Neural Networks I

Lecture 18

What's the hype around neural networks?

Character/handwriting recognition

Image compression (autoencoders)

Stock market prediction

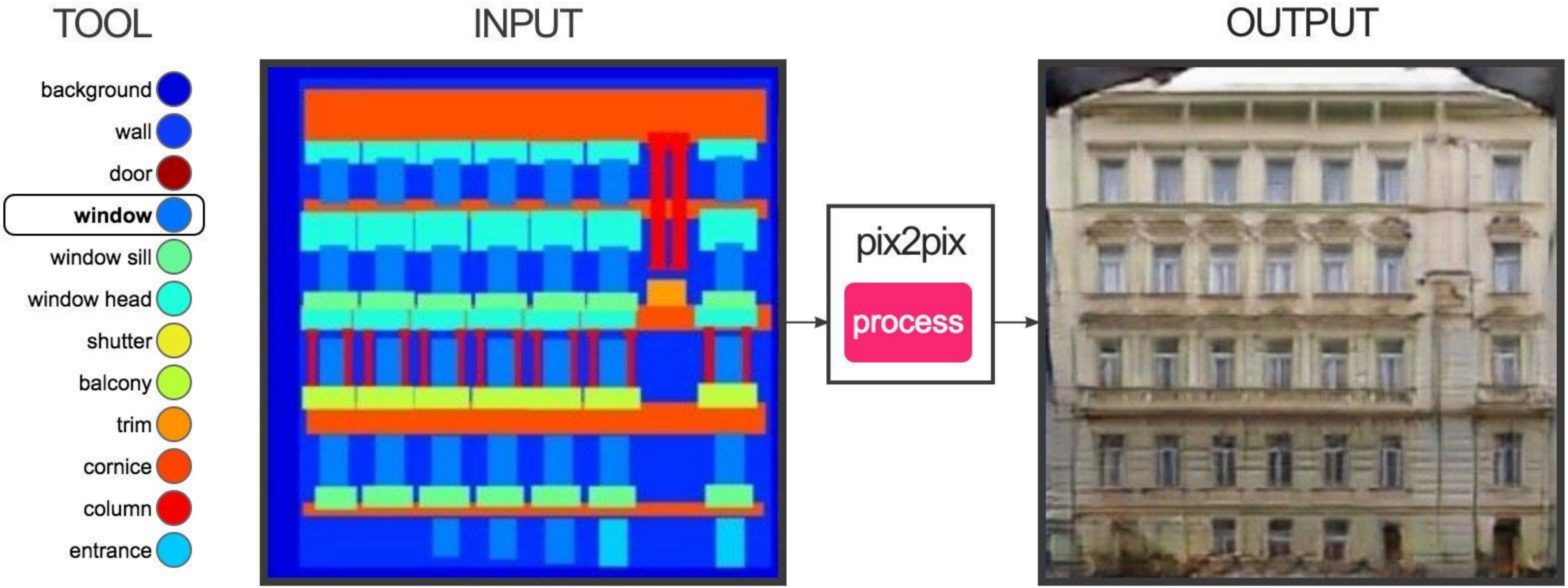
Credit approval

And some very recent interesting deep learning applications…

Image Style Transfer

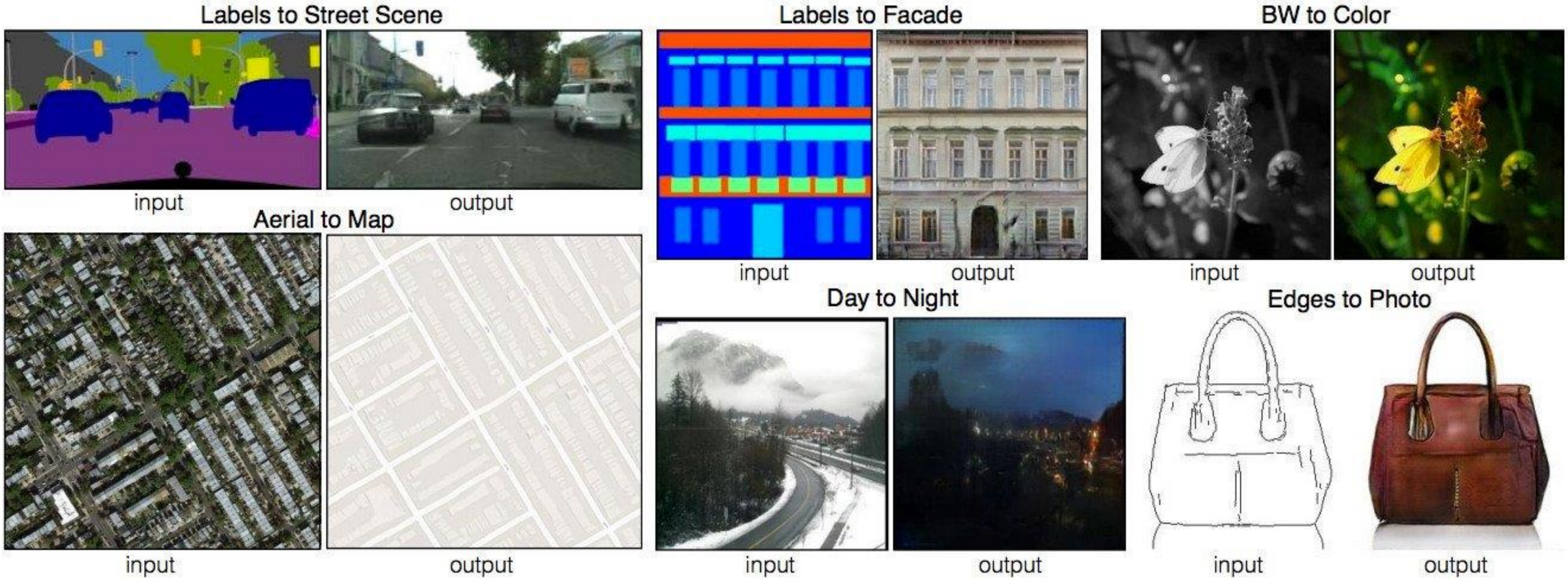
Dumoulin, Vincent, Jonathon Shlens, and Manjunath Kudlur. "A learned representation for artistic style." CoRR, abs/1610.07629 2.4 (2016): 5.

Image-to-image translation



Isola, Phillip, et al. "Image-to-image translation with conditional adversarial networks." arXiv preprint (2017).

Image-to-image translation



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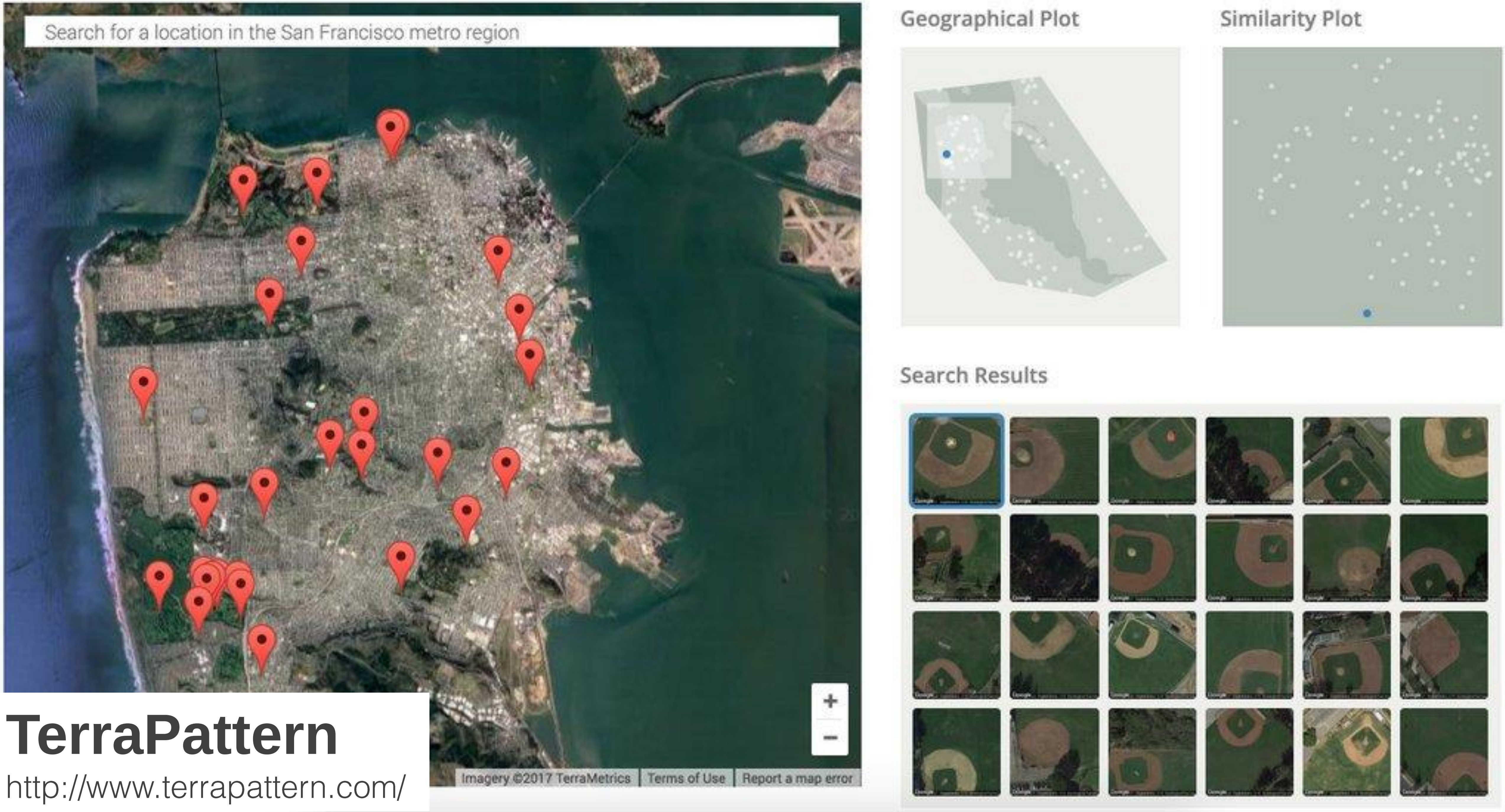


TerraPattern

http://www.terrapattern.com/

What is a neural network and **how does it work**?

How do we **choose model weights**?
(i.e. how do we fit our model to data)

What are the challenges of using neural networks?

Recall our goal in supervised learning

$$y = f(\boldsymbol{x}, \boldsymbol{w})$$

Labels

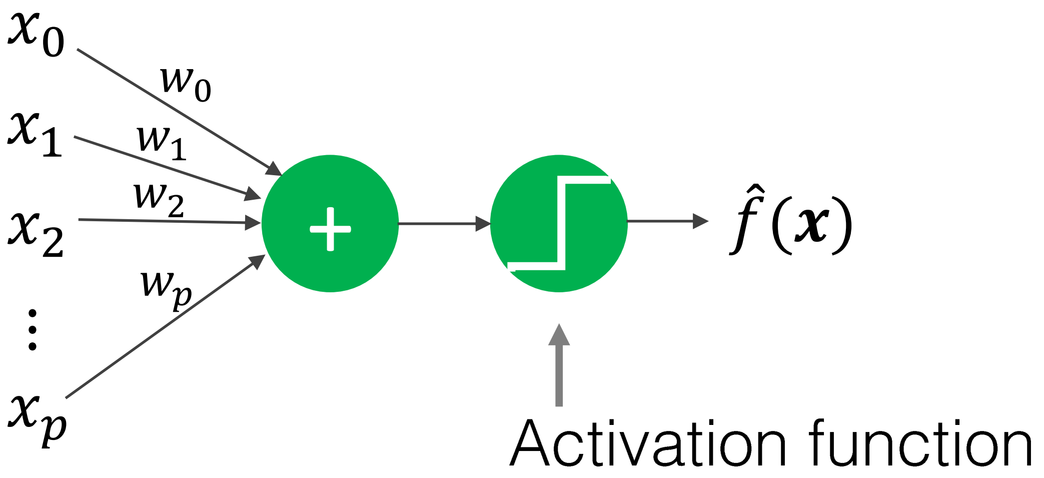
Model

Input Data

Parameter(s)

Perceptron

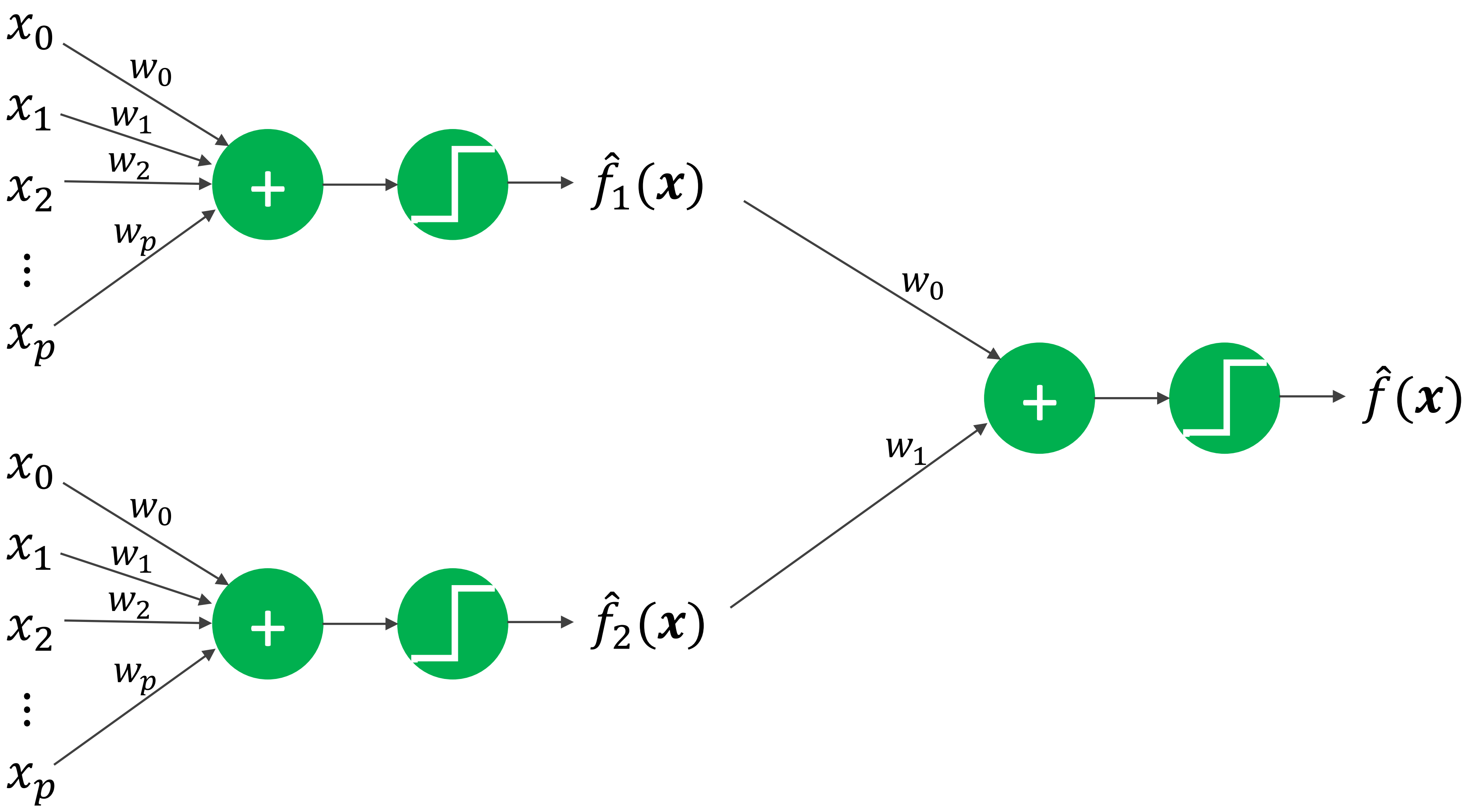
$$\hat{f}(\boldsymbol{x}) = sign\left(\sum_{i=0}^{p} w_i x_i\right)$$



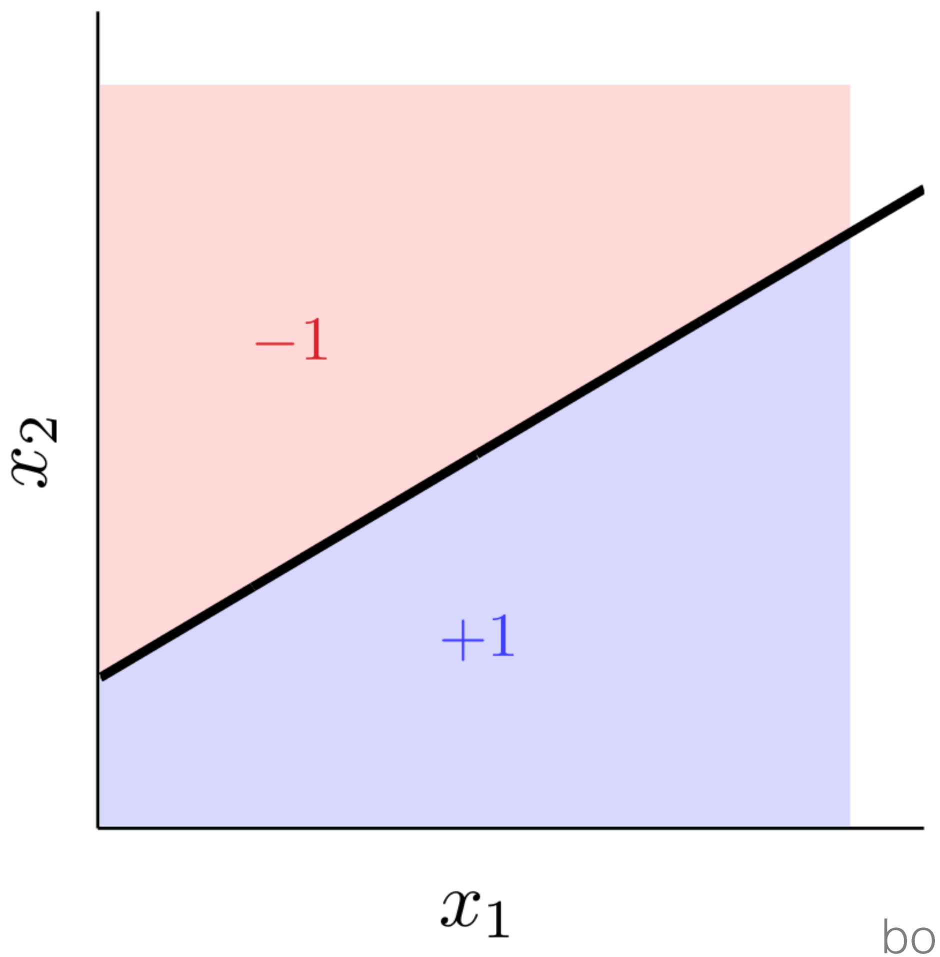
Source: Abu-Mostafa, Learning from Data, Caltech

Multilayer Perceptron

What if we stuck multiple perceptrons together?

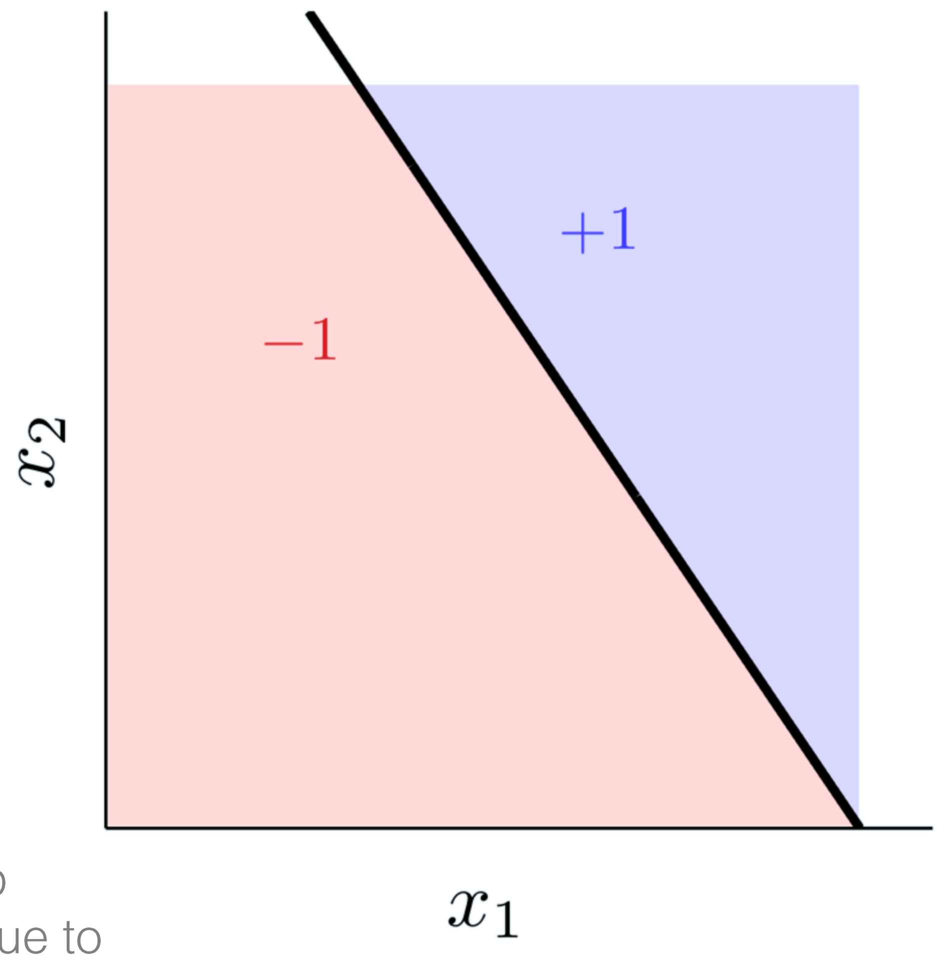


Perceptron #1



$$\hat{f}_1(\boldsymbol{x}) = sign(\boldsymbol{w}_1^T\boldsymbol{x})$$

Perceptron #2

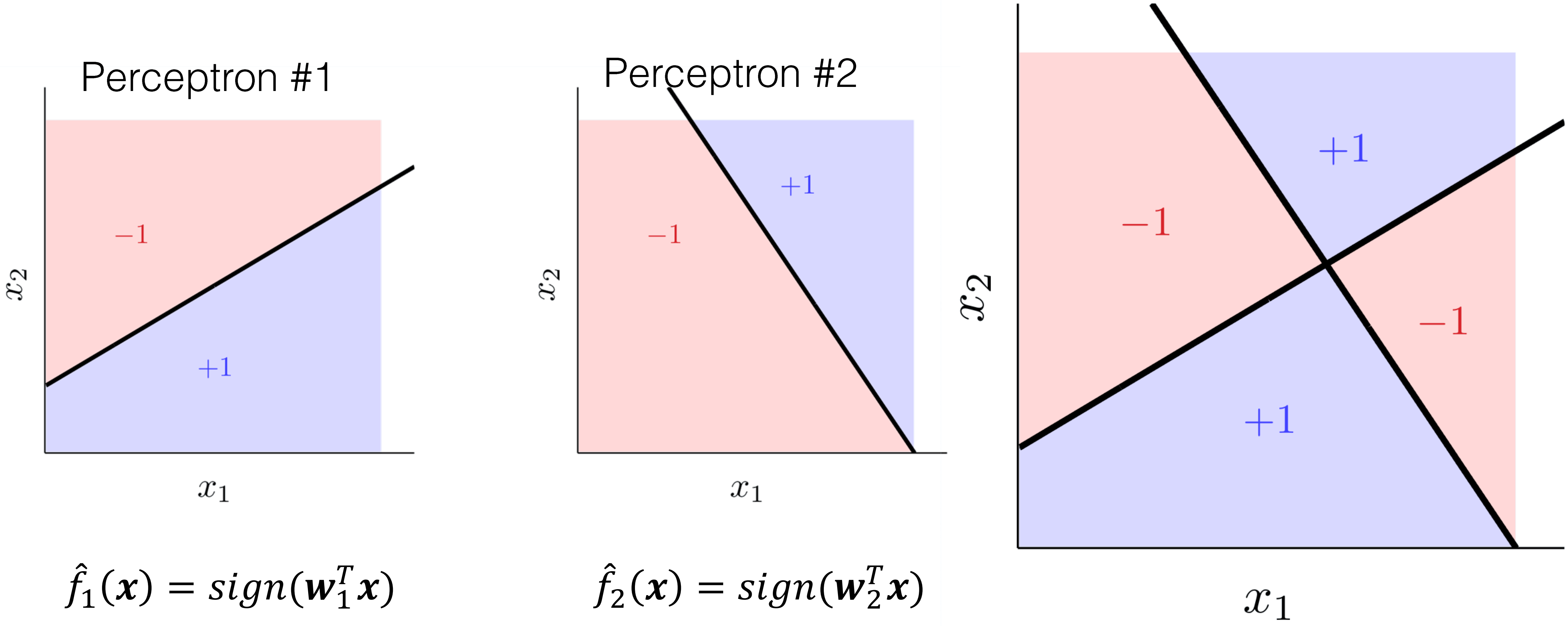


$$\hat{f}_2(\boldsymbol{x}) = sign(\boldsymbol{w}_2^T\boldsymbol{x})$$

The sharp boundary is due to our sign function

Source: Abu-Mostafa, Learning from Data, Caltech

Multilayer perceptron: $\hat{f}(\boldsymbol{x}) = \begin{cases} +1 & \hat{f}_1(-\hat{f}_2) + (-\hat{f}_1)\hat{f}_2 > 0 \\ -1 & \text{else} \end{cases}$

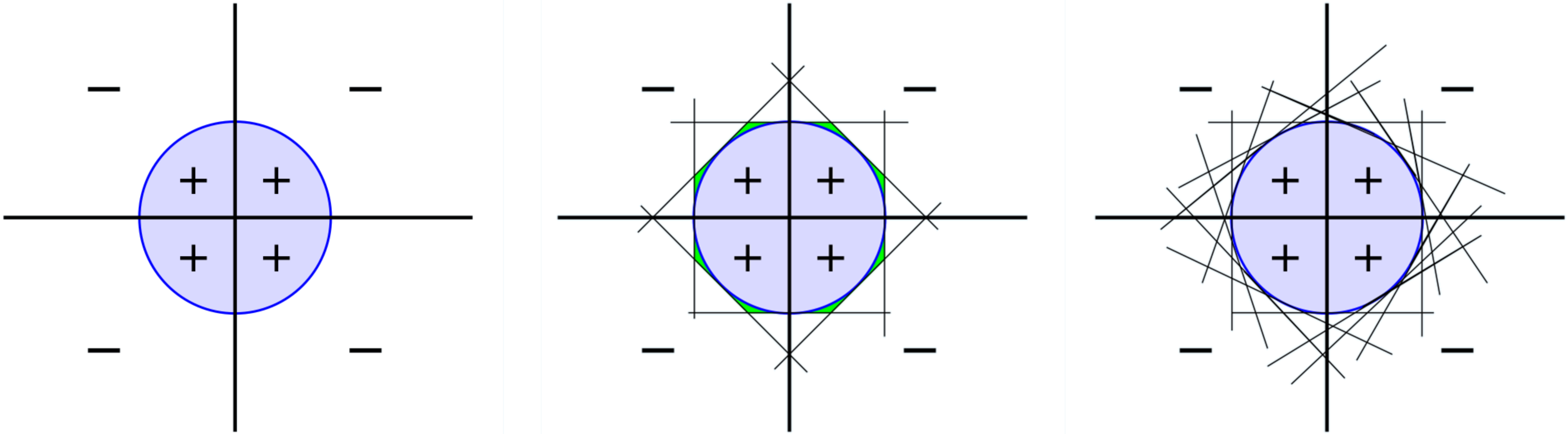


$\hat{f}_1(\boldsymbol{x}) = sign(\boldsymbol{w}_1^T \boldsymbol{x})$

$\hat{f}_2(\boldsymbol{x}) = sign(\boldsymbol{w}_2^T \boldsymbol{x})$

Source: Abu-Mostafa, Learning from Data, Caltech

Multilayer Perceptron



Target | 8 perceptrons | 16 perceptrons

The more nodes/neurons, the more flexible is the model

Source: Abu-Mostafa, Learning from Data, Caltech

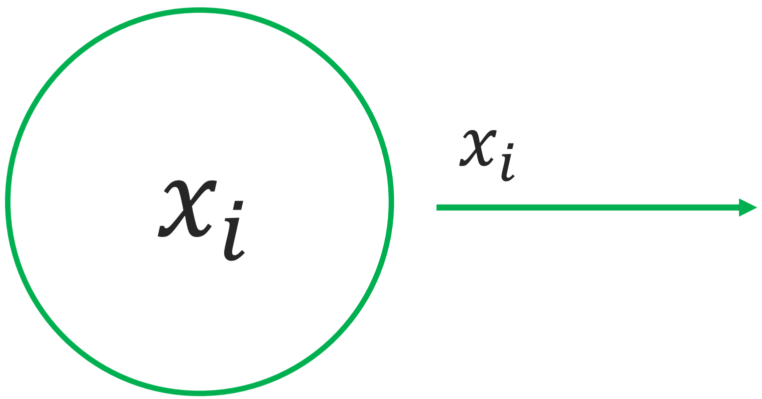
Universal function approximation

"A **feedforward network** with a single layer is sufficient to represent **any function**, but the layer may be infeasibly large and may fail to learn and generalize correctly."

Ian Goodfellow, Deep Learning

Creator of generative adversarial networks

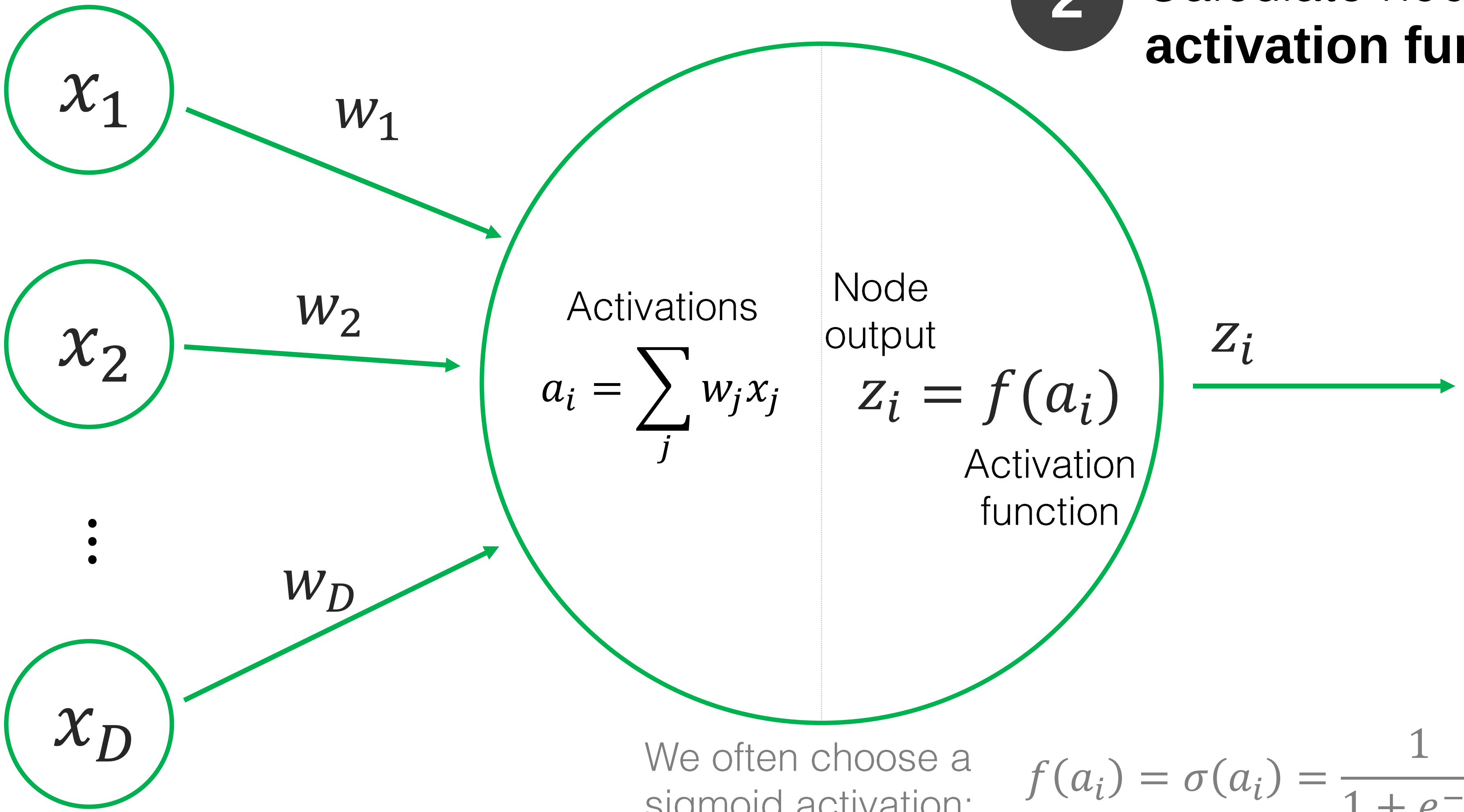
Input nodes / neurons



Simply passes the input value to the next layer

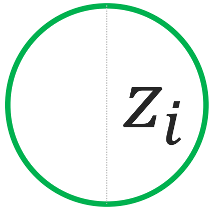
Hidden & output nodes

1. Calculate the **activations**: linear combinations of weights and the last layer's output
2. Calculate node output: apply the **activation function** to the activations

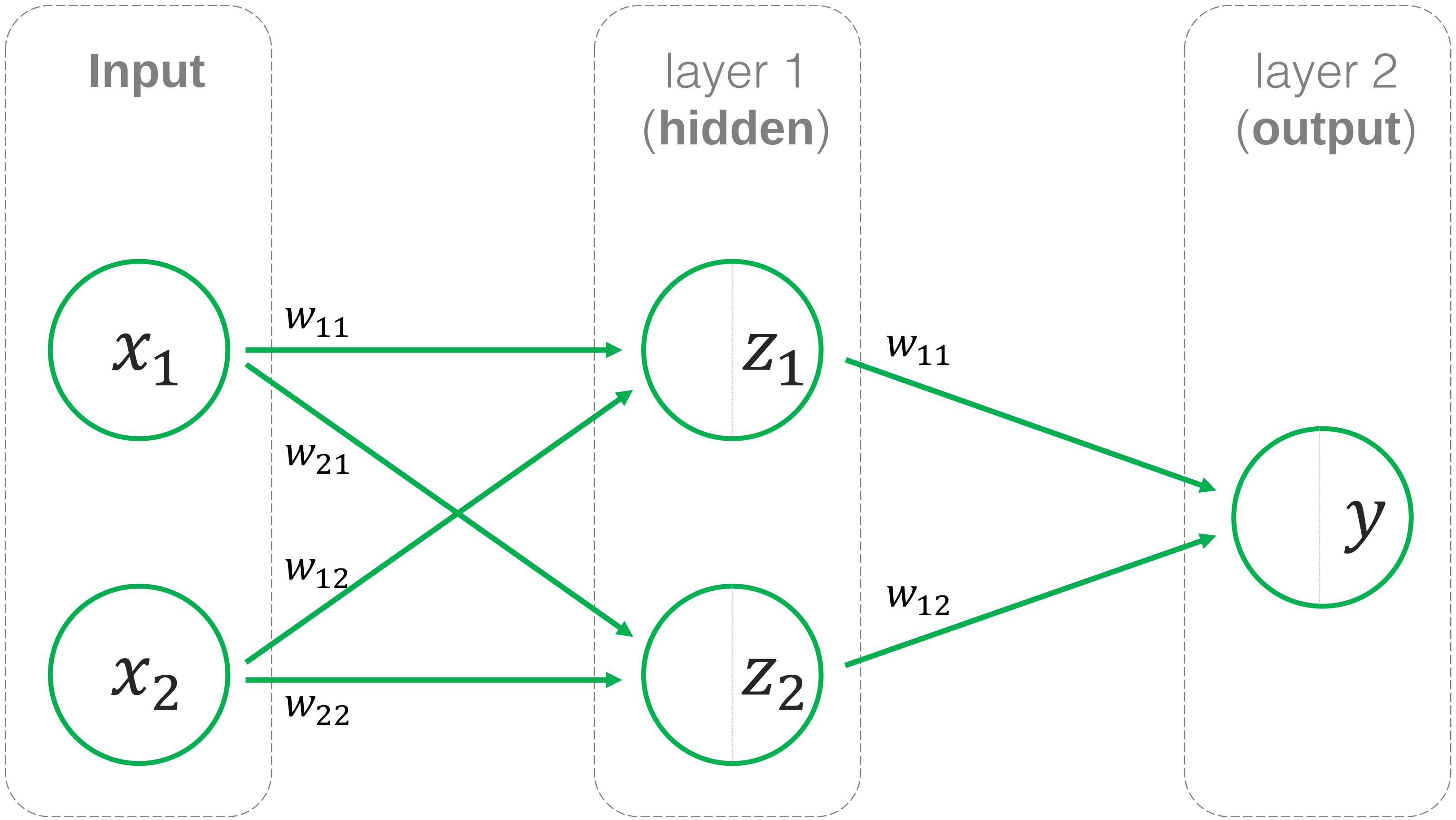


We often choose a sigmoid activation: $f(a_i) = \sigma(a_i) = \frac{1}{1 + e^{-a_i}}$

Represented as:



Simple Neural Network



$w_{ij}^{(k)}$

Layer k

From node j
(in the last layer)

to node i
(in the next layer)

Notational shorthand:
(a more precise
alternative notation)

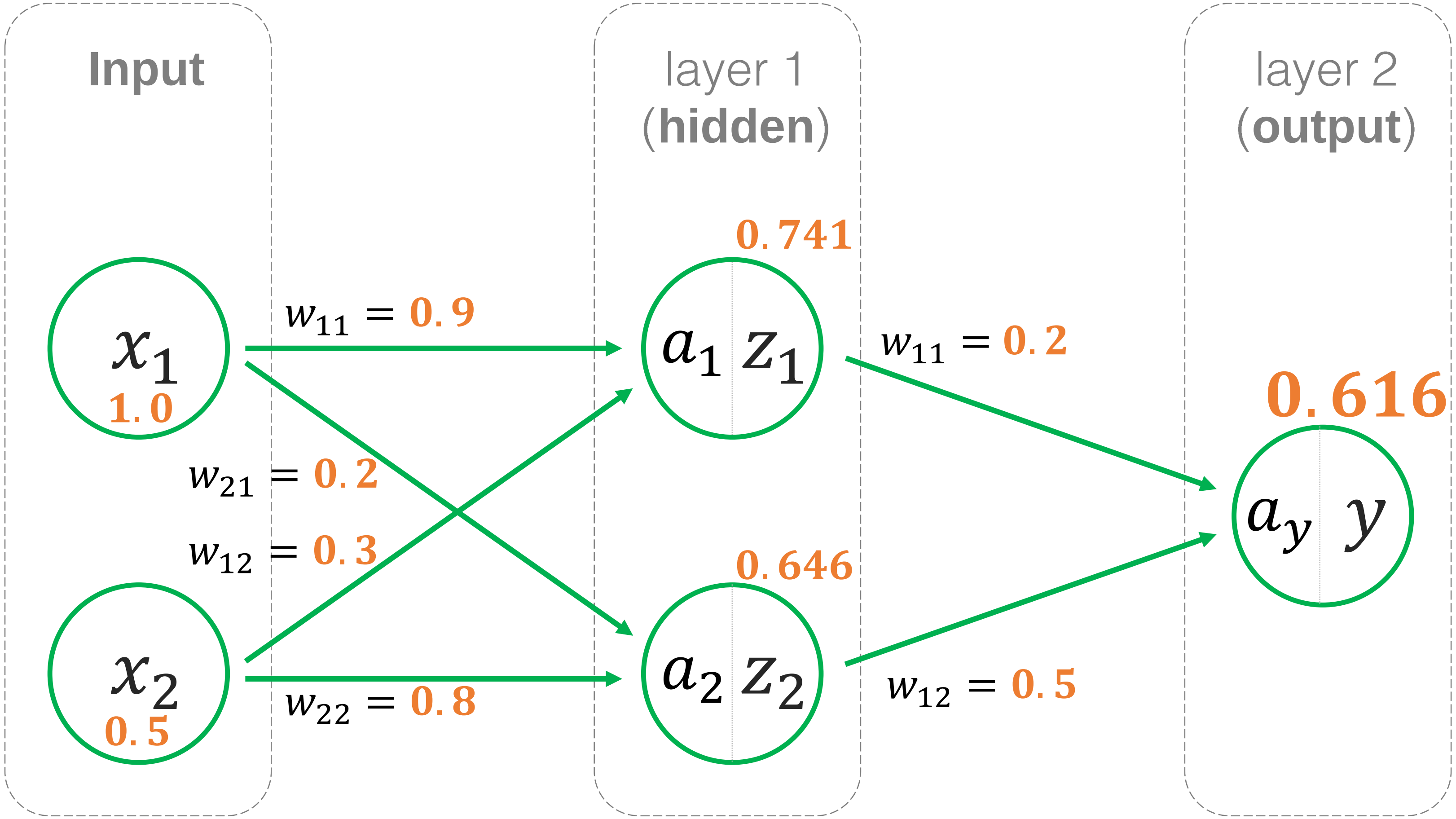
$w_{ij} = w_{ij}^{(1)}$ $\quad z_i = z_i^{(1)}$ $\quad w_{ij} = w_{ij}^{(2)}$ $\quad y = z_1^{(2)}$

Forward Propagation

Calculating the output from input

$$a_1 = (0.9)(1.0) + (0.3)(0.5) = 1.05$$
$$a_2 = (0.2)(1.0) + (0.8)(0.5) = 0.6$$
$$z_1 = \sigma(a_1) = \sigma(1.05) = 0.741$$
$$z_2 = \sigma(a_2) = \sigma(0.6) = 0.646$$

Hidden layer calculations



Output layer calculations

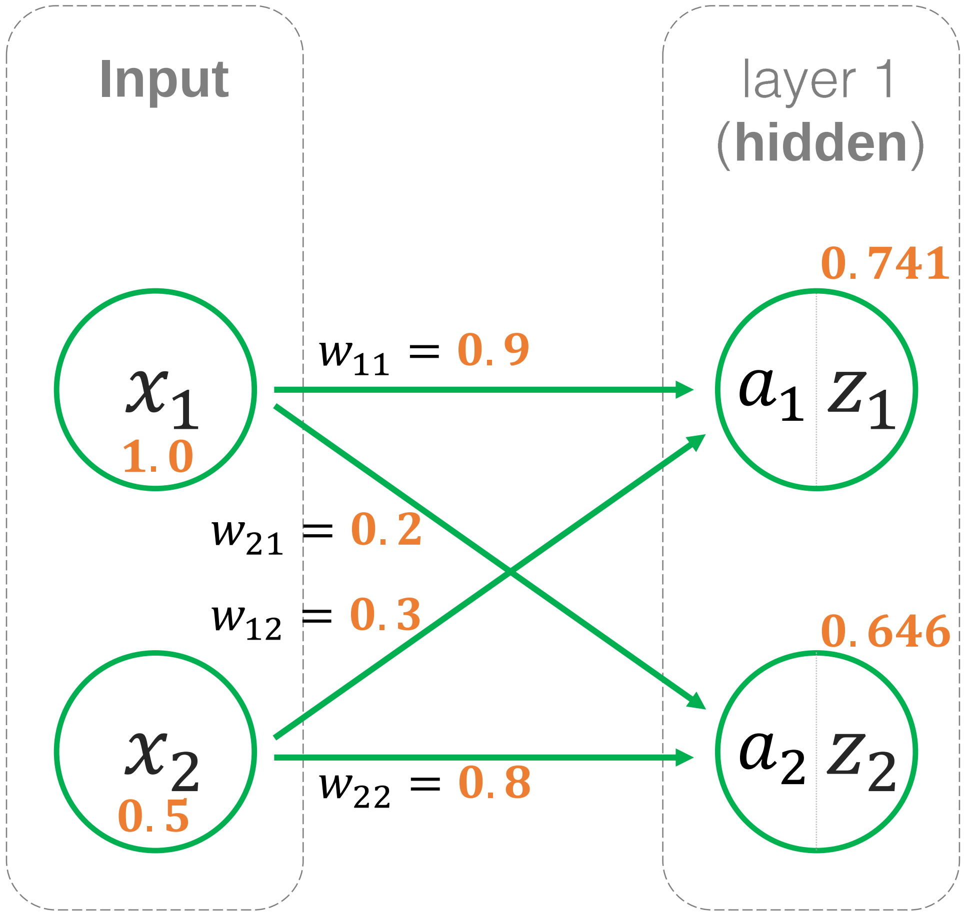
$$a_y = (0.2)(0.741) + (0.5)(0.646)$$
$$= 0.471$$
$$y = \sigma(a_y) = \sigma(0.471) = 0.616$$

$$\sigma(a_i) = \frac{1}{1 + e^{-a_i}}$$

Rashid, Make Your Own Neural Network

Forward Propagation

Calculating the output from input



Hidden layer matrix calculations

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \boldsymbol{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \boldsymbol{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\boldsymbol{W} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{array}{l} \rightarrow \text{The weights INTO node } z_1 \\ \rightarrow \text{The weights INTO node } z_2 \end{array}$$

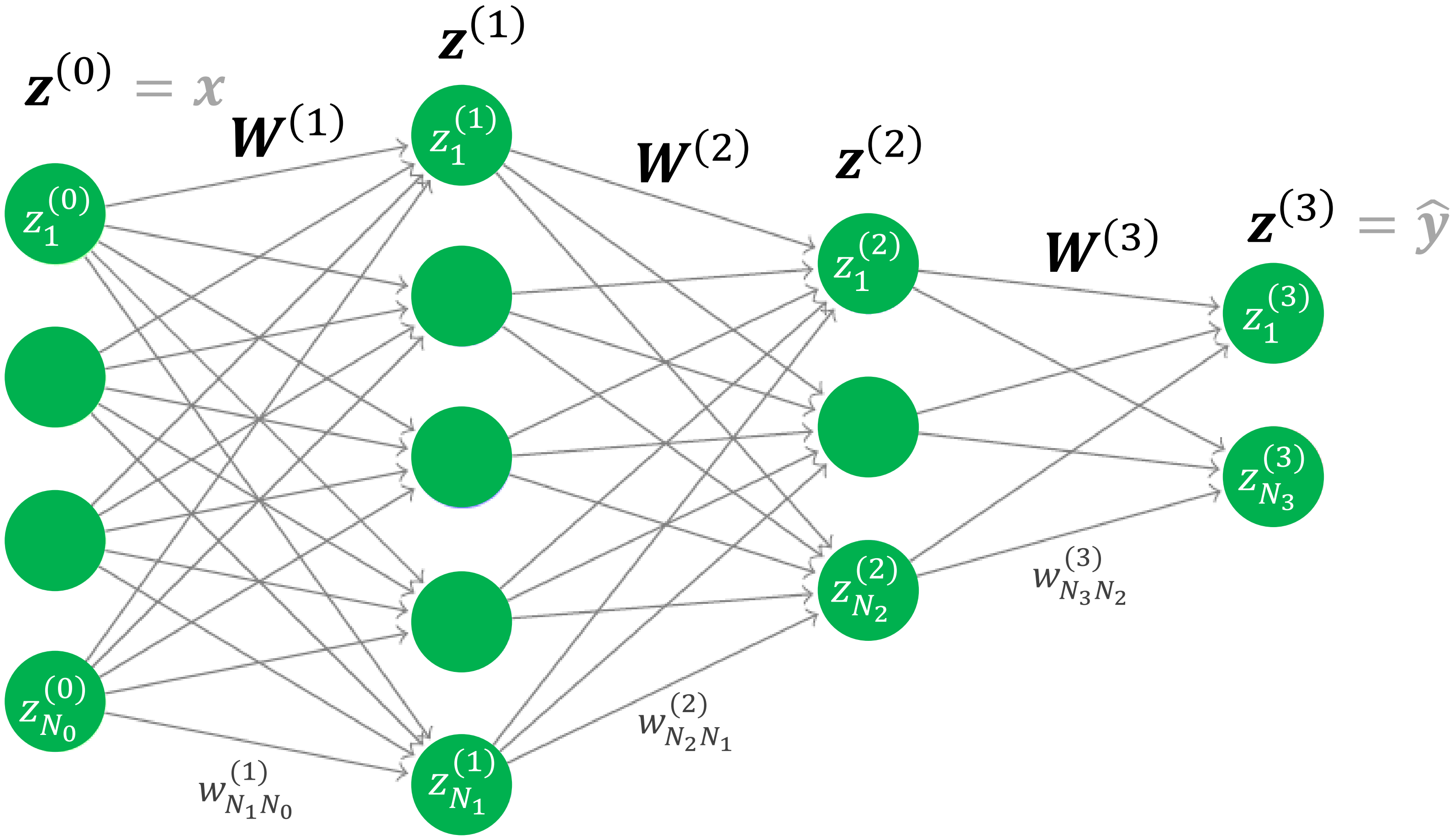
$$\boldsymbol{a} = \boldsymbol{W}\boldsymbol{x} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} w_{11}x_1 + w_{12}x_2 \\ w_{21}x_1 + w_{22}x_2 \end{bmatrix}$$

$$\boldsymbol{z} = \sigma(\boldsymbol{a}) = \begin{bmatrix} \sigma(w_{11}x_1 + w_{12}x_2) \\ \sigma(w_{21}x_1 + w_{22}x_2) \end{bmatrix}$$

Forward Propagation

Example neural network with $L = 3$ layers and the ith layer has N_i nodes



Simple steps for forward propagation:

For $i = 1$ to $L - 1$:

$$\mathbf{z}^{(i)} = \sigma\left(\boldsymbol{W}^{(i)} \mathbf{z}^{(i-1)}\right)$$

Where:

$$\mathbf{z}^{(0)} = \boldsymbol{x}$$

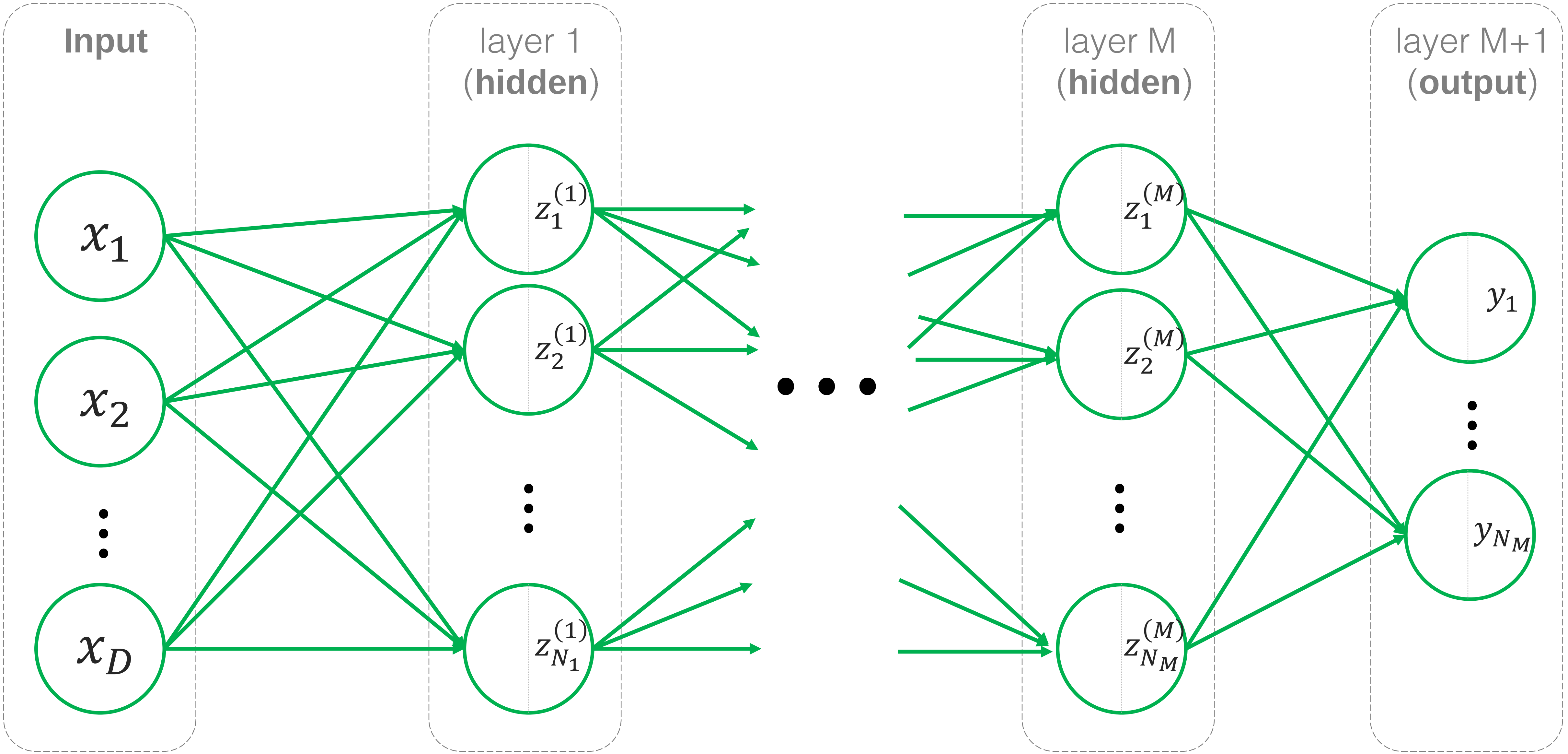
$$\hat{\boldsymbol{y}} = \mathbf{z}^{(L)}$$

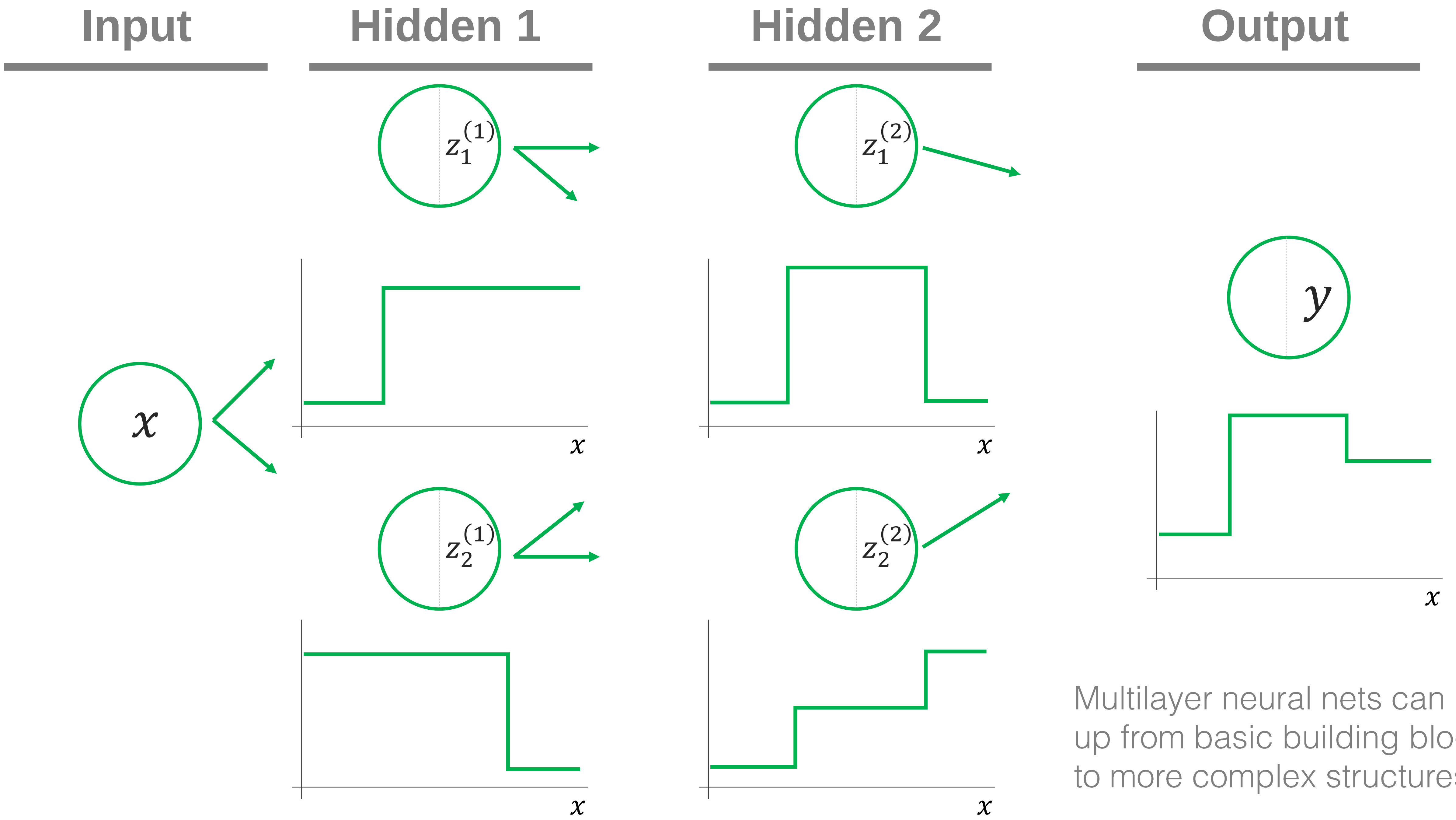
Prediction error is measured:

$$E_n = \frac{1}{2}(\hat{y}_n - y_n)^2$$

Sudeep Raja, A Derivation of Backpropagation in Matrix Form

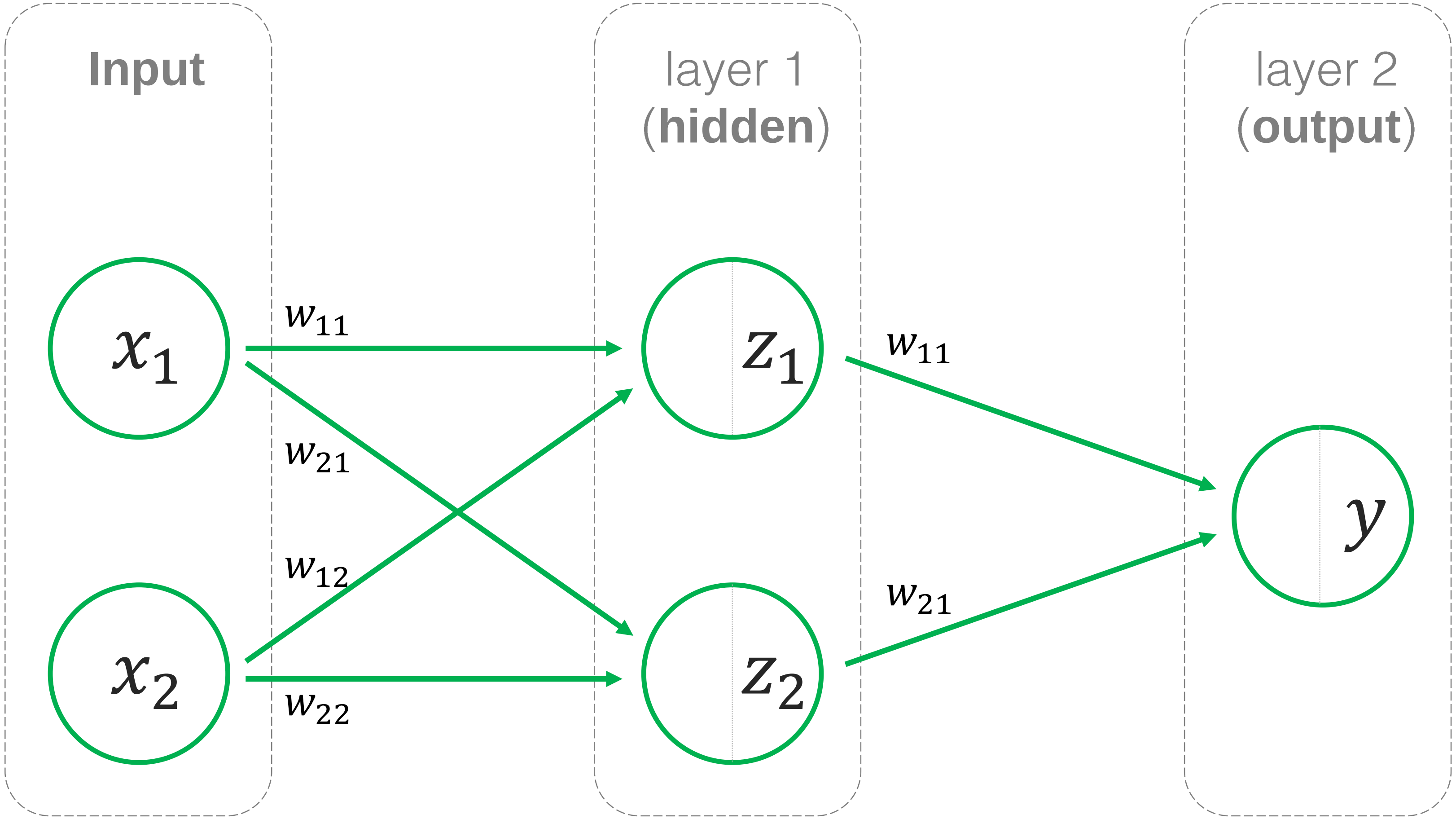
Neural networks can be customized





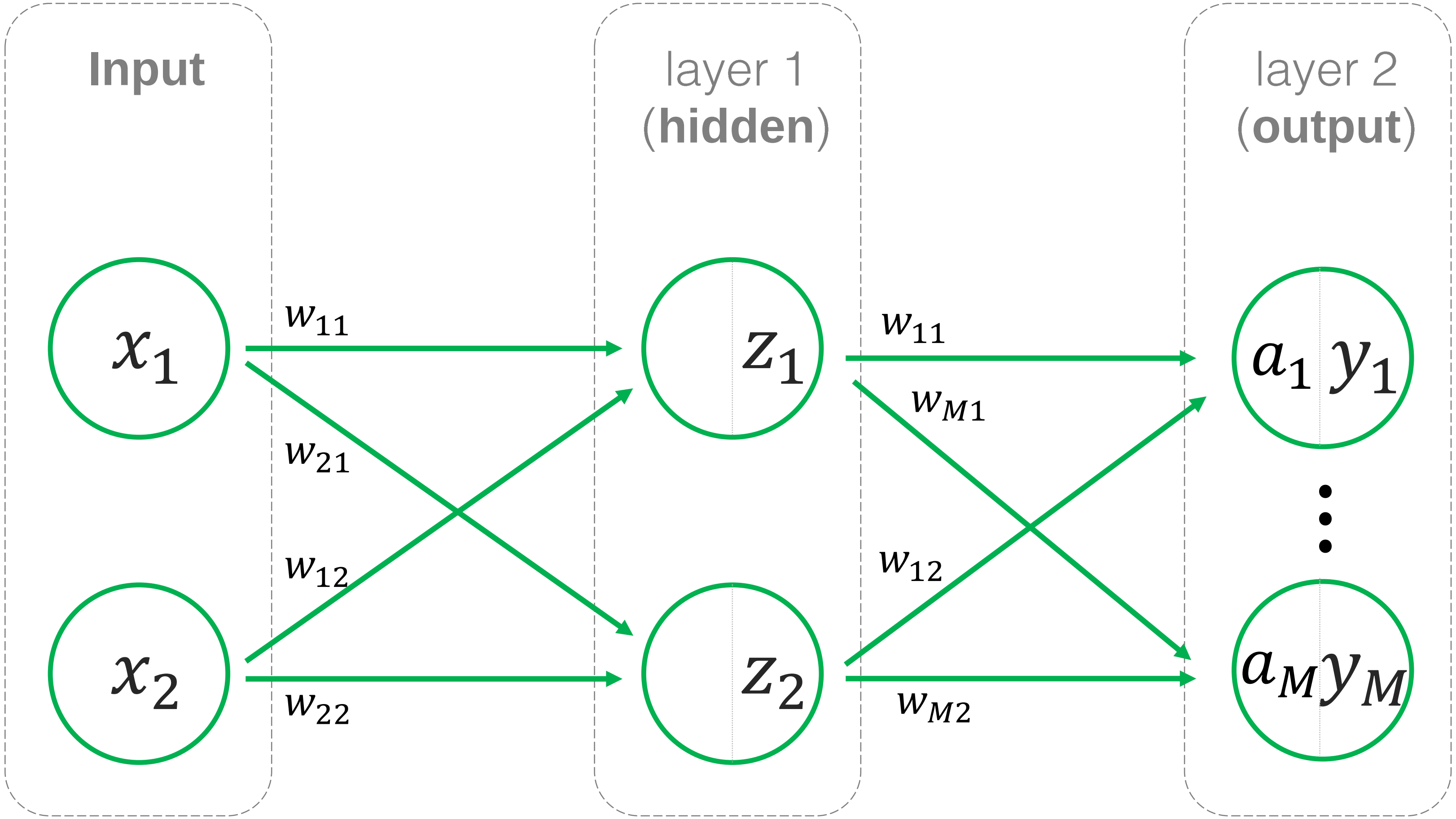
Multilayer neural nets can build up from basic building blocks to more complex structures

From regression to classification



For **binary classification** with a sigmoid activation function, the output is between zero and one, so threshold this value to assign the class

From regression to classification



For **multiclass problems**, we can have multiple outputs and use a softmax function:

$$y_i = g(a_i) = \frac{e^{a_i}}{\sum_{n=1}^{M} e^{a_n}}$$

Choose the largest y value as the predicted class

As with many aspects of neural networks this is on of a number of approaches

Next time…

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(i.e. how do we fit our model to data)

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