$$u = \begin{pmatrix} T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix}, \quad y_{out} = \begin{pmatrix} x \\ y \\ z \\ \psi \end{pmatrix}$$

$$\xi = (x, y, z, V_x^b, V_y^b, V_z^b, \phi, \theta, \psi, p, q, r)$$

$$\dot\xi = \begin{pmatrix} \dot P \\ \dot V^b \\ \dot\Theta \\ \dot\omega \end{pmatrix} = \begin{pmatrix} RV^b \\ \frac{1}{m}[F^b_{thrust} + F^b_{gravity} + F^b_{drag}] - \omega \times V^b \\ W(\Theta)\omega \\ I^{-1}(\tau^b_{control} + \tau^b_{drag} - \omega \times I\omega) \end{pmatrix}$$

<u>yaw</u> $\dot y_4 = \dot\psi = \frac{\sin\phi}{\cos\theta} q + \frac{\cos\phi}{\cos\theta} r$

$$\ddot y_4 = \ddot\psi = \underbrace{(s_\phi \sec\theta)\dot q + (c_\phi \sec\theta)\dot r}_{\text{input-dependent part}} + \underbrace{(\dot\phi c_\phi \sec\theta + s_\phi \dot\theta \sec\theta\tan\theta) q + (-\dot\phi s_\phi \sec\theta + c_\phi \dot\theta \sec\theta \tan\theta) r}_{\text{state-dependent part } f_\psi(\xi) \text{ [drift]}}$$

$$\hookrightarrow (0 \quad s_\phi \sec\theta \quad c_\phi \sec\theta) \underbrace{\begin{pmatrix} \dot p \\ \dot q \\ \dot r \end{pmatrix}}_{\dot\omega = I^{-1}[\tau^b_{tot} - \omega \times I\omega]}$$

$$= f_\psi(\xi) + (0 \quad s_\phi \sec\theta \quad c_\phi \sec\theta) I^{-1} [\underbrace{\tau^b_{tot}}_{\hookrightarrow \tau^b_{control} + \tau^b_{drag} = \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} - A_{rot}\omega} - \omega \times I\omega] = f_\psi(\xi) + (0 \quad s_\phi \sec\theta \quad c_\phi \sec\theta) I^{-1} \left[\begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} - A_{rot}\omega - \omega \times I\omega \right]$$

$$\Rightarrow \ddot\psi = f_\psi(\xi) + g_{\psi,\phi}(\xi)\tau_\phi + g_{\psi,\theta}(\xi)\tau_\theta + g_{\psi,\psi}(\xi)\tau_\psi = F(\xi) + G(\xi)u$$

<u>position</u>

$$\dot P = R(\Theta)V^b = f(V^b, \Theta) = f(\phi, \theta, \psi, V_x^b, V_y^b, V_z^b) \quad \text{function of the state}$$

$$\ddot P = \underbrace{\dot R}_{\hookrightarrow RS(\omega)} V^b + R\dot V^b = \underbrace{R\cancel{S(\omega)V^b}}_{R(\omega \times V^b)} + R\left[\frac{1}{m}(F^b_{thrust} + F^b_{gravity} + F^b_{drag}) - \cancel{\omega \times V^b}\right] = \frac{1}{m} R \begin{pmatrix} 0 \\ 0 \\ \underset{u_1}{F_T} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} - \frac{1}{m} A_{trans} V^b$$

$$P^{(3)} = \frac{d}{dt}\left[\overbrace{\frac{F_T}{m} R \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}^{①} + \overbrace{\begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}}^{②} + \overbrace{\frac{1}{m} R F^b_{drag}}^{③} \right] = \frac{\dot F_T}{m} R e_3 + \frac{F_T}{m} RS(\omega) e_3 + \frac{1}{m}\left[\dot R F^b_{drag} + R\dot F^b_{drag}\right]$$

① $\frac{d}{dt}\left[\frac{F_T}{m} R \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right] = \frac{\dot F_T}{m} R \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \frac{F_T}{m} \dot R \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

↳ not a control

⇒ define dynamic compensator state $\zeta_1 = F_T \Rightarrow \dot\zeta_1 = \dot F_T = v_T$ new input

② $\frac{d}{dt}\left[\begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} = const\right] = 0$

③ $\frac{d}{dt}\left[\frac{1}{m} R F^b_{drag}\right] = \frac{1}{m}\left[\underbrace{\dot R F^b_{drag}}_{R(\omega \times F^b_{drag})} + R\dot F^b_{drag}\right] = \frac{1}{m}\left(R(\omega \times F^b_{drag}) - RA_{trans}\left(\frac{1}{m}(F^b_{thrust} + F^b_{gravity} + F^b_{drag}) - \omega \times V^b\right)\right)$

↳ contains $\dot V_b$ ⇒ does not introduce any new input

$$P^{(4)} = \frac{d}{dt}\left[\overbrace{\frac{\dot F_T}{m} R e_3}^{①} + \overbrace{\frac{F_T}{m} RS(\omega) e_3}^{②} + \dots\right]$$

① $\frac{d}{dt}\left[\frac{\dot F_T}{m} R e_3\right] = \frac{\ddot F_T}{m} R e_3 + \frac{\dot F_T}{m} \dot R e_3$

↳ $\zeta_2 = \dot F_T \Rightarrow \dot\zeta_2 = \ddot F_T = v_T$ new input

② $\frac{d}{dt}\left[\frac{F_T}{m} RS(\omega) e_3\right] = \frac{\dot F_T}{m} RS(\omega) e_3 + \frac{F_T}{m} \dot R S(\omega) e_3 + \frac{F_T}{m} R \underbrace{S(\dot\omega) e_3}_{\dot\omega \times e_3}$

$$I^{-1}(\underbrace{\tau^b_{control} + \tau^b_{drag}}_{\hookrightarrow \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix}} - \omega \times I\omega)$$

$$\Rightarrow \begin{pmatrix} P^{(4)} \\ \ddot\psi \end{pmatrix} = J(\xi, \zeta)\tilde u + \ell(\xi, \zeta)$$

$$\Rightarrow \underbrace{\tilde u}_{\hookrightarrow \begin{pmatrix} \ddot F_T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix}} = J^{-1}(\xi, \zeta)[\underbrace{v}_{\hookrightarrow \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix}} - \ell(\xi, \zeta)] \quad \leftarrow \text{linearizing feedback law}$$

linearized system →
$x^{(4)} = v_1$
$y^{(4)} = v_2$
$z^{(4)} = v_3$
$\ddot\psi = v_4$

$$r = r_x + r_y + r_z + r_\psi = 4 + 4 + 4 + 2 = 14$$

$$\left.\begin{matrix} n = 12 \\ \nu = 2 \end{matrix}\right\} n_{ext} = 12 + 2 = 14 \Rightarrow r = n_{ext} \Rightarrow \text{fully state linearizable}$$

$m = 4$