STA 601: Fall 2019, Quiz I

September 19, 2019

Community Standard

To uphold the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

I have adhered to the Duke Community Standard in completing this exam.

Name: ______________________________

NetID: ______________________________

Signature: ______________________________

Common distributions

Normal with mean θ and variance σ^2: $p(y|\theta, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{1}{2\sigma^2}(y-\theta)^2)$ for $y \in \mathbb{R}$

Exponential with mean λ and variance λ^2: $p(y|\lambda) = \frac{1}{\lambda} \exp(-\frac{y}{\lambda})$ for $y, \lambda > 0$

Gamma with mean $\frac{\alpha}{\beta}$ and variance $\frac{\alpha}{\beta^2}$: $p(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y)$ for $y, \alpha, \beta > 0$

Inverse Gamma with mean $\frac{\beta}{\alpha-1}$ and variance $\frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$: $p(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{-(\alpha+1)} \exp(-\frac{\beta}{y})$ for $y, \alpha, \beta > 0$

Uniform distribution on $(0, 1)$: $p(y) = 1$ for $y \in [0, 1]$

Poisson distribution with mean θ: $p(y|\theta) = \frac{\exp(-\theta)\theta^y}{y!}$ for y a non-negative integer, $\theta > 0$.

1. (1 point each) State/Define the following (or answer TRUE/FALSE):

   (a) State Bayes' theorem.

   (b) Define finite exchangeability.

   (c) Define the notion of conjugate families.

   (d) TRUE or FALSE: Let $p_1(\theta), p_2(\theta)$ be conjugate priors for parameter θ in sampling model $p(y|\theta)$. The mixture distribution $p(\theta) = \frac{1}{2}p_1(\theta) + \frac{1}{2}p_2(\theta)$ is also conjugate for θ.

   (e) TRUE or FALSE: The predictive distribution of a model does not depend on any unknown quantities.

2. (8 points) Extreme events are frequently described by the double exponential distribution. Consider the parametrization in this case $p(y|\theta) = \frac{1}{2\theta}\exp(-\frac{|y|}{\theta}))$

   (a) Find the MLE for θ.

   (b) Derive the conjugate family of priors for this sampling model.

   (c) Write the posterior mean as a weighted sum of the prior mean and the MLE. Provide an interpretation to the parameters in the prior.

3. (7 points) Let X be a Poisson distribution with mean θ. Your goal is to study the mean without injecting too much prior information.

   (a) Derive Jeffrey's prior for θ and state if it is a proper distribution.

   (b) Derive the posterior and state if it is proper.