~~Previously~~ sampling a cont time you got discrete signal &

$\boxed{t = nT}$

$$x_a(t)\Big|_{t=nT} = x_a(nT) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(\Omega)\, e^{j\Omega nT}\, d\Omega$$

$\boxed{\Omega = \frac{2\pi}{T} \text{ is the Total width in freq domain}}$

$$= \frac{1}{2\pi}\left[\cdots + \int_{-\frac{3\pi}{T}}^{-\pi/T} + \int_{-\frac{\pi}{T}}^{\pi/T} + \int_{\pi/T}^{\frac{3\pi}{T}} + \cdots\right]$$

$$= \frac{1}{2\pi}\left[\sum_{r=-\infty}^{\infty}\int_{\frac{(2r-1)\pi}{T}}^{\left(\frac{2r+1}{T}\right)\pi} X_a(\Omega)\, e^{j\Omega nT}\, d\Omega\right]$$

$$= \frac{1}{2\pi}\left[\sum_{r=-\infty}^{\infty}\int_{-\frac{\pi}{T}}^{(\pi/T)} X_a\left(\Omega - \frac{2\pi r}{T}\right) e^{j\Omega nT}\, d\Omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\frac{\pi}{T}}^{\pi/T} \left[\sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi r}{T}\right) e^{j\Omega nT}\right] d\Omega\right]$$

$$= \left(e^{j\omega n}\right)$$

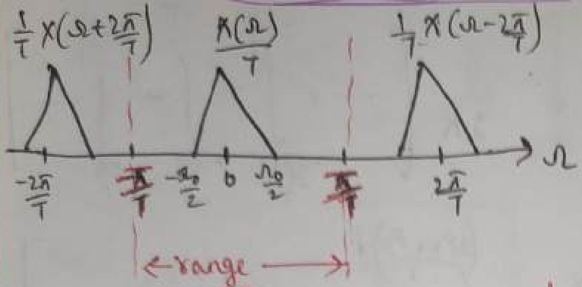
as $\omega = \Omega T$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} \sum_{r=-\infty}^{\infty} \frac{X\left(F_s(\omega - 2\pi r)\right)}{T}\, e^{j\omega n}\, d\omega\right]$$

$$(T)\, X(e^{j\omega}) = \sum_{r=-\infty}^{\infty} \frac{X\left(F_s(\omega-2\pi r)\right)}{T} = \sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi r}{T}\right)$$

$$X(e^{j\omega}) = \frac{1}{T}\sum_{r=-\infty}^{\infty} \frac{X\left(F_s(\omega-2\pi r)\right)}{T} = \frac{1}{T}\sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi r}{T}\right)$$

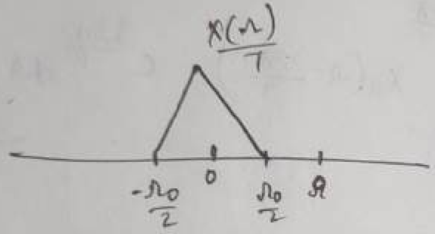
Reconstruction of x(t) from x[nT]



$$X(e^{j\Omega T}) = \frac{1}{T}\sum_{r=-\infty}^{\infty} X\left(\Omega - \frac{2\pi r}{T}\right)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)\, e^{j\Omega t}\, d\Omega$$

$$x(t) = \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} X(e^{j\Omega})\, e^{j\Omega t}\, d\Omega$$



$$x(t) = \frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\pi/T}\left(\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}\right) e^{j\Omega t}\, d\Omega$$

$$X(e^{j\omega}) = \frac{X(\Omega)}{T}$$

$$X(\Omega) = T\, X(e^{j\Omega})$$

for $-\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$

$$x(t) = \frac{T}{2\pi}\sum_{n=-\infty}^{\infty} x[n]\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} e^{-j\Omega nT}\, e^{j\Omega t}\, d\Omega$$

$$x(t) = \frac{T}{2\pi}\sum_{n=-\infty}^{\infty} x[n]\int_{-\frac{\pi}{T}}^{\pi/T} e^{j\Omega(t-nT)}\, d\Omega$$

$$x(t) = \sum_{n=-\infty}^{\infty} x[n]\left[\frac{e^{j\frac{\pi}{T}(t-nT)} - e^{-j\frac{\pi}{T}(t-nT)}}{(2j)\,\frac{\pi}{T}(t-nT)}\right]$$

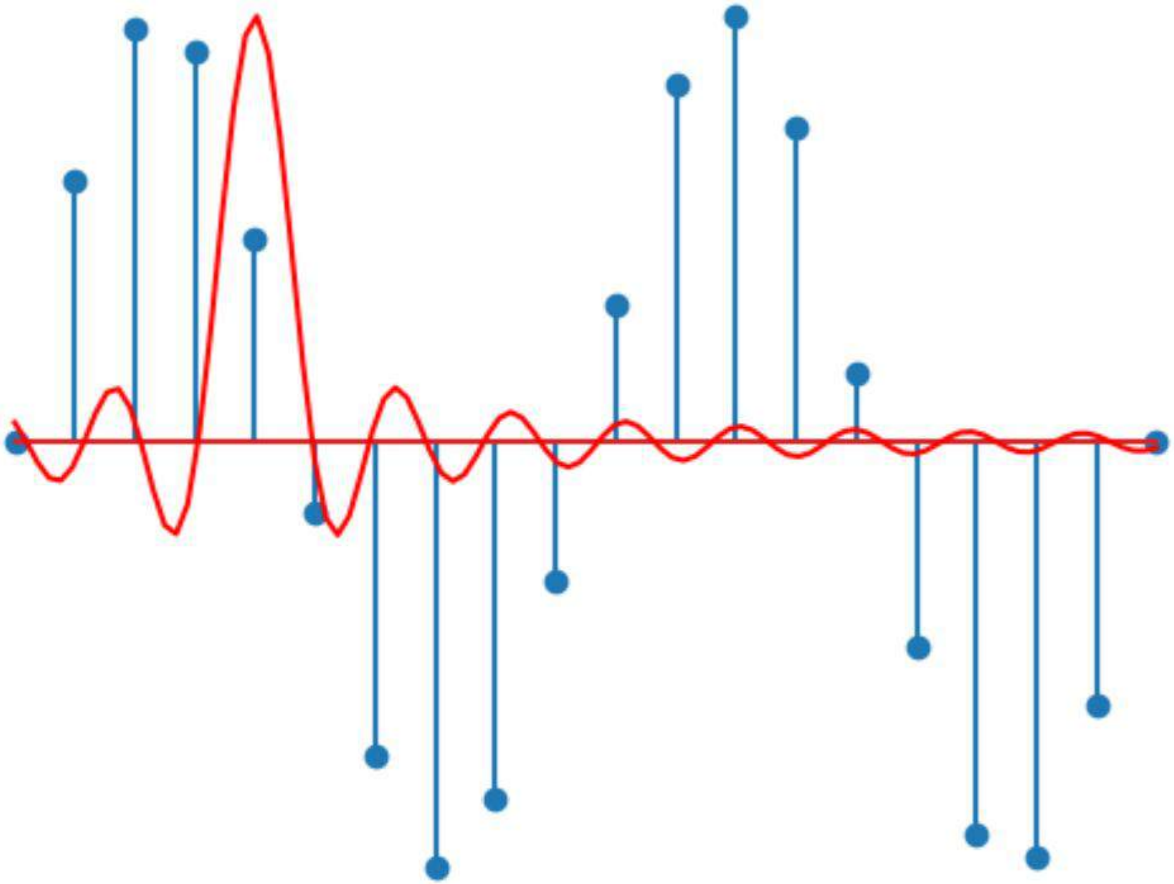
$$x(t) = \sum_{n=-\infty}^{\infty} x[n]\,\frac{\sin\frac{\pi}{T}(t-nT)}{\frac{\pi}{T}(t-nT)}$$

as

$$y(n) = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

here $k = nT$
$n = t$

$$x(t) = x[n] * \frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}}$$



Now the sinc function moves throughout the signal at t=nT and gives x(t)

Discrete

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$\boxed{x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega}$$

$\omega = \Omega T$

$d\omega = d\Omega\, T$

$=$

continious

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, d\Omega$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t}\, d\Omega \quad \leftarrow \quad \boxed{\left(\because\ t = nT = \frac{n}{F_s}\right)}$$

$$\boxed{x(nT) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega nT}\, d\Omega}$$

$$x(n) = \frac{T}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega T})\, e^{j\Omega nT}\, d\Omega$$

$$\cancel{X(e^{j\Omega T}) = \sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi r}{T}\right)}$$

so

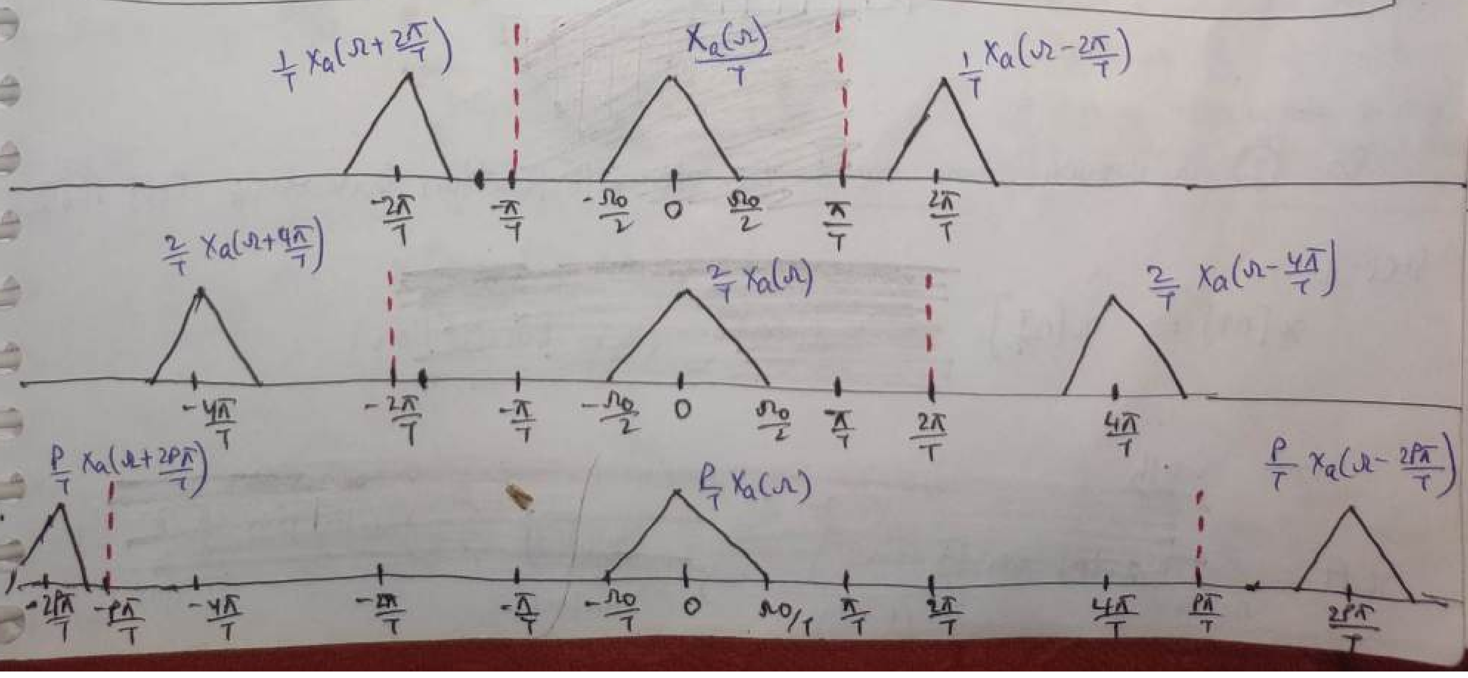
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi r}{T}\right) e^{j\Omega nT} = \frac{T}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega T})\, e^{j\Omega nT}\, d\Omega$$

$$\text{so}\quad T\, X(e^{j\Omega T}) = \sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi r}{T}\right)$$

$$\boxed{X(e^{j\Omega T}) = \frac{1}{T}\sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi r}{T}\right)}$$

$\approx$

$$\boxed{X(e^{j\Omega \frac{T}{2}}) = \frac{2}{T}\sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{4\pi r}{T}\right)} \qquad \boxed{X(e^{j\Omega \frac{T}{P}}) = \frac{P}{T}\sum_{r=-\infty}^{\infty} X_a\left(\Omega - \frac{2\pi P r}{T}\right)}$$



Understanding $x(t) = \sum_{n=-\infty}^{\infty} x[n] \dfrac{\sin \frac{\pi}{T}(t-nT)}{\frac{\pi}{T}(t-nT)}$

$$x(t) = \cdots\cdots + \underbrace{x(0)}_{c_i} \underbrace{\frac{\sin \frac{\pi}{T} t}{\frac{\pi t}{T}}}_{v_{iz}} + \underbrace{x(1)}_{c_i} \underbrace{\frac{\sin \frac{\pi}{T}(t-1)}{\frac{\pi}{T}(t-1)}}_{v_{iz}} + \underbrace{x(2)}_{c_i} \underbrace{\frac{\sin \frac{\pi}{T}(t-2)}{\frac{\pi}{T}(t-2)}}_{v_{iz}} + \cdots\cdots$$

So $x(t) = \sum_{i=-\infty}^{\infty} c_i v_{iz}$

$c_i = x[i]$

$v_{iz} = \dfrac{\sin \frac{\pi}{T}(z-i)}{\frac{\pi}{T}(z-i)}$

let $x(t) = w_z$

So $w_z = \sum_{i=-\infty}^{\infty} c_i v_{iz}$

$$w_z = \cdots\cdots + c_0 v_{0z} + c_1 v_{1z} + c_2 v_{2z} + \cdots \quad \text{— (1)}$$

- From matrix theory we know that any vector can be represented as linear combination of Basis vectors.

- Here c_i = Constants are discrete samples
- Here v_{iz} = basis functions

$$\int_{-\infty}^{\infty} \frac{\sin \frac{\pi}{T}(t-i)}{\frac{\pi}{T}(t-i)} \cdot \frac{\sin \frac{\pi}{T}(t-(i+1))}{\frac{\pi}{T}(t-(i+1))} \, dt = 0$$

So independent

Note

$\dfrac{\sin \frac{\pi}{T}(t-1)}{\frac{\pi}{T}(t-1)}$ and $\dfrac{\sin \frac{\pi}{T}(t-2)}{\frac{\pi}{T}(t-2)}$ are lin independent

Question : Given a even sample sequence, construct odd samples.

$$x[nT] \longrightarrow \boxed{S/m} \longrightarrow x\left[\frac{nT}{2}\right]$$

available, even $F_s = F_{s_{I/P}}$

$F_s = 2F_{s_{I/P}}$

<u>Sol</u>

$x_{even}[n] \longrightarrow$ Construct $x_{odd}[n]$ using $\cancel{x_{odd}[m]}$ $x[mT+\frac{T}{2}] = \sum_{n=-\infty}^{\infty} x_{even}[n] \dfrac{\sin \frac{\pi}{T}(mT-nT+\frac{T}{2})}{\frac{\pi}{T}(mT-nT+\frac{T}{2})}$

$$\boxed{x\left[mT+\frac{T}{2}\right] = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin \frac{\pi}{T}(mT-nT+\frac{T}{2})}{\frac{\pi}{T}(mT-nT+\frac{T}{2})}}$$

now

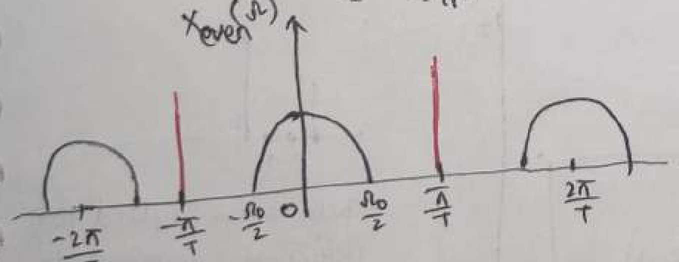
$\tilde{x}_e = \{ x_{even}(0),\ 0,\ x_{even}(1),\ 0,\ x_{even}(2),\ 0,\ \cdots \}$

$\tilde{x}_o = \{ \cancel{x_{odd}}\ 0,\ x_{odd}(0),\ 0,\ x_{odd}(1),\ 0,\ x_{odd}(2),\ \cdots \}$

$\tilde{x}_e + \tilde{x}_o = \{ x_{even}(0),\ x_{odd}(0),\ x_{even}(1),\ x_{odd}(1),\ x_{even}(2),\ x_{odd}(2),\ \cdots \}$

now this $\tilde{x}_e + \tilde{x}_o \Rightarrow F_s = 2F_{s_{I/P}}$

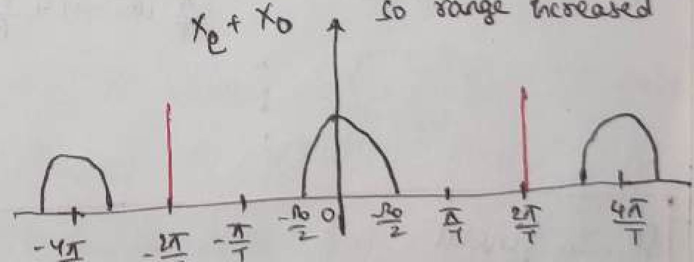
$F_s = F_{s_{I/P}}$



$-\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$

Total range $= \frac{2\pi}{T}$

$F_s = 2F_{s_{I/P}}$ so range increased



$-\frac{2\pi}{T} \le \Omega \le \frac{2\pi}{T}$

Total range $= \frac{4\pi}{T}$

$x[mT+\frac{T}{2}] = \cancel{x_{odd}[mT+\frac{T}{2}]} \cancel{\sum_{n=-\infty}^{\infty} x[nT] \frac{\sin\frac{\pi}{T}(mT-nT+\frac{T}{2})}{\frac{\pi}{T}(mT-nT+\frac{T}{2})}}$

now we did direct Substitution of $t = mT + \frac{T}{2}$.

Now lets ~~do~~ find the eq^n using DTFT

Frequency domain analysis.

$$x_a(t)\Big|_{t=nT} = X_a(\Omega)$$

$$x_a(t)\Big|_{t=nT+\frac{T}{2}} = X_a(\Omega)\, e^{j\frac{\Omega T}{2}}$$

In general

$$x(t)\Big|_{t=nT} \longleftrightarrow \frac{X_a(\Omega)}{T}$$

$$x(t)\Big|_{t=nT+\frac{T}{2}} \longleftrightarrow \frac{X_a(\Omega)}{T} \cdot e^{j\frac{\Omega T}{2}}$$

So that means If I Sample a Signal at $F_s = \frac{1}{T}$ then I can find the Signal Points at $F_s = \frac{2}{T} = 2F_s$ (or) in other words I can reconstruct other signal Points

---

$$F[x_1[n]] = F[x_{odd}[n]] = \frac{X_a(\Omega)}{T}$$

$$F[x_2[n]] = \; e^{j\frac{\Omega T}{2}} \qquad -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$$

(or)

$$e^{j\frac{\omega}{2}} \qquad -\pi \le \omega \le \pi$$

$$F[x_{even}[n]] = F(x_{odd}[n])\, F(x_2[n])$$

now need to calculate $x_2[n]$

$$\text{IDTFT}[x_2[n]] = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} e^{j\Omega T/2}\, e^{j\Omega n}\, d\Omega \;\text{(or)}\; \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\frac{\omega}{2}}\, e^{j\omega n}\, d\omega$$

$$x_2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\frac{w}{2}} e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jw(n+\frac{1}{2})}\, dw$$

$$= \frac{1}{\cancel{\ }} \frac{e^{j\pi(n+\frac{1}{2})} - e^{-j\pi(n+\frac{1}{2})}}{(2j)\pi(n+\frac{1}{2})}$$

$$= \frac{\sin \pi(n+\frac{1}{2})}{\pi(n+\frac{1}{2})}$$

$$\therefore x_2[n] = \frac{\sin \pi(n+\frac{1}{2})}{\pi(n+\frac{1}{2})}$$

$$x_{even}[n] = x_{odd}[n] * \frac{\sin \pi(n+\frac{1}{2})}{\pi(n+\frac{1}{2})}$$

$$x(t) = x[n] * \frac{\sin\left(\frac{\pi t}{T}\right)}{\left(\frac{\pi t}{T}\right)}$$

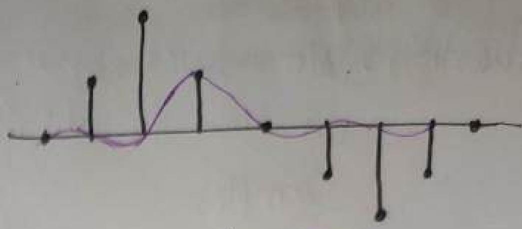


fig ①

$$x[mT] = x[n] * \frac{\sin\left(\frac{\pi m T}{T}\right)}{\frac{\pi m T}{T}}$$

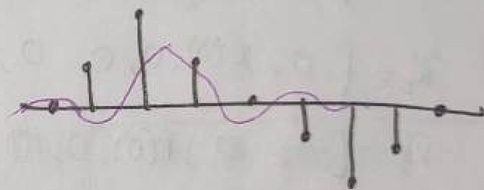
$$= x[n] * \frac{\sin \pi m}{\pi m} \quad (\text{fig } ①)$$

$\boxed{x_{odd}}$

$$x(t) = x[n] * \frac{\sin \frac{\pi}{T}\left(t + \frac{T}{2}\right)}{\frac{\pi}{T}\left(t + \frac{T}{2}\right)}$$

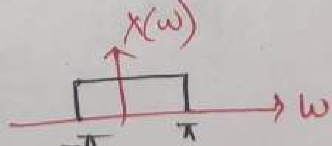
$\boxed{x_{even}}$

$$x[mT] = x[n] * \frac{\sin \frac{\pi}{T}\left(mT + \frac{T}{2}\right)}{\frac{\pi}{T}\left(mT + \frac{T}{2}\right)}$$



$$= x[n] * \frac{\sin \pi(m + 1/2)}{\pi(m + 1/2)}$$

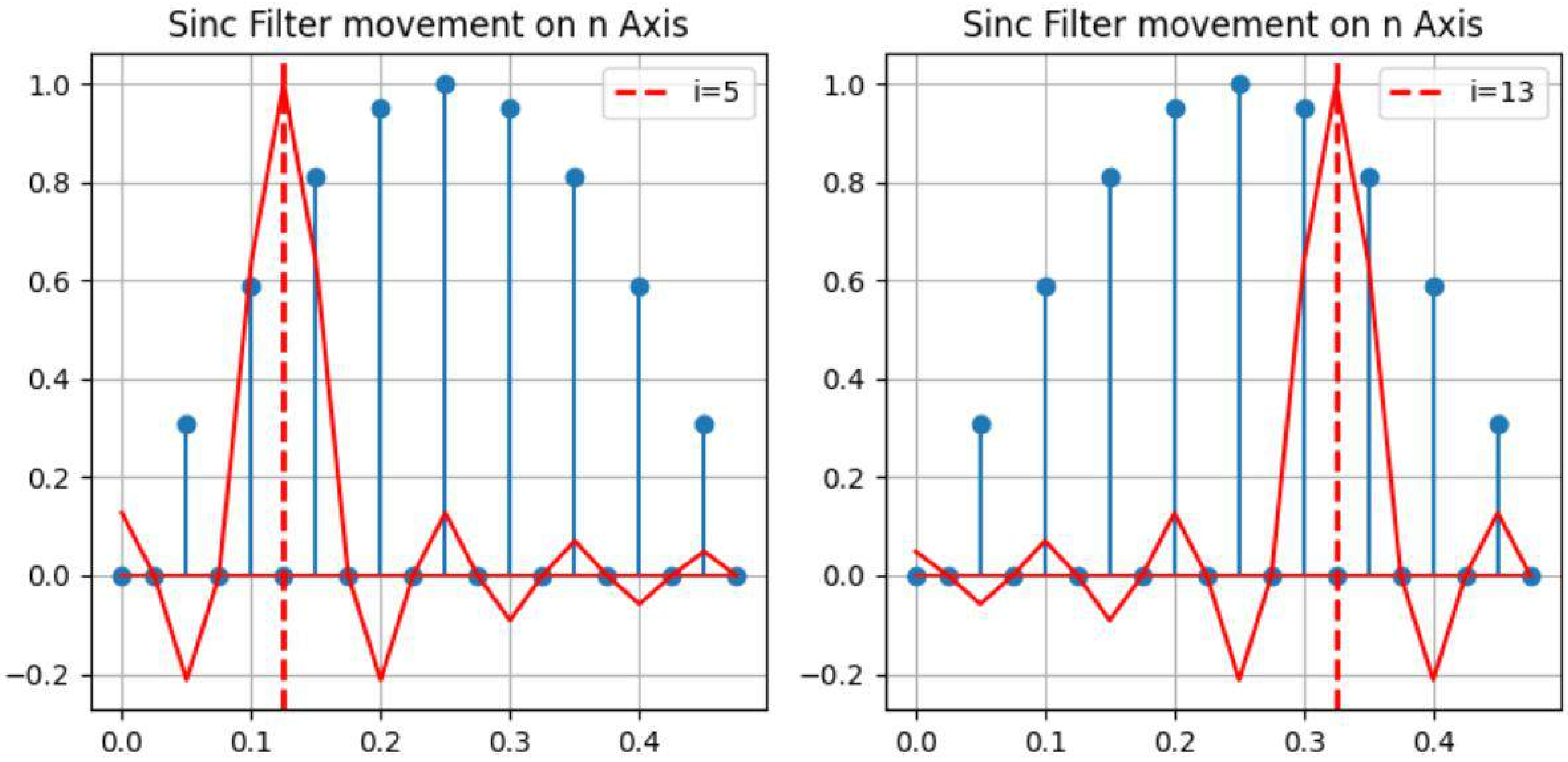
$$\frac{\sin \pi n}{\pi n} \leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right) \Rightarrow$$



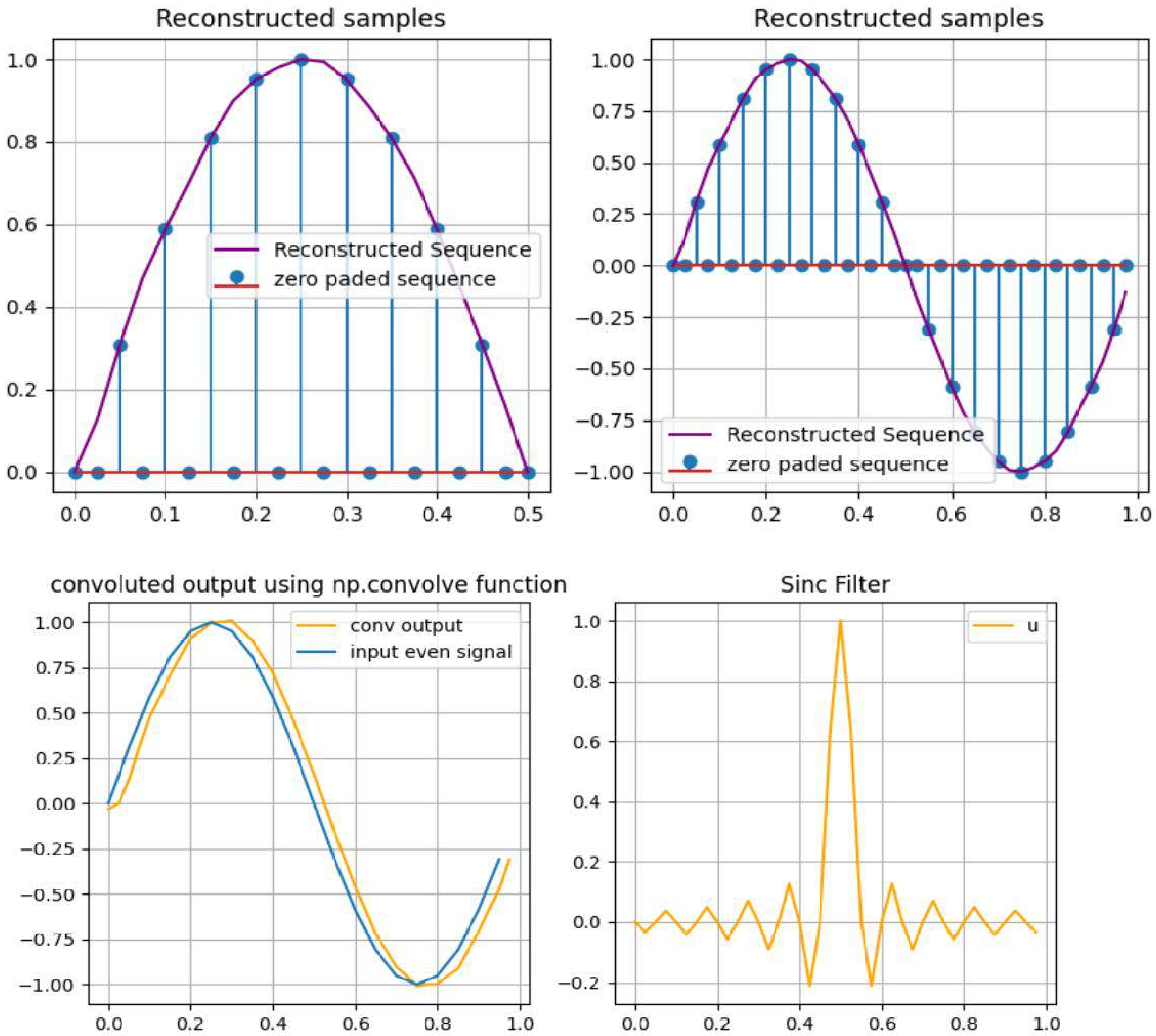
$$\left(\because \frac{\sin a n}{\pi n} \leftrightarrow \text{rect}\left(\frac{\omega}{2a}\right)\right)$$

as $t\big|_{t=nT} \Rightarrow \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} \Rightarrow \frac{\sin \pi n}{\pi n}$

Method -1 Padding 0 between 2 samples and convolving



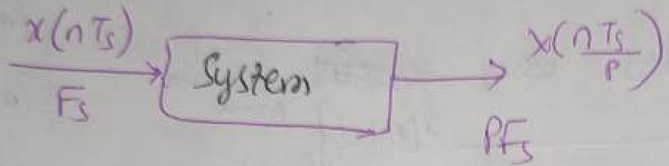
As I took sampling freq low to visualize properly, the sinc function is not smooth


Reconstructed samples
Reconstructed Sequence
zero paded sequence
Reconstructed samples
Reconstructed Sequence
zero paded sequence
convoluted output using np.convolve function
conv output
input even signal
Sinc Filter
u

Question: Now wrt even odd sample reconstruction, Now generalize on how to obtain polyphase components b/w two samples

(or)

Upsample the signal so that $F_s \to z F_s$ $\quad z = \{1, 2, \cdots\}$

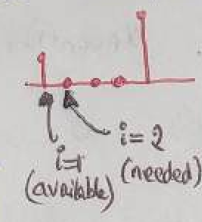


Sol

$x[nT_s] \longrightarrow$ reconstruct i^{th} polyphase sample

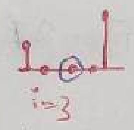
for $i=2$

$$x\left[mT + \frac{T}{P}(1)\right] = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin \frac{\pi}{T}\left(mT - nT + \frac{T}{P}(1)\right)}{\frac{\pi}{T}\left(mT - nT + \frac{T}{P}(1)\right)}$$



for $i=3$

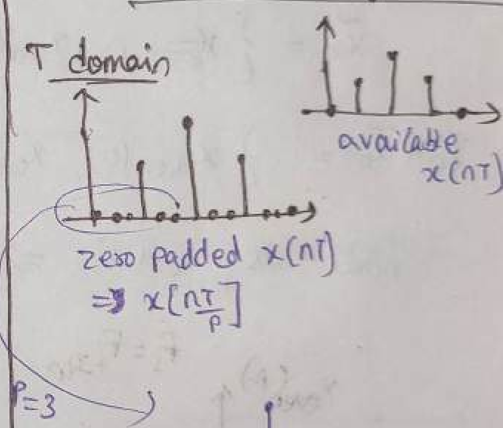
$$x\left[mT + \frac{T}{P}(2)\right] = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin \frac{\pi}{T}\left(mT - nT + \frac{T}{P}(2)\right)}{\frac{\pi}{T}\left(mT - nT + \frac{T}{P}(2)\right)}$$

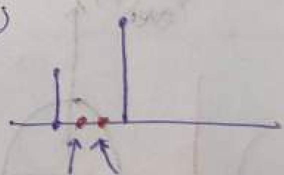


⋮

What is Polyphase

T domain





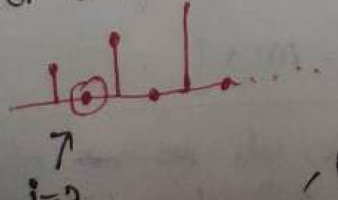
These 2 are called ~~Poly~~ Polyphase components

So, In general

$$\boxed{x\left[mT + \frac{T}{P}(i-1)\right] = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin \frac{\pi}{T}\left(mT - nT + \frac{T}{P}(i-1)\right)}{\frac{\pi}{T}\left(mT - nT + \frac{T}{P}(i-1)\right)}}$$

to get any i^{th} sample

For even odd Question, we had only one sample with '0' b/w 2 samples

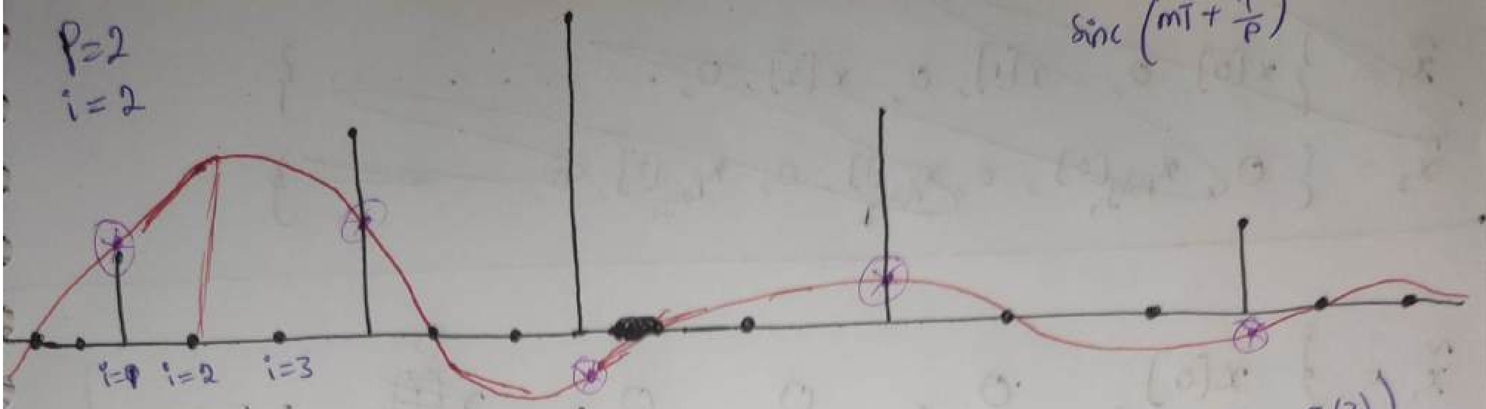


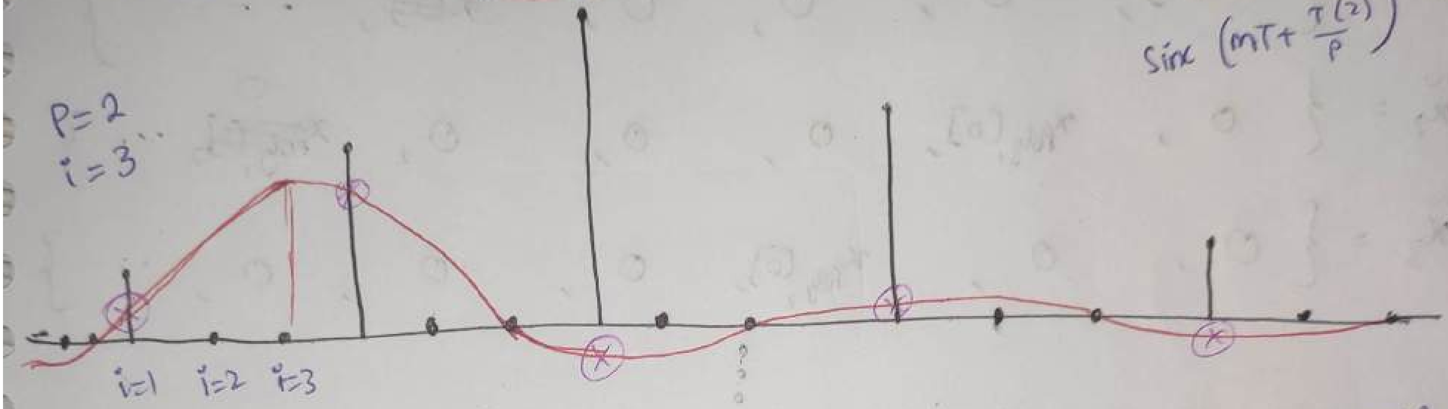
, P=2

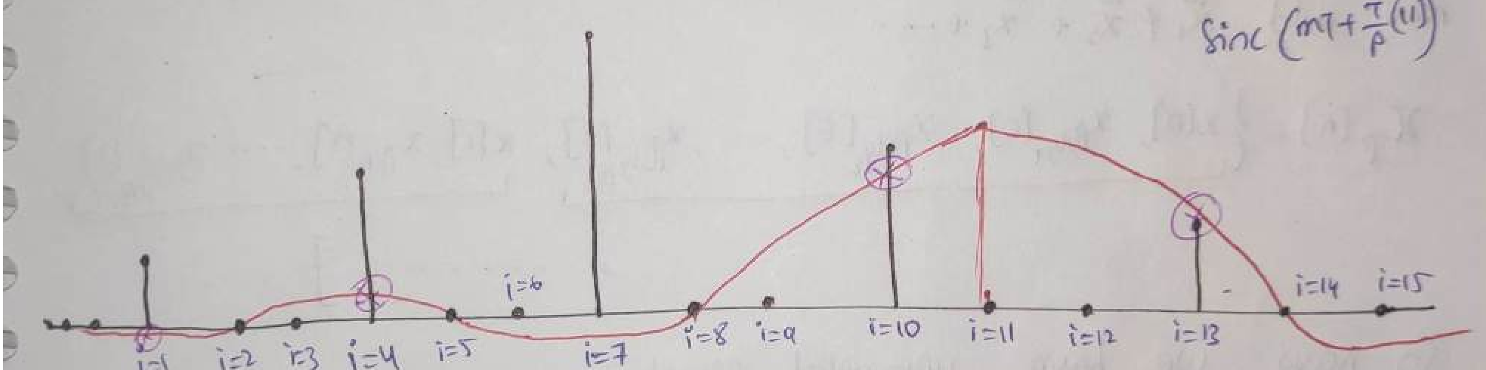
So

$$x\left[mT + \frac{T}{2}(1)\right] = \sum_{n=-\infty}^{\infty} \cdots$$

now sinc peak is shifted to $i=1$ position.







Now $\sum$(multiply the sample values with (sine values at $\otimes$)) i.e dot Prod
& this result gives $x\left[n\frac{T}{2}\right]$ at i^{th} Point.

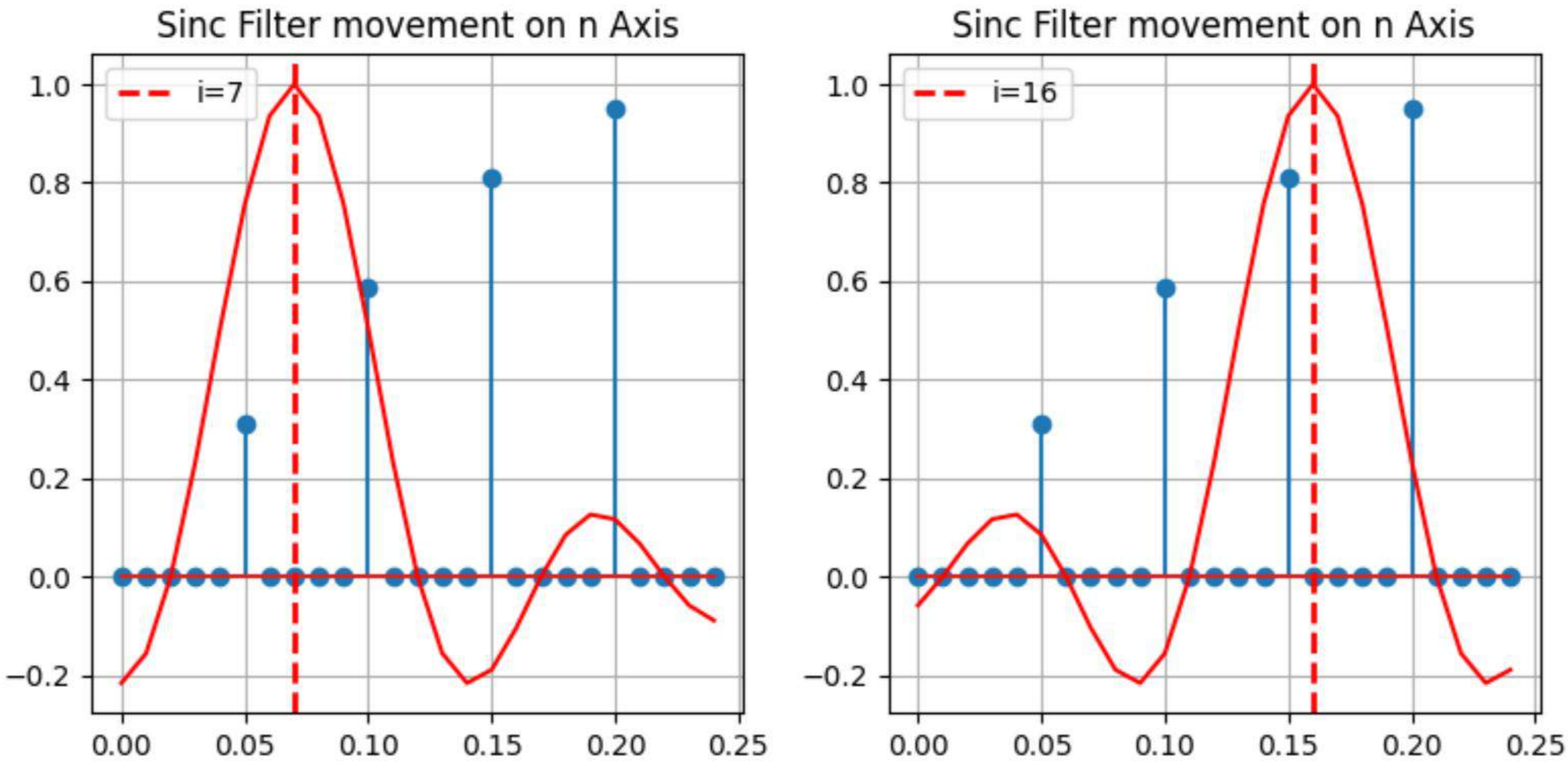
i.e for 1^{st} fig $\sum \otimes$ multiplied by sample value gives $x\left[n\frac{T}{P}\right]$ at i=2

i.e for 2^{nd} fig $\sum \otimes$ multiplied by sample value gives $x\left[n\frac{T}{P}\right]$ at i=3

for 3^{rd} fig $\sum \otimes$ multiplied by sample value gives $x\left[n\frac{T}{P}\right]$ at i=11

now we have $x[n]$ (given), $x\left[n+\frac{T}{P}(1)\right]$, $x\left[n+\frac{T}{P}(2)\right]$, ... so
Pad (P-1) 0's in each case.

Method -1 p=5, so padding 4 0's between 2 samples and convolving



$$\tilde{x}_1 = \{\, x[0],\ 0,\ 0,\ 0,\ \overset{0}{\cancel{x[1]}},\ \cdots \,\}$$

$$\tilde{x}_2 = \{\, 0,\ x_{Poly_1}[0],\ 0,\ 0,\ 0,\ \overset{0}{\cancel{x_{Poly_1}[1]}},\ \cdots \,\}$$

$$\tilde{x}_3 = \{\, 0,\ 0,\ x_{Poly_2}[0],\ 0,\ 0,\ 0,\ \cdots \,\}$$

$$\vdots$$

now add $\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 + \cdots$

$$x_p[n] = \{\, \underbrace{x[0],\ x_{Poly_1}[0],\ x_{Poly_2}[0],\ \cdots,\ x_{Poly_{P-1}}[0]}\, ,\ \underbrace{x[1],\ x_{Poly_1}[1],\ \cdots\ x_{Poly_{P-1}}[1]}\, ,\ \cdots\cdots \,\}$$

So now we have upsampled signal.

---

now we will analyze in freq spectrum

$$x[m] \longleftrightarrow \frac{X_a(\Omega)}{T}$$

$$x\left[mT + \frac{T}{P}(i-1)\right] \longleftrightarrow \frac{X_a(\Omega)}{T} e^{j\Omega \frac{T}{P}(i-1)} \ \text{——} \begin{cases} \text{for } i=2 & \frac{X_a(\Omega)}{T} e^{j\Omega \frac{T}{P}(1)} \\ \text{for } i=3 & \frac{X_a(\Omega)}{T} e^{j\Omega \frac{T}{P}(2)} \\ \vdots & \\ \text{for } i=P-1 & \frac{X_a(\Omega)}{T} e^{j\Omega \frac{T}{P}(P-2)} \end{cases}$$

Frequency domain analysis

$i = i^{th}$ poly phase component

$$x(t)\Big|_{t=nT} \longleftrightarrow \frac{X_a(\Omega)}{T}$$

$$x(t)\Big|_{t=nT+\frac{T}{P}(i-1)} \longleftrightarrow \frac{X_a(\Omega)}{T} e^{j\frac{T}{P}\Omega(i-1)}$$

In general

$$x(t)\Big|_{t=nT} \longleftrightarrow \frac{X_a(\Omega)}{T}$$

$$x(t)\Big|_{t=nT+\frac{T}{P}(i-1)} \longleftrightarrow \frac{X_a(\Omega)}{T} e^{j\frac{T}{P}(i-1)\Omega}$$

So that means if I Sample a Signal at $F_s = \frac{1}{T}$ then I can find the signal Points at $F_s = \frac{P}{T}$. (or)

In another words I can reconstruct ~~at~~ other sample Points.

$\boxed{\text{Limits of } \Omega / \omega}$

Please note that the signal is still sampled at $t = nT$. Just the signal is sampled at $nT + \frac{T}{P}(i-1)^{th}$ Position (Time shifting doesn't change the periodicity thereby freq limits)

$$x\left(m\frac{T}{P}\right) \Rightarrow -\frac{\pi P}{T} \le \Omega \le \frac{\pi P}{T}$$

$$\pi \le \omega \le -\pi$$

$$\boxed{\omega = \frac{\Omega T}{P}}$$

$\Rightarrow$ sampled at $t = \frac{mT}{P}$

$$x\left[nT + \frac{T(i-1)}{P}\right] \Rightarrow \quad -\frac{\pi}{T} \le \Omega \le \frac{\pi}{T} \qquad \boxed{\Omega T = \omega} \quad \Rightarrow \text{Sampled at } t = nT$$

$$-\pi \le \omega \le \pi$$

$$FT\left[x_1[n]\right] = \frac{X_a(\Omega)}{T}$$

$$FT\left[x_2[n]\right] = e^{j\frac{T}{P}\Omega(i-1)}$$

$$e^{j\omega\frac{(i-1)}{P}} \qquad \boxed{\omega = T\Omega}$$

so that $\frac{x_a(t)}{T} e^{j\frac{T}{P}(i-1)}$

$$= x_1[n] * x_2[n]$$

$$x_2[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega\frac{(i-1)}{P}} \cdot e^{j\omega n}\, d\omega$$

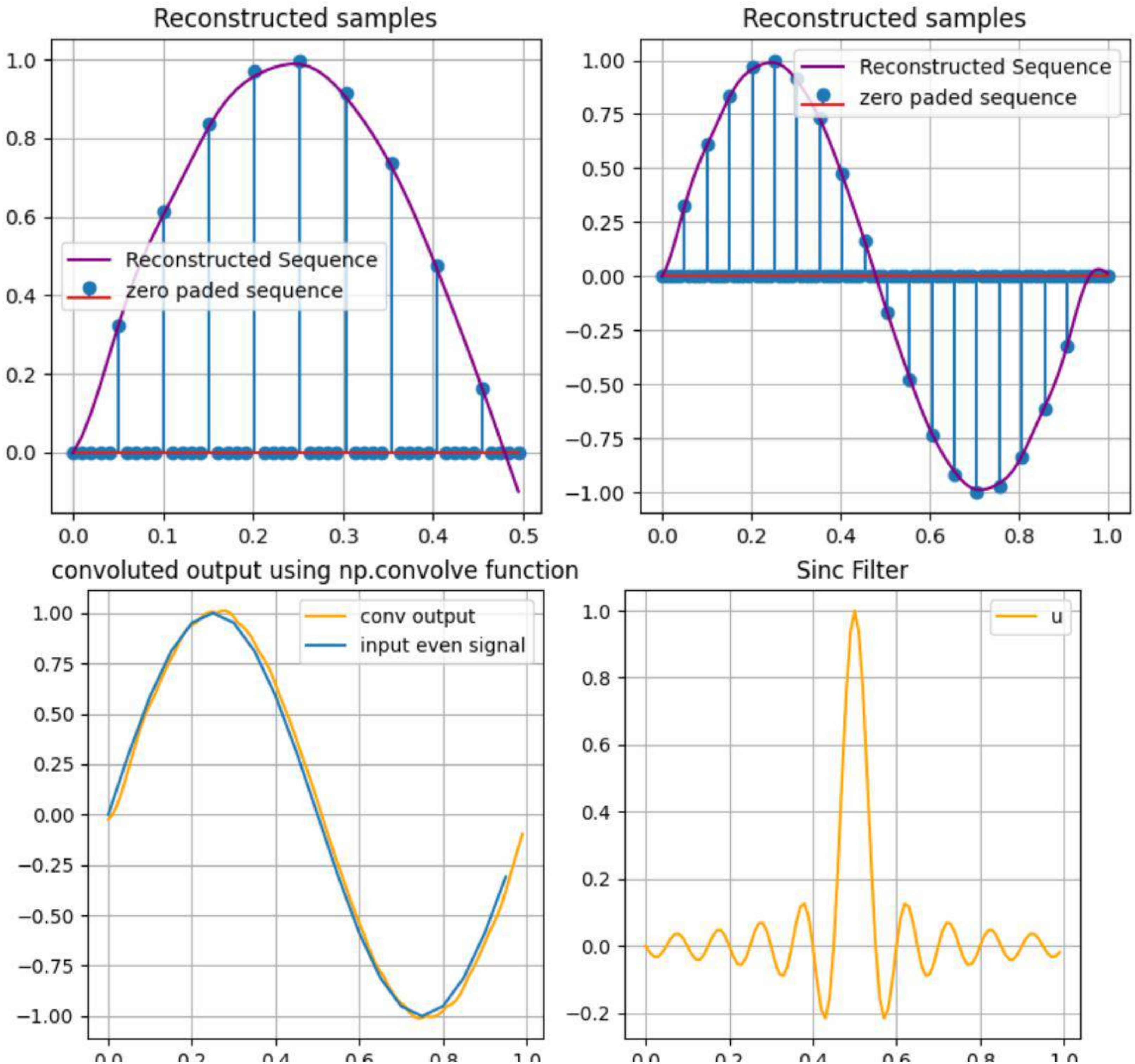
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega\left(n + \frac{i-1}{P}\right)}\, d\omega$$

$$= \left[\frac{e^{j\pi\left(n+\frac{i-1}{P}\right)} - e^{-j\pi\left(n+\frac{i-1}{P}\right)}}{(2j)\ \pi\left(n+\frac{i-1}{P}\right)}\right]$$

$$= \frac{\sin \pi\left(n+\frac{(i-1)}{P}\right)}{\pi\left(n+\frac{(i-1)}{P}\right)}$$

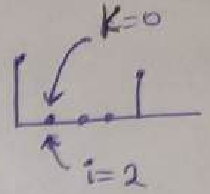
$$x_{\text{Polyphase}}[n] = x_1[n] * \frac{\sin \pi\left(n+\frac{(i-1)}{P}\right)}{\pi\left(n+\frac{(i-1)}{P}\right)}$$

Available i/p signal


Reconstructed samples
Reconstructed Sequence
zero paded sequence
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
Reconstructed samples
Reconstructed Sequence
zero paded sequence
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.2
0.4
0.6
0.8
1.0
convoluted output using np.convolve function
conv output
input even signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Sinc Filter
u
1.0
0.8
0.6
0.4
0.2
0.0
−0.2

Now we had obtained polyphase components from i/p. But there is a problem.

$$x_{polyphase}[k] = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin \pi \left[k - \left(n + \frac{i+1}{p}\right)\right]}{\pi\left(k - \left(n + \frac{i-1}{p}\right)\right)}$$



$$x_{polyphase}[0] = \underbrace{\cdots + x[-1]}_{} \frac{\sin \pi \left(0 - \left(-1 + \frac{1}{p}\right)\right)}{\pi\left(0 - \left(-1 + \frac{1}{p}\right)\right)} + x(0) \_\_ + x(1) \_\_ + \cdots$$

Here $x[-1], x[-2], \ldots$ are not there as we assumed the signal to start with $x[0], x[1], \ldots$.

So the $x_{poly}[k=0]$ value / sample will have error.

- We need to fix the error at $x_{poly}[k=0]$ sample. Similarly we need to fix error at $x_{poly}\left[k = \frac{N}{2} - 1\right]$ 1st & last poly samples

- So to solve this issue we need some signal representation for $n<0$ so that $x[n]\ \forall n<0$ & $x[n]\ \forall n > \frac{N}{2}$ exists.

- For aperiodic signals we can't do anything as they start at $n=0$ & go to $n=N-1$ & there ends the signal.

- But for periodic signals we have signal $x[n]\ \forall n<0$ &
$x[n]\ \forall n>N$

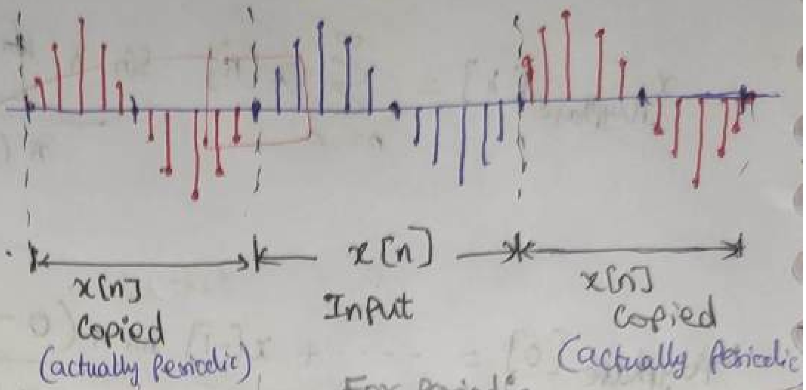
- So we could use the periodicity property of periodic signals & correct 1st & last poly samples i.e $x_{poly}(0)$, $x_{poly}(N-1)$

Periodic Property

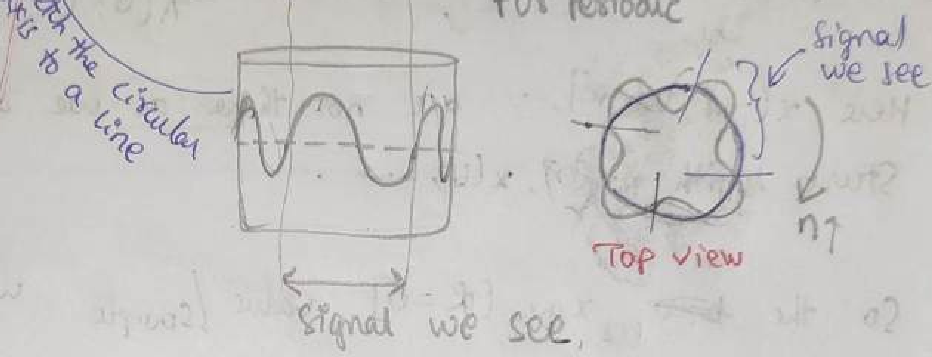
$$x(t) = x(t+T)$$

$$x(nT) = x(nT + lT)$$

$l \in \{1, 2, \cdots N\}$



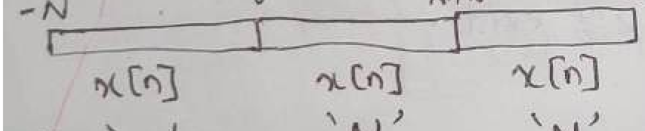
For periodic



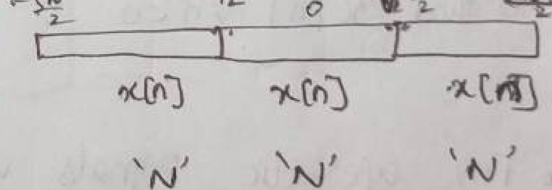
But aperiodic signals are not periodic, they cant wrap around itself so we need to do linear convolution.

The signal wraps around itself hence for periodic signals we use circular convolution

now the range,



(or)

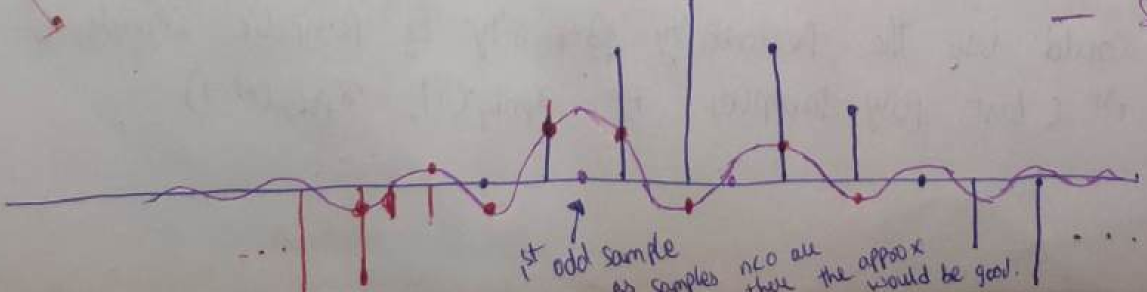


convinent to follow

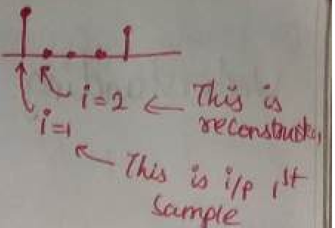
so lets take $x_p[n] = \{x[n]\ x[n]\ x[n]\}_{3N}$

now lets apply sinc filter of length L such that $L \leq N$

— Sinc filter



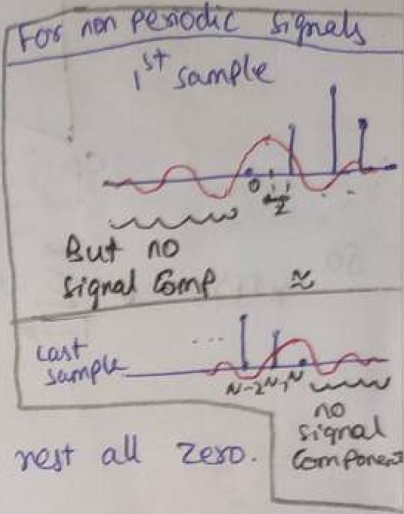
To revise, Problem with 1st sample (2nd Poly sample) is that its approximation is not accurate.



$$\underbrace{x_{Poly}[K]} = \sum_{m=-\infty}^{\infty} \underbrace{x[n]} \; \underbrace{\frac{\sin \pi\left(k-\left(n+\frac{i-1}{P}\right)\right)}{\pi\left(k-\left(n+\frac{(i-1)}{P}\right)\right)}}$$

this is 0 for $n<0$ & $n>N$
N: length $(x[n])$

this is zero for $m \geq \frac{L}{2}$ & $m < -\frac{L}{2}$



→ So 1st & last sample have some errors & for interval $-\frac{L}{2}$ to $\frac{L-1}{2}$ $x_{Poly}(k)$ is non zero, rest all zero.

For finite length sequence, the edges do not experience filter properly hence we use periodicity property to tackle them.

$$x_{Poly}(m) = x_{i/p}[n] \odot \phi(m) \quad \text{where } \phi(m) = \frac{\sin \pi\left(n+\frac{(i-1)}{P}\right)}{\pi\left(n+\frac{(i-1)}{P}\right)}$$

for periodic; $\odot$ ↑ circular Conv

As we know circular Convolution in time domain ⇒ multiplication in ω domain.

Here $n \in \left[-\frac{N}{2}, \ldots, \frac{N}{2}-1\right]$ (time axis)

$k \in \left[-\frac{N}{2}, \ldots, \frac{N}{2}-1\right]$ (freq axis)

$$\boxed{X_{i/p}(k) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} x_{i/p}[n]\, e^{-j\frac{2\pi k n}{N}}}$$
(DFT)

$$\boxed{X(e^{j\omega}) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} x[n]\, e^{-j\omega n}}$$
(DTFT)

DFT = DTFT only for $\omega = \frac{2\pi k}{N}$,

As circular Conv is highly computational we are using DFT (FFT thereby reducing complexity).