finding the eigen values yields

$\lambda_1 \simeq -21.125$, $\lambda_2 \simeq -5.604$, $\lambda_3 \simeq 2.675$
$\lambda_4 \simeq 11.054$

Part 2) finding the eigen vector the value $\lambda_3 = 2.675$

$$\Rightarrow (A - \lambda I)\vec{v} = 0$$

substituting $\lambda = 2.675$

$$\Rightarrow A - 2.675 I = \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.67 \end{bmatrix}$$

let $\vec{v} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$

· Setting up the Augmented Matrix

$$\left[\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right]$$

performing the Gaussian elimination

1) Divide Row 1 (R_1) by 1.325

$$\cancel{R_1 \div 1.325 \Rightarrow [\,1 \quad 6.0377 \quad -0.7547 \cdots}$$

$$R_1 \div 1.325 \Rightarrow \left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right]$$

2) Elimating below Row 1 (R_1):

- $R_2 + 2 \cdot R_1$
- $R_4 + R_1$

New matrix:

$$\left.\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 0.4004 & -3.5094 & -7.0189 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.4623 & -14.7547 & -17.1844 & 0 \end{array}\right.$$

3) Divide R_2 by 0.4004:

$$R_2 \div 0.4004 \Rightarrow \begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 1 & -8.766 & -17.533 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.4623 & -14.7547 & -17.444 & 0 \end{array}$$

Eliminating using new R_2

$R_3 - 10 \cdot R_2$

$R_4 + 6.9623 \cdot R_2$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5044 & 0 \\ 0 & 1 & -8.768 & -17.533 & 0 \\ 0 & 0 & 90.005 & 165.33 & 0 \\ 0 & 0 & 46.257 & 105.3 & 0 \end{array}\right)$$

Back substitution

from the row reduced System,

Row 3: $90.005z + 165.33w = 0 \Rightarrow z = \frac{-165.33}{90.005}w$

$\Rightarrow z \simeq 1.837w$

Row 2: $y - 8.768z - 17.533w = 0$

$\Rightarrow y = 8.768z + 17.533w$

Substituting $z = 1.837w$

$\Rightarrow y = 8.768(-1.837w) + 17.533w = 16.107w$

Row 1: $x + 6.0377y - 0.754z - 1.5094w = 0$

$\Rightarrow x = -6.0377y + 0.7547z + 1.5044w$

$\Rightarrow x = -6.0377(-16.107w) + 0.7547(-1.837w) + 1.5094w$

$x \simeq 97.23w - 1.387w + 1.504w$ ~~$\simeq 97.350w$~~

$x \simeq 97.352w$

General solution

let $w = 1$, then

$$\vec{V_3} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$$

Normalizing eigen vector $\vec{V_3}$

$$\|\vec{V_3}\| = \sqrt{(97.352)^2 + (-16.107)^2 + (-1.837)^2 + 1^2} = \sqrt{9741.1}$$

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$\sqrt{9741.1} \simeq 48.7$

Normalized eigen Vector $\vec{V_3}^{norm}$

$$\vec{V_3}^{norm} \simeq \frac{1}{48.7}\begin{pmatrix}97.352\\-16.107\\-1.837\\1\end{pmatrix} \simeq \begin{pmatrix}0.487\\-0.1683\\-0.019\\0.010\end{pmatrix}$$

In conclusion, the eigen Vector for the eigen value $\lambda_3 = 2.675$, ~~and~~ is $\begin{pmatrix}97.352\\-16.107\\-1.837\\1\end{pmatrix}$

$$\vec{V_3} = \begin{pmatrix}x\\y\\z\\w\end{pmatrix} = \begin{pmatrix}97.352\\-16.107\\-1.837\\1\end{pmatrix}$$

and the Normalized eigen vector is

$$\vec{V_3}^{norm} = \begin{pmatrix}0.487\\-0.163\\-0.019\\0.010\end{pmatrix}$$

$\lambda_3 = 2.675$ $\quad \vec{V_3} = \begin{pmatrix}97.352\\-16.107\\-1.837\\1\end{pmatrix}$ $\quad \vec{V_3}^{norm} = \begin{pmatrix}0.987\\-0.163\\-0.019\\0.010\end{pmatrix}$

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