Machine Learning

Announcements

Prediction

Regression

Regression

Given a set of (x, y) pairs, find a function f(x) that returns good y values

Regression

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Data from home sales records in Ames, Iowa

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(Demo)

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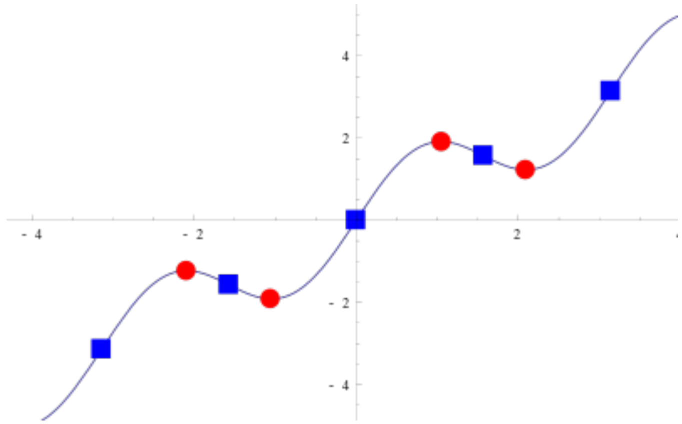

http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg

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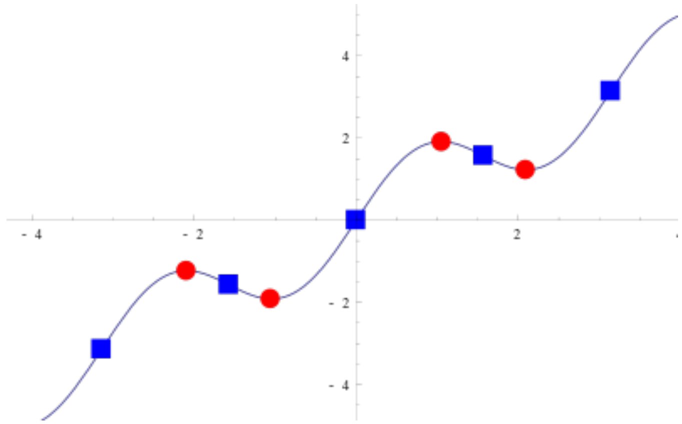

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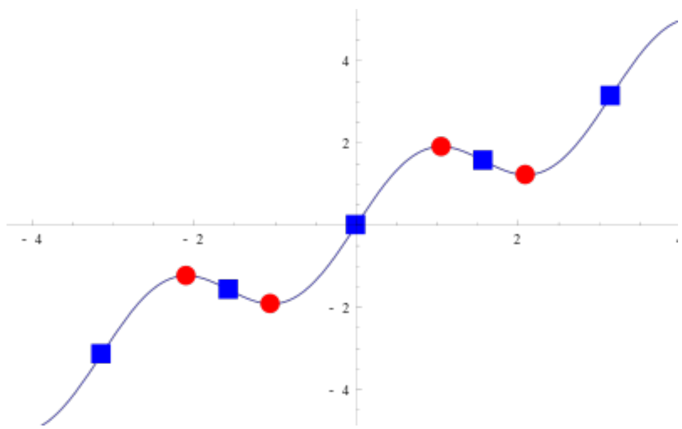

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Note: Root mean squared error can be optimized through linear algebra alone, but numerical optimization works for a much larger class of related error measures