Space-Charge Induced Distortion of Short-Pulse Beams in a Vacuum Drift Space

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Summary

- **We investigate the impact of space charge on the dynamics of short-pulse beams in a vacuum drift space, focusing on variations in beam profiles, charge densities, and pulse widths. Our analysis includes electron sheet phase-space trajectories and the evolution of pulse profiles during drift.**

Multiple-Sheet Model

A one-dimensional (1D) drift space with distance d and electron injection energy V_0, with M sheets inside.

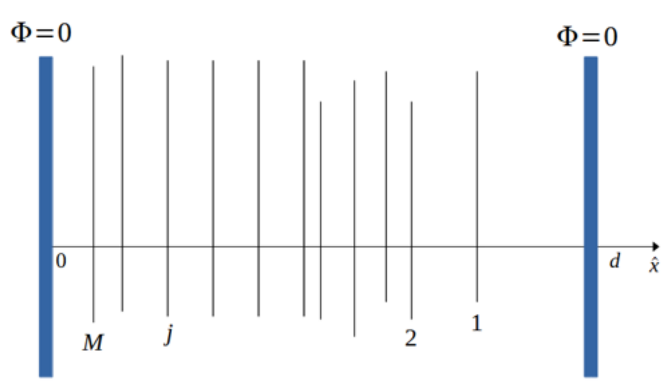

Figure 1: Sheet numbering inside the drift space

- Sheet j at position $\bar{x}_j = x_j/d$ has a normalized charge density $\bar{\rho}_j = \rho_j/\sigma_1$, where $\sigma_1 = -4\varepsilon_0 V_0/d$
- The Electric field on sheet j is

$$E_j = \frac{1}{\varepsilon_0}\left(\sum_{i=1}^{M}\rho_i\frac{x_i}{d} - \sum_{i=1}^{j-1}\rho_i + \frac{1}{2}\rho_j\right) \quad (1)$$

- The electron transit time is $T_0 = d/\left(\frac{2eV_0}{m_e}\right)^{1/2}$
- The normalized initial pulse length is $k = \frac{\tau_p}{T_0}$.
- The normalized beam charge density $f = \sum_j \bar{\rho}_j(1-\bar{x}_{j0})$.
- $\bar{x}_{j0}$ is the position of sheet j after the beam injection.

Model Parameters

Symbol	Meaning	Value
d	Drift space distance	1 cm
V_0	Electron injection energy	1 kV
M	Number of sheets	75
η	Electron q/m ratio	1.7588×10^{11}C/kg

Square-top Profile

- Sheets have **equal charge density** $\bar{\rho}$

$$\bar{\rho}_j = \frac{1}{M}\left[\frac{1}{1-\delta\bar{x}\left(\frac{M-1}{2}\right)}\right] \quad (2)$$

- We simulate 75 injected sheets inside the drift space. The initial pulse intervals $\delta\bar{x} = \bar{x}_n - \bar{x}_{n+1}$ are assumed to be uniform.

Trapezoidal Profile

- A **more general square-top model** that include a time of rise and a time of fall

$$\bar{\rho}_j = \begin{cases} \bar{\rho}_0 + (j-1)\frac{\bar{\rho}_1-\bar{\rho}_0}{n_r}, \ 1 \le j < n_r \\ \bar{\rho}_1, \ n_r \le j < (M-n_f)) \\ \bar{\rho}_0 + (M-j)\frac{\bar{\rho}_1-\bar{\rho}_0}{n_f}, \ (M-n_f)) \le j \le M \end{cases} \quad (3)$$

- $\bar{\rho}_0$ and $\bar{\rho}_1$ are respectively the lowest and highest charge density.

Gaussian Profile

- Sheet j has a **charge density**

$$\bar{\rho}_j = a\exp\left[-\frac{(j-\mu)^2}{b}\right] \quad (3)$$

where $\mu = (M+1)/2$, a and b are defined by the maximum charge density and the pulse duration.

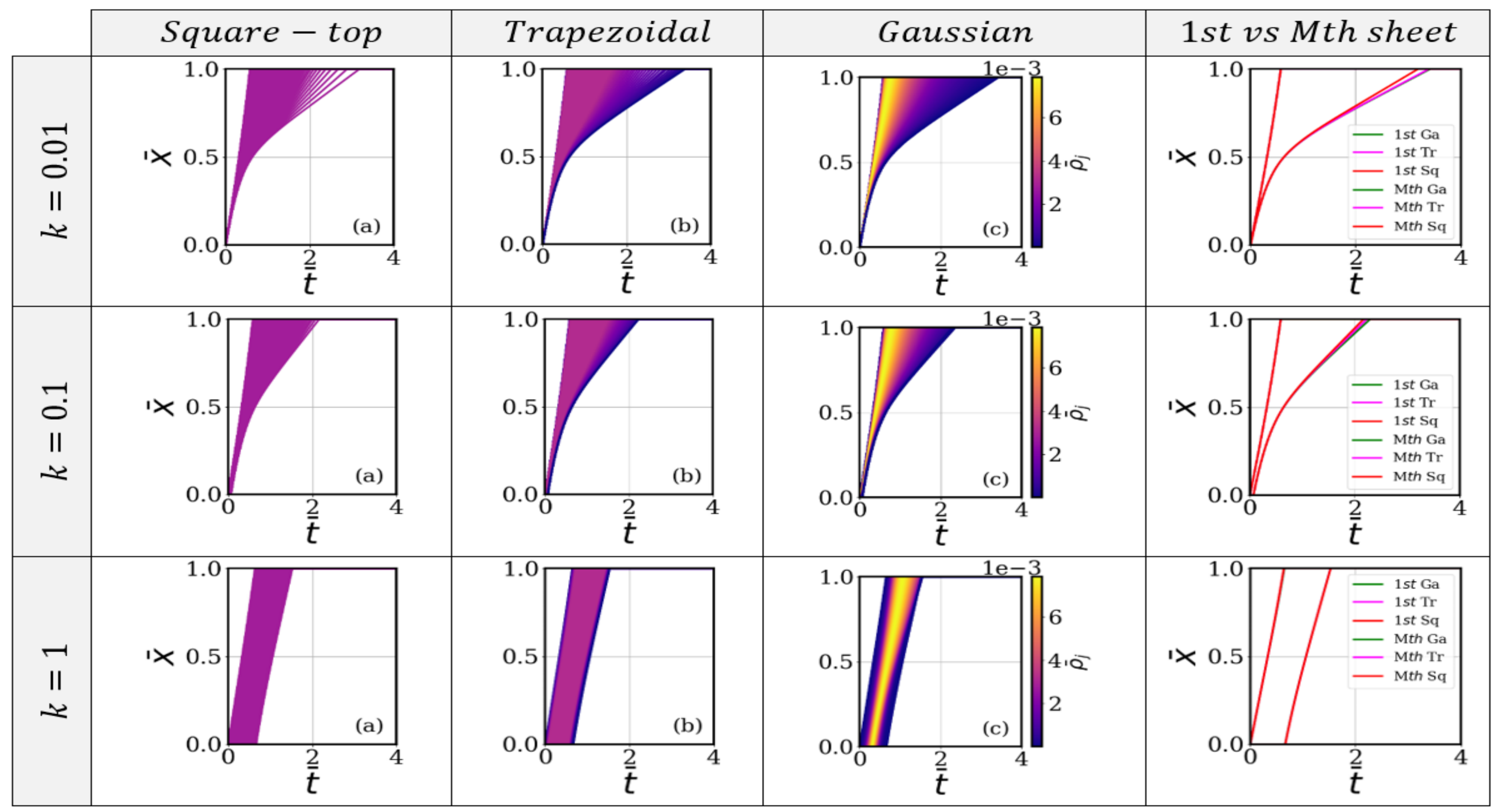

Figure 2: Electron sheet trajectories

Evolution of Pulse Profiles Inside the Gap

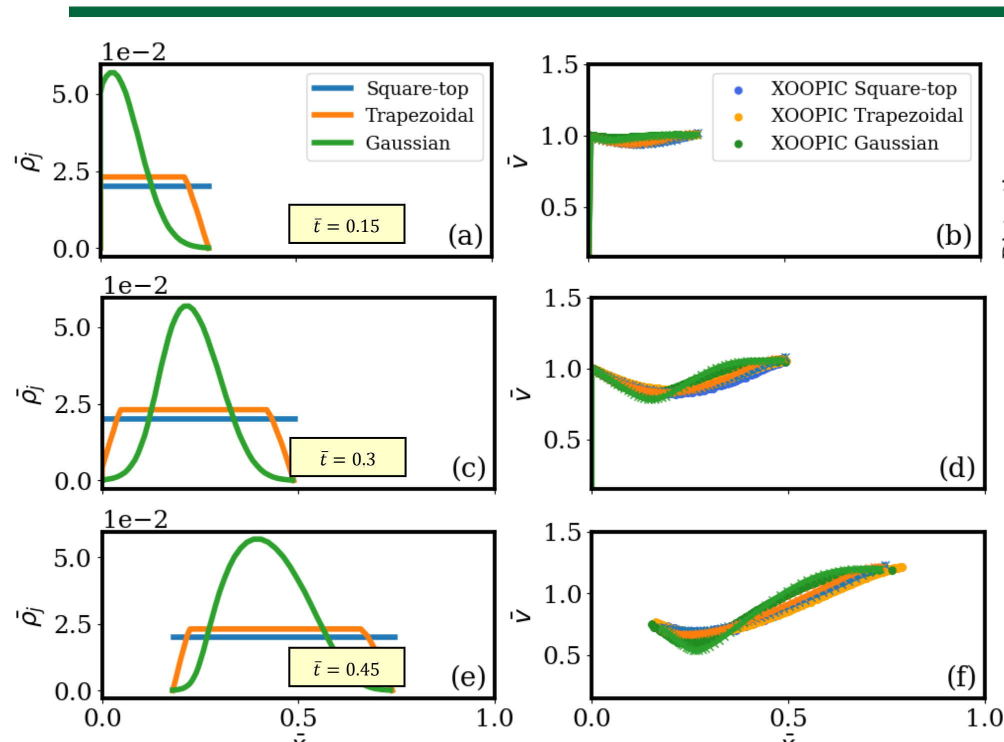

Figure 3: Evolution of pulse profile, and velocity inside the drift space and comparison to PIC simulations using XOOPIC.

Pulse profiles distortion and current limit

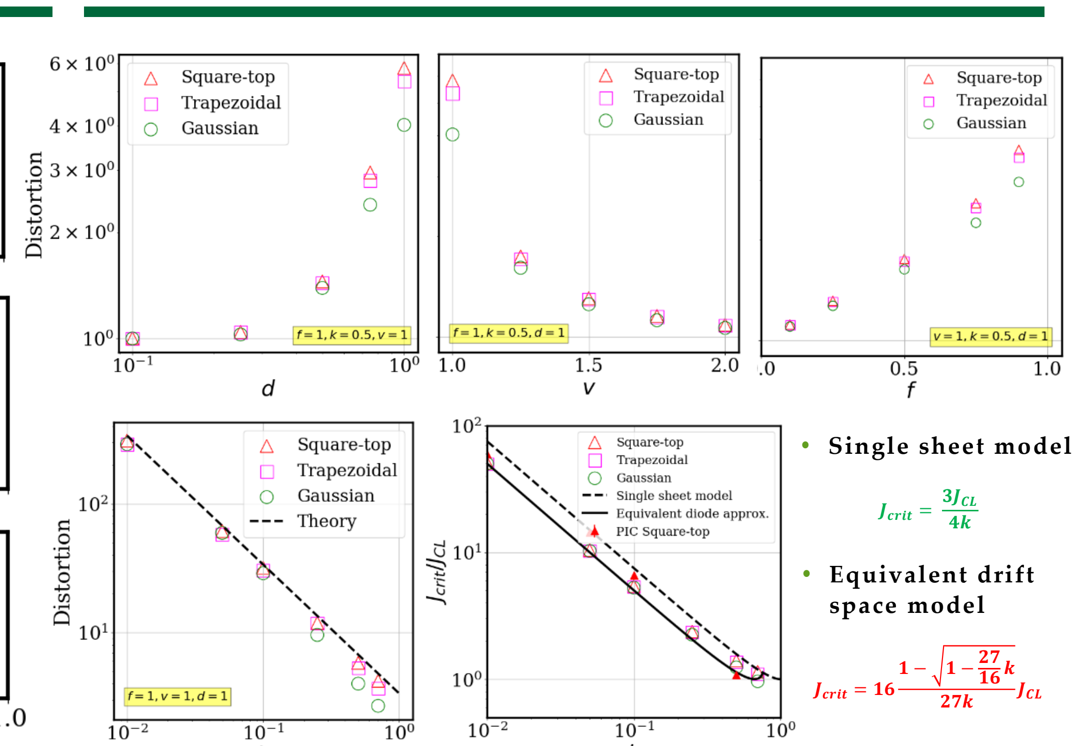

Figure 4: Beam distortion with comparison to PIC simulations using XOOPIC and the theoretical solution.

$$Distortion = \frac{\Delta t_{sp}}{\tau_p} \quad (4)$$

where Δt_{sp} is the temporal broadening due to space-charge effects, and τ_p is the initial pulse length.

Algorithm 1 Calculation of distortion

Input: $M, \delta\bar{x}$

1: $\delta\bar{x}_{init} \leftarrow (M-1)\delta\bar{x}$

2: $t \leftarrow 0$

3: **while** $\bar{x}_1(t) < 1$ **do**

4: $\quad t \leftarrow t+1$

5: **end while**

6: $\delta\bar{x}_{final} \leftarrow \bar{x}_1(t) - \bar{x}_M(t)$

7: $\Delta \leftarrow \delta\bar{x}_{final}/\delta\bar{x}_{final}$

8: **return** Δ

Conclusion

1 Figure 2 demonstrates that shorter pulse have higher distortion. The pulse shape influences the dynamics, for example the rear sheet of the Gaussian pulse is slower than the Trapezoidal or Square-top.

2 The initial profile and space charge strength are critical factors in determining short-pulse beam behavior in a drift space.

References & Acknowledgement

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