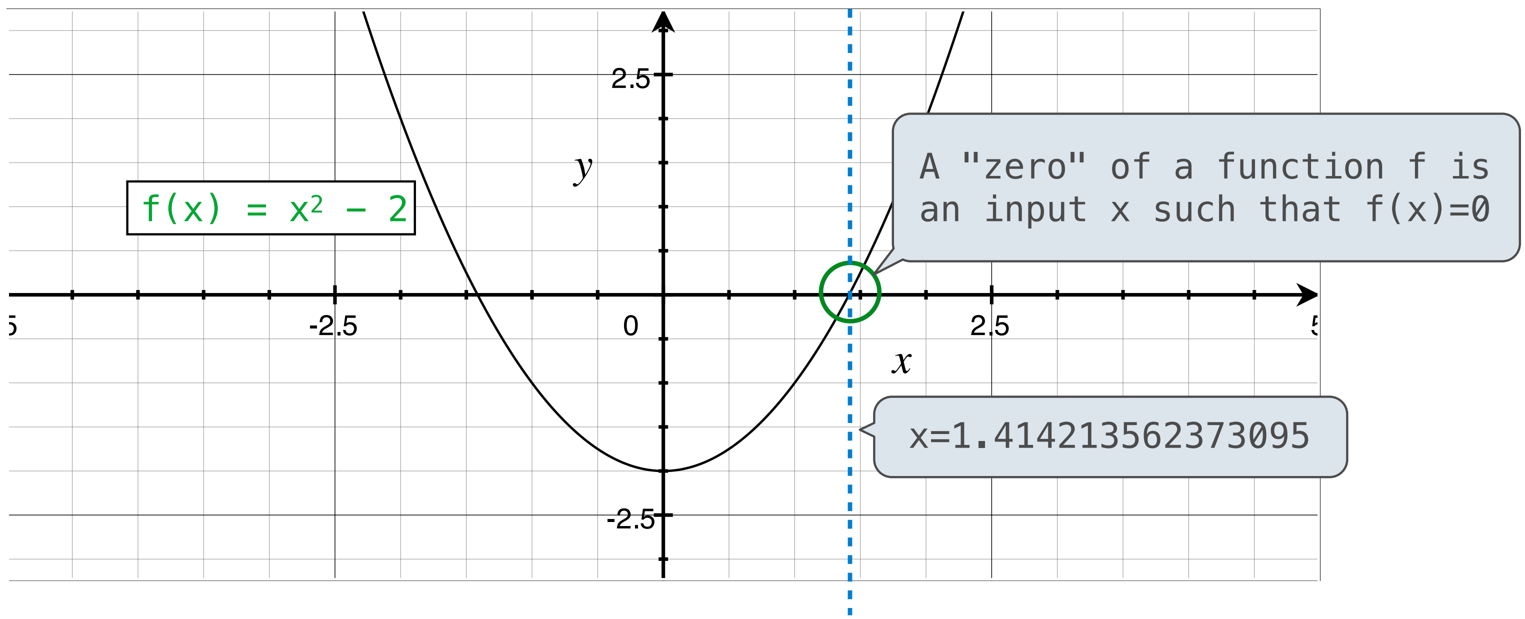

Application: a method for computing square roots, cube roots, etc.

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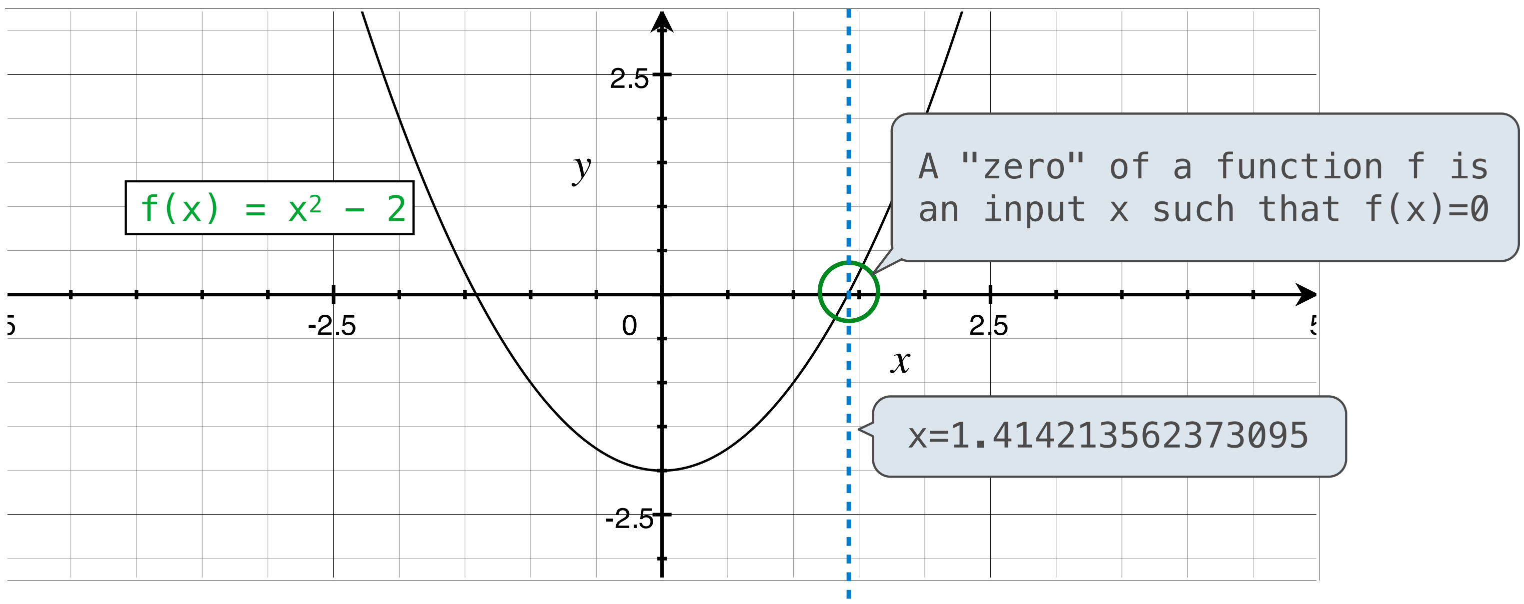

Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

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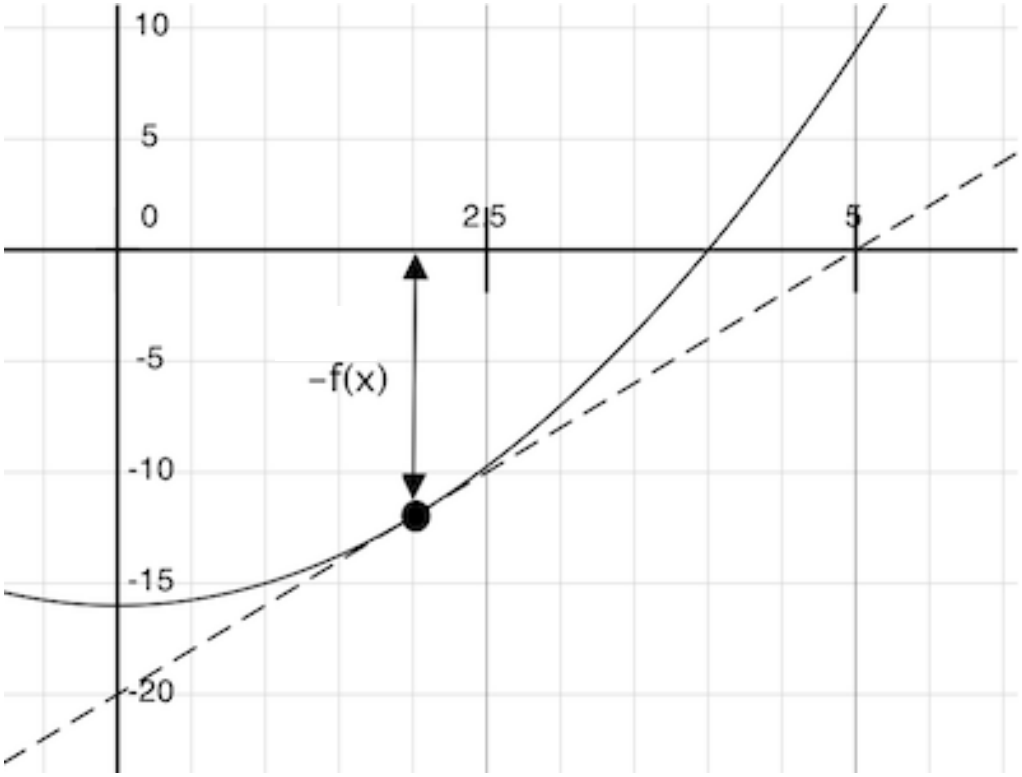

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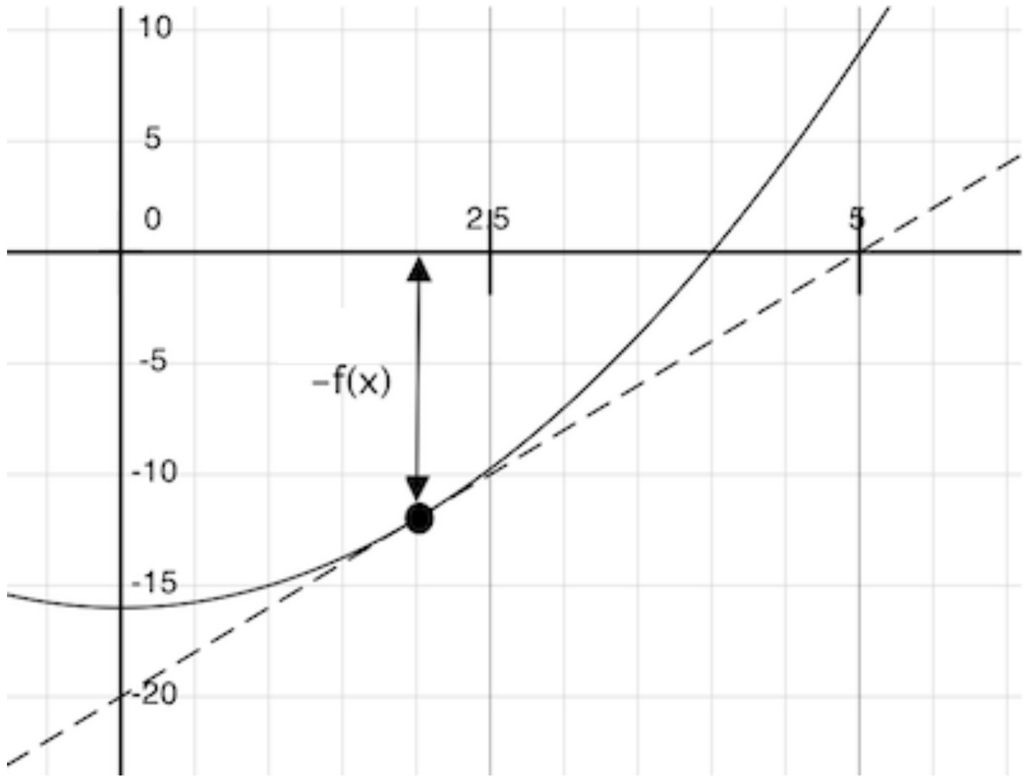

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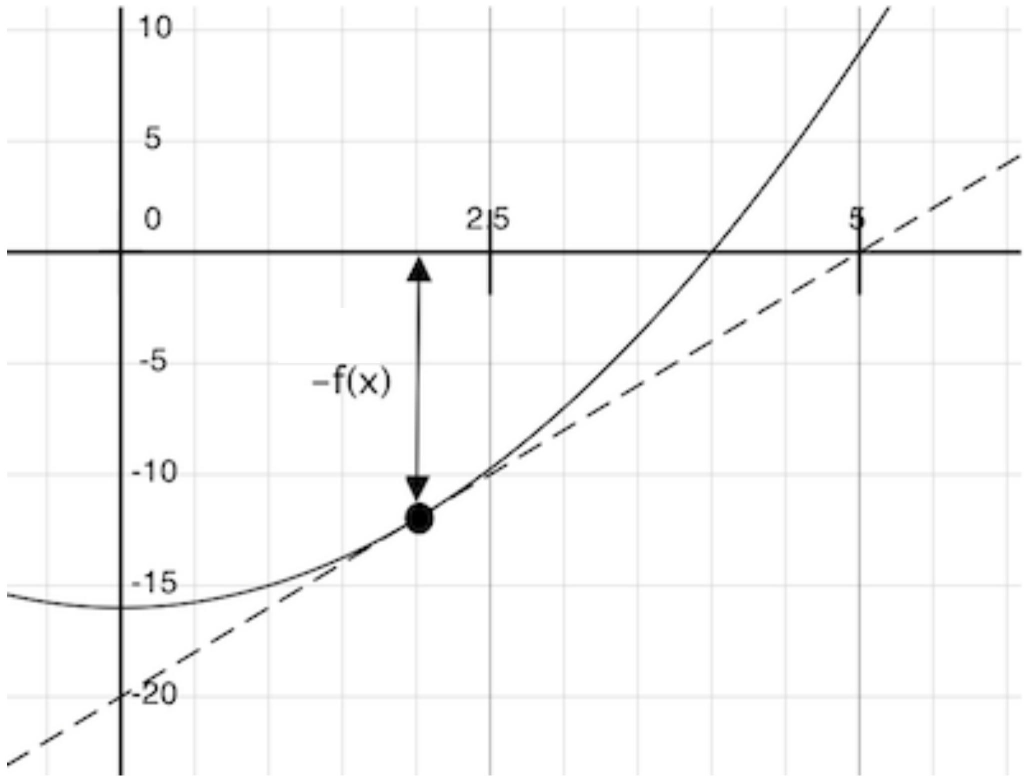

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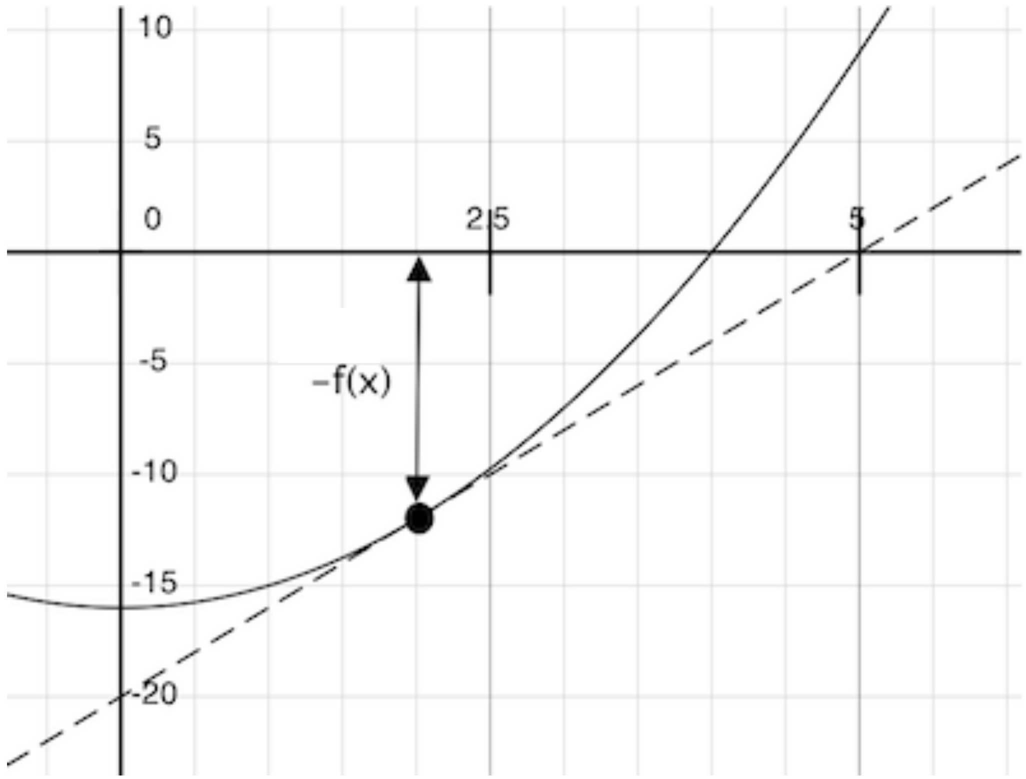

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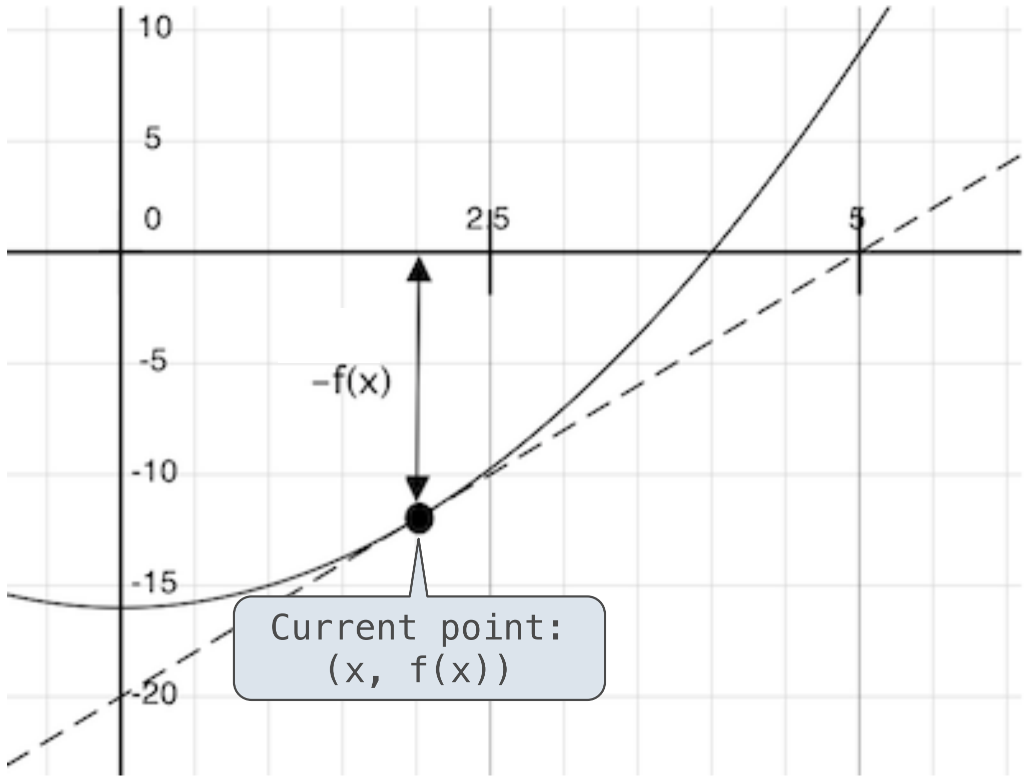

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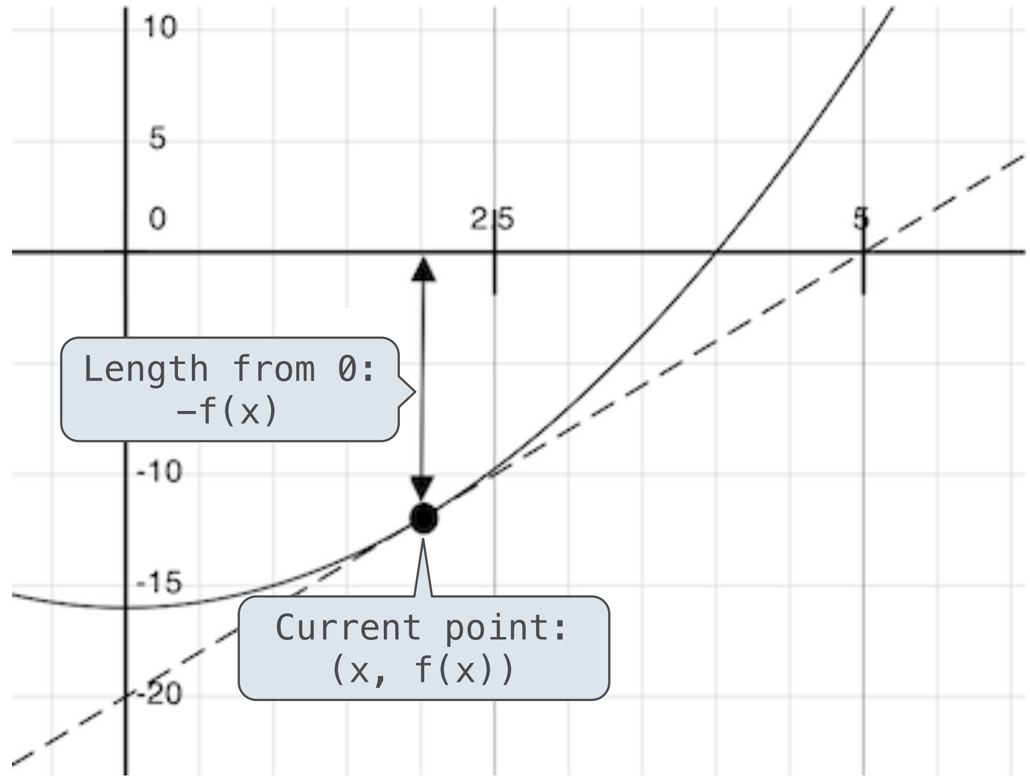

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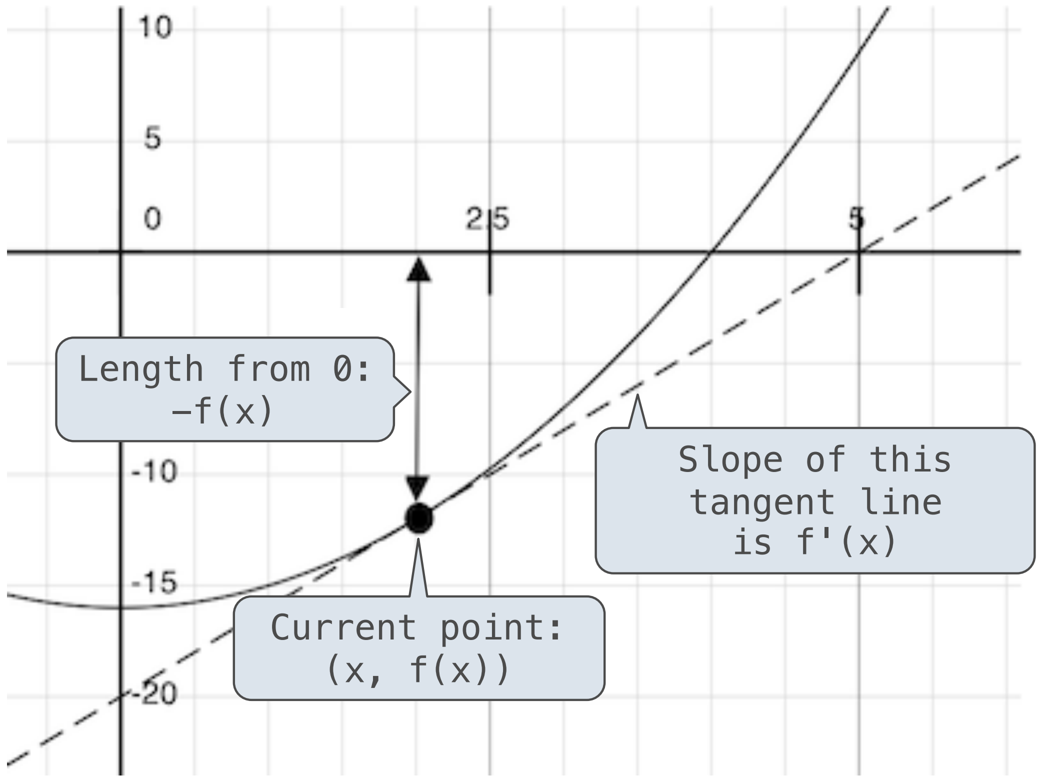

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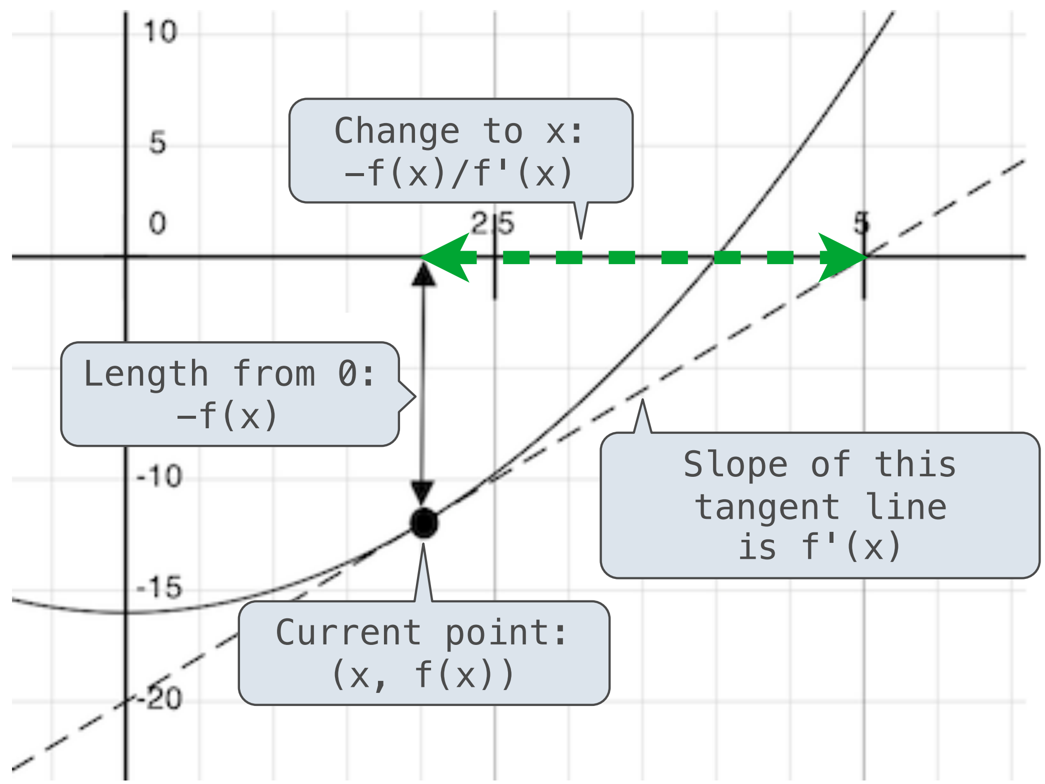

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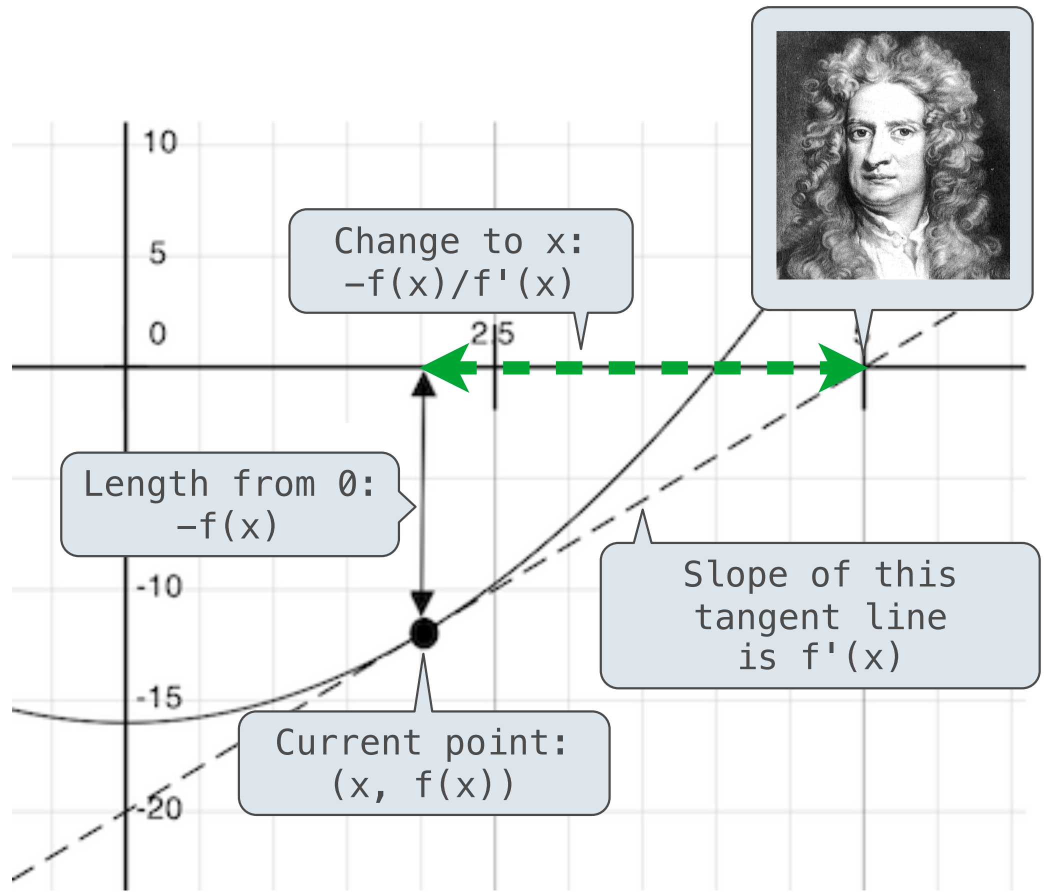

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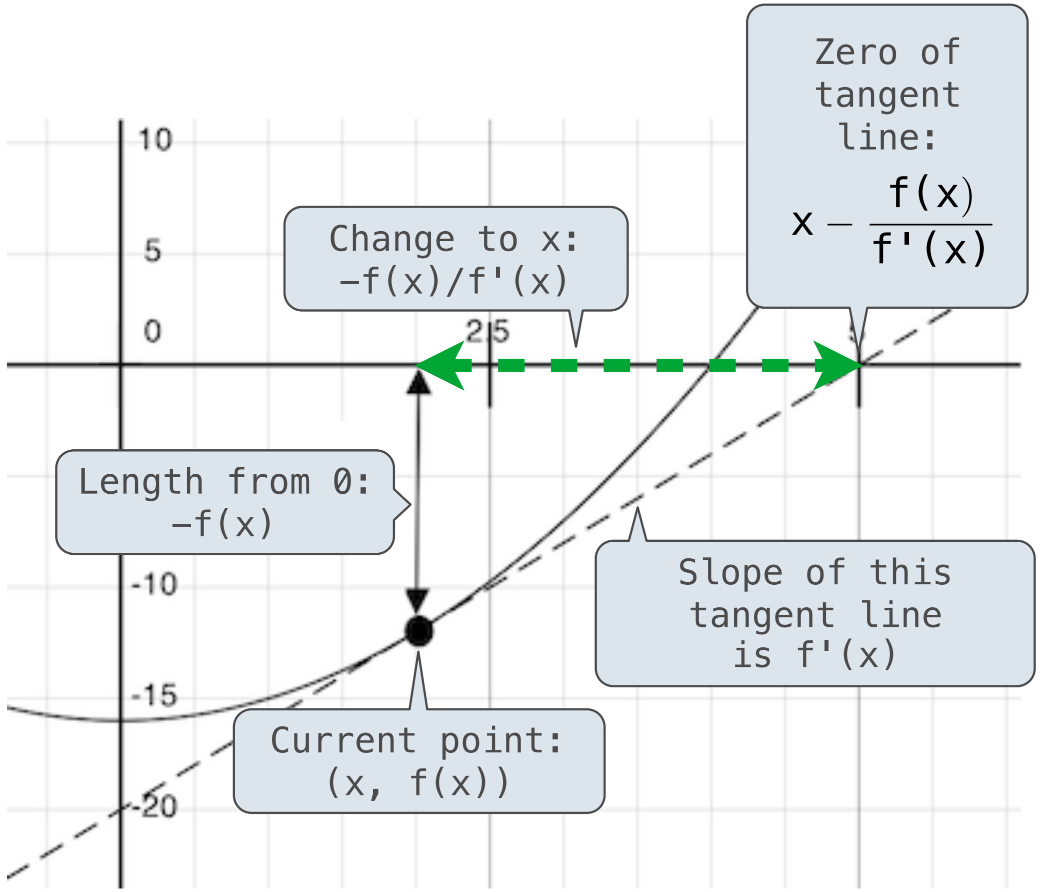

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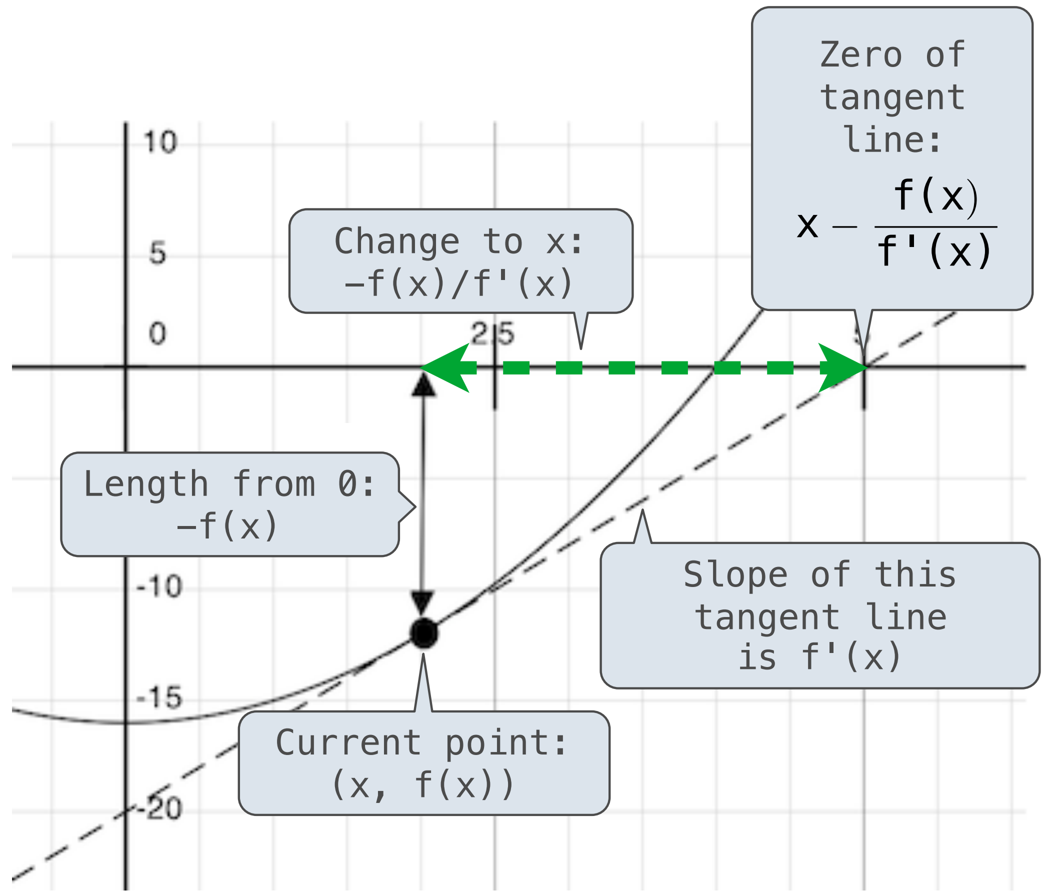

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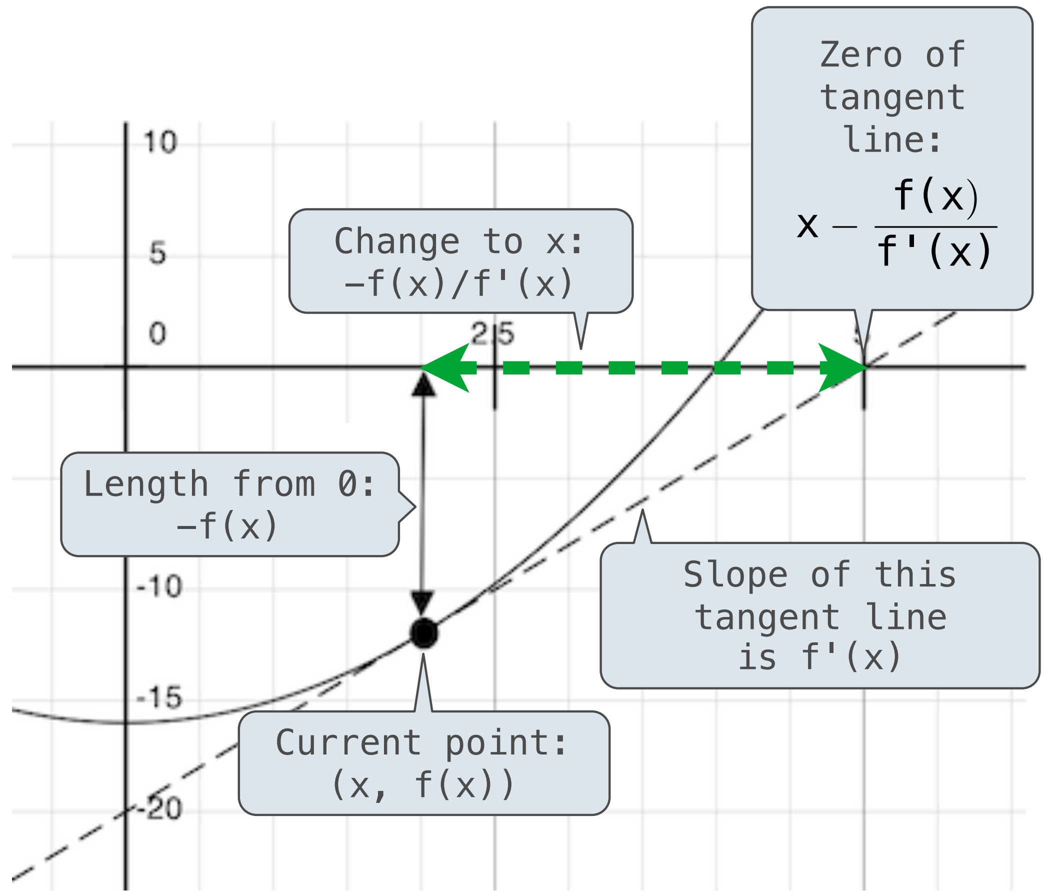

http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

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How to find the square root of 2?

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>>> f  = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
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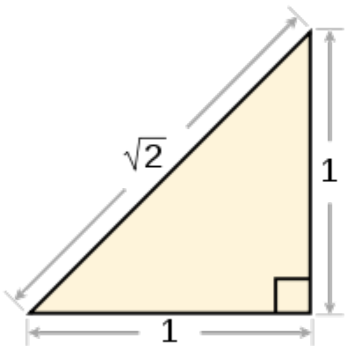


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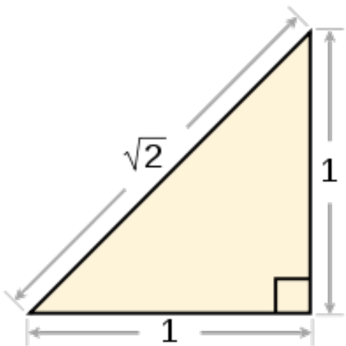


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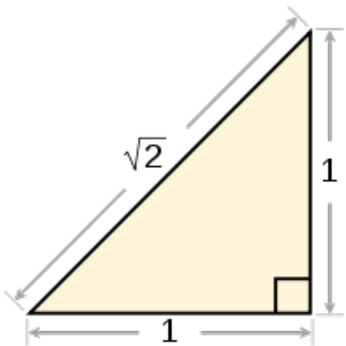


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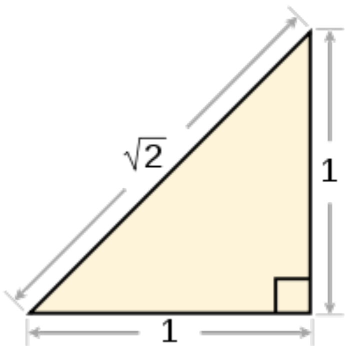


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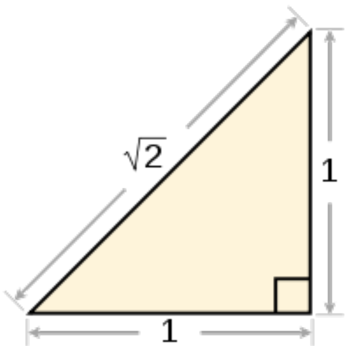


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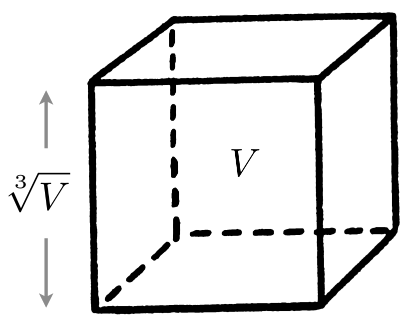

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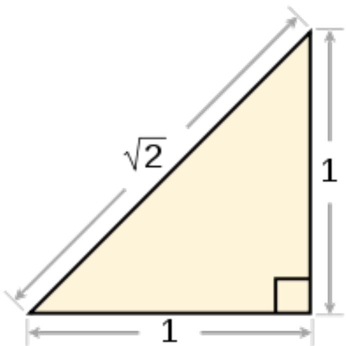


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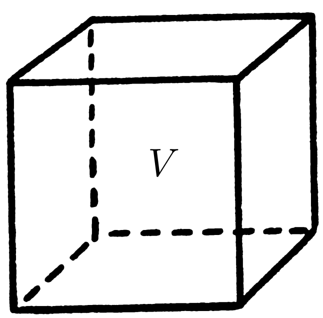

$\sqrt[3]{V}$

```
>>> g  = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

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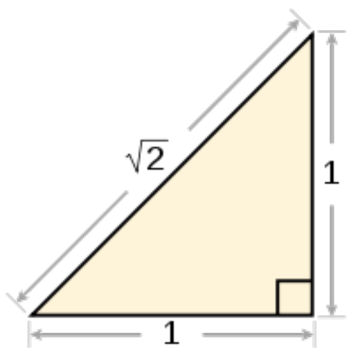


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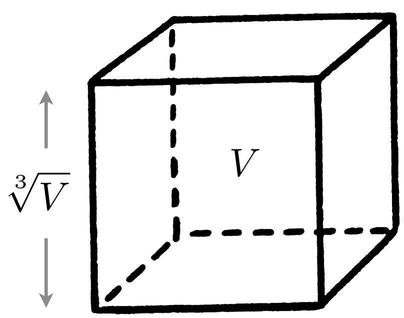


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Iterative Improvement

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Babylonian Method

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(Demo)

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Implementing Newton's Method

(Demo)

Extensions

Approximate Differentiation

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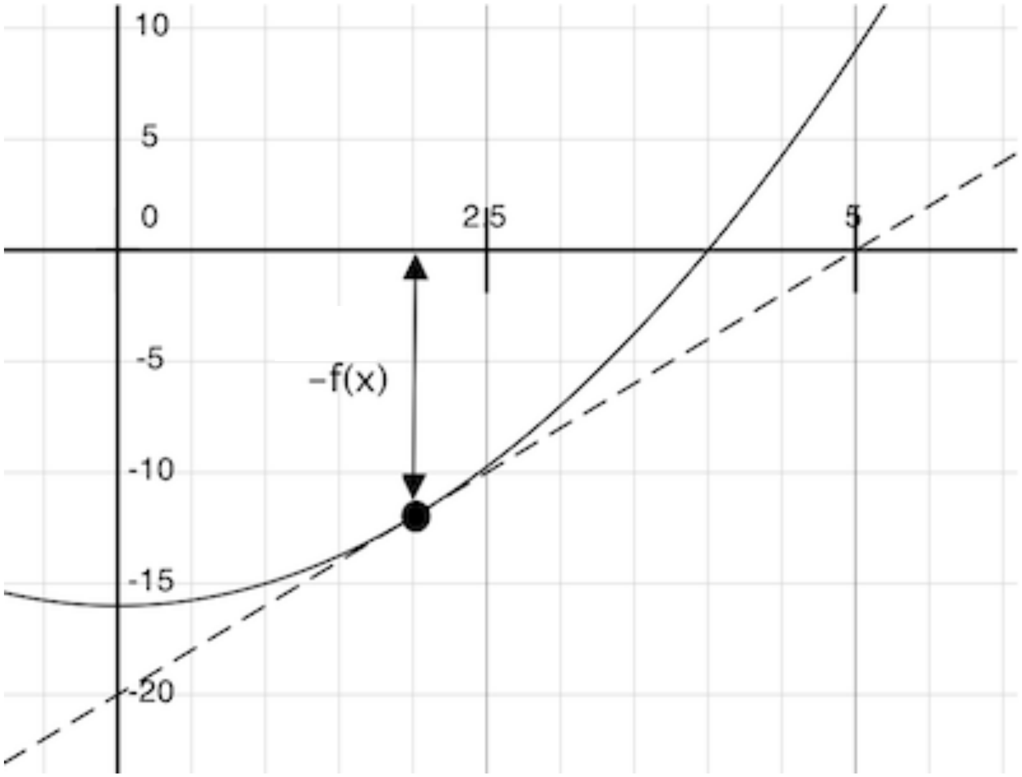

Approximate Differentiation

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Differentiation can be performed
symbolically or numerically
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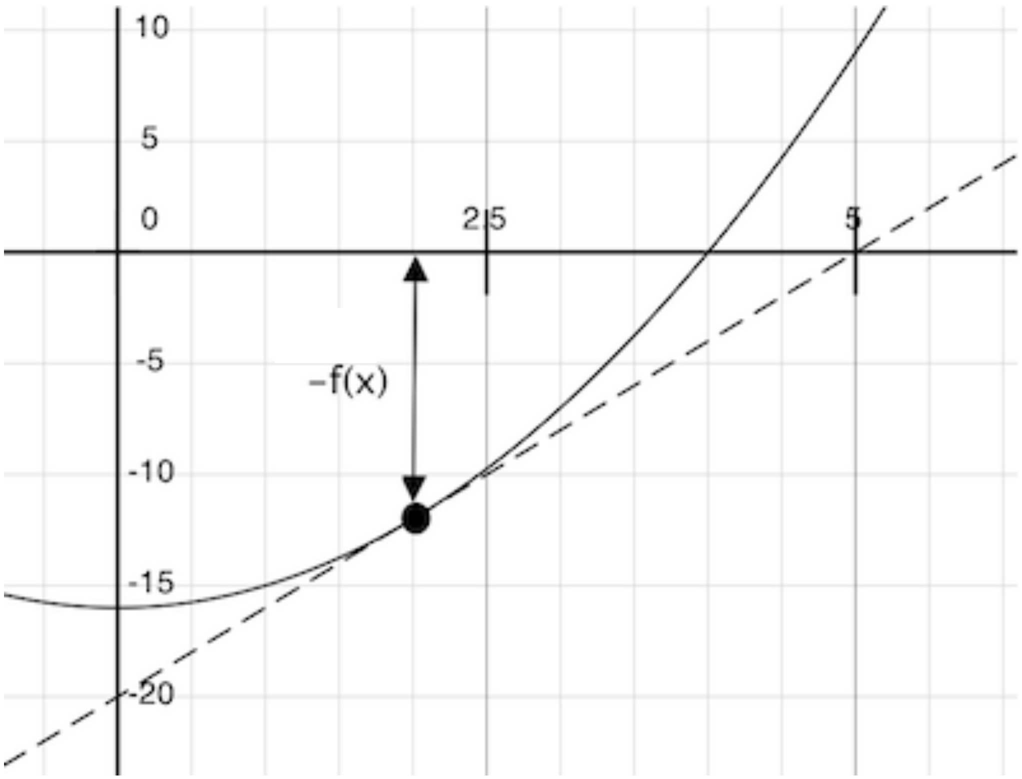

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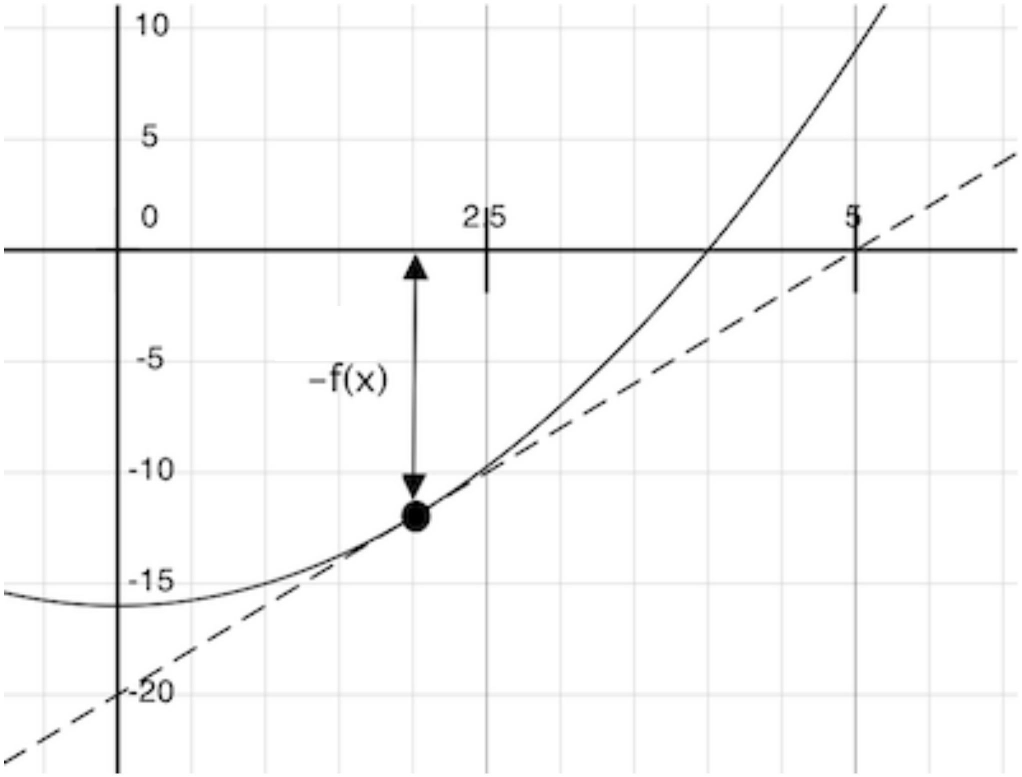
$f(x) = x^2 - 16$



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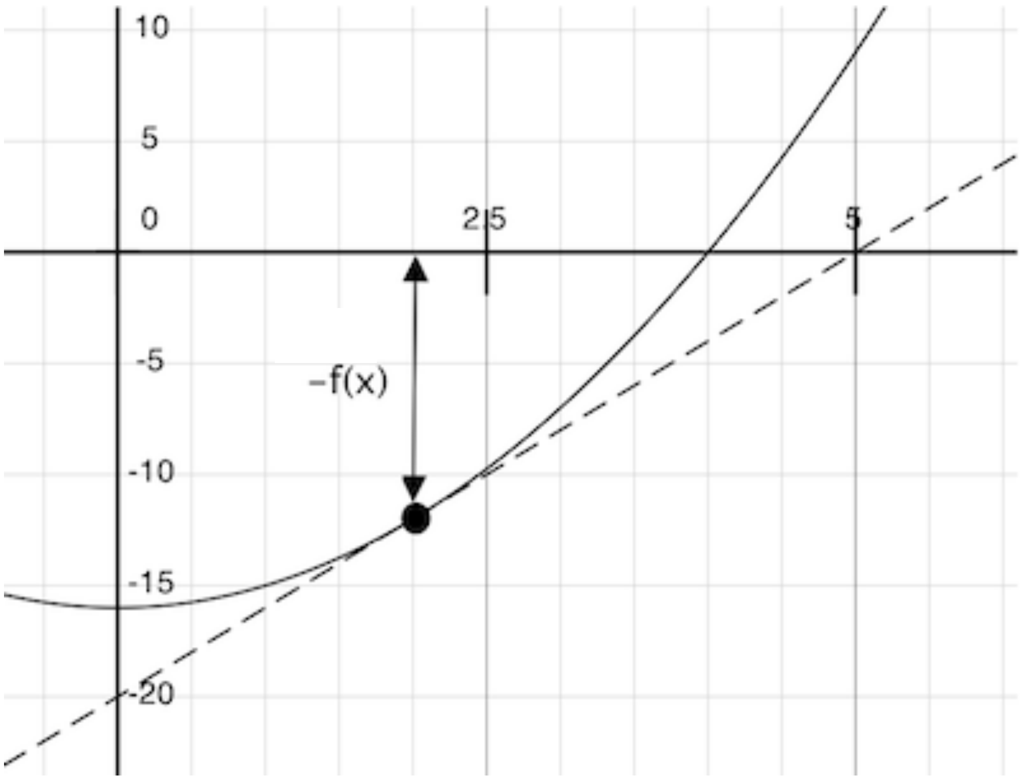

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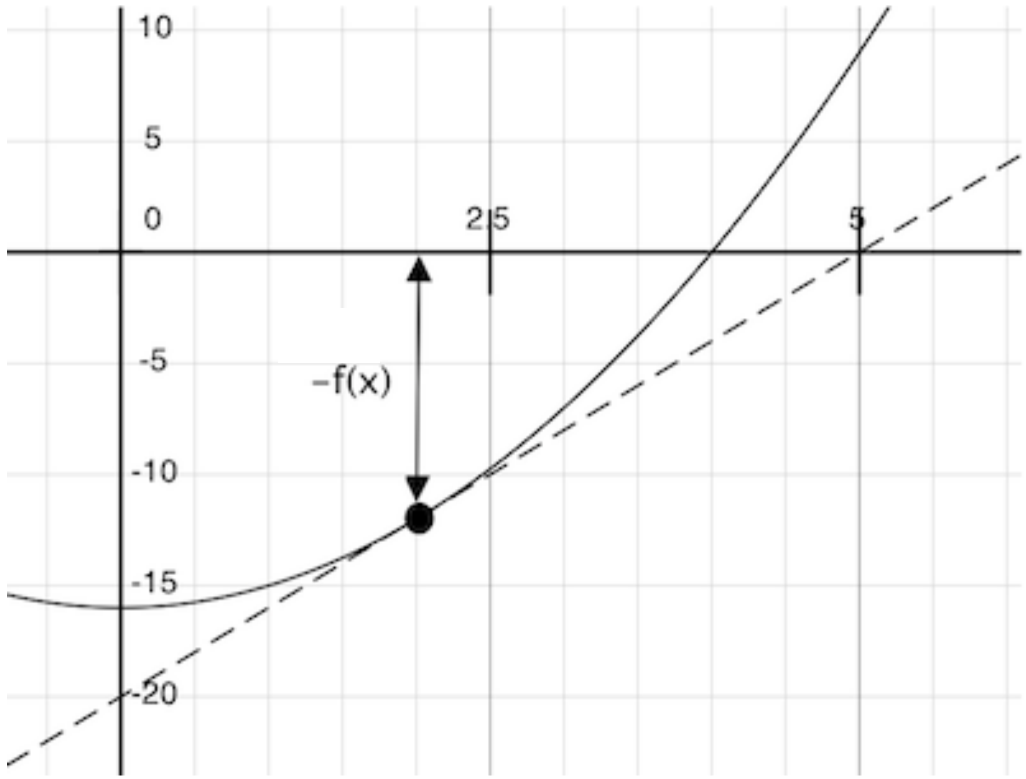

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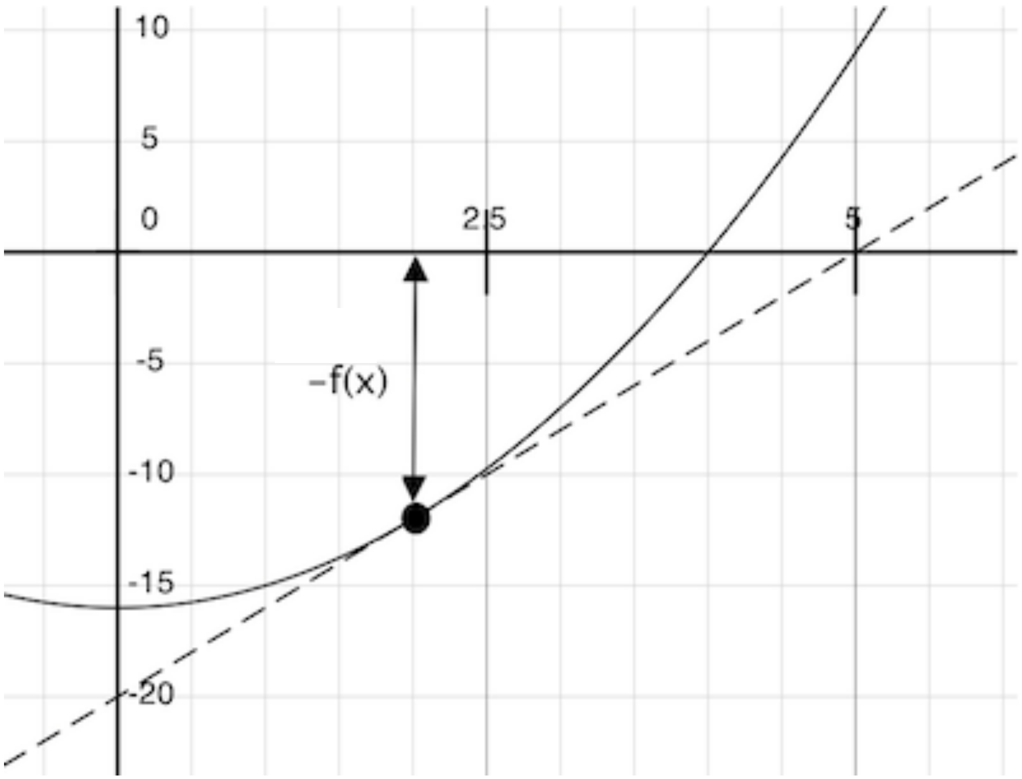

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f(x) = $\boxed{x^2}$ − 16

f'(x) = $\boxed{2x}$

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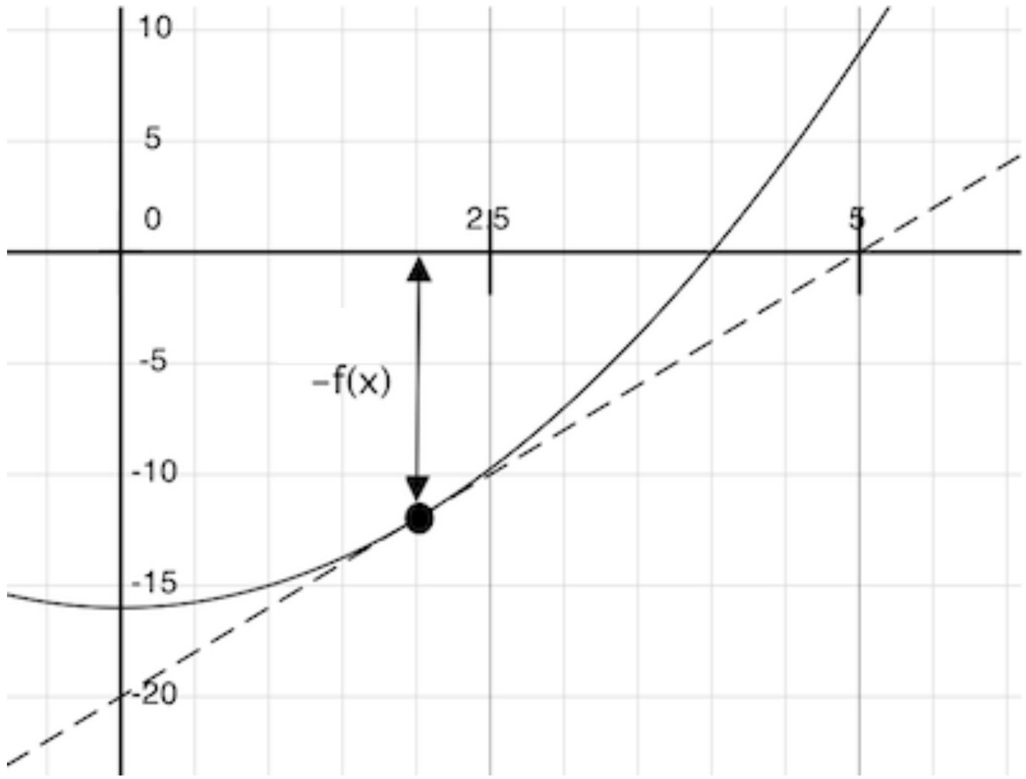
$$f'(x) = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a}$$



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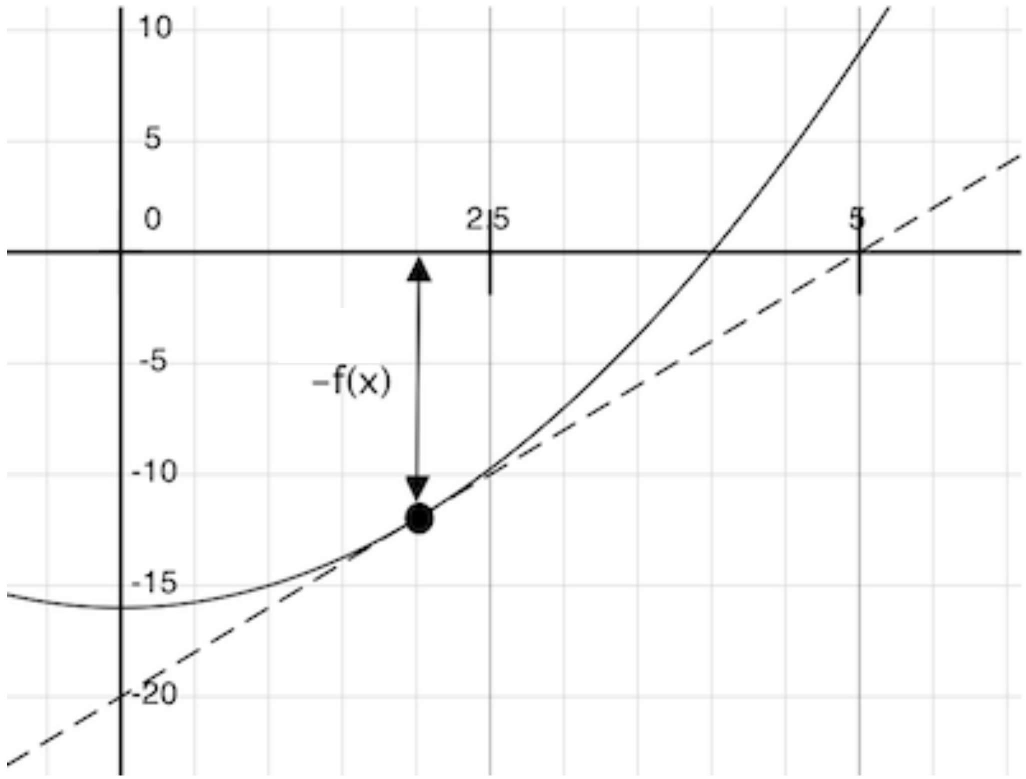
$f'(2) = 4$

$$f'(x) = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a}$$

$$f'(x) \approx \frac{f(x+a) - f(x)}{a}$$



Approximate Differentiation

Differentiation can be performed symbolically or numerically

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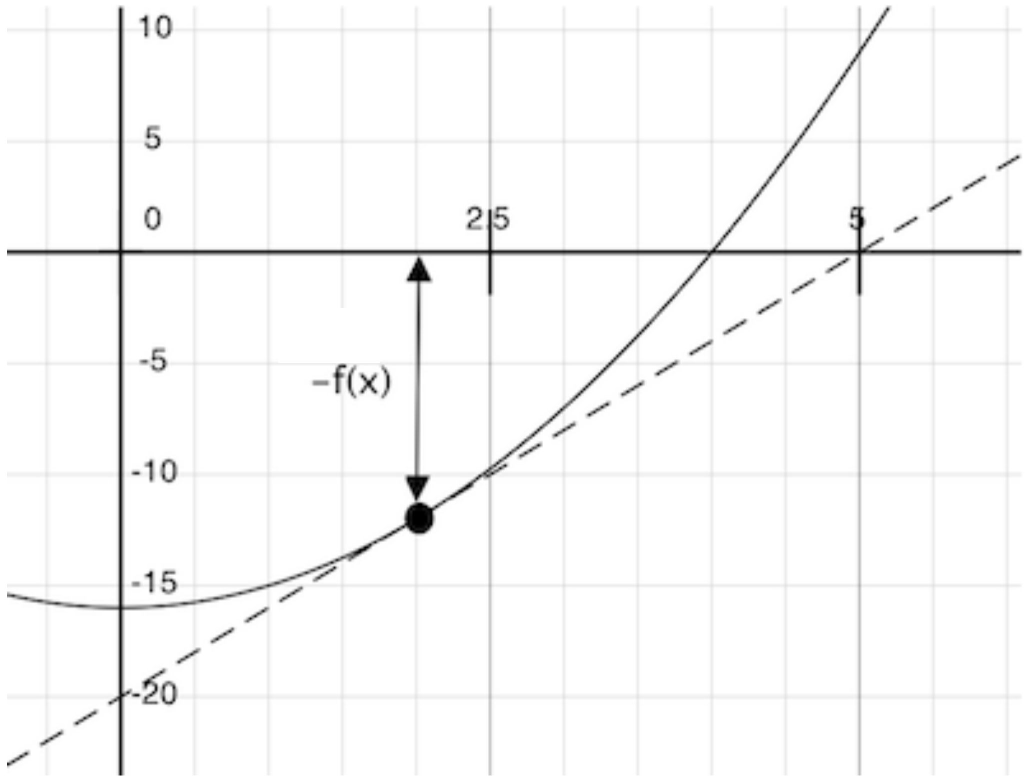

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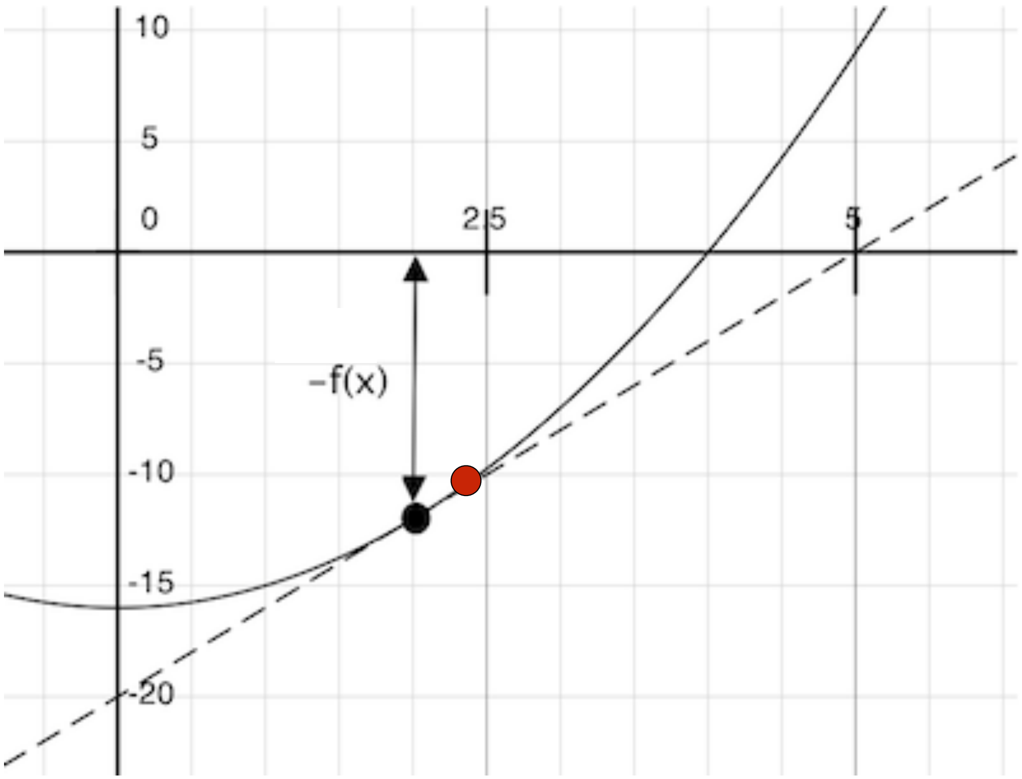

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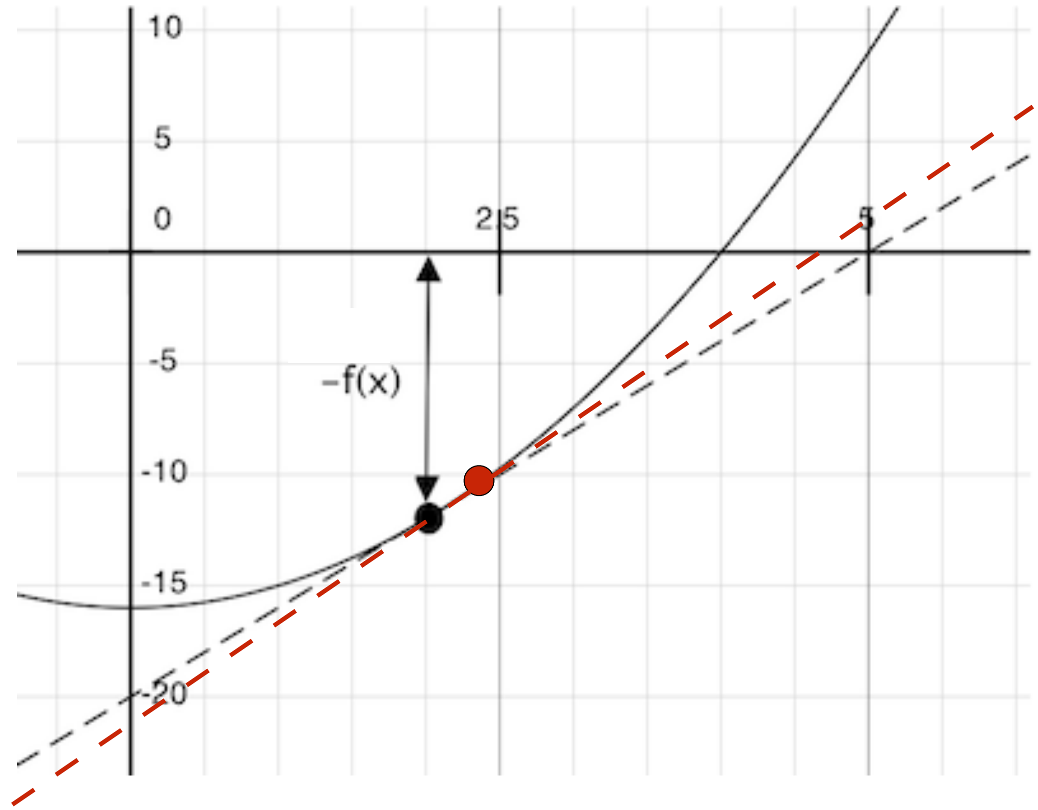

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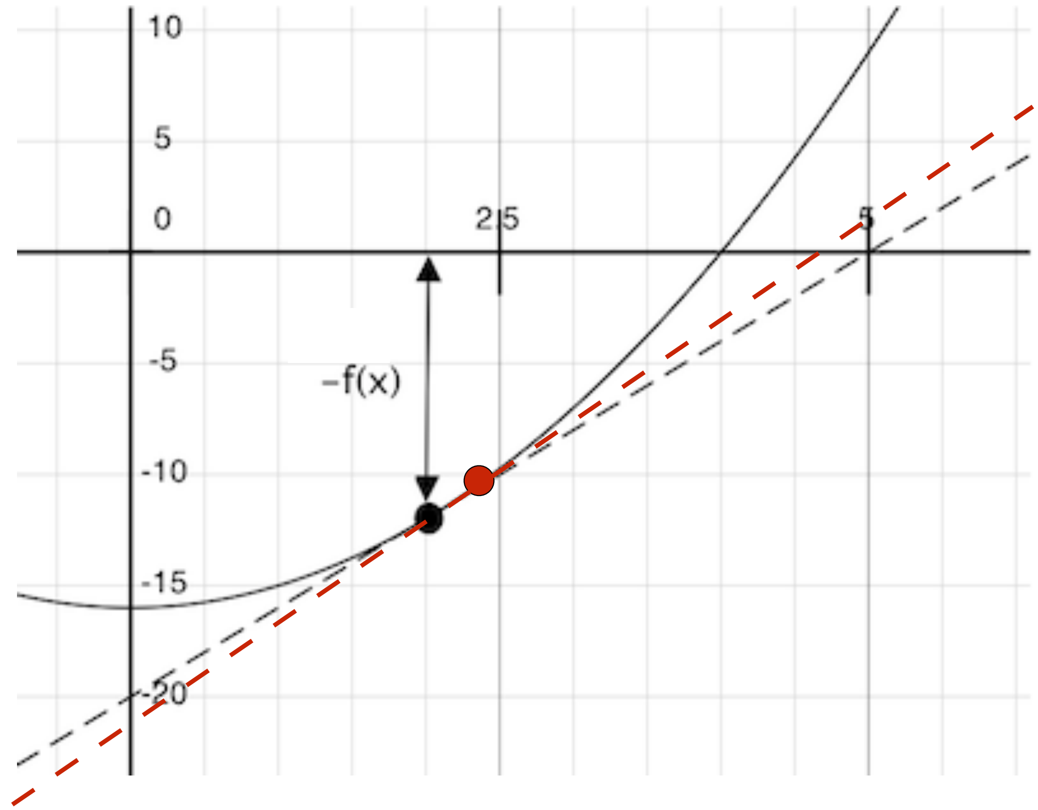
(Demo)



Inverse Function

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The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value x such that $f(x) = y$

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