Theory. Pdf

(append (append xs ys) zs) = (append xs (append ys zs))

part 1. null case

given: xs = '()

(append (append xs ys) zs)

= {by assumption, xs = '()}

(append (append '() ys) zs)

= {append-nil law}

(append ys zs)

= {append-nil right to left}

(append '() (append ys zs))

= {substitute xs = '()}

(append xs (append ys zs))

part 2: cons x case

given: xs = (cons p ps)

(append (append xs ys) zs)

= {by assumption, xs = (cons p ps)}

(append (append (cons p ps) ys) zs)

= {append-cons law}

(append (cons p (append ps ys)) zs)

= {append-cons law}

(cons p (append (append ps ys) zs))

= {apply induction hypothesis}

(cons p (append ps (append ys zs)))

= {append-cons law right to left}

(append (cons p ps) (append ys zs))

= {substitute xs = (cons p ps)}

(append xs (append ys zs))

By induction: (append (append xs ys) zs) = (append xs (append ys zs))

A. (cdr (cons x xs)) == xs

(cons x xs)

$$\frac{\langle e, \rho, \sigma_0 \rangle \Downarrow \langle PRIMITIVE(cons), \sigma_1 \rangle \quad \langle e_1, \rho, \sigma_1 \rangle \Downarrow \langle v_1, \sigma_2 \rangle \; v_1 = x \quad \langle e_2, \rho, \sigma_2 \rangle \Downarrow \langle v_2, \sigma_3 \rangle \; v_2 = xs}{\langle APPLY(e, e_1, e_2), \rho, \sigma_0 \rangle \Downarrow \langle PAIR\langle l_1, l_2 \rangle, \sigma_3\{l_1 \to x, l_2 \to xs\}}$$

$$\Downarrow$$

(cdr)

$$\frac{\langle e, \rho, \sigma_3 \rangle \Downarrow \langle PRIMITIVE(cdr), \sigma_4 \rangle \quad \langle e_1, \rho, \sigma_4 \rangle \Downarrow \langle PAIR\langle l_1, l_2 \rangle, \sigma_5 \rangle}{\langle APPLY(e, e_1), \rho, \sigma_0 \rangle \Downarrow \langle \sigma_5(l_2), \sigma_5 \rangle}$$

$$\sigma_5(l_2) = xs$$

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b. Assume e_1 and e_2 are the same function. They take the same value stored in σ and increment that by 1 and returns it:

(define increment (x) $\rho(x) = 0$ for this example
(+ x 1))

$e_2 = e_1$

(Assign) $$\frac{x \in dom\ \rho \quad \rho(x) = \ell = 0 \quad \langle e, \rho, \sigma \rangle \Downarrow \langle 0+1, \sigma' \rangle}{\langle SET(x, increment), \rho, \sigma \rangle \Downarrow \langle v, \sigma'\{0 \mapsto 1\} \rangle}$$

(cons e_1 e_2)

$$\frac{\begin{array}{l} \langle e, \rho, \sigma_0 \rangle \Downarrow \langle PRIMITIVE(cons), \sigma_1 \rangle \\ \langle e_1, \rho, \sigma_1 \rangle \Downarrow \langle 1, \sigma_2 \rangle \quad \sigma_2(\rho(x)) = 1 \\ \langle e_2, \rho, \sigma_2 \rangle \Downarrow \langle 2, \sigma_3 \rangle \quad \sigma_3(\rho(x)) = 2 \end{array}}{\langle APPLY(e, e_1, e_2), \rho, \sigma_0 \rangle \Downarrow \langle PAIR\langle \ell_1, \ell_2 \rangle, \sigma_3\{0 \to 1, 1 \to 2\} \rangle}$$

Assume: $x \in dom\ \rho$
$\rho(x) = 0$
$\sigma_0(\rho(x)) = 0$

The location of $\rho(x)$ starts as val 0. As the cons function is evaluated the new locations σ_2 and σ_3 hold different values of $\rho(x)$

(cdr (cons e_1 e_2))

$$\frac{\begin{array}{l} \langle e, \rho, \sigma_3 \rangle \Downarrow \langle PRIMITIVE(cdr), \sigma_4 \rangle \\ \langle e_1, \rho, \sigma_4 \rangle \Downarrow \langle PAIR\langle \ell_1, \ell_2 \rangle, \sigma_5 \rangle \end{array}}{\langle APPLY(e, e_1), \rho, \sigma_3 \rangle \Downarrow \langle \sigma_5(\ell_2), \sigma_2 \rangle}$$

$\sigma_5(\ell_2) = 2$, but we saw from e_2 (Assign) that $\sigma'(\ell) = 1$. thus the left hand does not equal the right hand.