# Project 2: Crispy Critters, Inc.

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# Problem Statement

**Crispy Critters, Inc. (CCI)** grows genetically engineered critters to break down automobiles. However, the technique used to grow the critters is imperfect, and with each step of production both good and bad critters will be produced.

CCI has options at each step of the production cycle; each choice is associated with a different cost and a different output. **We want to find the optimal way to produce critters to maximize profit with an acceptable ratio of good-to-bad critter output abiding by out constraints**. We also want to know the profit will change if we modify the most vital constraints.

We will be breaking down the problem in the following steps:

1. Identify constraints, build linear programming model in Python with Gurobi to identify the optimal solution.
2. Explore the current optimal solution based on the results of our Gurobi optimization.
3. Sensitivity Analysis based on the results of our Gurobi optimization.
4. Investigate improving yield of the harvesting process.
5. Investigate increasing allowable range of bad critters.
6. Investigate increasing staffing for both harvesting and baking.

# 1. Linear Programming Formulation

Completed Linear Programming Formulation:

Objective function:

$$\max \quad 100\,(Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B}) - (11X_A + 17.5X_B + 18Q_{1A} + 18Q_{1B} + 12.5Q_{2A} + 12.5Q_{2B} + 4T_H + 6.5T_B)$$

Subject to:

$X_A + X_B >= 1000$

$X_A + X_B <= 1000 + T_H$

$0.44X_A = Q_{1A} + Q_{1B}$

$0.72X_B = Q_{2A} + Q_{2B}$

$Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B} >= 800$

$Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B} <= 800 + T_B$

$0.5Q_{1A} + 0.4Q_{1B} + 0.6Q_{2A} + 0.3Q_{2B} >= .4(Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B})$

$0.05Q_{1A} + 0.06Q_{1B} + 0.1Q_{2A} + 0.06Q_{2B} <= .07(Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B})$

$T_H <= 400$

$T_B <= 300$

# 1. LP Formulation Explained

Decision variables are variables that will decide the output and represent the ultimate solution. For Crispy Critters, we will use the following decision variables to determine our objective function and constraints:

$X_A$: grams of environment A used to grow critters
$X_B$: grams of environment B used to grow critters
$Q_{1A}$: grams of critter mix harvested from environment A and baked with Agent 1
$Q_{1B}$: grams of critter mix harvested from environment B and baked with Agent 1
$Q_{2A}$: grams of critter mix harvested from environment A and baked with Agent 2
$Q_{2B}$: grams of critter mix harvested from environment B and baked with Agent 2
$T_H$: grams of environment harvested by technicians through overtime
$T_B$: grams of critter mix baked by technicians through overtime

Object functions refer to the outcome that we want to optimize. In the objective function

$$\max \quad 100\,(Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B}) - (11X_A + 17.5X_B + 18Q_{1A} + 18Q_{1B} + 12.5Q_{2A} + 12.5Q_{2B} + 4T_H + 6.5T_B)$$

We are essentially looking for the maximum profit (total revenue - total cost).

# 1. LP Formulation Explained

**Subject to refers to the constraints of the formulation. More specifically:**

- The total grams of environment used to grow the critters should exceed 1000. $X_A + X_B >= 1000$
- The total grams of environment used should not exceed 1000 + additional grams technicians are able to harvest from overtime. $X_A + X_B <= 1000 + T_H$
- All grams of critter mix produced from environment A should be baked using one of the two chemical agents. $0.44X_A = Q_{1A} + Q_{1B}$
- All grams of critter mix produced from environment B should be baked using one of the two chemical agents. $0.72X_B = Q_{2A} + Q_{2B}$
- The total grams of critter mix to be baked should exceed 800. $Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B} >= 800$
- The total grams of critter mix to be baked should not exceed 800 + additional grams technicians are able to bake from overtime. $Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B} <= 800 + T_B$
- The final product mix should contain at least 40% of “good” critters.
  $0.5Q_{1A} + 0.4Q_{1B} + 0.6Q_{2A} + 0.3Q_{2B} >= .4(Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B})$
- The final product mix should contain no more than 7% of “bad” critters.
  $0.05Q_{1A} + 0.06Q_{1B} + 0.1Q_{2A} + 0.06Q_{2B} <= .07(Q_{1A} + Q_{1B} + Q_{2A} + Q_{2B})$
- The total grams that the technicians harvest during overtime should not exceed 400. $T_H <= 400$
- The total grams that the technicians bake during overtime should not exceed 300. $T_B <= 300$

# 1. Python Implementation

We will be implementing the formulation in Python through Gurobi. First, we declare the model:

```
model = gp.Model()
```

Then, we declare the decision variables. For instance,

```
X_A = model.addVar(name="X_A")
```

Next, we add the constraints that we have listed in the formulation. For instance,

```
model.addConstr(X_A + X_B >= 1000)
```

The next step is then to set up the objective function

```
model.setObjective(
    total_revenue - total_cost,
    GRB.MAXIMIZE
)
```

And have Gurobi run the optimization from there.

```
model.optimize()
```

# 2. Optimal Result

The optimal result has a **maximized profit of $55,656.81.**

## Step 1

**Environment**: CCI should use 197 grams of environment A and 1203 grams of environment B.

**Harvesting**: CCI should pay technicians to harvest an additional 400 grams overtime.

## Step 2

**Agent**: CCI should apply Agent 1 to 87 grams of critter mix from environment A and 260 grams from environment B. Agent 2 should be applied to 606 grams from environment B and none from environment A.

**Baking**: CCI should pay technicians to bake an additional 153 grams over time.

# 2. Implications of Optimal Result

01 Environment

**Most of the critters would be grown in environment B.** The optimal solution has a rough ratio of 1:6 for critters grown in environment A to environment B.

02 Chemical Agent 1

**All of the critter mix harvested from environment A** and around **30% of the critter mix harvested from environment B** should be applied with chemical agent 1.

03 Chemical Agent 2

**None of the critter mix harvested from environment A** and around **70% of the critter mix harvested from environment B** should be applied with chemical agent 2.

04 Overtime

CCI should pay its workers to work overtime. More specifically, assign the technicians to **harvest 400 additional grams** and **bake 153 additional grams**.

05 Outcome

The above assignments would produce a total revenue $95,300 of and total cost of $39,635, resulting in a **total profit of $55,665.**

# 3. Sensitivity Analysis - Variables

Allowed Increase and Decrease represent change in variable value without altering optimal solution.

| Variable | Final Value | Objective Value | Reduced Cost | Allowable Decrease | Allowable Increase |
|---|---|---|---|---|---|
| Grams in Env. A | 197 | -11 | 0 | -246 | 2.79 |
| Grams in Env. B | 1203 | -17.5 | 0 | -31 | +infinity |
| Harvest A, Bake 1 | 87 | 82 | 0 | 35 | 119 |
| Harvest B, Bake 1 | 260 | 82 | 0 | 39 | 553 |
| Harvest A, Bake 2 | 0 | 87.5 | -18.86 | -infinity | 106 |
| Harvest B, Bake 2 | 606 | 87.5 | 0 | 64 | +infinity |
| Harvested grams overtime | 400 | -4 | 0 | -37 | +infinity |
| Baked grams overtime | 153 | -6.5 | 0 | -55 | 0 |

# 3. Sensitivity Analysis - Variables

Sensitivity analysis can help us understand how robust our optimal solution is. It informs us about how much each decision variable needs to change in order for the optimal solution to change, and which decision variables are most in danger of causing a suboptimal scenario.

In the sensitivity report, we see that most of the **reduced cost** column is 0. This means that currently, **most of our resource utilizations are optimized**. In the current solution, no grams harvested from A are baked with Agent 2. If we did bake some grams from environment A with Agent 2, profit would decrease.

Allowable Increase and Decrease represent change in variable value without altering the optimal solution. **Very small increases and decreases indicate a sensitive objective coefficient.** From this, we identify the following **sensitive variables to investigate:**

- Grams of **environment A** used
- Grams harvested from A baked in **Agent 1**
- Grams harvested from B baked in **Agent 1**
- Paying technicians **overtime to bake** more grams

# 3. Sensitivity Analysis - Constraints

Constraint Results for the optimal model:

| Constraint | Slack | Shadow Price |
|---|---|---|
| Min environment use | -400 | 0 |
| Max environment use | 0 | 37.18 |
| Min baking use | -153 | 0 |
| Max baking use | 0 | 6.5 |
| Harvested from A | 0 | -110 |
| Harvested from B | 0 | -76 |
| Max harvesting overtime | 0 | 33 |
| Min baking overtime | 147 | 0 |
| Good ratio | 0 | -147 |
| Bad ratio | 0 | 965 |

# 3. Sensitivity Analysis - Constraints

**Shadow price** indicates sensitivity of the constraint values, and a higher shadow price indicates profit is more sensitive to a change in that constraint.

If a constraint is **slack**, it will have a shadow price of 0 and is not close to being violated with current optimal solution.

**The minimum environment use constraint is not sensitive**, as indicated by a shadow price of 0.

- All reasonable solutions utilize more than 1000 grams produced between environments A + B. Changing the constraint will not affect the optimal solution.

**The required percentage of good critters and allowable percentage of bad critters are sensitive to change,** as indicated by their high shadow prices.

- If the required percentage of good critters increases by 1%, profit decreases by $147.
- If the acceptable percentage of bad critters increases by 1%, profit increases by $965.
- These constraints are very sensitive.

The relative large shadow prices for harvest yield from environments A and B indicate that changing yield would lead to changes in profit as well. These constraints are sensitive; but not as sensitive as the percentages of good and bad critter ratios in the final mix. Ultimately, **we would recommend CCI to look closely at the good and bad critter ratios constraint if the provided assumptions do not hold true**.

# 4. Profit Variations with Changes in A/B Harvest Yield

| % Increase in Harvest Yield A | Max Profit |
|---|---|
| 1% | $55868.3 |
| 2% | $56071.8 |
| 3% | $56267.8 |
| 4% | $56456.7 |
| 5% | $56638.9 |
| 6% | $56814.7 |
| 7% | $56984.5 |
| 8% | $57148.5 |
| 9% | $57307.1 |
| 10% | $57460.5 |

| % Increase in Harvest Yield B | Max Profit |
|---|---|
| 1% | $56568.8 |
| 2% | $57477.2 |
| 3% | $58382.1 |
| 4% | $59283.5 |
| 5% | $60181.4 |

## Interpretation & Suggestion

The left two tables show how maximum profit varies with each 1% change in A harvest yields and B. From a profitability standpoint, **focusing on increasing harvest yields of environment B appears to be more beneficial.** This is evident when comparing the impact of a 5% increase in each yield: the maximum profit for a 5% increase in yield B is **$3542.5 greater** than that for the same increase in yield A.

While an increase in harvest yield of both environments would drive an increase in profits, our final recommendation would be for CCI to **concentrate on enhancing the harvest yield of environment B** as our model shows a more significant increase in profit.

# 5. Profit Variations with Changes in Bad/Good Critter Thresholds

| % Good Critter | % Bad Critter | Max Profit |
|---|---|---|
| 40% | 7% | $55656.8 |
| 39% | 7% | $57089.2 |
| 38% | 7% | $58587.1 |
| 37% | 7% | $59454.4 |
| 36% | 7% | $59639.2 |
| 35% | 7% | $59824.0 |

| % Good Critter | % Bad Critter | Max Profit |
|---|---|---|
| 40% | 8% | $58900.0 |
| 39% | 8% | $59084.8 |
| 38% | 8% | $59269.6 |
| 37% | 8% | $59454.4 |
| 36% | 8% | $59639.2 |
| 35% | 8% | $59824.0 |

| % Good Critter | % Bad Critter | Max Profit |
|---|---|---|
| 40% | 9% | $58900.0 |
| 39% | 9% | $59084.8 |
| 38% | 9% | $59269.6 |
| 37% | 9% | $59454.4 |
| 36% | 9% | $59639.2 |
| 35% | 9% | $59824.0 |

| % Good Critter | % Bad Critter | Max Profit |
|---|---|---|
| 40% | 10% | $58900.0 |
| 39% | 10% | $59084.8 |
| 38% | 10% | $59269.6 |
| 37% | 10% | $59454.4 |
| 36% | 10% | $59639.2 |
| 35% | 10% | $59824.0 |

The tables present an optimization model that demonstrates the effects of a 1% change for each type (good and bad) of critter in the final mix.

# 5. Discussion: Impact Analysis

**The optimizations that we ran suggest that an increase the allowable bad critter percentage and a decrease in the required good critter percentage increases our profits. However, this decision should be considered carefully as it may impact the user experience and therefore the ultimate success of the product.**

### a) Increasing Bad Critters Percentage

As the percentage of bad critters increase (from 7% to 10%), the profit tends to increase or stay same for the same percentage of good critters.

**When we increased the allowable percentage of bad critters, the profit increased, but only when we increased the allowable percentage from 7% to 8%. This behavior is expected since more bad critters would generally mean more efficiency in the process.**

### b) Decreasing Good Critters Percentage

As the percentage of good critters decreases (from 40% to 35%), the profit tends to increase for the same percentage of bad critters.

**This could indicate that the cost of good critters higher than the revenue they generate, or it could reflect a diminishing return on additional good critters.**

### c) Optimal Combination

The highest profit of $59824.0 is achieved at the lowest good critters percentage (35%) for any given bad critters percentage.

**This suggests that the model finds it more profitable to operate with a lower ratio of good critters, possibly due to cost considerations.**

### d) Trade-offs

The model results suggest a trade-off between the percentages of good and bad critters.

**Keeping the required percentage of good critters at 40% while increasing the allowable percentage of bad critters to 8% would be a reasonable consideration for CC as we would increase total profits by over $3000 while making minimal modifications to the critter mix ratio.**

# 6. Optimized Result with Additional Staffing Availability

| | |
|---|---|
| Environment A | 0.0 |
| Environment B | 1528 |
| Grams baked with Agent 1, from A | 0.0 |
| Grams baked with Agent 1, from B | 937 |
| Grams baked with Agent 2, from A | 0.0 |
| Grams baked with Agent 2, from B | 163 |
| Overtime for Harvesting | 400 |
| Overtime for Baking | 300 |
| Staffing for Harvesting | 128 |
| Staffing for Baking | 0.0 |
| Total Profit | $56971.07 |

With more staffing availability into consideration, we found that we can optimize the total profit further by increasing staffing for harvesting to **harvest an additional 128 grams**. This will result in a profit increase of **$1,315**.

# 6. Implications of Increased Staffing

The results show us that the optimal staffing strategy would be to **increase the staffing for harvesting only** and by hiring staffs to harvest an additional 128 grams. We found that no additional staffs were needed for baking, which aligns with our findings in the sensitivity analysis.

Other implications worth noting as a result of increased staffing include:

- **Environment A will not be used as all.** While using environment B only will result in simpler coordination in step 1, this should be carefully considered as it introduces dependency risks and supply chain vulnerability.
- **The new maximized total profit will be $56,971,** a $1,315 increase from our original optimized result.
- **Both the overtime upper limits will be met.** Our previous optimization only met the upper limit for harvesting overtime, and this change aligns with our finding to increase harvest staffing, which was the previous bottleneck.

Our final suggestion is as follows:
Given that Crispy Critters has relied more on Environment B than A in the original plan, it is logical that increasing staff for harvesting results in a reduced dependence on Environment A. The consistent overtime in harvesting, as predicted by the LP model, suggests that this area is the critical bottleneck. Ultimately, our findings indicate that **improvements or changes in staffing should focus primarily on the harvesting process.**

# Executive Summary

**Building Linear Programing Model:** We built a linear programming model based on the provided assumptions and found an optimal maximum profit of $55656.81. In this solution, most of the critters would be grown in environment B and CCI should pay its workers to work overtime.

**Maximizing Profits through Enhanced Harvest Yields:** If we could improve harvest yields, CCI should focus on enhancing the harvest yield of environment B as our model indicates a substantially higher potential for increased profit through enhanced harvest yields in this particular environment.

**Optimizing Critter Mix for Profit Gain:** If we are to change the critter mix proportions, raising the allowable proportion of bad critters to 8% while maintaining a 40% threshold for good critters could be a viable strategy for Crispy Critters to boost profits by over $3,000 with only slight adjustments to the critter ratio.

**Strategic Staffing to Alleviate Harvesting Bottleneck**: If CCI is to increase staffing, they should do so in the harvesting department. The previous findings consisted a maximum usage of overtime in the harvesting department, indicating it as a critical bottleneck. Thus, any staffing enhancements should be concentrated on optimizing the harvesting process, and will increase our expected profit by $1,315.