Theory. PDF

10.a. The variable x is a global variable or a formal parameter.

d. The expression evaluates to a value and the evaluation process does not alter global variables nor formal parameters.

11.a. There exists ξ, ϕ, ρ, v, ξ', and ρ' such that $x \notin dom(\xi)$ where if $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle$ then v is valid

c. IF $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi_1, \phi, \rho' \rangle$ Then $\xi = \xi_1$

d. If $\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho \rangle$ then $dom\ \xi = dom\ \xi'$

R. WHILEEND and WHILEEND' are effectively the same. e1 evaluating to zero is equivalent in both these rules.

FORMALASSIGN and FORMALASSIGN' are effectively not the same. The conclusion of the FORMALASSIGN' rule states that ρ may change to ρ' but we know for a fact that environment ρ exhibits a mapping change:

ex.

$$D = \frac{n \in dom\ \rho \qquad \overline{\langle LITERAL(0), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho \rangle}}{\langle set(n, LITERAL(0)), \xi, \phi, \rho \rangle \Downarrow \langle 0, \xi, \phi, \rho\{n \mapsto 0\} \rangle}$$

16. a.

awk

Global Var $\dfrac{x \notin \text{dom } P \quad x \notin \text{dom } \xi}{\langle \text{var}(x), \xi, \phi, P \rangle \Downarrow \langle 0, \xi\{x \mapsto 0\}, \phi, P \rangle}$

Global Assign $\dfrac{x \notin \text{dom } P \quad x \notin \text{dom } \xi \quad \langle e, \xi, \phi, P \rangle \Downarrow \langle v, \xi', \phi, P' \rangle}{\langle \text{set}(x, e), \xi, \phi, P \rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, P' \rangle}$

b.

icon

Formal Var $\dfrac{x \notin \text{dom } P \quad x \notin \text{dom } \xi}{\langle \text{var}(x), \xi, \phi, P \rangle \Downarrow \langle 0, \xi, \phi, P'\{x \mapsto 0\} \rangle}$

Formal Assign $\dfrac{x \notin \text{dom } P \quad x \notin \text{dom } \xi \quad \langle e, \xi, \phi, P \rangle \Downarrow \langle v, \xi, \phi, P' \rangle}{\langle \text{set}(x, e), \xi, \phi, P \rangle \Downarrow \langle v, \xi, \phi, P'\{x \mapsto v\} \rangle}$

c. I would prefer Icon because if local variables were created, it will only stay in the scope of that function and not affect other functions. Awk would instantiate new variables so when you write more functions, you will be limited on variable names.

$$\text{FORMAL VAR}\ \frac{x \in dom\ \rho \quad \rho(x)=99}{\langle x, \xi, \phi, \rho\rangle \Downarrow \langle 99, \xi, \phi, \rho\rangle} \qquad \text{LITERAL}\ \frac{}{\langle LITERAL(3), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\rangle}$$

12. $$\text{FORMAL ASSIGN}\ \langle set(x, LIT(3)), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho'\{x \mapsto 3\}\rangle$$

$$\text{FORMAL VAR}\ \frac{x \in dom\ \rho' \quad \rho'(x)=3}{\langle x, \xi, \phi, \rho'\rangle \Downarrow \langle \rho'(x), \xi, \phi, \rho'\rangle}$$

$$\text{BEGIN}\ \frac{}{\langle BEGIN((set\ x\ 3)\ x), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho'\rangle}$$

13.a. IF True ($x \neq 0$) Global Var x

$$\text{GLOBAL VAR}\ \frac{\frac{x \notin dom\ \rho \quad x \in dom\ \xi}{\langle x, \xi, \phi, \rho\rangle \Downarrow \langle x, \xi, \phi, \rho\rangle} \quad x \neq 0 \quad \frac{x \notin dom\ \rho \quad x \in dom\ \xi \quad V_1 = \xi(x)}{\langle x, \xi, \phi, \rho\rangle \Downarrow \langle \xi(x), \xi, \phi, \rho\rangle}\ \text{GLOBAL VAR}}{\text{IF TRUE}\ \langle IF(x, x, 0), \xi, \phi, \rho\rangle \Downarrow \langle \xi(x), \xi, \phi, \rho\rangle}$$

$VAR(x) = \xi(x) = V_1$

IF False ($x=0$) Global Var X

GLOBAL VAR

$$\text{IF FALSE}\ \frac{\frac{x \notin dom\ \rho \quad x \in dom\ \xi}{\langle x, \xi, \phi, \rho\rangle \Downarrow \langle x, \xi, \phi, \rho\rangle} \quad x=0 \quad \frac{}{\langle 0, \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}\ \text{LITERAL}}{\langle IF(x, x, 0), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}$$

$VAR(x) = \xi(x) = V_1$

$$\text{Global VAR}\ \frac{x \notin dom\ \rho, x \in dom\ \xi}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle V_2, \xi'', \phi, \rho''\rangle} \quad VAR(x) = \xi''(x) = V_2$$

Since $VAR(x) = V_1$ and $VAR(x) = V_2$ then $V_1 = V_2$
Shown by derivation tree.

If x was in the environment of a formal var, the result will be the same since a value V is determined within the expression and has no global side effects.

meta theory

a. Environments: $\xi, \rho, \phi, \xi', \rho'$

ϕ includes initial basis plus user-defined function: Incrementy where ρ is x and y

$Y \in dom(\xi)$ and $\xi(Y) = 3$

$dom\ \rho = \{1, 2\}$

(define Increment y (x) (Y) (set (Y (+ Y 1))))

$e =$ APPLY(Incrementy, LIT(1), LIT(2))

$fv(e) = \{1\} \cup \{2\}$ thus $Y \notin fv(e)$

$$\frac{\xi(Y)=3,\ Y \in dom(\xi) \quad \phi(f) = Increment\,y(\langle x_1, x_2 \rangle, e)}{\begin{array}{c}\langle LIT(1), \xi, \phi, \rho \rangle \Downarrow \langle 1, \xi, \phi, \rho \rangle \\ \langle LIT(2), \xi, \phi, \rho \rangle \Downarrow \langle 2, \xi, \phi, \rho \\ \vdots \\ \langle e, \xi, \phi, \{x_1 \mapsto 1, x_2 \mapsto 2\} \rangle \Downarrow \langle 4, \xi'\{Y \mapsto 4\}, \phi\, \rho \rangle\end{array}}$$

APPLYUSER

$$\langle APPLY(Incrementy, LIT(1), LIT(2)), \xi, \phi, \rho \rangle \Downarrow \langle 4, \xi', \phi, \rho \rangle$$

$\boxed{\text{Counter example}}$

$\xi'(Y) = 4$ but $\xi(Y) = 3$

thus $\xi(Y) \neq \xi'(Y)$

even though

$Y \notin fv(e) = \{1, 2\}$

b. Formal Assign: D_r is my inductive hypothesis:

$$D = \frac{x \in dom\ \rho \quad Y \in dom\ \xi \quad \dfrac{Y \notin fv(e) \quad D_r}{\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}}{\langle set(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho'\{x \mapsto v\} \rangle}$$

by induction on D_r $\xi(Y) = \xi'(Y)$

Base Case:

$$\frac{Y \notin dom\ \rho \quad Y \in dom\ \xi}{\langle var(Y), \xi, \phi, \rho \rangle \Downarrow \langle \xi(Y), \xi, \phi, \rho \rangle}$$

The environment ξ does not change

Global Assign: D_r is my inductive hypothesis:

$$D = \frac{x \notin dom\ \rho \quad x, Y \in dom\ \xi \quad \dfrac{Y \notin fv(e) \quad D_r}{\langle e, \xi, \phi, \rho \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle}}{\langle set(x, e), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi'\{x \mapsto v\}, \phi, \rho \rangle}$$

Base Case:

$$\frac{Y \in dom\ \rho}{\langle var(Y), \xi, \phi, \rho \rangle \Downarrow \langle \rho(Y), \xi, \phi, \rho \rangle}$$

environment ξ does not change

by implication, the evaluation of e sets var x to map to v and Y is left unchanged thus: $\xi(Y) = \xi'(Y)$

$$\phi(f) = USER(\langle x_1, \ldots, x_n \rangle, e)$$

C.

$$\frac{\begin{array}{c}\langle e_1, \xi_0, \phi, \rho_0 \rangle \Downarrow \langle v_1, \xi_1, \phi, \rho_1 \rangle \\ \langle e_2, \xi_1, \phi, \rho_1 \rangle \Downarrow \langle v_2, \xi_2, \phi, \rho_2 \rangle \\ \langle e, \xi_2, \phi, \{x_1 \mapsto v_1, x_2 \mapsto v_2\} \rangle \Downarrow \langle v, \xi', \phi, \rho' \rangle\end{array}}{\langle APPLY(f, e_1, e_2), \xi_0, \phi, \rho_0 \rangle \Downarrow \langle v, \xi', \phi, \rho_n \rangle} \qquad \begin{array}{l} y \notin fv(e_1) \cup fv(e_2) \\ y \in dom\ \xi_n \\ y \in dom\ \xi' \\ \xi_2(y) \neq \xi'(y) \end{array}$$

The free variables in $fv(APPLY(f, e_1, e_2))$ appear only as the fv of e1 and fv of e2. Thus when evaluating $\{x_1 \to v_1\}^{(e1)}$ and $\{x_2 \to v_2\}^{(e2)}$, y is not a free var in e1 and e2. Yet the conjecture allows y to be a global variable when e calls function f, since f's global variables are not considered free in e. Thus e can have $y \in dom\ \xi_2$ and after evaluating f to conclusion $\langle v, \xi', \phi, \rho_n \rangle$, y may have changed in ξ'. Thus $\xi_2(y) \neq \xi'(y)$ in certain cases and the proof doesn't go through.

D. Global var y may not be included as parameters to a user defined function but evaluating the function body can still change the Global var y.