Convolutional neural network layers

In this notebook, we will build the convolutional neural network layers. This will be followed by a spatial batchnorm, and then in the final notebook of this assignment, we will train a CNN to further improve the validation accuracy on CIFAR-10.

In [1]:

```
## Import and setups

import time

import numpy as np
import matplotlib.pyplot as plt
from nndl.conv_layers import *
from utils.data_utils import get_CIFAR10_data
from utils.gradient_check import eval_numerical_gradient, eval_numerical_gradient_array
from utils.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

Implementing CNN layers

Just as we implemented modular layers for fully connected networks, batch normalization, and dropout, we'll want to implement modular layers for convolutional neural networks. These layers are in `nndl/conv_layers.py` .

Convolutional forward pass

Begin by implementing a naive version of the forward pass of the CNN that uses `for` loops. This function is `conv_forward_naive` in `nndl/conv_layers.py` . Don't worry about efficiency of implementation. Later on, we provide a fast implementation of these layers. This version ought to test your understanding of convolution. In our implementation, there is a triple `for` loop.

After you implement `conv_forward_naive` , test your implementation by running the cell below.

In [2]:

```
x_shape = (2, 3, 4, 4)
w_shape = (3, 3, 4, 4)
x = np.linspace(-0.1, 0.5, num=np.prod(x_shape)).reshape(x_shape)
w = np.linspace(-0.2, 0.3, num=np.prod(w_shape)).reshape(w_shape)
b = np.linspace(-0.1, 0.2, num=3)

conv_param = {'stride': 2, 'pad': 1}
out, _ = conv_forward_naive(x, w, b, conv_param)
correct_out = np.array([[[[-0.08759809, -0.10987781],
                           [-0.18387192, -0.2109216 ]],
                          [[ 0.21027089,  0.21661097],
                           [ 0.22847626,  0.23004637]],
                          [[ 0.50813986,  0.54309974],
                           [ 0.64082444,  0.67101435]]],
                         [[[-0.98053589, -1.03143541],
                           [-1.19128892, -1.24695841]],
                          [[ 0.69108355,  0.66880383],
                           [ 0.59480972,  0.56776003]],
                          [[ 2.36270298,  2.36904306],
                           [ 2.38090835,  2.38247847]]]])

# Compare your output to ours; difference should be around 1e-8
print('Testing conv_forward_naive')
print('difference: ', rel_error(out, correct_out))
```

```
Testing conv_forward_naive
difference:  2.2121476417505994e-08
```

Convolutional backward pass

Now, implement a naive version of the backward pass of the CNN. The function is `conv_backward_naive` in `nndl/conv_layers.py` . Don't worry about efficiency of implementation. Later on, we provide a fast implementation of these layers. This version ought to test your understanding of convolution. In our implementation, there is a quadruple `for` loop.

After you implement `conv_backward_naive` , test your implementation by running the cell below.

In [3]:

```
x = np.random.randn(4, 3, 5, 5)
w = np.random.randn(2, 3, 3, 3)
b = np.random.randn(2,)
dout = np.random.randn(4, 2, 5, 5)
conv_param = {'stride': 1, 'pad': 1}

out, cache = conv_forward_naive(x,w,b,conv_param)

dx_num = eval_numerical_gradient_array(lambda x: conv_forward_naive(x, w, b, conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_forward_naive(x, w, b, conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b, conv_param)[0], b, dout)

out, cache = conv_forward_naive(x, w, b, conv_param)
dx, dw, db = conv_backward_naive(dout, cache)

# Your errors should be around 1e-9'
print('Testing conv_backward_naive function')
print('dx error: ', rel_error(dx, dx_num))
print('dw error: ', rel_error(dw, dw_num))
print('db error: ', rel_error(db, db_num))
```

```
Testing conv_backward_naive function
dx error:  7.062629979321658e-09
dw error:  8.294887090081975e-11
db error:  2.522745337869892e-11
```

Max pool forward pass

In this section, we will implement the forward pass of the max pool. The function is `max_pool_forward_naive` in `nndl/conv_layers.py`. Do not worry about the efficiency of implementation.

After you implement `max_pool_forward_naive`, test your implementation by running the cell below.

In [4]:

```
x_shape = (2, 3, 4, 4)
x = np.linspace(-0.3, 0.4, num=np.prod(x_shape)).reshape(x_shape)
pool_param = {'pool_width': 2, 'pool_height': 2, 'stride': 2}

out, _ = max_pool_forward_naive(x, pool_param)

correct_out = np.array([[[[-0.26315789, -0.24842105],
                          [-0.20421053, -0.18947368]],
                         [[-0.14526316, -0.13052632],
                          [-0.08631579, -0.07157895]],
                         [[-0.02736842, -0.01263158],
                          [ 0.03157895,  0.04631579]]],
                        [[[ 0.09052632,  0.10526316],
                          [ 0.14947368,  0.16421053]],
                         [[ 0.20842105,  0.22315789],
                          [ 0.26736842,  0.28210526]],
                         [[ 0.32631579,  0.34105263],
                          [ 0.38526316,  0.4       ]]]])

# Compare your output with ours. Difference should be around 1e-8.
print('Testing max_pool_forward_naive function:')
print('difference: ', rel_error(out, correct_out))
```

```
Testing max_pool_forward_naive function:
difference:  4.1666665157267834e-08
```

Max pool backward pass

In this section, you will implement the backward pass of the max pool. The function is `max_pool_backward_naive` in `nndl/conv_layers.py`. Do not worry about the efficiency of implementation.

After you implement `max_pool_backward_naive`, test your implementation by running the cell below.

In [5]:

```
x = np.random.randn(3, 2, 8, 8)
dout = np.random.randn(3, 2, 4, 4)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

dx_num = eval_numerical_gradient_array(lambda x: max_pool_forward_naive(x, pool_param)[0], x, dout)

out, cache = max_pool_forward_naive(x, pool_param)
dx = max_pool_backward_naive(dout, cache)

# Your error should be around 1e-12
print('Testing max_pool_backward_naive function:')
print('dx error: ', rel_error(dx, dx_num))
```

```
Testing max_pool_backward_naive function:
dx error:  3.275628827601359e-12
```

Fast implementation of the CNN layers

Implementing fast versions of the CNN layers can be difficult. We will provide you with the fast layers implemented by utils. They are provided in `utils/fast_layers.py` .

The fast convolution implementation depends on a Cython extension ('pip install Cython' to your virtual environment); to compile it you need to run the following from the `utils` directory:

```
python setup.py build_ext --inplace
```

NOTE: The fast implementation for pooling will only perform optimally if the pooling regions are non-overlapping and tile the input. If these conditions are not met then the fast pooling implementation will not be much faster than the naive implementation.

You can compare the performance of the naive and fast versions of these layers by running the cell below.

You should see pretty drastic speedups in the implementation of these layers. On our machine, the forward pass speeds up by 17x and the backward pass speeds up by 840x. Of course, these numbers will vary from machine to machine, as well as on your precise implementation of the naive layers.

In [7]:

```
from utils.fast_layers import conv_forward_fast, conv_backward_fast
from time import time

x = np.random.randn(100, 3, 31, 31)
w = np.random.randn(25, 3, 3, 3)
b = np.random.randn(25,)
dout = np.random.randn(100, 25, 16, 16)
conv_param = {'stride': 2, 'pad': 1}

t0 = time()
out_naive, cache_naive = conv_forward_naive(x, w, b, conv_param)
t1 = time()
out_fast, cache_fast = conv_forward_fast(x, w, b, conv_param)
t2 = time()

print('Testing conv_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('Difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive, dw_naive, db_naive = conv_backward_naive(dout, cache_naive)
t1 = time()
dx_fast, dw_fast, db_fast = conv_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting conv_backward_fast:')
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('dx difference: ', rel_error(dx_naive, dx_fast))
print('dw difference: ', rel_error(dw_naive, dw_fast))
print('db difference: ', rel_error(db_naive, db_fast))
```

```
Testing conv_forward_fast:
Naive: 4.797780s
Fast: 0.016260s
Speedup: 295.063739x
Difference:  3.358910813299561e-10

Testing conv_backward_fast:
Naive: 6.411861s
Fast: 0.015437s
Speedup: 415.353277x
dx difference:  3.312114063590566e-11
dw difference:  8.591711550812164e-13
db difference:  1.2210168488921396e-15
```

In [8]:

```
from utils.fast_layers import max_pool_forward_fast, max_pool_backward_fast

x = np.random.randn(100, 3, 32, 32)
dout = np.random.randn(100, 3, 16, 16)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

t0 = time()
out_naive, cache_naive = max_pool_forward_naive(x, pool_param)
t1 = time()
out_fast, cache_fast = max_pool_forward_fast(x, pool_param)
t2 = time()

print('Testing pool_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('fast: %fs' % (t2 - t1))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive = max_pool_backward_naive(dout, cache_naive)
t1 = time()
dx_fast = max_pool_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting pool_backward_fast:')
print('Naive: %fs' % (t1 - t0))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('dx difference: ', rel_error(dx_naive, dx_fast))
```

```
Testing pool_forward_fast:
Naive: 0.383168s
fast: 0.003068s
speedup: 124.892990x
difference:  0.0

Testing pool_backward_fast:
Naive: 0.472052s
speedup: 43.260116x
dx difference:  0.0
```

Implementation of cascaded layers

We've provided the following functions in `nndl/conv_layer_utils.py` : - conv_relu_forward - conv_relu_backward - conv_relu_pool_forward - conv_relu_pool_backward

These use the fast implementations of the conv net layers. You can test them below:

In [9]:

```
from nndl.conv_layer_utils import conv_relu_pool_forward, conv_relu_pool_backward

x = np.random.randn(2, 3, 16, 16)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

out, cache = conv_relu_pool_forward(x, w, b, conv_param, pool_param)
dx, dw, db = conv_relu_pool_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_pool_forward(x, w, b, conv_param, pool_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_pool_forward(x, w, b, conv_param, pool_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_pool_forward(x, w, b, conv_param, pool_param)[0], b, dout)

print('Testing conv_relu_pool')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing conv_relu_pool
dx error:  1.1909928367748963e-08
dw error:  6.429429858441912e-09
db error:  3.2342564403597503e-12
```

In [10]:

```
from nndl.conv_layer_utils import conv_relu_forward, conv_relu_backward

x = np.random.randn(2, 3, 8, 8)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}

out, cache = conv_relu_forward(x, w, b, conv_param)
dx, dw, db = conv_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_forward(x, w, b, conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_forward(x, w, b, conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_forward(x, w, b, conv_param)[0], b, dout)

print('Testing conv_relu:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing conv_relu:
dx error:  6.463163991826658e-09
dw error:  3.327983168314795e-09
db error:  8.305987802894473e-11
```

What next?

We saw how helpful batch normalization was for training FC nets. In the next notebook, we'll implement a batch normalization for convolutional neural networks, and then finish off by implementing a CNN to improve our validation accuracy on CIFAR-10.

In [ ]:

```
def conv_forward_naive(x, w, b, conv_param):
  """
  A naive implementation of the forward pass for a convolutional layer.

  The input consists of N data points, each with C channels, height H and width
  W. We convolve each input with F different filters, where each filter spans
  all C channels and has height HH and width HH.

  Input:
  - x: Input data of shape (N, C, H, W)
  - w: Filter weights of shape (F, C, HH, WW)
  - b: Biases, of shape (F,)
  - conv_param: A dictionary with the following keys:
    - 'stride': The number of pixels between adjacent receptive fields in the
      horizontal and vertical directions.
    - 'pad': The number of pixels that will be used to zero-pad the input.

  Returns a tuple of:
  - out: Output data, of shape (N, F, H', W') where H' and W' are given by
    H' = 1 + (H + 2 * pad - HH) / stride
    W' = 1 + (W + 2 * pad - WW) / stride
  - cache: (x, w, b, conv_param)
  """
  out = None
  pad = conv_param['pad']
  stride = conv_param['stride']

  # ================================================================ #
  # YOUR CODE HERE:
  #   Implement the forward pass of a convolutional neural network.
  #   Store the output as 'out'.
  #   Hint: to pad the array, you can use the function np.pad.
  # ================================================================ #

  N, C, H, W = x.shape #get the shape of x
  F, C, HH, WW = w.shape #get the shape of w

  #calculate the output size
  Hout = int(1 + (H + 2 * pad - HH) / stride)
  Wout = int(1 + (W + 2 * pad - WW) / stride)

  #x pad
  padx = np.pad(x, ((0, 0), (0, 0), (pad, pad), (pad, pad)), mode = 'constant')

  #make the output to be the correct size
  out = np.zeros([N, F, Hout, Wout])

  #compute the out
  for data in np.arange(N): #for each data point
    for filter in np.arange(F): #for each filter
      for height in np.arange(Hout): #for each row
        for width in np.arange(Wout): #for each column
          out[data, filter, height, width] = np.sum(padx[data, :, height * stride : height * stride + HH, width * stride : w

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  cache = (x, w, b, conv_param)
  return out, cache


def conv_backward_naive(dout, cache):
  """
  A naive implementation of the backward pass for a convolutional layer.

  Inputs:
  - dout: Upstream derivatives.
  - cache: A tuple of (x, w, b, conv_param) as in conv_forward_naive

  Returns a tuple of:
  - dx: Gradient with respect to x
  - dw: Gradient with respect to w
  - db: Gradient with respect to b
  """
  dx, dw, db = None, None, None

  N, F, out_height, out_width = dout.shape
  x, w, b, conv_param = cache

  stride, pad = [conv_param['stride'], conv_param['pad']]
  xpad = np.pad(x, ((0,0), (0,0), (pad,pad), (pad,pad)), mode='constant')
  num_filts, _, f_height, f_width = w.shape

  # ================================================================ #
  # YOUR CODE HERE:
  #   Implement the backward pass of a convolutional neural network.
  #   Calculate the gradients: dx, dw, and db.
  # ================================================================ #
```

```
  N, C, H, W = x.shape #get the shape of x
  F, C, HH, WW = w.shape #get the shape of w

  #make the output to be the correct size
  dxpad = np.zeros(xpad.shape)
  dx = np.zeros(x.shape)
  dw = np.zeros(w.shape)
  db = np.zeros(b.shape)

  #calculate db
  for filter in np.arange(F):
      db[filter] += np.sum(dout[:, filter, :, :])  # sum all data point's filters

  for data in np.arange(N): #for each data point
      for filter in np.arange(F): #for each filter
          for height in np.arange(out_height): #for each row
              for width in np.arange(out_width): #for each column
                  #dw = xpad[] * dout
                  dw[filter] += xpad[data, :, height * stride : height * stride + HH, width * stride : width * stride + WW] * d
                  #dx = w * dout
                  dxpad[data, :, height * stride : height * stride + HH, width * stride : width * stride + WW] += w[filter] * d

  #update dx
  dx = dxpad[:, :, pad : -pad, pad : -pad]

  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #

  return dx, dw, db


def max_pool_forward_naive(x, pool_param):
  """
  A naive implementation of the forward pass for a max pooling layer.

  Inputs:
  - x: Input data, of shape (N, C, H, W)
  - pool_param: dictionary with the following keys:
    - 'pool_height': The height of each pooling region
    - 'pool_width': The width of each pooling region
    - 'stride': The distance between adjacent pooling regions

  Returns a tuple of:
  - out: Output data
  - cache: (x, pool_param)
  """
  out = None

  # ================================================================ #
  # YOUR CODE HERE:
  #   Implement the max pooling forward pass.
  # ================================================================ #

  #get the params of pool filter
  pool_height = pool_param.get('pool_height')
  pool_width = pool_param.get('pool_width')
  stride = pool_param.get('stride')

  N, C, H, W = x.shape #get the shape of x

  #calculate the output size
  Hout = int(1 + (H - pool_height) / stride)
  Wout = int(1 + (W - pool_width) / stride)

  #make the output to be the correct size
  out = np.zeros([N, C, Hout, Wout])

  for data in np.arange(N): # for each data point
    for channel in np.arange(C): # for each channel
        for height in np.arange(Hout): #for each row
            for width in np.arange(Wout): #for each column
                out[data, channel, height, width] = np.max(x[data, channel, height * stride : height * stride + pool_height, wi


  # ================================================================ #
  # END YOUR CODE HERE
  # ================================================================ #
  cache = (x, pool_param)
  return out, cache

def max_pool_backward_naive(dout, cache):
  """
  A naive implementation of the backward pass for a max pooling layer.

  Inputs:
  - dout: Upstream derivatives
  - cache: A tuple of (x, pool_param) as in the forward pass.

  Returns:
  - dx: Gradient with respect to x
  """
```

```
dx = None
x, pool_param = cache
pool_height, pool_width, stride = pool_param['pool_height'], pool_param['pool_width'], pool_param['stride']

# ================================================================ #
# YOUR CODE HERE:
#   Implement the max pooling backward pass.
# ================================================================ #

N, C, H, W = x.shape #get the shape of x
_, _, dout_height, dout_width = dout.shape #get the shape of dout

#make the output to be the correct size
dx = np.zeros(x.shape)

for data in np.arange(N): #for each data point
  for channel in np.arange(C): #for each channel
      for height in np.arange(dout_height): #for each row
          for width in np.arange(dout_width): #for each column
              maxnum = np.argmax(x[data, channel, height * stride : height * stride + pool_height, width * stride : width * s
              maxfield = np.unravel_index(maxnum, [pool_height, pool_width])
              dx[data, channel, height * stride : height * stride + pool_height, width * stride : width * stride + pool_widt

# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #

return dx
```