Space Charge Effects on the Evolution of Gaussian Short-Pulse Beam Profiles

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Summary

- We investigate **space charge effects** on the dynamics of Gaussian short pulse beam profile.
- We consider **short pulses** of different profiles for different charge densities and pulse widths.
- We analyze the electron sheet **phase-space trajectories** and pulse profile evolution during gap transit.

Multiple-Sheet Model

A one-dimensional (1D) planar diode with gap distance d and gap voltage V_g, with M sheets inside.

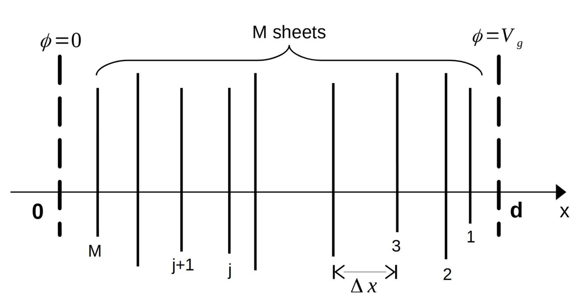

Figure 1: Sheet numbering inside the diode gap [1]

- Sheet j at position $\bar{x}_j = x_j/d$ has a normalized charge density $\bar{\rho}_j = \rho/\sigma_1$.
- The normalized electric field on sheet j is

$$\bar{E}_j = \frac{E_j}{E_0} = 1 + \left[\sum_{i=1}^{M} \bar{\rho}_i \bar{x}_i - \left(\sum_{i=1}^{j-1} \bar{\rho}_i + \frac{1}{2}\bar{\rho}_j\right)\right] \tag{1}$$

- The normalized Electric field at the cathode ($\bar{x} = 0$)

$$\bar{E}_K = 1 + \sum_{i=1}^{M} \bar{\rho}_i (\bar{x}_i - 1) \tag{2}$$

- The Space Charge Limited (SCL) charge density $\bar{\rho}_j^*$ is found for $\bar{E}_K = 0$

$$1 + \sum_{i=1}^{M} \bar{\rho}_j^* (\bar{x}_i - 1) = 0 \tag{3}$$

Model Parameters

Symbol	Meaning	Formula/Value
E_0	Applied field	$-V_g/d$
σ_1	SCL density	$\varepsilon_0 E_0$
τ_p	Pulse length	$[0.1, 1] \times T_0$
T_0	Transit time	$\sqrt{2d/(eE_0/m)}$
Δ	Distortion	$\delta\bar{x}_{final}/\delta\bar{x}_{init}$
J	Current density	$J = 3\sum_{i=1}^{M} \bar{\rho}_i \bar{v}_i$

Gaussian Pulse Profiles

- Sheet j has a **charge density**

$$\bar{\rho}_j = a \exp\left(-\frac{(j-\mu)^2}{b}\right) \tag{4}$$

where $\mu = (M+1)/2$, a and b are found by solving (3) with $\sum_{i=1}^{M} \bar{\rho}_i^* = 1$.
- We simulate for $M = 30$ preloaded sheets, $a = 1/12.5$, and $b = 50$.
- The initial pulse intervals $\delta\bar{x} = \bar{x}_n - \bar{x}_{n+1}$ are assumed to be uniform. $\delta x \approx (1/2)(eE_0/m)\tau_p^2$.

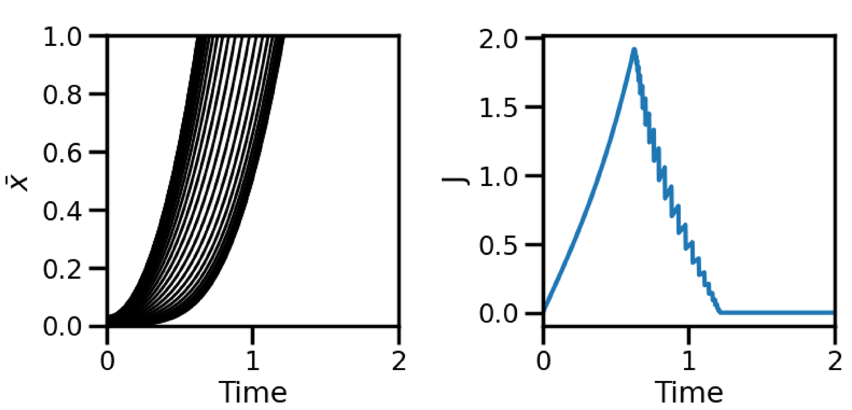

Figure 2: Sheets' trajectories & current density for T_0 and $\bar{\rho}^*$

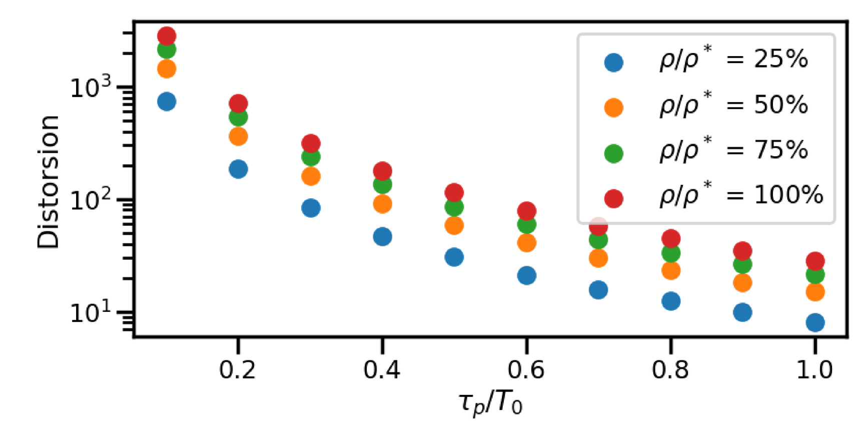

Figure 3: Gaussian profile distortion with initial pulse length

Comparison of pulse profiles

- We compare pulses of different charges and shapes.

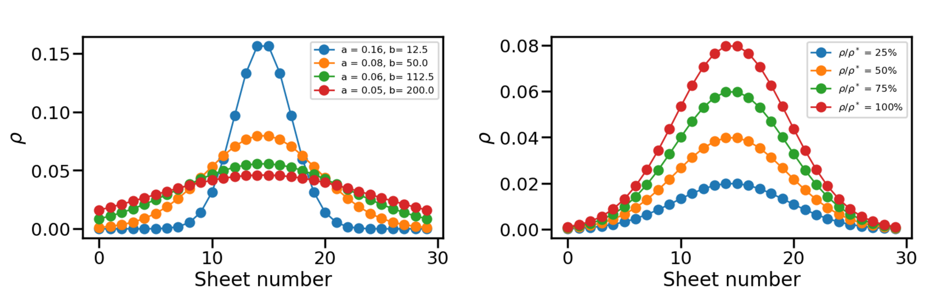

Figure 4: Comparison Case 1 and Case 2

Simulation details

- We simulate Gaussian pulses with $M = 30$ preloaded sheets. We fixed $\delta\bar{x} = 1/M^2 = 1/900$ and $\bar{x}_{30} = 0$.
- **Case 1**: We maintain the total charge but vary the shape of the pulse.
- **Case 2**: We vary the total charge but keep the pulse shape unchanged.

Evolution of the Gaussian Pulse Profiles Inside the Gap

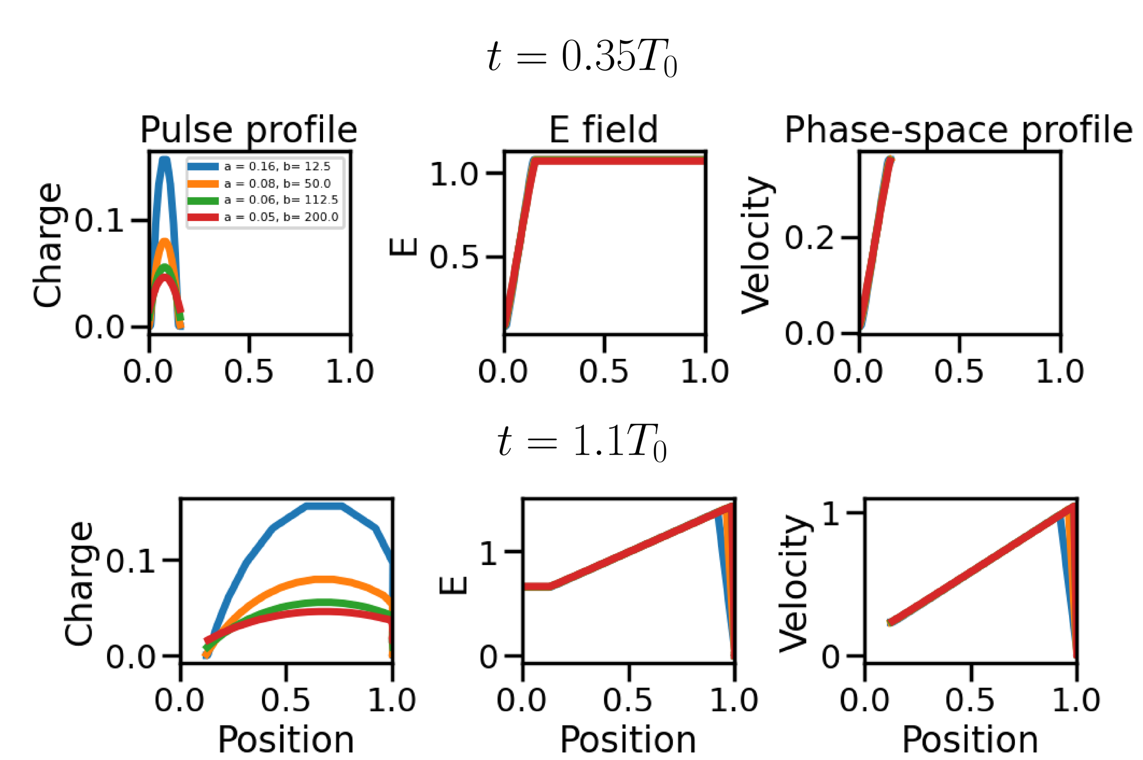

Figure 5: Evolution of pulse profile, electric field, and velocity for Case 1.

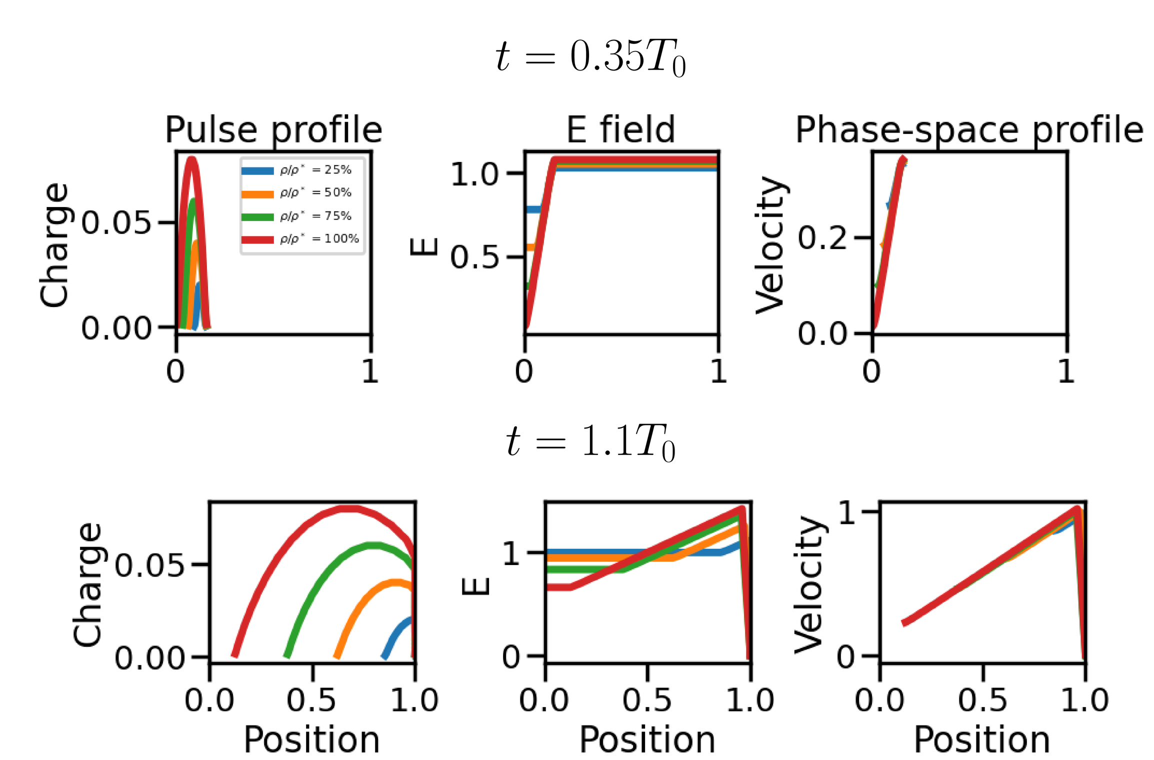

Figure 6: Evolution of pulse profile, electric field, and velocity for Case 2.

Algorithm 1 Calculation of distortion

Input: M, $\delta\bar{x}$

```
δx̄_init ← (M − 1)δx̄
t ← 0
while x̄_1(t) < 1 do
    t ← t + 1
end while
δx̄_final ← x̄_1(t) − x̄_M(t)
Δ ← δx̄_final/δx̄_init
return Δ
```

Conclusion & Future Work

1. For the same total charge all Gaussian pulses undergo similar distortion (fig. 5).
2. The shorter the pulse length, the more significant the distortion becomes.
3. The smaller the charge, the faster the tail of the pulse travels through the gap (fig. 6).
4. In future work, we will assess the Child-Langmuir limit as the pulse length decreases.

References & Acknowledgement

[1] C. Birdsall and W. Bridges (1966). Electron dynamics of diode regions. New York: Academic Press

[2] P. Zhang et *al.*, Applied Physics Reviews 4, 011304 (2017).

[3] A. Valfells et *al.*, Physics of Plasmas 9(5), 2377-2382 (2002).

[4] C. Kaur et *al.*, Physical Review E 106(5-2), 055203 (2002).

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