Part 2.

Danitza Bermejo

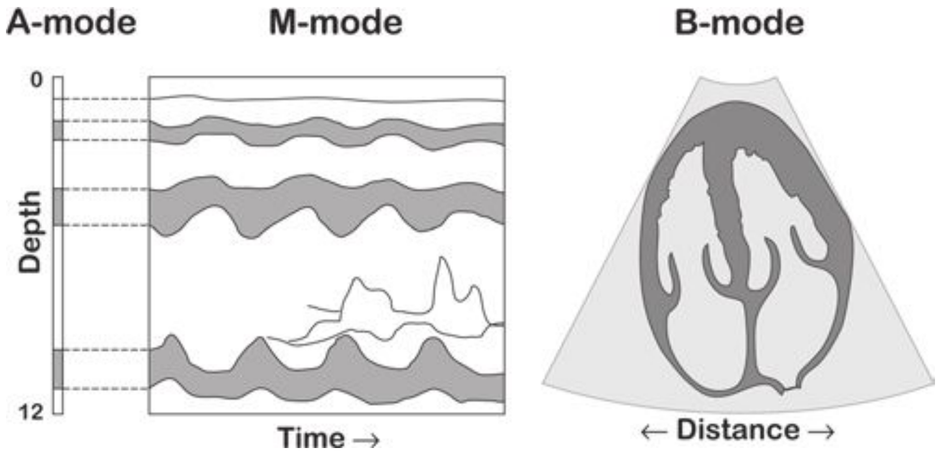
Overview

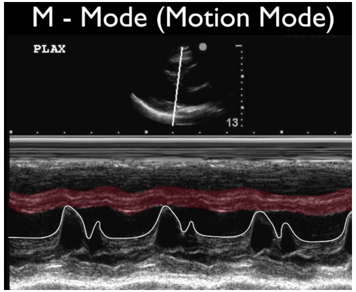
- **Ultrasound imaging**
- **FWI**
- **Diffusion model + FWI**
- **PINN + FWI**

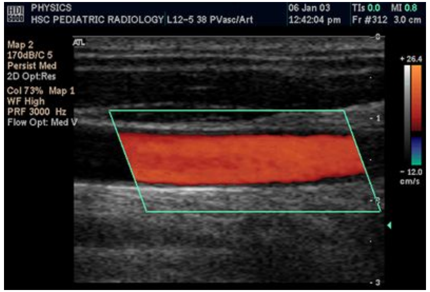
Ultrasound Imaging

Visualize internal body structures by different modes:

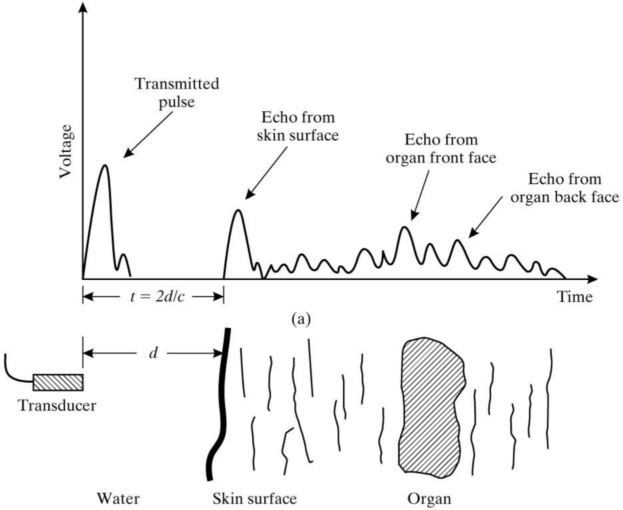
- A-mode (Amplitude mode)
- B-mode (Brightness mode)
- M-mode (Motion mode)
- Doppler mode
  - i. Continuous wave
  - ii. Pulsed wave
  - iii. Color
  - iv. Power

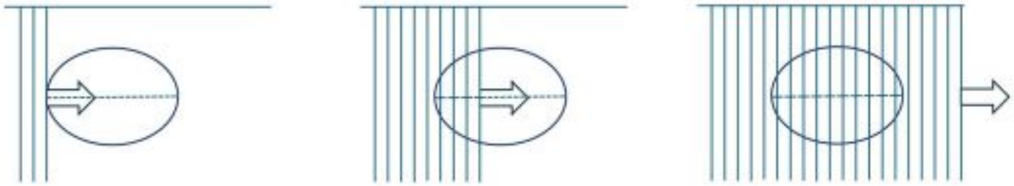


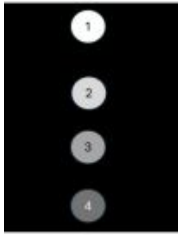




Ultrasound Imaging (B-mode)







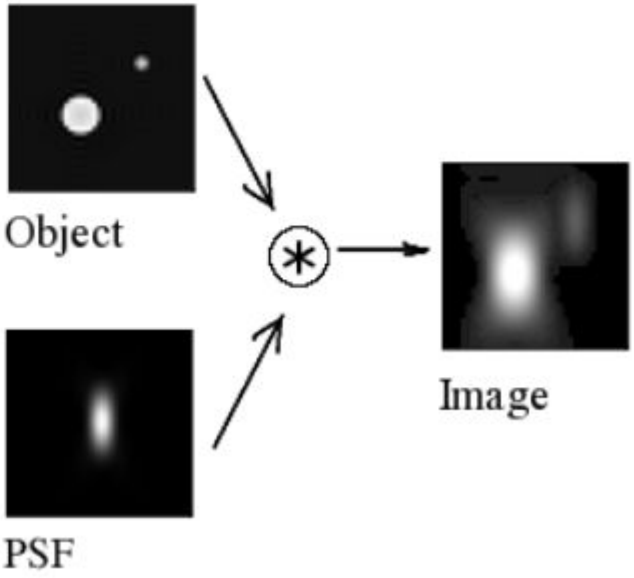
B-mode

Image resolution

Spatial resolution:

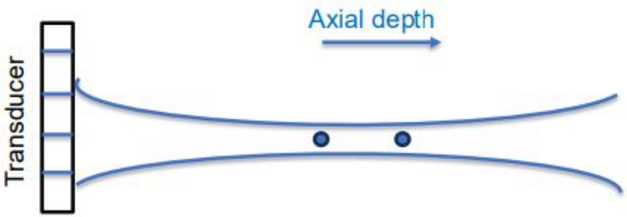
Ability to differentiate structures located close by in space

- Point Spread Function is analyzed.
- Determined by transducer aperture, element directivity, pitch, imaging position, angle.



Spatial resolution

Axial

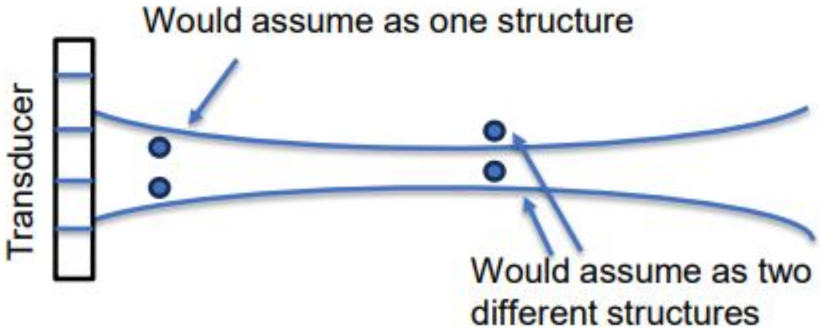


Number of cycles in pulse

$$r_A = \frac{\lambda N}{2} = \frac{c \times \Delta T}{2}$$

Pulse length

Lateral



Axial distance

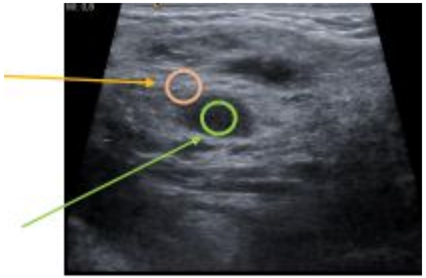
$$r_L = \frac{\lambda z}{D}$$

Transducer diameter

Signal-to-noise ratio

Noise caused by thermal motion of electrons

- Digitization with high number of bits
- Spatial filtering (beamforming)
- Spatial/temporal averaging (compounding)
- Post-processing for noise reduction
- Speckle reduction algorithms

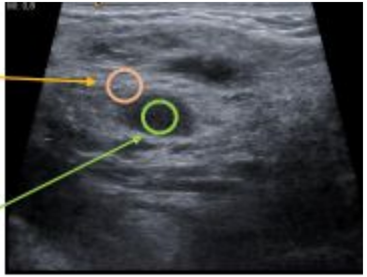


Contrast resolution

$$Contrast = 20log_{10}\left(\frac{S_r}{S_b}\right)$$

Echo signal amplitude of region of interest

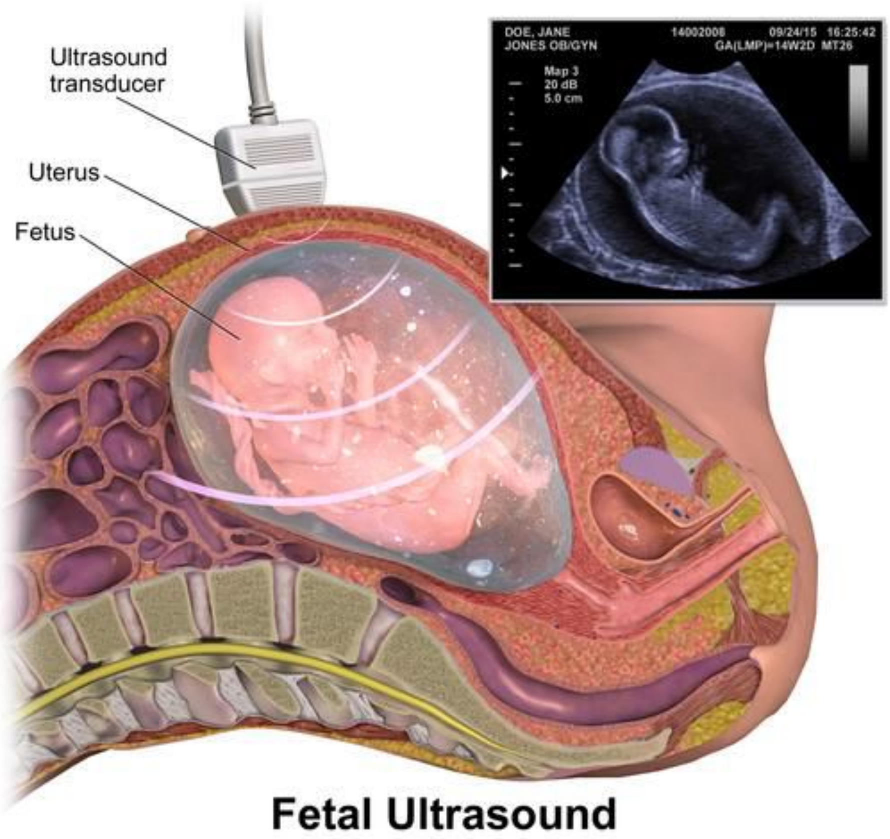
Echo signal amplitude of background



Ability to differentiate two scatterers that differ in intensity

- Compression of dynamic range improves contrast by highlighting weaker scatterers
- Presence of speckle degrades contrast
- Ultrasound contrast agents used to improve contrast from vascular organs

Examples - ultrasound imaging



Renal cyst

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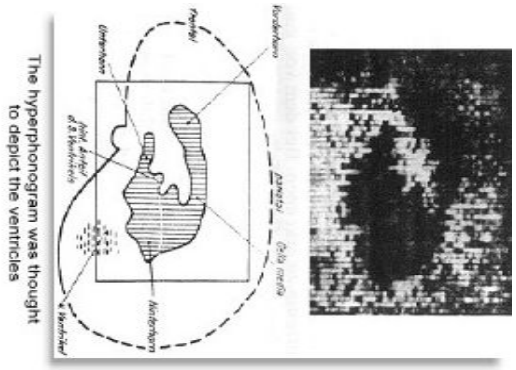
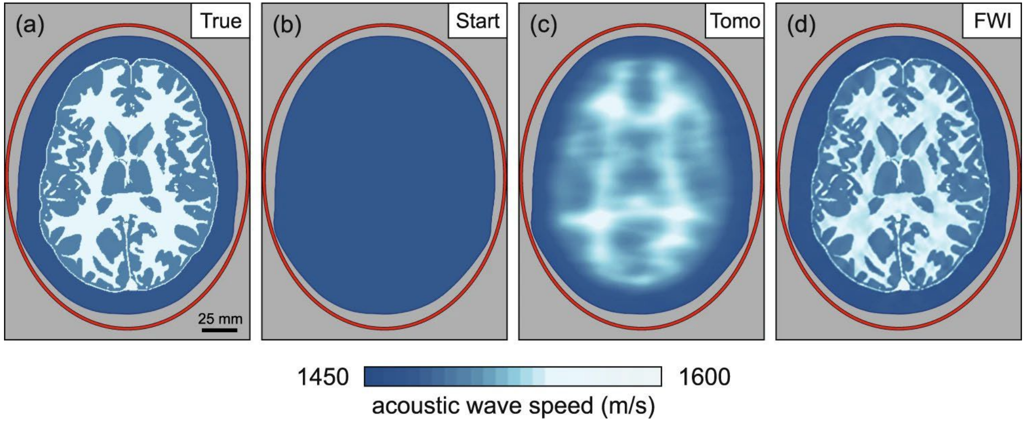
From imaging to inversion

1942



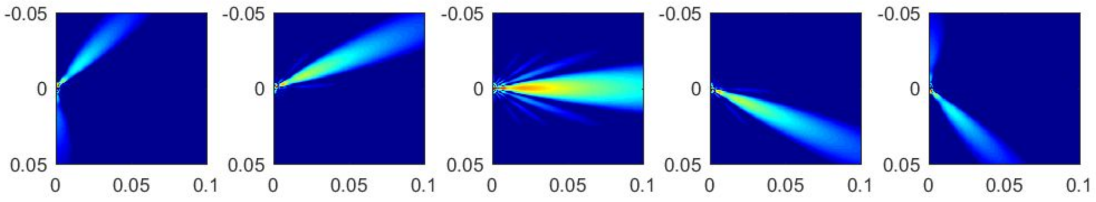
2020

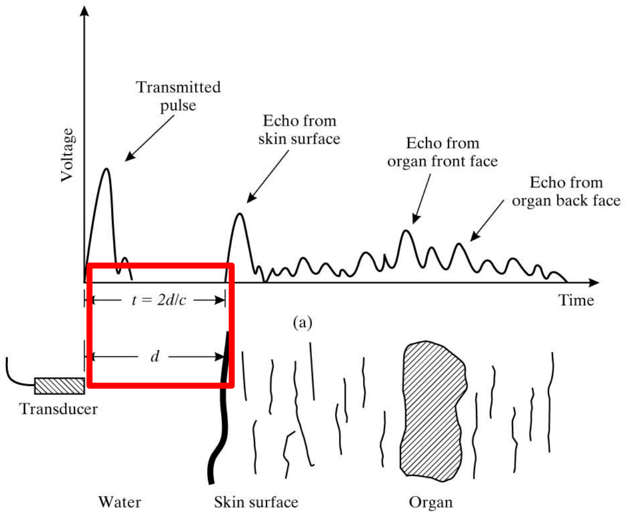
Full-Waveform Inversion (FWI)



Conventional Ultrasound Imaging

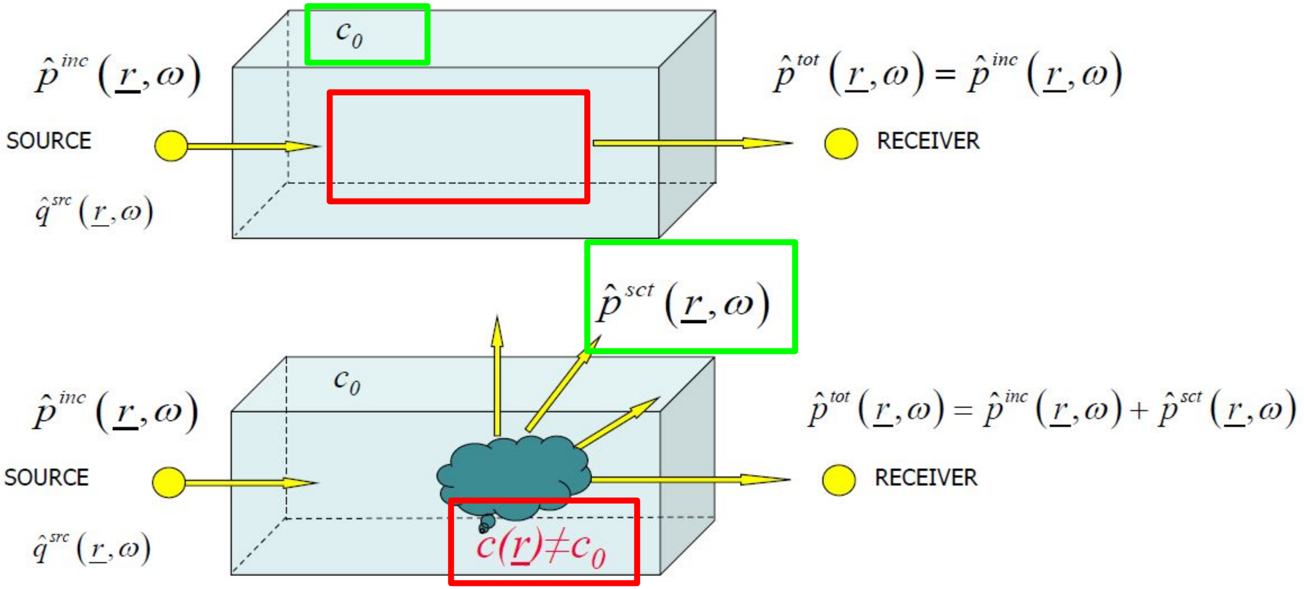
wave propagation





Media

Forward and inverse problem



Acoustic wave equation – forward problem

Wave equation: $$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})}\partial_t^2 p(\vec{x},t) = -S^{pr}(\vec{x},t)$$

Helmholtz equation: $$\nabla^2 p(\vec{x}) + \frac{\omega^2}{c_{bg}^2} p(\vec{x}) = -S^{pr}(\vec{x}) + \omega^2 \underbrace{\left(\frac{1}{c_{bg}^2} - \frac{1}{c^2(\vec{x})}\right)}_{=\chi(\vec{x})} p(\vec{x})$$

Radon transform: $$\Delta t_\beta(\gamma) = \int \frac{1}{c(\vec{x})}\,\mathrm{d}\vec{s}(\vec{x})$$

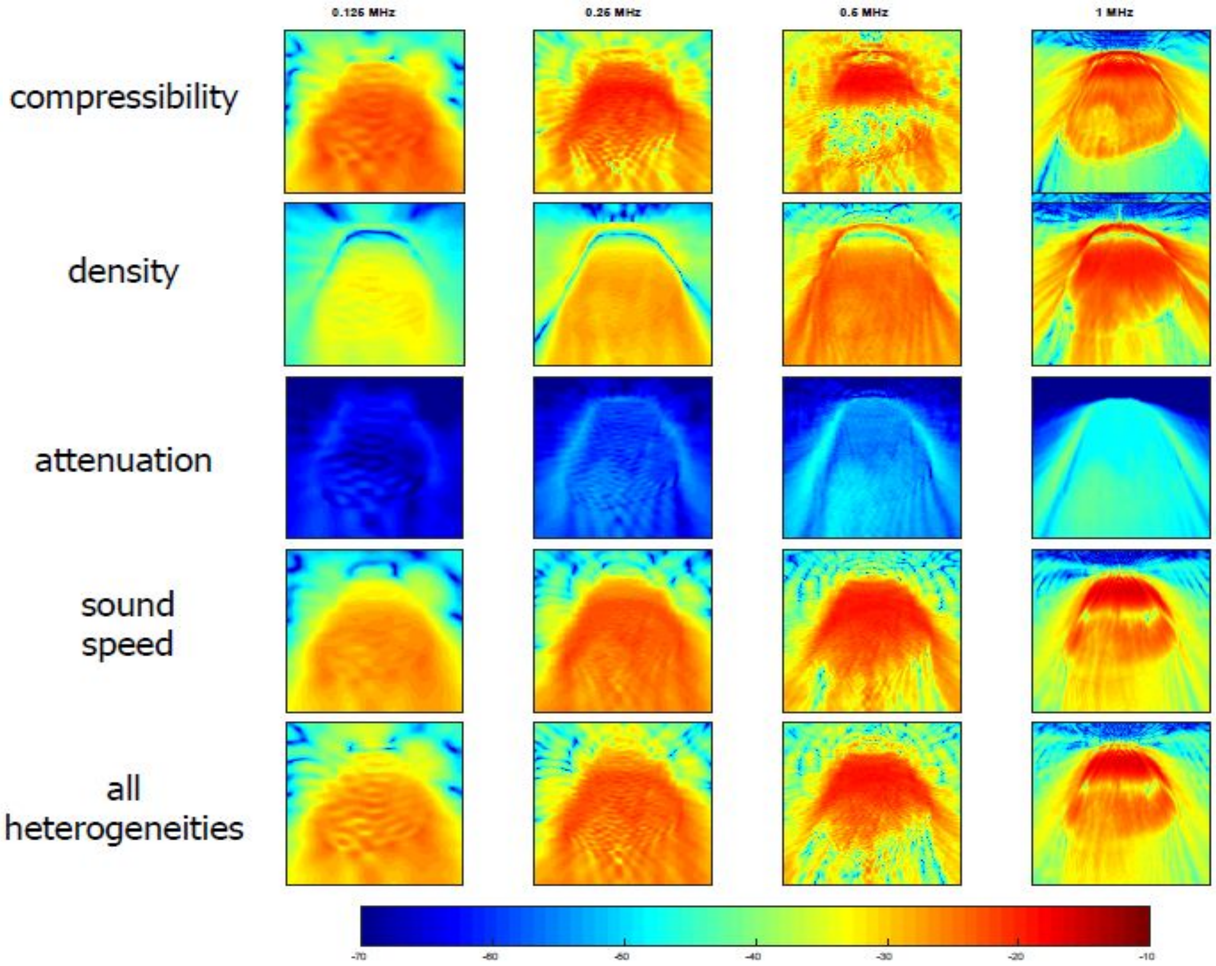
Parabolic approximation:

$$p(k_x,k_y,z_0+\Delta) = p(k_x,k_y,z_0)\,e^{-ik_z\Delta} \quad \text{with} \quad k_z = k_{mean} + (k_x^2+k_y^2)/2k_{mean}$$

Integral equation:

$$\boxed{\text{Born approx.: } p \rightarrow p^{inc}}$$

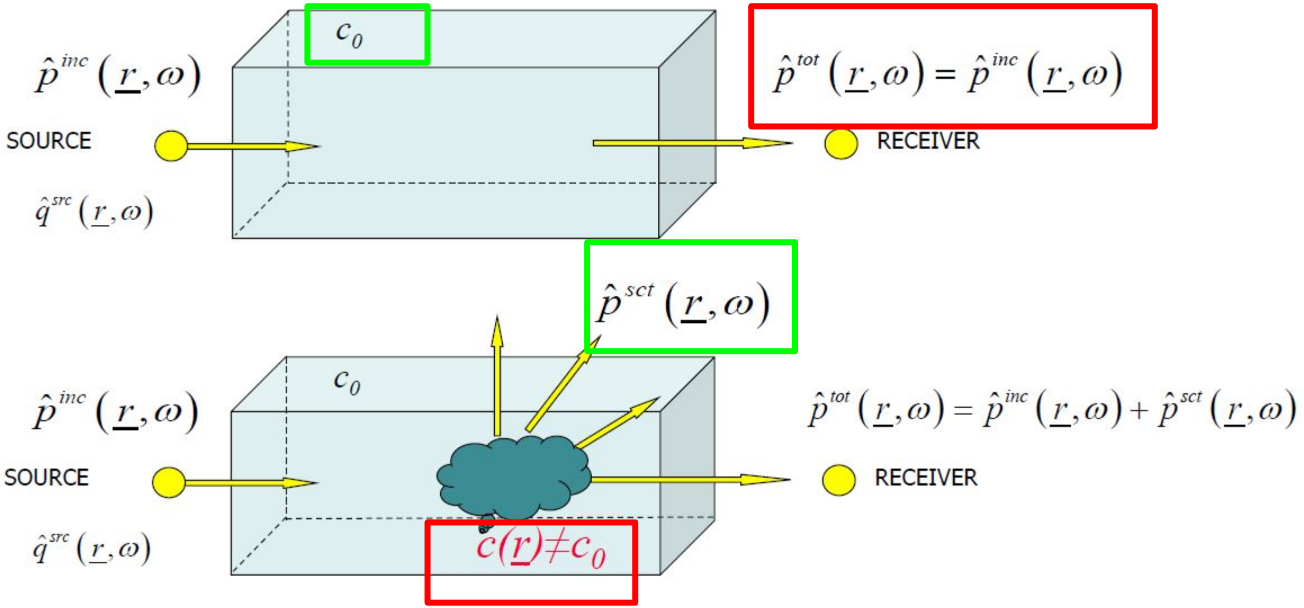
$$p(\vec{x}) = p^{inc}(\vec{x}) - \omega^2 \int G(\vec{x}-\vec{x}\,')\,\chi(\vec{x}\,')\,p(\vec{x}\,')\,\mathrm{d}V(\vec{x}\,') \quad \text{with} \quad G(\vec{x}) = \frac{e^{-ik|\vec{x}|}}{4\pi|\vec{x}|}$$



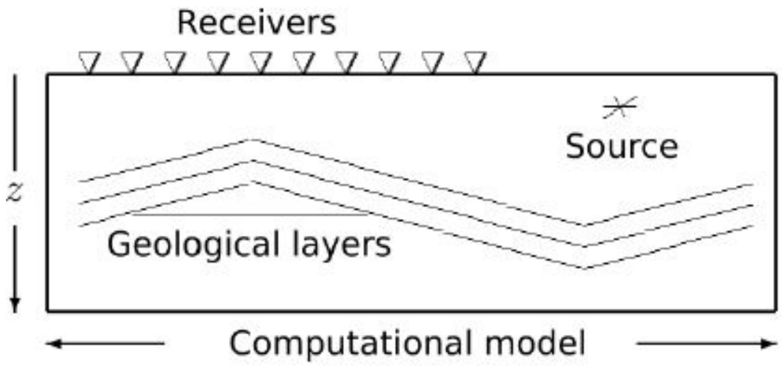
0.125 MHz
0.25 MHz
0.5 MHz
1 MHz
compressibility
density
attenuation
sound speed
all heterogeneities
-70
-60
-50
-40
-30
-20
-10

Media

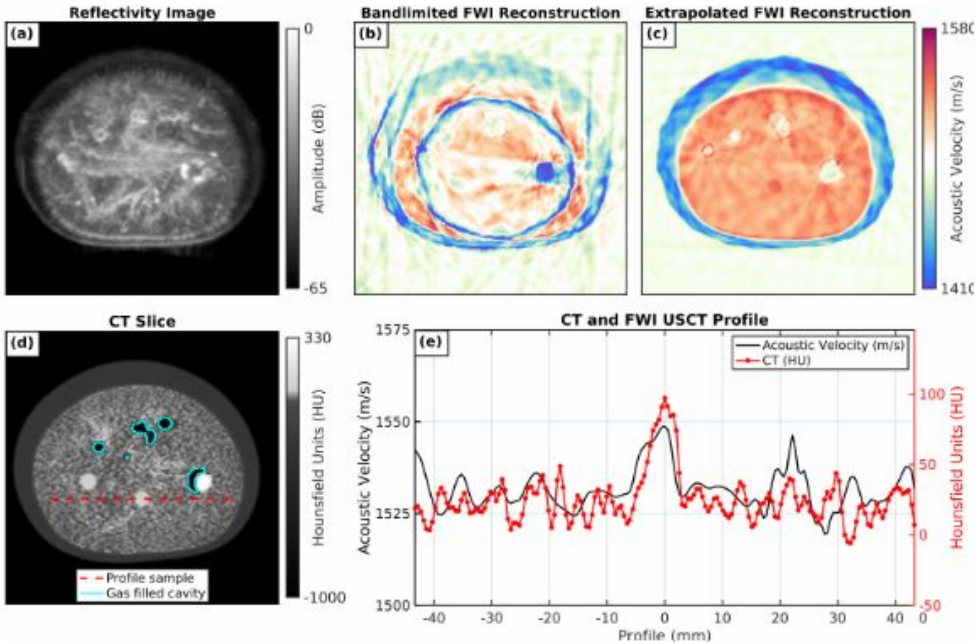
Forward and inverse problem



Example

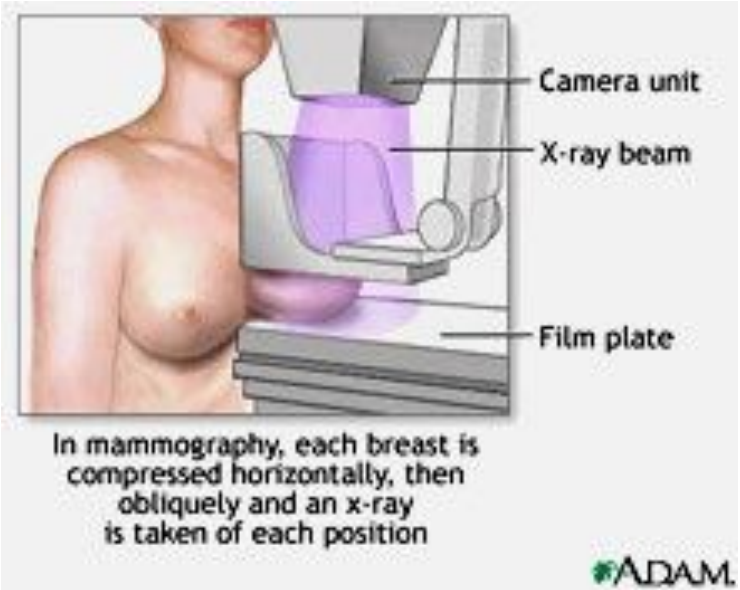


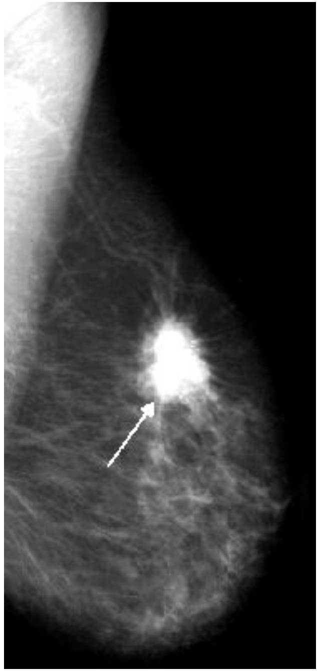
Receivers
Source
z
Geological layers
Computational model



Reflectivity Image
(a)
Amplitude (dB)
0
-65
Bandlimited FWI Reconstruction
(b)
Extrapolated FWI Reconstruction
(c)
Acoustic Velocity (m/s)
CT Slice
(d)
Profile sample
Gas filled cavity
Hounsfield Units (HU)
330
-1000
CT and FWI USCT Profile
(e)
Acoustic Velocity (m/s)
CT (HU)
Profile (mm)
1575
1550
1525
1500
100
50
0
-50
-40
-30
-20
-10
0
10
20
30
40

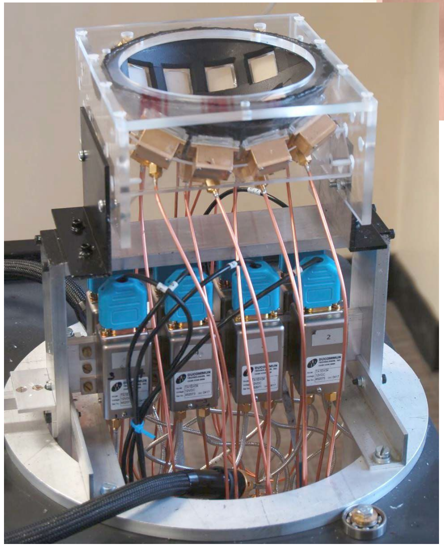
Breast ultrasound



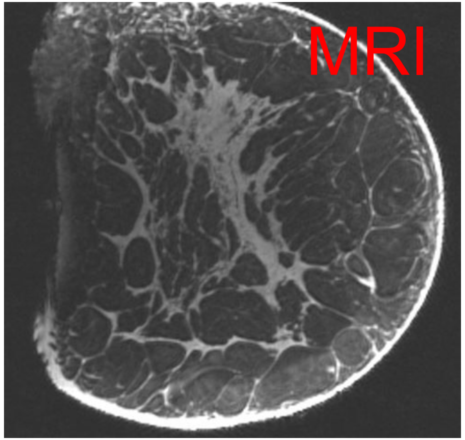


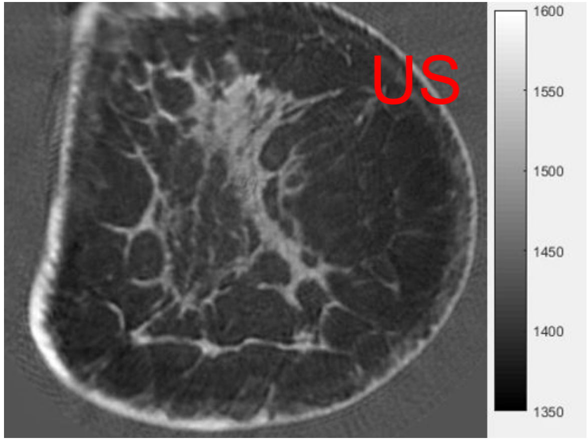
Breast ultrasound

Breast ultrasound

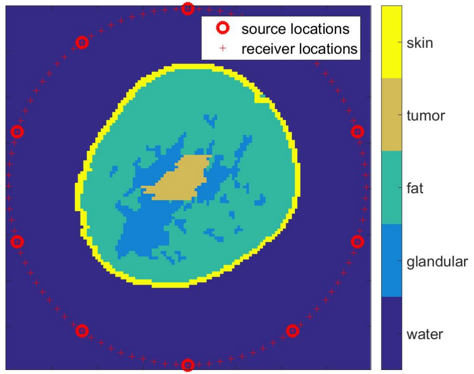


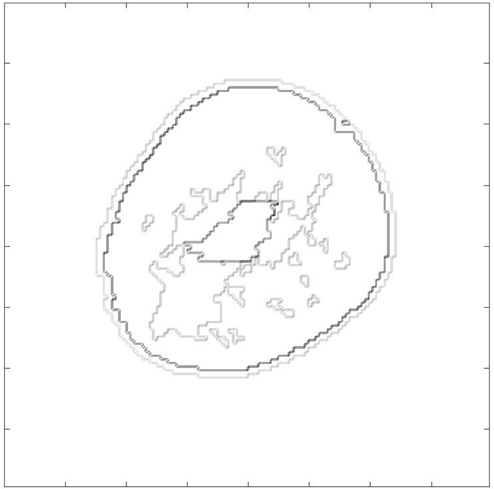
Breast ultrasound

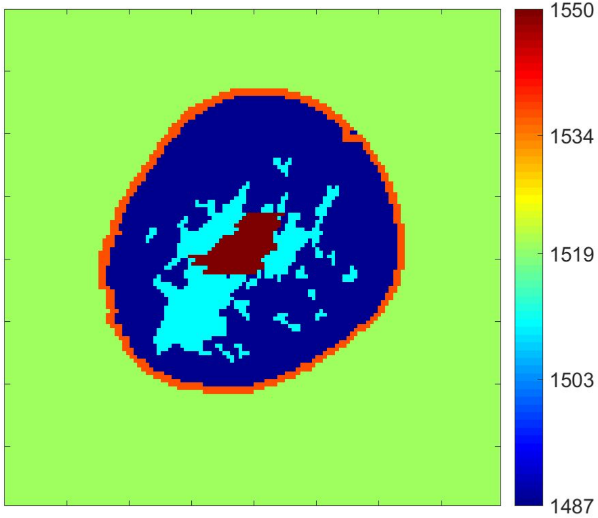




Reflectivity versus quantitative imaging







Speed of sound
Density
Compressibility
Absorption

Tissue characterisation

Overview

- Ultrasound imaging
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- **Diffusion model + FWI**
- PINN + FWI

Denoising Diffusion Probabilistic Models (DDPMs)

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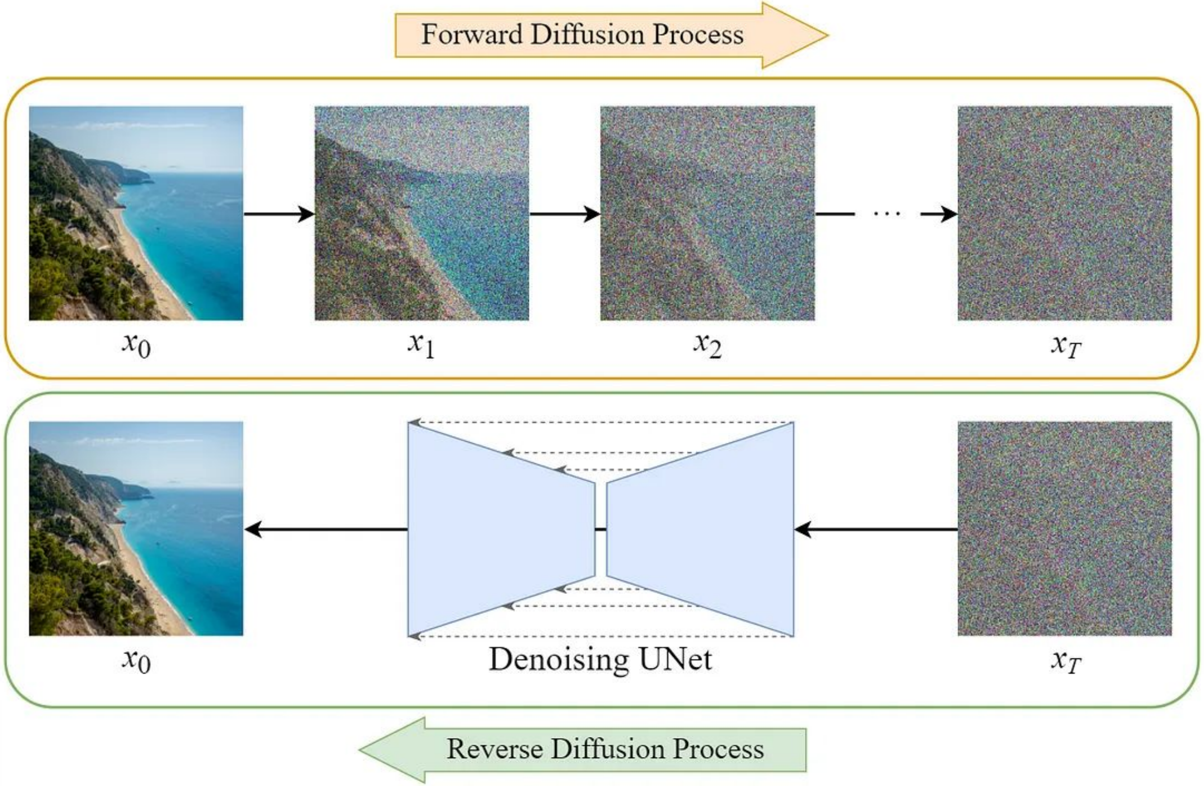
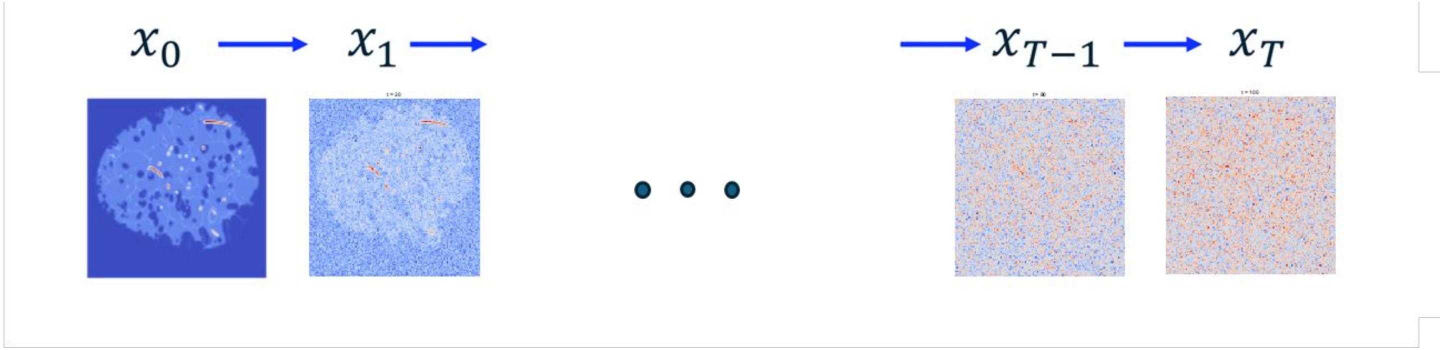
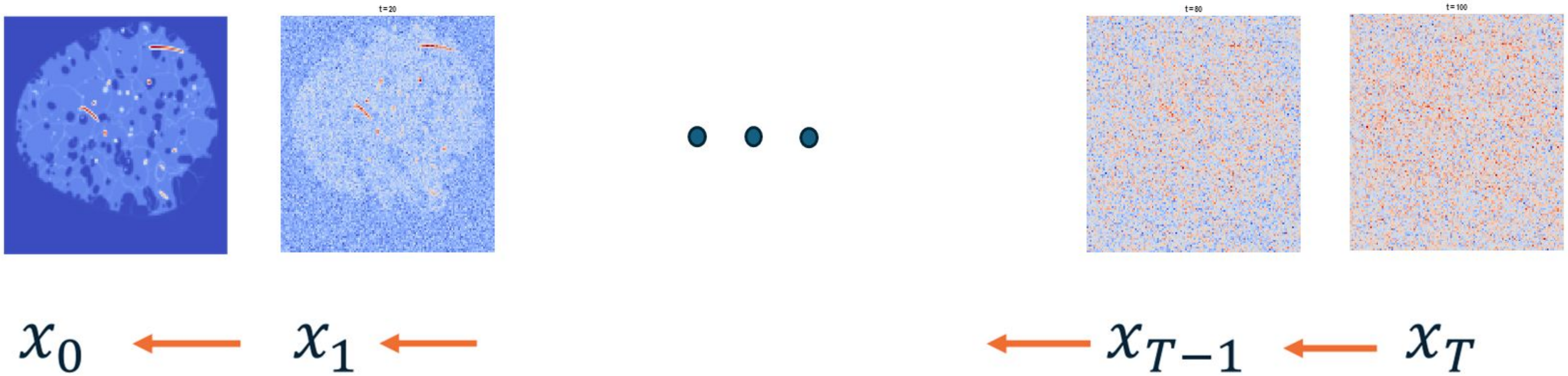


1. Forward Diffusion Process (enconding)

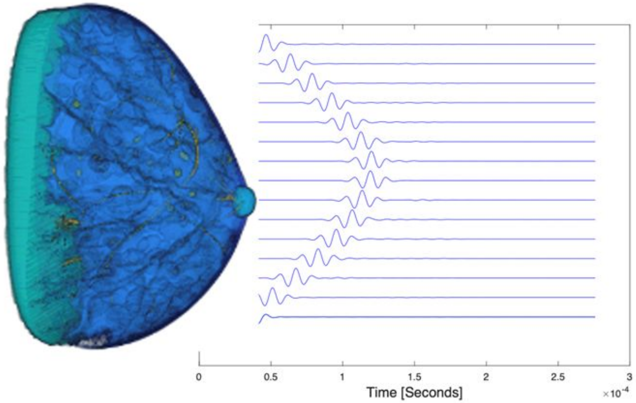


Take an image from a dataset and add noise with Gaussian distribution: **N(0,I),** such that the structure of the data is destroyed

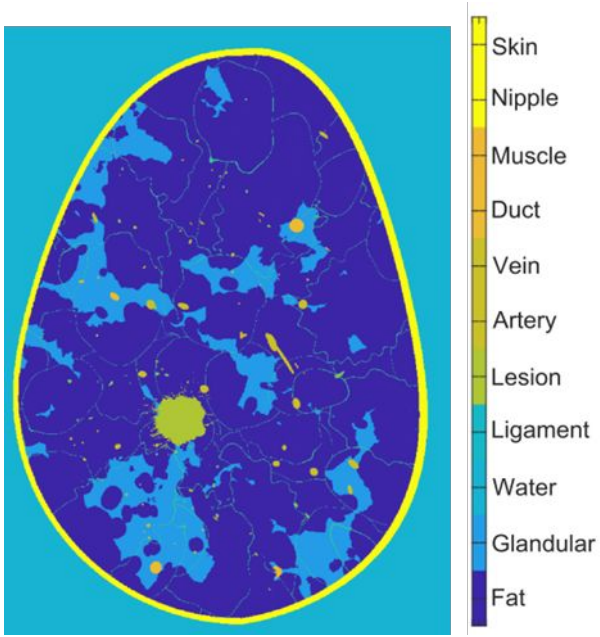
2. Denoising Diffusion Process (decoding)



US breast imaging using Full Waveform Inversion (FWI)



Time-domain signals

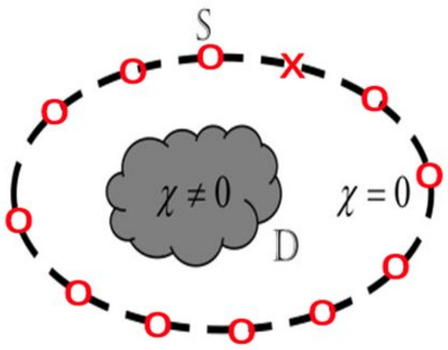


Tissue	Speed-of-sound propagation [m/s]
Fat	1440.0
Glandular	1505.0
Water	1520.0
Ligament	1525.0
Lesion	1572.0
Artery	1578.2
Vein	1578.2
Duct	1588.0
Muscle	1588.4
Nipple	1624.0
Skin	1624.0

Quantitative image of:
Speed of sound or density

US breast imaging using FWI

Forward Model



$$\frac{1}{c^2(x,z)}\frac{\partial^2 p(x,z,t)}{\partial^2 t} + \frac{\partial^2 p(x,z,t)}{\partial^2 x} + \frac{\partial^2 p(x,z,t)}{\partial^2 z} = f(x_0,z_0,t).$$

(PDE)

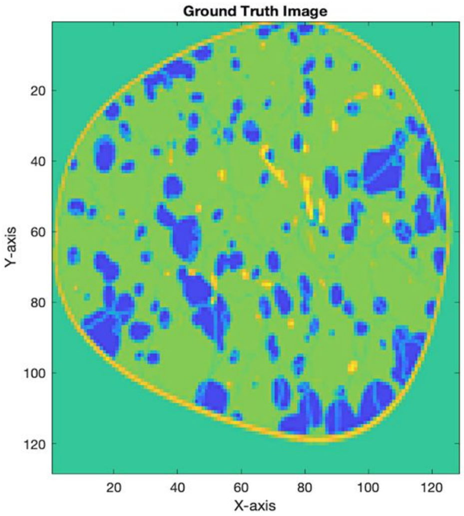
- $p(x,z,t)$ pressure field
- $c(x,z)$ speed-of-sound (SoS) of the breast
- $f(x_0,z_0,t)$ source wavelet

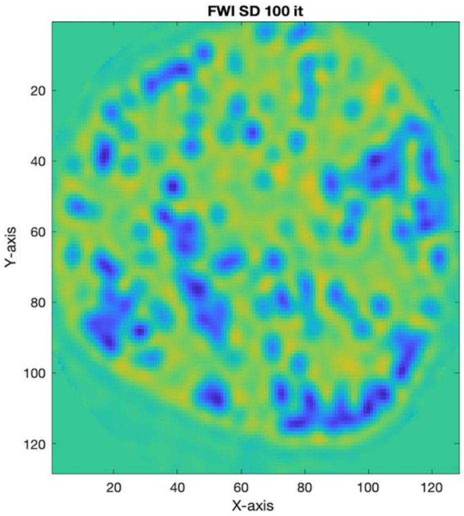
$$\boldsymbol{p}^r = \boldsymbol{G}(\boldsymbol{c}) + \boldsymbol{\eta}$$

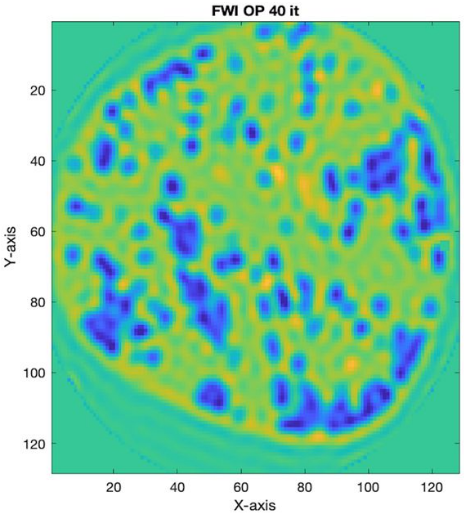
- $\boldsymbol{p}^r \in R^{N_R \times N_t}$ pressure field at the receiver's location
- $\boldsymbol{G}$ non-linear operator representing the AWE
- $\boldsymbol{c} \in R^{N_x \times N_z}$ SoS of the breast
- $\boldsymbol{\eta} \in R^{N_R \times N_t}$ noise

US breast imaging using FWI

Ground – Truth Image	FWI – steepest descent	FWI – LBFGS opt. descent







FWI – steepest descent:

$$\mathbf{c}^{k+1} = \mathbf{c}^k - \beta \cdot g(\mathbf{c}^k)$$

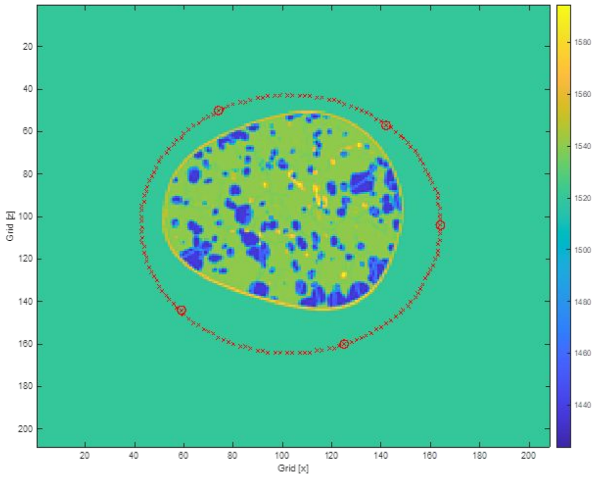
PSNR =39.98dB
SSIM = 0.43

FWI – LBFGS opt. descent:

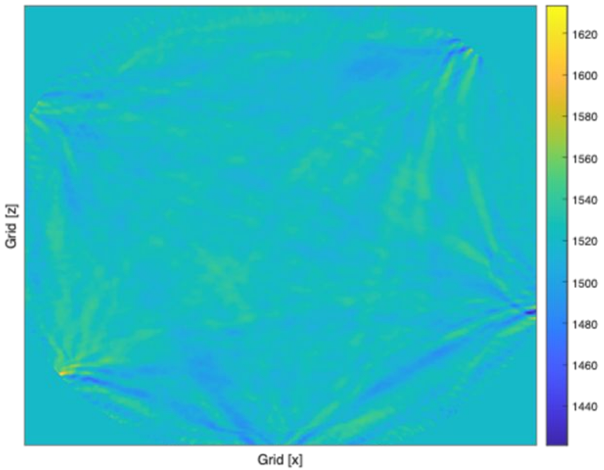
$$\mathbf{c}^{k+1} = \mathbf{c}^k - \beta^k(\mathbf{c}^k) \cdot g(\mathbf{c}^k)$$

PSNR =41.49dB
SSIM = 0.52

US breast imaging using FWI



Δx ≈ 0.5 mm
Δy ≈ 0.5 mm
Δt = 0.08 μs
N_t = 3200
F = 0.2MHz



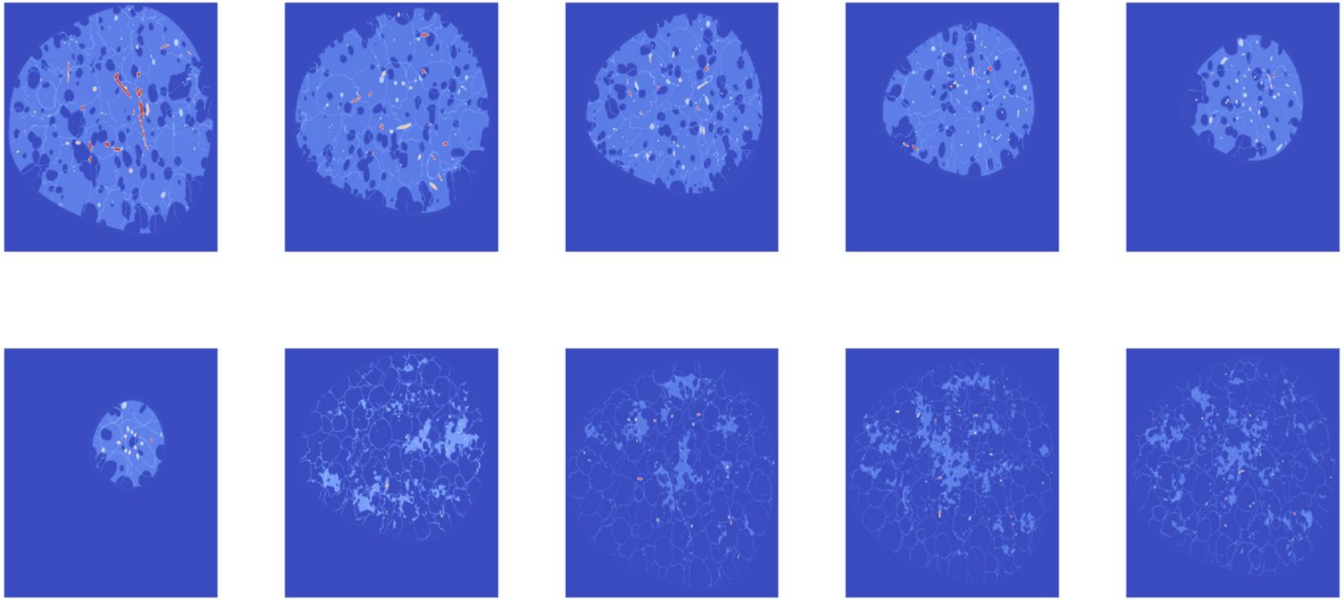
FWI – steepest descent/optimal descent

- ✓ Cycle-skipping
- ✓ Local optimization
- ✓ Very-few number of transducers
- ✓ Requires low-frequency reconstruction first

Proposed breast imaging: FWI guided - Generative AI

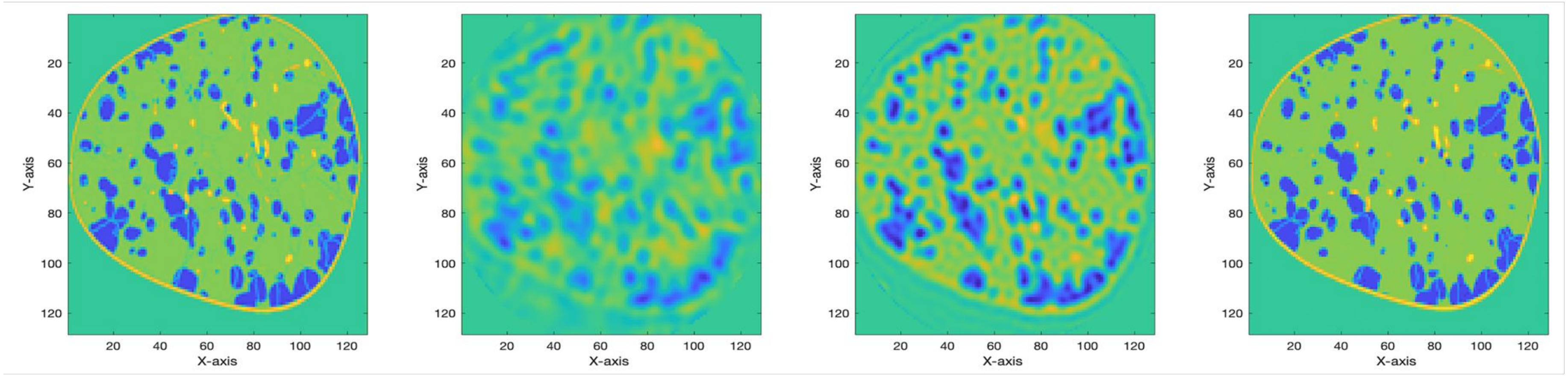
Set of 2,599 images of MRI breast - data augmentation = 6,000 images

- original size= 1790x2000
- downsample image = 128x128



Proposed breast imaging: FWI guided - Generative AI

Ground – Truth Image | FWI – steepest descent | FWI – LBFGS opt. descent | FWI +Diffusion



PSNR =39.98dB
SSIM = 0.43

PSNR =41.49dB
SSIM = 0.52

PSNR =40.44dB
SSIM = 0.64

Overview

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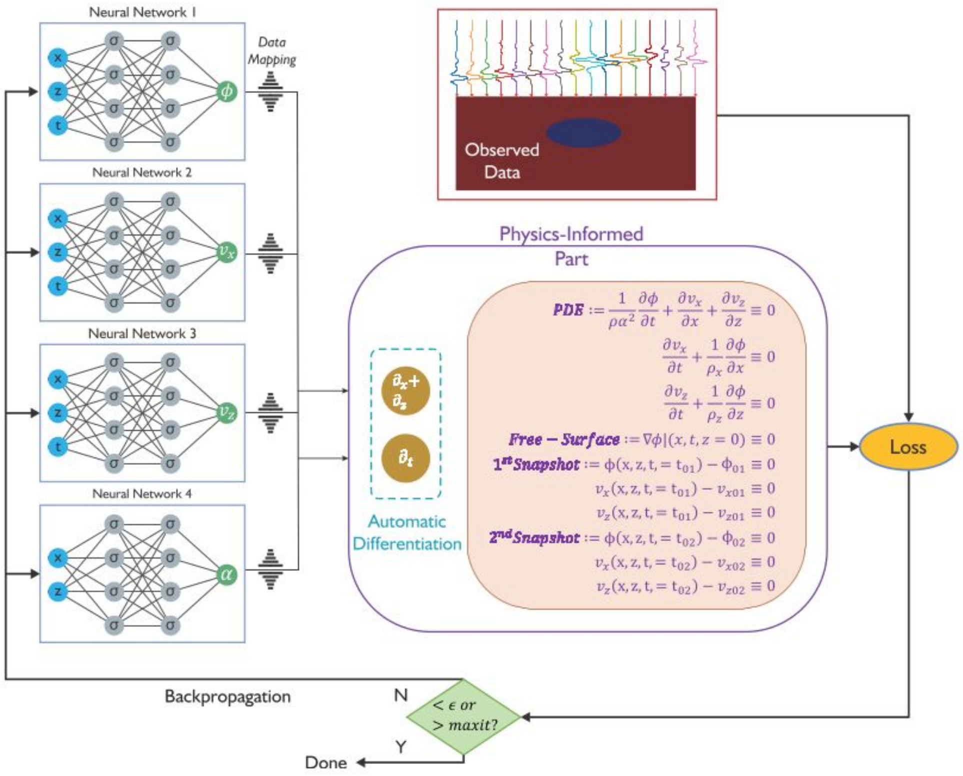
Physics-Informed Neural Networks (PINNs)

Physics-Informed Neural Network for the Seismic Velocity Problem using Neural Tangent Kernels [Lopez et al., 2024].

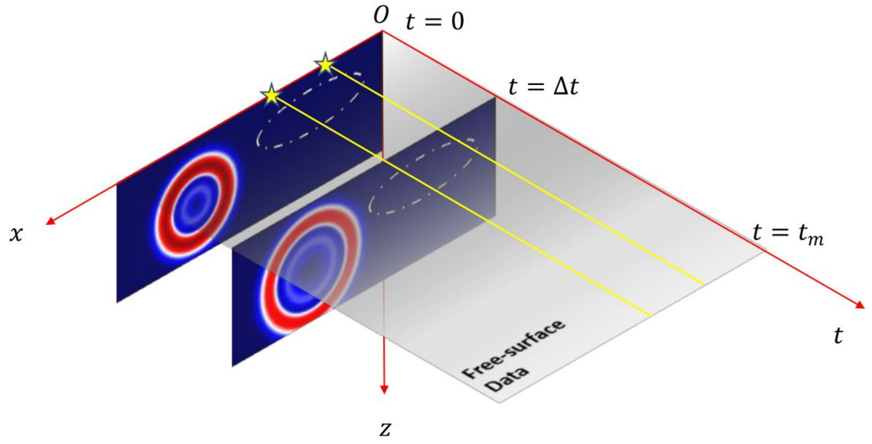
Seismological.



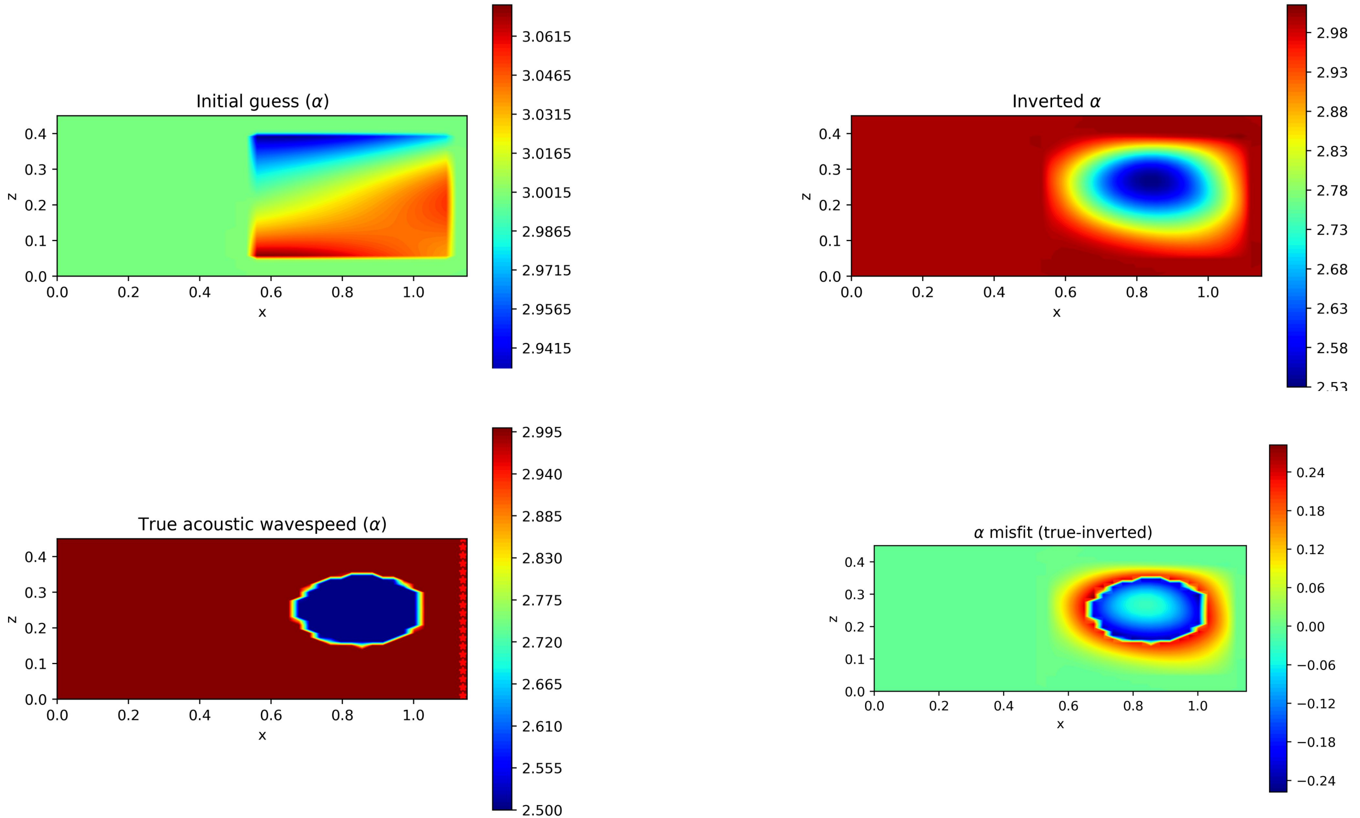
Physics-Informed Neural Networks (PINNs)

[Rasht-Behesht et al., 2022] present the first FWI for seismological applications using PINNs.

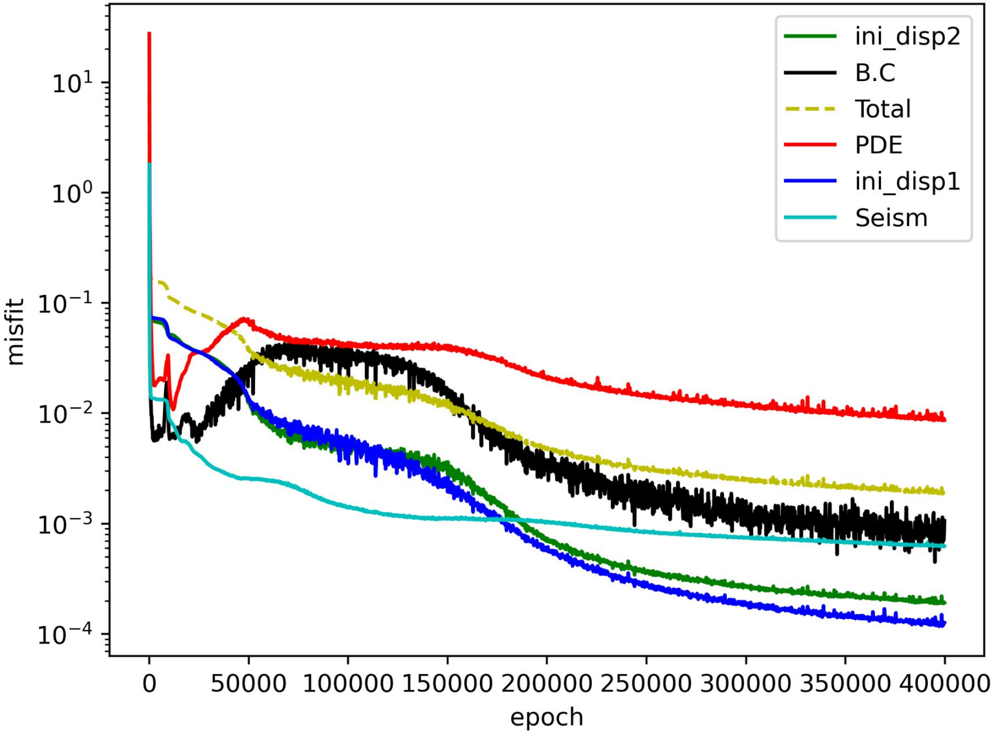
- They define a fully connected feed-forward NN with an input layer consisting of the physical coordinates x, z and time t, L hidden layers and an output layer representing the scalar acoustic wave potential $\phi \subset R$.
- The various other physical variables, such as displacement or pressure, are obtained through the automatic differentiation of the wave potential NN using TensorFlow.
- They choose $\sigma = \tanh(\cdot)$ or $\sin(\cdot)$ as the nonlinear activation function for all NNs.



Physics-Informed Neural Networks (PINNs)



Physics-Informed Neural Networks (PINNs)



Más …

https://colab.research.google.com/drive/117X5Lvmb99h6nR8lKN1R3t-FyW1R-Nb4?usp=sharing

Data: https://we.tl/t-kZYRDXzQ2C

GitHub: https://github.com/yvid27/DSP_Bucaramanga