Space Charge Effects on the Evolution of Short Pulse Beam Profiles

Yves Heri, Peng Zhang*

Michigan State University, Electrical and Computer Engineering, East Lansing, MI, USA

*pz@egr.msu.edu

Summary

- We investigate **space charge effects** on the dynamics of short pulse beam profile.
- We consider **short pulses** of different profiles for different charge densities and pulse widths.
- We analyze the electron sheet **phase-space trajectories** and pulse profile evolution during gap transit.

Multiple-Sheet Model

A one-dimensional (1D) planar diode with gap distance d and gap voltage V_g, with M sheets inside.

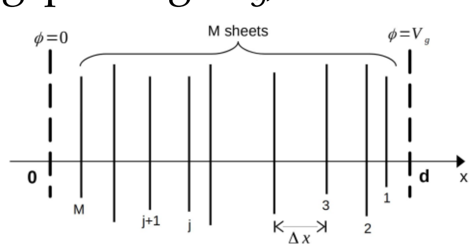


Figure 1: Sheet numbering inside the diode gap [1]

- Sheet j at position $\bar{x}_j = x_j/d$ has a normalized charge density $\bar{\rho}_j = \rho/\sigma_1$.
- The normalized electric field on sheet j is

$$\bar{E}_j = \frac{E_j}{E_0} = 1 + \left[\sum_{i=1}^{M} \bar{\rho}_i \bar{x}_i - \left(\sum_{i=1}^{j-1} \bar{\rho}_i + \frac{1}{2}\bar{\rho}_j\right)\right] \quad (1)$$

- The normalized Electric field at the cathode $(\bar{x} = 0)$

$$\bar{E}_K = 1 + \sum_{i=1}^{M} \bar{\rho}_i(\bar{x}_i - 1) \quad (2)$$

- The Space Charge Limited (SCL) charge density $\bar{\rho}_j^*$ is found for $\bar{E}_K = 0$

$$1 + \sum_{i=1}^{M} \bar{\rho}_i(\bar{x}_i - 1) = 0 \quad (3)$$

Model Parameters

Symbol	Meaning	Formula/Value
E_0	*Applied field*	$-V_g/d$
σ_1	*SCL density*	$\varepsilon_0 E_0$
τ_p	*Pulse length*	$[0.1, 1] \times T_0$
T_0	*Transit time*	$\sqrt{2d/(eE_0/m)}$
Δ	*Distortion*	$\delta\bar{x}_{final}/\delta\bar{x}_{init}$
J	*Current density*	$3\sum_{i=1}^{M} \bar{\rho}_i \bar{v}_i$

Square-top Profile

- All sheets have **equal charge density** $\bar{\rho}$
- The SCL charge density $\bar{\rho}^*$ is found from (3)

$$\bar{\rho}^* = \frac{1}{M}\left[\frac{1}{1 - \delta\bar{x}\left(\frac{M-1}{2}\right)}\right] \quad (4)$$

- We simulate 30 preloaded sheets inside the gap. The initial pulse intervals $\delta\bar{x} = \bar{x}_n - \bar{x}_{n+1}$ are assumed to be uniform.

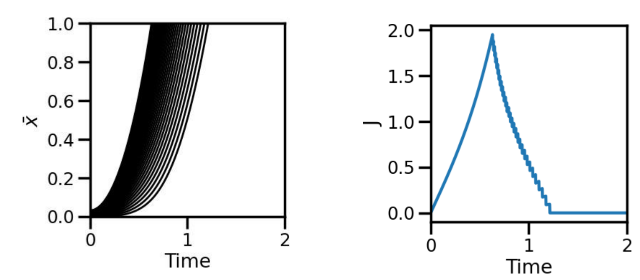


Figure 2: Sheets' trajectories & current density for T_0 and $\bar{\rho}^*$

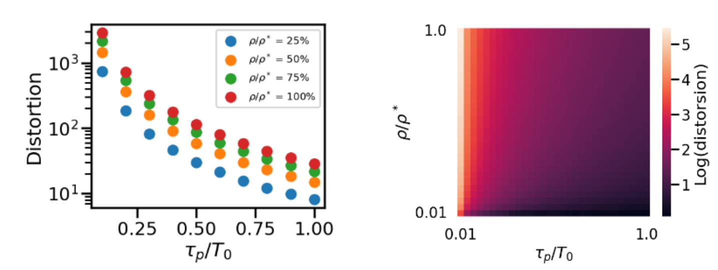


Figure 3: Square-top profile distortion with initial pulse length

Comparison of pulse profiles

- We compare pulses of different charges and shapes.

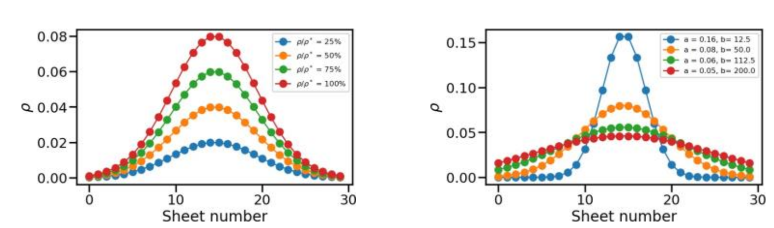


Figure 6: Comparison Case 1 and Case 2

- **Case 1**: We vary the total charge but keep the pulse shape unchanged.
- **Case 2**: We maintain the total charge but vary the shape

Gaussian Profile

- Sheet j has a **charge density**

$$\bar{\rho}_j = a \exp\left[-\frac{(j-\mu)^2}{b}\right] \quad (5)$$

where $\mu = (M+1)/2$, a and b are found by solving (3) with $\sum_{i=1}^{M} \bar{\rho}_i^* = 1$. (as the bulk of sheets tends to combine, looking like a single sheet).

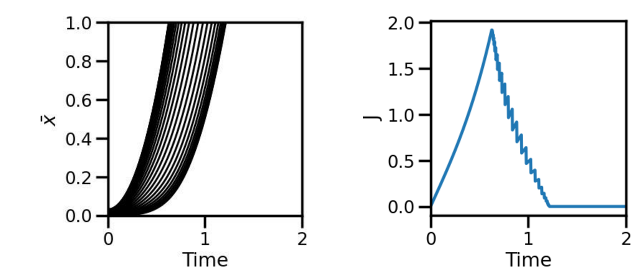


Figure 4: Sheets' trajectories & current density for T_0 and $\bar{\rho}^*$

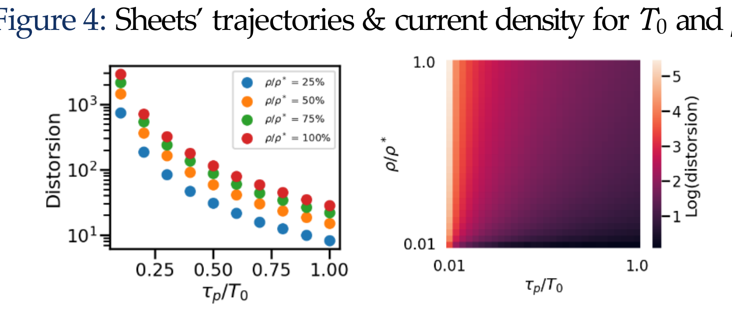
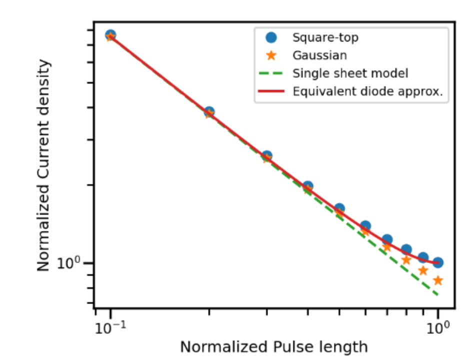


Figure 5: Gaussian profile distortion with initial pulse length

Child-Langmuir limit

- We simulate Gaussian pulses with $M = 30$ preloaded sheets. We fixed $\delta\bar{x} = 1/M^2 = 1/900$.



- **Single sheet model**

$$J_{crit} = \frac{3J_{CL}}{4X_{CL}}$$

- **Equivalent diode approximation**

$$J_{crit} = 2\frac{1 - \sqrt{1 - \frac{3}{4}X_{CL}^2}}{X_{CL}^3} J_{CL}$$

Figure 7: Normalized critical current density, J/J_{CL} as a function of the normalized pulse length.

Evolution of the Gaussian Pulse Profiles Inside the Gap

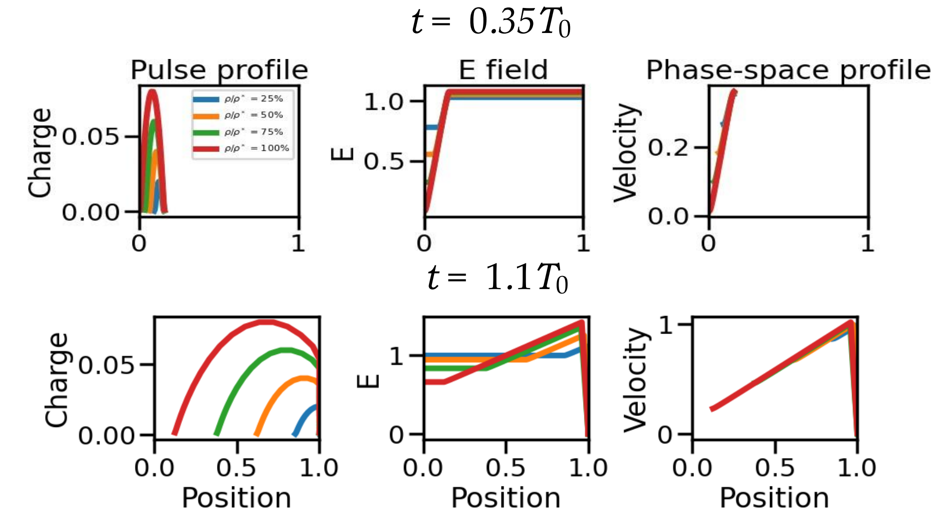


Figure 8: Evolution of pulse profile, electric field, and velocity for Case 1.

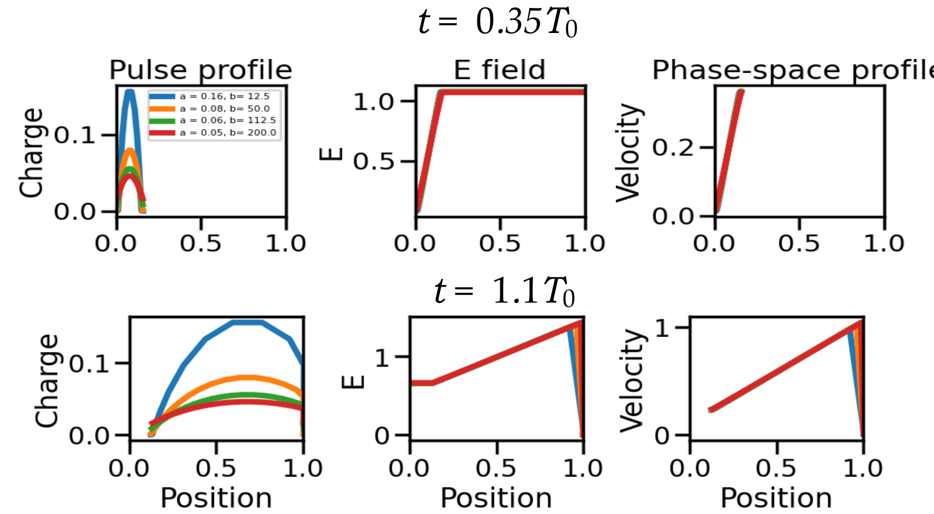


Figure 9: Evolution of pulse profile, electric field, and velocity for Case 2.

Algorithm 1 Calculation of distortion

Input: $M, \delta\bar{x}$
$\delta\bar{x}_{init} \leftarrow (M-1)\delta\bar{x}$
$t \leftarrow 0$
while $\bar{x}_1(t) < 1$ **do**
  $t \leftarrow t + 1$
end while
$\delta\bar{x}_{final} \leftarrow \bar{x}_1(t) - \bar{x}_M(t)$
$\Delta \leftarrow \delta\bar{x}_{final}/\delta\bar{x}_{final}$
return Δ

Conclusion & Future Work

1. For the same total charge, square-top and Gaussian pulses undergo similar distortion (fig. 3 and 5).
2. The shorter the pulse length, the more significant the distortion becomes.
3. The smaller the charge, the faster the tail of the pulse travels through the gap (fig. 8).
4. The Child-Langmuir limit increases as the pulse length decreases.

References & Acknowledgement

1 C. Birdsall and W. Bridges (1966). Electron dynamics of diode regions. New York: Academic Press

2 P. Zhang et *al.*, Applied Physics Reviews 4, 011304 (2017).

3 A. Valfells et *al.*, Physics of Plasmas 9(5), 2377-2382 (2002).

4 C. Kaur et *al.*, Physical Review E 106(5-2), 055203 (2002).

Acknowledgement: Work is supported by the Office of Naval Research (ONR) YIP Grant No. N00014-20-1-2681, the Air Force Office of Scientific Research (AFOSR) Grant No. FA9550-20-1-0409, and the Air Force Office of Scientific Research (AFOSR) Award No. FA9550-22-1-0523

Download the poster:
https://tinyurl.com/2yka3v4h

MICHIGAN STATE UNIVERSITY