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Section: Online

CS-402(Intro to Advance Studies-II)

Home Work -2

Question:1.5. Consider three different processors P1, P2, and P3 executing the same instruction set. P1 has a 3 GHz clock rate and a CPI of 1.5. P2 has a 2.5 GHz clock rate and a CPI of 1.0. P3 has a 4.0 GHz clock rate and has a CPI of 2.2.

- a.** Which processor has the highest performance expressed in instructions per second?
- b.** If the processors each execute a program in 10 seconds, find the number of cycles and the number of instructions.
- c.** We are trying to reduce the execution time by 30% but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

Solution:

* Homework-2

Question: 1.5:-

a) Given,

	P ₁	P ₂	P ₃
clock rate	3GHz	2.5GHz	4.0GHz
CPI	1.5	1.0	2.2

Processor (P) = performance (clock rate / CPI)

$$P_1 = \frac{3 \times 10^9}{1.5} = 2 \times 10^9 \text{ instruction per sec.}$$

$$P_2 = \frac{2.5 \times 10^9}{1.0} = 2.5 \times 10^9 \text{ instruction per sec.}$$

$$P_3 = \frac{4 \times 10^9}{2.2} = 1.82 \times 10^9 \text{ instruction per sec.}$$

Therefore; $P_2 = 2.5 \times 10^9$ has highest instructions per sec.

b) Cycles:-

$$\text{For } P_1: 3\text{GHz} \times 10 = 3 \times 10^{10} \text{ cycles.}$$

$$\text{For } P_2: 2.5\text{GHz} \times 10 = 2.5 \times 10^{10} \text{ cycles.}$$

$$\text{For } P_3: 4\text{GHz} \times 10 = 4 \times 10^{10} \text{ cycles.}$$

* Number of Instructions:-

$$\text{For } P_1: \frac{3\text{GHz} \times 10}{1.5} = 2 \times 10^{10} \text{ instructions.}$$

$$\text{For } P_2: \frac{2.5\text{GHz} \times 10}{1.0} = 2.5 \times 10^{10} \text{ instructions}$$

$$\text{for } P_3: \frac{4.0\text{GHz} \times 10}{2.2} = 1.82 \times 10^{10} \text{ instructions}$$

$$c) \text{ Execution Time} = \frac{\text{Number of instructions} \times \text{CPI}}{\text{clock rate.}}$$

$$* \text{ Execution Time} \times 0.7 = \frac{\text{No of instructions} \times \text{CPI} \times 1.2}{\text{New clock rate.}}$$

$$\rightarrow \text{New clock rate} = \frac{\text{clock rate} \times 1.2}{0.7.}$$

$$= 1.71 \times \text{clock rate.}$$

The new clock rate for each processor will be:

$$\begin{array}{l|l|l} P_1 = 3 \text{ GHz} \times 1.71 & P_2 = 2.5 \text{ GHz} \times 1.71 & P_3 = 4.0 \text{ GHz} \times 1.71 \\ = 5.13 \text{ GHz} & = 4.27 \text{ GHz} & = 6.84 \text{ GHz.} \end{array}$$

Hence, The new clock rate will be: $1.71 \times \text{clock rate.}$

Therefore, The clock rate must increase by 71%.

Question:1.6. Consider two different implementations of the same instruction set architecture. The instructions can be divided into four classes according to their CPI (class A, B, C, and D). P1 with a clock rate of 2.5 GHz and CPIs of 1, 2, 3, and 3, and P2 with a clock rate of 3 GHz and CPIs of 2, 2, 2, and 2.

Given a program with a dynamic instruction count of $1.0E6$ instructions divided into classes as follows: 10% class A, 20% class B, 50% class C, and 20% class D, which implementation is faster?

- a. What is the global CPI for each implementation?
- b. Find the clock cycles required in both cases.

Solution:

Question 1.6

Given,

	P_1	P_2
clockrate:	2.5GHz	3GHz
CPI's :	1, 2, 3, 3	2, 2, 2, 2

→ Dynamic instruction count = 1.0×10^6

Class A: 10%

Class B: 20%

Class C: 50%

Class D: 20%

$$* P_1 \text{ total time} = \frac{(1 \times 10^5 + (2 \times 10^5 \times 2) + (5 \times 10^5 \times 3) + (2 \times 10^5 \times 3))}{2.5 \times 10^9}$$

$$= 10.4 \times 10^{-4} \text{ s}$$

$$= 1.04 \text{ ms}$$

$$* P_2 \text{ total time} = \frac{((2 \times 10^5) + (2 \times 10^5 \times 2) + (5 \times 10^5 \times 2) + (2 \times 10^5 \times 2))}{3 \times 10^9}$$

$$= 6.66 \times 10^{-4} \text{ s}$$

$$= 666.67 \text{ ns}$$

Therefore, the processor P_2 implementation is faster.

a) Global CPI for each implementation is:

$$\text{CPI of } P_1 = \frac{10.4 \times 10^{-4} \times 2.5 \times 10^9}{10^6} = \boxed{2.6}$$

$$* \text{ CPI of } P_2 = \frac{6.66 \times 10^{-4} \times 3 \times 10^9}{10^6} = \boxed{2.0}$$

b) clock cycles?

$$\text{for } P_1 = 10^6 \times ((1 \times 10\%) + (2 \times 20\%) + (3 \times 50\%) + (3 \times 20\%)) \\ = 2.6 \times 10^6 \text{ clock cycles.}$$

$$\text{for } P_2 = 10^6 \times ((2 \times 10\%) + (2 \times 20\%) + (2 \times 50\%) + (2 \times 20\%)) \\ = \boxed{2.0 \times 10^6 \text{ clock cycles.}}$$

Question: 1.7. Compilers can have a profound impact on the performance of an application.

Assume that for a program, compiler A results in a dynamic instruction count of $1.0E9$ and has an execution time of 1.1 s, while compiler B results in a dynamic instruction count of $1.2E9$ and an execution time of 1.5 s.

- a. Find the average CPI for each program given that the processor has a clock cycle time of 1 ns.
- b. Assume the compiled programs run on two different processors. If the execution times on the two processors are the same, how much faster is the clock of the processor running compiler A's code versus the clock of the processor running compiler B's code?
- c. A new compiler is developed that uses only $6.0E8$ instructions and has an average CPI of 1.1 . What is the speedup of using this new compiler versus using compiler A or B on the original processor?

Solution:

3

Question 1.7 :-

Given,

	Compiler A	Compiler B.
Dynamic instruction count	$1.0E9$	$1.2E9$
Execution time	1.1 s	1.5 s

a) Average CPI for each program with given processor has a clock cycle time of 1 ns .

Formulae:

$$\text{CPI} = (\text{CPU clock cycle} / \text{Instruction time})$$

for

$$\text{for A: } \text{CPI}_A = (1.1\text{ s} / (1 \times 10^{-9})) (1 / (1 \times 10^9))$$

$$= \underline{\underline{1.1}}$$

for

$$\text{B: } \text{CPI}_B = (1.5\text{ s} / (1 \times 10^{-9})) (1 / (1.2 \times 10^9))$$

$$= \underline{\underline{1.25}}$$

b) Execution time = CPU time = (Instructions \times CPI) / clock rate.

execution time 1 = execution time 2.

$$\therefore \text{clock rate}_1 = [(Inst_1 \times CPI_1) / (Inst_2 \times CPI_2)] \text{ clock rate}_2$$

$$= [(10^9 \times 1.1) / (1.2 \times 10^9 \times 1.25)] \times \text{clock rate}_2$$

$$\text{Clock rate}_1 = 0.73 \text{ clock rate}_2$$

So, the clock rate of processor 1 is approximately 27% slower than clock rate of processor 2.

17. c) for the original processor write a clock cycle time of Ins.

$$\begin{aligned} (\text{CPU time})_A \mid (\text{CPU time})_{\text{new}} &= \frac{(\text{instruction count} \times (\text{CPI})_A)}{(\text{instruction count} \times (\text{CPI})_{\text{new}})} \\ &= (1.0E9 \times 1.1)_A \mid (6.0E8 \times 1.1)_{\text{new}} \\ &= \boxed{1.67} \end{aligned}$$

$$\begin{aligned} * (\text{CPU time})_B \mid (\text{CPU time})_{\text{new}} &= \frac{\text{instruction count} \times (\text{CPI})_B}{(\text{instruction count} \times (\text{CPI})_{\text{new}})} \\ &= (1.2E9 \times 1.25)_B \mid (6.0E8 \times 1.1)_{\text{new}} \\ &= \boxed{2.27} \end{aligned}$$

Question: 1.9. Assume for arithmetic, load/store, and branch instructions, a processor has CPIs of 1, 12, and 5, respectively. Also assume that on a single processor a program requires the execution of 2.56×10^9 arithmetic instructions, 1.28×10^9 load/store instructions, and 256 million branch instructions. Assume that each processor has a 2 GHz clock frequency.

Assume that, as the program is parallelized to run over multiple cores, the number of arithmetic and load/store instructions per processor is divided by $0.7 \times p$ (where p is the number of processors) but the number of branch instructions per processor remains the same.

1.9.1: Find the total execution time for this program on 1, 2, 4, and 8 processors, and show the relative speedup of the 2, 4, and 8 processor result relative to the single processor result.

1.9.2 : If the CPI of the arithmetic instructions was doubled, what would the impact be on the execution time of the program on 1, 2, 4, or 8 processors?

1.9.3: To what should the CPI of load/store instructions be reduced in order for a single processor to match the performance of four processors using the original CPI values?

Solution:

* Question 1.9:-

Given,

- Processor = 1, 2, 4, 8
- CPI of Arithmetic instruction = 1 \rightarrow if doubled = 2.
- CPI of L/S instructions = 12.
- CPI of branch instructions = 5.
- No. of Arithmetic instructions = 2.56×10^9 .
- No. of L/S instructions = 1.28×10^9 .
- No. of branch instructions = 256 million.

$$\left. \begin{array}{l} 1 \text{ million} = 1 \times 10^6 \\ 1 \text{ GHz} = 1 \times 10^9 \end{array} \right\}$$

1.9.2 - clock frequency = $2 \text{ GHz} = 2 \times 10^9$
 Arithmetic instruction is doubled
 * Total clock cycle = $\sum \text{no. of instructions} \times \text{CPI}$

$$\begin{aligned} \text{clock cycle of } P_1 &= (2.56 \times 10^9 \times 2) + (1.28 \times 10^9 \times 12) + (256 \times 10^6 \times 5) \\ &= 21.76 \times 10^9 \\ &= 2176 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{clock cycle of } P_2 &= \left(\frac{2.56 \times 10^9}{2 \times 0.7} \times 2 \right) + \left(\frac{1.28 \times 10^9}{2 \times 0.7} \times 12 \right) + (256 \times 10^6 \times 5) \\ &= (1.83 \times 10^9 \times 2) + (0.91 \times 10^9 \times 12) + (1.28 \times 10^9) \\ &= 15.86 \times 10^9 \\ &= 1586 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{clock cycle of } P_4 &= \left(\frac{2.56 \times 10^9}{4 \times 0.7} \times 2 \right) + \left(\frac{1.28 \times 10^9}{4 \times 0.7} \times 12 \right) + (256 \times 10^6 \times 5) \\ &= 1.82 \times 10^9 + 5.52 \times 10^9 + 1.28 \times 10^9 \\ &= 8.62 \times 10^9 \\ &= 862 \times 10^7 \end{aligned}$$

$$\begin{aligned} \text{clock cycle of } P_8 &= \left(\frac{2.56 \times 10^9}{8 \times 0.7} \times 2 \right) + \left(\frac{1.28 \times 10^9}{8 \times 0.7} \times 12 \right) + (256 \times 10^6 \times 5) \\ &= 0.92 \times 10^9 + 2.76 \times 10^9 + 1.28 \times 10^9 \\ &= 4.96 \times 10^9 \\ &= 496 \times 10^7 \end{aligned}$$

$$\star \text{ Execution time} = \frac{\text{clock cycle}}{\text{clock frequency}}$$

$$\rightarrow \text{Execution time } (P_1) = \frac{2176 \times 10^7}{2 \times 10^9} = 10.88 \text{ seconds.}$$

$$\rightarrow \text{Execution time } (P_2) = \frac{1586 \times 10^7}{2 \times 10^9} = 7.93 \text{ seconds.}$$

$$\rightarrow \text{Execution time } (P_4) = \frac{862 \times 10^7}{2 \times 10^9} = 4.31 \text{ seconds.}$$

$$\rightarrow \text{Execution time } (P_8) = \frac{496 \times 10^7}{2 \times 10^9} = 2.48 \text{ seconds.}$$

Ex 1.9.1 Single Arithmetic instruction.

$$\star \text{ clock cycle of } P_1 = (2.56 \times 10^9 \times 1) + (1.28 \times 10^9 \times 12) + (256 \times 10^6 \times 5)$$

$$= 19.2 \times 10^9$$

$$= 192 \times 10^8$$

$$\star \text{ clock cycle of } P_2 = \left(\frac{2.56 \times 10^9}{2 \times 0.7} \times 1 \right) + \left(\frac{1.28 \times 10^9}{2 \times 0.7} \times 12 \right) + (256 \times 10^6 \times 5)$$

$$= (1.83 \times 10^9) + (10.92 \times 10^9) + (1.28 \times 10^9)$$

$$= 14 \times 10^9$$

$$\star \text{ clock cycle of } P_4 = \left(\frac{2.56 \times 10^9}{4 \times 0.7} \times 1 \right) + \left(\frac{1.28 \times 10^9}{4 \times 0.7} \times 12 \right) + (256 \times 10^6 \times 5)$$

$$= 77 \times 10^8$$

$$= 77 \times 10^8$$

$$\star \text{ clock cycle of } P_8 = \left(\frac{2.56 \times 10^9}{8 \times 0.7} \times 1 \right) + \left(\frac{1.28 \times 10^9}{8 \times 0.7} \times 12 \right) + (256 \times 10^6 \times 5)$$

$$= 4.5 \times 10^9$$

$$= 45 \times 10^8$$

$$\star \text{ Execution time } (P_1) = \frac{192 \times 10^8}{2 \times 10^9} = 9.6 \text{ seconds.}$$

$$\text{Execution time } (P_2) = \frac{14 \times 10^9}{2 \times 10^9} = 7 \text{ seconds}$$

$$\therefore \therefore (P_4) = \frac{77 \times 10^8}{2 \times 10^9} = 3.85 \text{ seconds.}$$

$$\therefore \therefore (P_8) = \frac{45 \times 10^8}{2 \times 10^9} = 2.25 \text{ seconds.}$$

* Relative Speedup:

$$\text{Relative Speedup} = \frac{\text{Execution time of the processor } P_1}{\text{execution time of current processor.}}$$

$$* \text{Speedup}(P_1) = 9.6 / 9.6 = \boxed{1}$$

$$* \text{Speedup}(P_2) = 9.6 / 7 = \boxed{1.37}$$

$$* \text{Speedup}(P_4) = 9.6 / 3.85 = \boxed{2.49}$$

$$* \text{Speedup}(P_8) = 9.6 / 2.25 = \boxed{4.26}$$

* 19.3

$$* \text{Clock cycle of } P_4 = \left(\frac{2.56 \times 10^9}{4 \times 0.7} \times 1 \right) + \left(\frac{1.28 \times 10^9}{4 \times 0.7} \times 12 \right) + (256 \times 10^6 \times 5)$$

$$= (0.91 \times 10^9) + (5.52 \times 10^9) + (1.28 \times 10^9)$$

$$= 7.7 \times 10^9$$

$$= \boxed{7.7 \times 10^8}$$

$$* \text{clock cycle of } P_1 = (2.56 \times 10^9 \times 1) + (1.28 \times 10^9 \times 12) + (256 \times 10^6 \times 5)$$

$$= \boxed{76.8 \times 10^8}$$

→ On trial & error basis, choose $CP1(L/s)$ as 1, 2, 3.

When $CP1(L/s)$ is 3, the clock cycle of both the processor match with each other.

∴ Therefore reduced $CP1(L/s)$ is '3' because it match with the Performance of four processors.

Question: 1.11: The results of the SPEC CPU2006 bzip2 benchmark running on an AMD Barcelona has an instruction count of 2.389×10^{12} , an execution time of 750 s, and a reference time of 9650 s.

1.11.1: Find the CPI if the clock cycle time is 0.333 ns.

1.11.2: Find the SPECratio.

1.11.3: Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% without affecting the CPI.

1.11.4: Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% and the CPI is increased by 5%.

1.11.5: Find the change in the SPECratio for this change.

1.11.6: Suppose that we are developing a new version of the AMD Barcelona processor with a 4 GHz clock rate. We have added some additional instructions to the instruction set in such a way that the number of instructions has been reduced by 15%. The execution time is reduced to 700 s and the new SPECratio is 13.7. Find the new CPI.

1.11.7: This CPI value is larger than obtained in 1.11.1 as the clock rate was increased from 3 GHz to 4 GHz. Determine whether the increase in the CPI is similar to that of the clock rate. If they are dissimilar, why?

1.11.8: By how much has the CPU time been reduced?

1.11.9: For a second benchmark, libquantum, assume an execution time of 960 ns, CPI of 1.61, and clock rate of 3 GHz. If the execution time is reduced by an additional 10% without affecting the CPI and with a clock rate of 4 GHz, determine the number of instructions.

1.11.10: Determine the clock rate required to give a further 10% reduction in CPU time while maintaining the number of instructions and with the CPI unchanged.

1.11.11: Determine the clock rate if the CPI is reduced by 15% and the CPU time by 20% while the number of instructions is unchanged.

Solution:

Question: 1.11

Given,

- Instruction count = 2.389×10^{12}
- Execution time = 750 sec.
- Reference time = 960 9650 s.

1.11.1

Given,

clock cycle time = 0.333 ns.

$$* CPI = \frac{\text{Execution time}}{(\text{Instruction count} \times \text{Clock cycle time})}$$

$$CPI = \frac{750 \text{ s}}{(2.389 \times 10^{12} \times 0.333 \text{ ns})}$$

$$CPI = 0.94$$

1.11.2

$$\text{SPEC ratio} = \frac{\text{Reference time}}{\text{Execution time}}$$
$$= \frac{9650 \text{ s}}{750 \text{ s}}$$

$$\text{SPEC ratio} = 12.89$$

1.11.3

Given,

no. of instructions of benchmark increased by 10%.

$$* \text{Increase in CPU Time} = 1 + \left(\frac{\text{Increase of } I}{\text{Execution time}} \right)$$

$$\text{New CPI} = 2.389 \times 10^{12} + 2.389 \times 10^{12} \times 0.1$$

$$= 2.389 \times 10^{12} + 2.389 \times 10^{11}$$

$$\text{New CPI} = 2.6279 \times 10^{12}$$

$$\text{Increase in CPU Time} = \frac{\text{CPI} \times \text{Instruction count}}{\text{clock rate}}$$

$$= \frac{2.6279612 \times 0.94}{3 \times 10^9}$$

$$= \frac{2.6279612 \times 940}{3 \times 10^9 \times 10^3}$$

$$= \frac{2.6279 \times 940}{3}$$

$$\therefore \text{CPU Time} = 823.41 \text{ Sec.}$$

$$\% \text{ Increase in CPU Time} = \frac{\text{New time} - \text{Old time}}{\text{New time}} \times 100$$

$$= \frac{823.41 - 750}{750} \times 100\%$$

$$= \frac{73.41}{750} \times 100\%$$

$$= 0.098 \times 100\%$$

$$= 9.8\%$$

Therefore, the increase in CPU Time is 9.8% (or) 73.41 Sec.

1.11.4

Given,

no. of instructions increased by 10% & CPI increased by 5%.

- Assume initial no. of instructions = 100.
- Assume initial CPI = 100
- After increase by 10% it will become = 110 (no. of inst)
- After increase of 5% CPI = 105.

$$\begin{aligned}
 * \text{ Increase in CPI Time} &= \frac{\text{CPU Time (after)}}{\text{CPU Time (before)}} \\
 &= \frac{110 \times 105}{100 \times 100} \\
 &= 1.155.
 \end{aligned}$$

Therefore, the increase in CPI Time Execution is by 15%.

1.11.5 Find change in SPEC ratio for this change.

— From the above data.

Old SPEC ratio = 12.9.

New Execution time = 866 s. (750 + 15.5% change = 866 s)

$$\begin{aligned}
 * \text{ New Specratio} &= \frac{\text{Reference time}}{\text{New Execution time}} \\
 &= \frac{9650 \text{ s}}{866 \text{ s}} \\
 &= 11.14.
 \end{aligned}$$

$$\begin{aligned}
 * \text{ Change in SPEC ratio} &= \frac{\text{New SPEC ratio} - \text{SPEC ratio}}{\text{SPEC ratio}} \\
 &= \frac{11.14 - 12.9}{12.9} \\
 &= -0.1364 \\
 &= -13.64\%.
 \end{aligned}$$

Therefore, the change in SPEC ratio is -13.64.

That is decreased by 13.64%.

1.11.6

Given,

- New SPEC ratio = 13.7.

- Execution time = 700

- clock rate = 4 GHz

- No. of Inst = 2.389×10^{12}

- Inst. reduced by 15%

$$\begin{aligned} * \text{ Final no. of instruction} &= 2.389 \times 10^{12} - 2.389 \times 10^{12} \times 15/100 \\ &= 238.9 \times 10^{10} - 35.8 \times 10^{10} \\ &= 2.03 \times 10^{12} \end{aligned}$$

$$* \text{ CPU Execution time} = \frac{\text{CPI} \times \text{Number of instructions}}{\text{clock rate.}}$$

$$700 = \frac{\text{CPI} \times 2.03 \times 10^{12}}{4 \times 10^9}$$

$$\text{CPI} = \frac{700 \times 4 \times 10^9}{2.03 \times 10^{12}}$$

$$\boxed{\text{CPI} = 1.37 \text{ cycles.}}$$

1.11.7

If the clock rate is 4 GHz, CPU time is 750, instruction count is 2.389×10^{12} , then CPI value is:

$$\text{CPI} = \frac{\text{clock rate} \times \text{CPU time}}{\text{Instruction count.}}$$

$$= 4 \times 10^9 \times 750 / 2.389 \times 10^{12}$$

$$= 3000 \times 10^9 / 2.389 \times 10^{12}$$

$$= 3 \times 10^{12} / 2.389 \times 10^{12}$$

$$= 3 / 2.389$$

$$\text{CPI} = 1.255$$

→ CPI values are increased from 0.94 to 1.25

Thus,

$\boxed{\text{Clock rate} \propto \text{CPI}}$

1.11.8

- Initial CPU time = 750
 - Final CPU time after decreasing instruction is 700.
 * Percentage of reduced instruction time = $\frac{750 - 700}{750} \times 100$
 $= \frac{50}{750} \times 100$
 $= 6.66\%$

1.11.9

Given,
 Assume Execution time = 960 ns.
 CPI = 1.61
 Clock rate = 3 GHz. A New clock rate = 4 GHz (4×10^9 Hz).

Number of instructions = $\frac{\text{CPU-time} \times \text{clock rate}}{\text{CPI}}$
 $= \frac{864 \times 10^{-9} \times 4 \times 10^9}{1.61}$
 $= 3456 / 1.61$
 $= 2146.58$

{ New execution time =
 initial execution time - 10% of
 ini. ex. time }
 $= 960 - (960 \times 10/100)$
 $= 960 - 96$
 $= 864 \text{ ns.}$
 $= 864 \times 10^{-9} \text{ s.}$

1.11.10

The Execution time has been reduced by further 10%.
 Final no. of instructions = $864 - 864 \times 10/100$
 $= 777.6$
 → Changed clock rate = 4 GHz
 → No. of instructions = 2146×10^9

$$* \text{CPU Execution time (reduced)} = \frac{\text{CPI} \times \text{no. of instructions}}{\text{clock rate (new)}}$$

$$777.6 = \frac{1.61 \times 2146 \times 10^9}{\text{clock rate}}$$

$$* \text{clock rate} = \frac{2146 \times 10^9 \times 1.61}{777.6}$$

$$= 4.44 \text{ GHz}$$

Therefore, the required clock rate for 10% reduction is 4.44 GHz

* 1.11.11 Reduced CPI for 15%.

$$\begin{aligned} (\text{Reduced CPI} &= \text{initial CPI} - 15\% (\text{initial CPI})) \\ &= (1.61 - 15/100 \times 1.61) = 1.61 - 0.15 \times 1.61 \\ &= 1.367 \approx 1.37 \end{aligned}$$

* Reduced CPU time by 20%.

$$\begin{aligned} \text{New CPU time} &= (\text{initial CPU time} - (20\% \text{ initial CPU time})) \\ &= (777.6 - (20/100 \times 777.6)) \\ &= (777.6 - 155.52) \\ &= 622.08 \text{ ns.} \quad | \quad 622.08 \times 10^{-9} \text{ s.} \end{aligned}$$

$$* \text{clock rate} = \left(\frac{\text{CPI} \times \text{no. of instructions}}{\text{CPU execution time}} \right)$$

$$= \frac{1.37 \times 2147}{622.08 \times 10^{-9}}$$

$$= 2941.39 \times 10^9 / 622.08$$

$$= 4.728 \text{ GHz}$$

Therefore, the clock rate is 4.728 GHz.

Question: 1.12: Section 1.10 cites as a pitfall the utilization of a subset of the performance equation as a performance metric. To illustrate this, consider the following two processors. P1 has a clock rate of 4 GHz, average CPI of 0.9, and requires the execution of 5.0×10^9 instructions. P2 has a clock rate of 3 GHz, an average CPI of 0.75, and requires the execution of 1.0×10^9 instructions.

1.12.1: One usual fallacy is to consider the computer with the largest clock rate as having the largest performance. Check if this is true for P1 and P2.

1.12.2: Another fallacy is to consider that the processor executing the largest number of instructions will need a larger CPU time. Considering that processor P1 is executing a sequence of 1.0×10^9 instructions and that the CPI of processors P1 and P2 do not change, determine the number of instructions that P2 can execute in the same time that P1 needs to execute 1.0×10^9 instructions.

1.12.3: A common fallacy is to use MIPS (millions of instructions per second) to compare the performance of two different processors, and consider that the processor with the largest MIPS has the largest performance. Check if this is true for P1 and P2.

1.12.4: Another common performance figure is MFLOPS (millions of floating-point operations per second), defined as $\text{MFLOPS} = \text{No. FP operations} / (\text{execution time} \times 10^6)$ but this figure has the same problems as MIPS. Assume that 40% of the instructions executed on both P1 and P2 are floating-point instructions. Find the MFLOPS figures for the programs.

Solution:

Question 1.12

Given,

	clock Rate	CPI	Instructions
P ₁	4GHz = 4×10^9 Hz	0.9	$5.0 \times 10^9 = 5 \times 10^9$
P ₂	3GHz = 3×10^9 Hz	0.75	$1.0 \times 10^9 = 1 \times 10^9$

* CPU time = $\frac{\text{CPI} \times \text{instruction count}}{\text{clock rate}}$

$$P_{1\text{CPU}} = \frac{0.9 \times 5 \times 10^9}{4 \times 10^9}$$

$$= 4.5/4$$

$$= 1.125 \text{ s}$$

$$P_{2\text{CPU}} = \frac{0.75 \times 1.0 \times 10^9}{3 \times 10^9}$$

$$= 0.75/3$$

$$= 0.25 \text{ s}$$

$$\therefore P_{1\text{CPU}} > P_{2\text{CPU}}$$

There fore P₂ performs better than P₁.

→ false.

* 1.12.2

* Execution Time = $\frac{\text{Instructions} \times \text{CPI}}{\text{clock rate}}$

→ execution time of P₁ = $10^9 \times 0.9 / 4 \times 10^9$

$$= 0.225 \text{ Sec.}$$

Execution time of P₂ = $\frac{I \times 0.75}{(3 \times 10^9)}$

Same execution time for both processor

$$0.225 = I \times 0.75 / 3 \times 10^9$$

$$I = (0.225 \times 3 \times 10^9) / 0.75$$

$$= 9 \times 10^8$$

∴ Thus, for the execution time to be equal the processor P₂ has to execute 9×10^8 instructions.

* 1.12.3

$$\rightarrow \text{Processor } P_1: \begin{cases} \text{clock rate} = 4 \text{ GHz} \\ \text{CPI} = 0.9 \\ \text{no. of inst} = 5 \times 10^9 \end{cases} \quad \text{Processor } P_2: \begin{cases} \text{clock rate} = 3 \text{ GHz} \\ \text{CPI} = 0.75 \\ \text{no. of instruc} = 1 \times 10^9 \end{cases}$$

$$* \text{Time taken} = \frac{\text{No. of inst} \times \text{CPI}}{\text{clock rate}}$$

$$T(P_1) = \frac{5 \times 10^9 \times 0.9}{4 \times 10^9} \\ = 1.125$$

$$T(P_2) = \frac{1 \times 10^9 \times 0.75}{3 \times 10^9} \\ = 0.25$$

$$* \text{Calculating MIPS} = \frac{\text{clock rate} \times 10^{-6}}{\text{CPI}}$$

$$P_{1 \text{ MIPS}} = \frac{4 \times 10^9 \times 10^{-6}}{0.9} \\ = 4.44 \times 10^3$$

$$P_{2 \text{ MIPS}} = \frac{3 \times 10^9 \times 10^{-6}}{0.75} \\ = 4 \times 10^3$$

→ performance of $P_2 > P_1$.

Therefore, it conclude that MIPS of the processor is inversely proportional its performance.

* 1.12.4

$$\text{MFLOPS} = \frac{\text{No. of FP operations}}{(\text{execution time} \times 10^6)}$$

$$\text{FP}_{op1} = \frac{5 \times 10^9 \times 0.4}{1} \\ = 2 \times 10^9$$

$$\text{FP}_{op2} = \frac{10^9 \times 0.4}{1} \\ = 4 \times 10^8$$

* MFLOPS for processor 1:

$$\begin{aligned}\text{Clock cycles} &= \text{CPI} \times \text{no. of FP inst.} \\ &= 0.9 \times 2 \times 10^9 \\ &= 1.8 \times 10^9\end{aligned}$$

$$\begin{aligned}\text{Execution time} &= \text{no. of instr.} \times \text{CPI} / \text{clock rate} \\ &= 2 \times 10^9 \times 0.9 / 4 \times 10^9 \\ &= 0.45\end{aligned}$$

$$\begin{aligned}\text{MFLOPS} &= \text{no. of FP operations} / (\text{execution time} \times 10^6) \\ &= 1.8 \times 10^9 / 0.45 \times 10^6 \\ &= 4 \times 10^3\end{aligned}$$

* MFLOPS for processor 2:

$$\begin{aligned}\text{Clock cycle} &= 0.75 \times 4 \times 10^8 \\ &= 3 \times 10^8\end{aligned}$$

$$\begin{aligned}\text{Exe. Time} &= 4 \times 10^8 \times 0.75 / 3 \times 10^9 \\ &= 0.1\end{aligned}$$

$$\begin{aligned}\text{MFLOPS} &= 3 \times 10^8 / 0.1 \times 10^6 \\ &= 3 \times 10^3\end{aligned}$$

Thus, the MFLOPS values for $P_1 = 4 \times 10^3$
 $P_2 = 3 \times 10^3$.

Question:1.13: Another pitfall cited in Section 1.10 is expecting to improve the overall performance of a computer by improving only one aspect of the computer. Consider a computer running a program that requires 250 s, with 70 s spent executing FP instructions, 85 s executed L/S instructions, and 40 s spent executing branch instructions.

1.13.1: By how much is the total time reduced if the time for FP operations is reduced by 20%?

1.13.2: By how much is the time for INT operations reduced if the total time is reduced by 20%?

1.13.3: Can the total time can be reduced by 20% by reducing only the time for branch instructions?

Solution:

* Question: 1.13

Given,

FP Instructions	LI5 Instructions	Branch Instructions	Time t_{tot}
70	85	40	250 s.

→ Time reduced for FP operations = 20%

* Calculate FP Instructions CPU time:

$$\begin{aligned}
 t_{FP\text{reduced}} &= t_{FP} - (t_{FP} \times 0.2) \\
 &= 70 - (70 \times 0.2) \\
 &= 56 \text{ Sec.}
 \end{aligned}$$

* Total time after reducing the FP is:

$$\begin{aligned}
 t_{\text{tot}FP\text{reduced}} &= t_{FP} + t_{INT} + t_{LI5} + t_{\text{branch}} \\
 &= 56s + 85s + 55s + 40s \\
 &= 236 \text{ Sec.}
 \end{aligned}$$

* Calculate Total reduced time:

$$\begin{aligned}
 t_{\text{total}reduced} &= t_{\text{tot}} - t_{\text{tot}FP\text{reduced}} \\
 &= 250s - 236s \\
 &= 14 \text{ Sec}
 \end{aligned}$$

* Percentage of total time reduced,

$$\begin{aligned} t_{\text{per}} &= \frac{t_{\text{tot red}}}{t_{\text{tot}}} \times 100 \\ &= 145 / 2505 \times 100 \\ &= 5.6\% \end{aligned}$$

∴ Therefore, the total time reduced for 20% of FP instruction is 5.6%

* 1.13.2

Assume total time = 250 sec.

If the INT is reduced by 20% = 0.2.

$$\therefore 0.2 \times 250 = 50$$

→ 50 seconds time is reduced for INT operations.

* 1.13.3

Acc. to Amdahl's Law:

$$* \left[\text{Exe. time after improvement} = \frac{\text{affected.}}{\text{Amount of impro.}} + \text{Exe. time unaffected.} \right]$$

→ Assume amount of improvement be 'n'.

$$\rightarrow \text{unaffected} = (250 - 40) = 210 \text{ s.}$$

$$* \text{Execution time after improvement} = \frac{40}{n} + 210.$$

The execution time after 20% reduction is:-

$$\begin{aligned} 250 - 250 \times 20\% &= 250 - 250 \times 20/100 \\ &= 250 - 50 \\ &= 200. \end{aligned}$$

Therefore,

$$200 = 40/n + 210$$

$$-10 = 40/n$$

→ Since the value on left is negative, the total time cannot be reduced by 20% by only reducing the branch inst. exe. time.

Question: 1.14: Assume a program requires the execution of 50×10^6 FP instructions, 110×10^6 INT instructions, 80×10^6 L/S instructions, and 16×10^6 branch instructions. The CPI for each type of instruction is 1, 1, 4, and 2, respectively. Assume that the processor has a 2 GHz clock rate.

1.14.1: By how much must we improve the CPI of FP instructions if we want the program to run two times faster?

1.14.2: By how much must we improve the CPI of L/S instructions if we want the program to run two times faster?

1.14.3: By how much is the execution time of the program improved if the CPI of INT and FP instructions is reduced by 40% and the CPI of L/S and Branch is reduced by 30%?

Solution:

* Question 1.14's

- Given, FP.
- no. of inst = 50×10^6
 - No. of INT inst = 110×10^6
 - No. of L/s inst = 80×10^6
 - No. of branch inst = 16×10^6
- CPI of FP inst = 1
 - " " INT " = 1
 - " " L/s " = 4
 - " " branch " = 2.
 - clock rate = 2GHz

* 1.14.1 improved CPI of FP instructions.

$$\begin{aligned} \text{Clock cycle} &= \left[\begin{aligned} &\text{CPI(FP)} \times \text{no. of FP inst} + \text{CPI(INT)} \times \text{no. of INT inst} + \\ &\text{CPI(L/s)} \times \text{no. of L/s inst} + \text{CPI(branch)} \times \text{no. of} \\ &\text{Inst.} \end{aligned} \right] \\ &= \left[(1 \times 50 \times 10^6) + (1 \times 110 \times 10^6) + (4 \times 80 \times 10^6) + (2 \times 16 \times 10^6) \right] \\ &= 50 \times 10^6 + 110 \times 10^6 + 320 \times 10^6 + 32 \times 10^6 \\ &= 512 \times 10^6 \end{aligned}$$

$$\begin{aligned} \text{Execution time} &= \frac{512 \times 10^6}{2 \times 10^9} \\ &= 0.256 \text{ seconds.} \end{aligned}$$

→ Two times faster:

$$\begin{aligned} \text{Clock cycle} &= \left[\begin{aligned} &\text{CPI(improved FP)} \times \text{no. of FP inst} + \text{CPI(INT)} \times \\ &\text{no. of INT inst} + \text{CPI(L/s)} \times \text{Num of L/s inst} + \\ &\text{CPI(Branch)} \times \text{no. of branch inst.} \end{aligned} \right] \\ \text{CPI(improved FP)} &= \frac{\frac{\text{Clock cycle}}{2} - \left[\begin{aligned} &\text{CPI(INT)} \times \text{no. of INT inst} + \\ &\text{CPI(L/s)} \times \dots \text{L/s} + \\ &\text{CPI(branch)} \times \dots \text{branch} \end{aligned} \right]}{\text{Num of FP instructions.}} \end{aligned}$$

$$\begin{aligned}
 * \text{ CPI (Improved FP)} &= \left[\frac{512 \times 10^6}{2} - \frac{[(1 \times 110 \times 10^6) + (4 \times 80 \times 10^6) + (2 \times 16 \times 10^6)]}{50 \times 10^6} \right] \\
 &= \left[\frac{256 \times 10^6 - 462 \times 10^6}{50 \times 10^6} \right] \\
 &= -4.12
 \end{aligned}$$

→ CPI (Improved FP) is less than zero, Hence, if CPI of floating point instruction are improved then the program cannot run two times faster.

Therefore, it is not possible to improve FP instruction to run the program two times faster.

Given,

- | | <u>CPI</u> |
|--|-----------------|
| - Number of FP instructions = 50×10^6 | $\rightarrow 1$ |
| - Number of INT " = 110×10^6 | $\rightarrow 1$ |
| - Number of L/S instructions = 80×10^6 | $\rightarrow 4$ |
| - Number of branch instructions = 16×10^6 | $\rightarrow 2$ |
| - clock rate = 2 GHz. and clock cycle is = 512×10^6 . | |

Formulae:-

$$\frac{\text{clock cycle}}{2} = \left\{ \begin{array}{l} \text{CPI(FP)} \times \text{Num of FP instructions} + \text{CPI(INT)} \times \text{Num. of INT Instr.} \\ + \text{CPI(Improved L/S)} \times \text{Num of L/S instructions} + \text{CPI(branch)} \times \text{Num} \\ \text{of branch instructions.} \end{array} \right\}$$

1.14.2

$$\begin{aligned} * \text{CPI(Improved L/S)} &= \left\{ \frac{512 \times 10^6}{2} - \frac{\{(1 \times 50 \times 10^6) + (1 \times 110 \times 10^6) + (2 \times 16 \times 10^6)\}}{80 \times 10^6} \right\} \\ &= \left\{ 256 \times 10^6 - \frac{\{(50 \times 10^6) + (110 \times 10^6) + (32 \times 10^6)\}}{80 \times 10^6} \right\} \\ &= \frac{256 \times 10^6 - 192 \times 10^6}{80 \times 10^6} \\ &= 0.8 \end{aligned}$$

Therefore, the CPI of Improved L/S = 0.8

~~1.14.3~~

1.14.3

- CPI of FP if reduced by 40% = 0.6.
- CPI " INR " " " 40% = 0.6
- " " LIS " " " 30% = 2.8.
- " " branch " " " 30% = 1.4.

$$\begin{aligned} * \text{ clock cycle} &= \left[(1 \times 50 \times 10^6) + (1 \times 110 \times 10^6) + (4 \times 80 \times 10^6) + (2 \times 16 \times 10^6) \right] \\ &= 50 \times 10^6 + 110 \times 10^6 + 320 \times 10^6 + 32 \times 10^6 \\ &= 512 \times 10^6 \end{aligned}$$

* Execution time (before improvement) = $\frac{\text{clock cycle}}{\text{clock rate}}$

$$\begin{aligned} &= \frac{512 \times 10^6}{2 \times 10^9} \\ &= 256 \times 10^{-3} \\ &= 0.256 \text{ seconds.} \end{aligned}$$

* Execution time after improvement:

$$\begin{aligned} \text{clock cycle (improved)} &= \left[(0.6 \times 50 \times 10^6) + (0.6 \times 110 \times 10^6) + (2.8 \times 80 \times 10^6) + (1.4 \times 16 \times 10^6) \right] \\ &= 30 \times 10^6 + 66 \times 10^6 + 224 \times 10^6 + 22.4 \times 10^6 \\ &= 342.4 \times 10^6 \end{aligned}$$

$$* \text{ Execution time (after improvement)} = \frac{\text{clock cycle (improved)}}{\text{clock rate.}}$$

$$= \frac{342.4 \times 10^6}{2 \times 10^9}$$

$$= 171.2 \times 10^{-3}$$

$$\boxed{= 0.1712 \text{ seconds.}}$$

Therefore, the execution time after improvement is "0.171 seconds."

==

Question:1.15: When a program is adapted to run on multiple processors in a multiprocessor system, the execution time on each processor is comprised of computing time and the overhead time required for locked critical sections and/or to send data from one processor to another. Assume a program requires $t = 100$ s of execution time on one processor. When run p processors, each processor requires t/p s, as well as an additional 4 s of overhead, irrespective of the number of processors. Compute the per-processor execution time for 2, 4, 8, 16, 32, 64, and 128 processors. For each case, list the corresponding speedup relative to a single processor and the ratio between actual speedup versus ideal speedup (speedup if there was no overhead).

Solution:

* Question: 1.15

Given,

Execution time for one processor = 100s.

Overhead = 4s + Execution time of processor.

- Number of processors in $P_1 = 2$

$P_2 = 4$

$P_3 = 8$

$P_4 = 16$

$P_5 = 32$

$P_6 = 64$

$P_7 = 128$

The Execution time for one processor = $\frac{\text{Execution time of one processor}}{\text{Number of processors}}$

- for P_1 : $100/2$

= 50 seconds.

- for P_2 : $100/4$

= 25 seconds.

- for P_3 : $100/8 = 12.5$ seconds.

- for P_4 : $100/16 = 6.25$ seconds.

- for P_5 : $100/32 = 3.125$ seconds.

- for P_6 : $100/64 = 1.5625$ seconds.

- for P_7 : $100/128 = 0.78125$ seconds.

* formula to calculate the overhead time of processor.

$$\left(\begin{array}{l} \text{Overhead time of} \\ \text{each processor} \end{array} = 4 + \text{Execution time of the processor.} \right)$$

$$\text{For } P_1 = 4 + 50 = 54 \text{ seconds.}$$

$$P_2 = 4 + 25 = 29 \text{ sec.}$$

$$P_3 = 4 + 12.5 = 16.5 \text{ sec.}$$

$$P_4 = 4 + 6.25 = 10.25 \text{ sec.}$$

$$P_5 = 4 + 3.125 = 7.125 \text{ sec.}$$

$$P_6 = 4 + 1.5625 = 5.5625 \text{ sec.}$$

$$P_7 = 4 + 0.7812 = 4.7812 \text{ sec.}$$

* Speedup with respect to overhead time:

Formulae:

$$(\text{Speedup} = \text{Execution time of one processor} / \text{overhead time})$$

$$\text{for } P_1 = 100 / 54 = 1.85$$

$$P_2 = 100 / 29 = 3.44$$

$$P_3 = 100 / 16.5 = 6.06$$

$$P_4 = 100 / 10.25 = 9.75$$

$$P_5 = 100 / 7.125 = 14.03$$

$$P_6 = 100 / 5.5625 = 17.97$$

$$P_7 = 100 / 4.7812 = 20.91$$

Continuation of Question 1.5

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$$* \text{ Speedup ratio} = \frac{\text{Speedup with respect to overhead time.}}{\text{Number of processors.}}$$

$$\text{for } P_1 = 1.85/2 = 0.925$$

$$\text{for } P_2 = 3.44/4 = 0.86$$

$$P_3 = 6.06/8 = 0.75$$

$$P_4 = 9.75/16 = 0.61$$

$$P_5 = 14.03/32 = 0.43$$

$$P_6 = 17.97/64 = 0.28$$

$$P_7 = 20.91/128 = 0.16$$