

Elective in Robotics

Quadrotor control via dynamic feedback linearization

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simplified model for control design

negligible

- aerodynamics
- gyroscopic effects

assuming

- small φ and $\vartheta \Rightarrow (\dot{\varphi}, \dot{\vartheta}, \dot{\psi}) \simeq (p, q, r)$
- symmetric shape and negligible disturbances

$$\ddot{x} = -(\cos(\psi) \sin(\vartheta) \cos(\varphi) + \sin(\psi) \sin(\varphi)) \frac{T}{m}$$

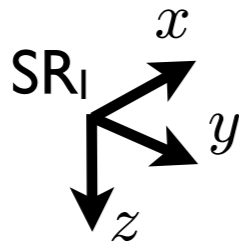
$$\ddot{y} = -(\sin(\psi) \sin(\vartheta) \cos(\varphi) - \sin(\varphi) \cos(\psi)) \frac{T}{m}$$

$$\ddot{z} = -\cos(\vartheta) \cos(\varphi) \frac{T}{m} + g$$

$$\ddot{\varphi} = \frac{\tau_{\varphi}}{I_x}$$

$$\ddot{\vartheta} = \frac{\tau_{\vartheta}}{I_y}$$

$$\ddot{\psi} = \frac{\tau_{\psi}}{I_z}$$



state

$$\xi = (x, y, z, v_x, v_y, v_z, \varphi, \vartheta, \psi, p, q, r)'$$

inputs

$$u = (T, \tau_{\varphi}, \tau_{\vartheta}, \tau_{\psi})'$$

feedback linearization

- due to underactuation, the quadrotor system cannot be transformed into an equivalent linear, controllable system by **static** state feedback
- it is however possible to resort to **dynamic** state feedback to obtain **full state** linearization
- given the nonlinear system

$$\dot{\xi} = f(\xi) + g(\xi) u \quad \xi \in R^n, \quad u \in R^m$$

- find, if possible, a dynamic compensator of the form

$$\begin{aligned} \dot{\zeta} &= a(\xi, \zeta) + b(\xi, \zeta)v & \zeta \in R^\nu, \quad v \in R^m \\ u &= c(\xi, \zeta) + d(\xi, \zeta)v \end{aligned}$$

- s.t. the c.l. system, under state transformation, is equivalent to a linear system

- first define an m -dimensional output

$$\eta = h(\xi)$$

- then proceed by successively **differentiating** the output until the input appears in a **nonsingular** way
- if the sum of the output differentiation orders equals the dimension of the extended state space $n+\nu$, full input–state–output linearization is obtained
- the closed-loop system is then equivalent to a set of decoupled input–output chains of integrators from the input v_i to the output η_i ($i=1,\dots,m$)

- the linearising quadrotor output is

$$\eta(\xi) = (x, y, z, \psi)^T$$

- the input u appears in a nonsingular way deriving 4 times the cartesian position $P = (x, y, z)^T$ and 2 times the yaw angle ψ

- \Rightarrow it is necessary to introduce two integrators on the input

channel $u_1 = T$

$$\zeta = \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} = \begin{pmatrix} T \\ \dot{T} \end{pmatrix}$$

- the output derivatives can be written as

$$\begin{pmatrix} P^{(4)} \\ \psi^{(2)} \end{pmatrix} = l(\xi, \zeta) + J(\xi, \zeta) \tilde{u}$$

with $\tilde{u} = (\ddot{T}, \tau_\varphi, \tau_\vartheta, \tau_\psi)^T$

- the linearizing and decoupling control law is

$$\tilde{u} = J(\xi, \zeta)^{-1} (-l(\xi, \zeta) + v)$$

- resulting in the closed loop system

$$\begin{aligned} x^{(4)} &= \eta_1^{(4)} = v_1 \\ y^{(4)} &= \eta_2^{(4)} = v_2 \\ z^{(4)} &= \eta_3^{(4)} = v_3 \\ \psi^{(2)} &= \eta_4^{(2)} = v_4 \end{aligned}$$

- **trajectory tracking** can be obtained by choosing

$$\begin{aligned} v_1 &= \eta_{1d}^{(4)} + \sum_{j=1}^4 c_{j-1}^1 e_1^{(j-1)} \\ v_2 &= \eta_{2d}^{(4)} + \sum_{j=1}^4 c_{j-1}^2 e_2^{(j-1)} \\ v_3 &= \eta_{3d}^{(4)} + \sum_{j=1}^4 c_{j-1}^3 e_3^{(j-1)} \\ v_4 &= \eta_{4d}^{(2)} + \sum_{j=1}^2 c_{j-1}^4 e_4^{(j-1)} \end{aligned}$$

with $e_i = (\eta_d - \eta)$

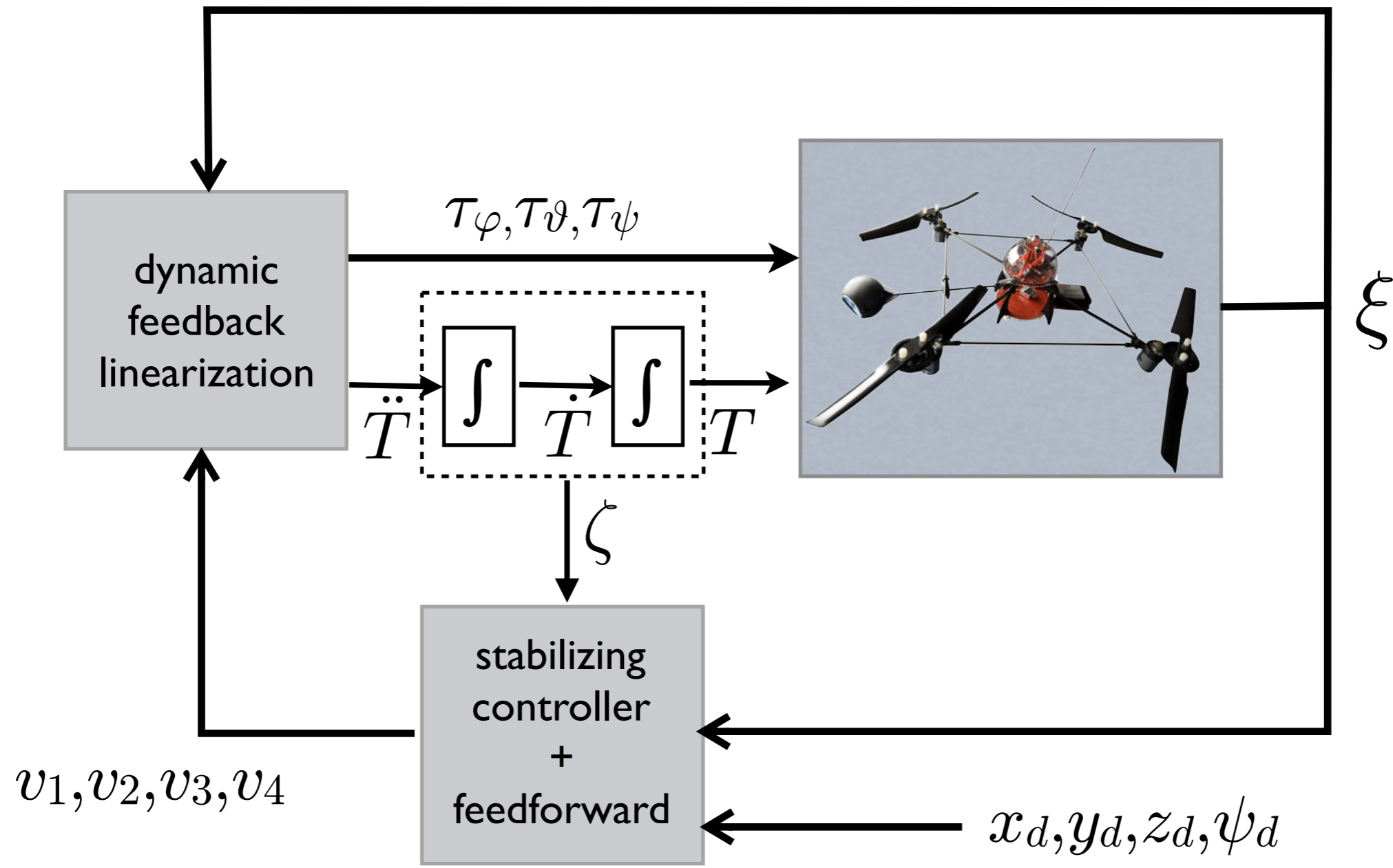
- the dynamic compensator takes the form

$$\begin{aligned}\dot{\zeta}_1 &= \zeta_2 \\ \dot{\zeta}_2 &= J_1(\xi, \zeta)^{-1}(-l(\xi, \zeta) + v)\end{aligned}$$

$$\begin{aligned}T &= \zeta_1 \\ \tau_\varphi &= J_2(\xi, \zeta)^{-1}(-l(\xi, \zeta) + v) \\ \tau_\vartheta &= J_3(\xi, \zeta)^{-1}(-l(\xi, \zeta) + v) \\ \tau_\psi &= J_4(\xi, \zeta)^{-1}(-l(\xi, \zeta) + v)\end{aligned}$$

where $J_i(\xi, \zeta)^{-1}$ ($i=1, \dots, 4$) denotes the i -th row of $J_i(\xi, \zeta)^{-1}$

control system



simulation results

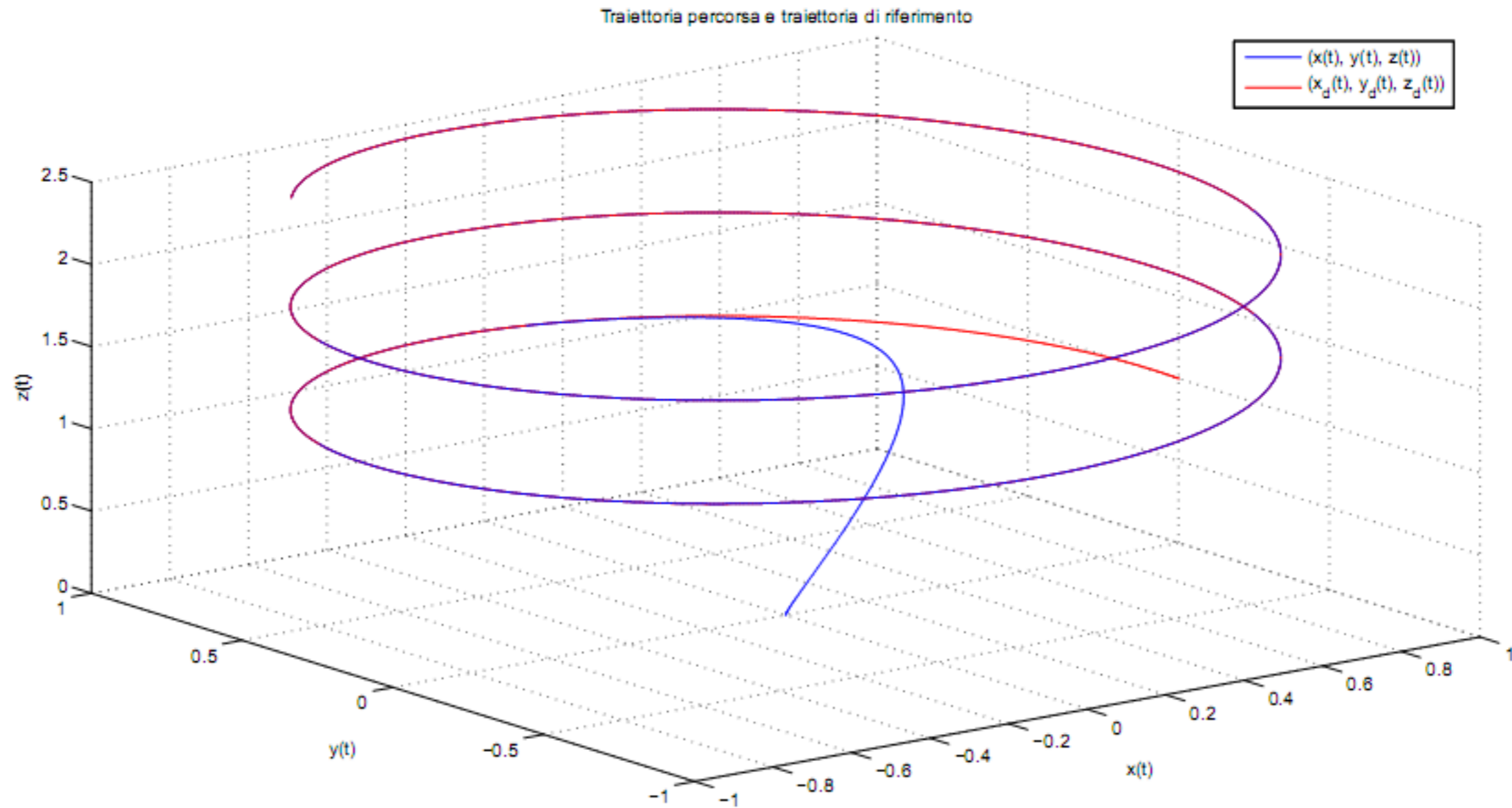
- **scenario 1**: controller validated on the *complete model*, including aerodynamic friction
- **scenario 2**: a *wind gust* is added as a *disturbance* on the X inertial axis, to test the *robustness* and *sensitivity* of the controller
- **scenario 3**: the same gust is then added also on Y and Z axes
- the reference trajectory chosen for the simulation is the spiral

$$x_d(t) = \cos t$$

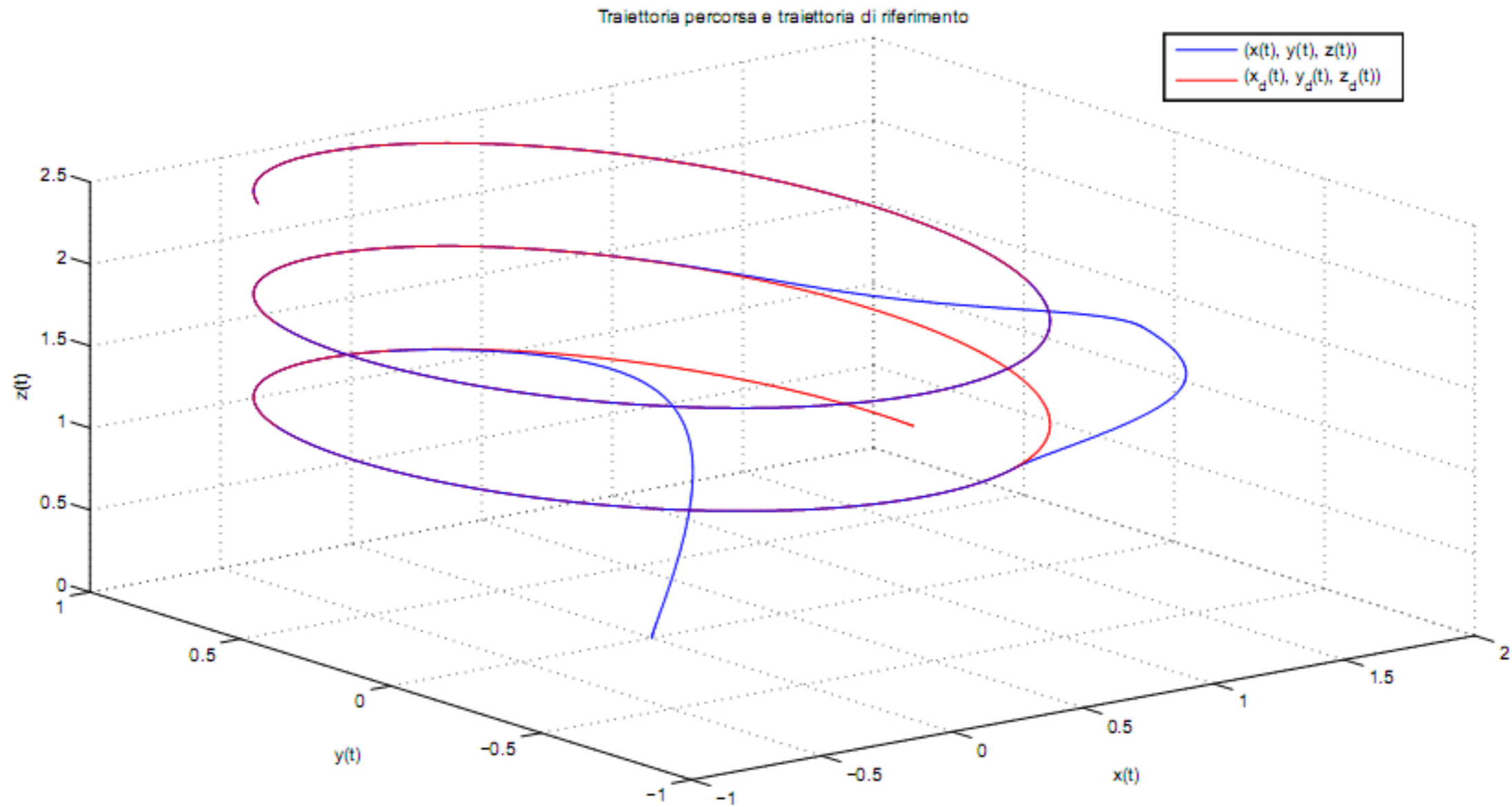
$$y_d(t) = \sin t$$

$$z_d(t) = 1 + \frac{t}{10}$$

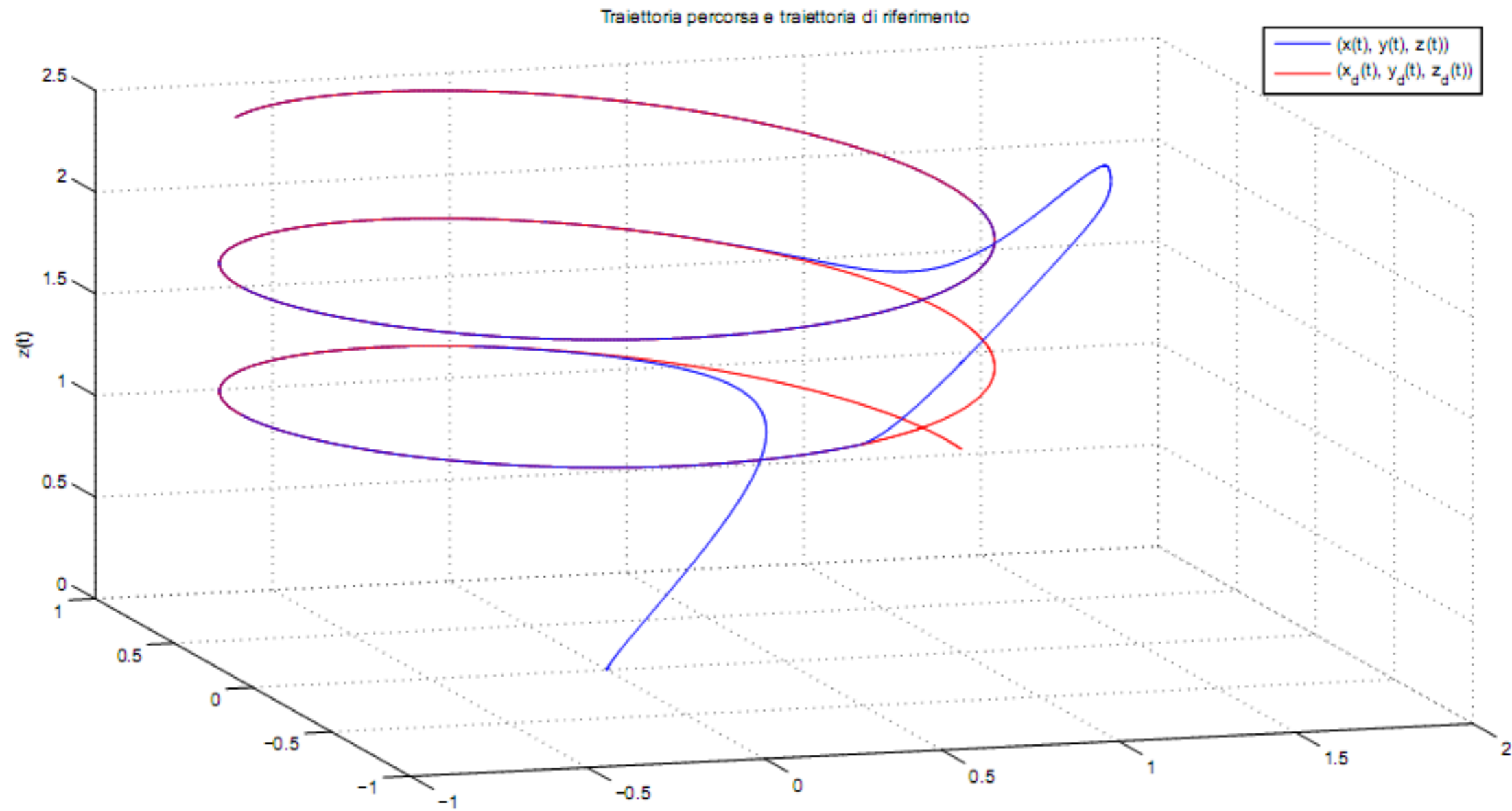
scenario I



scenario 2



scenario 3



removing the 'small angles' assumption

$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

$$\dot{z} = v_z$$

$$\dot{v}_x = \cancel{F_{A,x}} - (\cos(\psi) \sin(\vartheta) \cos(\varphi) + \sin(\psi) \sin(\varphi)) \frac{T}{m}$$

$$\dot{v}_y = \cancel{F_{A,y}} - (\sin(\psi) \sin(\vartheta) \cos(\varphi) - \sin(\varphi) \cos(\psi)) \frac{T}{m}$$

$$\dot{v}_z = \cancel{F_{A,z}} + g - \cos(\vartheta) \cos(\varphi) \frac{T}{m}$$

$$\dot{\varphi} = p + \sin(\varphi) \tan(\vartheta) q + \cos(\varphi) \tan(\vartheta) r$$

$$\dot{\vartheta} = \cos(\varphi) q - \sin(\varphi) r$$

$$\dot{\psi} = \sin(\varphi) \sec(\vartheta) q + \cos(\varphi) \sec(\vartheta) r$$

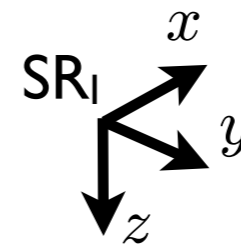
$$\dot{p} = \cancel{\tau_{A,x}} + \frac{I_r}{I_x} q \Omega_r + \frac{I_y - I_z}{I_x} q r + \frac{\tau_\varphi}{I_x}$$

$$\dot{q} = \cancel{\tau_{A,y}} + \frac{I_r}{I_y} p \Omega_r + \frac{I_z - I_x}{I_y} p r + \frac{\tau_\vartheta}{I_y}$$

$$\dot{r} = \cancel{\tau_{A,z}} + \frac{I_x - I_y}{I_z} p q + \frac{\tau_\psi}{I_z}$$

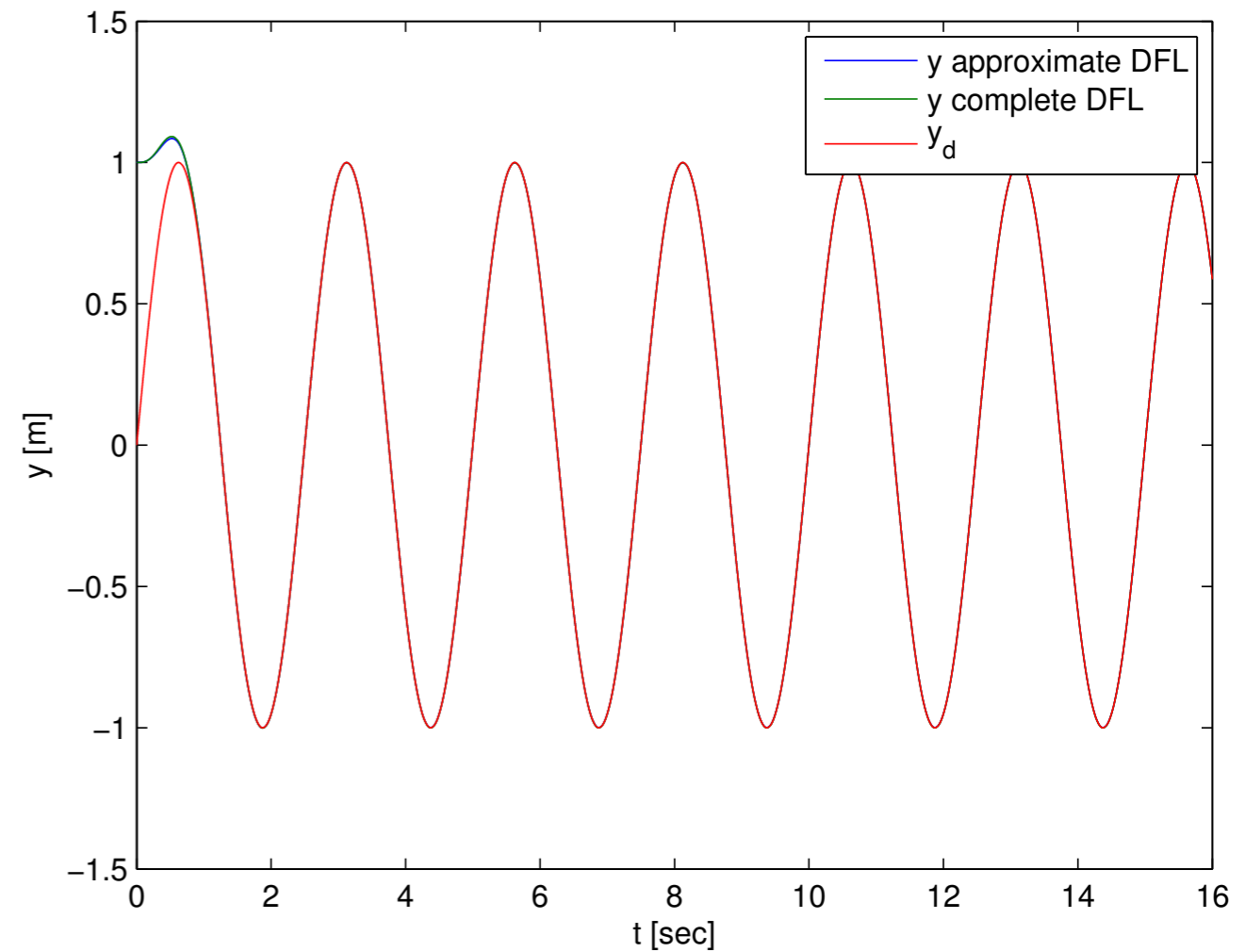
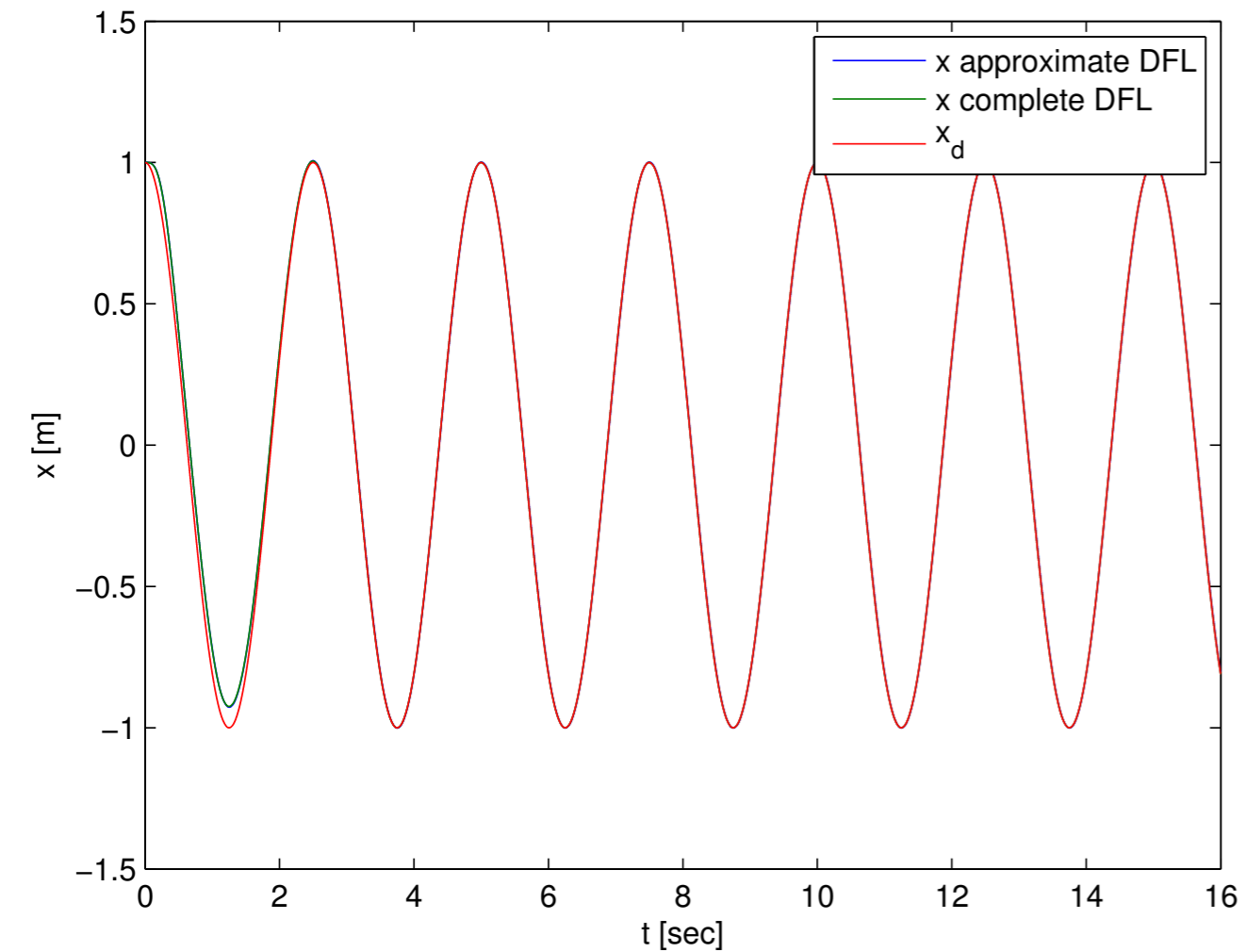
- consider the complete model neglecting aerodynamics, gyroscopic effects due to rotors velocity, disturbances

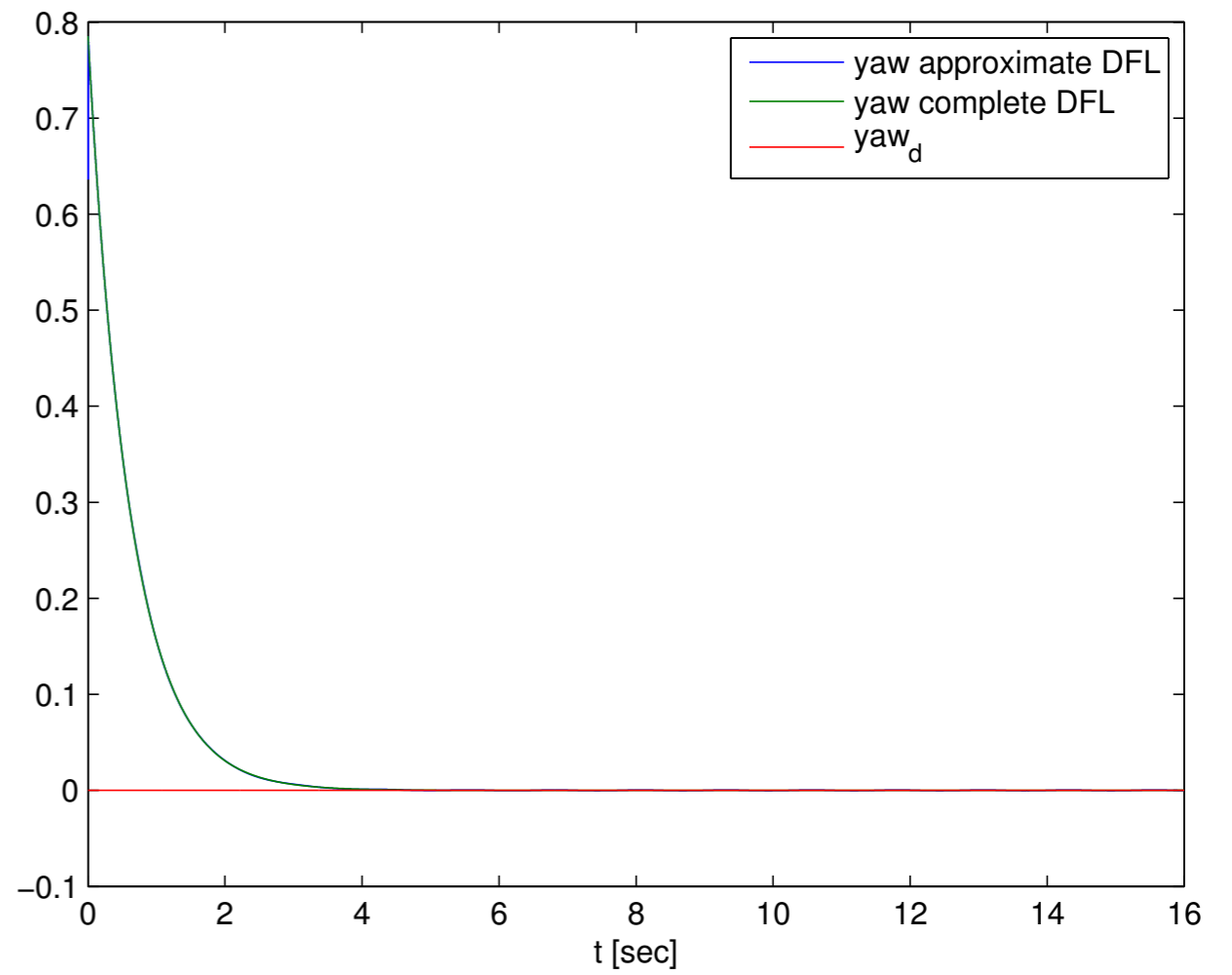
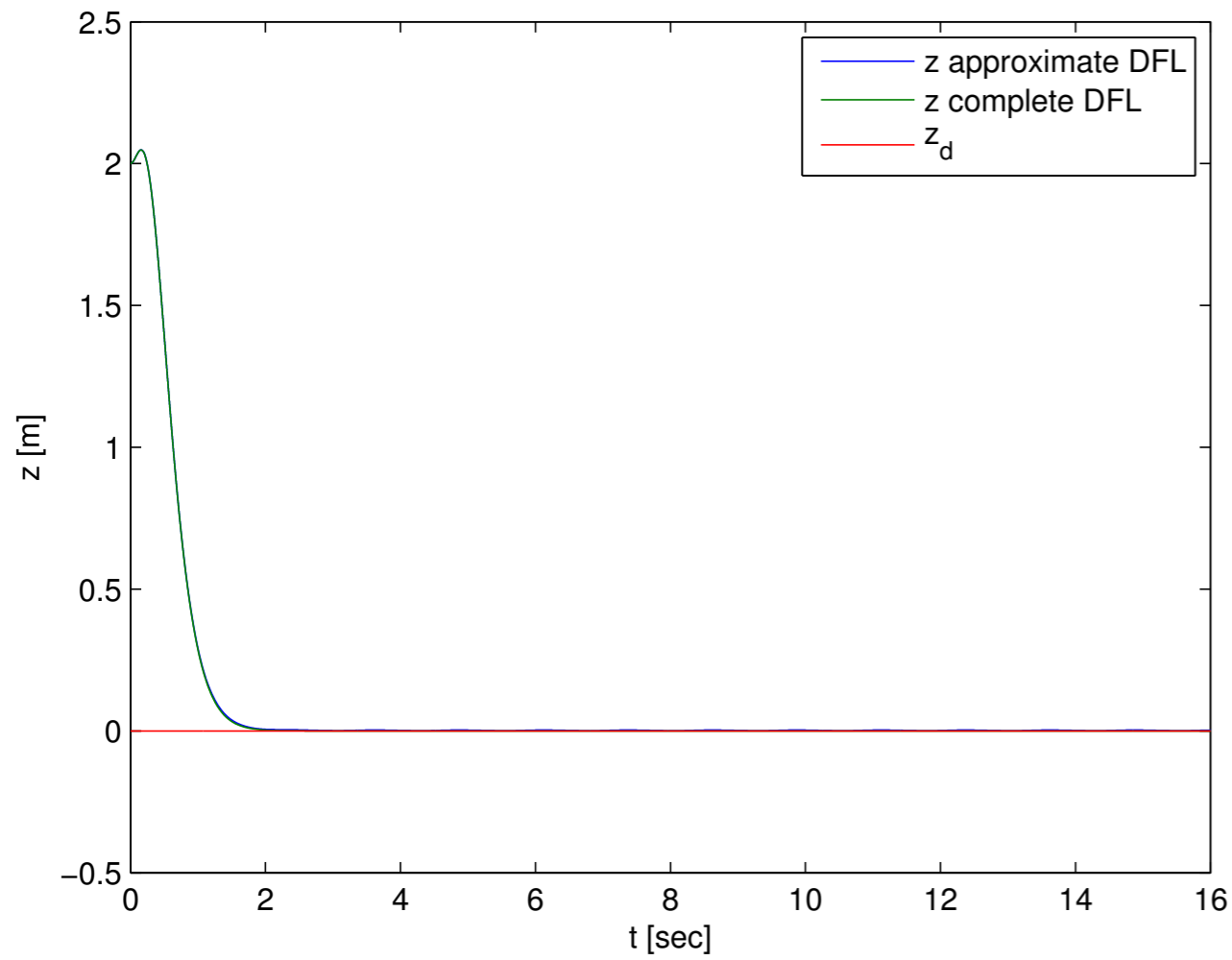
- compute DFL



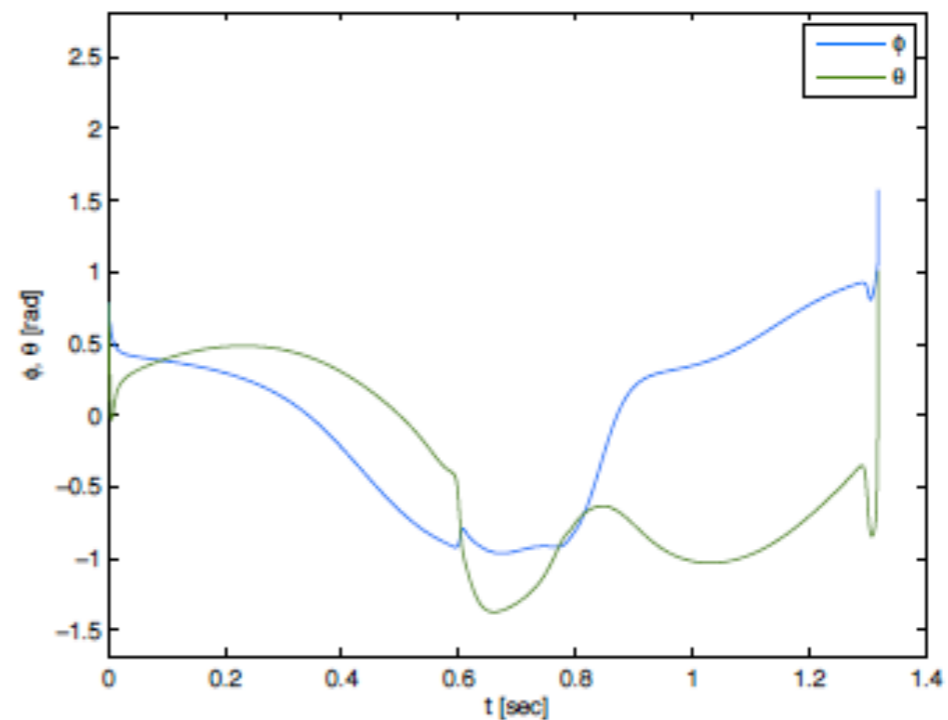
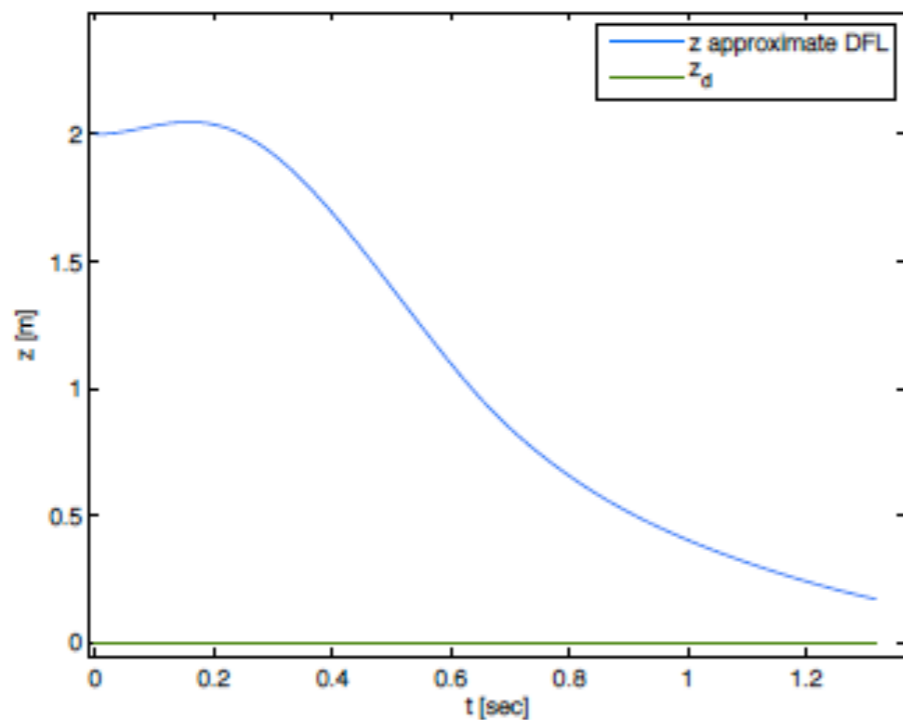
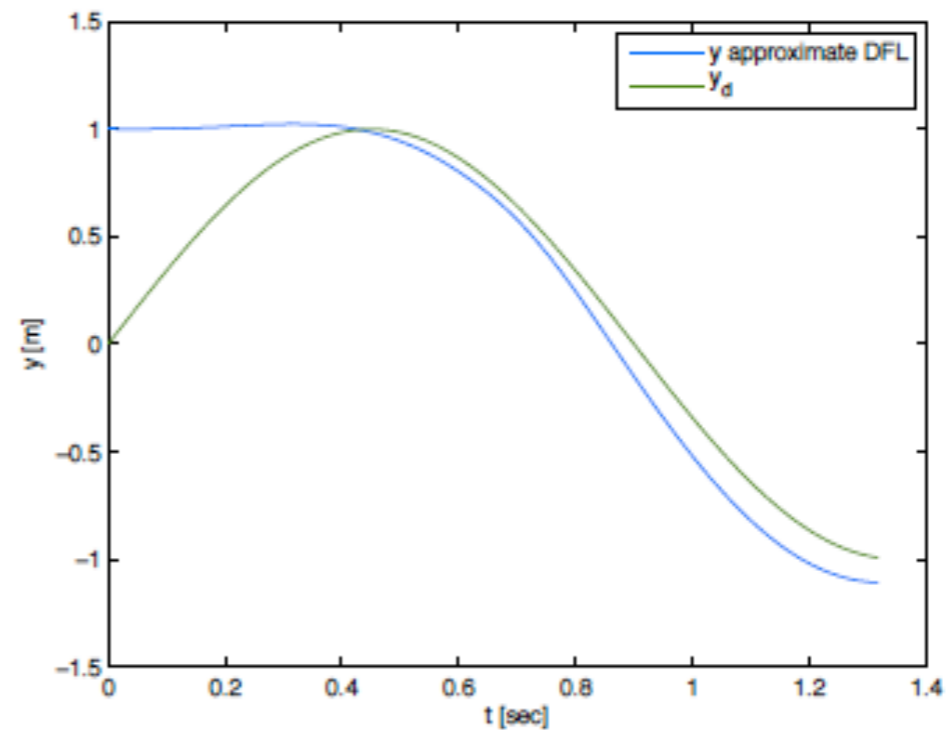
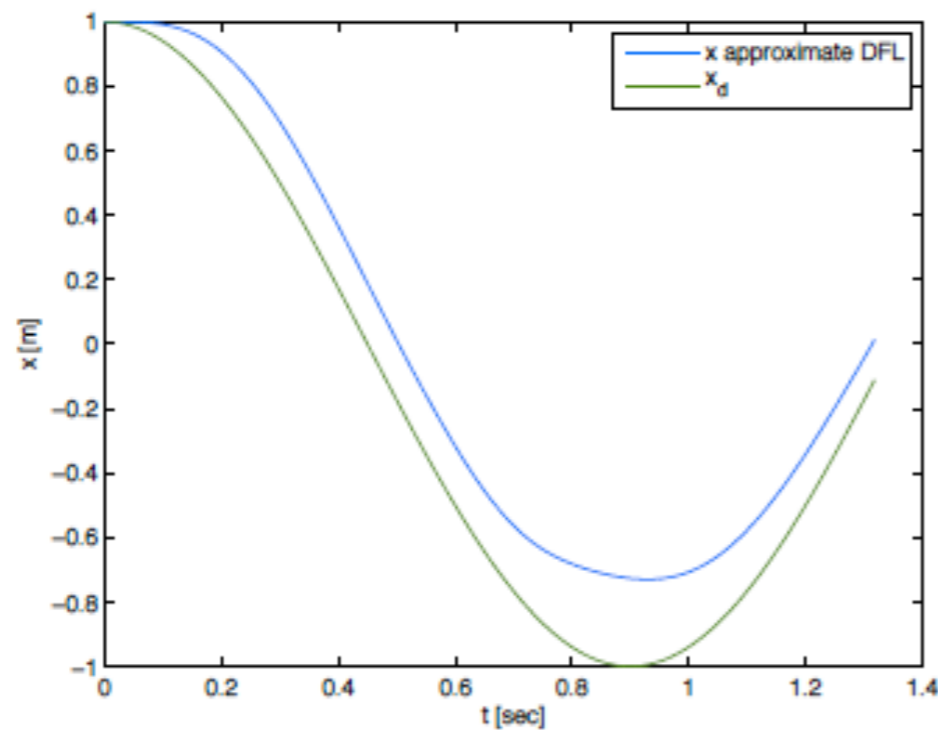
comparative simulations

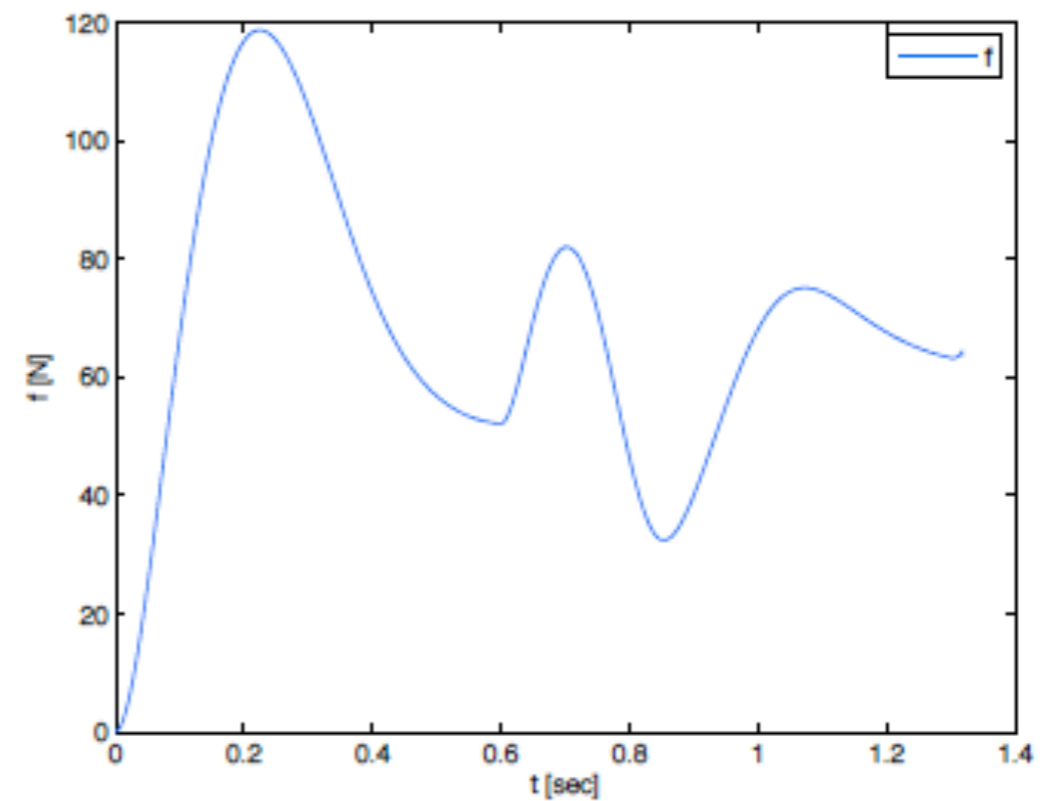
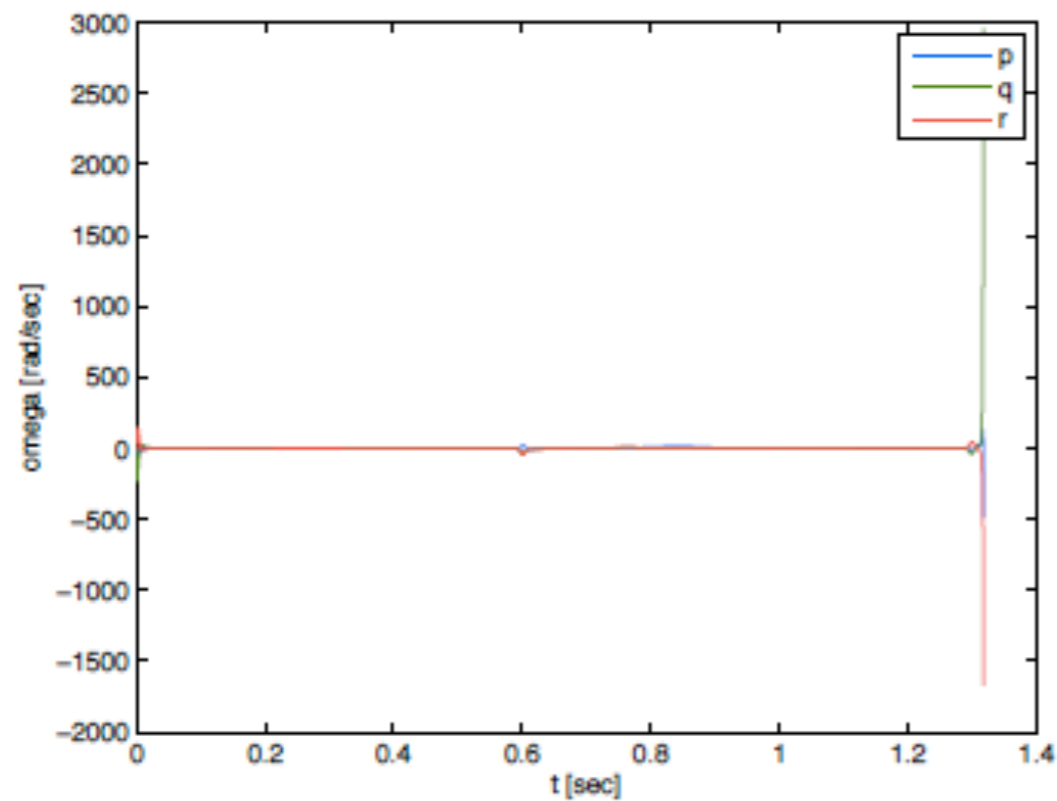
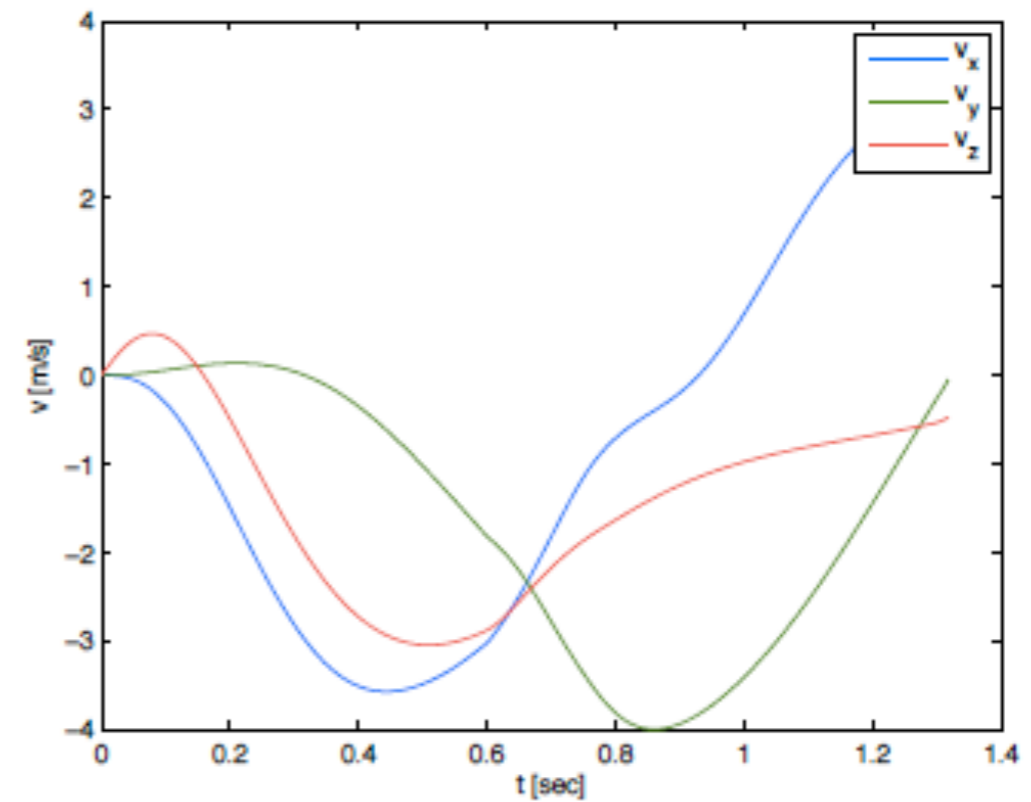
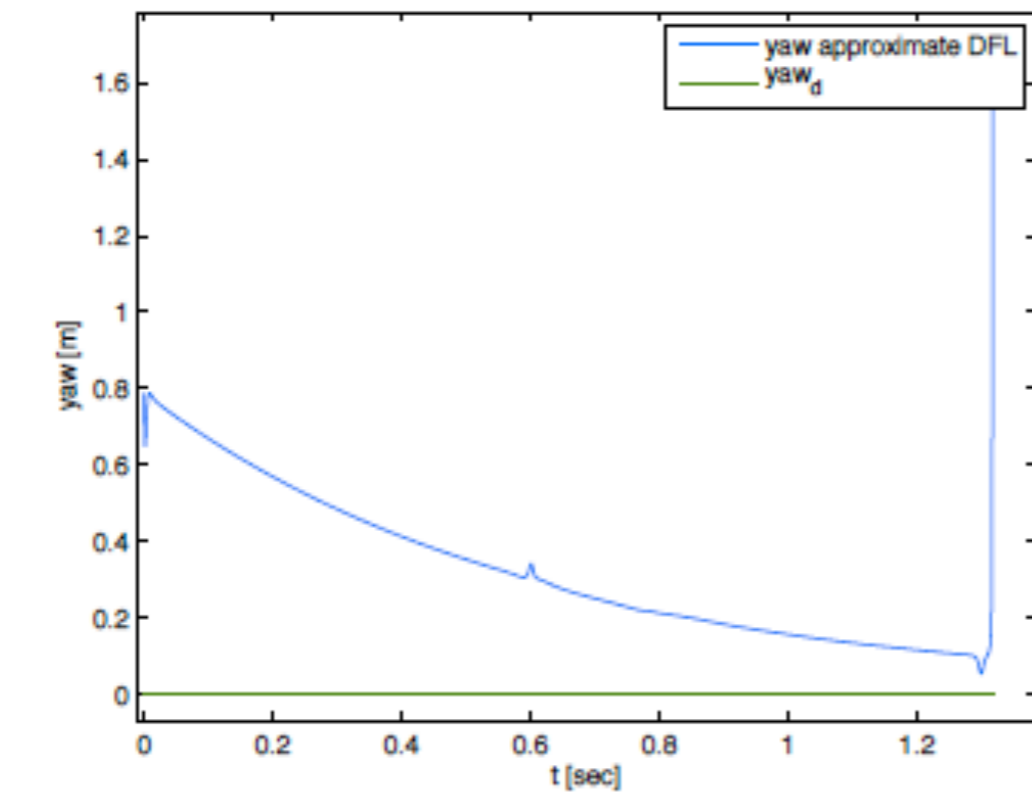
- **simulation 1**: circle with period $T=2.5s$ and radius $R=1m$

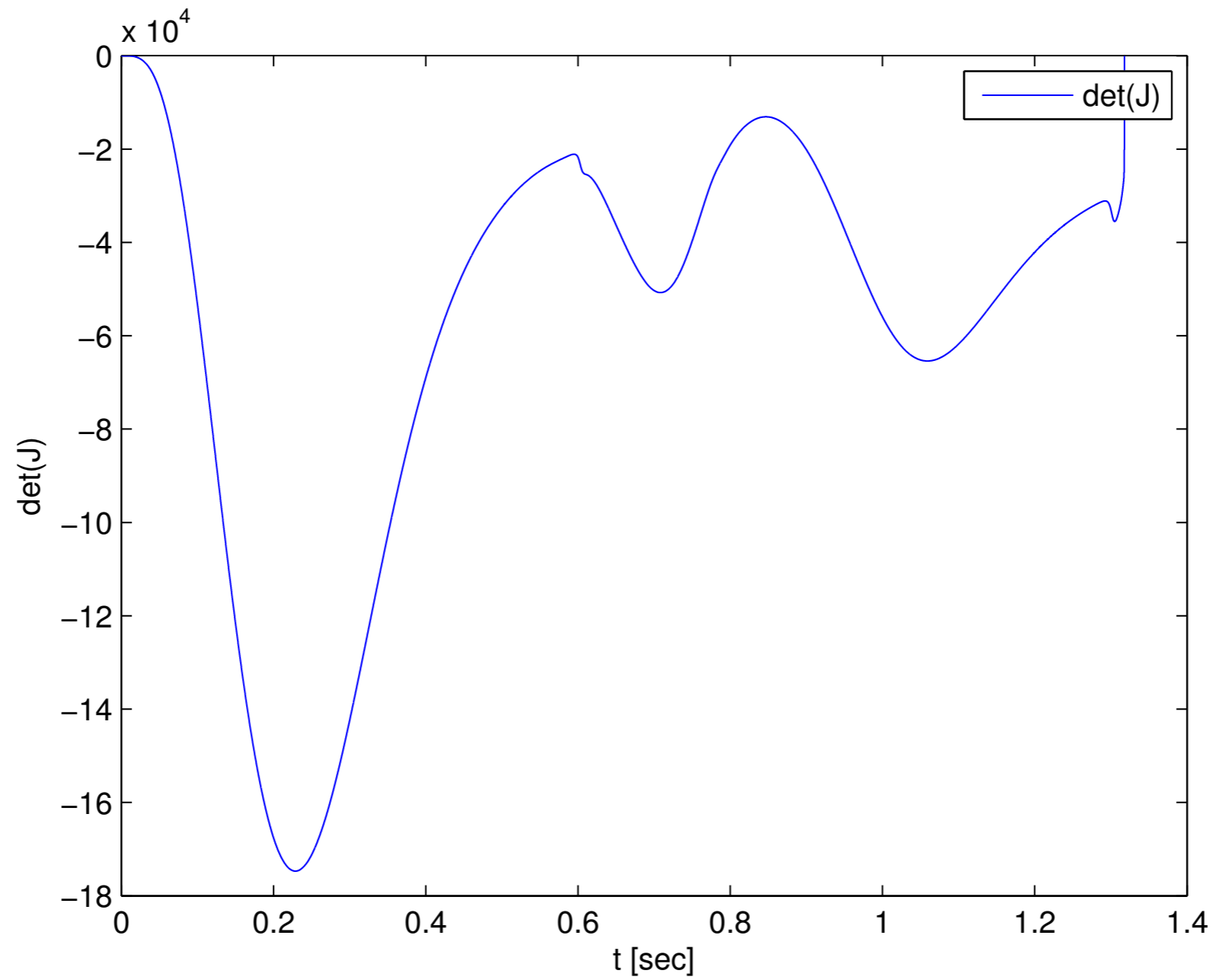




- **simulation 2**: circle with period $T=1.8s$ and radius $R=1m$
 - approximate model







- **simulation 2:** circle with period $T=1.8s$ and radius $R=1m$

- **complete model**

