Space Charge Limited Current Scaling for Short-Pulse Beam in a Vacuum Diode with Different Pulse Shapes

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Summary

- We study how **space charge** influences short pulse **beam dynamics**, examining different **profiles**, charge densities, and widths. We analyze electron sheet trajectories and pulse profile changes during gap transit.

Multiple-Sheet Model

A one-dimensional (1D) planar diode with gap distance d and gap voltage V_g, with M sheets inside.

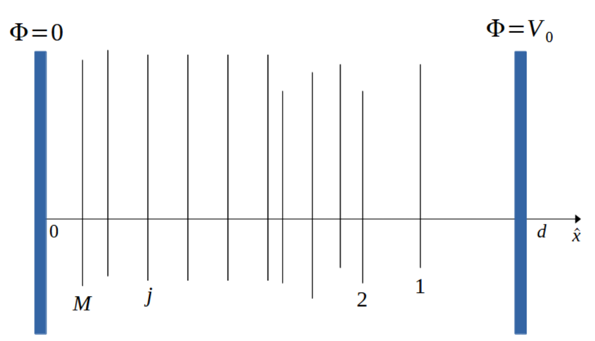

Figure 1: Sheet numbering inside the diode gap [1]

- Sheet j at position $\bar{x}_j = x_j/d$ has a normalized charge density $\bar{\rho}_j = \rho/\sigma_1$.
- The Electric field on sheet j is

$$E_j = E_0 + \frac{1}{\varepsilon_0}\left(\sum_{i=1}^{M} \rho_i \frac{x_i}{d} - \sum_{i=1}^{j-1} \rho_i + \frac{1}{2}\rho_j\right)$$

- The Electric field at the cathode ($x = 0$)

$$E_c = E_0 - \frac{1}{\varepsilon_0}\sum_{i=1}^{M} \rho_i \left(1 - \frac{x_i}{d}\right)$$

- The Space Charge Limited (SCL) charge density $\rho_{i,SCL}$ is found E

$$\sum_{i=1}^{M} \rho_{i,SCL}\left(1 - \frac{x_i}{d}\right) = \rho_{SCL}$$

Model Parameters

Symbol	Meaning	Value
d	Gap distance	1.5 mm
V_0	Applied voltage	30 kV
M	Number of sheets	40
η	Electron q/m ratio	1.7588×10^{11}C/kg

Square-top Profile

- Sheets have **equal charge density** $\bar{\rho}$
- The charge density $\bar{\rho}^*$ is found from (3)

$$\bar{\rho}^* = \frac{1}{M}\left[\frac{1}{1 - \delta\bar{x}\left(\frac{M-1}{2}\right)}\right] \quad (4)$$

- We simulate 40 preloaded sheets inside the gap. The initial pulse intervals $\delta\bar{x} = \bar{x}_n - \bar{x}_{n+1}$ are assumed to be uniform

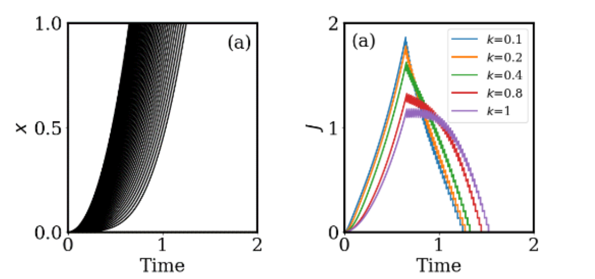

Figure 2: Trajectories & current density for T_0 and $\bar{\rho}^*$

Trapezoidal Profile

- A **more general square-top model** that include a time of rise and a time of fall

$$\rho_j = \begin{cases} \rho_0 + (j-1)\dfrac{\rho_1 - \rho_0}{n_r} & 1 \le j < n_r \\ \rho_1, & n_r \le j < (M - n_f) \\ \rho_0 + (M-j)\dfrac{\rho_1 - \rho_0}{n_f}, & (M - n_f) \le j \le M \end{cases}$$

- ρ_0 and ρ_1 are respectively the lowest and highest charge density.

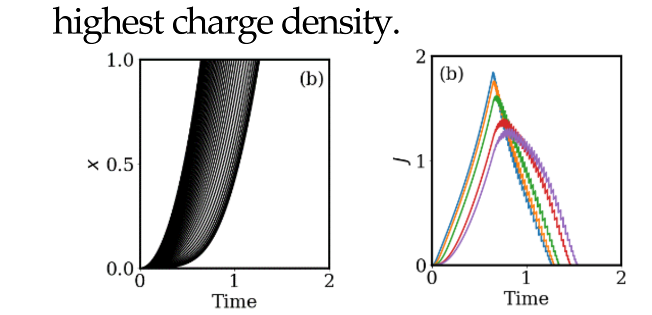

Figure 3: Trajectories & current density for T_0 and $\bar{\rho}^*$

Gaussian Profile

- Sheet j has a **charge density**

$$\bar{\rho}_j = a \exp\left[-\frac{(j-\mu)^2}{b}\right] \quad (5)$$

where $\mu = (M+1)/2$, a and b are found by solving (3) with $\sum_{i=1}^{M}\bar{\rho}_i^* = 1$. (as the bulk of sheets tends to combine, looking like a single sheet).

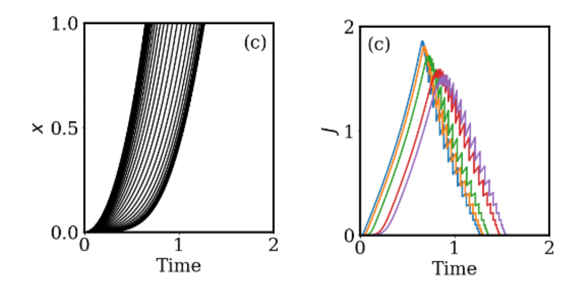

Figure 4: Trajectories & current density for T_0 and $\bar{\rho}^*$

Pulse profiles distortion

Distortion $\propto f/k$

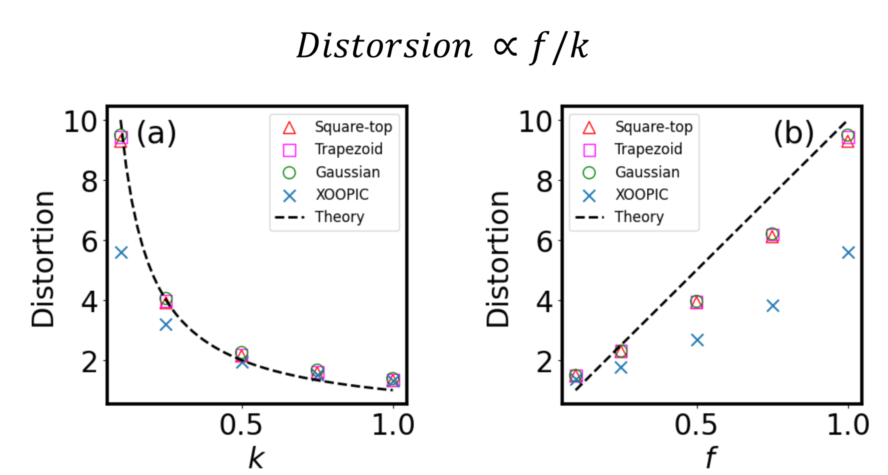

Figure 5: Beam distortion as a function of normalized pulse length for $f = 1$ (a) and normalized charge density for $k = 0.1$ (b), with comparison to PIC simulations using XOOPIC and the theoretical solution.

Child-Langmuir limit

- We simulate Gaussian pulses with $M = 30$ preloaded sheets. We fixed $\delta\bar{x} = 1/M^2 = 1/900$.

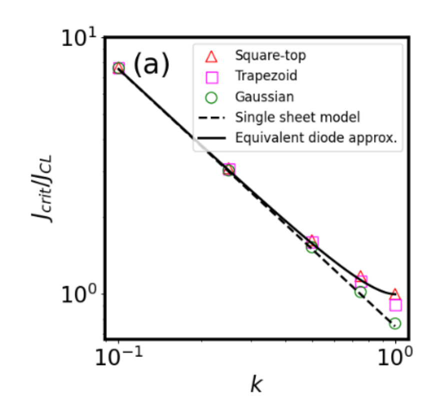

Figure 6: Normalized critical current density, J/J_{CL} as a function of the normalized pulse length.

- **Single sheet model**

$$J_{crit} = \frac{3J_{CL}}{4X_{CL}}$$

- **Equivalent diode approximation**

$$J_{crit} = 2\frac{1 - \sqrt{1 - \frac{3}{4}X_{CL}^2}}{X_{CL}^3} J_{CL}$$

Electron energy distribution at the anode

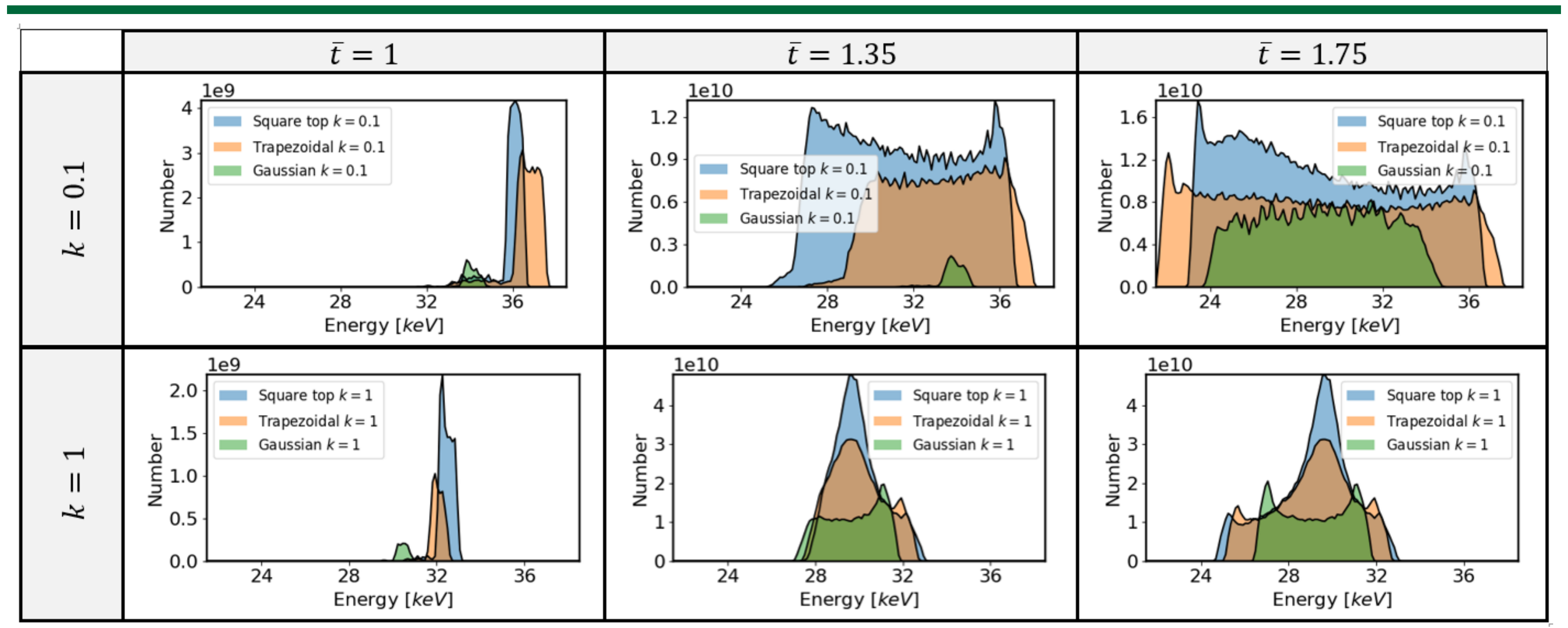

Figure 7: Electron energy distribution at the anode where $\bar{t} = t/T_{CL}$

Algorithm 1 Calculation of distortion

```
Input: M, δx̄
δx̄_init ← (M − 1)δx̄
t ← 0
while x̄_1(t) < 1 do
    t ← t + 1
end while
δx̄_final ← x̄_1(t) − x̄_M(t)
Δ ← δx̄_final/δx̄_final
return Δ
```

Conclusion & Future Work

1. For the same total charge, square-top and Gaussian pulses undergo similar distortion (fig. 5).
2. The shorter the pulse length, the more significant the distortion becomes.
3. The smaller the charge, the faster the tail of the pulse travels through the gap .
4. The Child-Langmuir limit increases as the pulse length decreases.

References & Acknowledgement

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