Joint Value Estimation and Bidding (JVEB) in First-Price Auctions (FPAs)

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POMS-HK, Shenzhen

Janurary 2026

Talk Outline

- Introduction
- An "online + causal" formulation for JVEB
- Main results
- Conclusion and future directions

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Digital advertising

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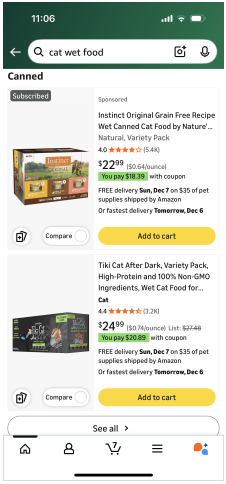

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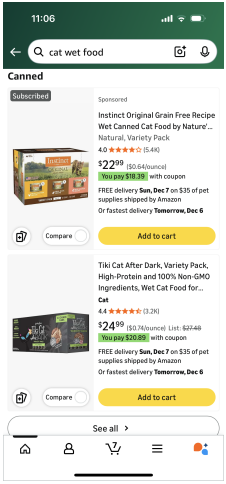

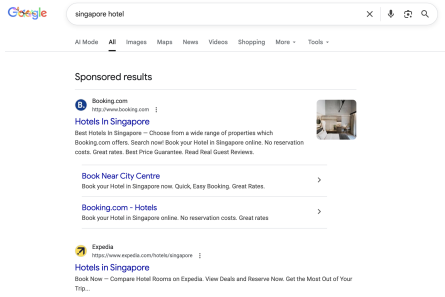

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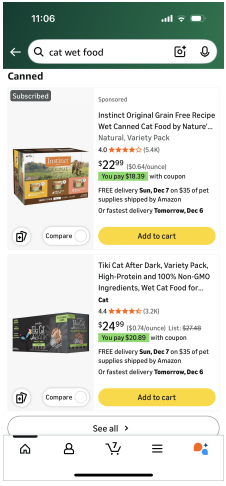

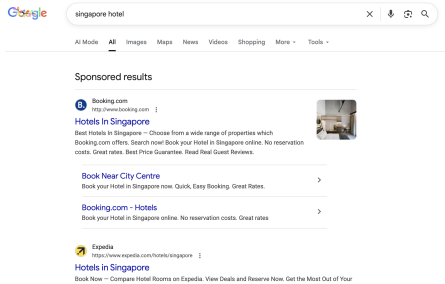

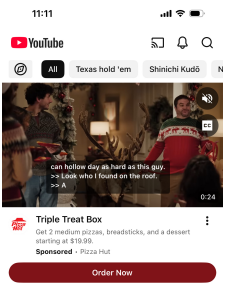

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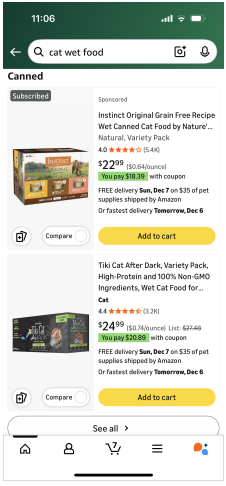

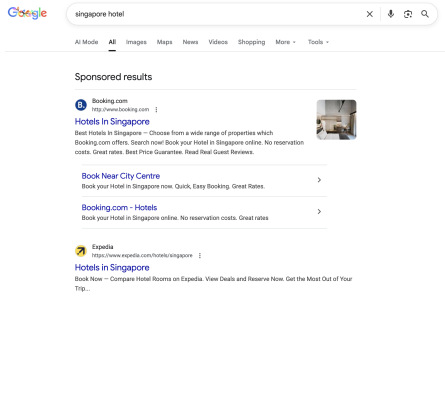

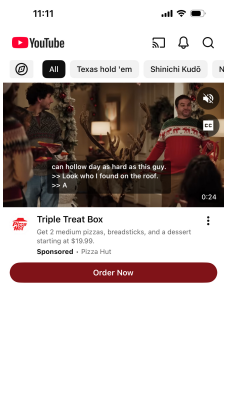

Digital advertising v.s. traditional advertising

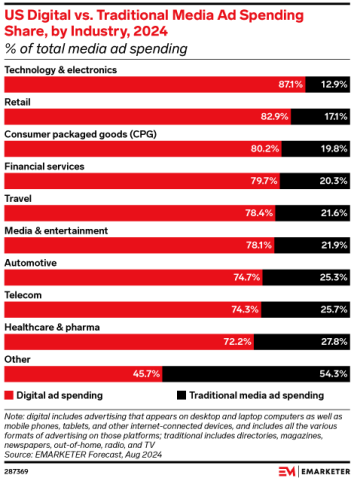

Figure: Digital advertising over industries.

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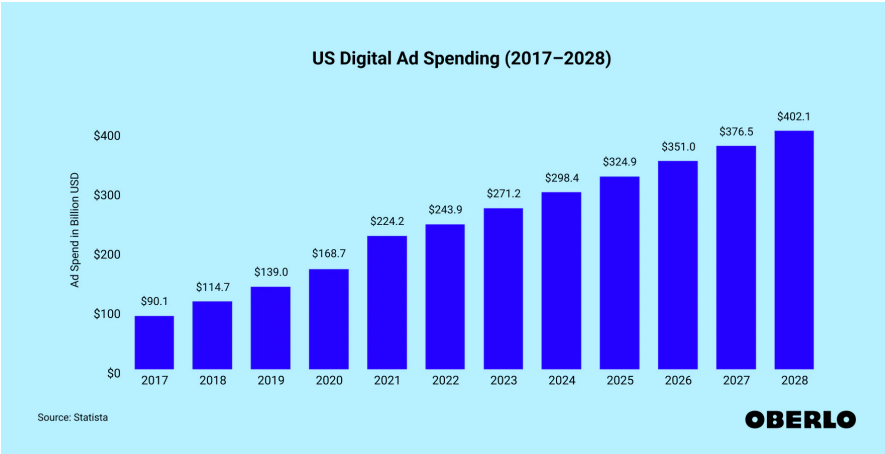

Figure: Media ad spending trend and forecast (updated 2024).

Digital advertising with first-price auctions (FPAs)

Publisher $\longleftrightarrow$ Ad Exchange (Auctions) $\longleftrightarrow$ Bidders.

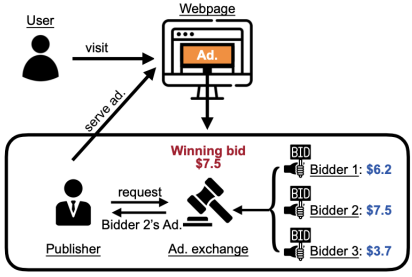

Figure: Example ad exchange diagram*. FPAs used by Google, Amazon, etc...

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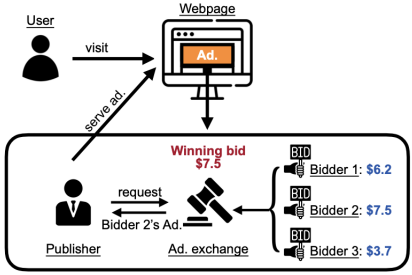

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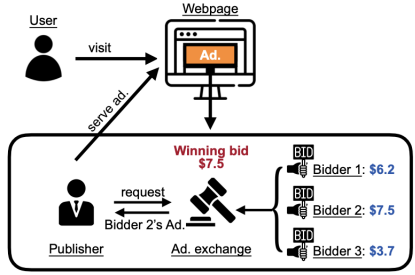

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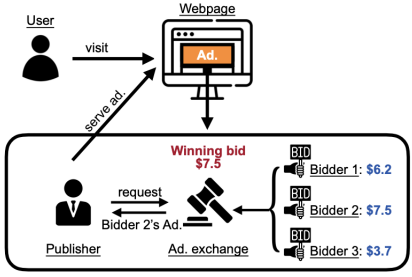

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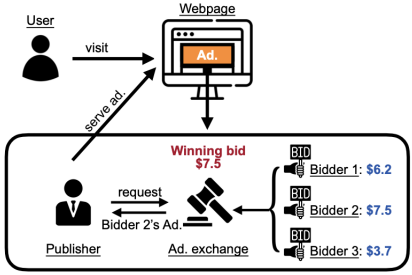

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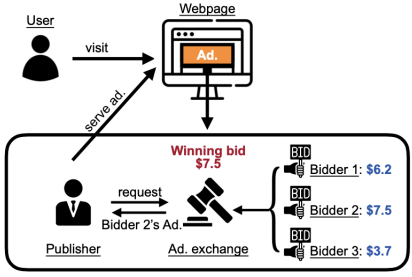

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Waisman and Nair and Narrion (2025): model user value as **treatment effect**.

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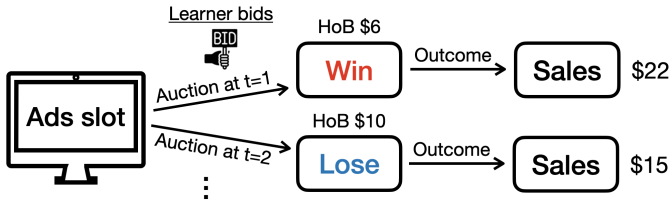

Figure: Value = outcome difference

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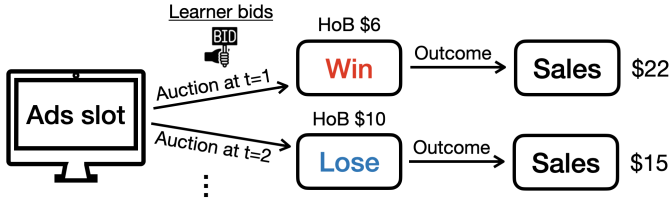

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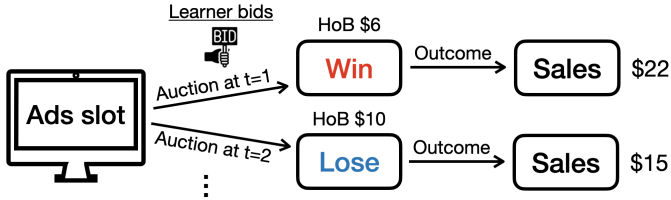

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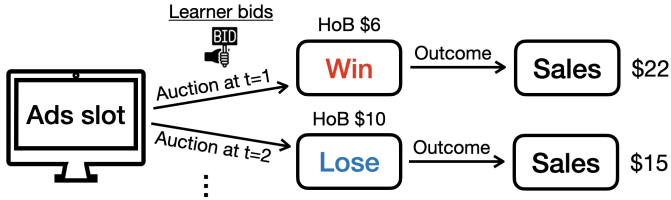

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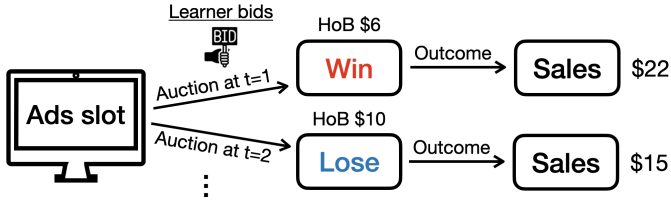

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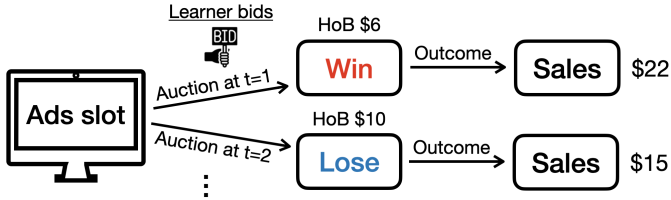

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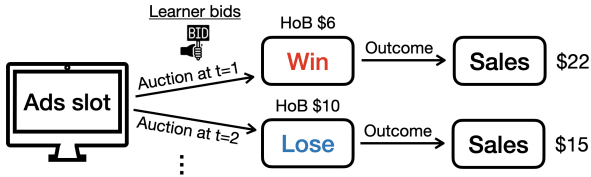

Over horizon $t = 1, 2, \ldots, T$:

- observe user context $x_t \in \mathbb{R}^d$,
- submit bid b_t,
- receive outcome $v_{t,1}$ if $b_t \geq m_t$ (the HOB), or $v_{t,0}$ otherwise,
- expected reward $r_t(b_t) = \mathbb{E}[\mathbb{1}[b_t \geq m_t](v_{t,1} - b_t) + \mathbb{1}[b_t < m_t]v_{t,0}]$.

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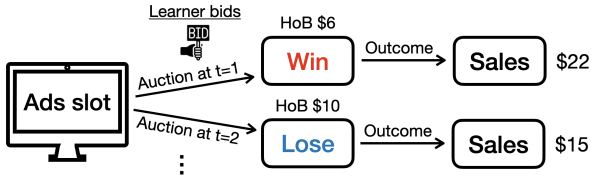

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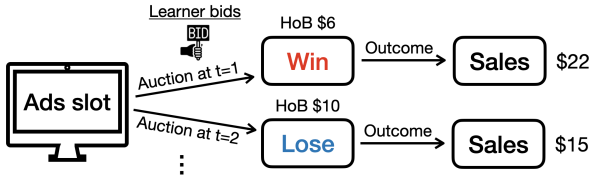

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Regret minimization:

$$R(\pi) = \mathbb{E}\left[\sum_{t=1}^{T} \max_{b_t^*} r_t(b_t^*) - r_t(b_t)\right].$$

A causal framework

Neyman-Rubin potential outcome model:

- Units $i = 1, 2, \ldots, N$
- Covariates X_i
- Treatments $\tau_i \in \{0, 1\}$
- Propensity $e_i = \mathbb{P}(\tau_i = 1 | X_i)$
- Potential outcomes $\{Y_i(0), Y_i(1)\}$
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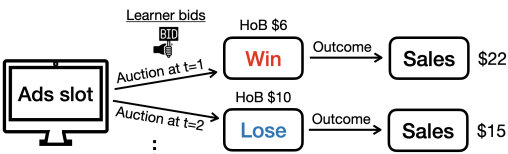

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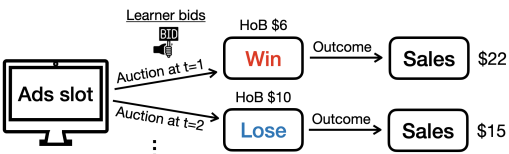

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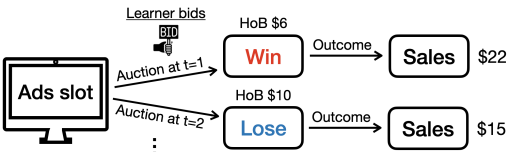

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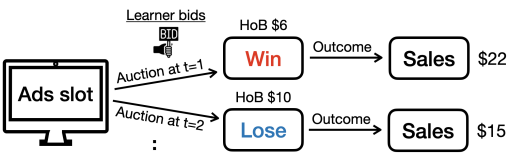

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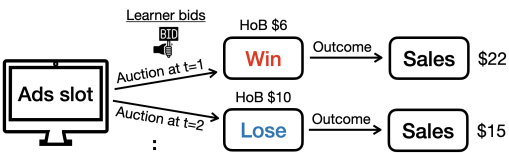

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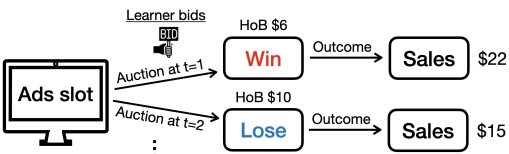

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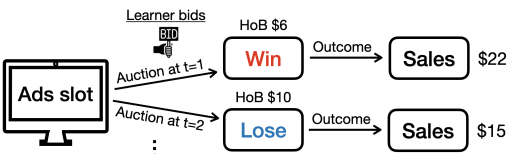

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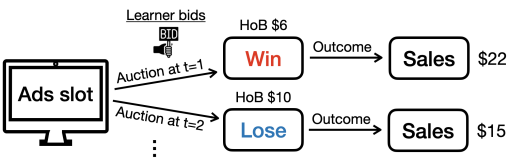

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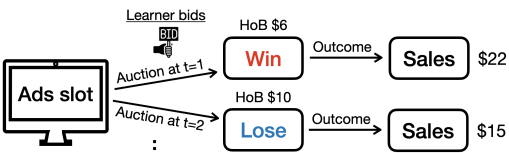

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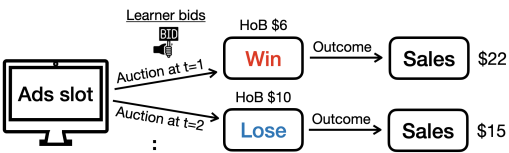

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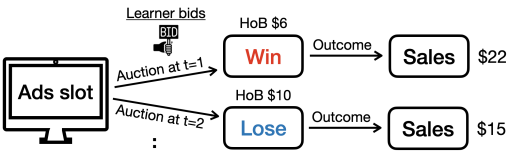

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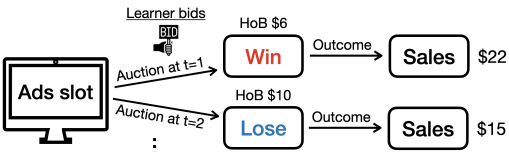

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Our result: **optimal** regret with **no/few** randomized experiments!

Model formulation

HOB modeling:

Over horizon $t = 1, 2, \ldots, T$:

- observe user context $x_t \in \mathbb{R}^d$,
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[a] This also aligns with recent causal literature assuming that the treatment effect has simpler structures than the baselines.

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Talk Outline

- Introduction
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- Main results
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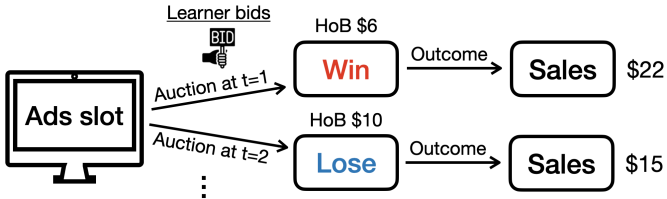

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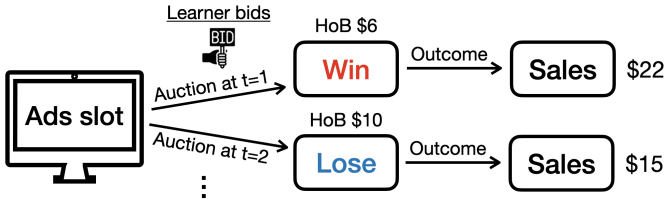

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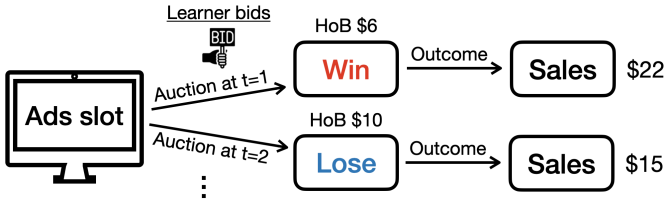

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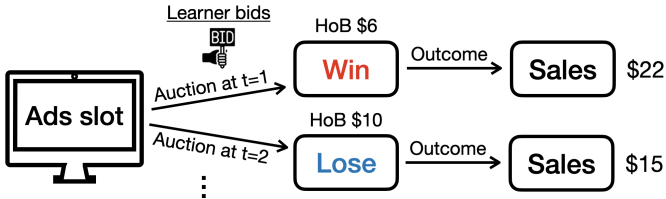
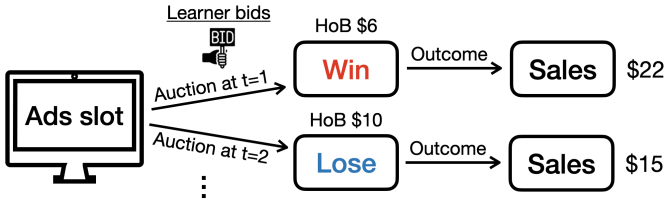


Summary

- User value = *marginal* gain from ad exposure
- Existing works cannot handle the two potential outcomes $v_{t,0}, v_{t,1} \implies$ wasteful bids.
- Bidders need an "online + causal" framework.
- Randomized experiments could be unnecessary (and suboptimal).

HOB feedback	Upper Bound	I.i.d. HOB	Linear HOB	Lower Bound
Full-info	$\sqrt{\Delta dT}$	$\sqrt{dT}$	$d\sqrt{T}$	$\sqrt{dT}$
Binary	$\sqrt{\Delta dT} + \xi T$	$d^{\frac{1}{3}} T^{\frac{2}{3}}$		



Future directions

- Is it optimal to consider oracle abstractions?
- Optimal regrets under second-price auctions (SPAs)?
- Bidding with constraints, e.g. budget, ROI, ...
- Bidding with delayed feedback?

θ_* estimation

Recall $\mathbb{E}[v_{t,1} - v_{t,0}] = \theta_*^\top x_t$; HOB CDF G_t.

Inverse-propensity-weighted (IPW) estimator: with propensity $G_t(b_t)$,

$$\widehat{\Delta v}_t(b_t) \coloneqq \frac{\mathbb{1}[b_t \geq m_t] v_{t,1}}{G_t(b_t)} - \frac{\mathbb{1}[b_t < m_t] v_{t,0}}{1 - G_t(b_t)}, \qquad \mathbb{E}[\widehat{\Delta v}_t(b_t)] = \theta_*^\top x_t.$$

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Exploration-exploitation trade-off: exploratory $G_t(b_t)$ v.s. good b_t

Better of two Upper Confidence Bounds (UCBs)

Linear Regression: $|\widehat{\theta}_t^\top x_t - \theta_*^\top x_t| \lesssim \rho_t$.

Given x_t, maximize reward

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Bad estimation when variance large

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