Unit - 6

Interpolation and Exterpolation

Differences : Let $y_0, y_1, y_2 \cdots y_n$ data values at $x_0, x_1, x_2, x_3 \ldots x_n$ respectively then $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2 \cdots y_n - y_{n-1}$ are called differences of y data values

Forward Difference

1) First Forward difference

$\Delta y_0 = y_1 - y_0$
$\Delta y_1 = y_2 - y_1$
$\Delta y_2 = y_3 - y_2$
$\Delta y_{n-1} = \Delta y_n - \Delta y_{n-1}$

where Δy_0, $\Delta y_1 \ldots \Delta y_{n-1}$ are called first forward difference and Δ is first forward difference operator.

2) Second forward difference

$\Delta^2 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
$\Delta^2 y_1 = \Delta^2 y_2 - \Delta^2 y_1$
$\Delta^2 y_2 = \Delta^2 y_3 - \Delta^2 y_2$
$\Delta^2 y_{n-2} = \Delta^2 y_{n-1} - \Delta^2 y_{n-2}$

where $\Delta^2 y_0$, $\Delta^2 y_1$, $\Delta^2 y_2 \ldots \Delta^2 y_n$ are called second forward difference and Δ^2 is second forward difference operator

Similarly for n^{th} forward difference:

$$\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i \qquad i = 0, 1, 2, 3$$

Forward difference table

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		$y_1 - y_0 = \Delta y_0$		
x_1	y_1		$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$	
		$y_2 - y_1 = \Delta y_1$		$\Delta^2 y_1 - \Delta^2 y_0 = \Delta^3 y_0$
x_2	y_2		$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$	
		$y_3 - y_2 = \Delta y_2$		
x_3	y_3			

Q Find forward difference table for $f(x) = x^2$; $x = 0, 1, 2, 3, 4, 5$ and calculate $\Delta f(0)$, $\Delta^2 f(1)$, $\Delta^3 f(0)$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	0				
		1			
1	1		2		
		3		0	
2	4		2		0
		5		0	
3	9		2		
		7			
4	16				

$\Delta f(0) = \Delta y_0 = 1$

$\Delta^2 f(1) = \Delta^2 y_1 = 2$

$\Delta^3 f(0) = \Delta^3 y_0 = 0$

2

x	1	3	5	7	9
$f(x)$	8	12	21	36	62

find values of : $\Delta f(5)$, $\Delta^2 f(3)$, $\Delta^3 f(3)$

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	8				
		4			
3	12		5		
		9		1	
5	21		6		4
		15		5	
7	36		11		
		26			
9	62				

$\Delta f(5) = \Delta y_2 = 15$

$\Delta^2 f(3) = \Delta^2 y_1 = 6$

$\Delta^3 f(3) = \Delta^3 y_1 = 5$

Backward difference table

x	$y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	∇^4
x_0	y_0				
		~~$y_1-y_0=\nabla y_1$~~ $y_1-y_0=\nabla y_1$			
x_1	y_1	~~y_2~~	$\nabla y_2-\nabla y_1=\nabla^2 y_2$		
		$y_2-y_1=\nabla y_2$		$\nabla^2 y_3-\nabla^2 y_2=\nabla^3 y_3$	$\nabla^3 y_4$
x_2	y_2		$\nabla y_3-\nabla y_2=\nabla^2 y_3$		$-\nabla$
		$y_3-y_2=\nabla y_3$		$\nabla^2 y_4-\nabla^2 y_3=\nabla^3 y_4$	$=\nabla$
x_3	y_3		$\nabla_4-\nabla_3=\nabla^2 y_4$		
		$y_4-y_3=\nabla y_4$			
x_4	y_4				

Q3 Find the difference table for the following data hence fi
$\Delta f(1)$, $\Delta^2 f(2)$ and $\nabla f(4)$, $\nabla^2 f(3)$, $\nabla^3 f(5)$

x	1	2	3	4	5
$f(x)$	3	18	83	256	627

x	y	Δ/∇	Δ^2/∇^2	Δ^3/∇^3	Δ^4/∇^4
1	3				
		15			
2	18		50		
		65		58	
3	83		108		32 ~~40~~
		173		90	
4	256		198		
		371			
5	627				

$\Delta f(1) \Rightarrow \Delta y_0 = 15$

$\Delta^2 f(2) \Rightarrow \Delta^2 y_1 = 108$

$\nabla f(4) \Rightarrow \nabla y_3 = 173$

$\nabla^2 f(3) \Rightarrow \nabla^2 y_2 = 50$

$\nabla^3 f(5) \Rightarrow \nabla^3 y_n = 90$

Interpolation with equal Intervals:

Newton's Forward difference interpolation.

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots$$

where $p = \dfrac{x - x_0}{h}$

where x = required value of given f^n.

x_0 : First value of x

h = common difference b/w x

Newton's Backward difference interpolation:

$$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n \ldots$$

where $p = \dfrac{x - x_n}{h}$

where

x_n = last value of x

Q The following table gives value of $f(x)$ for equally space value of x

x_i: 0 1 2 3 4
f_i: 1 7 23 55 109

Construct the table of finite difference and use N... to estimate $f(0.5)$

	x	f_i	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	0	1				
			6			
x_1	1	7		10		
			16		6	
x_2	2	23		16		0
			32		6	
x_3	3	55		22		
			54			
x_4	4	109				

Using NFDI

~~$f(x)$~~ $p = \frac{x - x_0}{h} = \frac{0.5 - 0}{1}$

$\boxed{= 0.5}$

$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}$

$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$

$$f(0.5) = 1 + (0.5)(6) + \frac{(0.5)(-0.5)(10)}{2}$$

$$+ \frac{(0.5)(-0.5)(-1.5)(6)}{6} + 0$$

~~$= 1 + 3 - 2.5 + 2.25$~~

$= 1 + 3 - 1.25 + 0.375$

$\boxed{= 3.125}$

Q (NBDI)

~~$f(x) = y_n +$~~

Using NBDI

$$p = \frac{x - x_n}{h} = \frac{0.5 - 4}{1} = \boxed{-3.5}$$

$$f(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$f(0.5) = \overset{109}{\cancel{4}} + (-3.5)(54) + \frac{(-3.5)(-2.5)}{2}(\overset{11}{\cancel{22}}) + \frac{(-3.5)(-2.5)(-1.5)}{6} \times 6$$

$= \overset{109}{\cancel{4}} \; \overset{-}{\cancel{+}} \; \overset{189}{\cancel{87.8}} + 96.25 - 13.125$

~~$= -0.375$~~ ~~$= 104.625$~~

$\boxed{= 3.125}$

H.W Find out interpolating polynomial for the datapoint of Q 1

Q Find interpolating polynomial which takes the following values $y(0)=1$, $y(1)=0$, $y(2)=1$ and $y(3)=10$ and also find $y(4)$

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	0	1			
			-1		
x_1	1	0		2	
			1		6
x_2	2	1		8	
			9		
x_3	3	10			

Using NFDI

$$p = \frac{x - x_0}{h} = \frac{x - 0}{1}$$

$$\boxed{= x}$$

$$f(x) = y_0 + p \cdot \Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}$$

$= 1 + x \cdot (-1) + x \cdot (x-1) \cdot (2) + x(x-1)(x-2) \cdot (6)$

$= 1 - x + 2x^2 - 2x + x^2 - x \cdot (6x - 12)$

$= 1 - x + 2x^2 - 2x + 6x^3 - 12x^2 - 6x^2 + 12x$

$$1 - x - 2x + 12x + 2x^2 - 12x^2 - 6x^2 + 6x^3$$

$$1 + 9x - 16x^2 + 6x^3$$

$$= 6x^3 - 16x^2 + 9x + 1$$

$$= 1 + x(-1) + \frac{x(x-1)(2)}{2} + \frac{x(x-1)(x-2)\,6}{6}$$

$$= 1 - x + x^2 - x + x^2 - x(x-2)$$

$$= 1 - \cancel{x} + \cancel{x^2} - \cancel{x} + x^3 - 2x^2 - \cancel{x^2} + \cancel{2x}$$

$$y(x) = x^3 - 2x^2 + 1$$

$$y(4) = (4)^3 - 2(4)^2 + 1$$

$$= 64 - 32 + 1$$

$$\boxed{= 33}$$

* Central Difference Interpolation

Based on central Difference Interpolation we have Five methods

(1) Gauss Forward
(2) Gauss Backward
(3) Stirling Interpolation
(4) Bessel's Interpolation
(5) Laplace Evertt's Interpolation

* Central Difference table

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	y_{-2}				
		~~Δy_{-2}~~ Δy_{-2}			
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		
		$\boxed{\Delta y_{-1}}$		$\boxed{\Delta^3 y_{-2}}$	
x_0	$\boxed{y_0}$		$\boxed{\Delta^2 y_{-1}}$		$\boxed{\Delta^4 y_{-2}}$
		$\boxed{\Delta y_0}$		$\boxed{\Delta^3 y_{-1}}$	
x_1	y_1		$\Delta^2 y_0$		
		Δy_1			
x_2	y_2				

(1) Gauss Forward Difference Interpolation.

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{p(p-1)(p+1)(p-2)}{4!}\Delta^4 y_{-2} + \ldots$$

where $p = \dfrac{x - x_0}{h}$

(2) Gauss Backward Difference Interpolation.

$$f(x) = y_0 + p\Delta y_{-1} + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{p(p+1)(p-1)}{3!}$$

$$+ \frac{p(p+1)(p-1)(p+2)}{4!}\Delta^4 y_{-2}$$

Find the values of y when $x = 3.2$ from the following table using Gauss forward difference

x	2.0	2.5	3.0	3.5	4.0
y	246.2	409.3	537.2	636.6	715.9

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	2.0	246.2				
			163.1			
x_{-1}	2.5	409.3		-35.2		
			127.9		63.4	
x_0	3.0	$\boxed{537.2}$		$\boxed{-28.5}$		$\boxed{1.7}$
			$\boxed{99.4}$		$\boxed{+8.4}$	
x_1	3.5	636.6		-20.1		
			79.3			
x_2	4.0	715.9				

Using GFDI

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{p(p-1)(p+1)(p-2)}{4!}\Delta^4 y_{-2}$$

$$p = \frac{x - x_0}{h} = \frac{3.2 - 3.0}{0.5} \boxed{= 0.4}$$

$$= 537.2 + (0.4 \times 99.4) + \left[\frac{0.4 \times (-0.6)}{2} \times (-28.\right.$$

$$+ \left[\frac{0.4 \times (-0.6) \times 1.4}{6} \times 8.4\right]$$

$$+ \left[\frac{0.4 \times (0.6) \times 1.4 \times (-1.6)}{24}\right.$$

$$= 537.2 + (39.76) + 3.42 + -0.47 + 0.03$$

$$= 579.94$$

Q The following data are taken from the steam table

Temp C°	140	150	160	170
pressure	3.685	4.854	6.302	8.076

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	140	3.685				
			1.169			
x_{-1}	150	4.854		0.2789		
			1.448		0.047	
x_0	160	6.302		0.326		0.002
			1.774		0.049	
x_1	170	8.076		0.375		
			2.149			
x_2	180	10.225				

Using GFDI:

$$p = \frac{x - x_0}{h} = \frac{142 - 160}{10} = \boxed{-1.8}$$

$$f(x) = y_0 + p\,\overset{1.774}{\cancel{\Delta y_0}} + \frac{p(p-1)}{2!}(0.326)$$

$$+ \frac{p(p-1)(p+1)}{3!}(0.049) + \frac{p(p-1)(p+1)(p-2)}{4!}(0.002)$$

$$= 6.302 + (-1.8)(1.774) + \frac{(-1.8)(-2.8)(0.326)}{2}$$

$$+ \frac{(-1.8)(-2.8)(-0.8)(0.049)}{6} + \frac{(-1.8)(-2.8)(-0.8)(-3.8)(0.002)}{24}$$

$$= 6.302 - 2.48 + \cancel{1.64}\ 0.82 - 0.03 + 0.0012$$

$$\cancel{\boxed{= 4.6132}} \quad \boxed{= 3.898} \text{ Ans}$$

Using GBDI:

$$p = \frac{x - x_0}{h} = \frac{175 - 160}{10} = \boxed{1.5}$$

$f(x) = y_0 + \cancel{p \Delta y_0}\ p(1.448) + \frac{p(p+1)}{2!}(0.326)$

$+ \frac{p(p+1)(p-1)}{3!}(0.047) + \frac{p(p+1)(p-1)(p+2)}{4!}(0.00$

$= 6.302 + (1.5)(1.448) + \frac{(1.5)(2.5)(0.326)}{2}$

$+ \frac{(1.5)(2.5)(0.5)(0.047)}{6} + \frac{(1.5)(2.5)(0.5)(3.5)(0}{\cancel{4}24}$

$= 6.302 + 2.172 + 0.611 + 0.014 + 0.00054$

$\boxed{= 9.099}$

Q Estimate the sales for 1966 using following table

$\boxed{= 32.34}$

Year	1931	1941	1951	1961	197
Sale	12	15	20	27	52

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	1931	12				
			3			
x_{-1}	1941	15		2		
			5		0	
x_0	1951	20		2		16
			7		16	
x_1	1961	27		18		
			25			
x_2	1971	52				

using GBDI :

$p = \frac{x - x_0}{h} = \frac{1966 - 1951}{10}$ $\boxed{= 1.5}$

$f(x) = \overset{y_0}{20} + (\overset{p}{\cancel{1.5}})(5) + \frac{p(p+1)}{2!}(2)$

$+ \frac{p(p+1)(p-1)}{3!}(0) + \frac{p(p+1)(p-1)(p+2)}{4!}(16)$

$= 20 + (1.5)(5) + \frac{(1.5)(2.5)(2)}{2} + 0 + \frac{(1.5)(2.5)(0.5)(3.5)(16)}{24}$

$= 20 + 7.5 + 3.75 + 0 + 4.375$

$= 35.625$

3. Stirling's Interpolation

$$\begin{array}{cccccc} & & \Delta y_{-1} & & \Delta^3 y_{-2} & \ldots \\ x_0 & y_0 & & \Delta^2 y_{-1} & & \Delta^4 y_{-2} \\ & & \Delta y_0 & & \Delta^3 y_{-1} & \ldots \end{array}$$

$f(x) = y_0 + p\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2}{2!}\Delta^2 y_{-1}$

$+ \frac{p(p^2-1)}{3!}\left(\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}\right) + \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2} \ldots$

Q Using Stirling formula to find U_{32} from the following

$U_{20} = 14.035$, $U_{25} = 13.674$, $U_{30} = 13.257$

$U_{35} = 12.734$, $U_{40} = 12.084$, $U_{45} = 11.309$

Q Determine the interpolating polynomial for the following data

x:	0	1	2	3	4
y:	41	43	47	53	61

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	0	41				
			2			
x_{-1}	1	43		2		
			4		0	
x_0	2	47		2		0
			6		0	
x_1	3	53		2		
			8			
x_2	4	61				

$$p = \frac{x - x_0}{h} = \frac{x-2}{1}$$

$$= \boxed{x-2}$$

$$f(x) = 47 + p\left(\frac{4+6}{2}\right) + \frac{p^2}{2!}(2) + 0 + 0$$

$$= 47 + (x-2)(5) + \frac{(x-2)^2(\cancel{2})}{\cancel{2}}$$

$$= 47 + 5x - 10 + x^2 - 4x + 4$$

$$= x^2 + x + 41$$

4. Bessele's Interpolation

x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$ ...
		Δy_0		$\Delta^3 y_{-1}$	
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$ ...

$$f(x) = \left(\frac{y_0 + y_1}{2}\right) + \left(p - \frac{1}{2}\right)\Delta y_0 + \frac{p(p-1)}{2!}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right)$$

$$+ \frac{p(p-1)(p-\frac{1}{2})}{3!}\Delta^3 y_{-1} + \frac{p(p-1)(p+1)(p-2)}{4!}\left(\frac{\Delta^4 y_{-1} + \Delta^4 y_{-2}}{2}\right) + \ldots$$

Q. From the following table find the values of $f(2.73)$ using besseles Interpolation

x	2.5	2.6	2.7	2.8	2.9	3.0
$f(x)$	0.4938	0.4953	0.4965	0.4974	0.4981	0.4987

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	2.5	0.4938				
			0.0015			
x_{-1}	2.6	0.4953		-0.0003		
			0.0012		0	
x_0	2.7	0.4965		-0.0003		0.0005
			0.0009		0.0001	
x_1	2.8	~~0.4974~~		-0.0002		~~-0.0004~~ 0
			0.0007		0.0001	
x_2	2.9	0.4981		-0.0001		
			0.0006			
x_3	3.0	0.4987				

$$p = \frac{x - x_0}{h} = \frac{2.73 - 2.7}{0.1} \quad \boxed{= 0.3}$$

$$f(x) = 0.49695 + \cancel{2} + (-0.2)(0.0009)$$
$$+ (0.3)(-0.35)(-0.00025) + \frac{(0.3)(-0.7)(-0.2)}{6}(0.00$$
$$+ \frac{(0.3)(-0.7)(1.3)(-1.7)}{24}(0.00025)$$

$$= 0.49695 - 0.00018 + 0.000026 + 0.000001$$
$$+ \cancel{0.019338}\ 0.000005$$
$$= 0.4968$$

Q. Find value of : sin 38° using bessele's.

x°	15	20	25	30
$\sin x$	0.2588190	0.3420209	0.4226183	0.5
	35	40		
	0.5735764	0.6427876		

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{-2}	15	0.2588190			
			0.083201		
x_{-1}	20	0.3420209		-0.002603	
			0.080598		-0.005819
x_0	25	0.4226183		-0.003216	
			0.077382		0.00059
x_1	30	0.5		-0.003806	
			0.073576		-0.008171
x_2	35	0.5735764		-0.004365	
			0.069211		
x_3	40	0.6427876			

	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
.25				
	0.0832011			
.34		-0.0026029		
	0.0805982		-0.0006136	
.4		-0.0032165		0.0000248
	0.0773817		0.0005888	
.5		-0.0038053		0.0000289
	0.0735764		-0.0000599	
0.57		-0.0043652		
	0.0692112			
0.64				

$\Delta^5 y$

0.0000041

(5) Laplace Everell's Interpolation.

$x_0 \quad y_0 \quad \Delta^2 y_{-1} \quad \Delta^4 y_{-2}$

$x_1 \quad y_1 \quad \Delta^2 y_0 \quad \Delta^4 y_{-1}$

$$f(x) = q y_0 + \frac{q(q^2-1^2)}{3!}\Delta^2 y_{-1} + \frac{q(q^2-1^2)(q^2-2^2)}{5!}\Delta^4 y_{-2}$$

$$+ p y_1 + \frac{p(p^2-1^2)}{3!}\Delta^2 y_0 + \frac{p(p^2-1^2)(p^2-2^2)}{5!}\Delta^4 y_{-1}$$

where $p = \frac{x - x_0}{h}$ and $q = 1-p$

Q Find value of e^x when $x = 0.644$ using Everell's

x	0.61	0.62	0.63	0.64
e^x	1.840431	1.858928	1.877610	1.896481

0.65	0.66	0.67
1.915541	1.934792	1.954237

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_{-3}	0.61	1.840431					
			0.018497				
x_{-2}	0.62	1.858928		0.000185			
			0.018682		0.000004		
x_{-1}	0.63	1.877610		0.000189		-0.000004	
			0.018871		0.0		0.000006
x_0	0.64	1.896481		0.000189		0.000002	
			0.019060		0.000002		-0.000001
x_1	0.65	1.915541		0.000191		0.000001	
			0.019251		0.000003		
x_2	0.66	1.934792		0.000194			
			0.019445				
x_3	0.67	1.954237					

$\Delta^6 y = -0.000007$

$$p = \frac{x - x_0}{h} = \frac{0.644 - 0.64}{0.01} \qquad \boxed{= 0.4}$$

$$\boxed{q = 0.6}$$

1.137888   -0.0000120

$$f(x) = (0.6)(1.896481) + \frac{(0.6)(-0.106)}{6}(0.000189)$$

$$+ \frac{(0.6)(-0.106)(-3.64)}{120}(0.000002) \quad 0.000000004$$

$$+ \frac{(0.6)(-0.106)(-3.64)(-8.64)}{5040}(0.000007) \quad -0.0000000003$$

0.76621640

$$+ (0.4)(1.915541) + \frac{(0.4)(-0.84)}{6}(0.000191) \quad 0.0000010696$$

$$+ \frac{(0.4)(-0.84)(-3.84)}{120}(0.000001) \quad 0.00000000011$$

$$= 1.904103108$$

Q2 Using everett's Interpolation compute y(35)

x	20	30	40	50
y	512	439	346	243

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{-1}	20	512			
			-7.3		
x_0	30	439		-20	
			-9.3		10
x_1	40	346		-10 ~~-43.7~~	
			-103		
x_2	50	243			

$p = \frac{x - x_0}{h} = \frac{35 - 30}{10} = \boxed{0.5}$

$q = 1 - p = \boxed{0.5}$

$f(x) = (0.5)(439) + \frac{(0.5)(-0.75)(-20)}{6}$

$+ (0.5)(346) + \frac{(0.5)(-0.75)(-10)}{6}$

$= 219.5 + 1.25 + 173 + 0.62$

$= 394.37$

Interpolation with unequal Intervals

(1) Langrange's Interpolation

(1) for $n=1$

x	x_0	x_1
y	y_0	y_1

$\rightarrow$ $$f(x) = \frac{(x-x_1)}{(x_0-x_1)} y_0 + \frac{(x-x_0)}{(x_1-x_0)} y_1$$

(2) for $n=2$

x	x_0	x_1	x_2
y	y_0	y_1	y_2

$\rightarrow$ $$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_2)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_3$$

(3) for n data values of x

x	x_0	x_1	x_2	---	x_n
y	y_0	y_1	y_2	---	y_n

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} y_n$$

Q Compute $f(0.4)$ using langrange's Interpolation

x	0.3	0.5	0.6
y	0.61	0.69	0.72

$$f(x) = \frac{[0.4 - 0.5][0.4 - 0.6]}{(0.3 - 0.5)(0.3 - 0.6)}(0.61)$$

$$+ \frac{(0.4 - 0.3)(0.4 - 0.6)}{(0.5 - 0.3)(0.5 - 0.6)}(0.69)$$

$$+ \frac{(0.4 - 0.3)(0.4 - 0.5)}{(0.6 - 0.3)(0.6 - 0.5)}(0.72)$$

$$\frac{(-0.1)(-0.2)}{(0.2)(-0.3)}(0.61) + \frac{(0.\cancel{1})(-0.2)}{(0.2)(-0.\cancel{1})}(0.69)$$

$$+ \frac{(0.\cancel{1})(-0.1)}{(0.3)(0.\cancel{1})}(0.72)$$

$$= -0.135 + 0.69 + 0.24$$

$$\boxed{= 0.795} \qquad \boxed{= 0.65}$$

Q2

x :	-1	0	1	3
$f(x)$:	2	1	0	-1

$$\cancel{f(x) = \frac{(x-0)(x-1)(x-3)}{(x^{-1}+0)(-1-1)(-1-3)} + \frac{(x+1)(x-1)(x-3)}{(0+1)(0-1)(0-3)}}$$

$$\cancel{+ \frac{(x+1)(x-0)(x-3)}{(1+1)(1-0)(1-3)} + \frac{(x+1)(x-0)(x-1)}{(3-2)(3-1)(3+1)}}$$

$$\cancel{f(x) = \frac{(x-0)(x-1)(x-3)}{(x_0-0)(x_0-1)(x_0-3)}}$$

$$f(x) = \frac{(x-0)(x-1)(x-3)}{(-1-0)(-1-1)(-1-3)} + \frac{(x+1)(x-1)(x-3)}{(0+1)(0-1)(0-3)}$$

$$\frac{(x+1)(x-0)(x-3)}{(1+1)(1-0)(1-3)} + \frac{(x+1)(x-0)(x-1)}{(3+1)(3-0)(3-1)}$$

$$\rightarrow \frac{(x-0)(x-1)(x-3)}{(-1)(-2)(-4)} + \frac{(x+1)(x-1)(x-3)}{(1)(-1)(-3)} + \frac{(x+1)(x-0)(x-3)}{(2)(1)(-2)}$$

$$+ \frac{(x+1)(x-0)(x-1)}{(4)(3)(2)}$$

$$\rightarrow \frac{(x^2-x)(x-3)}{-8} + \frac{x^2-1(x-3)}{3} + \frac{(x^2+x)(x-3)}{-4}$$

$$+ \frac{(x^2+x)(x-1)}{24}$$

$$\frac{(x^2-x)(x-3)}{-8} + \frac{(x^2-1)(x-3)}{3} + \frac{(x^2-x)(x-3)}{-4} + \frac{(x^2+x)(\;)(x-1)}{24}$$

$\times 3$ $\quad\times -8$ $\quad\times 6$ $\quad\times(-1)$

$$\left[\frac{x^3-3x^2+x^2+3x}{-8}\right] + \left[\frac{x^3-3x^2-x+3}{3}\right] + \left[\frac{x^3-3x^2+x^2+3x}{-4}\right] + \left[\frac{x^3-x^2+x^2}{24}\right]$$

$$\frac{3x^3+9x^2+3x^2+9x-8x^3+24x^2+8x-24+6x^3-18x^2+6x^2+18x-x^3+x^2-x^2+x}{24}$$

$$3x^3-8x^3+6x^3-x^3+9x^2+3x^2+24x^2-18x^2+16x^2$$
$$+9x+8x+18x+8x-24$$

$$34x^2+43x-24$$

$$=\frac{1}{24}\left[x^3-25x+24\right]$$ //

Q Using Langrange's interpolation find $f(0)$

x :	-1	-2	2	4
$f(x)$:	-1	-9	11	69

$\rightarrow$ $\frac{(0+2)(0-2)(0-4)}{(-1+2)(-1-2)(-1-4)} + \frac{(0+1)(0-2)(0-4)}{(-2+1)(-2-2)(-2-4)}$

$\frac{(0+1)(0+2)(0-4)}{(2+1)(2+2)(2-4)} + \frac{(0+1)(0+2)(0-2)}{(4+1)(4+2)(4-2)}$

$\rightarrow$ $\frac{2(-2)(-4)}{(-1)(-3)(-5)} + \frac{(1)(-2)(-4)}{(-1)(-4)(-6)} + \frac{(1)(2)(-4)}{(3)(4)(-2)} + \frac{(1)(2)(-2)}{(5)(6)(2)}$

$= \frac{16}{15}(-1) + \frac{\cancel{8}}{\cancel{24}}(-9) + \frac{\cancel{8}}{\cancel{24}}(11) + \frac{4}{60}(69)$

$\cancel{= 1.066 - \cancel{0.333} + -0.0666}$

$\cancel{\boxed{= -1.1326}}$

$1.066 + 3 + 3.666 - 4.6$

$\boxed{\cancel{-3.132}}$ $= \boxed{1}$

2. Newton's Divided difference Interpolation

*) Divided difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		$\frac{y_1 - y_0}{x_1 - x_0} = \Delta y_0 = [x_0, x_1]$	$\frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} = \Delta^2 y_0 = [x_0 x_1, x_2]$	
x_1	y_1			$\frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0} = \Delta^3 y_0$
		$\frac{y_2 - y_1}{x_2 - x_1} = \Delta y_1 = [x_1, x_2]$	$\frac{\Delta y_2 - \Delta y_1}{x_3 - x_1} = \Delta^2 y_1 [x_1, x_2, x_3]$	$= [x_0\, x_1\, x_2\, x_3]$
x_2	y_2			
		$\frac{y_3 - y_2}{x_3 - x_2} = \Delta y_2 = [x_2\, x_3]$		
x_3	y_3			

Q. If $f(x) = \frac{1}{x}$ find divided differences $[a, b]$ and $[a, b, c]$

x	$f(x)$	Δy	$\Delta^2 y$
a	$1/a$		
		$\frac{1/b - 1/a}{b - a}$	
			$\frac{-1/bc + 1/ab}{c - a}$
b	$1/b$		
		$\frac{1/c - 1/b}{c - b}$	
c	$1/c$		

$$[a,b] = \frac{\frac{a-b}{ab}}{b-a} = -\frac{1}{ab}$$

$$[b,c] = \frac{\frac{b-c}{bc}}{c-b} = -\frac{1}{bc}$$

$$[a,b,c] = \frac{\frac{-a+c}{abc}}{c-a} = \frac{1}{abc}$$