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$$\det(A-\lambda I)=0$$

$$\begin{vmatrix} 4-x & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{vmatrix} = 0$$

$$(4-x)\begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix} -$$

$$8\begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix} +$$

$$-1\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -13-\lambda \end{vmatrix} -$$

$$-2\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

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Suppose

$$\begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix} = M_\lambda$$

suppose $\lambda = x$

formula det of (M_λ)

$= aei + bfg + cdh - ceg - bdi - afh$

where

$a = -9-x$, $b = -2$, $c = -4$, $d = 10$

$e = 5-x$, $f = -10$, $g = -13$, $h = -14$

$i = -13-x$

$aei = (-9-x)(5-x)(-13-x)$

$(5-x)(-13-x) = -65 - 5x + 13x + x^2 = -65 + 8x + x^2$

$aei = \cancel{-8}(-9-x)(-65+8x+x^2)$

$= 585 - 72x - 9x^2 + 65x - 8x^2 - x^3$

$= \underline{\underline{585 - 7x - 17x^2 - x^3}}$

$bfg = (-2)(-10)(-13) = \underline{\underline{-260}}$

$cdh = (-4)(10)(-14) = \underline{\underline{560}}$

$ceg = (-4)(5-x)(-13)$

$= \underline{\underline{260 - 52x}}$

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$bdi = (-2)(10)(-13-x)$
$= \underline{\underline{260 + 20x}}$

$afh = (-9-x)(-10)(-14)$
$= \underline{\underline{-1260 - 140x}}$

using the determinat formular we get

$\det(M_\lambda) = (585 - 7x - 17x^2 - x^3) + (-260) + 560 - (260 - 52x) - (260 + 20x) - (-1260 - 140x)$

$= \underline{\underline{1625 + 165x - 17x^2 - x^3}}$

Suppose

$$M_2 = \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix} \text{ and } \lambda = x$$

using the formular we have used above let's identifie our labels (a, b, c, ...)

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$a = -2$, $b = -2$, $c = -4$, $d = 0$

$e = 5-x$, $f = -10$, $g = -1$, $h = -14$

$i = -13-10$

now let's compute formula's terms

$$aei = (-2)(5-x)(-13+x)$$
$$= (-2)(-65 - \underbrace{5x + 13x}_{8x} + x^2)$$
$$= \underline{\underline{130 - 16x - 2x^2}}$$

$$bfg = (-2)(-10)(-1) = \underline{\underline{-20}}$$
$$cbh = (-4)(0)(-14) = \underline{\underline{0}}$$

$$ceg = (-4)(5-x)(-1)$$
$$= \underline{\underline{20 - 4x}}$$

$$bdi = (-2)(0)(-13-x) = \underline{\underline{0}}$$
$$afh = (-2)(-10)(-14) = \underline{\underline{-280}}$$

using the determinant formula we get

$$\det(M_2) = (130 - 16x - 2x^2) + (-20) - (20 - 4x)$$
$$+ (-280)$$

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$\det(M_2) = \underline{\underline{370 - 12x - 2x^2}}$

Suppose

$$M_3 = \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix} \text{ and } \lambda = x$$

then the table for the formula will be

$a = -2,\ b = -9-x,\ c = -4,\ d = 0$

$e = 10,\ f = 5-x,\ g = -1$

$h = -13,\ i = -14$

then the terms will be

$aei = (-2)(10)(-14) = \underline{\underline{280}}$

$bfg = (-9-x)(5-x)(-1)$

$= (-45 + \underbrace{9x - 5x}_{4x} + x^2) - 1$

$= \underline{\underline{45 - 4x - x^2}}$

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$ceh = (-4)(-13)(0) = \underline{\underline{0}}$

$cdg = (-4)(10)(-1) = \underline{\underline{40}}$

$bdi = (-9-x)(0)(-14) = \underline{\underline{0}}$

$afh = (-2)(5-x)(-13)$

$= \underline{\underline{130-26x}}$

now let's combine all terms

$det(M_3) = (280) + (45-4x-x^2) - (40)$

$- (130-25x)$

$= \underline{\underline{155 + 22x - x^2}}$

Suppose

$$M_4 = \begin{vmatrix} -2 & -9-x & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix} \text{ and } \lambda = x$$

then labels will be

$a = (-2)$, $b = -9-x$, $c = -2$, $d = 0$

$e = 10$, $f = 5-x$, $g = -1$, $h = -13$

$i = -14$

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and terms will be

$aei = (-2)(10)(-14) = \underline{\underline{280}}$

$bfg = (-9-x)(5-x)(-1)$

$= (-45 + \underbrace{9x - 5x}_{4x} + x^2)(-1)$

$= \underline{\underline{45 - 4x - x^2}}$

$cdh = (-2)(0)(-13) = \underline{\underline{0}}$

$ceg = (-2)(10)(-1) = 20$

$bdi = (-9-x)(0)(-14) = 0$

$afh = (-2)(5-x)(-13)$

$= \underline{\underline{130 - 25x}}$

now $\det(M_4) = (280) + (45 - 4x - x^2) + (20)$
$- (130 - 25x)$

$= 175 + 22x - x^2$

finally $\det(A - \lambda I) = (4-x)(1625 + 165x - 17x^2 - x^3)$
$- 8(370 - 12x - 2x^2) - 1(155 + 22x - x^2)$
$+ 2(x^2 + 22x - x^2)$

$= x^4 + 13x^3 - 219x^2 - 835x + 3500.$

~~to find~~ now we have this equation

$$\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$$

because the leading coefficient is 1 the the possible rational roots are the factors of 3500

Since the factors 3500 are many we decided to use a library to compute this equation and we got

$\lambda_1 \simeq 2.6$

$\lambda_2 \simeq -5.6$

$\lambda_3 \simeq 11.05$

$\lambda_4 \simeq -21.12$

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Finding eigenvector for eigen value $\lambda \simeq -21.12$

formular $\vec{V}(A - \lambda I) = 0$

$\vec{V}(A - 21.12 I) = 0$

$$\cancel{A} - 21.12 I = \begin{pmatrix} 25.12 & 8 & -1 & -2 \\ -2 & 12.12 & -2 & -4 \\ 0 & 10 & 26.12 & -10 \\ -1 & -13 & -14 & 8.12 \end{pmatrix}$$

then

$$\begin{pmatrix} 25.12 & 8 & -1 & -2 \\ -2 & 12.12 & -2 & -4 \\ 0 & 10 & 26.12 & -10 \\ -1 & -13 & -14 & 8.12 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

suppose $V_4 = 1$

from row (3)

$10 V_2 + 26.12 V_3 - 10 V_4 = 0$

$10 V_2 = -26.12 V_3 + 10$

$V_2 = -2.612 V_3 + 1$

from Row 1

$$25.12x_1 + 8(-2.612x_3 + 1) - x_3 - 2 = 0$$

$$25.12x_1 - 20.896x_3 + 8 - x_3 - 2 = 0$$

$$25.12x_1 - 21.896x_3 + 6 = 0$$

$$x_1 = \frac{21.896x_3 - 6}{25.12}$$

$$= 0.87x_3 - 0.2388$$

from Row 3

$$-(0.87x_3 - 0.2388) - 13(-2.612x_3 + 1) - 14x_3 + 8.12 = 0$$

$$\Rightarrow -0.87x_3 + 0.2388 - 33.956x_3 - 13 - 14x_3 + 8.12 = 0$$

$$\Rightarrow \cancel{-48.826} \; x_3 = -20.7612$$

$$\cancel{= -95.5}$$

$$-48.826 \quad x_3 = \frac{-20.7612}{-48.826}$$

$$x_3 \approx 0.42$$

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then

$$v_1 = 0.87\, v_3 - 0.2388$$
$$= 0.87\,(0.242) - 0.2388$$
$$= -0.028$$

$$v_2 = -2.612\, v_3 + 1$$
$$= -2.612\,(0.242) + 1$$
$$= 0.3678$$

So

eigen vector for eigenvalue $\lambda = 21.12$

is $\vec{V}_2 = \begin{pmatrix} -0.028 \\ 0.3678 \\ 0.242 \\ 1 \end{pmatrix}$