A stochastic model for the continuous-time dynamics of a limit orderbook

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1 Overview

In order-driven markets, models of orderbook dynamics can provide insights into the market supply and demand, and short-term market price movements. My project is to use current states of the limit orderbook to predict its short-term features: sizes and depths of bid and ask side, and price dynamics. The goal is to support traders with simulated limit orderbook model, in execution of algo-trading strategies (VWAP, TWAP, etc.) and extract features from orderbook snapshots including bid-ask spread, order imbalances, order flow, etc.

2 Model

There are two types of orders: market orders and limit orders. A limit order is an order to buy or sell a stock at a specific price or better. A buy limit order can only be executed at the limit price or lower, and a sell limit order can only be executed at the limit price or higher. A market order is an order to buy or sell at the best available price in the limit order book. When market order arrives and matches with best available price in the limit order, an trade executed.

• Quantities for each price level of the limit order book is a random event. $X(t) \equiv (X_1(t), X_2(t), ..., X_n(t)), t0$, $X_p(t)$ is the quantity of limit orders at price p. Here negative sign as bid side and positive sign as ask side.

• With prices of orders at time t, the ask price $p_A(t)$ is the smallest price among prices of all orders with positive sizes, $X_p(t) > 0$; the bid price $p_B(t)$ is the largest price among prices of all orders with negative sizes, $X_p(t) < 0$.

Middle Price $p_M = \frac{(p_B(t)+p_A(t))}{2}$

Spread Price $s(t) = p_A(t) - p_B(t)$

• The state of limit order book is updated with events of new market:

The state x is a n-dimensional vector, with n as levels of prices.
A limit buy order at price p (smaller than best ask) increases the quantity at price level p;
A limit sell order at price p (larger than best bid) increases the quantity at price level p;
A cancellation of limit buy order at price p (smaller than best ask) decreases the quantity at price level p;
A cancellation of limit sell order at price p (larger than best bid) decreases the quantity at price level p;
A market buy order decreases the quantity at the ask price;
A market sell order increases the quantity at the bid price.

• The stochastic model driven by incoming flow of these six events are modeled with independent Poisson processes.
Limit buy orders arrive at a distance of i ticks from the opposite best quote at independent, exponential times with rate $\lambda(i)$.
Market buy orders arrive at independent, exponential times with rate μ,
And cancellation rate of limit orders at a distance of i ticks from the opposite best quote is $\theta(i)$ times number of outstanding limit orders.

• After building models in java virtual machine, tune parameters with respect to historical data, and obtain the probability of each bid/ask price is going up or down in the next state.
• Exploratory Data Analysis of midprices, including drift and volatilities. If the volume of limit orders posted at the best bid price is significantly larger than the volume of limit orders at the best ask price, the midprice will be pushed towards the ask price.

• Extract features of basic set and export into python for SVM midprice prediction. Execute an algorithm accordingly.

There are numerous research papers devoted to modeling limit orderbooks, one of which is done by Rama Cont, Sasha Stoikov and Rishi Talreja. They provide an analytically tractable framework with parameters calculated from empirical data (five best prices on each side of the order book with stocks traded on the Tokyo stock exchange).

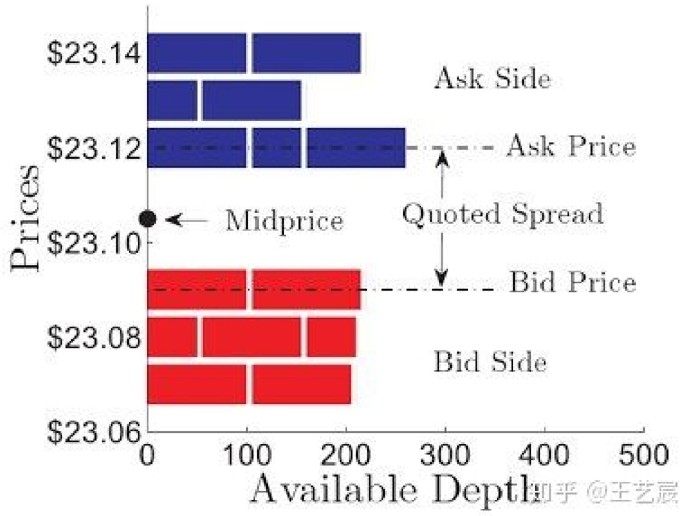

Figure 1: Orderbook dynamics

Figure 2: Orderbook dynamics

3 Programming Language and Library

The model was coded in Java; Data was fed into visual VML Launcher and midprice changes and quantities were analyzed in R.

Packages include algoOrder, and Stochastic Orderbook. Algoorder include 12 classes of dashBoard, Logger(getmessage and time), Marketstructure(instrumentexecutor, spread and tradesize factory), Tradingsignals, orderscheduler, simulation, priceengine.

Stochastic Orderbook: Tickfactory, L2 stochastic orderbook, Test of L2 orderbook and Report.

4 Data and Methodology

I studied the set of data of E-mini futures 2016 order book generated with Deltix running from 2016-01-09 to 2016-12-15 at millisecond-level (1000 per second). Class Parameters include Random generator, fullorderbook, marketDepth, ticksize, and three rates, insertionRates, cancelationRatesPerVolumeUnit and marketRate.

Initially parameters are taken values as follows: Rate of sending market orders: 0.35, rate of cancellation market orders are 0.5 at midprice, $\frac{0.25}{2*ordersize}$, $\frac{0.25}{2*ordersize}$ at best bid and ask price, $\frac{0.25*0.5}{2*ordersize}$, and $\frac{0.25*0.5}{2*ordersize}$ at second best prices, and rate of sending limit orders are 0.5, 0.25 for both sides at best prices and $0.25*0.5$ at second best prices.

Coded models in stochastic book class to identify event to send, updated books with respect to rates with quantities and prices at five best ask and bid levels, and updated next state rates.

Parameters Tuning: I made a grid of Arraylists of Doubles for each initial parameters. Then minimizing the differences between simulated drifts and volatilities of midprice with realized drifts and volatilities of quoted market prices.

In both cases, we adopted the optimal holding period of 22 months since a spin-off occurred as suggested in McConnell and Ovtchinnikov (2004)(4). That is to say, any spin-off stock whose ex date is longer than 22 months in the past will be removed from the portfolio. To be more realistic, we introduced a 10-day lag after the spinoff's completion date in order to take into account the fact that the newly-listed companies might not be accessible for trading immediately. The performance benchmark used in the backtests is S&P 500 (SPY500).

Quantopian's Research and Algorithm environments were used to replicate the strategies. To note, due to spin-off subsidiaries not meeting the criteria necessary, the QTradableUS universe could not be used. Additionally, the Quantopian's Eventvestor Spin-offs dataset was used to obtain information on completed spin-offs, as compared to the CRSP distribution file and Mergent Dividend Record in the case of McConnell and Ovtchinnikov (2004)(4) and McConnell et al. (2015)(1). A comparison of the datasets is shown in Table 1. Furthermore, Quantopian's Morningstar dataset was used to find sector-specific information on individual stocks in order to choose the appropriate hedging security.

To note, as highlighted in Table 1, the Quantopian Eventvestor dataset differs slightly from the dataset used in the studies being replicated. Moreover, another concern is that while Quantopian makes the dataset

Market Rate Differences between Mean of simulated Midprice and market price	Rateofcanc Differences between drift of simulatec
0.1	0.5
$5.125 * 10^{-5}$	0.00125
0.2	0.5
$1.625 * 10^{-5}$	0.077
0.3	0.7
$5.627 * 10^{-5}$	0.033
0.4	0.8
$3.125 * 10^{-4}$	0.01718
0.5	0.8
$3.17 * 10^{-3}$	0.019
Total	**140**

Table 1: The number of Spin-off events studied between 2010 and 2016. A detailed list can be found in Appendix.

available for 2007-2016, the first completed spin-off is recorded in 2010. These issues present evidence that contradict the findings of are replications strategies when compared to previous studies.

5 Backtest Results and Analysis

5.1 Strategy 1

ReturnBreakdown1.PNG

(a) The Return Breakdown

Overviewtable1.PNG

(b) Overview Performance

CumulativeReturns1.PNG

(c) Cumulative Returns

RollingBeta1.PNG

(d) Rolling Beta

Sharpe1.PNG

(e) Sharpe Ratio

RollingSharpe1.PNG

(f) Aolling Sharpe Ratio subfigure

TotalReturns1.PNG

(g) Total Returns

Figure 3: Backtest Results From Strategy 1

The equally weighted portfolio of spin-off stocks achieved a cumulative return of 131.7%, and an annual buy-and-hold return of 12.8%. While slightly lower than the statistics obtained by the 2015 Paper(1), the strategy did outperform the benchmark in terms of return. That being said, we observed considerably high fluctuations in the 6-month rolling Sharpe ratio, with an overall average of 0.72. The Sharpe ratios indicate less than acceptable returns when risk is taken into account.

5.2 Strategy 2

ReturnBreakdown2.PNG

(a) The Return Breakdown

Overviewtable2.PNG

(b) Overview Performance

CumulativeReturns2.PNG

(c) Cumulative Returns

RollingBeta2.PNG

(d) Rolling Beta

Sharpe2.PNG

(e) Sharpe Ratio

RollingSharpe2.PNG

(f) Rolling Sharpe Ratio

TotalReturns2.PNG

(g) Total Returns

Figure 4: Strategy 2 Backtest Results

When exposure to the spin-offs' industries were hedged out, the cumulative return became significantly less, and the Sharpe ratio dropped to 0.30. Backtest results from both strategies have maximum drawdowns larger than 35% in magnitude, indicating great downside risks. This is also reflected in the low returns stability in both algorithms (0.72 and 0.01 respectively). Note that the equal-weighted strategy's Sharpe ratio decays in both cases as a result of an increase in slippage cost (Quantopian's backtesting framework takes a 5 basis point fixed trading cost into account by default). For example, Strategy 1 had a very high Sharpe ratio at low capital deployments, which decayed rapidly and became negative in the period of $2015 - 2016$.

6 Conclusion

Based on the backtest results, we concluded that the Spin-off strategy suggested by McConnel, Sibley, and Xu (2015)(1) is not a tradable strategy. While the strategy outperforms in terms of return over certain periods, the outperformance appears dependent on the backtesting time frame. Similarly, McConnel et. al (2015)(1) does not take risk-adjusted measures of return into account, like max drawdowns, stability, alpha or Sharpe ratios.

An assessment of these risk-adjusted measures, as depicted in Figures 2 and 3, show that overall risk adjusted return is low. Moreover, the strategy does not hold up to any of Quantopian's more advanced backtesting performance measures. Furthermore, while Quantopian's backtesting results take into account a default slippage model, it is possible that actual slippage would be much greater, due to the fact that trading of the companies takes place over the first few months that they are listed, a time that typically is associated with higher volatility.

While our study does not rule out the possibility that the 'Spinoff Anomaly' can be traded, an assessment of the strategy suggested by McConnell et. al (2015)(1) indicates that a significant amount of work must be done in order to design a realistic quantitative strategy that captures the anomaly's returns. That being said, specific ETFs, such as Guggenheim SP Spin-Off ETF (CSD) and the VanEck Vectors Spin-Off ETF (SPUN), have been designed to trade Spin-offs. Unfortunately, the risk-adjusted performance of these ETFs cannot be measured at this time on Quantopian due to the resources required to access the series.

References

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[2] Gerald F. Davis, Kristina A. Diekmann, and Catherine H. Tinsley. *The Decline and Fall of the Conglomerate Firm in the 1980s: The Deinstitutionalization of an Organizational Form.* American Sociological Review, Vol. 59, No. 4. (Aug., 1994), pp. 547-570.

[3] Patrick J.Cusatis, James A.Miles, and J.RandallWoolridge. *Restructuring through spinoffs: The stock market evidence.* Journal of Financial Economics, Volume 33, Issue 3, June 1993, Pages 293-311.

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7 Appendix

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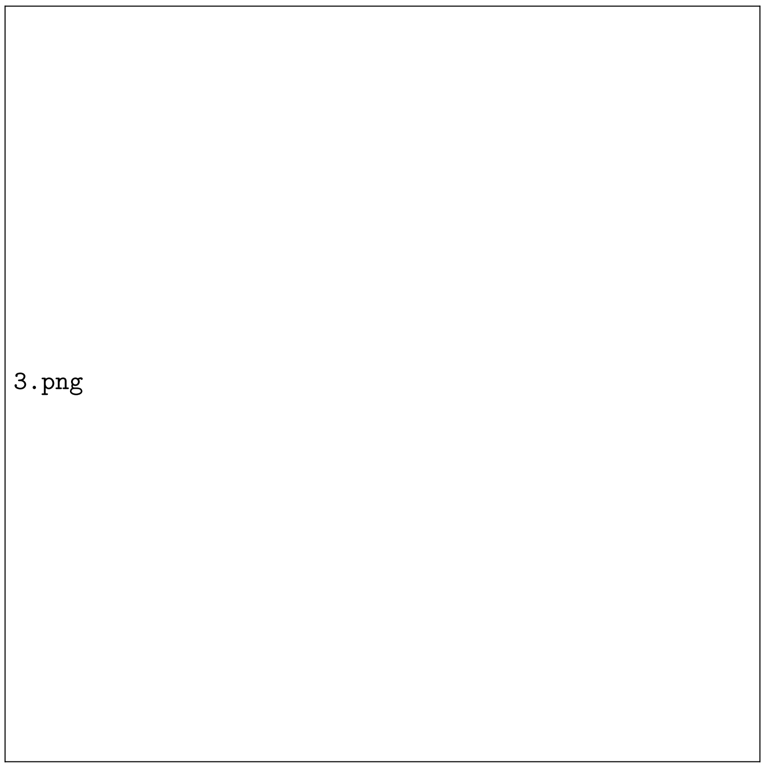
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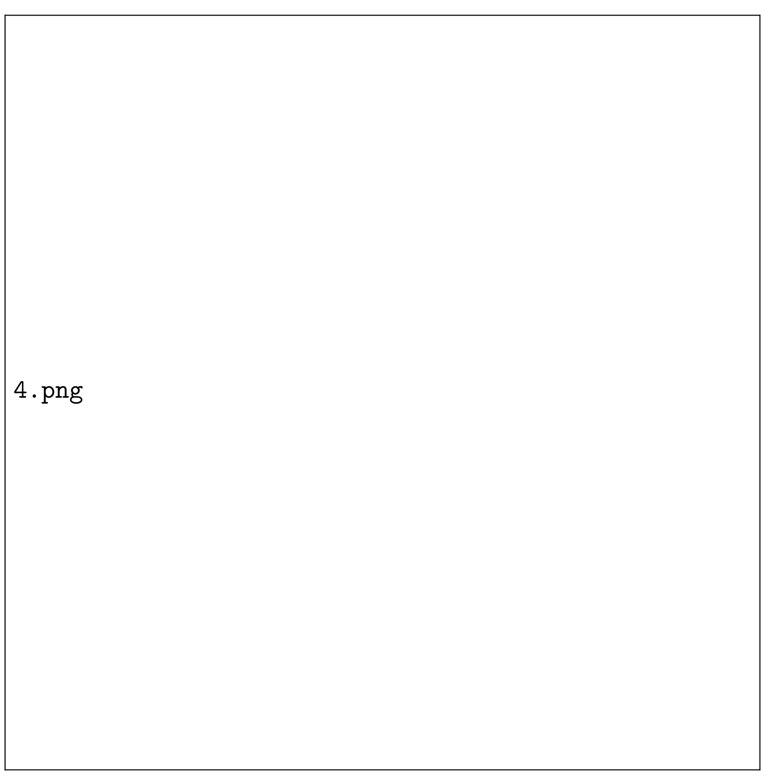
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Figure 5: List of Spin-off events by Date