Circuit Theory and Electronics Fundamentals

Laboratory 5: Operational Amplifier Circuit

Integrated Masters in Aerospace and Technological Physics Engineering, Técnico, University of Lisbon

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1 Introduction

The objective of this laboratory assignment is to simulate and study a circuit that amplifies a given range of frequencies (a band-pass filter, BDF). The main goal was to achieve the best possible value for the merit of the circuit. This merit value depends on the cost of the components that we used, namely capacitors, transistors and resistors. The formula for the merit M is given by

$$M = \frac{1}{cost(G_{dev} + freq_{dev} + 10^{-6})} \tag{1}$$

where cost = cost of (resistors + capacitors + transistors), which includes the cost of the operational amplifier, in this case the 741 OP-AMP model, G_{dev} is the (linear) gain deviation at the central frequency and $freq_{dev}$ is the central frequency deviation.

The cost of one resistor is one monetary unit (MU) per kΩ; of one capacitor is 1 MU per μF and of one transistor is 0.1 MU. Although the objective was to increase the merit value obtained, the assignment also imposed restrictions on the type and amount of components to use. Thus, unlike previous assignments were it was possible to test effectively infinite combinations of components so as too achieve the best merit possible, that was not possible in this particular case. The main focus was keeping the frequency and gain deviation as low as possible. Since the cost of the OP-AMP was fixed and significantly higher than the cost of the rest of the circuit, as can be seen from its internal structure, keeping the cost low was not as significant. The circuit used is printed in Figure 1.

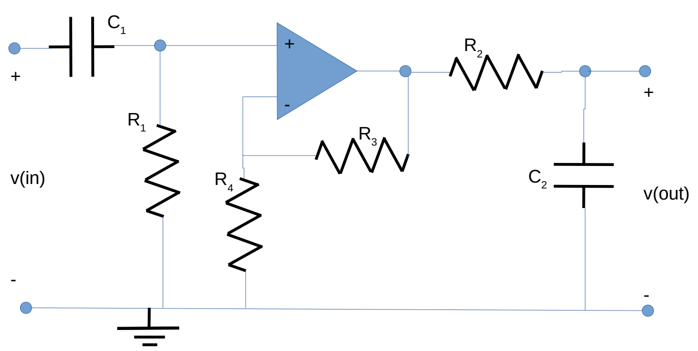


Figure 1: Circuit T5

The total cost of the OP-AMP was determined to be 1.332×10^4 MU, which was a lot higher than the cost of the other components, four resistors and two capacitors. The values of the components in circuit 1 are in Table 1.

Name	Value
R_1	1.000000e+03
R_2	1.000000e+03
R_3	3.150000e+05
R_4	1.000000e+03
C_1	1.100000e-07
C_2	2.200000e-07

Table 1: Values for the components in the circuit. Resistances in Ω, capacitances in F.

The assignment imposed a restriction of at most three 1kΩ, 10kΩ and 100kΩ resistors and at most three 220nF and three 1μF capacitors. Hence, R_3 is a series of three 100kΩ, one 10kΩ and two parallel 10kΩ resistors and C_1 a series of two 220nF capacitors.

2 Analysis

2.1 Theoretical explanation

The circuit fulfills two main roles. The first is to only let a particular range of frequencies through (a *bandwidth*) - this type of circuit is known as a band-pass filter and has been thoroughly studied previously. The other role is to amplify the input signal. The component that allows this kind of behaviour is called the operational amplifier. Combining these two roles we get a circuit that amplifies a given set of frequencies.

Since this is a BDF, there are two cutoff frequencies (gain 3dB below maximum gain), one below the central frequency (lower cutoff - radian - frequency, ω_L) and one above (higher cutoff - radian - frequency, ω_H). These frequencies can be determined (with the particular combination of components chosen) according to two very similar formulae:

$$\omega_H = \frac{1}{R_1 C_1} \tag{2}$$

$$\omega_L = \frac{1}{R_2 C_2} \tag{3}$$

The central frequency can be deduced from these two cutoff frequencies:

$$\omega_0 = \sqrt{\omega_L \omega_H} \tag{4}$$

Note that these are *angular* frequencies, the actual frequency is easy to compute knowing that $\omega = 2\pi f$.

As for amplifying the signal, we need to study the transfer function of this circuit, given by equation 5, to compute its gain, which should be close to 40 dB for the purpose of this assignment.

$$T = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \cdot (1 + \frac{R_3}{R_4}) \cdot \frac{1}{1 + R_2 C_2 s} \tag{5}$$

A plot of this transfer function should show the wanted gain near f=1kHz, the central frequency we want the circuit to operate in.

Finally, we also need to compute the input and output impedances, given by the formulae:

$$Z_{in} = \frac{1 + j\omega R_1 C_1}{j\omega C_1} \tag{6}$$

$$Z_{out} = \frac{R_2}{1 + j\omega R_2 C_2} \tag{7}$$

2.2 Lab Experience

We used a breadboard to assemble the circuit with resistors, capacitors and the µA741 General-Purpose OP-AMP, that we later connected to the VCC and -VCC entries. We connected the breadboard circuit to an AC signal generator and we also used an oscilloscope to observe the wave functions in real time and retrieve the data needed, namely the output and input voltages. Following the equations outlined in the previous section, we could easily calculate the angular cutoff frequencies and the corresponding value of R_2.

Therefore, given f_0=1 kHz and $w_0 = 2\pi f_0$, establishing that R_1=1 kΩ and $C_1 = C_2$= 220 nF, we get that

1. $w_L = \frac{1}{R_1 C_1}$= 4545.455 rad/s

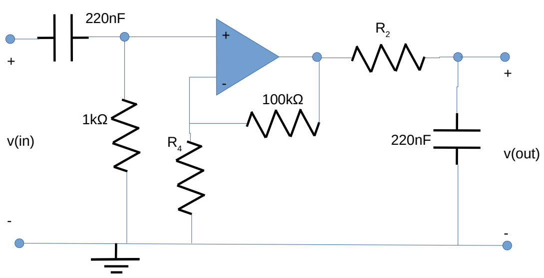
2. w_0= 6283.185 rad/s

3. $w_H = \frac{w_0^2}{w_l}$= 8685.251 rad/s

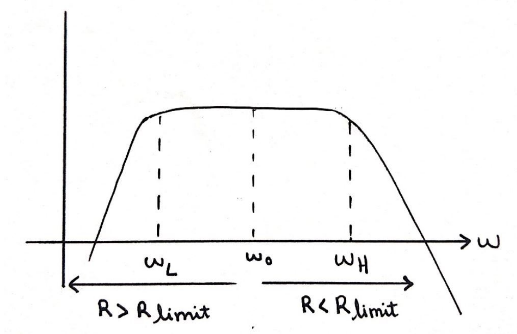
4. $R_2 = \frac{1}{w_H C_2}$= 523.353 Ω

We obviously could not simulate a resistor with value of exactly 523.353 Ω, so we approximated this value to 500 Ω so that R_2 could be made by two parallel 1 kΩ resistors.

We calculated the critical resistance by $R_{limit} = \frac{1}{C w_0}$. This means that from w_0 on the resistance, in this case R_2, had to be lower than R_{limit} and from 0 to w_0 the resistance R_1 had to be higher. In the lab, we used C=220nF, so that R_{limit}=723.43Ω. These results are consistent with the ones that we found, since we established R_1=1kΩ ($> R_{limit}$) and calculated R_2= 523.353 Ω ($< R_{limit}$).



(a) Circuit with the used values



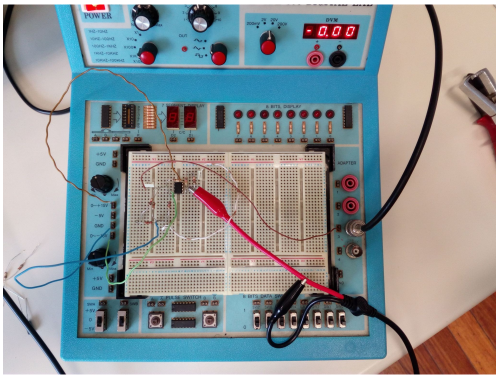
(b) Band pass illustration

Figure 2: Pre-circuit studies

Now that we had R_1, R_2 and have R_3 fixed at 100 kΩ, as suggested by the lab instructor, we had to obtain the best R_4 so that the gain would be 40 dB. Having started with R_4=1 kΩ, we realized that the voltage gain could be improved by decreasing R_4, lowering it a little bit more than half. We obtain 40,5 dB of gain with R_4=1 kΩ // 3 kΩ and 39,40074 dB of gain with R_4=1 kΩ // 4 kΩ. The figure 3a shows the assembled circuit and the figure 3b shows a detail of the oscilloscope interface, where v_i=90 mV and v_o=9.4 V:

1. $\frac{v_{output}}{v_{input}} = \frac{9.4V}{90mV}$= 104.(4)

2. $gain$ = $20 \log_{10} \frac{v_{output}}{v_{input}}$ $\approx$ 40.4 dB



(a) Assembled circuit

(b) Oscilloscope data

Figure 3: Details

2.3 Comparing the theoretical and the simulated results

In this study of the OP-AMP the results that are worthy of being compared are presented in tables 2 to 5. We can immediately see that the impedance of the input, Z_{in}, is roughly the same in both cases. The same can be said about the output impedance, Z_{out}. The values of Z_{out} obtained from the theoretical and the simulated calculations differ approximately 5Ω, which is not a relevant difference in this context. The theoretical results presented in these tables are the absolute values of the calculations made from the expressions 6 and 7.

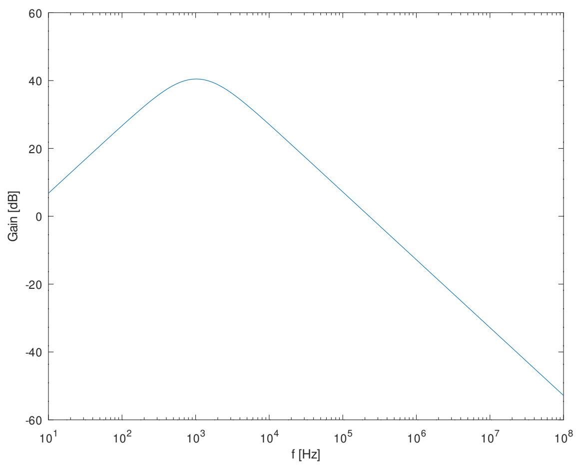
Name	Value
Z_{in}	1758.810076
Z_{out}	586.134309

Table 2: Theoretical input and output impedances in Ω.

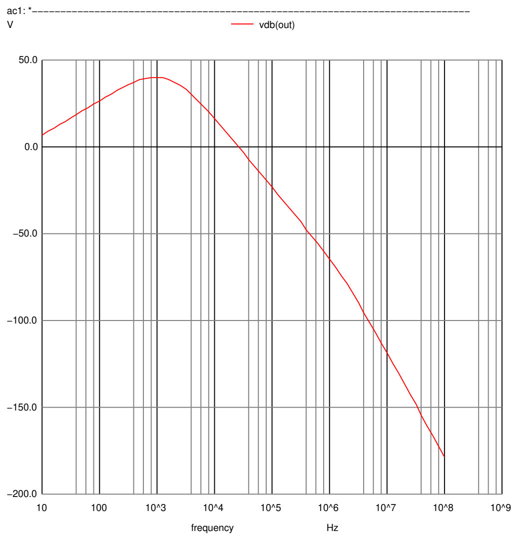
Name	Value
zin	1.759004e+03
zout	5.911203e+02

Table 3: Simulation input and output impedances in Ω.

The gain plots are presented below in figure 4.



(a) Octave plot



(b) NgSpice plot

Figure 4: V_{gain} as a function of f (dB)

As proposed in the objectives of this laboratory assignment, the voltage gain of our circuit reached 40 dB, as proven by the theoretical and simulated calculations.

Name	Value
f_L	723.431560
f_H	1446.863119
f_0	1023.086723
G_{1kHz}	40.449306

Table 4: Theoretical results

Name	Value
fl	3.867110e+02
fh	2.058741e+03
fo	1.000000e+03
gaindb	4.005178e+01
merit	1.224679e-04
cost	1.365633e+04

Table 5: Simulated results

Given that for us to obtain a central frequency of f_0=1 kHz in the theoretical model we would have had to use resistors and capacitors that we didn't have, we achieved f_0= 1023.086723 Hz with the allowed components. These results gave us a pass-band circuit with a bandwidth of 723 Hz.

The results of the NgSpice[2] model are somewhat similar to the ones mentioned before. The model itself provided the ideal central frequency (1 kHz), which was also achieved in the Octave[1] analysis, but doubled the bandwidth of the pass-band circuit. Even though we cannot explain with certainty the cause of the discrepancy between models, we can draw a few conclusions from this. Firstly, both results were obtained with the exact number and type of components, leading us to believe that, given the complexity and completeness of the NgSpice model, the real values would be much more similar to the NgSpice ones presented in table 5. We couldn't prove this first conclusion as the laboratory experience was long before the beginning of the theoretical and simulation study and in the lab we assembled a slightly different circuit. The last explanation we came up with to justify the difference between these values is that the type of OP-AMP used could have had an influence in the calculations, due to the approximations used.

Furthermore, the cost of the circuit was obtained from the NgSpice since it was computed using the model provided to us in the script. The merit was also obtained from NgSpice model.

Finally, as mentioned before, the voltage gain was approximately 40 dB in both models.

3 Conclusion

To conclude, the values obtained from both the theoretical and the simulation analysis were, in general, similar, with most of the objectives achieved, namely the gain of 40 dB and the central frequency of 1 kHz. The value of the merit is very low because of the high cost, which is mostly due to the OP-AMP circuit components.

Regarding the physical laboratory class, while it did provide valuable insight for the assignment and the course in general, it would perhaps be better if it were closer to the date of the assignment, so as to already have some kind of comparison with the theoretical models.

References

[1] GNU Octave, version 6.2.0 (February 20 2021)

[2] NgSpice, *open-source spice simulator*, version 31