Double Diffusion and Oxygen Mixing

Overview

1. Background Knowledge about Double Diffusion
    a. Navier Stokes Equations
    b. Salt-Finger Convection and Oscillatory Diffusion
2. Simulations with different parameters.
    a. Nek5000 Solver
    b. Using different Rp value with both types of diffusion
3. Oxygen distribution and observations
    a. Oxygen mixing
    b. Oxygen re-initialized with layers formed

Double Diffusion Equations

Non-dimensionalized Boussinesq Equation

1. $\frac{1}{Pr}\left(\frac{\partial \hat{\mathbf{v}}}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \hat{\mathbf{v}}\right) = -\nabla \hat{p} + (\hat{T}' - \hat{S}')\mathbf{k} + \nabla^2 \hat{\mathbf{v}},$
2. $\frac{\partial \hat{T}'}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \hat{T}' \pm \hat{w} = \nabla^2 \hat{T}',$
3. $\frac{\partial \hat{S}'}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \hat{S}' \pm \frac{\hat{w}}{R_p} = \tau \nabla^2 \hat{S}',$
4. $\nabla \cdot \mathbf{v} = 0.$

Linear Equation of State

$$\frac{\rho - \rho_0}{\rho_0} = -\alpha(T - T_0) + \beta(S - S_0).$$

Non Dimensionalized with $d = \left(\frac{\kappa_T \nu}{g\alpha|\overline{T_z}|}\right)^{1/4},$

Two types of Double Diffusion

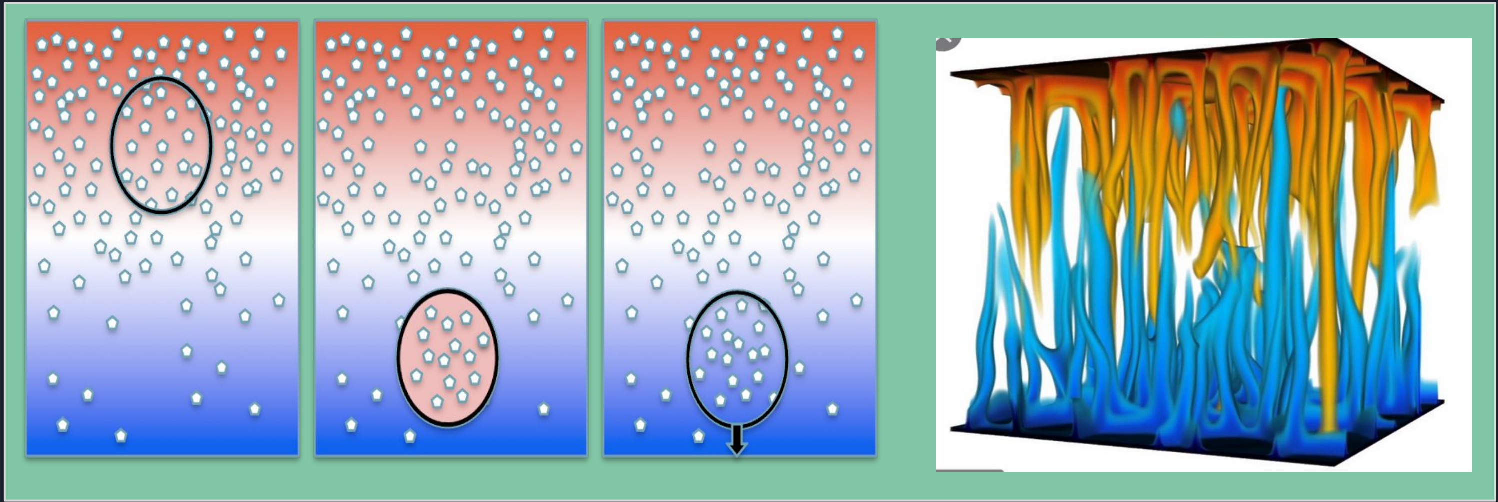
1. Salt-Finger Convection
2. Oscillatory double diffusion

From the linear equation of state, we can derive an equation for the density gradient

$$\frac{\partial \rho}{\partial y} = -\alpha\left(\frac{\partial \mathbf{T}}{\partial y}\right) + \beta\left(\frac{\partial \mathbf{S}}{\partial y}\right)$$

We can achieve double diffusion if $\frac{\partial \rho}{\partial y} < 0$, with the convention that the value of y (depth) decreases as we go deeper. (so density would increase with depth)

Salt-Finger Diffusion



This situation occurs when:

$$\frac{\partial \mathbf{T}}{\partial y} > 0 \qquad \frac{\partial \mathbf{S}}{\partial y} > 0$$

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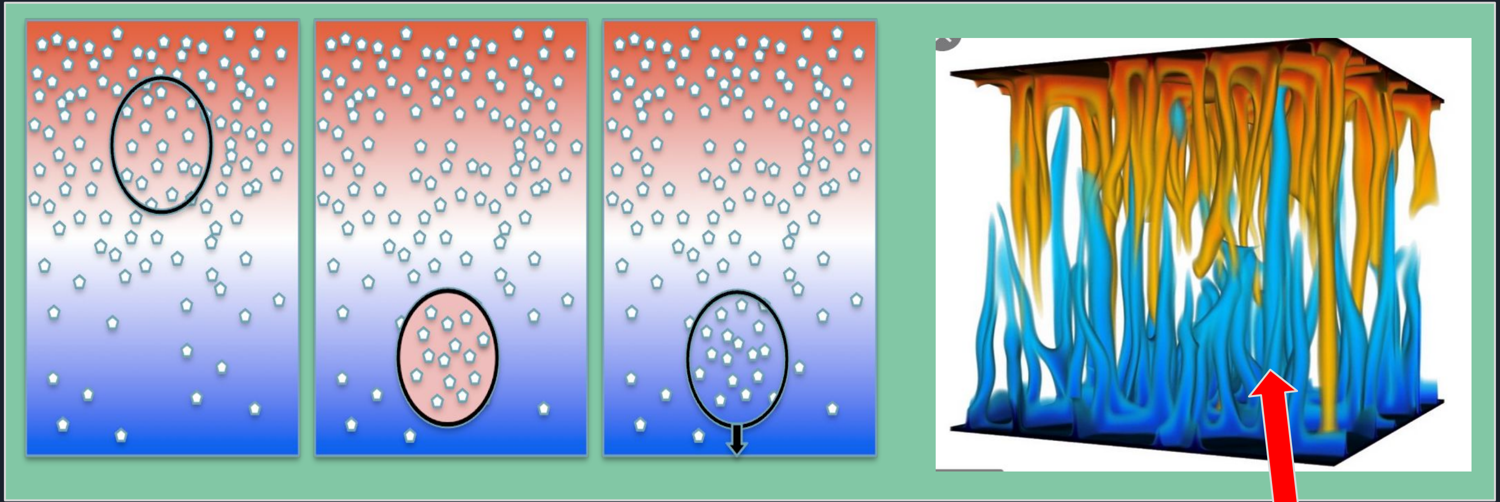

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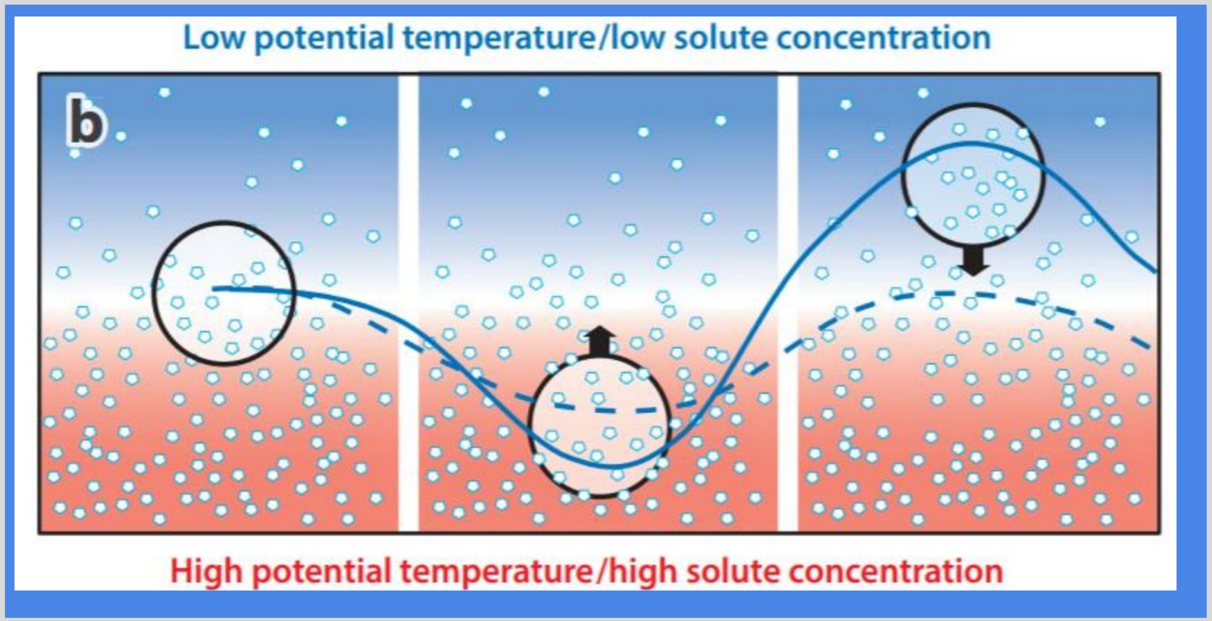
Salt Fingers

Oscillatory Diffusion



Occurs when T and S are increasing with depth.

$$\frac{\partial \mathbf{T}}{\partial y} < 0 \qquad \frac{\partial \mathbf{S}}{\partial y} < 0$$

The Density Ratio (Rp)

If we recall the non-dimensionalized Boussinesq equations, we can derive several non-dimensionalized numbers. One of those numbers is the Density Ratio:

Salt Fingers:

$$\boxed{R_p = (\alpha/\beta)(\overline{T_z}/\overline{S_z})}$$

Osc. Diff. :

$$\boxed{R_p = (\beta/\alpha)(\bar{S}_z/\bar{T}_z)}$$

Simulations

1. Various Simulations using Nek5000 (solver) and VisIt/Python (For visualization)
   a. Rp = 1.1 (Salt-Fingers)
   b. Rp = 1.3 (Salt-Fingers)
   c. Rp = 1.8 (Salt-Fingers)
   d. Rp = 1.05 (Oscillative Diffusion)
   e. Rp = 2.0 (Oscillative Diffusion)

With these simulations we can observe different regimes of the model with regards to the temperature and the salinity, how certain parameters behave and how oxygen is distributed.

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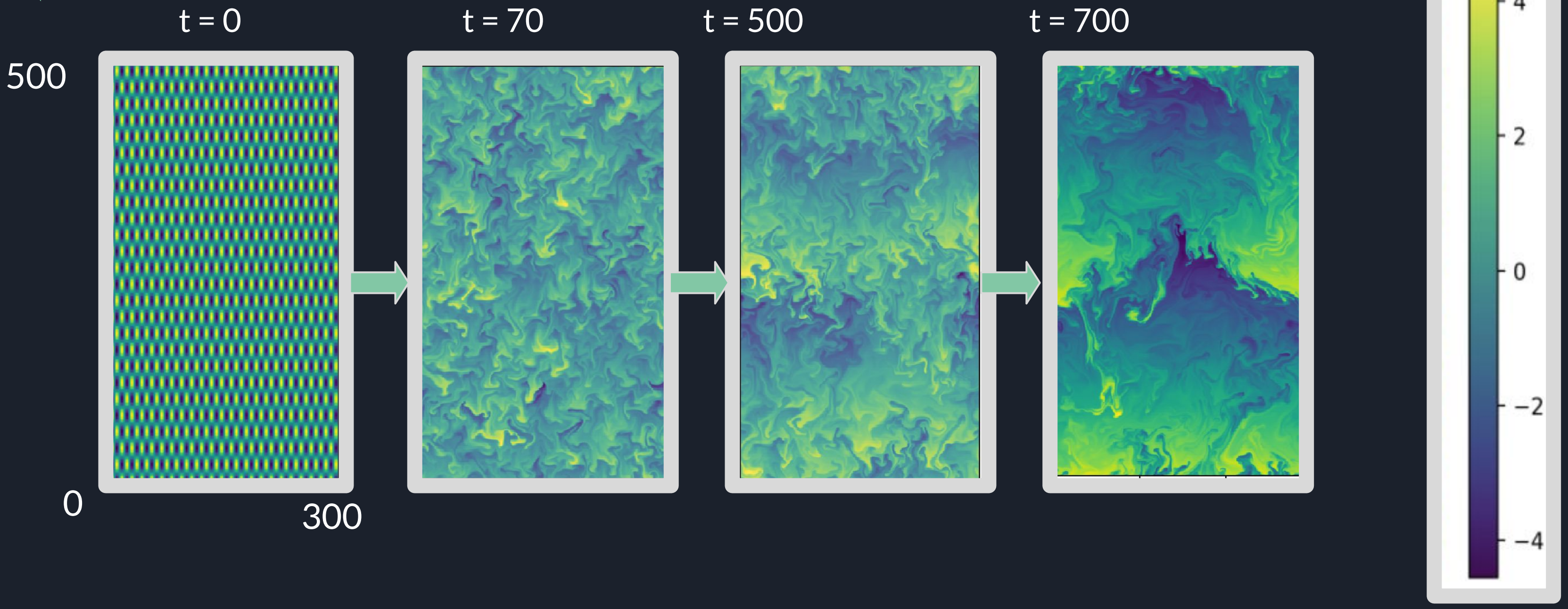

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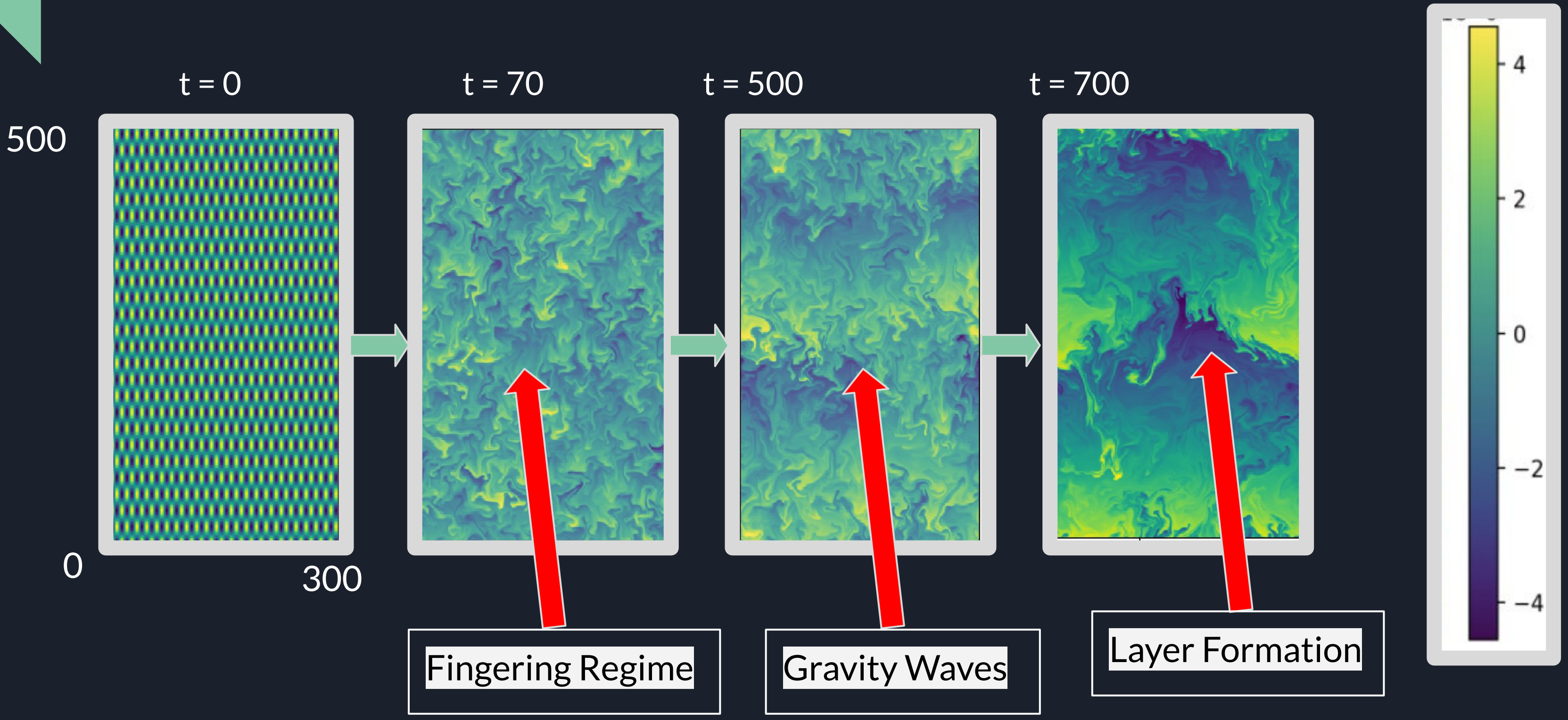
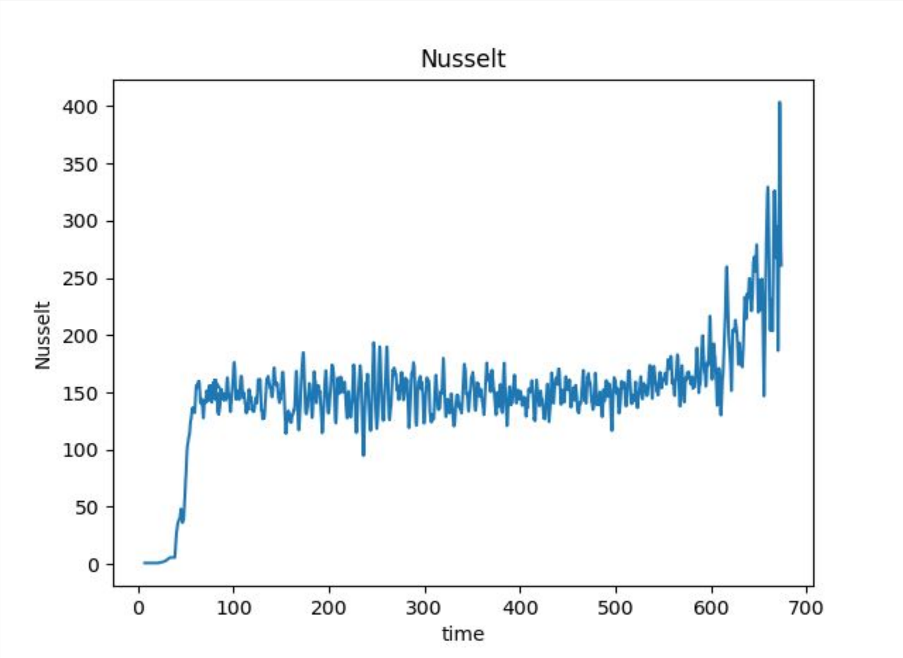


Nusselt Number



Nu = 1 - <wT>

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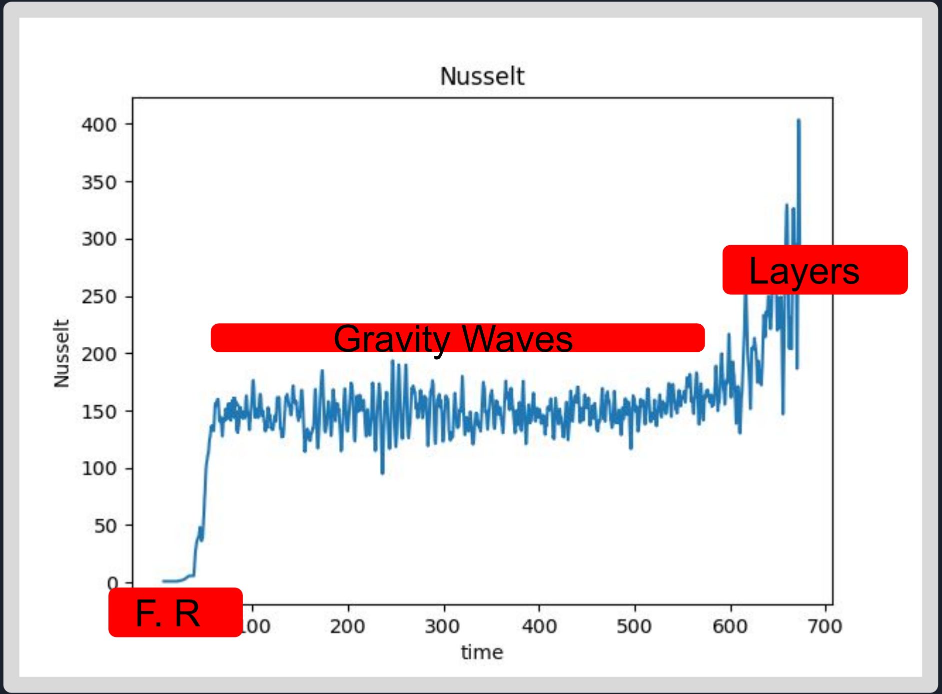

Nu = 1 - <wT> (or 1 - <wS>)

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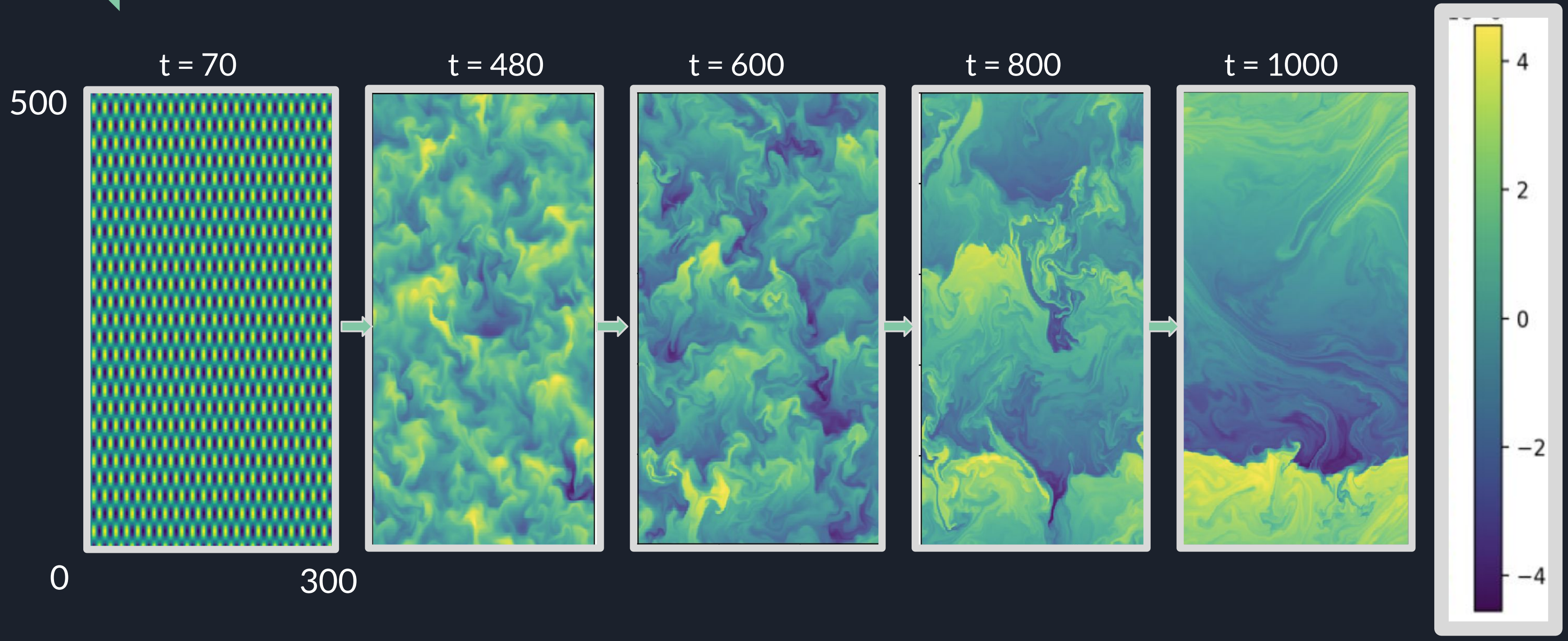

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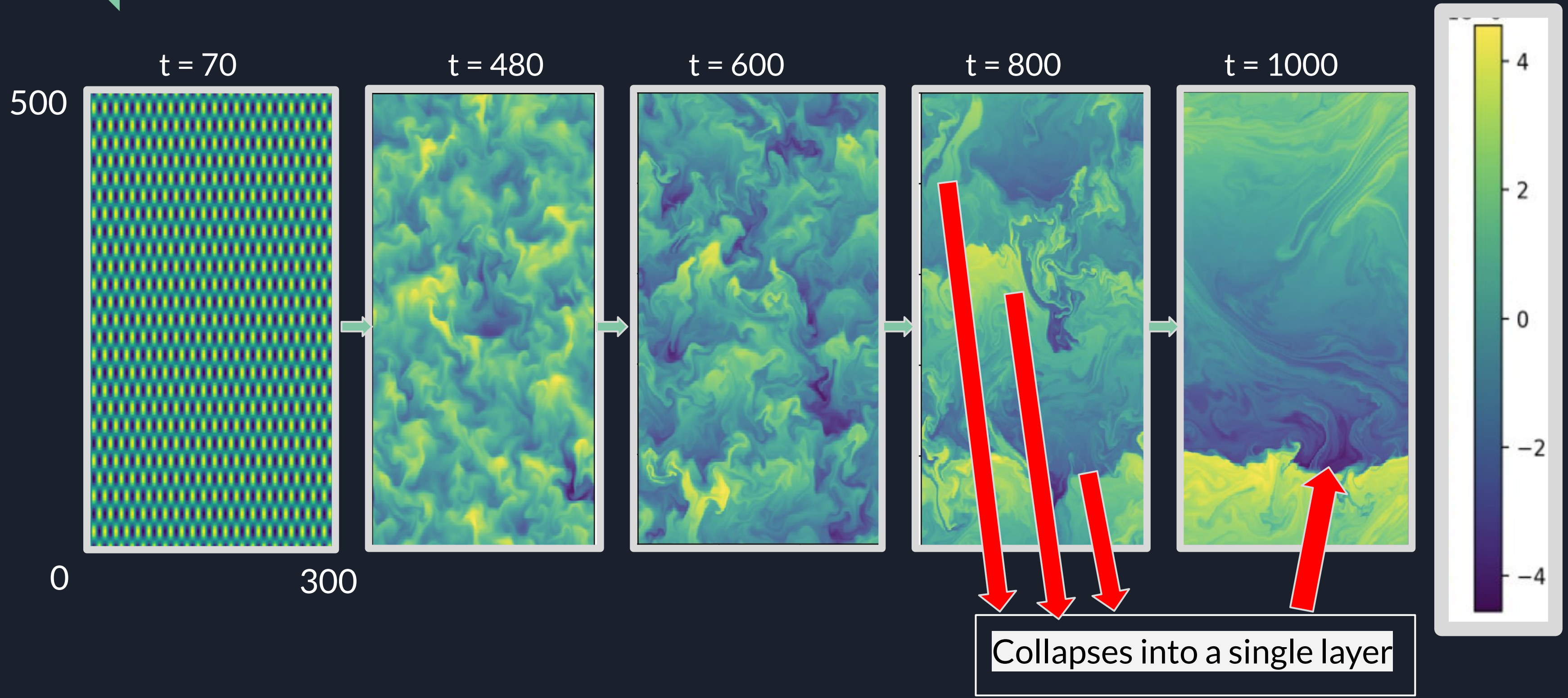

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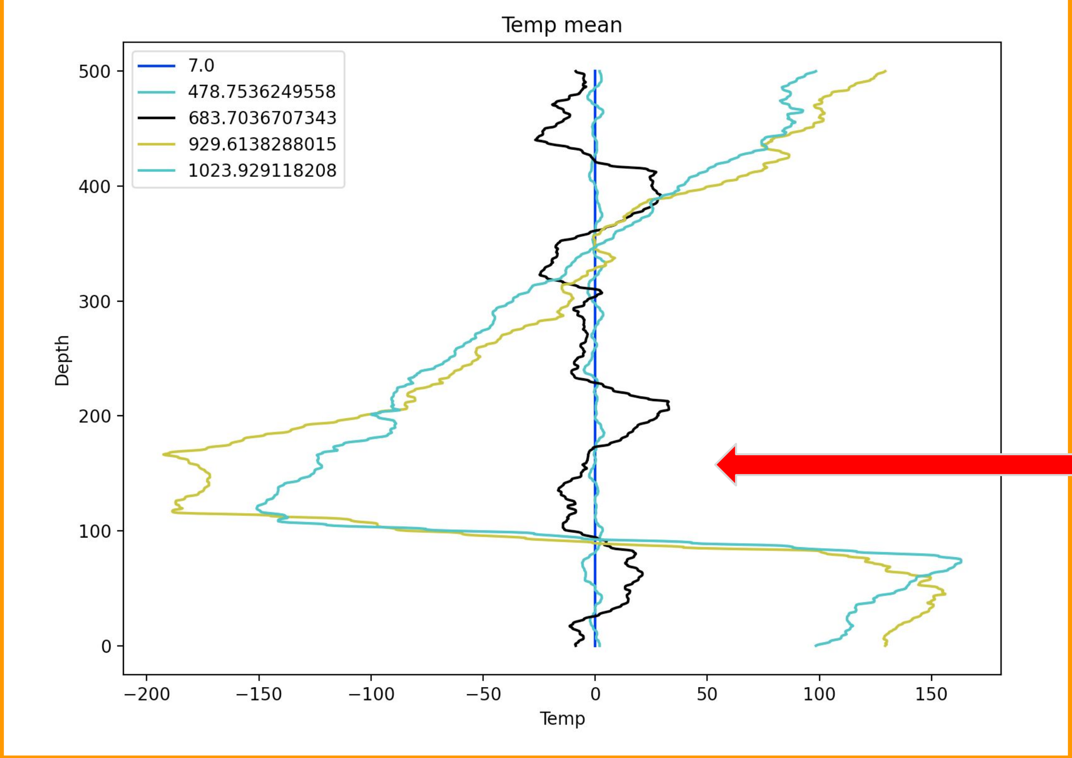
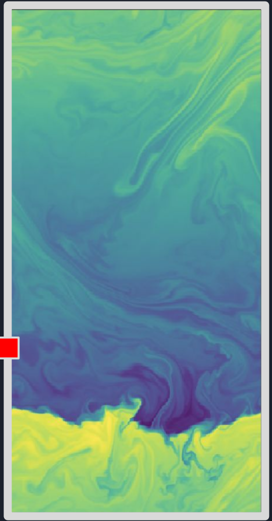
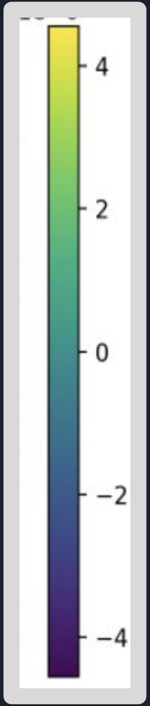


t = 1000





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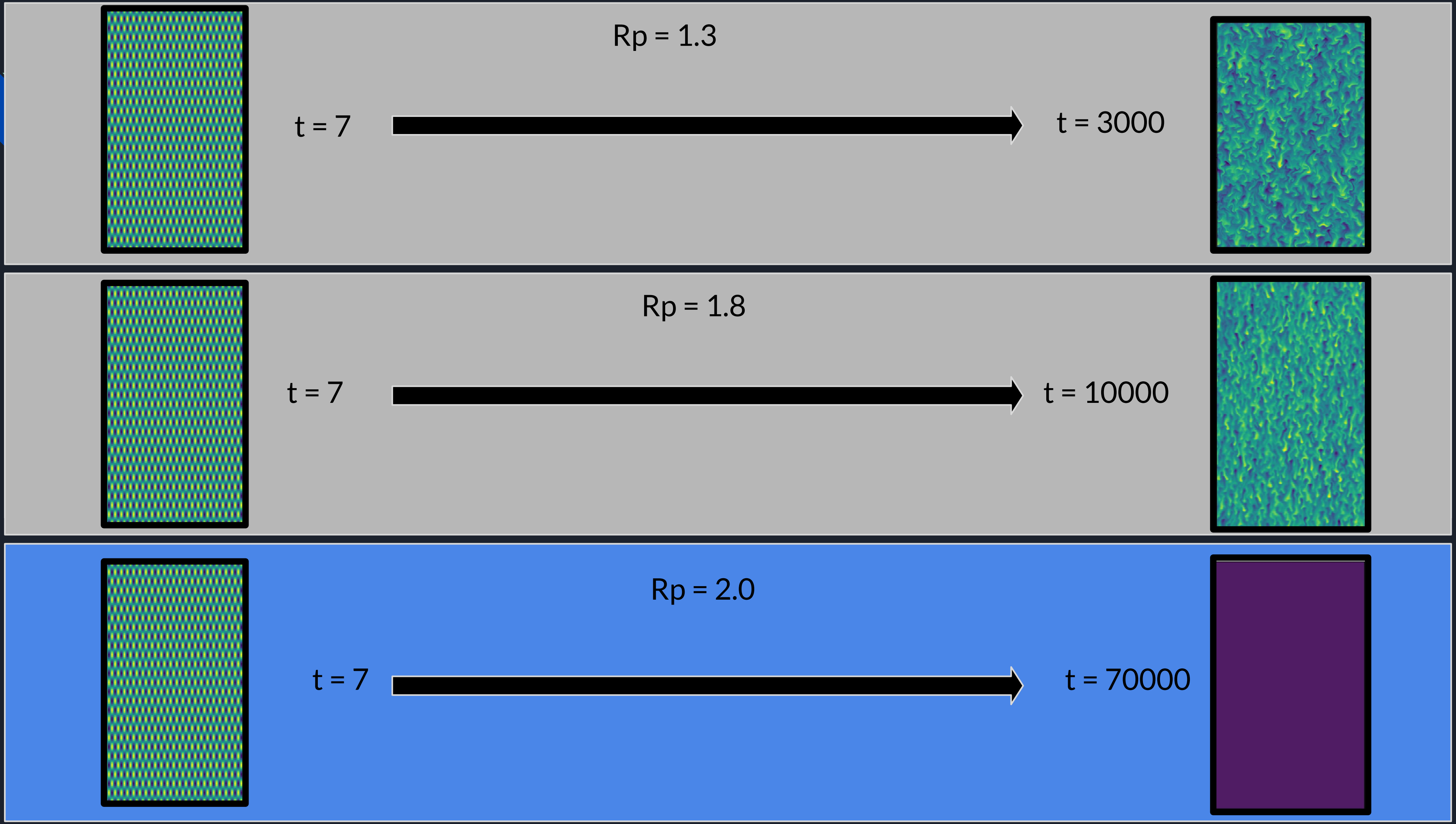
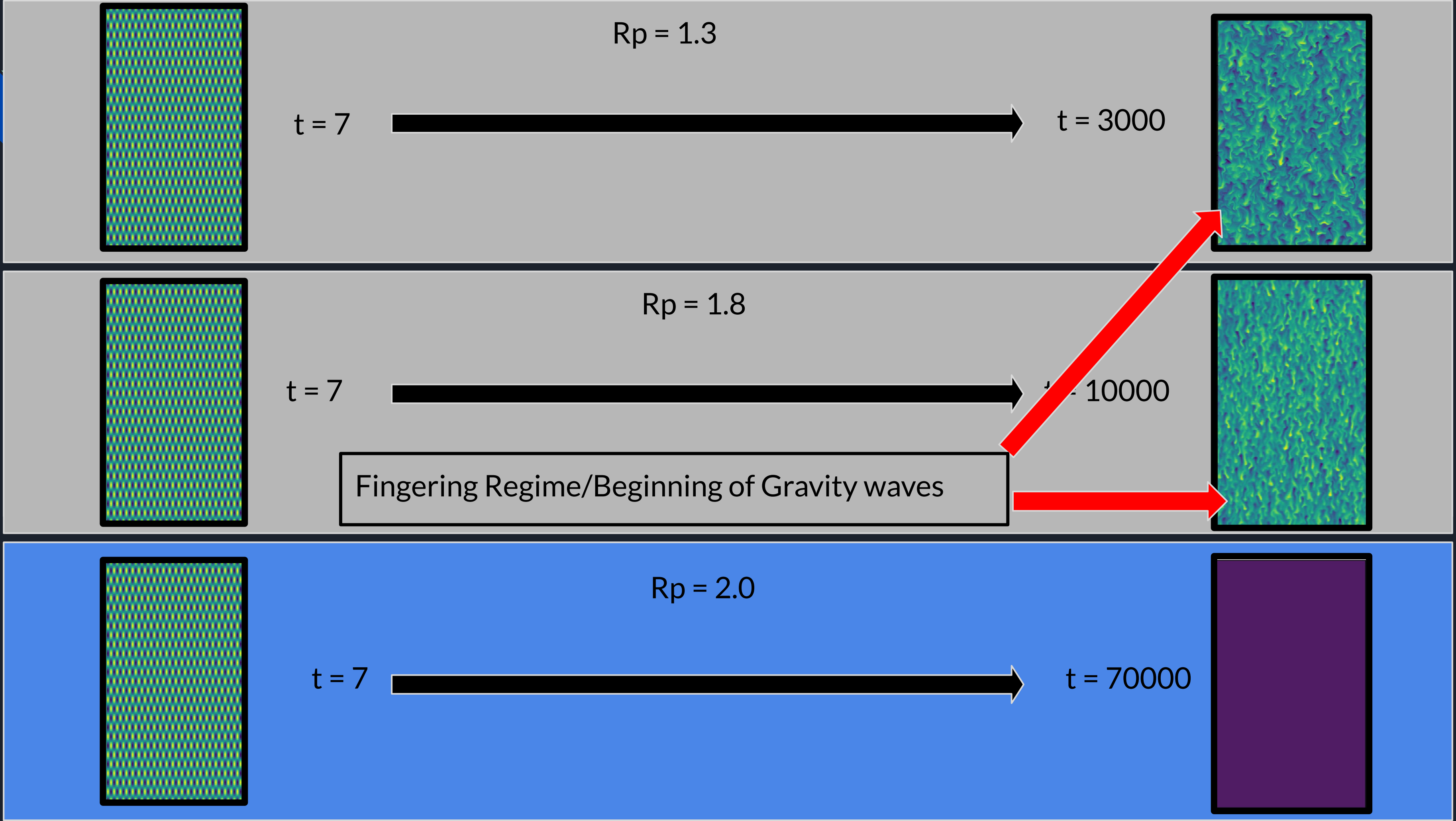

Rp = 1.3
t = 7
t = 3000
Rp = 1.8
t = 7
t = 10000
Rp = 2.0
t = 7
t = 70000


Rp = 1.3
t = 7
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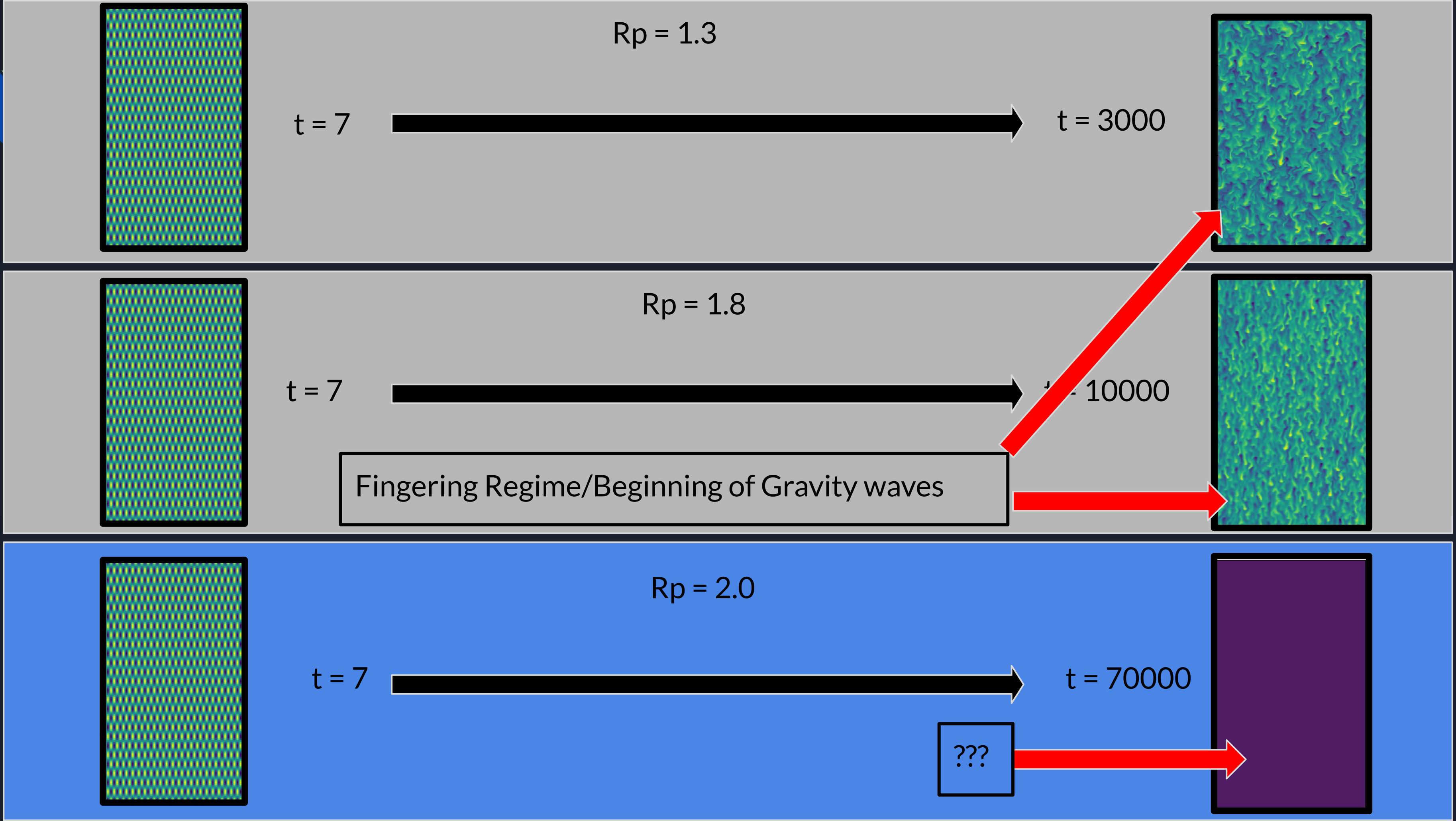
Rp = 1.8
t = 7
10000
Fingering Regime/Beginning of Gravity waves
Rp = 2.0
t = 7
t = 70000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t = 3000
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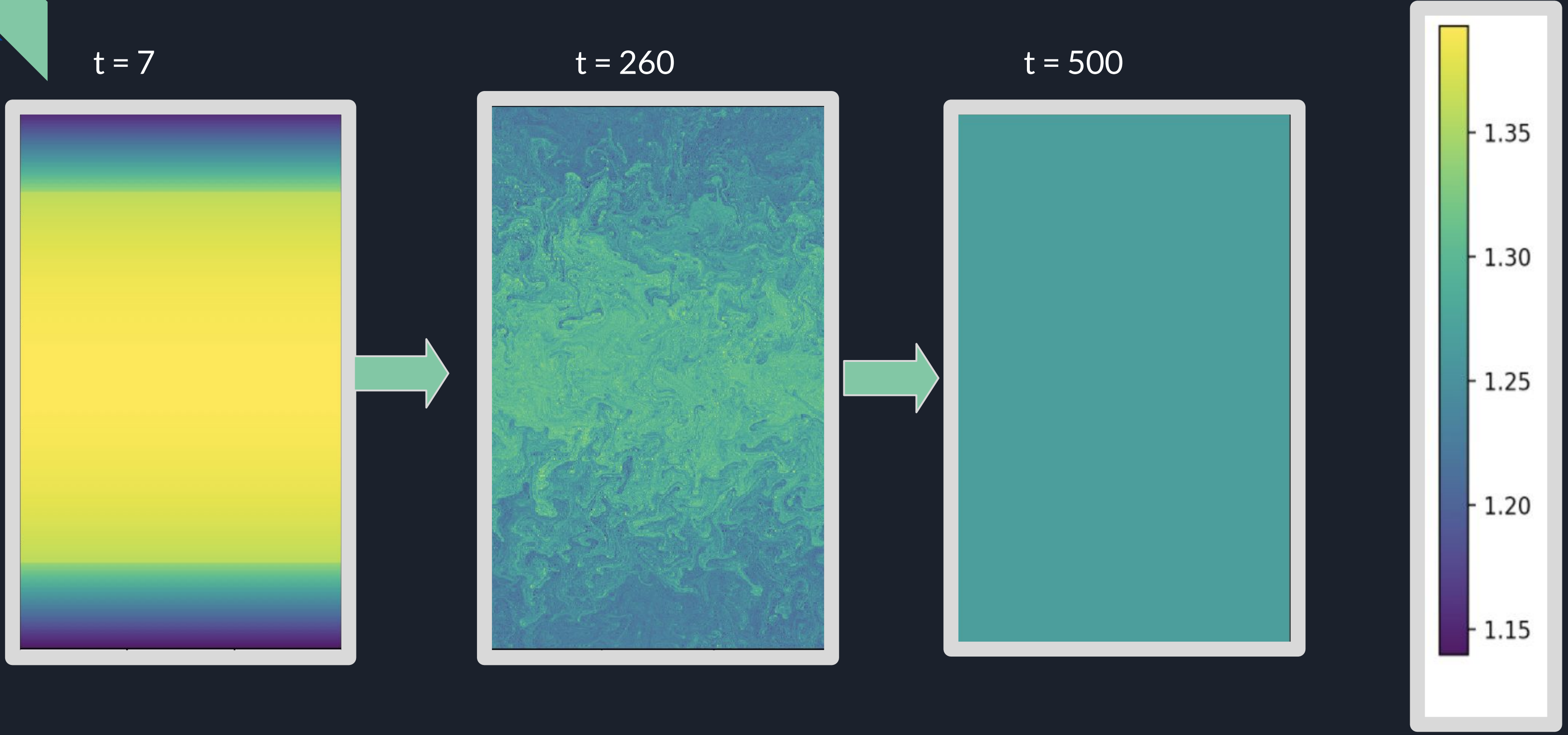
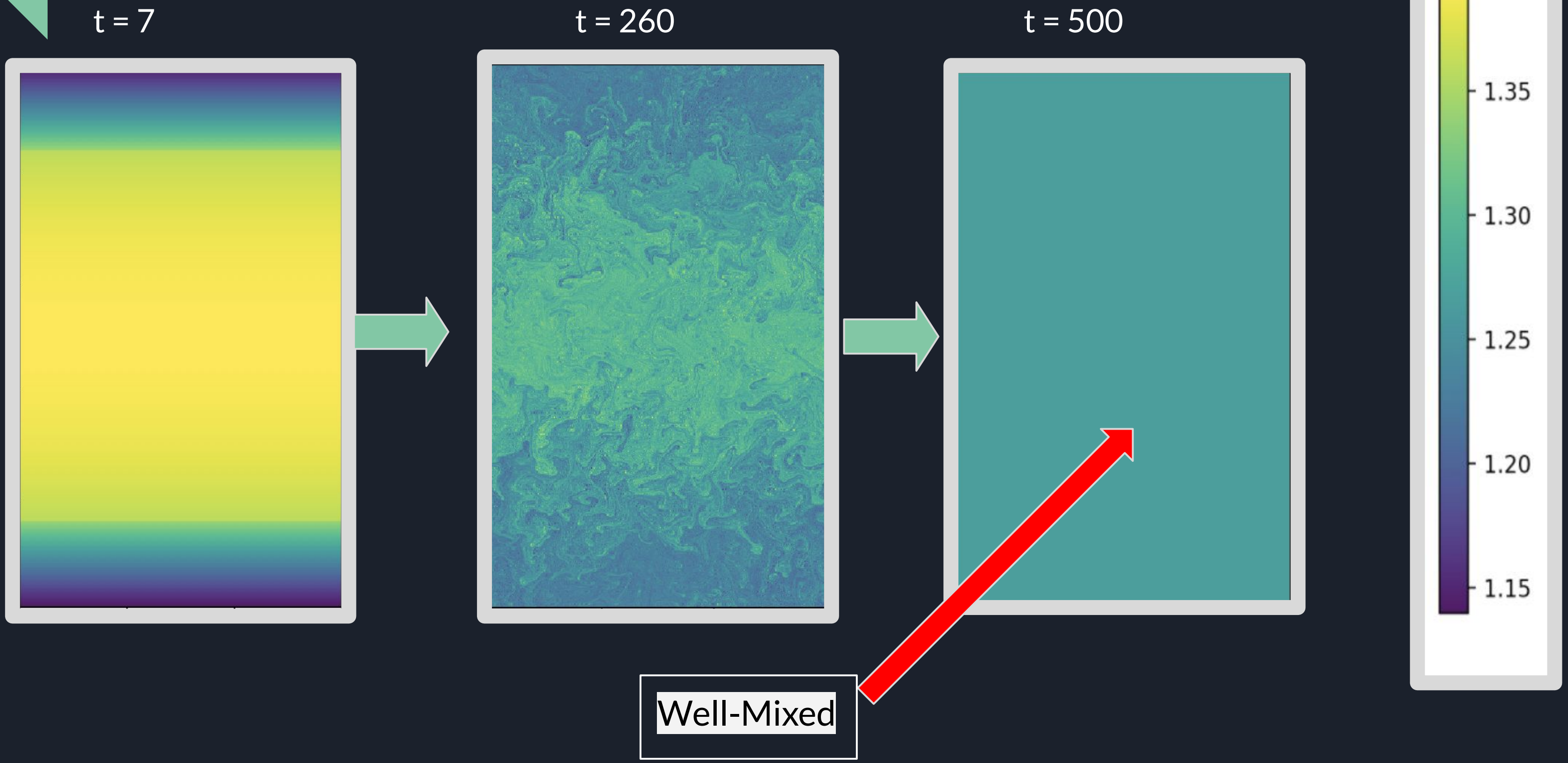
Rp = 2.0
t = 7
t = 70000
???

Oxygen Distribution (Rp=1.1)

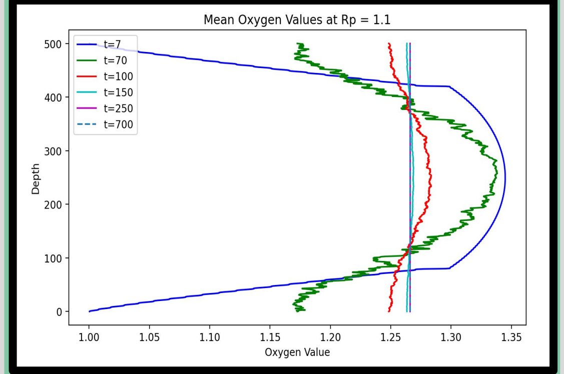


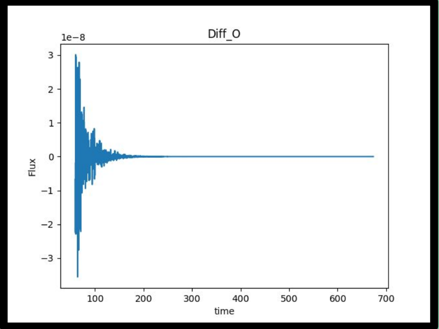
Oxygen Distribution (Rp=1.1)



Oxygen Mean for Salt-Finger Convection

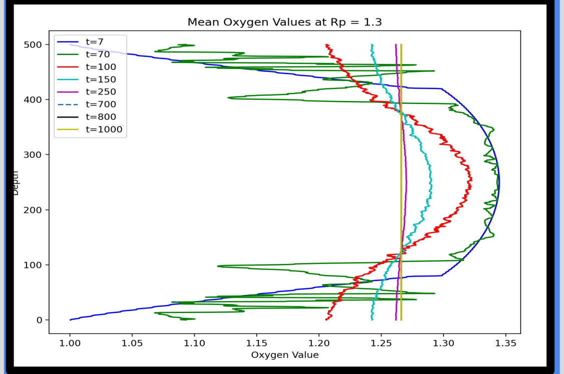
Rp = 1.1

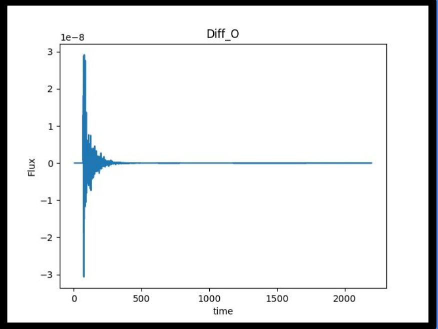




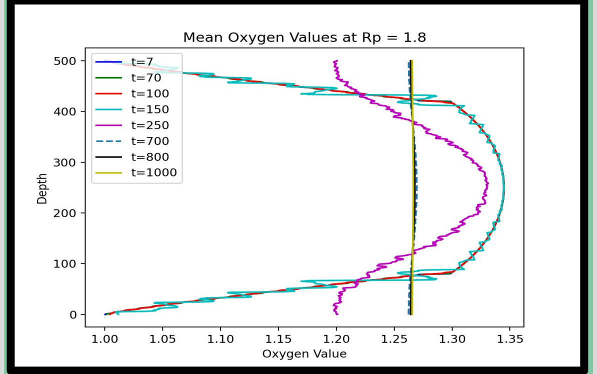
Diff = <k_o [dO/dy]>

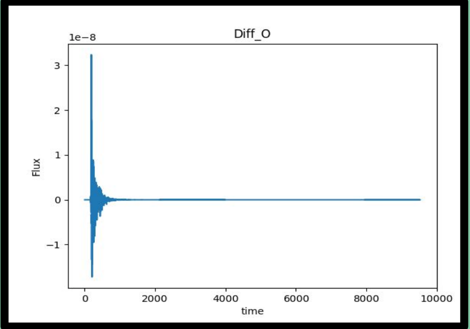
Rp = 1.3

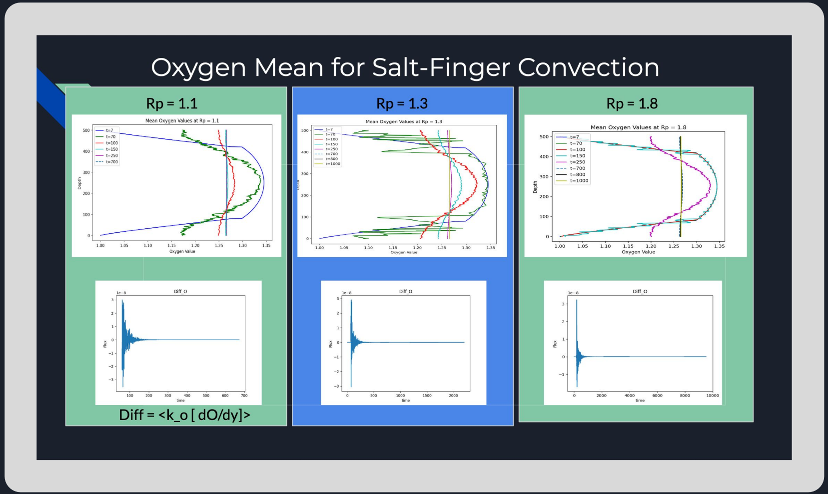




Rp = 1.8





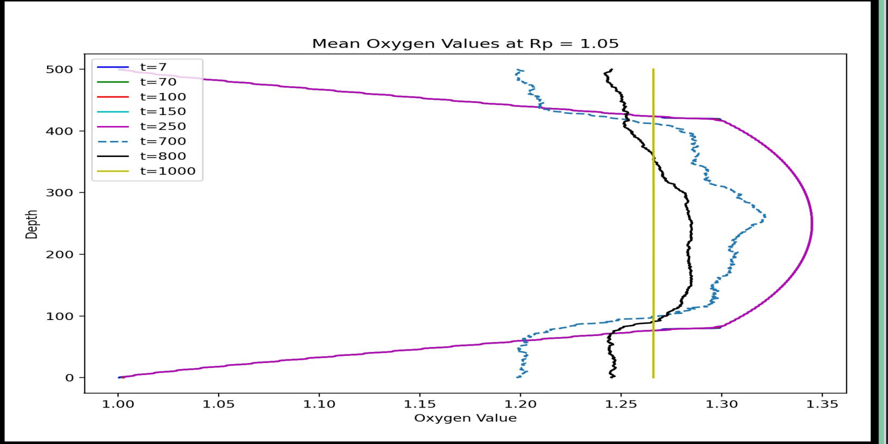


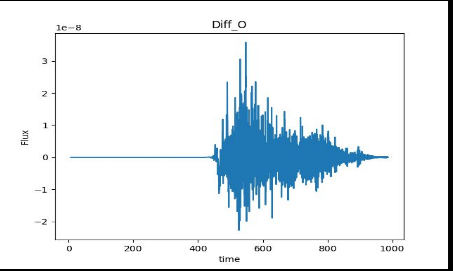
Note:

- The higher the Rp, the slower the oxygen mixes (as long as 1.1<Rp<1.8)
- The Diffusive flux exponentially decrease from t = 0. This suggests the oxygen mixes well very quickly in the beginning.
- All of them stabilize to O = 1.26.

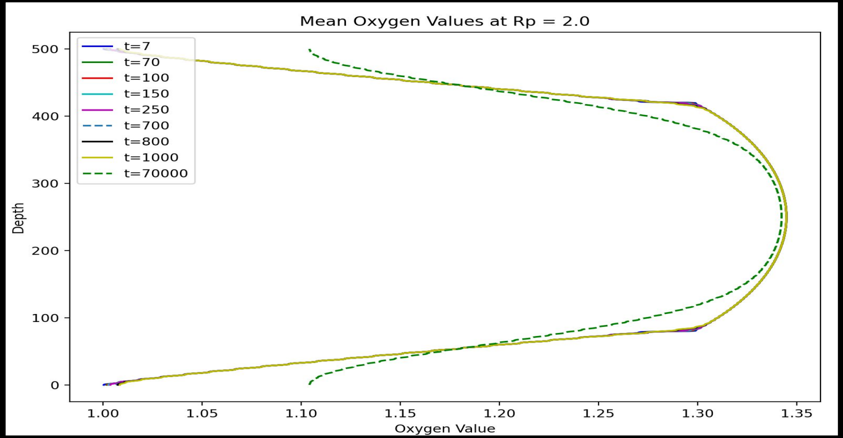
Oxygen Mean Distribution For Osc. Diff.

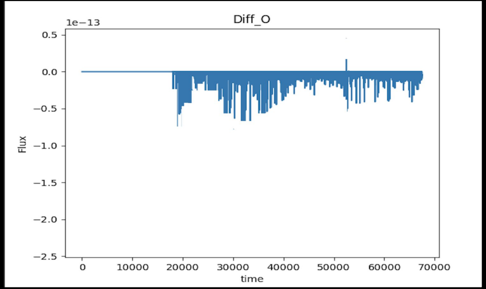
Rp = 1.05



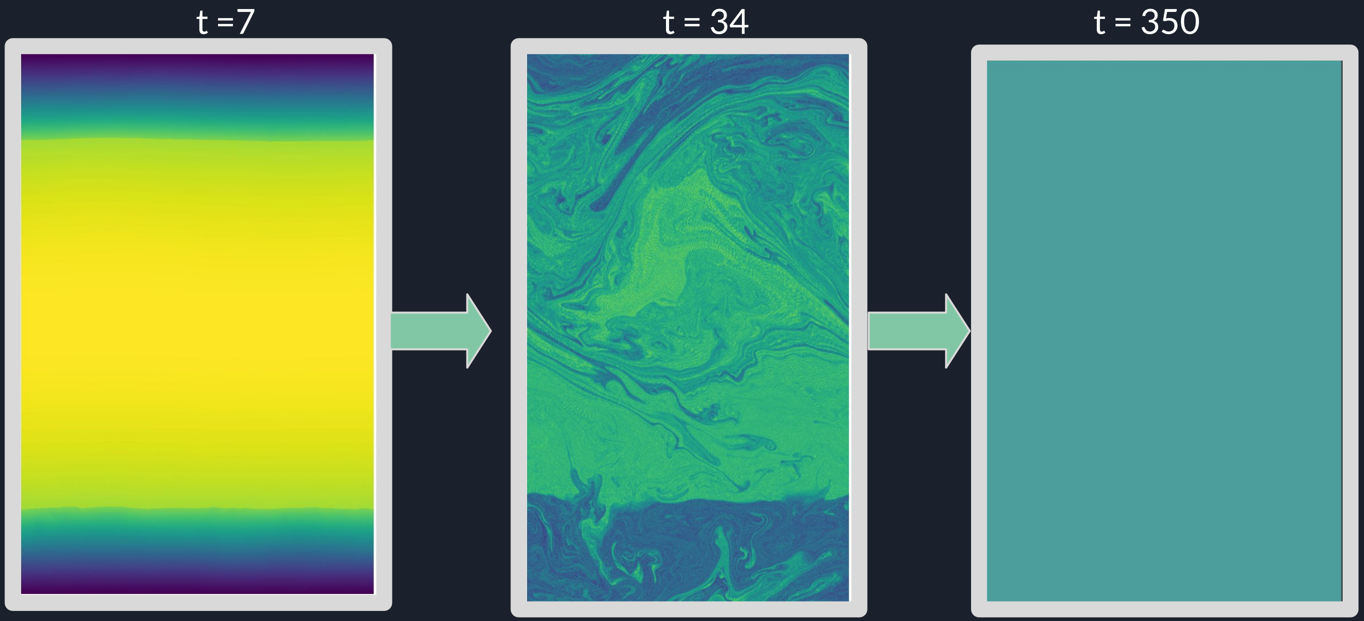


Rp = 2.0



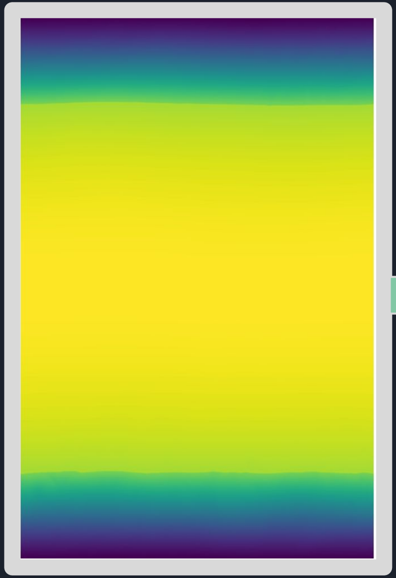


Oxygen re-initialized with layers already formed (For Rp =2.0)

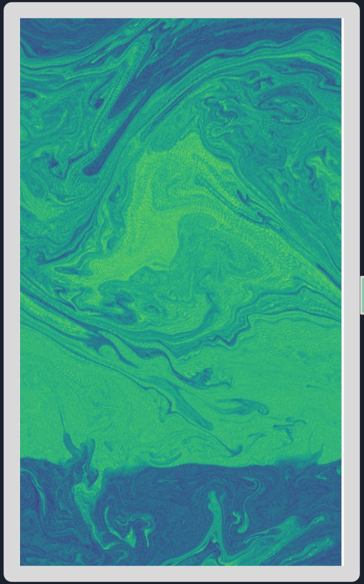


Oxygen re-initialized with layers already formed (For Rp =2.0)

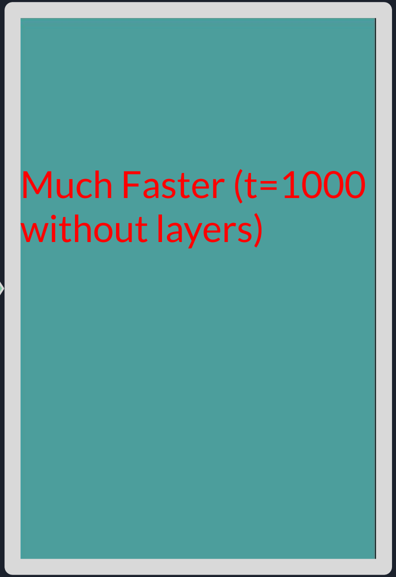
t =7



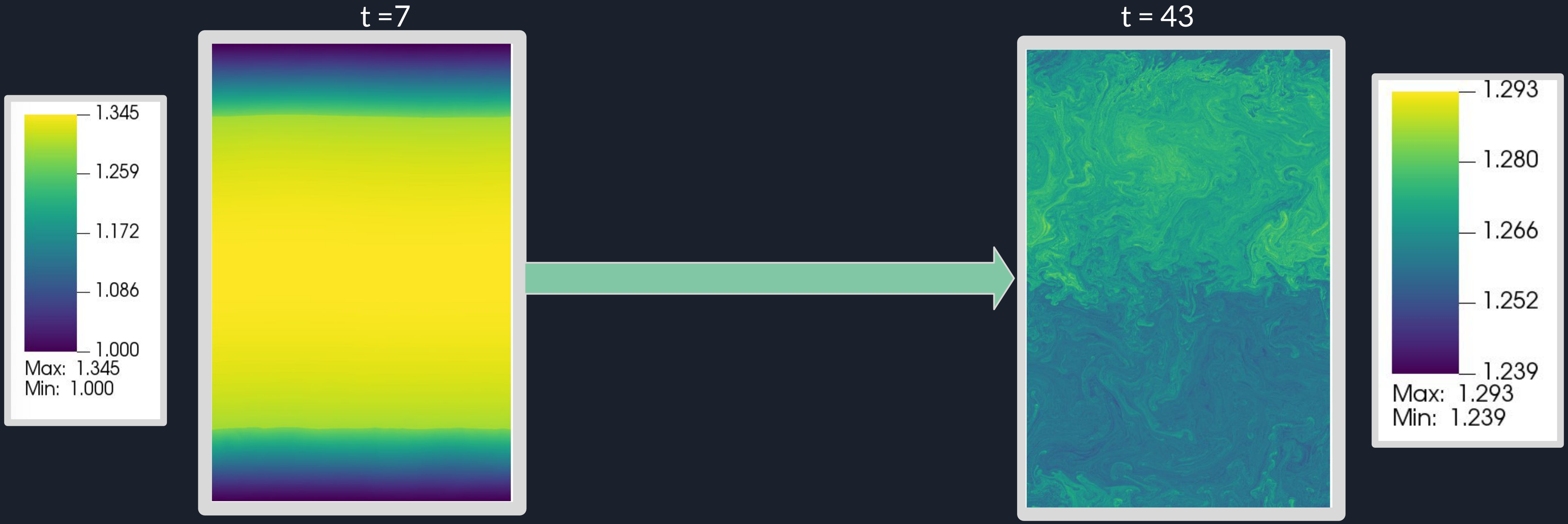
t = 34



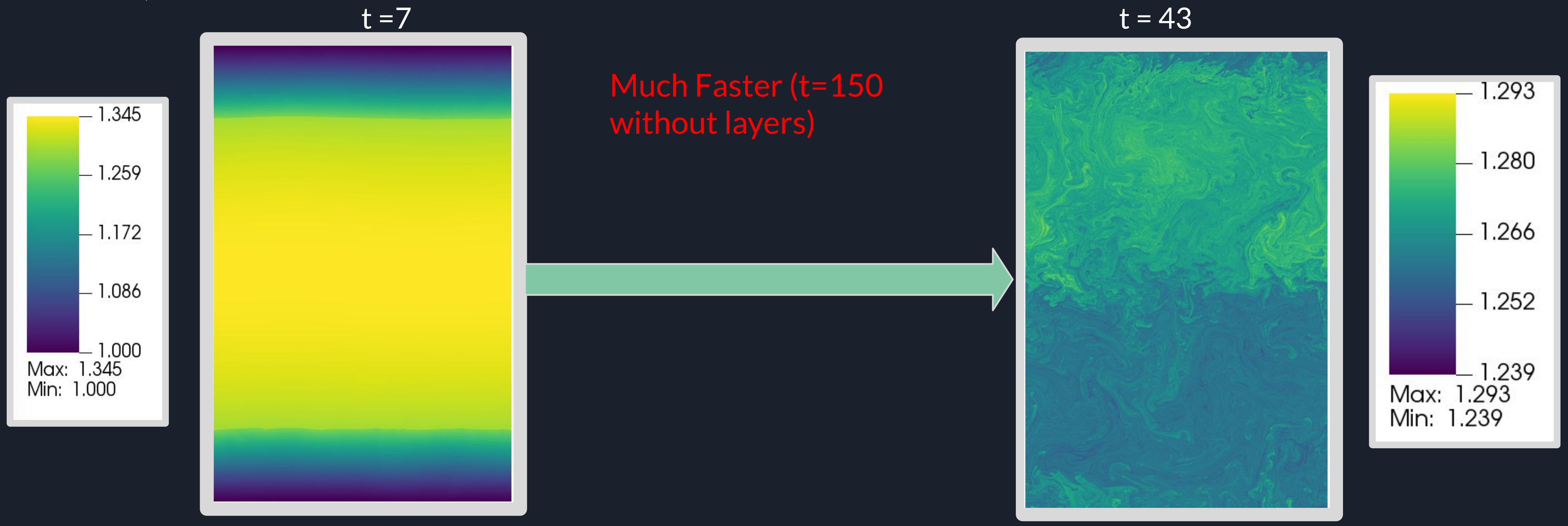
t = 350



Oxygen re-initialized with layers already formed (For Rp = 1.1)



Oxygen re-initialized with layers already formed (For Rp = 1.1)



The End