Introduction, Logic

June 15, 2014

What are discrete structures...?

- discrete $\neq$ discreet
- Discrete math deals with objects that can only take on distinct, separated values (e.g., number of people in a room).
- ...in contrast to values that can vary continuously (e.g., how far you are from a wall).

...and what are they good for?

- Shortest path problem: How do we find the fastest route from Chicago to Bloomington?
- Graph coloring: How many colors does it take to color a map such that no two bordering countries share the same color?
- Zero-knowledge proofs: How can a user prove to her bank that she knows her password without ever revealing the password itself?
- Algorithm analysis: What is the fastest sorting algorithm possible?
- Proof theory: Can we ever prove statements that are false? Are there true statements that are impossible to prove?
- Set theory: How do we compare sizes of infinite sets?

Course Staff

- Instructor - Yipu Wang
- TA - Noah Chartoff

Schedule

- 8 weeks
- Monday through Thursday 10-11:15
- MTW lecture, Th problem session

Coursework

- Moodle reading quiz due before every lecture (M-W). First one due this Wednesday morning.
- Moodle Mini-homework due once a week on Wednesdays at 11:59pm. First one due on 6/25.
- Longform HW due in class on Wednesdays. First one due on 6/25.
- 2 midterms and final exam.

Grading

- Midterms 20% each
- Final 25%
- Reading quizzes 5% (lowest two quizzes dropped)
- Mini-homeworks 10% (lowest mini-HW dropped)
- Homeworks 20% (lowest HW dropped)

Grading Scale

- Thresholds for guaranteed grades:
  - A 94%
  - A- 90%
  - B- 80%
  - C- 70%
  - D- 60%
- In previous terms, this course has given about 20% A's, 30% B's, 30% C's, 15% D's, and 5% F's.

Textbook

- Margaret Fleck's *Building Blocks for Theoretical Computer Science*. Link on course website.
- CS 173 Discussion Problems. Please pick up at end of class today.
- Optional: Kenneth Rosen's *Discrete Mathematics and its Applications*

What is logic?

- **Logic** is the study of reasoning.
- *"'Contrariwise,' continued Tweedledee, 'if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic.'" - Alice in Wonderland*

Lewis Carroll Logic Puzzle

Suppose

- All babies are illogical.
- Nobody is despised who can manage a crocodile.
- Illogical persons are despised.

Can babies manage crocodiles?

Atoms

- A **proposition** is a statement that is either true or false.
- Examples: "It is raining" and "$8 < 4$."
- Propositions cannot be questions or contain variables.
- Non-examples: "Is the sky blue?" and "$x < 9$"

Complex propositions

- Propositions can be joined together.
- e.g., "Springfield is the capital of IL and the sky is blue."
- **Atoms** are the simplest propositions, and we use variables to denote them.
- e.g., if p is "Springfield is the capital of IL" and q is "the sky is blue," then the previous example can be written as "p and q" or "$p \wedge q$."

AND ($\wedge$)

$p \wedge q$ means "p and q" and is true when p and q are both true.
We can express this using a **truth table**

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

NOT ($\neg$)

$\neg p$ means "not p."

p	$\neg p$
T	F
F	T

OR ($\vee$)

$p \vee q$ means "p or q"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication ($\rightarrow$)

$p \rightarrow q$ means "p implies q."
In other words, "if p, then q."
In other words, "When p is true, then so is q."

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Chaining Implications

If $p \to q$ and $q \to r$ are true, then so is $p \to r$.

Proof:

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Bi-implication ($\leftrightarrow$)

$p \leftrightarrow q$ means "q implies p and p implies q."
In other words, "p if and only if q."

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

In other words, "p and q have the same truth value."

Logical Equivalence

Two propositions s and t are **logically equivalent** if they have the same truth values for all cases, and we denote this by $s \equiv t$.

Examples:

- $p \vee q \equiv q \vee p$ and $p \wedge q \equiv q \wedge p$
- $p \rightarrow q \equiv \neg p \vee q$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Converse and Contrapositive

- The converse of $p \to q$ is $q \to p$.
- The contrapositive of $p \to q$ is $\neg q \to \neg p$
- Any statement is equivalent to its contrapositive.

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

Now, back to the logic puzzle...

Lewis Carroll Logic Puzzle

- All babies are illogical.
  $B \to I$
- Nobody is despised who can manage a crocodile.
  $C \to \neg D$
- Illogical persons are despised.
  $I \to D$

Thus

$$B \to I \to D \to \neg C.$$

Negation

To negate expressions, we must use the following three equivalences.

- $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- $\neg(p \vee q) \equiv \neg p \wedge \neg q$.

English examples:

- The opposite of "I will eat dinner and shower" is "I will not eat dinner or I will not take a bath."
- The opposite of "I will go to the movies or go to the park" is "I will not go to the movies and I will not go to the park."

Thus

$$\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q$$

Negation example

Formula to negate

$(p \to q) \vee r$

$$\begin{align}
\neg((p \to q) \vee r) &\equiv \neg(p \to q) \wedge \neg r \tag{1}\\
&\equiv (p \wedge \neg q) \wedge \neg r \tag{2}
\end{align}$$

To do

1. Pick up discussion problems.
2. Enroll in Piazza.
3. Make sure you can access Moodle. First reading quiz due right before Wednesday's class.