Project 1. The Gauss-Newton Method.

i) From Gauss-Newton Method

We have that $\beta = \beta - (J^TJ)^{-1}J^Tr$

Let $J = QR$

$$\Rightarrow (J^TJ)^{-1}J^Tr$$
$$= ((QR)^TQR)^{-1}(QR)^Tr$$
$$= (R^TQ^TQR)^{-1}\cdot R^TQ^Tr$$
$$= (R^TR)^{-1}R^TQ^Tr$$
$$= R^{-1}(R^T)^{-1}R^TQ^Tr = R^{-1}Q^Tr.$$

$$\Rightarrow \beta = \beta - R^{-1}Q^Tr$$

We say it's justified to modify the algorithm to set $\beta = \beta - R^{-1}Q^Tr$

ii) We use QR factorization to minimize the error.

The error amplification of QR-fac is determined by

$k(Q)\cdot k(R)$ where k is the condition number.

$\because$ Q is orthogonal matrix

$$\Rightarrow k(Q) = 1.$$

$$\Rightarrow \begin{cases} \|J\| = \|Q\|\cdot\|R\| \\ \|J^{-1}\| = \|R^{-1}\|\cdot\|Q^{-1}\| \end{cases}$$

$$\Rightarrow k(A) = \|A\|\cdot\|A^{-1}\| = \|R\|\cdot\|R^{-1}\| = k(R).$$

$\Rightarrow$ QR factorization let us have the same condition number, which give us the minimum conditioning error.