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by

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## ABSTRACT

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Degree: Master of Science - Financial Engineering

This thesis is dedicated to all Stevens students. Dedication is optional.

**Acknowledgments**

The acknowledgements section recognizes anyone that provided significant help in producing your thesis or dissertation. Frequently acknowledged people are your advisor, colleagues, and family. Sometimes companies or outside groups have contributed to the research done for a dissertation, and they can be thanked her as well. The Acknowledgements page is optional.

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# Chapter 1

# Introduction

Intro to be added

# Chapter 2

# Method Description

In this section, the origin DOI method, JDOI method and approximation method based on DOI method are described. In section 2.1,, we introduce the origin DOI method. In section 2.2, we illustrate the approximation method proposed by Kristensen and Mele (2011). In section 2.3, we discuss our estimator to price American options based on JDOI method.

## 2.1 The DOI Variance Reduction Method

Consider a multi-factor model, in which a $d$-dimensional vector of state variables $X(t)$ on a filtered probability space$(\Omega, \mathcal{F}, \mathbb{Q})$ satisfies the following Stochastic Differential Equations(SDEs)

$$dX(t) = \mu(t, X(t))dt + \sigma(t, X(t))dW(t) \tag{2.1.1}$$

where $\mu(t, X(t))$ and $\sigma(t, X(t))$ are drift and diffusion functions under the risk-neutral measure $\mathbb{Q}$, which also satisfies appropriate growth and Lipschiz conditions such that equation(2.1.1) admits a unique strong solution and is Markovian; $W(t)$ is a $d$-dimensional standard Brownian Motion and $t \in [0, T]$.

Let $V(t, x)$ be the value function of European option written on $X(T)$ with current state $X(t) = x$, $G(t, x)$ be the payoff function. We define the infinitesimal generator $\mathcal{L}$ associated with equation(2.1.1)to be

$$(\mathcal{L}V)(t,x) = \frac{\partial V}{\partial t} + \sum_{i=1}^{d} \mu_i(t,x)\frac{\partial V}{\partial x} + \frac{1}{2}\sum_{i=1}^{d}\sum_{j=1}^{d}(\sigma(t,x)\sigma^{\intercal}(t,x))_{i,j}\frac{\partial^2 V}{\partial x_i x_k} \tag{2.1.2}$$

Let $R(t,x)$ be the instantaneous short-term interest rate, $Q(t,x)$ the instantaneous coupon rate, combining with equation(2.1.1) and equation(2.1.2), the price of European option $V$ is a solution to the following partial differential equation(PDE)

$$LV(x,t) = (R(x,t) - Q(x,t))V(x,t) \tag{2.1.3}$$

with boundary condition $V(T,x(T)) = G(T,X(T))$. It's easily seen that under risk neutral measure $\mathbb{Q}$, the instantaneous option price change is equal to the price gain in saving account minus paid coupon.

Next we consider to use a $m$-dimensional($m \leq d$) process $\bar{X}(t)$ which is a simpler process to approximate the price of option. $\bar{X}(t)$ satisfies the following SDE

$$d\bar{X}(t) = \begin{cases} \bar{\mu}_i(t,\bar{X}(t))dt + \bar{\sigma}_i(t,\bar{X}(t))dW(t) & 1 \leq i \leq m \\ 0 & \text{otherwise} \end{cases} \tag{2.1.4}$$

where $\bar{\mu}(t,\bar{X}(t))$ and $\bar{\sigma}(t,\bar{X}(t))$ are drift and diffusion functions, and they are also assumed to satisfy appropriate conditions such that equation(2.1.4) admits a unique strong solution and is Markovian.

With this new process, the European option written on $\bar{X}(t)$ is given by

$$\bar{V} = \mathbb{E}_{t,x}\left[e^{-\int_t^{t+u} R(s,\bar{X}(s))ds}G(T,\bar{X}(t+u))\right] \tag{2.1.5}$$

and assume the following additional integrability condition

$$\mathbb{E}_{t,x}\left[\sup_{0\le u\le T-t}\left|e^{-\int_t^{t+u}R(s,\bar{X}(s))ds}G(T,\bar{X}(t+u))\right|\right]<\infty \tag{2.1.6}$$

Additionally, strong Markovian arguments imply that the European-style option $\bar{V}$ satisfies

$$L\bar{V}(x,t)+q(t,x)=R(x,t)\bar{V}(x,t) \tag{2.1.7}$$

It's easily seen that

$$V(t,x)=\bar{V}+\mathbb{E}_{t,x}\left[\right] \tag{2.1.8}$$

## 2.2 Approximation Method based on DOI method

## 2.3 JDOI method

**Appendix A**

**Appendix A Title**

Appendices at the end of a dissertation are optional, and depend on the content of the dissertation. There can be one or more appendicies, however they should retain the page numbering requirements for dissertations. Any concerns about the formatting of an appendix should be brought to Doris Oliver, who can direct you how to format your appendix if you have questions.

| Theoretical Dissertation Timeline | | |
|---|---|---|
| Taskt | Time to Finish | Notes |
| Problem statement | 10 hours | Initially very upbeat. |
| Research | 3 days | Literature search to very previous studies. |
| Reformulation | 4 hours | Presented and accepted by advisor |
| Research | 20 days | Literature search to very previous studies. |
| Experiments | 14 days | Do some experiments and get results. |
| Format | 1 day | Understand format guidelines for paper. |
| Write | years | Write the paper. |
| Revise | not too long | Proof read, etc. |
| Format | 1-3 days | Verify correct report format is used. |
| See Library | 1 hour | Meet with Doris to verify formatting. |
| Defend | 1 day | Defend your research. |
| Revise | 0 hours | It was perfect the first time. |
| Submit | 1 day | Submit final dissertation to the library. |

## Bibliography

Kristensen, D. and A. Mele (2011, November). Adding and subtracting Black-Scholes: A new approach to approximating derivative prices in continuous-time models. *Journal of Financial Economics 102*(2), 390–415.