🔍 What problem does this paper address?

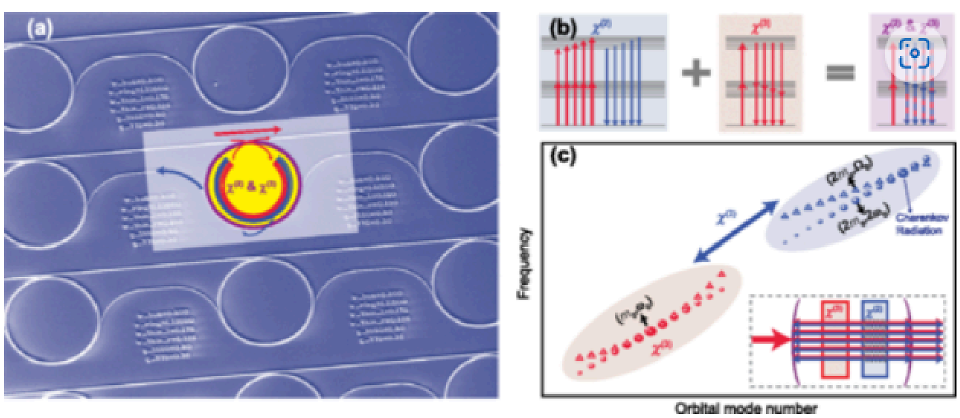
-Generate Combs in short wavelength range limited by large material dispersion and loss
-Use Pockels effect couple infrade and near visible
-22% conversion efficiency from pulse pumped laser to near visible comb but not phase locked
strong chi2 coupling
-tune the visible comb by more than one FSR robustly. by dispersive wave
可以强调“近可见波段是传统Kerr梳难以覆盖的区域”，本工作**拓展了可集成光梳的波段范围。**

🧪 What are the key methods or experimental techniques?

- Experimental setup / materials used
- Mode dispersion parameters:
  - IR: (d_1 = 727,\text{GHz},\ d_2 = 140,\text{MHz})
  - VIS: (D_1 = 703.7,\text{GHz},\ D_2 = 43,\text{MHz})
  - χ(2) coupling strength: (g^{(2)} = 0.08,\text{MHz})

series of rings. shifting the radius of each ring, the resonance shifted by 9nm each.

- AlN resonator eight rings are cascaded using one set of bus waveguides



-
- Theoretical model (if any)
- Couple mode equation in the supplimentary

$j=-N_1 \qquad\qquad j=-N_2$

we have the total system Hamiltonian (time-independent) as

$$\mathscr{H}_{tot}=\sum_{j=-N_1}^{N_1}\hbar\left(d_2j^2-p^2d_2-\delta\right)a_j^{\dagger}a_j+\sum_{j=-N_2}^{N_2}\hbar\left[\Omega_0+\left(D_1-d_1\right)j+D_2j^2-2\left(\omega_0+p^2d_2+\delta\right)\right]b_j^{\dagger}b_j$$
$$+\hbar\sqrt{\frac{2\kappa_{p,1}P_{in}}{\hbar\left(\omega_p+\delta\right)}}\left(a_p+a_p^{\dagger}\right)$$
$$+\mathscr{H}_{\chi^{(2)}}+\mathscr{H}_{\chi^{(3)}}, \tag{S.11}$$

where

$$\mathscr{H}_{\chi^{(2)}}=\sum_{j,k,l}\hbar g_{jkl}^{(2)}\left(a_ja_kb_l^{\dagger}+a_j^{\dagger}a_k^{\dagger}b_l\right) \tag{S.12}$$

is the three-wave mixing interaction due to second-order nonlinear optical effect $\left(\chi^{(2)}\right)$, and

$$\mathscr{H}_{\chi^{(3)}}=\sum_{j,k,l,n}\hbar g_{jkln}^{(3)aa}a_j^{\dagger}a_k^{\dagger}a_la_n+\sum_{j,k,l,n}\hbar g_{jkln}^{(3)bb}b_j^{\dagger}b_k^{\dagger}b_lb_n+\sum_{j,k,l,n}\hbar g_{jkln}^{(3)ab}a_j^{\dagger}a_kb_l^{\dagger}b_n \tag{S.13}$$

is the four-wave mixing interactions due to Kerr and cross-Kerr effects $\left(\chi^{(3)}\right)$. $g_{jkl}^{(2)}$, $g_{jkln}^{(3)aa}$, $g_{jkln}^{(3)bb}$ and $g_{jkln}^{(3)ab}$ are the coupling strengths.

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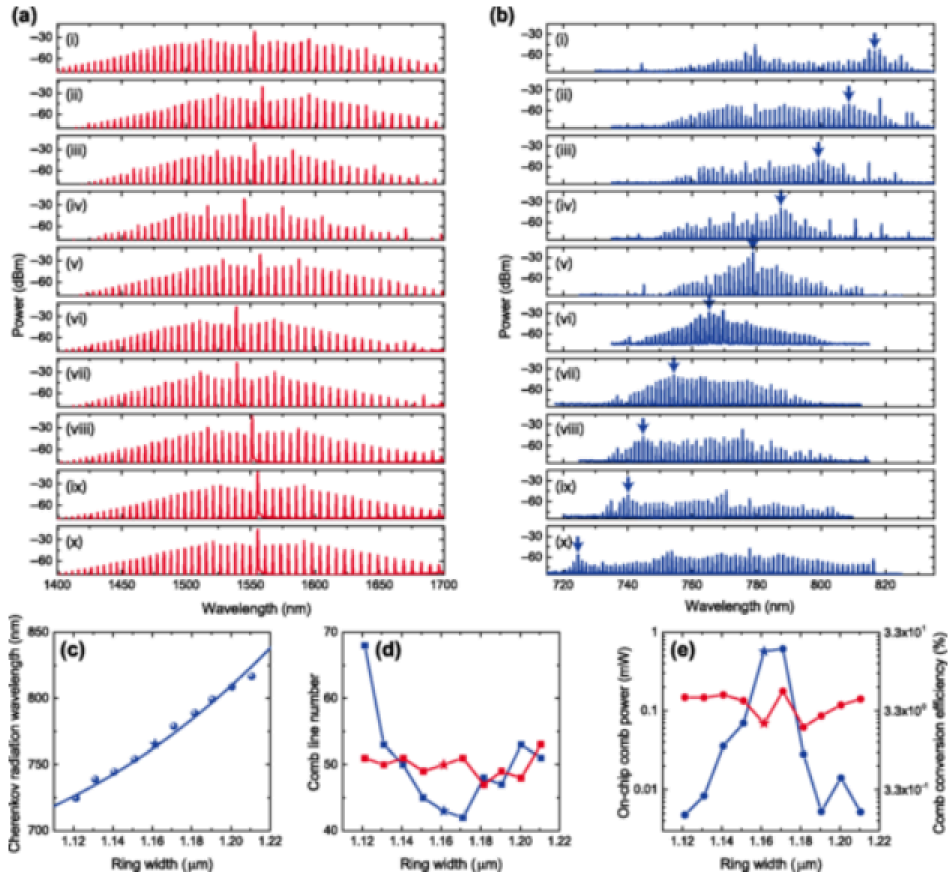
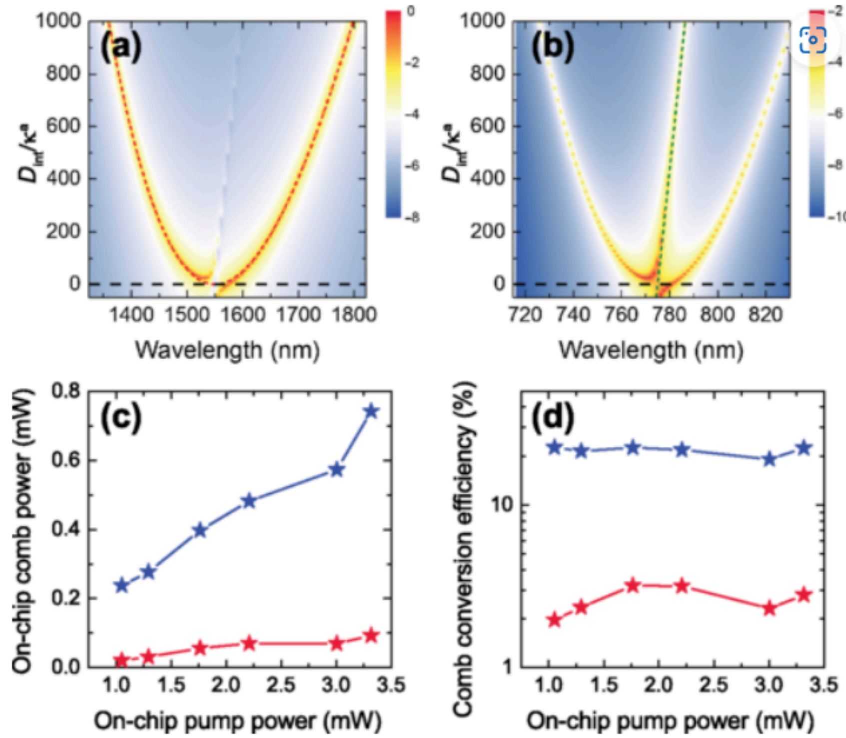
📈 What are the main results?

- What does the paper demonstrate?
  dispersive wave caused by visble and infrade phase matching. And the position could be shifed by wg width when dispersive wave near pump efficiency increase
- Cherenkov radiation ≠ SHG dispersive wave，它是由 **χ(2)耦合构成的 hybrid mode 对 comb 产生共振增强**，类似高阶色散引发的 DW，但这里机制不同。
- 所以可以写成:
- The Cherenkov-like radiation is a dispersive wave caused by phase matching between the IR comb line and a hybridized VIS-IR mode, **not** traditional SHG.
- Any key figures (Fig. X) to note
  -



(a)
(b)
(i)
(ii)
(iii)
(iv)
(v)
(vi)
(vii)
(viii)
(ix)
(x)
Power (dBm)
Wavelength (nm)
(c)
Cherenkov radiation wavelength (nm)
Ring width (μm)
(d)
Comb line number
Ring width (μm)
(e)
On-chip comb power (mW)
Comb conversion efficiency (%)
Ring width (μm)



Dual-band comb generation efficiency under different pump powers. (a) The density of states for the infrared modes (with a pump in the a_0 infrared mode) when the Cherenkov-like radiation is close to the second-harmonic wavelength of the pump. (b) The corresponding density of states for the near-visible mode. (c) Infrared (red) and near-visible (blue) comb powers under different pump powers. (d) On-chip conversion efficiency of the infrared (red) and near-visible (blue) combs. Here both the pump and the comb powers refer to the on-chip average powers. The on-chip peak power is around 1000 times higher than the average power considering a pulse duty cycle of 1/1000.

🌟 What is novel or interesting about this work?

- Technical innovations
  Theory frame work about explain DW with DOS
  identify the infrade and near visble hybridization by the chi2 effects from resonance thermal shifts
- “Theory framework explain DW with DOS” 是一个特别好的总结点，别的总结一般只说“observed DW”，你能从理论角度说出来很专业；
- 你提到 thermal shift 识别 hybridization，也说明你注意到他们在频率 domain 做 tuning 的巧妙方法。

💡**建议细化比较**:

- 再补一句“以往做 visible comb 要外部倍频、或者弱 intracavity SHG，而本文靠 χ(2)-χ(3) 强耦合实现”会更完整。
- Compared to prior work, what’s new?
  different tuning method by tuning the width of the wg change phase matching not by tune the pump

📌 Connections to Gong Zheng's PhD thesis

- Which chapter does this relate to?
  not his work
- Is this part of a larger research trajectory?
  build the theory back ground towards pockles comb on AlN

💡 My thoughts & extensions

- Can I use this technique/idea?
  -Learn the theory frame work and combine thermal and photorefractive components inside. compare with the LLE method
- How could I adapt this concept to my experiment or simulation?
- Yes may be we can learn the multiplixing method?
-
- What are my questions after reading?
- The chrenkov radiation is the DW of the SHG?
- Could implement a simplified modal-expansion + thermal detuning model to test whether DW can appear in my SHG simulation.
- Try extracting the DOS structure from QuTiP simulated hybrid-mode Hamiltonian?

🌈 Cherenkov-like Radiation 与 Hybrid Mode 的关系整理

🧠 核心结论：

> **Cherenkov-like radiation 出现在 near-visible 波段，是因为某个 IR comb line 与 hybrid mode 的频率共振对齐，导致 DOS 增强，从而产生频谱尖峰。**

🔗 逻辑链梳理：

1️⃣ 什么是 Hybrid Mode?

- 在 AlN 微环中，由于强 χ(2) 非线性（Pockels 效应），红外模 (a_j) 和可见模 (b_j) 被强耦合起来。
- 二者形成两个 **混合模式（hybrid modes）**，分别记作 (A_j, B_j)，是以下线性组合：
  $A_j = \alpha_j a_j + \beta_j b_j, \quad B_j = \alpha'_j a_j + \beta'_j b_j$
- 每个 hybrid mode 有自己新的本征频率，记为 (\lambda_j^\pm)，其解析表达式为：

$$\lambda_j^\pm = \frac{1}{2}(\chi_j^a + \chi_j^b \pm \sqrt{(\chi_j^a - \chi_j^b)^2 + 4G_j^2})$$

其中 $\chi^{a,b}_j$ 为 detuning + loss，G_j 是 $\chi^{(2)}$ 耦合强度。

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2 什么是 DOS（Density of States）？

- 描述某一频率是否与某个模式频率共振增强的程度；
- 近似可用 Lorentzian 表示：

$$
\text{DOS}_j(\omega) \propto \frac{1}{(\omega - \omega_j)^2 + \kappa_j^2}
$$

- 在本文中，**DOS 是基于上述 hybrid mode 的本征频率 $\lambda^\pm_j$** 计算出来的。

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3 为什么 near-visible 会出现 Cherenkov-like radiation?

- IR comb 是 Kerr 效应在 IR 波段产生的，其频率分布为：

$$
f_n = f_0 + n \cdot \text{FSR}
$$

- 当某一个 comb line 的频率刚好等于一个 hybrid mode 的频率（尤其是以可见光为主成分的 hybrid mode）：

$$
f_n^{\text{comb}} \approx \lambda_j^\pm
$$

→ **DOS 增强** → 光谱中该点能量急剧上升 → 出现“Cherenkov-like radiation”。

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4 图示说明（参考 Fig. 2b）：

- 图中绿虚线表示 $D_{\text{int}} = 0$ 的位置；
- 该点即为 comb line 与 hybrid mode 共振的频率；
- 可视为 **非线性光学版本的 Cherenkov radiation（色散波）**。

✅ 小结：

概念	含义
Hybrid mode	红外 + 可见光模式通过 $\chi(2)$ 强耦合混合出的新模式
DOS	基于 hybrid mode 频率计算，衡量 comb line 是否落在共振点上
Cherenkov-like radiation	来自 comb line 与 hybrid mode 的频率对齐共振增强
可调性	改变波导宽度可调节 hybrid mode 的频率，从而调控 radiation 位置
