

# ECE 590, Fall 2019

## Problem Set 1

- **### Important :** You are only allowed to use the Python built in function for generating uniform random variables.

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```
In [1]: from random import random
import numpy as np
import matplotlib.pyplot as plt
```

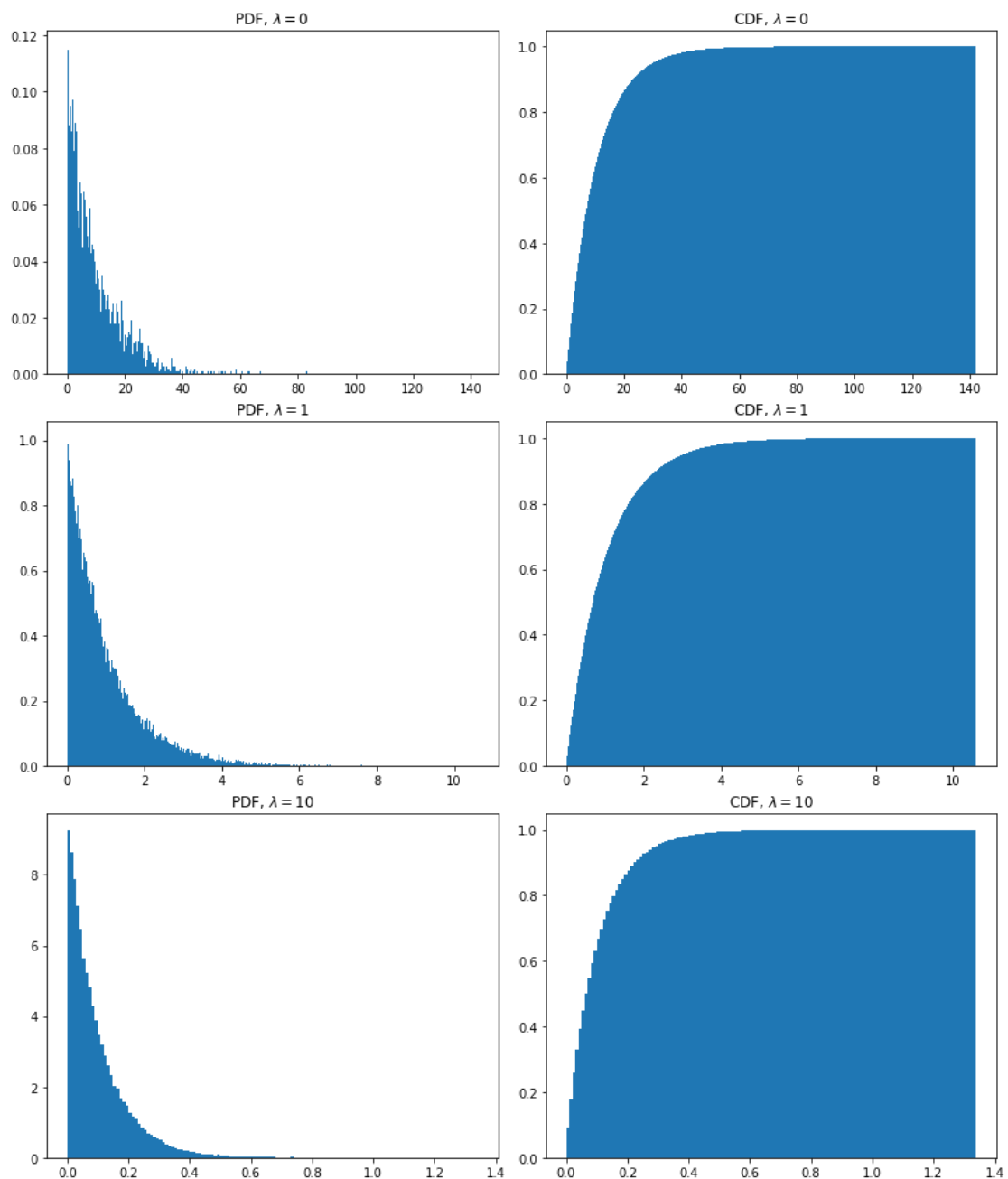
### Problem 1 (Exponential distribution)

```

In [2]: lambda_ = [0.1, 1, 10]
fig, axs = plt.subplots(3, 2, figsize=(13, 15))
for i in range(3):
    Unif = np.array([random() for _ in range(10**5)])
    Exp = -np.log(1 - Unif) / lambda_[i]
    axs[i, 0].hist(Exp, bins=np.arange(0, Exp.max()+0.01, 0.01), density
= True)
    axs[i, 0].set_title(r"PDF, $\lambda = %d$" % lambda_[i])
    axs[i, 1].hist(Exp, bins=np.arange(0, Exp.max()+0.01, 0.01), density
= True, cumulative= True)
    axs[i, 1].set_title(r"CDF, $\lambda = %d$" % lambda_[i])
fig.suptitle(r"Histograms of Generated Exponentially Distributed Random
Variables with Parameter $\lambda$", fontsize=20)
plt.tight_layout(pad=6, h_pad=0.5, w_pad=1)
plt.show()

```

Histograms of Generated Exponentially Distributed Random Variables with Parameter  $\lambda$



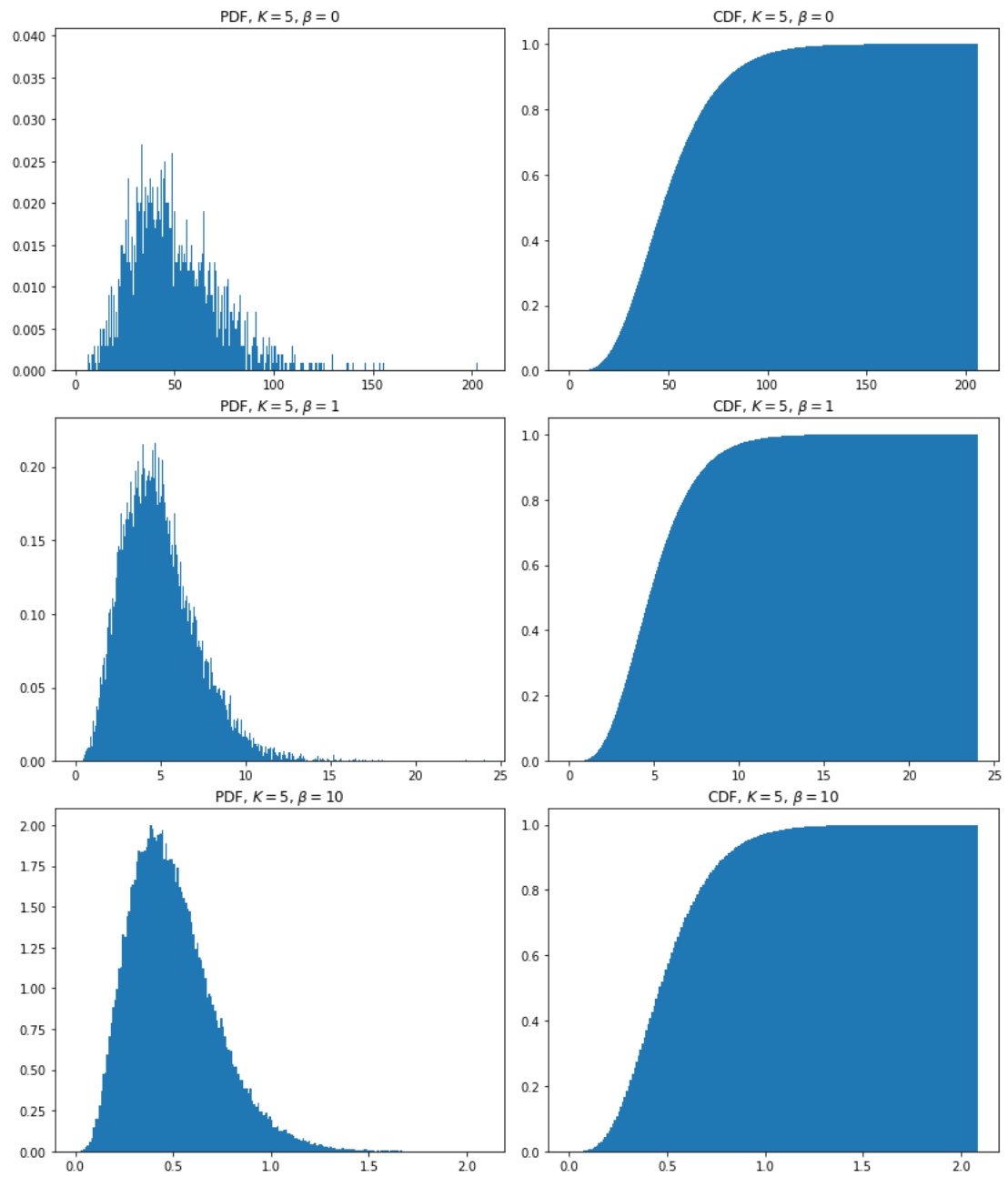
## Problem 2 (Gamma distribution)

```

In [3]: beta = [0.1, 1, 10]
K = 5
fig, axs = plt.subplots(3, 2, figsize=(13, 15))
for i in range(3):
    Gam = np.zeros(10**5)
    for _ in range(K):
        Unif = np.array([random() for _ in range(10**5)])
        Exp = -np.log(1 - Unif) / beta[i]
        Gam += Exp
    axs[i, 0].hist(Gam, bins=np.arange(0, Gam.max()+0.01, 0.01), density
= True)
    axs[i, 0].set_title(r"PDF, $K = 5$, $\beta = %d$" % beta[i])
    axs[i, 1].hist(Gam, bins=np.arange(0, Gam.max()+0.01, 0.01), density
= True, cumulative= True)
    axs[i, 1].set_title(r"CDF, $K = 5$, $\beta = %d$" % beta[i])
fig.suptitle(r"Histograms of Generated Gamma Distributed Random Variable
s with Parameter $K$ and $\beta$", fontsize=20)
plt.tight_layout(pad=6, h_pad=0.5, w_pad=1)
plt.show()

```

## Histograms of Generated Gamma Distributed Random Variables with Parameter $K$ and $\beta$



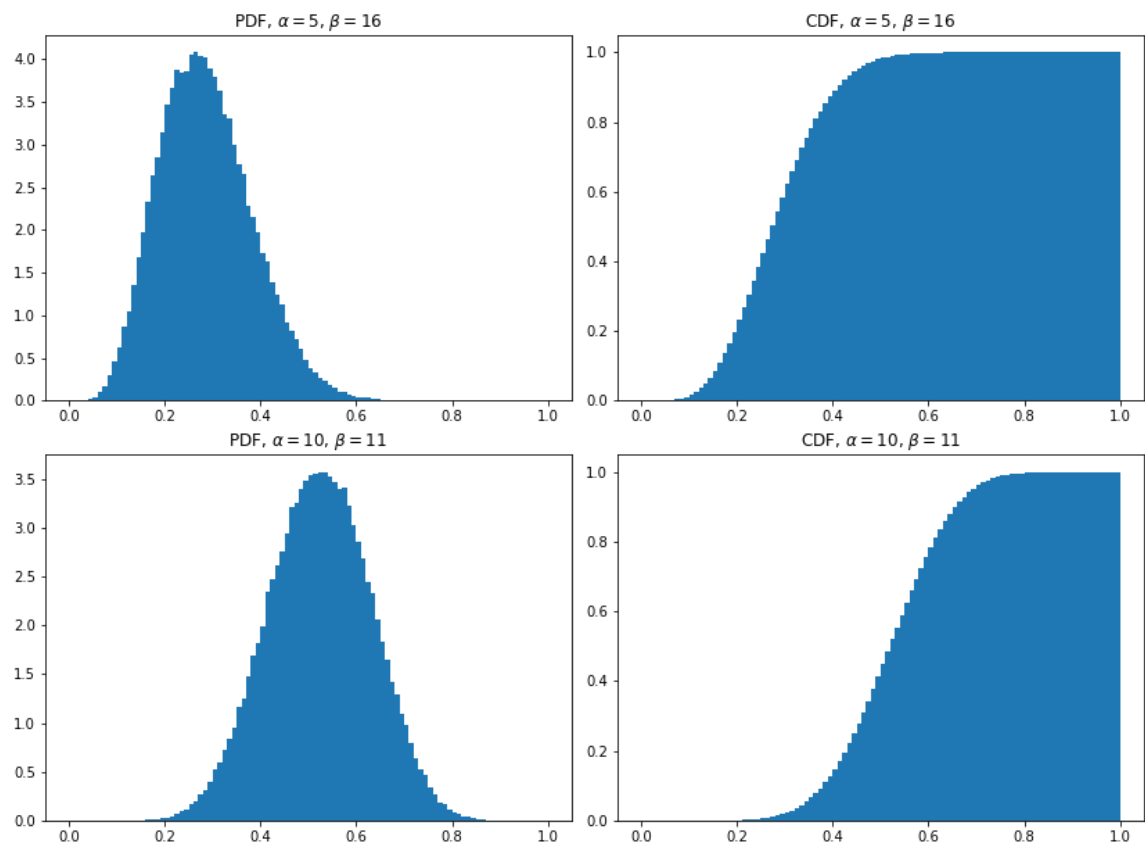
### Problem 3 (Beta distribution)

```

In [4]: alpha = [5, 10]
beta = [16, 11]
fig, axs = plt.subplots(2, 2, figsize=(13, 10))
for i in range(2):
    a = alpha[i]
    b = beta[i]
    k = a
    n = b + k - 1
    Unif = np.array([[random() for _ in range(10**5)] for _ in range(n)
    ])
    Beta = np.sort(Unif, axis=0)[k]
    axs[i, 0].hist(Beta, bins=np.arange(0, 1+0.01, 0.01), density=True)
    axs[i, 0].set_title(r"PDF, $\alpha = %d$, $\beta = %d$" % (a, b))
    axs[i, 1].hist(Beta, bins=np.arange(0, 1+0.01, 0.01), density=True,
    cumulative=True)
    axs[i, 1].set_title(r"CDF, $\alpha = %d$, $\beta = %d$" % (a, b))
fig.suptitle(r"Histograms of Generated Beta Distributed Random Variables
with Parameter $\alpha$ and $\beta$", fontsize=20)
plt.tight_layout(pad=6, h_pad=0.5, w_pad=1)
plt.show()

```

Histograms of Generated Beta Distributed Random Variables with Parameter  $\alpha$  and  $\beta$



#### Problem 4 (Dirichlet distribution)

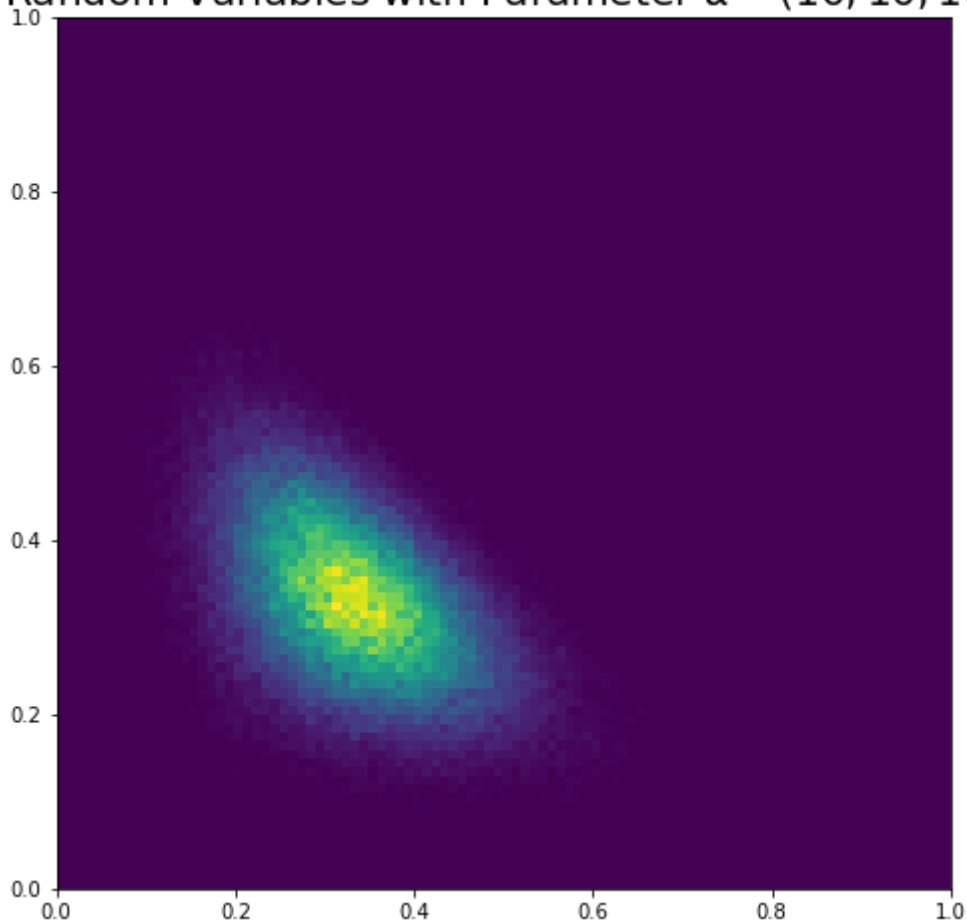
```

In [5]: alpha = [10, 10, 10]
N = 3
beta = 1

Gam = np.zeros((N, 10**5))
for n in range(N):
    for _ in range(alpha[n]):
        Unif = np.array([random() for _ in range(10**5)])
        Exp = -np.log(1 - Unif) / beta
        Gam[n] += Exp
Dir = Gam / Gam.sum(axis=0)
plt.figure(figsize=(8, 8))
plt.hist2d(Dir[0, :], Dir[1, :], bins=[np.arange(0, 1.01, 0.01), np.arange(0, 1.01, 0.01)], density=True)
plt.title("2-d Histograms of Generated Dirichlet Distributed\n" + r"Random Variables with Parameter $\alpha=(10,10,10)$", fontsize=20)
plt.show()

```

2-d Histograms of Generated Dirichlet Distributed Random Variables with Parameter  $\alpha = (10, 10, 10)$



```

In [6]: # X1Beta(10 ,20)
#

alpha = [10, 10, 10]

Dir = np.zeros((3, 10**5))
for i in range(len(alpha)-1):
    a = alpha[i]
    b = sum(alpha[(i+1):])
    k = a
    n = b + k - 1
    Unif = np.array([[random() for _ in range(10**5)] for _ in range(n
)])
    Beta = np.sort(Unif, axis=0)[k]
    Dir[i] = (1 - Dir[:i].sum(axis=0)) * Beta
Dir[2] = 1 - Dir.sum(axis=0)

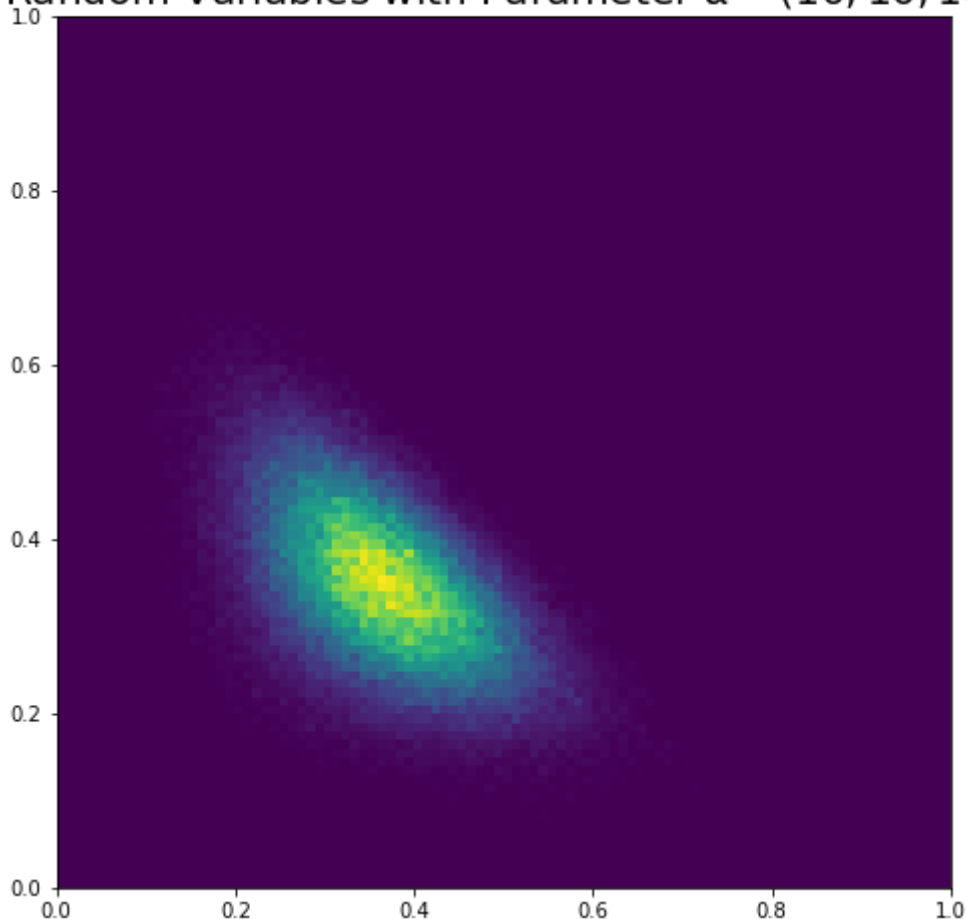
```

```

In [7]: plt.figure(figsize=(8, 8))
plt.hist2d(Dir[0, :], Dir[1, :], bins=[np.arange(0, 1.01, 0.01), np.arange(0, 1.01, 0.01)], density=True)
plt.title("2-d Histograms of Generated Dirichlet Distributed\n" + r"Random Variables with Parameter $\alpha=(10,10,10)$", fontsize=20)
plt.show()

```

2-d Histograms of Generated Dirichlet Distributed  
Random Variables with Parameter  $\alpha = (10, 10, 10)$





In [ ]: