Midterm Examination

FE621 Sections A and WS

from Saturday March 27 to Sunday March 28, 2021

Name:

- This is an online examination. Some problems require use of mathematical derivations, some require use of a computer program.
- For the written parts please scan or take pictures of the derivations and upload them.
- For the computer parts please upload the source files used. Also detail your results and comment in a separate report or the main report. The problems you are supposed to use a computer for are accompanied by text in italics.
- There are 4 problems worth a total of 100 points. Please double check you solved all problems.
- Be very specific with your definitions and derivations. Showcase your work.
- Communication with other students either physical or virtual is strictly forbidden.

Problem 1. (Binomial Tree) In this question, consider the following setup: the initial stock price is \$50, the strike price is \$49, and risk free interest rate is 5% per annum, and the volatility is 30% per annum. The time to maturity is 9 months, and you are asked to use a three-step binomial tree.

(a) Compute the price of an European put option. How many shares does the investor need to hold in the hedging portfolio at the beginning of the contract, in order to fully hedge the risks?

(b) Compute the price of an American put option. How many shares does the investor need to hold in the hedging portfolio at the beginning of the contract, in order to fully hedge the risks?

(c) Given the previous information, in addition, assume that there is a barrier level at L=38. Price an European down and out put option.

Problem 2. (Trinomial Tree) For this problem you need to use the code in problem 2 assignment 2. You need to modify the code to price an American put option.

Consider the following parameters: the initial stock price is \$50, the strike price is \$49, risk free interest rate is 5% per annum, and the volatility is 30% per annum. The time to maturity is 9 months.

(a) Calculate the price of an American Put option using a trinomial tree with three steps. Compare with the answer in part (b) of the previous problem.

(b) Estimate the actual price of an American option using a tree with 200 steps.

Problem 3. (Finite Difference Method)

We know that an option price under a certain stochastic model satisfies the following PDE:

$$\frac{\partial V}{\partial t} + 2\tan(S)\frac{\partial V}{\partial S} + S^3\frac{\partial^2 V}{\partial S^2} - rV = 0.$$

Assume you have an equidistant grid with points of the form $(i, j) = (i\Delta t, j\Delta x)$, where $i \in \{1, 2, \ldots, N\}$ and $j \in \{-N_S, N_S\}$. Let $V_{i,j} = (i\Delta t, j\Delta x)$.

(a) Discretize the above PDE and give the finite difference equation for an Explicit scheme. Use the notation introduced above. Please simplify the final expressions so that the unknown quantity is on the left hand side and all the known quantities are on the right hand side.

(b) Derive the discretized equation for the Implicit scheme. Please simplify the final expressions so that the unknown quantities are on the left hand side and the known quantity is on the right hand side. You do not need to solve the corresponding matrix equation.

Please note that neither the type of the option nor the boundary conditions are needed to derive the discretized equation.

Problem 4. In the following statements please circle (or indicate in your submission) only the statements that are always true.

(a) The price of American Call options is always equal to the price of European Call options when written on an continuous asset that does not pay dividends.

(b) The price of a Put option calculated using an Explicit Finite Difference scheme with 100 time steps is identical with the price of the same option calculated using a trinomial tree with 100 steps.

(c) Suppose we are using an Explicit finite difference scheme for a European Call with strike $K = 100$ with $\Delta t = 0.01$ and $\Delta x = 0.1$. The stock price is $S = 80$, and we use $r = 0.05$ and $\sigma = 0.4$. The distance between the approximated option value and the true value depends only on S and K.

(d) Using a Finite Difference scheme to calculate the option price we have to discount the final value we find.

(e) The value of the log return is always lower than or equal to the value of the simple return.

(f) The sum of three increments of a Brownian motion over three different one minute intervals is a normal random variable with standard deviation three times the length of the one minute interval.

(g) A call option may be hedged by holding a certain number of shares of the underlying

(h) We can price Up and Out Barrier Options using a Binomial tree when the volatility of the underlying process is constant.

Bonus We can use two independent normal random numbers to create two correlated normals with correlation 0.1.