Yuchen Wang HW5

1.

I will use S for Smoke; F for Fire; H for Heat

(a) $(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F) = (\neg S \vee F) \Rightarrow (S \vee \neg F)$

$= [\neg(\neg S \vee F)] \vee (S \vee \neg F)$

$= (S \wedge \neg F) \vee S \vee \neg F$

S	F	result
T	T	T
T	F	T
F	T	F
F	F	T

Since the results are not all Ts neither all Fs, it is neither valid nor unsatisfiable.

(b) $(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F) = (\neg S \vee F) \Rightarrow [\neg(S \vee H) \vee F]$

$= (\neg S \vee F) \Rightarrow [(\neg S \wedge \neg H) \vee F]$

$= [\neg(\neg S \vee F)] \vee [(\neg S \wedge \neg H) \vee F]$

$= (S \wedge \neg F) \vee (\neg S \wedge \neg H) \vee F$

S	F	H	Result
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	T

Since the results are not all Ts neither all Fs, it is neither valid nor unsatisfiable.

(4) $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

① $((S \wedge H) \Rightarrow F) \rightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

$= [\neg(S \wedge H) \vee F] \Rightarrow (\neg S \vee F \vee \neg H)$

$= (\neg S \vee \neg H \vee F) \Rightarrow (\neg S \vee F \vee \neg H)$

$= [\neg(\neg S \vee \neg H \vee F)] \vee (\neg S \vee F \vee \neg H)$

$= (S \wedge H \wedge \neg F) \vee \neg S \vee F \vee \neg H$

② $((S \Rightarrow F) \vee (H \Rightarrow F)) \Rightarrow ((S \wedge H) \Rightarrow F)$

$= (\neg S \vee F \vee \neg H) \Rightarrow (\neg S \vee \neg H \vee F)$

$= [\neg(\neg S \vee F \vee \neg H)] \vee \neg S \vee \neg H \vee F$

$= (S \wedge \neg F \wedge H) \vee \neg S \vee \neg H \vee F$

Therefore, $[(S \wedge H \wedge \neg F) \vee \neg S \vee F \vee \neg H] \wedge [(S \wedge \neg F \wedge H) \vee \neg S \vee \neg H \vee F]$

S	F	H	Result
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Since the results are all Ts, this sentence is valid.

2.

I will use U for unicorn; ML for mythical; MT for mortal; MM for mammal; H for horned; MG for magical

(a) KB: $ML \Rightarrow \neg MT$ ①

$\neg ML \Rightarrow MT \wedge MM$ ②

$\neg MT \vee MM \Rightarrow H$ ③

$H \Rightarrow MG$ ④

(b) ①: $\neg ML \vee \neg MT$

②: $ML \vee (MT \wedge MM) = (ML \vee MT) \wedge (ML \vee MM)$

③ $\neg(\neg MT \vee MM) \vee H = (MT \wedge \neg MM) \vee H$

$= (MT \vee H) \wedge (\neg MM \vee H)$

④ $\neg H \vee MG$

So, the CNF is $(\neg ML \vee \neg MT) \wedge (ML \vee MT) \wedge (ML \vee MM) \wedge (MT \vee H) \wedge (\neg MM \vee H) \wedge (\neg H \vee MG)$

(c) $\neg ML \vee \neg MT$ ①

$ML \vee MT$ ②

$ML \vee MM$ ③

$MT \vee H$ ④

$\neg MM \vee H$ ⑤

$\neg H \vee MG$ ⑥

(i) Since we want to test $KB \models ML$, we will test whether $KB \wedge \neg ML$ is inconsistent with resolution.

$\neg ML$ ⑦

② and ⑦: MT ⑧

③ and ⑦: MM ⑨

⑤ and ⑨: H ⑩

⑥ and ⑩ MG ⑪

After trying resolutions, we can show exhaustively that $KB \wedge \neg ML$ is not inconsistent. Therefore, it is impossible to derive that the unicorn is mythical.

(ii) Since we want to test $KB \stackrel{?}{\models} MM$, we will test whether $KB \wedge \neg MM$ is inconsistent with resolution.

$\neg MM$		⑫
⑫ and ⑥:	$\neg H$	⑬
⑬ and ⑨:	$\neg MM$	⑭
⑭ and ⑨:	ML	⑮
⑬ and ⑭:	MJ	⑯
⑯ and ①:	$\neg ML$	⑰
⑮ and ⑰:	$\square$	

After resolution, we find that $KB \wedge \neg MM$ is inconsistent. Therefore, we can conclude that $KB \models MM$

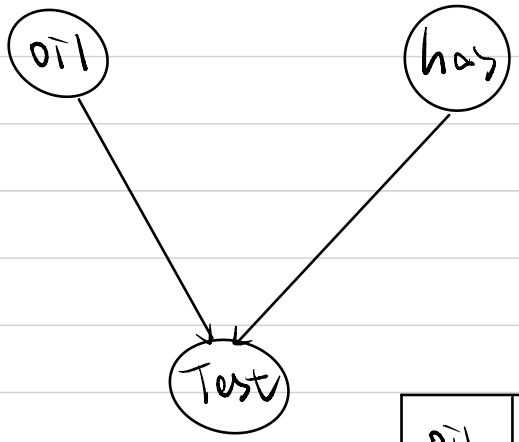
(iii) Since we want to test $KB \stackrel{?}{\models} H$, we will test whether $KB \wedge \neg H$ is inconsistent with resolution.

$\neg H$		⑱
⑱ and ④:	MJ	⑲
⑲ and ⑤:	$\neg MM$	⑳
⑳ and ③:	ML	㉑
⑲ and ①:	$\neg ML$	㉒
㉑ and ㉒:	$\square$	

After resolution, we find that $KB \wedge \neg H$ is inconsistent. Therefore, we can conclude that $KB \models H$.

3. Based on this paragraph, we can have a Bayesian Network.

T	F
0.5	0.5



T	F
0.2	0.8

oil	gas	T
T	T	x
T	F	0.9
F	T	0.3
F	F	0.1

← since they can't be occured at the same time

$$P(Oil \mid Test = Positive) = \frac{Pr(Test = Positive \mid Oil) * Pr(Oil)}{Pr(Test = Positive)}$$

$$= \frac{0.9 \times 0.8 \times 0.5}{0.9 \times 0.5 \times 0.8 + 0.3 \times 0.5 \times 0.2 + 0.1 \times 0.5 \times 0.8}$$

$$= \frac{0.36}{0.36 + 0.03 + 0.04}$$

$$= 0.8372$$

The probability that oil is present when test is positive is 83.72%

I have discussed this assignment with Chang Xie.
But we write by ourselves.