Dynamic Feedback Linearizability of Ingenuity

A.A. 2025 - 2026

Luca Franzin
Andrea Gravili
Giuseppe D'Addario
Federico Tranzocchi
Sapienza University of Rome

Contents

Chapter 1

Introduction

The exploration of Mars has been significantly advanced with the success of the NASA Perseverance rover mission, whose primary objective is to search for signs of past microbial life and collect samples of Martian rock and soil. A key component of this mission is the *Ingenuity* helicopter (Figure 1.1), which demonstrates the potential of aerial vehicles for planetary exploration. Ingenuity's flights provide valuable data on the Martian atmosphere and, in general, environmental conditions that can be leveraged for future missions.

Figure 1.1: Ingenuity at Wright Brothers Field on 6 April 2021, its third day of deployment on Mars. Image source: [1].

1.1 The Ingenuity helicopter

Unlike the more common quadrotors, Ingenuity is a *co-axial* helicopter with two counter-rotating rotors stacked on a single mast. This design choice allows for a more compact structure that does not need a tail rotor to counteract torque. Instead of controlling motion through varying the speed of the two rotors, Ingenuity uses a mechanism called *swashplate* [2] (see Figure 1.2) to change the pitch of the rotor blades cyclically and collectively.

- *Cyclic pitch* changes the angle of the blades as they rotate, which tilts the thrust vector allowing to generate the forces and moments needed for horizontal motion (roll and pitch).

- *Collective pitch* adjusts the angle of all blades of a rotor simultaneously, controlling the total magnitude of the thrust for vertical motion.

This actuation method yields flight dynamics that are fundamentally different from those of multi-rotor drones.

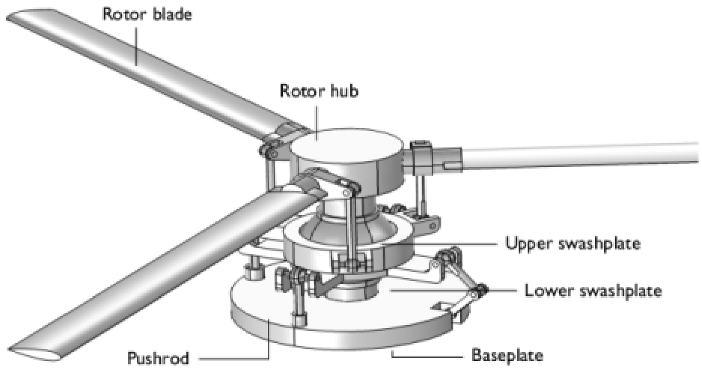

Figure 1.2: A helicopter swashplate mechanism. It translates non-rotating control inputs from the servos into the rotating frame of the rotor blades, controlling both collective and cyclic pitch. Image source: [3].

1.1.1 Control

Controlling Ingenuity is a challenging task due to two main reasons. First, the Martian atmosphere is much thinner than Earth's, with a density of about 1% that of sea level on Earth. This results in reduced aerodynamic forces and moments, making it difficult to generate sufficient lift and, in general, it alters the helicopter's flight dynamics. This requires the two rotors to spin at very high speeds (over 2500 RPM) to generate enough lift for takeoff and maneuvering, leading to complex aerodynamic effects [4].

Second, the helicopter's dynamics are nonlinear and coupled. In fact, the forces generated by the rotors depend on the vehicle's orintation, and the translational and rotational motions are linked. While linear controllers have been succesfully used for Ingenuity's flight control [2], their performance can be limited, especially during aggressive maneuvers or in the presence of disturbances such as wind gusts. In general, standard linear control techniques tend to struggle in handling these nonlinearities.

Dynamic feedback linearization

We adopt an approach from nonlinear control theory known as *dynamic feedback linearization* (DFL) [5] that aims to cancel out the coupled nonlinearities in the system dynamics through feedback, resulting in a decoupled linearized system that can be controlled using linear control methods. The main idea is to find a suitable change of coordinates and a nonlinear control law that cancels out the unwanted nonlinear terms in the dynamics.

The ”dynamic” aspect of DFL refers to an extension of the method that is necessary when the control inputs do not immediately affect the outputs, requiring the controller to have its own internal dynamics. As we will show, this is precisely the case for controlling the position of a co-axial helicopter like Ingenuity using the swashplate mechanism.

1.2 Objective

The main goal of this project is to formally analyze the dynamic feedback linearizability of the Ingenuity helicopter, design a state feedback controller for *TODO: add task*, and validate its performance through simulations. Specifically, we aim to:

1. Develop a dynamic model of Ingenuity based on Newton-Euler equations, capturing the key dynamics and couplings introduced by the co-axial rotor configuration;
2. analyze such dynamics to determine the conditions under which the system can be linearized using dynamic feedback;
3. design the nonlinear control law that achieves linearization and decoupling;
4. *TODO: implement a controller to carry out some task*;
5. evaluate the performance of our controller in simulation under various scenarios and for different maneuvers.

1.3 Structure

The report is organized as follows. In Chapter 2 we present the detailed dynamic model of the Ingenuity helicopter deriving the equations of motion. In Chapter 3 we provide a mathematical formulation of the dynamic feedback linearization technique and derive the specific control law for our system. In Chapter 4 we describe the simulation environment, the implementation details of the controller, and the test scenarios. In Chapter 5 we present the results of our experiments and evaluate the controller's performance. Finally, in Chapter 6 we summarize our findings and discuss potential future work.

Chapter 2

Ingenuity dynamic model

In this chapter, we develop the mathematical model of the Ingenuity helicopter using Newton-Euler equations. We start by defining the reference frames and the notation used throughout the report, the state and input vectors of the system, and derive the translational and rotational equations of motion governing the vehicle's flight. The dynamic model derived here will serve as the basis for the control system design analyzed in Chapter 3.

2.1 Coordinate frames

To describe the motion of Ingenuity, we define two main right-handed coordinate frames, as illustrated in Figure **??**:

- The *inertial frame* $\{I\}$, which is a fixed non-accelerating frame used as a global reference. Its origin is located at the takeoff point on the Martian surface, and its axes are denoted by (x_i, y_i, z_i). We adopt a z-up convention, with the z_i axis pointing vertically upwards in the opposite direction of gravity.

- The *body frame* $\{B\}$, which is a frame attached to the vehicle with its origin at the center of mass. Its axes (x_b, y_b, z_b) are aligned with the principal axes of Ingenuity, with x_b pointing forward, y_b pointing to the right, and z_b pointing up along the rotor mast.

The orientation of the body frame with respect to the inertial frame is described by a rotation matrix $\boldsymbol{R} \in SO(3)$ which transforms vectors from the body frame to the inertial frame. Such a matrix is paramterized using the ZYX Euler angles convention, defined by the yaw (ψ), pitch (θ), and roll (ϕ) angles:

$$\boldsymbol{R} = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \tag{2.1}$$

where $c_\alpha = \cos(\alpha)$ and $s_\alpha = \sin(\alpha)$, for any angle α.

2.2 State and input

The *state* of the system captures its complete dynamic condition at any given time. The *input* vector represents the control commands used to control the system.

State vector The state of the Ingenuity helicopter is described by a 12-dimensional vector $\boldsymbol{\xi} \in \mathbb{R}^{12}$, which includes its position, linear velocity, orientation, and angular velocity:

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{P} \\ \boldsymbol{V}^b \\ \boldsymbol{\Theta} \\ \boldsymbol{\omega} \end{pmatrix} \in \mathbb{R}^{12} \tag{2.2}$$

where:

- $\boldsymbol{P} = (x, y, z)^T \in \mathbb{R}^3$ is the position of the vehicle's center of mass in the inertial frame $\{I\}$;
- $\boldsymbol{V}^b = (V_x^b, V_y^b, V_z^b)^T = (u, v, w)^T \in \mathbb{R}^3$ is the linear velocity of the center of mass expressed in the body frame $\{B\}$;
- $\boldsymbol{\Theta} = (\phi, \theta, \psi)^T \in \mathbb{R}^3$ are the Euler angles representing the orientation of the body frame $\{B\}$ with respect to the inertial frame $\{I\}$;
- $\boldsymbol{\omega} = (p, q, r)^T \in \mathbb{R}^3$ is the angular velocity of the vehicle expressed in the body frame $\{B\}$.

Input vector Ingenuity is controlled via the net thrust produced by its two rotors and the net torques applied to the body. We define a 4-dimensional input vector $\boldsymbol{u} \in \mathbb{R}^4$ that abstracts the complex swashplate mechanism and rotor dynamics into the following components:

$$\boldsymbol{u} = \begin{pmatrix} F_T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} \in \mathbb{R}^4 \tag{2.3}$$

where:

- F_T is the magnitude of the total thrust force acting along the body z_b axis;
- τ_ϕ, τ_θ, and τ_ψ are the net control torques applied around the body axes x_b, y_b, z_b, respectively.

2.3 Equations of motion

The dynamics are separated into *kinematic* and *dynamic* equations: kinematics describe how the position and orientation evolve based on the velocities, while dynamics describe how the velocities change based on the applied forces and torques.

2.3.1 Kinematics

The kinematic equations describe the geometry of motion without considering the forces that cause it.

The rate of change of the inertial position is the linear velocity in the inertial frame, which is obtained by transforming the body-frame velocity using the rotation matrix in Eq. (2.1):

$$\dot{\boldsymbol{P}} = \boldsymbol{R}\boldsymbol{V}^b \tag{2.4}$$

The rate of change of the Euler angles is related to the body-frame angular velocity through the following transformation:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \dot{\boldsymbol{\Theta}} = \boldsymbol{W}(\boldsymbol{\Theta})\boldsymbol{\omega} = \begin{pmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{2.5}$$

where $t_\alpha = \tan(\alpha)$, for any angle α.

2.3.2 Translational dynamics

The translational dynamics are derived from *Newton's second law*, which we express in the body frame $\{B\}$:

$$m\dot{\boldsymbol{V}}^b = \boldsymbol{F}^b_{\text{tot}} - m\boldsymbol{\omega} \times \boldsymbol{V}^b \tag{2.6}$$

where m is the mass of the vehicle, and the term $m(\boldsymbol{\omega} \times \boldsymbol{V}^b)$ is the Coriolis force that arises from differentiating the velocity in a rotating frame. The total force $\boldsymbol{F}^b_{\text{tot}}$ in the body frame is th sum of the thrust force, gravity, and aerodynamic drag:

$$\boldsymbol{F}^b_{\text{tot}} = \boldsymbol{F}^b_{\text{thrust}} + \boldsymbol{F}^b_{\text{gravity}} + \boldsymbol{F}^b_{\text{drag}} \tag{2.7}$$

In detail:

- The *thrust force* is the force generated by the two rotors acting along the positive z_b axis of the body frame $\{B\}$:

$$\boldsymbol{F}^b_{\text{thrust}} = \begin{pmatrix} 0 \\ 0 \\ F_T \end{pmatrix} \tag{2.8}$$

- The *gravitational force* is the weight of the vehicle acting downwards in the inertial frame $\{I\}$ rotated into the body frame $\{B\}$ using the transpose of the rotation matrix in Eq. (2.1):

$$\boldsymbol{F}^b_{\text{gravity}} = \boldsymbol{R}^T \boldsymbol{F}^i_{\text{gravity}} = \boldsymbol{R}^T \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \tag{2.9}$$

- The *aerodynamic drag force* is modeled as a simple linear damping proportional to the body-frame velocity:

$$\boldsymbol{F}^b_{\text{drag}} = -\boldsymbol{A}_{\text{trans}} \boldsymbol{V}^b \tag{2.10}$$

  where $\boldsymbol{A}_{\text{trans}}$ is a diagonal matrix containing the translational drag coefficients along each body axis (see Chapter 4).

2.3.3 Rotational dynamics

The rotational dynamics are derived from *Euler's equation* for rigid body rotation, expressed in the body frame $\{B\}$:

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} = \boldsymbol{\tau}^b_{\text{tot}} - \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega} \tag{2.11}$$

where $\boldsymbol{I}$ is the inertia matrix, and the term $\boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega}$ represents the gyroscopic torques due to the rotating frame. The total torque $\boldsymbol{\tau}^b_{\text{tot}}$ in the body frame is the sum of the control torques and aerodynamic damping torques:

$$\boldsymbol{\tau}^b_{\text{tot}} = \boldsymbol{\tau}^b_{\text{control}} + \boldsymbol{\tau}^b_{\text{drag}} \tag{2.12}$$

In detail:

- The *control torques* are the torques generated by the swashplate mechanism around each body axis, and are given by the control inputs in Eq. (2.3):

$$\boldsymbol{\tau}^b_{\text{control}} = \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} \tag{2.13}$$

- The *aerodynamic damping torques* are modeled as linear damping opposing the angular velocity:

$$\boldsymbol{\tau}^b_{\text{drag}} = -\boldsymbol{A}_{\text{rot}}\boldsymbol{\omega} \tag{2.14}$$

  where $\boldsymbol{A}_{\text{rot}}$ is a diagonal matrix containing the rotational drag coefficients around each body axis (see Chapter 4).

2.4 Complete state-space model

Combining the kinematic and dynamic equations, we obtain the complete *nonlinear* state-space model of the Ingenuity helicopter in the form $\dot{\boldsymbol{\xi}} = \boldsymbol{f}(\boldsymbol{\xi}, \boldsymbol{u})$:

$$\dot{\boldsymbol{\xi}} = \begin{pmatrix} \dot{\boldsymbol{P}} \\ \dot{\boldsymbol{V}}^b \\ \dot{\boldsymbol{\Theta}} \\ \dot{\boldsymbol{\omega}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{R}\boldsymbol{V}^b \\ \frac{1}{m}\left(\boldsymbol{F}^b_{\text{thrust}} + \boldsymbol{F}^b_{\text{gravity}} + \boldsymbol{F}^b_{\text{drag}}\right) - \boldsymbol{\omega} \times \boldsymbol{V}^b \\ \boldsymbol{W}(\boldsymbol{\Theta})\boldsymbol{\omega} \\ \boldsymbol{I}^{-1}\left(\boldsymbol{\tau}^b_{\text{control}} + \boldsymbol{\tau}^b_{\text{drag}} - \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega}\right) \end{pmatrix} \tag{2.15}$$

We will be using this set of 12 coupled nonlinear differential equations as the basis for our analysis of dynamic feedback linearization in Chapter 3.

Chapter 3

Control design

3.1 Objective

3.2 Relative degree calculation

3.3 Dynamic compensator design

3.4 Linearizing control law

3.5 Linear tracking controller

Chapter 4

Simulation setup

4.1 Model parameters

4.2 Simulation scenarios

4.3 Performance metrics

Chapter 5

Results

5.1 TODO: task

5.2 Analysis

Chapter 6

Conclusion

6.1 Summary

6.2 Future work

Bibliography

[1] Wikipedia contributors, *Ingenuity (helicopter) — Wikipedia, the free encyclopedia*, [Online; accessed 25-Nov-2025], 2024. [Online]. Available: https://en.wikipedia.org/wiki/Ingenuity_(helicopter).

[2] H. F. Grip, B. Balaram, T. Canham, *et al.*, "Flight control system for nasa's mars helicopter," in *AIAA Scitech 2019 Forum*, 2019. DOI: 10.2514/6.2019-1289.

[3] COMSOL Multiphysics, *Helicopter swashplate*, COMSOL Application Gallery, [Online; version 5.6; accessed 25-Nov-2025], 2020. [Online]. Available: https://doc.comsol.com/5.6/doc/com.comsol.help.models.mbd.helicopter_swashplate/helicopter_swashplate.html.

[4] B. Balaram, T. Canham, C. Clark, *et al.*, "Mars helicopter technology demonstrator," in *2018 AIAA Atmospheric Flight Mechanics Conference*, 2018. DOI: 10.2514/6.2018-0023.

[5] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, N.J.: Prentice Hall, 1991.