Space Charge Effects on the Evolution of Short Pulse Beam Profiles

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Summary

- We investigate **space charge effects** on the dynamics of short pulse beam profile.
- We consider **short pulses** of different profiles for different charge densities and pulse widths.
- We analyze the electron sheet **phase-space trajectories** and pulse profile evolution during gap transit.

Multiple-Sheet Model

A one-dimensional (1D) planar diode with gap distance d and gap voltage V_g, with M sheets inside.

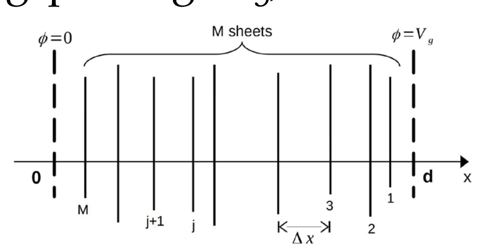

Figure 1: Sheet numbering inside the diode gap [1]

- Sheet j at position $\bar{x}_j = x_j/d$ has a normalized charge density $\bar{\rho}_j = \rho/\sigma_1$.
- The normalized electric field on sheet j is

$$\bar{E}_j = \frac{E_j}{E_0} = 1 + \left[\sum_{i=1}^{M} \bar{\rho}_i \bar{x}_i - \left(\sum_{i=1}^{j-1} \bar{\rho}_i + \frac{1}{2}\bar{\rho}_j\right)\right] \quad (1)$$

- The normalized Electric field at the cathode ($\bar{x} = 0$)

$$\bar{E}_K = 1 + \sum_{i=1}^{M} \bar{\rho}_i(\bar{x}_i - 1) \quad (2)$$

- The Space Charge Limited (SCL) charge density $\bar{\rho}_j^*$ is found for $\bar{E}_K = 0$

$$1 + \sum_{i=1}^{M} \bar{\rho}_i(\bar{x}_i - 1) = 0 \quad (3)$$

Model Parameters

Symbol	Meaning	Formula/Value
E_0	Applied field	$-V_g/d$
σ_1	SCL density	$\varepsilon_0 E_0$
τ_p	Pulse length	$[0.1, 1] \times T_0$
T_0	Transit time	$\sqrt{2d/(eE_0/m)}$
Δ	Distortion	$\delta\bar{x}_{final}/\delta\bar{x}_{init}$
J	Current density	$3\sum_{i=1}^{M} \bar{\rho}_i \bar{v}_i$

Square-top Profile

- All sheets have **equal charge density** $\bar{\rho}$
- The SCL charge density $\bar{\rho}^*$ is found from (3)

$$\bar{\rho}^* = \frac{1}{M}\left[\frac{1}{1 - \delta\bar{x}\left(\frac{M-1}{2}\right)}\right] \quad (4)$$

- We simulate 30 preloaded sheets inside the gap. The initial pulse intervals $\delta\bar{x} = \bar{x}_n - \bar{x}_{n+1}$ are assumed to be uniform.

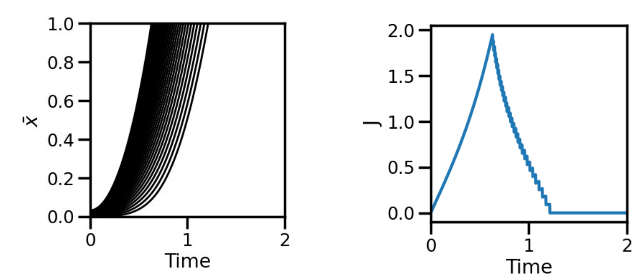

Figure 2: Sheets' trajectories & current density for T_0 and $\bar{\rho}^*$

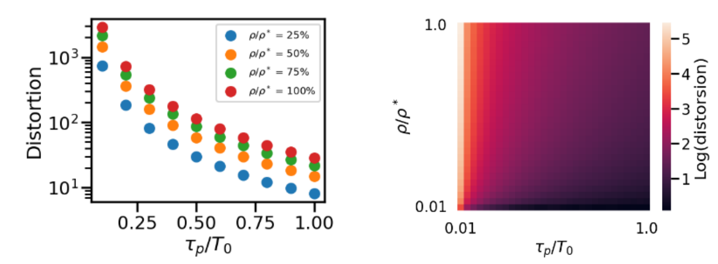

Figure 3: Square-top profile distortion with initial pulse length

Comparison of pulse profiles

- We compare pulses of different charges and shapes.

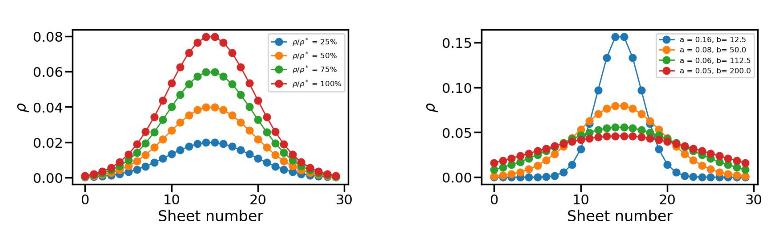

Figure 6: Comparison Case 1 and Case 2

- **Case 1**: We vary the total charge but keep the pulse shape unchanged.
- **Case 2**: We maintain the total charge but vary the shape

Gaussian Profile

- Sheet j has a **charge density**

$$\bar{\rho}_j = a \exp\left[-\frac{(j-\mu)^2}{b}\right] \quad (5)$$

where $\mu = (M+1)/2$, a and b are found by solving (3) with $\sum_{i=1}^{M} \bar{\rho}_i^* = 1$. (as the bulk of sheets tends to combine, looking like a single sheet).

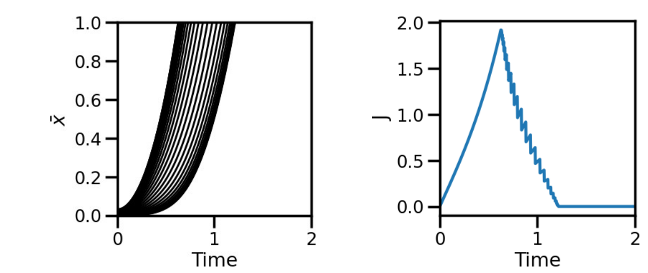

Figure 4: Sheets' trajectories & current density for T_0 and $\bar{\rho}^*$

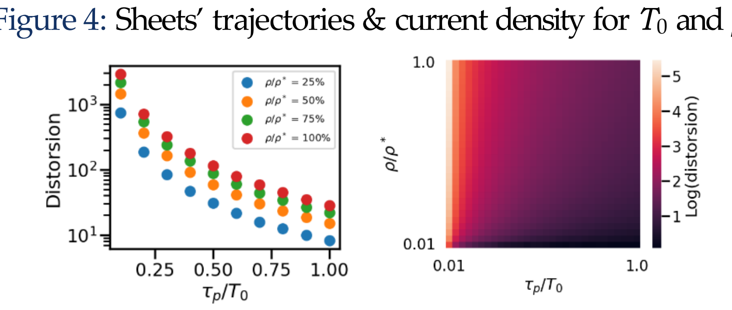

Figure 5: Gaussian profile distortion with initial pulse length

Child-Langmuir limit

- We simulate Gaussian pulses with $M = 30$ preloaded sheets. We fixed $\delta\bar{x} = 1/M^2 = 1/900$.

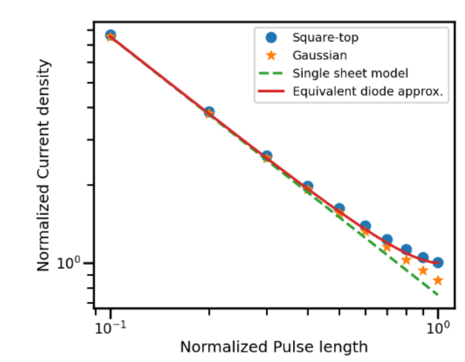

Figure 7: Normalized critical current density, J/J_{CL} as a function of the normalized pulse length.

- **Single sheet model**

$$J_{crit} = \frac{3J_{CL}}{4X_{CL}}$$

- **Equivalent diode approximation**

$$J_{crit} = 2\frac{1 - \sqrt{1 - \frac{3}{4}X_{CL}^2}}{X_{CL}^3} J_{CL}$$

Evolution of the Gaussian Pulse Profiles Inside the Gap

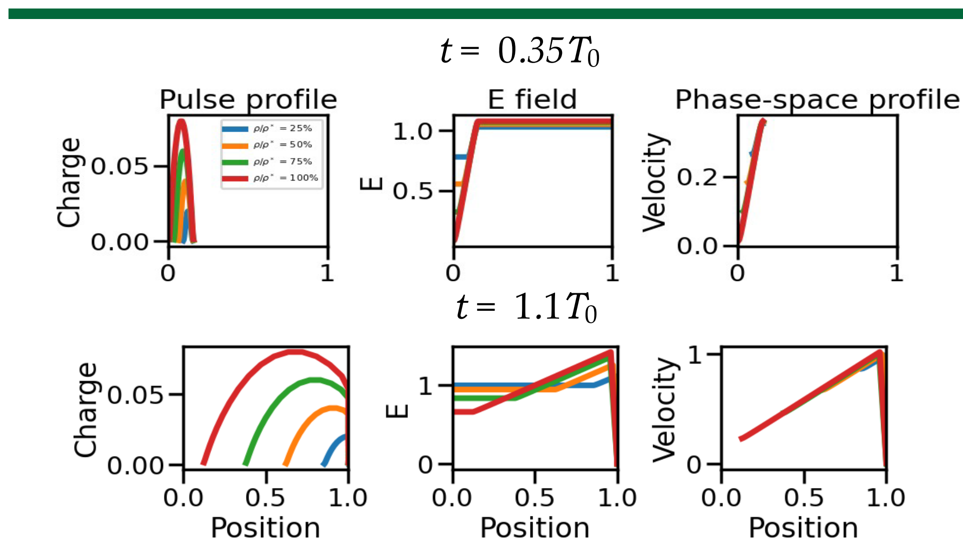

Figure 8: Evolution of pulse profile, electric field, and velocity for Case 1.

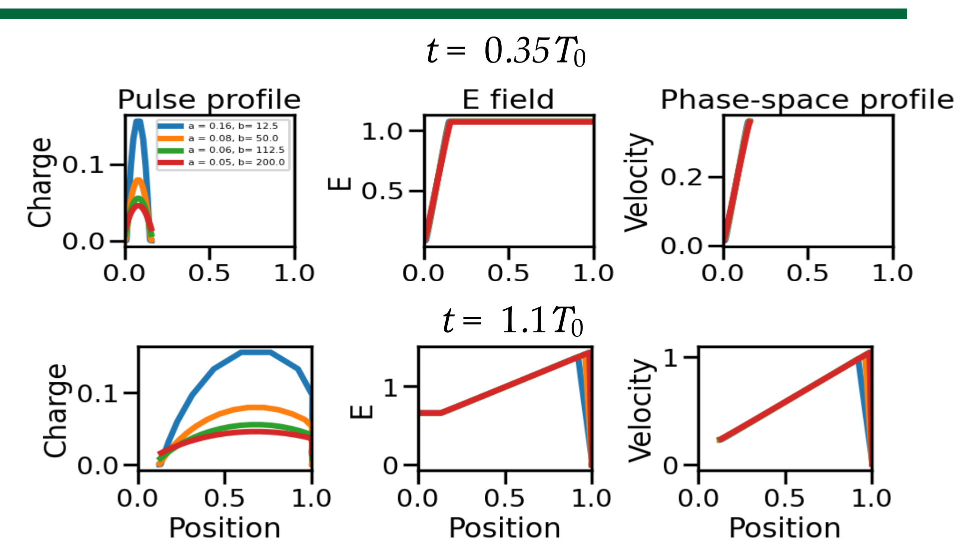

Figure 9: Evolution of pulse profile, electric field, and velocity for Case 2.

Algorithm 1 Calculation of distortion

```
    Input: M, δx̄
1:  δx̄_init ← (M − 1)δx̄
2:  t ← 0
3:  while x̄_1(t) < 1 do
4:      t ← t + 1
5:  end while
6:  δx̄_final ← x̄_1(t) − x̄_M(t)
7:  Δ ← δx̄_final/δx̄_final
8:  return Δ
```

Conclusion & Future Work

1. For the same total charge, square-top and Gaussian pulses undergo similar distortion (fig. 3 and 5).
2. The shorter the pulse length, the more significant the distortion becomes.
3. The smaller the charge, the faster the tail of the pulse travels through the gap (fig. 8).
4. The Child-Langmuir limit increases as the pulse length decreases.

References & Acknowledgement

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