Acoustical Wavefield Imaging Course

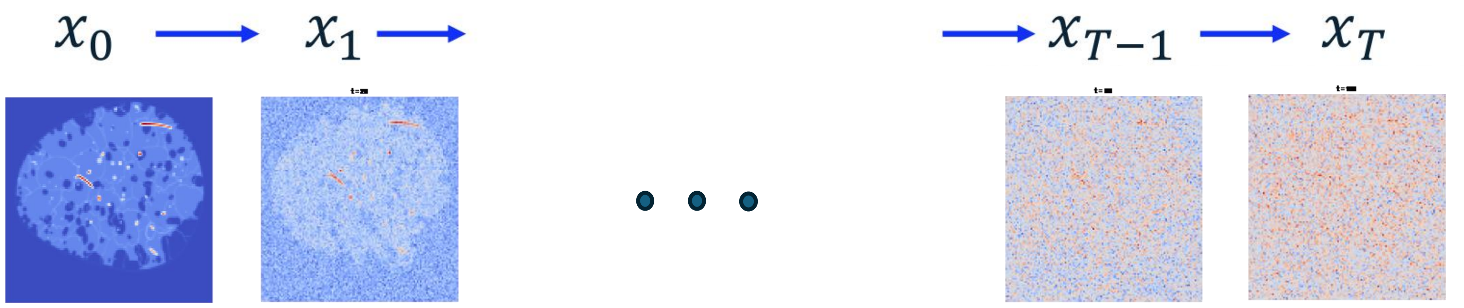
Generative AI guided by Full Waveform Inversion for Ultrasound Breast Imaging

Ana Ramírez, Ph.D.
Professor- Universidad Industrial de Santander

Universidad Industrial de Santander

Denoising Diffusion Probabilistic Models (DDPMs)

1. Forward Diffusion Process (enconding)



$$q(x_1, \dots, x_T | x_0) = \Pi_{t=1}^{T} q(x_t \, | x_{t-1})$$

$$q(x_t | x_{t-1}) \sim N(\sqrt{1-\beta_t} x_{t-1}, \beta_t \boldsymbol{I})$$

β_t is a scheduled variance previously selected

Take an image from a dataset and add noise with Gaussian distribution: **N(0,I),** such that the structure of the data is destroyed

1. Forward Diffusion Process (enconding)

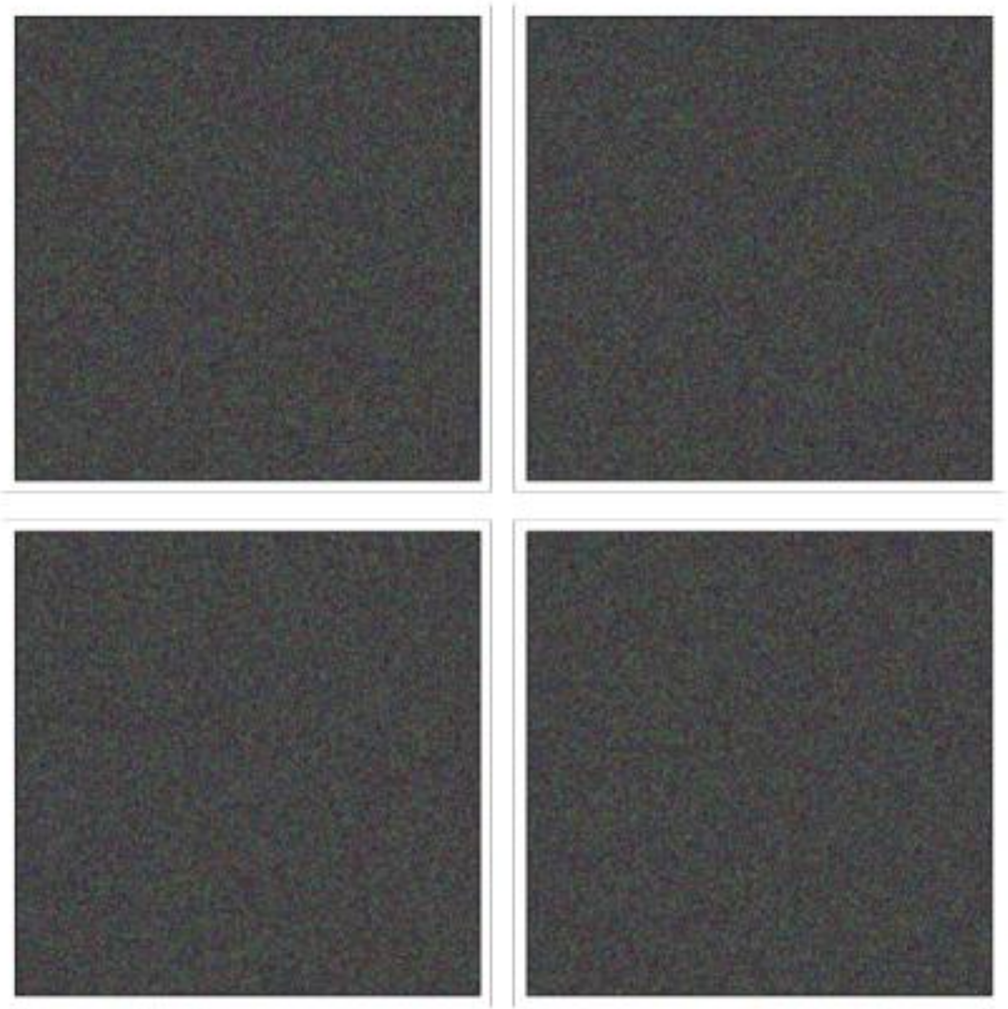
- What happens when we add multiple stages of noise?

let $\alpha_t = 1 - \beta_t \quad \bar{\alpha}_t = \prod_{i=1}^{t} \alpha_i$

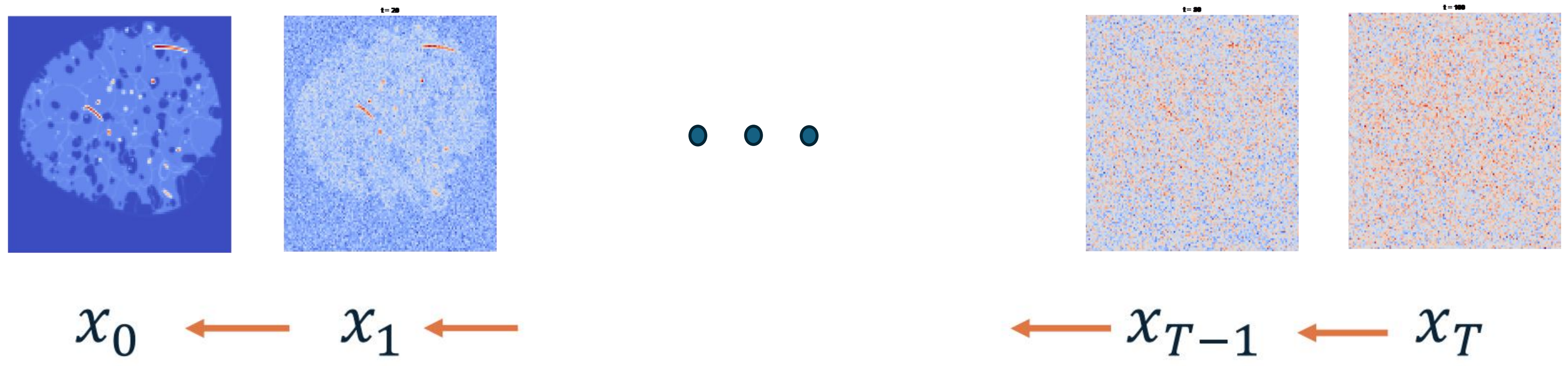
$$\begin{aligned}
\mathbf{x}_t &= \sqrt{\alpha_t}\mathbf{x}_{t-1} + \sqrt{1-\alpha_t}\boldsymbol{\epsilon}_{t-1} && \text{;where } \boldsymbol{\epsilon}_{t-1}, \boldsymbol{\epsilon}_{t-2}, \dots \sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \\
&= \sqrt{\alpha_t \alpha_{t-1}}\mathbf{x}_{t-2} + \sqrt{1-\alpha_t \alpha_{t-1}}\bar{\boldsymbol{\epsilon}}_{t-2} && \text{;where } \bar{\boldsymbol{\epsilon}}_{t-2} \text{ merges two Gaussians (*).} \\
&= \dots \\
&= \sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}
\end{aligned}$$

$$q(\mathbf{x}_t|\mathbf{x}_0) = \mathcal{N}(\mathbf{x}_t; \sqrt{\bar{\alpha}_t}\mathbf{x}_0, (1-\bar{\alpha}_t)\mathbf{I})$$

Decoding: One distribution to another



2. Denoising Diffusion Process (decoding)



$q(x_{t-1}|x_t)$ is unknown!

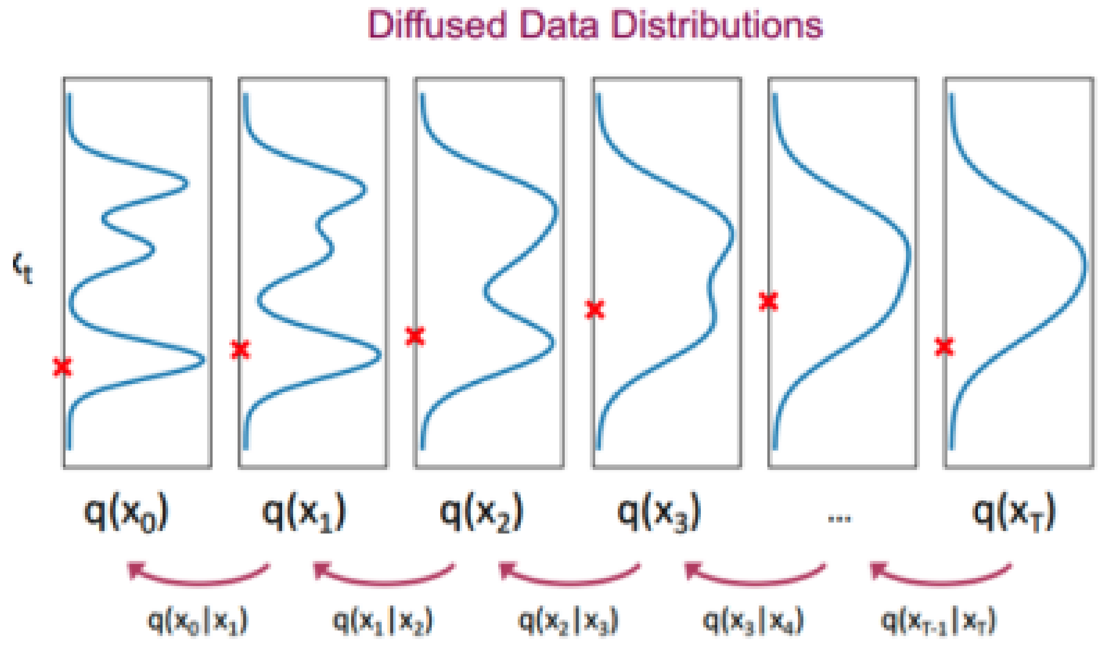
But why?

2. Denoising Diffusion Process (decoding)

- Why is it unknown?

  $$q(\mathbf{x}_{t-1}|\mathbf{x}_t) \propto q(\mathbf{x}_{t-1})q(\mathbf{x}_t|\mathbf{x}_{t-1})$$

- Intractable! It requires the whole data distribution
- Approximate it
- What would be our best guess?



$$p(\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_t; 0, 1) \quad p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \beta_t \mathbf{I})$$

2. Denoising Diffusion Process (decoding)

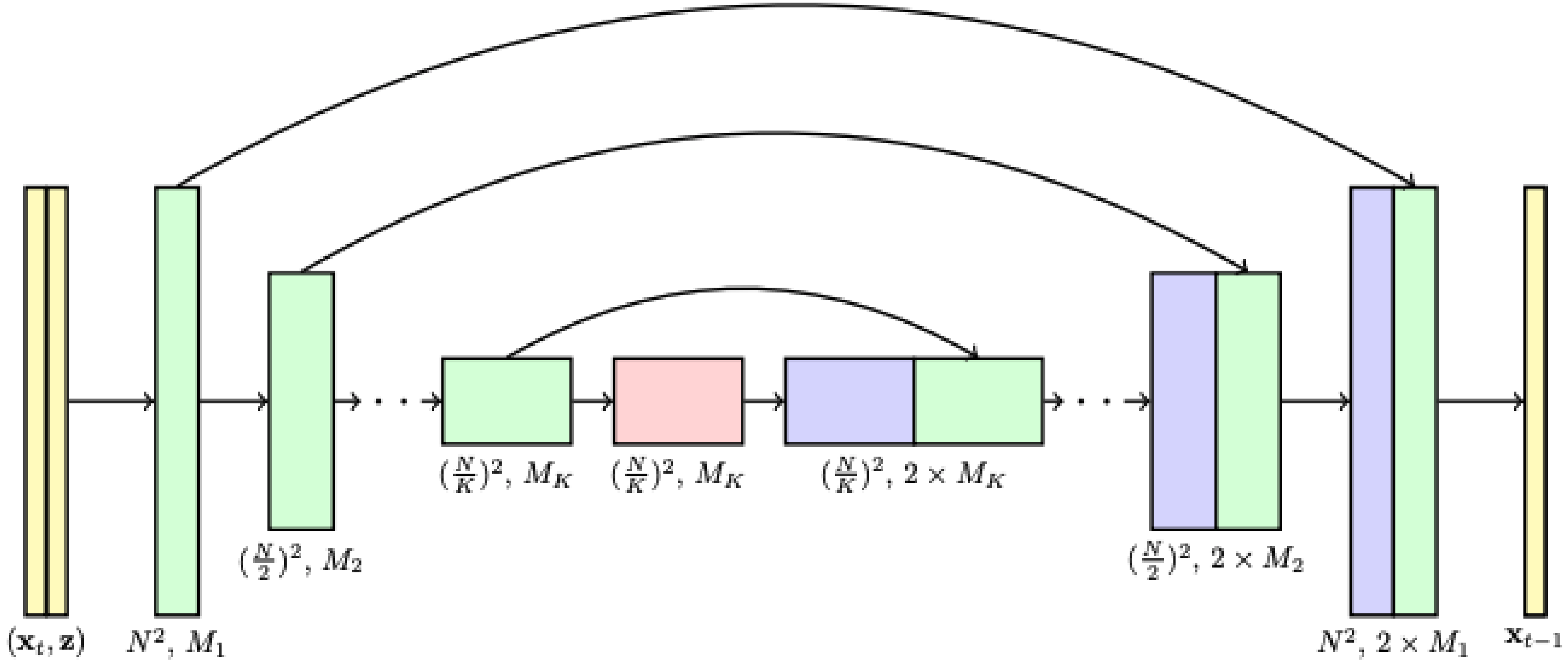
¿Can we learn a function that performs the reverse process?

$$p(\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_t; 0, 1) \quad p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t) = \mathcal{N}(\mathbf{x}_{t-1}; \boldsymbol{\mu}_\theta(\mathbf{x}_t, t), \beta_t \mathbf{I})$$

$$\boldsymbol{\mu}_\theta(\mathbf{x}_t, t) \rightarrow \text{Neural Network}$$

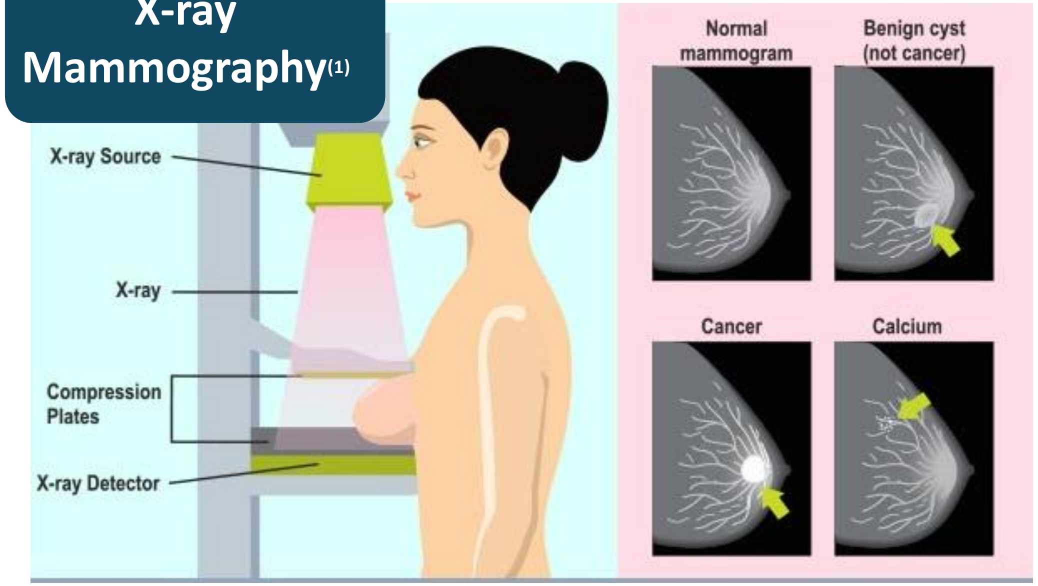
Generate an image from a noise distribution **N(0,I)** by using a NN that learns the reverse process such that the structure of the data is restored.

Neural Network - Unet



Motivation

X-ray Mammography(1)

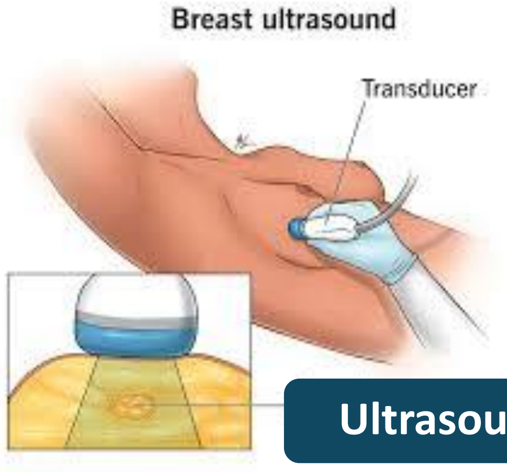


Disadvantages:

1. Requires ionizing radiation or intravenous contrast injection.
2. The sensitivity declines to 30%–48% in extremely dense breasts(2).

Supplemental Screening Methods

MRI



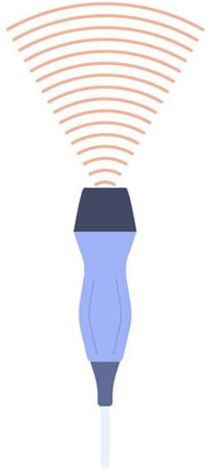
Ultrasound(3)

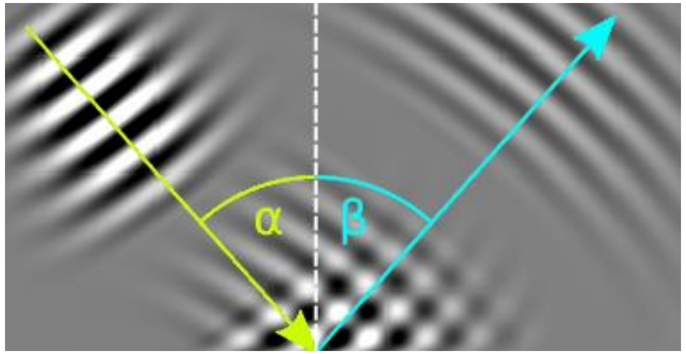
MRI: American Cancer Society recommends screening MRI for women at high risk for breast cancer. Drawbacks: HIgh-costs and fear to claustrophia.

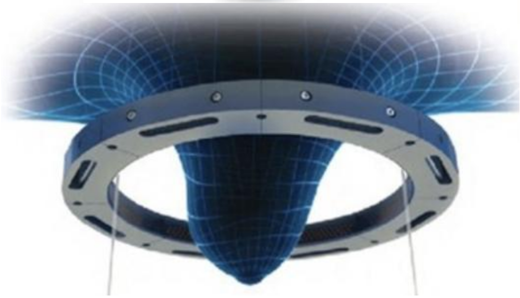
Ultrasound: Adding screening US to mammography increases specificity for women with dense breasts. Drawback: Low-resolution imagery.

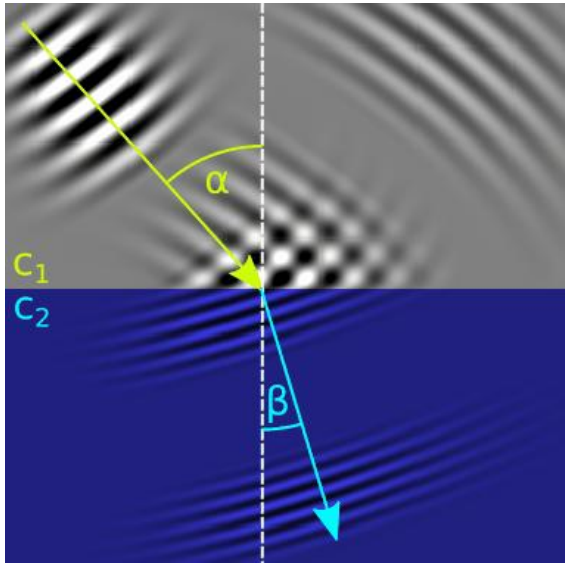
(1) https://www.netmeds.com/health-library/post/mammography-what-is-it-and-what-to-expect
(2) Hussein H, Abbas E, Keshavarzi S, Fazelzad R, Bukhanov K, Kulkarni S, Au F, Ghai S, Alabousi A, Freitas V. Supplemental Breast Cancer Screening in Women with Dense Breasts and Negative Mammography: A Systematic Review and Meta-Analysis. Radiology. 2023 Mar; 306(3):e221785. doi: 10.1148/radiol.221785. Epub 2023 Jan 31. PMID: 36719288.
(3) https://my.clevelandclinic.org/health/diagnostics/21496-breast-ultrasound

Motivation









Phased-array transducer where the imaging is based only on reflected waves

circular-array transducer where the imaging is based on reflected and refracted waves

<u>Ultrasound resolution limits:</u>

- ✓ **Short wavelength (high frequency of the transducer)**
- ✓ **Higher frequencies of the transducers implies lower depth due to the attenuation coefficient**

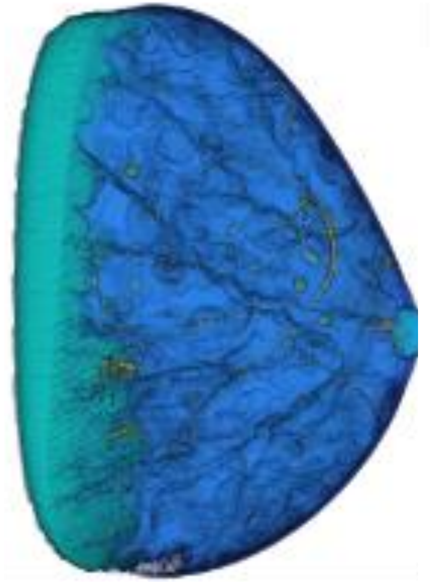
Kratkiewicz, K.; Pattyn, A.; Alijabbari, N.; Mehrmohammadi, M. Ultrasound and Photoacoustic Imaging of Breast Cancer: Clinical Systems, Challenges, and Future Outlook. *J. Clin. Med.* **2022**, *11*, 1165. https://doi.org/10.3390/jcm11051165

https://www.delphinusmt.com/softvue/how-it-works

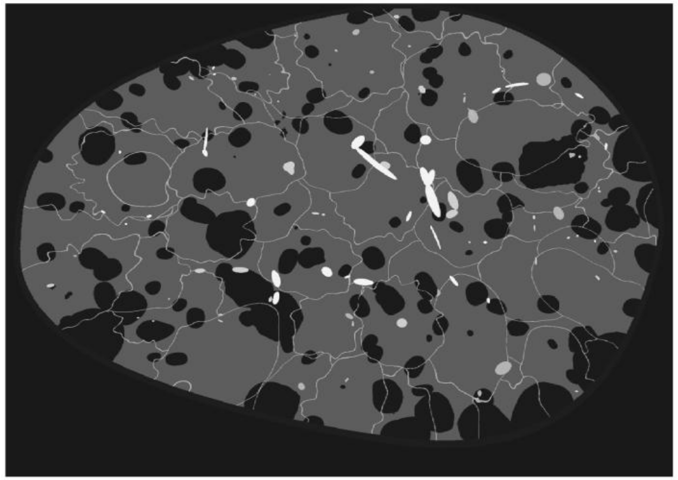
Motivation

The goal of our research is:

1. To develop an ultrasound imaging method that uses Generative AI to retrieve high-resolution images (from MRI databases).
2. The breast images should be consistent with the physics-phenomena of the acoustic source propagating through the breast.



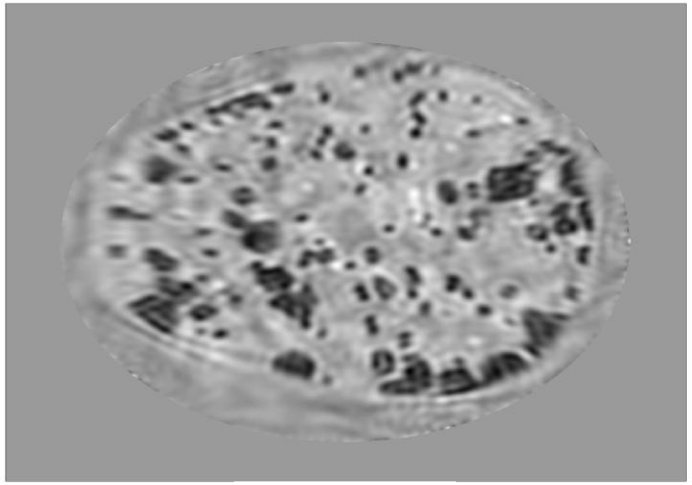
Virtual dense breast phantom



2D synthetic MRI image

Resolution: 10µm

Intensity Image

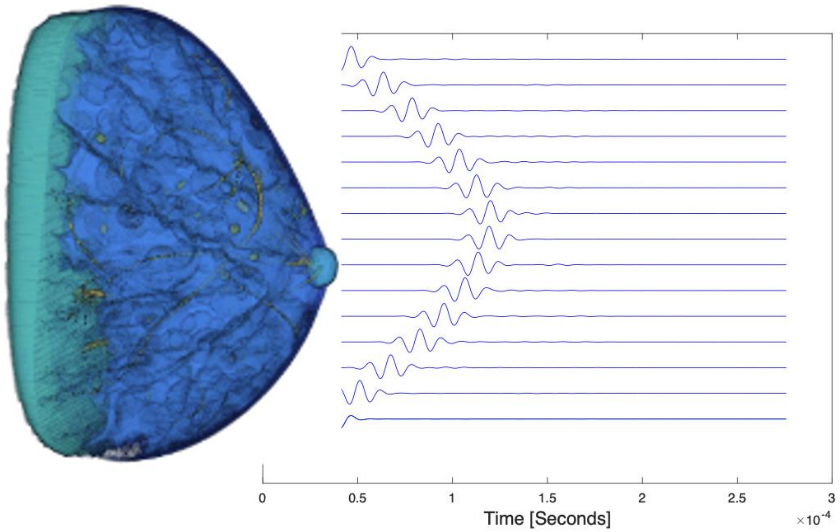


2D reconstructed US image

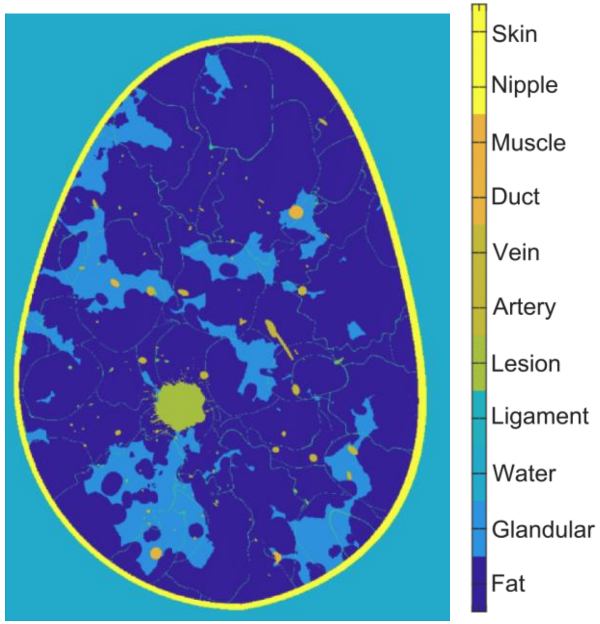
Resolution: 1mm

Speed of Sound Image

US breast imaging using Full Waveform Inversion (FWI)



Time-domain signals

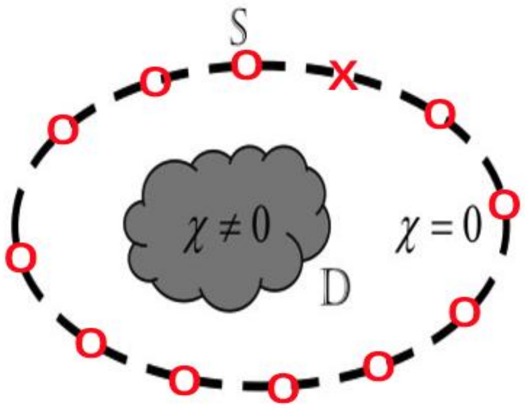


Quantitative image of:
Speed of sound or density

Tissue	Speed-of-sound propagation [m/s]
Fat	1440.0
Glandular	1505.0
Water	1520.0
Ligament	1525.0
Lesion	1572.0
Artery	1578.2
Vein	1578.2
Duct	1588.0
Muscle	1588.4
Nipple	1624.0
Skin	1624.0

US breast imaging using Full Waveform Inversion (FWI)

Forward Model



$$\frac{1}{c^2(x,z)}\frac{\partial^2 p(x,z,t)}{\partial^2 t} + \frac{\partial^2 p(x,z,t)}{\partial^2 x} + \frac{\partial^2 p(x,z,t)}{\partial^2 z} = f(x_0,z_0,t). \quad \text{(PDE)}$$

- $p(x,z,t)$ pressure field
- $c(x,z)$ speed-of-sound (SoS) of the breast
- $f(x_0,z_0,t)$ source wavelet

$$\boldsymbol{p}^r = \boldsymbol{G}(\boldsymbol{c}) + \boldsymbol{\eta}$$

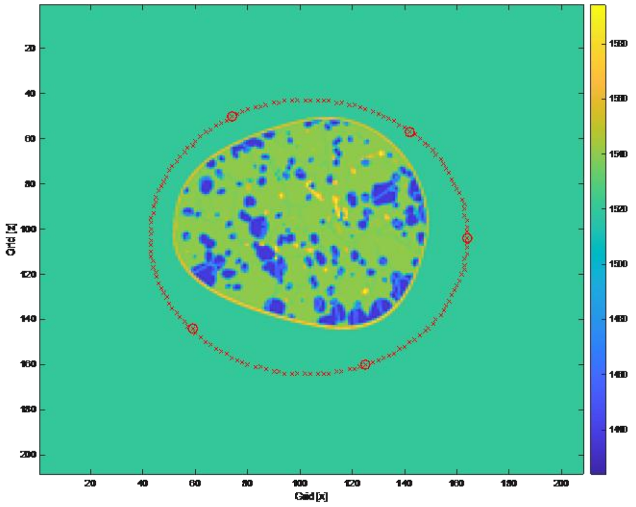
- $\boldsymbol{p}^r \in R^{N_R \times N_t}$ pressure field at the receiver's location
- $\boldsymbol{G}$ non-linear operator representing the AWE
- $\boldsymbol{c} \in R^{N_x \times N_z}$ SoS of the breast
- $\boldsymbol{\eta} \in R^{N_R \times N_t}$ noise

US breast imaging using Full Waveform Inversion (FWI)

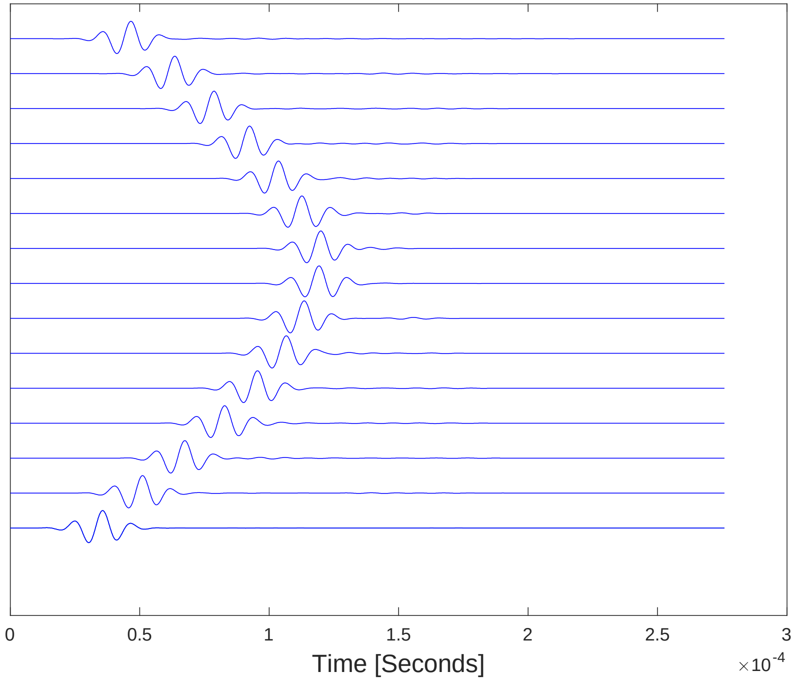
Inverse Problem: knowing $\mathbf{p}^r$ we need to estimate *c(x,z)*

$$\min_{\boldsymbol{c}} ||\boldsymbol{p}^r - \boldsymbol{G}(\boldsymbol{c})||_2^2$$

1. Initial velocity image $c^0(x,z)$
2. Find the modeled field at the receivers $p^m = G(c^0)$
3. Find the gradient of the l2-error between $p^r - p^m$
4. Update velocity image iteratively $c^k(x,z)$

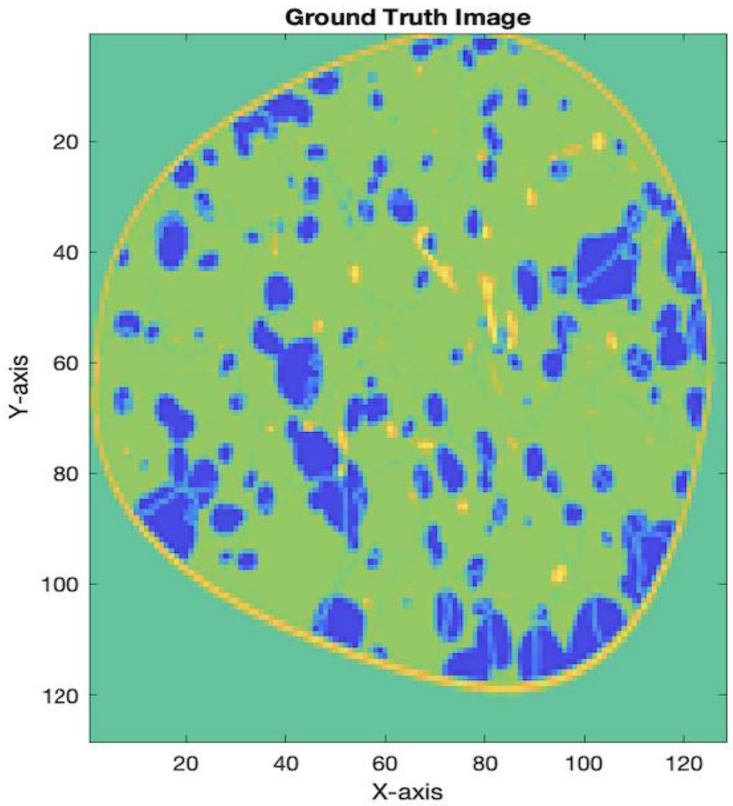


$\Delta x \approx 1.1$ mm
$\Delta y \approx 1.1$ mm
$\Delta t = 0.33\ \mu s$
$N_t = 1600$
F = 90KHz

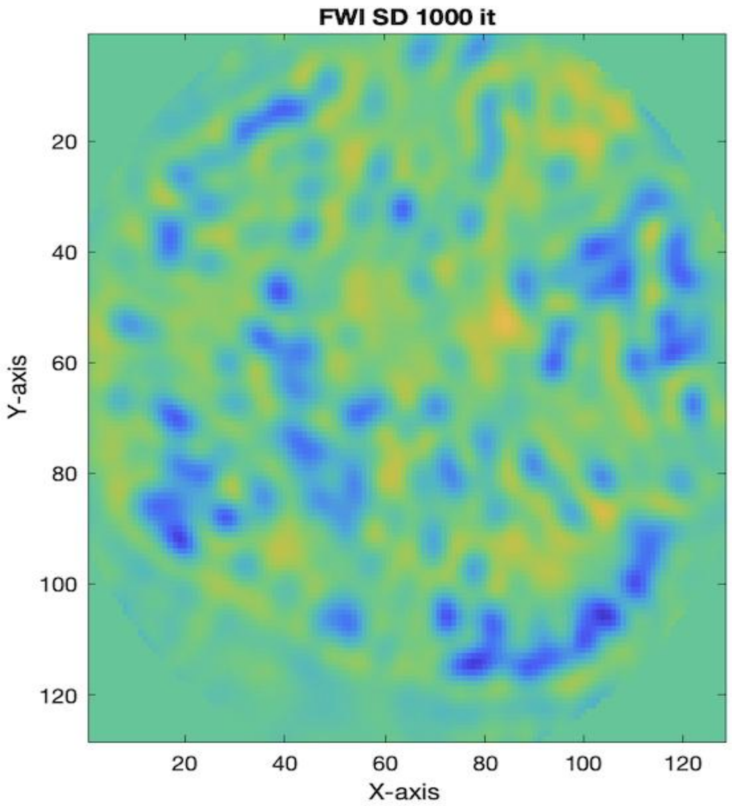


US breast imaging using Full Waveform Inversion (FWI) - 5 sources

Ground – Truth Image



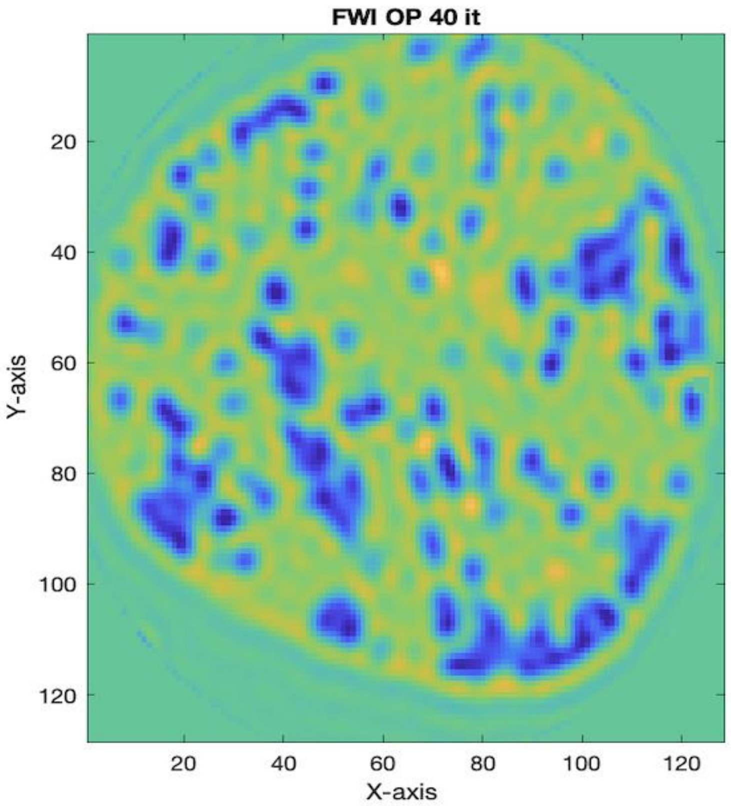
FWI – steepest descent



$$\mathbf{c}^{k+1} = \mathbf{c}^k - \beta \cdot g(\mathbf{c}^k)$$

PSNR =38.09dB
SSIM = 0.33

FWI – LBFGS opt. descent

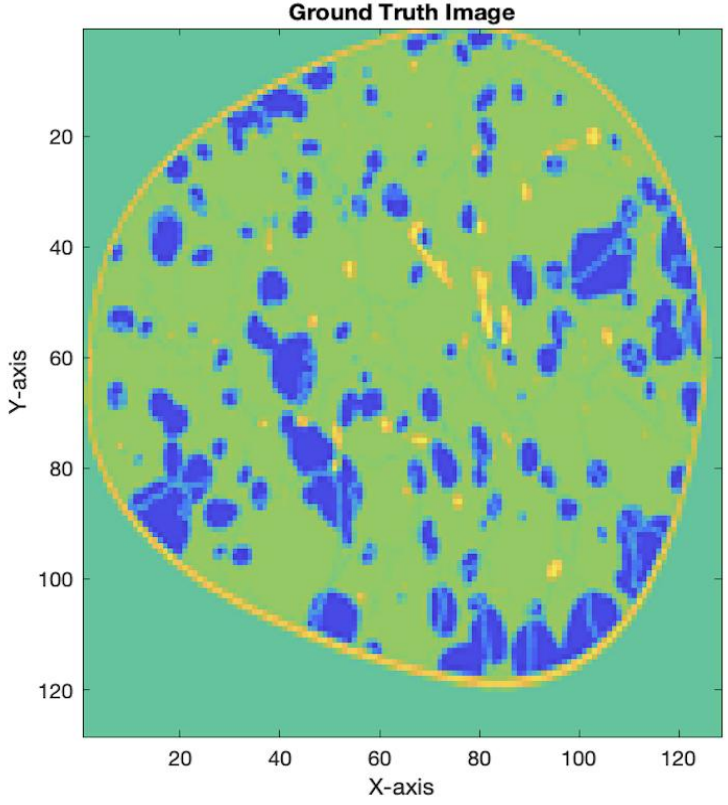


$$\mathbf{c}^{k+1} = \mathbf{c}^k - \beta^k(\mathbf{c}^k) \cdot g(\mathbf{c}^k)$$

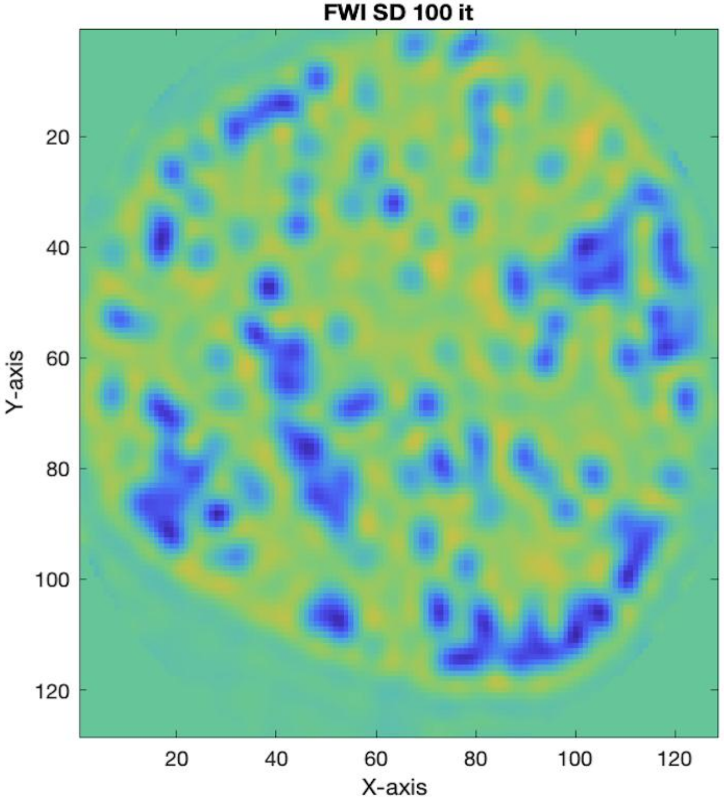
PSNR =41.26dB
SSIM = 0.50

US breast imaging using Full Waveform Inversion (FWI) - 10 sources

Ground – Truth Image



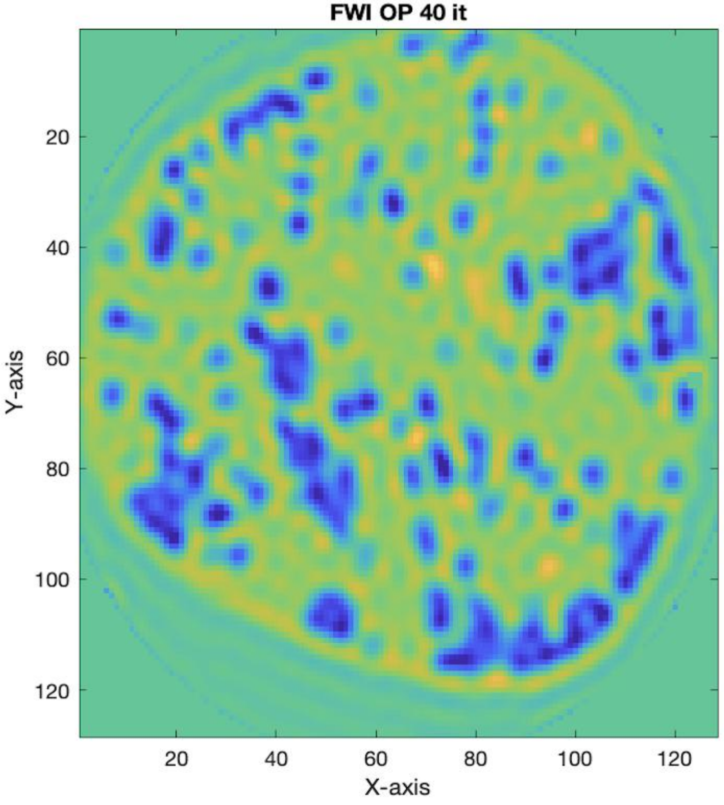
FWI – steepest descent



$$\mathbf{c}^{k+1} = \mathbf{c}^{k} - \beta \cdot g(\mathbf{c}^{k})$$

PSNR =39.98dB
SSIM = 0.43

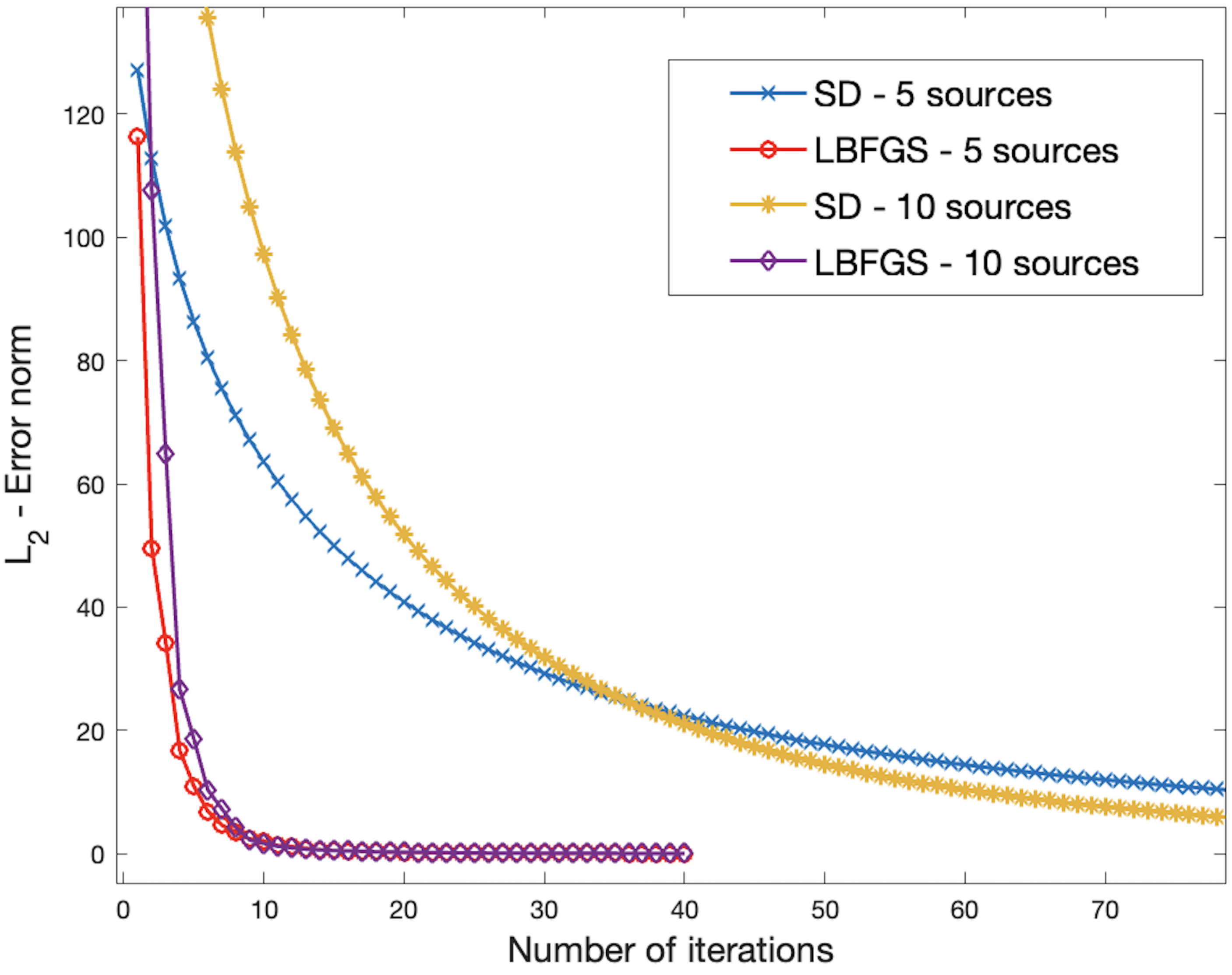
FWI – LBFGS opt. descent



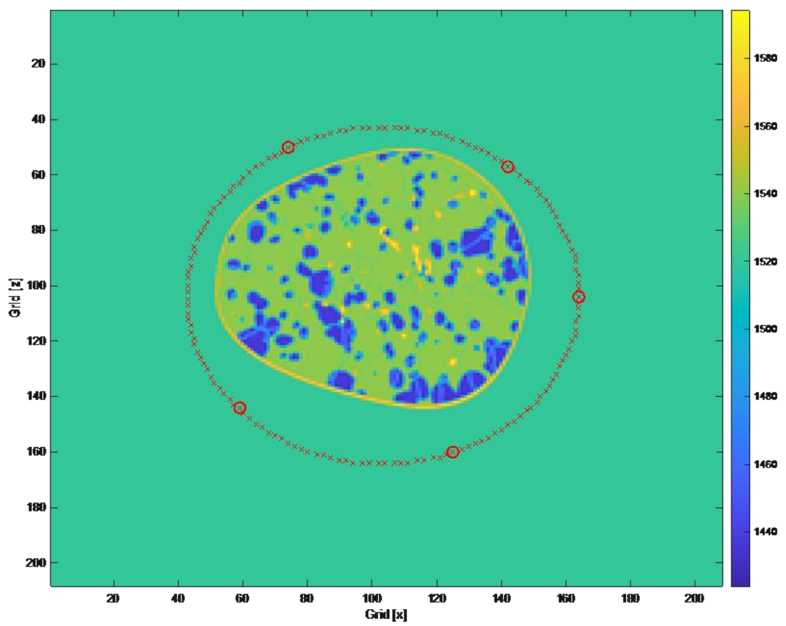
$$\mathbf{c}^{k+1} = \mathbf{c}^{k} - \beta^{k}(\mathbf{c}^{k}) \cdot g(\mathbf{c}^{k})$$

PSNR =41.49dB
SSIM = 0.52

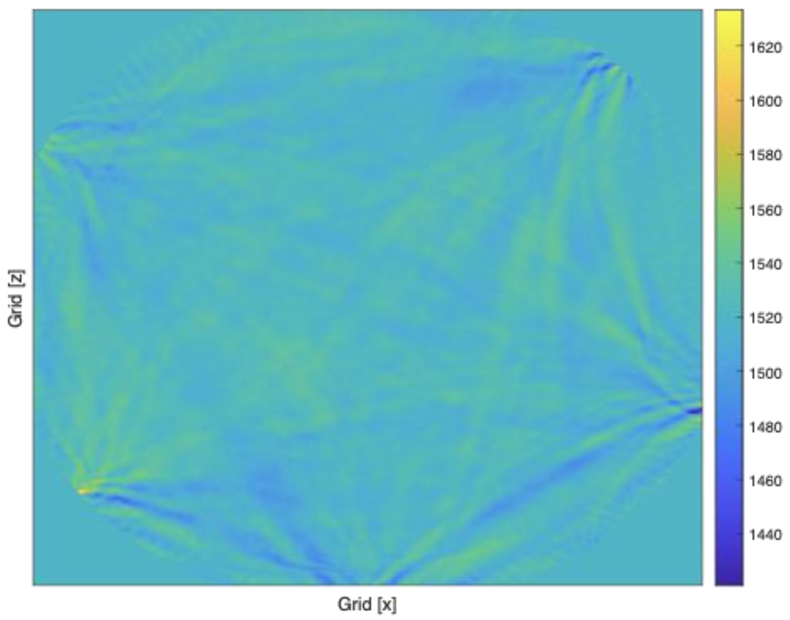
US breast imaging using Full Waveform Inversion (FWI)



US breast imaging using Full Waveform Inversion (FWI)



Δx ≈ 0.5 mm
Δy ≈ 0.5 mm
Δt = 0.08 μs
$N_t = 3200$
F = 0.2MHz



FWI – steepest descent/optimal descent

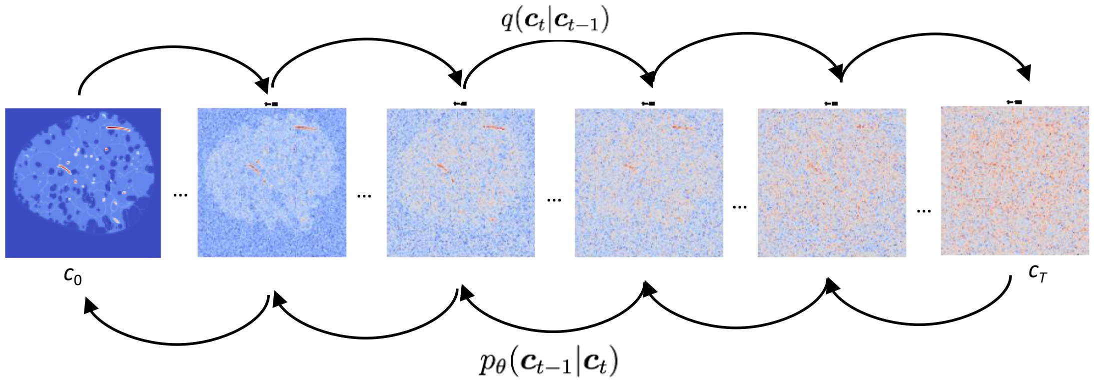
- ✓ Cycle-skipping
- ✓ Local optimization
- ✓ Very-few number of transducers
- ✓ Requires low-frequency reconstruction first

Proposed breast imaging: FWI guided - Generative AI

Inverse Model

$$\min_{\boldsymbol{c}} ||\boldsymbol{p}^r - \boldsymbol{G}(\boldsymbol{c})||_2^2 - \lambda \log p(\boldsymbol{c}).$$

- $p(\boldsymbol{c})$ is the prior distribution function of $\boldsymbol{c}$



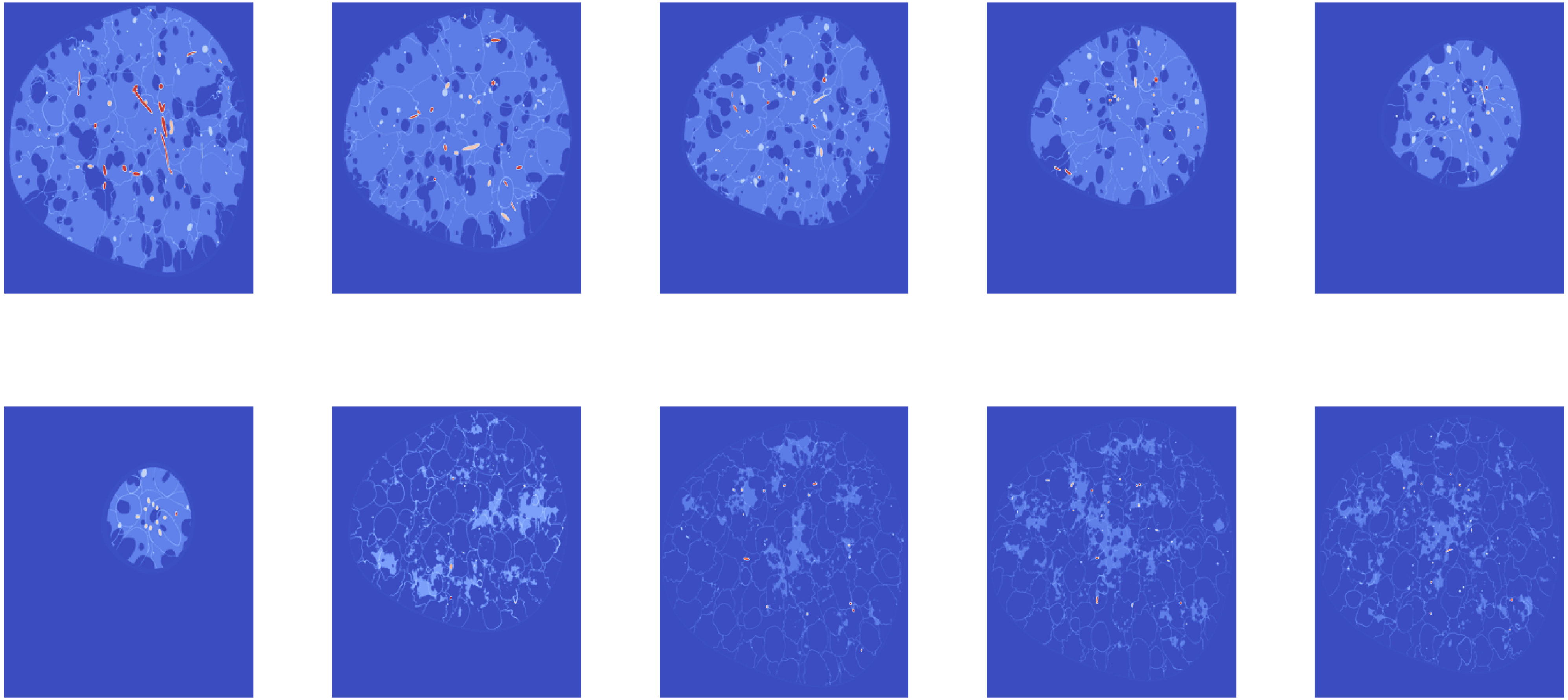
$$p_\theta(\boldsymbol{c}_{t-1}|\boldsymbol{c}_t) = \mathcal{N}(\boldsymbol{\mu}_\theta(\boldsymbol{c}_t, t), \boldsymbol{\Sigma}_\theta(\boldsymbol{c}_t, t)).$$

Jonathan Ho and Ajay Jain and Pieter Abbeel. Denoising Diffusion Probabilistic Models. 2020.
https://doi.org/10.48550/arXiv.2006.11239

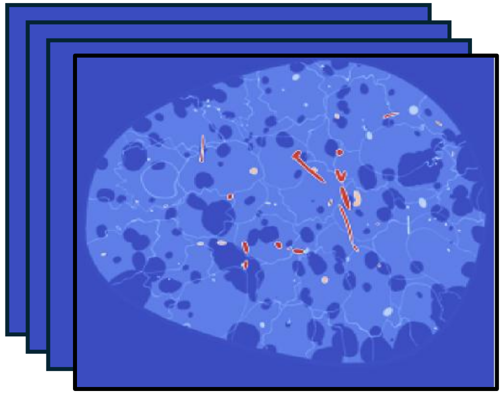
Proposed breast imaging: FWI guided - Generative AI

Set of 2,599 images of MRI breast - data augmentation = 6,000 images

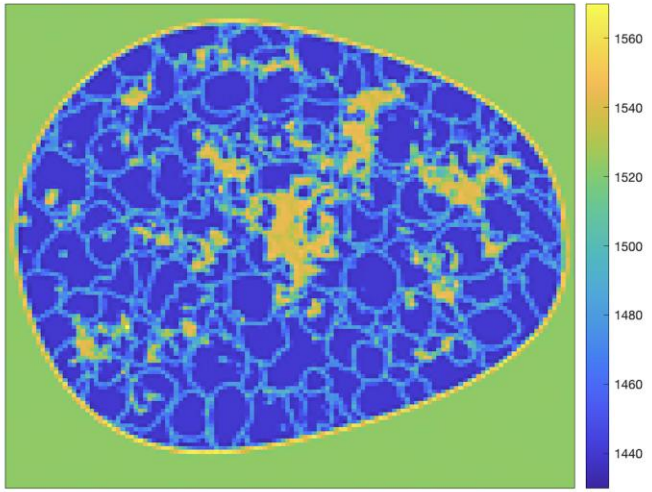
- original size= 1790x2000
- downsample image = 128x128

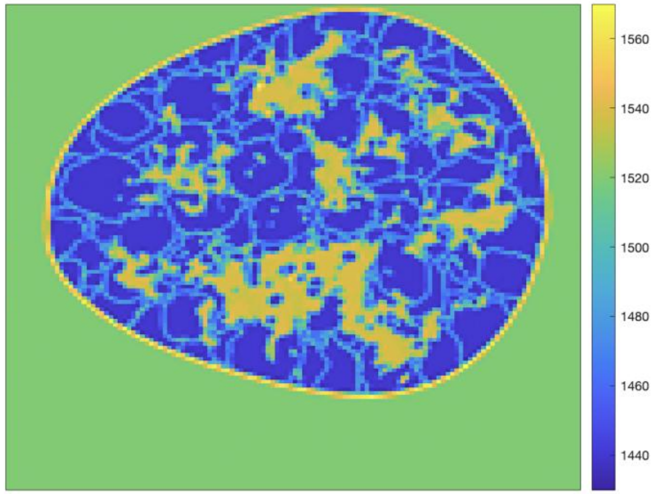


Proposed breast imaging: FWI guided - Generative AI



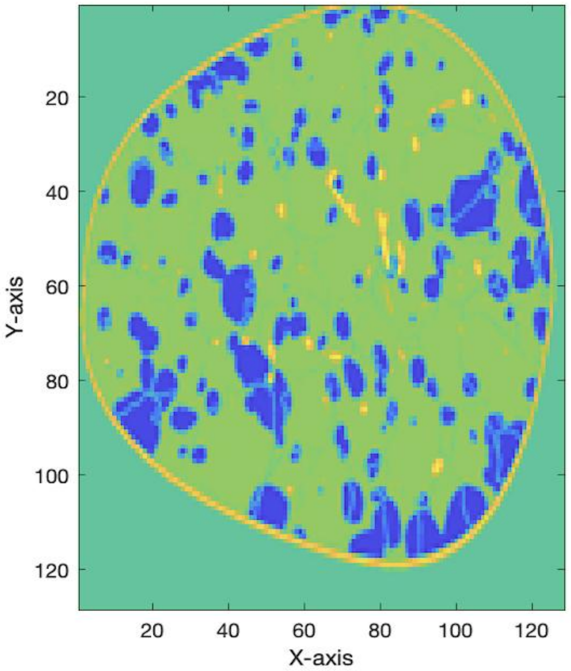
Batch size = 32
Size = 128x128
Timesteps = 1000
lr = 8e-5
Total training timesteps = 10000



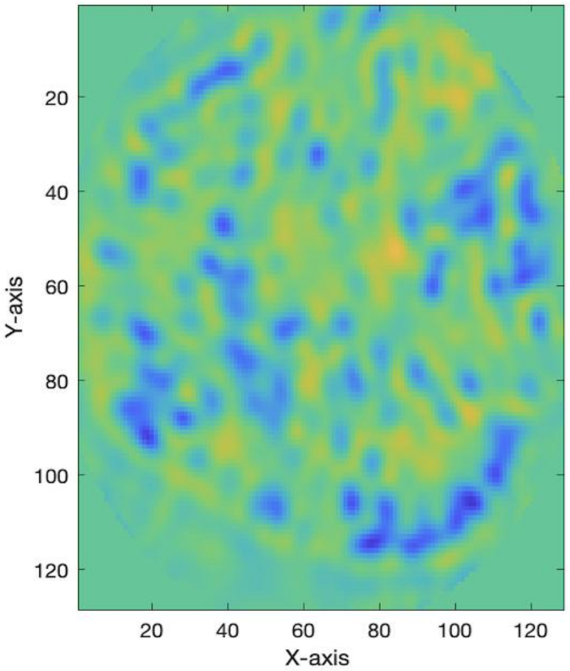


Proposed breast imaging: FWI guided - Generative AI (5 sources)

Ground – Truth Image

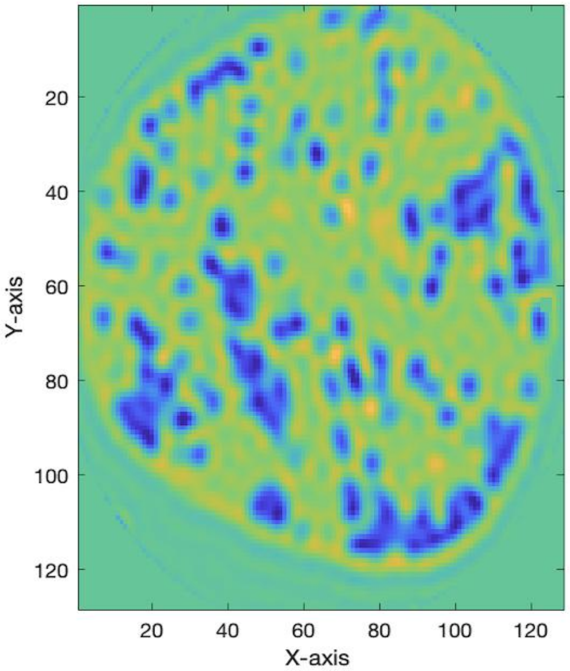


FWI – steepest descent



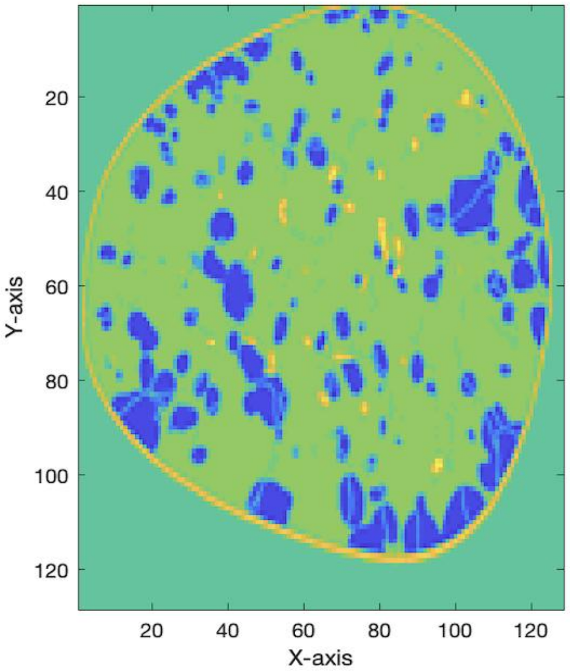
PSNR =38.09dB
SSIM = 0.33

FWI – LBFGS opt. descent



PSNR =41.26dB
SSIM = 0.50

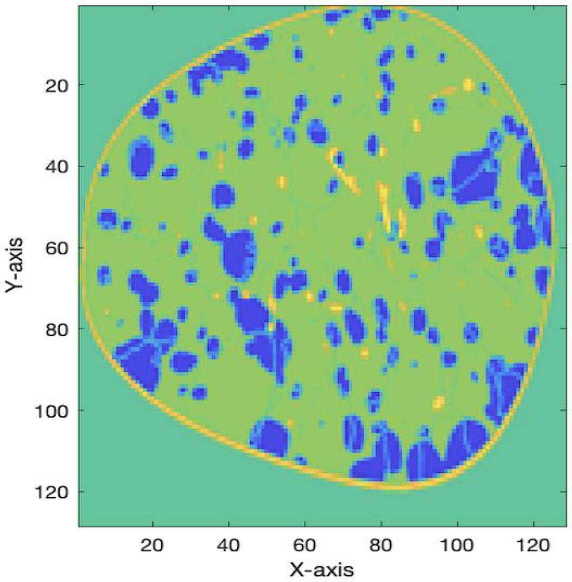
FWI +Diffusion



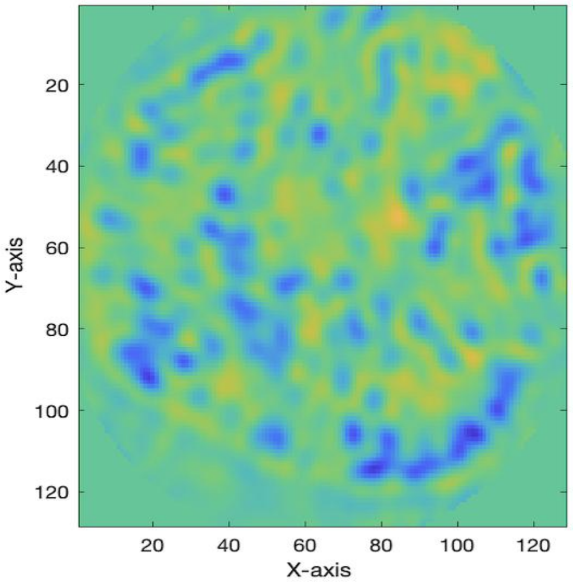
PSNR =39.82dB
SSIM = 0.66

Proposed breast imaging: FWI guided - Generative AI (10 sources)

Ground – Truth Image

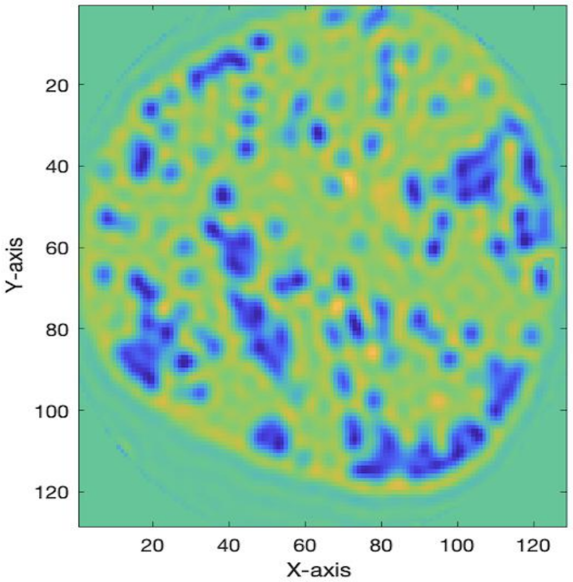


FWI – steepest descent



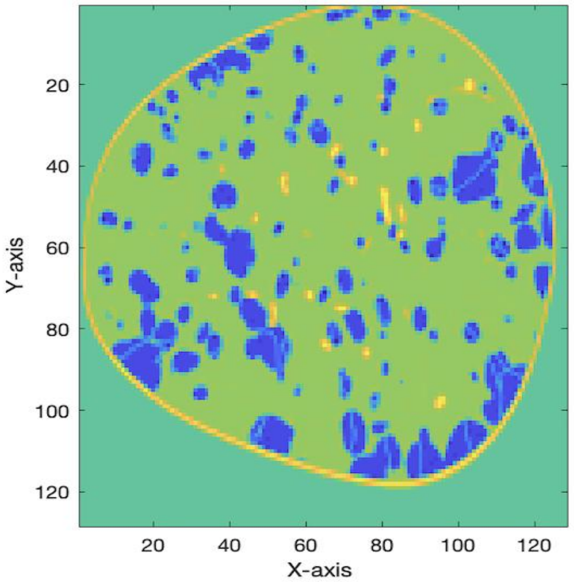
PSNR =39.98dB
SSIM = 0.43

FWI – LBFGS opt. descent



PSNR =41.49dB
SSIM = 0.52

FWI +Diffusion



PSNR =40.44dB
SSIM = 0.64