Date: / /

$$\det(A-\lambda I)=0$$

$$\begin{vmatrix} 4-x & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{vmatrix} = 0$$

$$(4-x)\begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix} -$$

$$8\begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix} +$$

$$-1\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -\cancel{13}14-\lambda \end{vmatrix} -$$

$$-2\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

Date: / /

Suppose

$$\begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix} = M_\lambda$$

suppose $\lambda = x$

formula det of (M_λ)

$= aei + bfg + cdh - ceg - bdi - afh$

where

$a = -9-x$, $b=-2$, $c=-4$, $d=10$

$e = 5-x$, $f=-10$, $g=-13$, $h=-14$

$i = -13-x$

$aei = (-9-x)(5-x)(-13-x)$

$(5-x)(-13-x) = -65-5x+13x+x^2 = -65+8x+x^2$

$aei = \cancel{x}\,(-9-x)(-65+8x+x^2)$

$= 585 - 72x - 9x^2 + 65x - 8x^2 - x^3$

$= \underline{\underline{585 - 7x - 17x^2 - x^3}}$

$bfg = (-2)(-10)(-13) = \underline{\underline{-260}}$

$cdh = (-4)(10)(-14) = \underline{\underline{560}}$

$ceg = (-4)(5-x)(-13)$

$= \underline{\underline{260 - 52x}}$

Date: / /

$bdi = (-2)(10)(-13-x)$
$= \underline{\underline{260 + 20x}}$

$afh = (-9-x)(-10)(-14)$
$= \underline{\underline{-1260 - 140x}}$

using the determinat formular we get

$\det(M_\lambda) = \left(585 - 7x - 17x^2 - x^3\right) + (-260)$
$+ 560 - (260 - 52x).$
$- (260 + 20x) - (-1260 - 140x)$

$= \underline{\underline{1625 + 165x - 17x^2 - x^3}}$

Suppose

$$M_2 = \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix} \text{ and } \lambda = x$$

using the folmular we have used above
let's identifie our labels (a,b,c,...)

Date: / /

$a = -2$, $b = -2$, $c = -4$, $d = 0$

$e = 5 - x$, $f = -10$, $g = -1$, $h = -14$

$i = -13 - 10$

now let's compute formula's terms

$$aei = (-2)(5-x)(-13+x)$$
$$= (-2)(-65 - \underbrace{5x + 13x}_{8x} + x^2)$$
$$= 130 - 16x - 2x^2$$

$$bfg = (-2)(-10)(-1) = -20$$
$$cbh = (-4)(0)(-14) = 0$$

$$ceg = (-4)(5-x)(-1)$$
$$= 20 - 4x$$

$$bdi = (-2)(0)(-13-x) = 0$$
$$afh = (-2)(-10)(-14) = -280$$

using the determinant formula we get

$$\det(M_2) = (130 - 16x - 2x^2) + (-20) - (20 - 4x)$$
$$+ (-280)$$

Date: / /

$$\det(M_2) = \underline{\underline{370 - 12x - 2x^2}}$$

suppose

$$M_3 = \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix} \text{ and } \lambda = x$$

then the lables for the formular will be

$a = -2$, $b = -9-x$, $c = -4$, $d = 0$
$e = 10$, $f = 5-x$, $g = -1$
$h = -13$, $i = -14$

then the terms will be

$$aei = (-2)(10)(-14) = \underline{\underline{280}}$$

$$bfg = (-9-x)(5-x)(-1)$$

$$= (-45 + \underbrace{9x - 5x}_{4x} + x^2) -1$$

$$= \underline{\underline{45 - 4x - x^2}}$$

Date: / /

$ceh = (-4)(-13)(0) = \underline{\underline{0}}$

$ceg = (-4)(10)(-1) = \underline{\underline{40}}$

$bdi = (-9-x)(0)(-14) = \underline{\underline{0}}$

$afh = (-2)(5-x)(-13)$

$= \underline{\underline{130 - 26x}}$

now let's combine all terms

$$\det(M_3) = (280) + (45 - 4x - x^2) - (40) - (130 - 25x)$$

$$= \underline{\underline{155 + 22x - x^2}}$$

Suppose

$$M_4 = \begin{vmatrix} -2 & -9-x & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix} \text{ and } \lambda = x$$

then labels will be

$a = (-2)$, $b = -9-x$, $c = -2$, $d = 0$

$e = 10$, $f = 5-x$, $g = -1$, $h = -13$

$i = -14$

Date: / /

and terms will be

$aei = (-2)(10)(-14) = \underline{\underline{280}}$

$bfg = (-9-x)(5-x)(-1)$

$= (-45 + \underbrace{9x - 5x}_{4x} + x^2)(-1)$

$= \underline{\underline{45 - 4x - x^2}}$

$cdh = (-2)(0)(-13) = \underline{\underline{0}}$

$ceg = (-2)(10)(-1) = 20$

$bdi = (-9-x)(0)(-14) = 0$

$afh = (-2)(5-x)(-13)$

$= \underline{\underline{130 - 25x}}$

now $\det(M_4) = (280) + (45 - 4x - x^2) + (20)$

$- (130 - 25x)$

$= 175 + 22x - x^2$

finally $\det(A - \lambda I) = (4-x)(1625 + 165x - 17x^2 - x^3)$

$- 8(370 - 12x - 2x^2) - 1(155 + 22x - x^2)$

$+ 2(x^2 + 22x - x^2)$

$= x^4 + 13x^3 - 219x^2 - 835x + 3500.$

~~to find~~ now we have this equation

$$\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$$

because the leading coefficient is 1 the the possible rational roots are the factors of 3500

Since the factors 3500 are many we decided to use a library to compute this equasion and we got

$$\lambda_1 \simeq 2.6$$

$$\lambda_2 \simeq -5.6$$

$$\lambda_3 \simeq 11.05$$

$$\lambda_4 \simeq -21.12$$

Date: / /

Finding eigenvector for eigen value $\lambda \simeq -21.12$

formular $\vec{V}(A-\lambda I)=0$

$\vec{V}(A-21.12\, I)=0$

$$A-21.12 I \simeq \begin{pmatrix} 25.12 & 8 & -1 & -2 \\ -2 & 12.12 & -2 & -4 \\ 0 & 10 & 26.12 & -10 \\ -1 & -13 & -14 & 8.12 \end{pmatrix}$$

then

$$\begin{pmatrix} 25.12 & 8 & -1 & -2 \\ -2 & 12.12 & -2 & -4 \\ 0 & 10 & 26.12 & -10 \\ -1 & -13 & -14 & 8.12 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

suppose $V_4 = 1$

from row (3)

$10V_2 + 26.12V_3 - 10V_4 = 0$

$10V_2 = -26.12V_3 + 10$

$V_2 = -2.612V_3 + 1$

Date: / /

From Row 1

$$25.12x_1 + 8(-2.612x_3 + 1) - x_3 - 2 = 0$$

$$25.12x_1 - 20.896x_3 + 8 - x_3 - 2 = 0$$

$$25.12x_1 - 21.896x_3 + 6 = 0$$

$$x_1 = \frac{21.896x_3 - 6}{25.12}$$

$$= 0.87x_3 - 0.2388$$

From Row 4

$$-(0.87x_3 - 0.2388) - 13(-2.612x_3 + 1) - 14x_3 + 8.12 = 0$$

$$\Rightarrow -0.87x_3 + 0.2388 - 33.956x_3 - 13 - 14x_3 + 8.12 = 0$$

$$\Rightarrow \cancel{-48.826}\, x_3 = -20.7612$$

$\cancel{-85.5}$

-48.826

$$x_3 = \frac{-20.7612}{-\text{[illegible]}}$$

$$x_3 \approx 0.42$$

Date: / /

then

$v_1 = 0.87 v_3 - 0.2388$

$= 0.87(0.242) - 0.2388$

$= -0.028$

$v_2 = -2.612 v_3 + 1$

$= -2.612(0.242) + 1$

$= 0.3678$

So

eigen vector for eigenvalue $\lambda = 21.12$

is $\vec{v}_2 \begin{pmatrix} -0.028 \\ 0.3678 \\ 0.242 \\ 1 \end{pmatrix}$

No.

Date

$\lambda_1 \simeq 2.6 \quad \lambda_2 = -5.6 \quad \lambda_3 \simeq 11 \quad \lambda_4 \simeq -21.$[illegible]

$\vec{v}\,[A - \lambda I] = 0 \Rightarrow A - (-5.6)\, I = A + 5.6 I$

$$A + 5.6 I = \begin{pmatrix} 4+5.6 & 8 & -1 & -2 \\ -2 & -9+5.6 & -2 & -4 \\ 0 & 10 & 5+5.6 & -10 \\ -1 & -13 & -14 & -13+5.6 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 9.6 & 8 & -1 & -2 \\ -2 & -3.4 & -2 & -4 \\ 0 & 10 & 10.6 & -10 \\ -1 & -13 & -14 & -7.4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0.5634 \\ -0.6161 \\ 0.5495 \\ -0.0334 \end{pmatrix}$$

4

No.
Date :

PART 1

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

finding the eigen values and eigen vector of the matrix A

1) Eigenvalues :

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \begin{vmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{vmatrix} = 0$$

using cofactor expansion along the first row ;

* First term: $(4-\lambda)\begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix}$

* Second term; $+8\begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix}$

* third term: $-1(-1)\begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix}$

PAPERLINE

* fourth term

$$-2\begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

$\Rightarrow$ substituting and simplifying the minors

$\Rightarrow (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) - 8(-2\lambda^2 - 22\lambda + 370) - 1(\lambda^3 + 390) + 2(\lambda^2 + 22\lambda + 275)$

$\Rightarrow \lambda^4 + 73\lambda^3 - 214\lambda^2 - 835\lambda + 3500 = 0$

finding the eigen values yields

$\lambda_1 \simeq -21.125$, $\lambda_2 \simeq -5.604$, $\lambda_3 \simeq 2.675$

$\lambda_4 \simeq 11.054$

Part 2) finding the eigen vector the value $\lambda_3 = 2.675$

$\Rightarrow (A - \lambda I)\vec{V} = 0$

substituting $\lambda = 2.675$

$$\Rightarrow A - 2.675 I = \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.67 \end{bmatrix}$$

let $\vec{V} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$

Setting up the Augmented Matrix

$$\left[\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right]$$

performing the Gaussian elimination

1) Divide Row 1 (R_1) by 1.325

~~$R_1 \div 1.325 \Rightarrow [\,1 \quad 6.0377 \quad -0.7547 \ldots$~~

$$R_1 \div 1.325 \Rightarrow \left[\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right]$$

2) Eliminating below Row 1 (R_1):

- $R_2 + 2 \cdot R_1$
- $R_4 + R_1$

New matrix:

$$\left.\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 0.4004 & -3.5094 & -7.0189 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.4623 & -14.7547 & -17.1844 & 0 \end{array}\right.$$

3) Divide R_2 by 0.4004:

$$R_2 \div 0.4004 \Rightarrow \begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5094 & 0 \\ 0 & 1 & -8.766 & -17.533 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.4623 & -14.7547 & -17.444 & 0 \end{array}$$

PAPERLINE

Eliminating using new R_2

$R_3 - 10 \cdot R_2$

$R_4 + 6.9623 \cdot R_2$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 6.0377 & -0.7547 & -1.5044 & 0 \\ 0 & 1 & -8.768 & -17.533 & 0 \\ 0 & 0 & 90.005 & 165.33 & 0 \\ 0 & 0 & 46.257 & 105.3 & 0 \end{array}\right)$$

Back substitution

from the row reduced system,

Row 3: $90.005z + 165.33w = 0 \Rightarrow z = \frac{-165.33}{90.005}w$

$\Rightarrow z \simeq 1.837w$

Row 2: $y - 8.768z - 17.533w = 0$

$\Rightarrow y = 8.768z + 17.533w$

Substituting $z = 1.837w$

$\Rightarrow y = 8.768(-1.837w) + 17.533w = 16.107w$

Row 1: $x + 6.0377y - 0.754z - 1.5044w = 0$

$\Rightarrow x = -6.0377y + 0.7547z + 1.5044w$

$\Rightarrow x = -6.0377(-16.107w) + 0.7547(-1.837w) + 1.5094w$

$x \simeq 97.23w - 1.387w + 1.504w \cancel{\simeq 97.350w}$

$x \simeq 97.352w$

General solution

let $w = 1$, then

$$\vec{V_3} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$$

Normalizing eigen vector $\vec{V_3}$

$$\|\vec{V_3}\| = \sqrt{(97.352)^2 + (-16.107)^2 + (-1.837)^2 + 1^2} = \sqrt{9742.1}$$

PAPERLINE

$$\sqrt{9741.1} \simeq 48.7$$

Normalized eigen Vector $\vec{V_3}^{norm}$

$$\vec{V_3}^{norm} \simeq \frac{1}{48.7}\begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix} \simeq \begin{pmatrix} 0.487 \\ -0.1683 \\ -0.019 \\ 0.010 \end{pmatrix}$$

In conclusion, the eigen Vector for the eigen value $\lambda_3 = 2.675$, ~~and~~ is $\begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$

$$\vec{V_3} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix}$$

and the Normalized eigen vector is

$$\vec{V_3}^{norm} = \begin{pmatrix} 0.487 \\ -0.163 \\ -0.019 \\ 0.010 \end{pmatrix}$$

$$\lambda_3 = 2.675 \quad \vec{V_3} = \begin{pmatrix} 97.352 \\ -16.107 \\ -1.837 \\ 1 \end{pmatrix} \quad \vec{V_3}^{norm} = \begin{pmatrix} 0.987 \\ -0.163 \\ -0.019 \\ 0.010 \end{pmatrix}$$

For $\lambda_2 = 11.05$

formula = $(A - \lambda I)$

$$A - 11.05I \Rightarrow \begin{bmatrix} 4-11.05 & 8 & -2 & -2 \\ -2 & -9-11.05 & -2 & -4 \\ 0 & 10 & 5-11.05 & -10 \\ 1 & 13 & -14 & -13-11.05 \end{bmatrix}$$

$$= \begin{bmatrix} -7.05 & 8 & -1 & -2 \\ -2 & -20.05 & -2 & -4 \\ 0 & 10 & -6.05 & -10 \\ 1 & -13 & -14 & -24.05 \end{bmatrix}$$

Vector : $(A - \lambda I)X = 0$

$$= \begin{bmatrix} -7.05 & 8 & -1 & -2 \\ -2 & -20.05 & -2 & -4 \\ 0 & 10 & -6.05 & -10 \\ 1 & -13 & -14 & -24.05 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

let use equation 3.

$$10x_2 - 6.05x_3 - 10x_4 = 0 \Rightarrow x_2 = 0.605x_3 + x_4$$

let's replace x_2 in equation 1

$$-7.05x_1 + 8(0.605x_3 + x_4) - x_3 - 2x_4 = 0$$

$-7.05x_1 + 4.84x_3 + 8x_4 - x_3 - 2x_4 = 0$

$-7.05x_1 + 3.84x_3 + 6x_4 = 0$

$$x_1 = \frac{3.84x_3 + 6x_4}{7.05}$$

Pick $x_4 = 0.5$, $x_3 = -0.8$

$x_2 =$

$$x_1 = \frac{3.84(0.5) + 6(-0.8)}{7.05}$$

$x_1 = -0.408$

$x_2 = 0.6x_3 + x_4$

$x_2 = 0.6(0.5) + -0.8$

$x_2 = -0.49$

$$\vec{V}_2 = \begin{bmatrix} -0.408 \\ -0.49 \\ -0.8 \\ -0.5 \end{bmatrix}$$

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