

Date: / /

$$\det(A - \lambda I) = 0$$

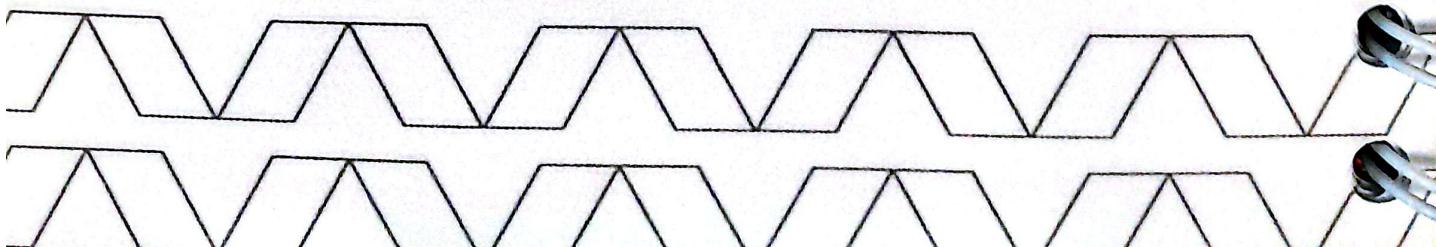
$$\left| \begin{array}{cccc} 4-x & x & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{array} \right| = 0$$

$$\begin{array}{c|ccc} (4-x) & -9-\lambda & -2 & -4 \\ \hline & 10 & 5-\lambda & -10 \\ & -13 & -14 & -13-\lambda \end{array} \quad -$$

$$\begin{array}{c|ccc} 8 & -2 & -2 & -4 \\ \hline & 0 & 5-\lambda & -10 \\ & -1 & -14 & -13-\lambda \end{array} \quad +$$

$$\begin{array}{c|ccc} -1 & -2 & -9-\lambda & -2 \\ \hline & 0 & 10 & 5-\lambda \\ & -1 & -13 & -13-\lambda \end{array} \quad -$$

$$\begin{array}{c|ccc} -2 & -2 & -9-\lambda & -2 \\ \hline & 0 & 10 & 5-\lambda \\ & -1 & -13 & -14 \end{array}$$



Suppose

$$\begin{vmatrix} -9-x & -2 & -4 \\ 10 & 5-x & -10 \\ -13 & -14 & -13-x \end{vmatrix} = M_1$$

Suppose $x = x$

formula \det of (M_1)

$$= aei + bfg + cdh - cei - bdi - afh$$

where

$$a = -9-x, b = -2, c = -4, d = 10$$

$$e = 5-x, f = -10, g = -13, h = -14$$

$$i = -13-x$$

$$aei = (-9-x)(5-x)(-13-x)$$

$$(5-x)(-13-x) = -65 - 5x + 13x + x^2 = -65 + 8x + x^2$$

$$aci = -8(-9-x)(-65+8x+x^2)$$

$$= 585 - 72x - 9x^2 + 65x - 8x^2 - x^3$$

$$= \underline{\underline{585 - 7x - 17x^2 - x^3}}$$

$$bfg = (-2)(-10)(-13) = \underline{\underline{-260}}$$

$$cdh = (-4)(10)(-14) = \underline{\underline{560}}$$

$$ceg = (-4)(5-x)(-13) = \underline{\underline{260 - 52x}}$$

$$\text{bedi} = (-2)(10)(-13-x)$$

$$= \underline{\underline{260 + 20x}}$$

$$\text{aph} = (-9-x)(-10)(-14)$$

$$= \underline{\underline{-1260 - 140x}}$$

using the determinant formula we get

$$\det(M_1) = (585 - 7x - 17x^2 - x^3) + (-260)$$

$$+ 560 - (260 - 52x) \cdot$$

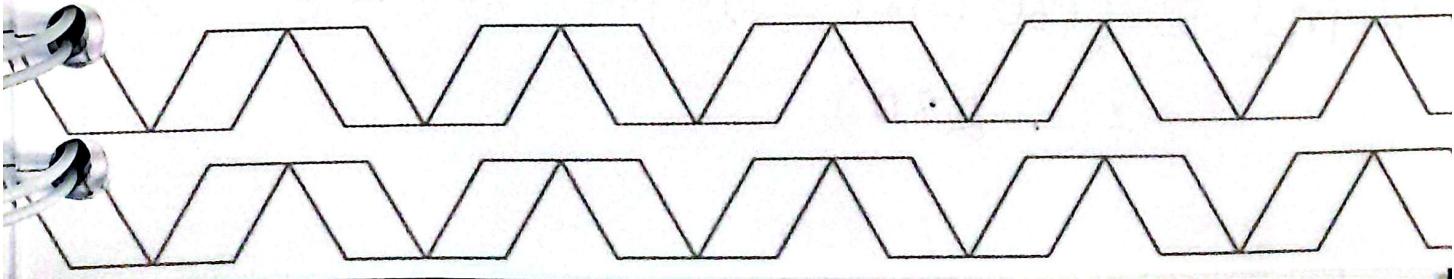
$$- (260 + 20x) - (-1260 - 140x)$$

$$= \underline{\underline{1625 + 165x - 17x^2 - x^3}}$$

Suppose

$$M_2 = \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix} \text{ and } \lambda = x$$

using the formula we have used above
let's identify our labels (a, b, c, ...)



$$a = -2, b = -2, c = -4, d = 0$$

$$e = 5-x, f = -10, g = -1, h = -14$$

$$i = -13 - 10$$

now let's compute determinant's terms

$$\begin{aligned}aei &= (-2)(5-x)(-13+x) \\&= (-2)(-65 - 5x + 13x + x^2) \\&= \underline{\underline{130 - 16x - 2x^2}}\end{aligned}$$

$$bef = (-2)(-10)(-1) = \underline{\underline{-20}}$$

$$cbh = (-4)(0)(-14) = \underline{\underline{0}}$$

$$\begin{aligned}ceg &= (-4)(5-x)(-1) \\&= \underline{\underline{20 - 4x}}\end{aligned}$$

$$bdj = (-2)(0)(-13-x) = \underline{\underline{0}}$$

$$ehf \cancel{= (-2)(-10)(-14)} = \underline{\underline{-280}}$$

using the determinant formula we get

$$\det(M_2) = (130 - 16x - 2x^2) + (-20) - (20 - 4x)$$

$$+ (-280)$$

$$\text{dif}(M_2) = \frac{370 - 12x - 2x^2}{\underline{\underline{}}}$$

Supposl

$$M_3 = \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix} \text{ and } \lambda = 2$$

then the labor for the formulator
will be

$$a = -2, b = -9-x, c = -4, d = 0$$

$$c = 10, f = r-x, g = z-1$$

$$R_2 = -13, \quad i = +4$$

then the terms will be

$$y_{\text{ei}} = (-2)(10)(-14) = \underline{\underline{280}}$$

$$b_{pq} = (-g-x)(r-x)(-1)$$

$$2 \left(-4x + \underbrace{3x - 5x + x^2}_{4x} \right) - 1$$

Digitized by srujanika@gmail.com

$$2 \quad \cancel{45} - 4x - x^2$$

$$\text{col}_h = (-4)(-15)(0) = \underline{\underline{0}}$$

$$\text{co}_g = (-4)(10)(-1) = \underline{\underline{40}}$$

$$\text{bot}_i = (-9-x)(0)(-14) = \underline{\underline{0}}$$

$$\alpha_{fh} = (-2)(5-x)(-13)$$

$$= \underline{\underline{130 - 26x}}$$

now let's combine all terms

$$\begin{aligned} \text{det}(M_3) &= (280) + (4r - 4x - x^2) - (40) \\ &\quad - (130 - 25x) \\ &= 155 + 22x - x^2 \end{aligned}$$

Suppose

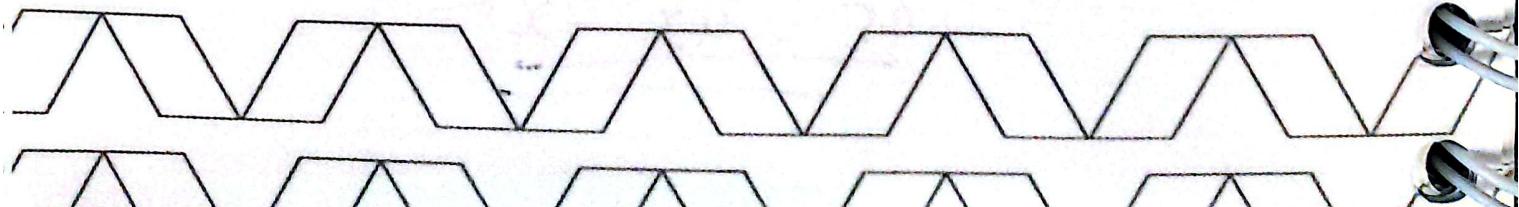
$$M_4 = \left| \begin{array}{ccc} -2 & -9-x & -2 \\ 0 & 10 & 5-x \\ -1 & -13 & -14 \end{array} \right| \text{ and } \lambda = x$$

then labels will be

$$a = (-2), b = -9-x, c = -2, d = \underline{\underline{0}}$$

$$e = 10, f = 5-x, g = -1, h = -13$$

$$i = -14$$



and terms will be

$$\text{axi} = (-2)(10)(-14) = 2 \underline{\underline{280}}$$

$$\text{bf}g = (-9-x)(5-x)(-1)$$

$$= (-45 + \cancel{9x} - \cancel{5x} + x^2) (-1)$$

~~9x~~

$$= \underline{\underline{45 - 4x - x^2}}$$

$$\text{cet} = (-2)(0)(-13) = \underline{\underline{0}}$$

$$\text{ceg} = (-2)(10)(-1) = 20$$

$$\text{bdi} = (-9-x)(0)(-14) = 0$$

$$\text{gfa} = (-2)(5-x)(-13)$$

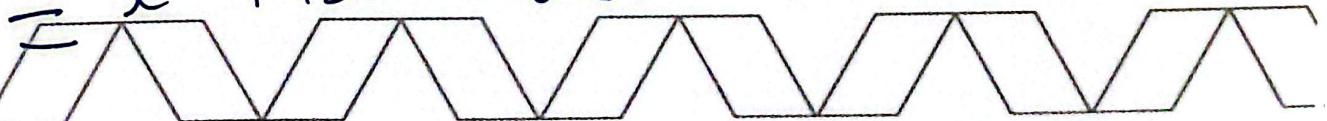
$$= \underline{\underline{130 - 25x}}$$

$$\text{now } \det(M_4) = (280) + (45 - 4x - x^2) - (20) \\ - (130 - 25x)$$

$$= 175 + 22x - x^2$$

$$\text{finally } \det(A - \lambda I) = (4-x)(1625 + 165x - x^2) \\ - 8(370 - 12x - 2x^2) - 1(155 + 22x - x^2) \\ + 2(x^2 + 22x - x^2)$$

$$= x^4 + 13x^3 - 219x^2 - 835x + 3500$$



so first now we have this equation

$$\lambda^4 + 13\lambda^3 - 215\lambda^2 - 835\lambda + 3500 = 0$$

because the leading coefficient is 1
 the possible rational roots
 are the factors of 3500

Since the factors 3500 are many
 we decided to use a library to
 compute this equation and we
 got

$$\lambda_1 \approx 2.6$$

$$\lambda_2 \approx -5.6$$

$$\lambda_3 \approx 11.05$$

$$\lambda_4 \approx -21.12$$

Date: / /

Finding eigenvector for eigen value

$$\lambda \approx -21.12$$

formular $\vec{v}^T (A - \lambda I) = 0$

$$\vec{v}^T (A - 21.12 I) = 0$$

$$A - 21.12 I \approx \begin{pmatrix} 25.12 & 8 & -1 & -2 \\ -2 & 12.12 & -2 & -4 \\ 0 & 10 & 26.12 & -10 \\ -1 & -13 & -14 & 8.12 \end{pmatrix}$$

then

$$\begin{pmatrix} 25.12 & 8 & -1 & -2 \\ -2 & 12.12 & -2 & -4 \\ 0 & 10 & 26.12 & -10 \\ -1 & -13 & -14 & 8.12 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Suppose } v_4 = 1$$

from row (3)

$$10v_2 + 26.12v_3 + 10v_4 = 0$$

$$10v_2 = -26.12v_3 + 10$$

$$v_2 = -2.612v_3 + 1$$

from Row 1

$$25.12x_1 + 8(-2.612x_3 + 1) - x_3 - 2 = 0$$

$$25.12x_1 - 20.896x_3 + 8 - x_3 - 2 = 0$$

$$25.12x_1 - 21.896x_3 + 6 = 0$$

$$x_1 = \frac{21.896x_3 - 6}{25.12}$$

$$= 0.87x_3 - 0.2388$$

From Row 3

$$-(0.87x_3 - 0.2388) - 13(-2.612x_3 + 1) - 14x_3 + 8.12 = 0$$

$$\Rightarrow -0.87x_3 + 0.2388 - 33.956x_3 - 13 - 14x_3 + 8.12 = 0$$

$$+ 8.12 = 0$$

$$\Rightarrow \cancel{-48.826} \cancel{x_3} = -20.7612$$

$$\therefore x_3 = \frac{-20.7612}{-48.826} = 0.4212$$

$$x_3 \approx 0.4212$$

then

$$\begin{aligned} \mathbf{v}_1 &= 0.87 \mathbf{v}_3 - 0.2388 \\ &= 0.87 (0.242) - 0.2388 \\ &= -0.028 \end{aligned}$$

$$\begin{aligned} \mathbf{v}_2 &= -2.612 \mathbf{v}_3 + 1 \\ &= -2.612 (0.242) + 1 \\ &= 0.3678 \end{aligned}$$

So

eigen vector for eigenvalue $\lambda = -21.12$

is $\vec{v}_2 = \begin{pmatrix} -0.028 \\ 0.3678 \\ 0.242 \\ 1 \end{pmatrix}$

