Induction

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Domino theory...

- Suppose I have a line of infinitely many dominoes numbered 0, 1, 2, 3, ...
- I want to topple all of the dominoes.
- I push over the domino labelled with 0.
- For all k, if domino k falls, then domino $k+1$ falls.
- So, every domino will eventually fall.

Induction

- Suppose $P(n)$ is a predicate, and I have infinitely many propositions $P(0), P(1), P(2), \ldots$
- I want to show that all the propositions are true.
- I prove $P(0)$.
- I prove that for all k, if $P(k)$ is true, then $P(k+1)$ is true.
- So, I can conclude that $P(n)$ is true for all $n \in \mathbb{N}$.

Induction Principle

Theorem

Suppose $P(n)$ is a predicate. If

1. (Base case) $P(0)$ is true, and
2. (Inductive step) for all $k \in \mathbb{N}$, if $P(k)$ is true, then $P(k+1)$ is true,

then $P(n)$ is true for all $n \in \mathbb{N}$.

Proof: Take a set theory class.

Examples

- The sum of the first n positive integers is $n(n+1)/2$.
- The sum of the first n odd integers is n^2.
- If $n \geq 4$, then $n! > 2^n$.
- If $x > 1$ and $n \in \mathbb{Z}^+$, then $(1+x)^n \geq 1 + nx$.

Example

Definition

An **ell** is a shape made up of three 1×1 squares connected edge-to-edge in the shape of an "L."

Proposition

Any $2^n \times 2^n$ square with a 1×1 square removed can be tiled by ells.

Coloring Example

Proposition

Any bipartite graph is 2-colorable.

Proposition

If the maximum degree in a graph is D, then the graph is $(D+1)$-colorable.

Strong induction

Theorem

Suppose $P(n)$ is a predicate. If

1. (Base case) $P(0)$ is true, and
2. (Inductive step) for all $k \in \mathbb{N}$, if $P(0), P(1), \ldots, P(k)$ are true, then $P(k+1)$ is true,

then $P(n)$ is true for all $n \in \mathbb{N}$.

Chicken McNugget Theorem

Theorem

If m and n are relatively prime positive integers, the greatest integer that cannot be written in the form $am + bn$ for non-negative integers a, b is $mn - m - n$.

Proposition

Suppose Chicken McNuggets come in packages of 9 and 20 pieces. We can buy any number of Chicken McNuggets we want if that number is at least 152.

Proof:

Chomp

- Chomp is a two-player game played with a rectangular chocolate bar made up of smaller square blocks, known as cells.
- Each player, on his or her turn, chooses a remaining cell and eats it along with all the cells that are above it and to its right.
- The lower left cell is poisoned.
- The first player has a winning strategy if the starting bar is not 1×1, although nobody knows what it is.

Proposition

Suppose it is player A's turn. If the remaining cells form an "L" that is n cells wide and n cells tall, then player B has a winning strategy.