

# Python for Finance

## Risk In Finance

ACADEMIC YEAR 2021-2022

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# Why Python ?

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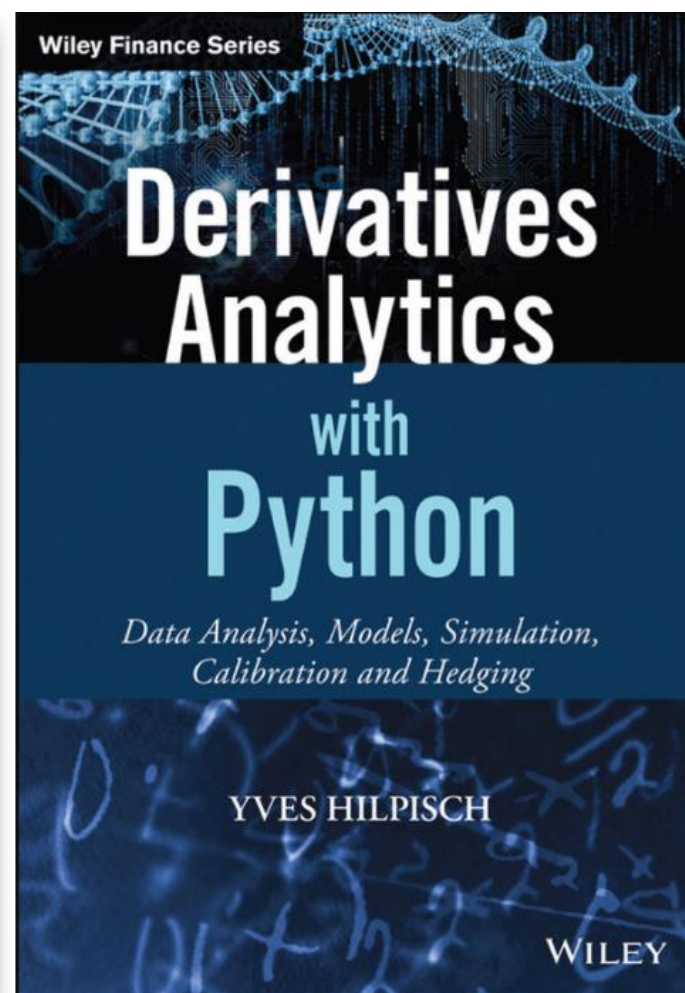
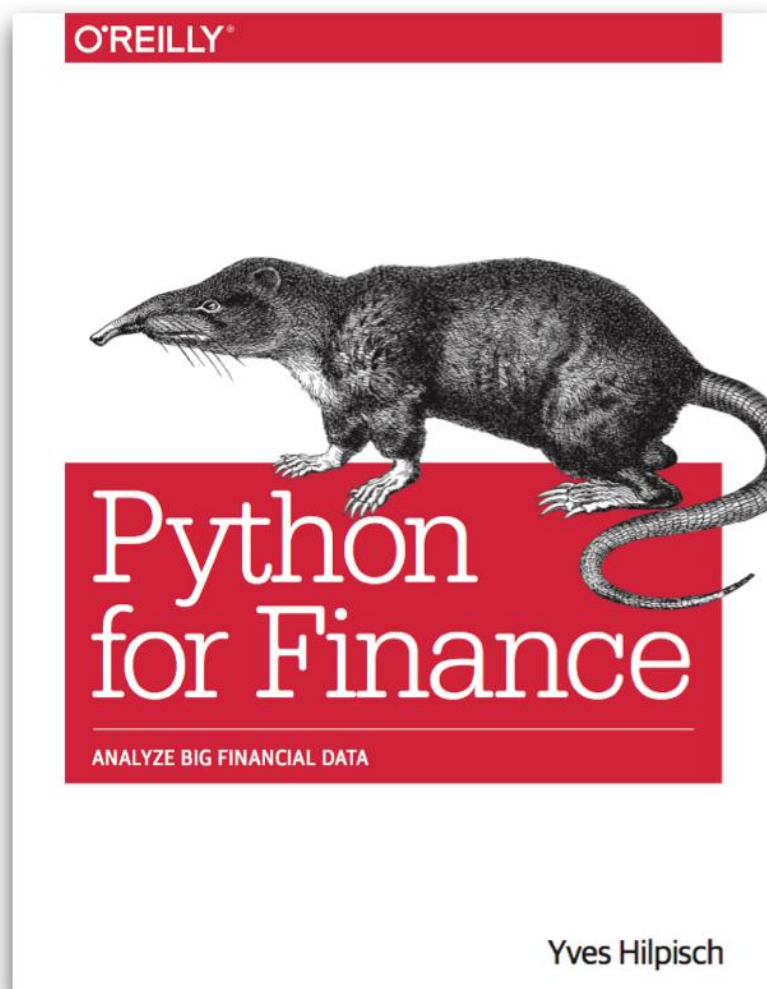
- ❑ Most commonly used programming language today
- ❑ It is easy to learn as it has a simple syntax
- ❑ Python is an open source general purpose language
- ❑ Python has a huge community of users especially in finance world
- ❑ Pure Python is not as fast as C++ or Fortran or even Matlab
- ❑ **But** this has been solved by libraries developed in C++ for Python

# What Libraries ?

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- ❑ **Numpy** = Library of very fast, basic numerical methods
- ❑ **Pandas** = Library for loading, manipulating and analysing data
- ❑ Matplotlib and Seaborn = Libraries for data visualization
- ❑ Scipy = Library of scientific mathematical functions
- ❑ Numba = Fast JIT compiler makes code run as fast as C++
- ❑ Scikit-Learn = Comprehensive machine learning library
- ❑ FinancePy = Derivative valuation in Python
- ❑ You get the best of both worlds
  - ❑ A simple interpreted language
  - ❑ Loads of pre-written functions
  - ❑ High speed execution

## References: Two Books by Yves Hilpisch



❑ See <https://github.com/yhilpisch/>

# Python Installation

# Where to get Python 3

- ❑ We are using Python 3 as Python 2 is no longer supported
- ❑ Continuum is a company that provides a simple install package
- ❑ Get it at <https://www.anaconda.com/products/individual>
- ❑ Then install it – it installs Python + All Important Packages



Individual Edition

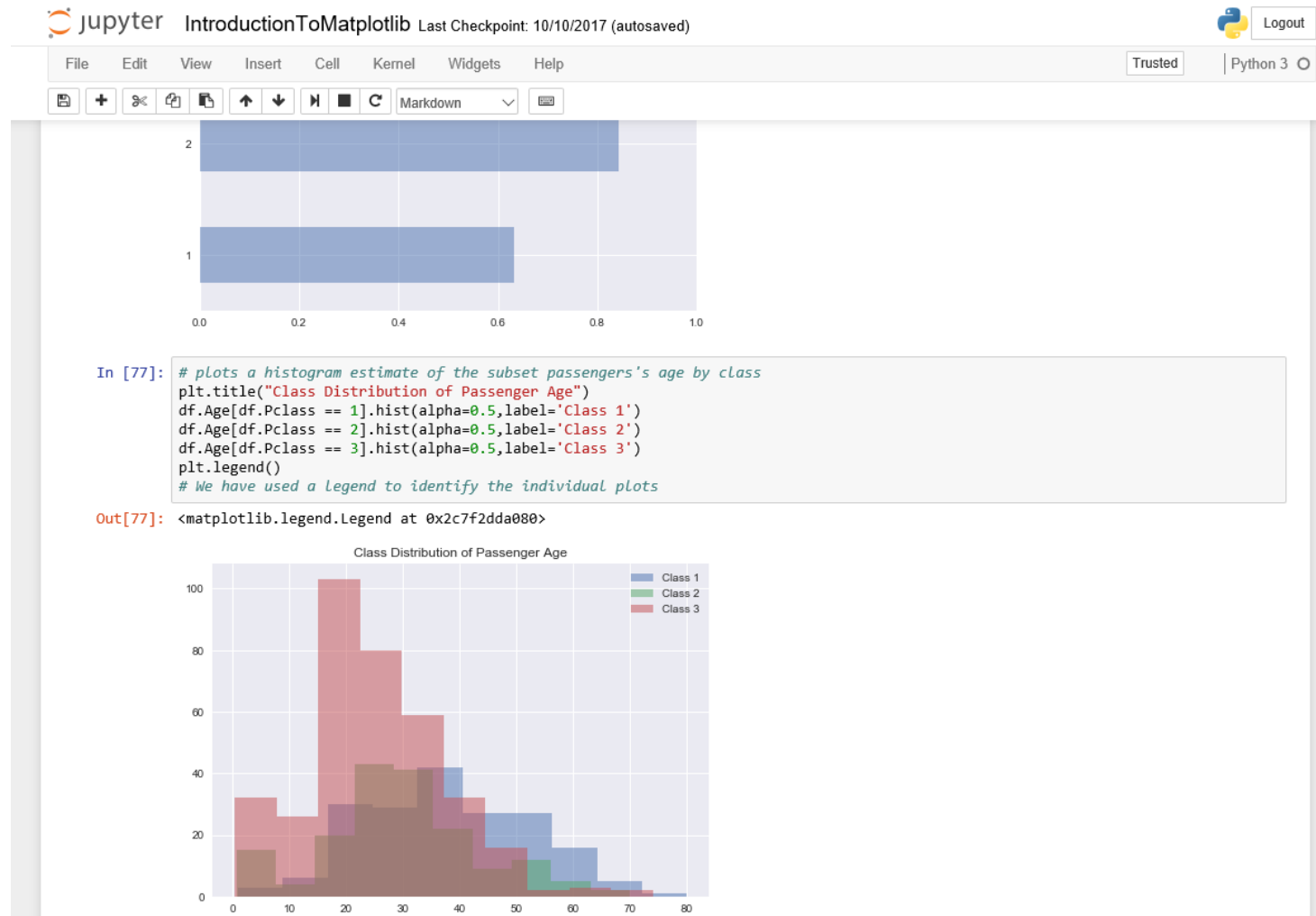
## Your data science toolkit

With over 25 million users worldwide, the open-source Individual Edition (Distribution) is the easiest way to perform Python/R data science and machine learning on a single machine. Developed for solo practitioners, it is the toolkit that equips you to work with thousands of open-source packages and libraries.



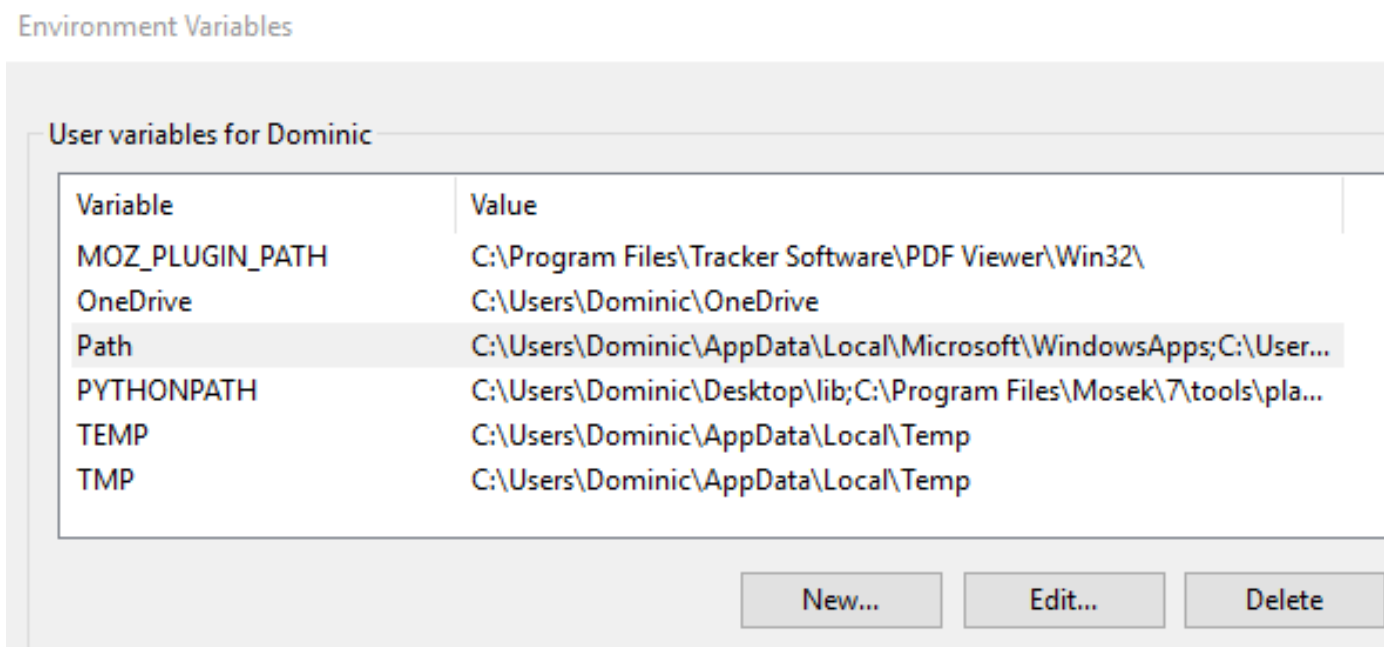
# We will use the Jupyter Notebook

- Very powerful environment for analysis e.g. Machine Learning



# Starting and Using the Jupyter Notebook

- ❑ Open Anaconda cmd and type **“jupyter notebook”**
- ❑ If this does not work, add the anaconda directory to your path
- ❑ To do this in windows, type ‘path’ into Cortana
- ❑ It should prompt you to edit environment files for user
- ❑ Click on this and an Environment variables gui will open





# Starting the Jupyter Notebook

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- ❑ You should edit the variable called 'path'
- ❑ On windows add the following directory path to the path

C:\Users\YOURNAME\Anaconda3\Scripts

- ❑ Close the path setting window and open a new command line tool using 'cmd'
- ❑ Create a folder for your work

# Using the Jupyter Notebook

- ❑ You type in a cell and press SHIFT + RETURN to calculate it
- ❑ Each calculation is numbered in the order of execution
- ❑ [\*] means that it is still calculating
- ❑ Use ESC M to make a cell into a comment
- ❑ # is a header 1, ## is a header 2, ### is a header 3 *Transfer Code model into Markdown*
- ❑ “%matplotlib inline” displays plots in notebook
- ❑ %time allows you to calculate the wall time (does not take into account other processes – like the clock on your wall)
- ❑ Allows you to use Latex formulas using \$ symbol

# Python Coding

# Python Datatypes

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- ❑ Integers

```
>>> z = 5/2 # This assigns 2.5 to z
```

- ❑ Floats

```
>>> x = 3.452
```

- ❑ Strings

```
>>> name = "EDHEC"
```

- ❑ In Python you can do multiple assignments

```
>>> x, y = 10,20
```

- ❑ This is useful as many functions can return more than one value

# Basic Printing in Python

- ❑ Can use the print command – it needs brackets

```
print('Hello!')  
Hello!
```

- ❑ Can combine a set of outputs and spaces are automatically added

```
x = 5.4233  
print('Hello!' , x)  
Hello! 5.4233
```

- ❑ You can add strings and then print them – for numerical values you can use str(x) but no space is added *str() transfer the numerical values into string format*

```
print("Hello" + str(x) + "EDHEC")  
Hello5.4233EDHEC
```

- ❑ Python is first adding the strings and then printing them

# Mathematics in Python

- ❑ Assignment uses = and comparison uses ==
- ❑ Variables are given a type as soon as they are assigned a value
- ❑ Python works out the type itself – you may need to help it !
- ❑ Standard operators + - \* / work as expected
- ❑ Remainder function is %
- ❑ The logical operators are **and**, **or** and **not**
- ❑ Python works out if something is an integer or a float from context – be careful

```
type(15.0)
```

```
float
```

```
type(1)
```

```
int
```

# We need Maths Functions

- ❑ Raw python does not know mathematical functions apart from +, -, \* and / and maybe a few more
- ❑ To have access to more functionality we need to import the Python Math library

*Instead:*

*$x = \text{math.exp}(5.0)$ , which means  $x = e^5$*

- ❑ To do this we can add the following at the top of our code

```
import math
```

- ❑ Now we need to call

```
math.exp(x)
```

- ❑ But this is ugly, so instead I write

```
from math import exp
```

- ❑ Now I can call it without any prefix – but be careful – if you have another library with an exp function there may be confusion

# Fancy Printing in Python

- ❑ Can use fancy printing format to control precision. Suppose

```
x = 3.342392392038982398; string = 'Hello'
```

- ❑ No formatting gives

```
print('x = `,x)
x = 3.3423923920389824
```

- ❑ **Formatting code – 'f' is floating** and 9.5 = 9 chars with 5 decimal places and **you have to drop the comma and use a % symbol**

```
print('x = %7.5f' % x)
x = 3.34239
```

- ❑ Combining strings – formatting code 's' and floats in a tuple

```
print('%s = %6.5f' % (string,x))
Hello = 3.34239
```



# Times and Dates in Python

# Dates in Python

- ❑ Python has its own built-in date and times library

```
import datetime as dt
print('Current date and time: ', dt.datetime.now())
print(dt.datetime.now().strftime('%y/%m/%d %H:%M'))
print('Current year:' , dt.datetime.now().strftime('%Y'))
print('Month of year: ', dt.datetime.now().strftime('%B'))
print('Day of the month : ', dt.datetime.now().strftime('%d'))
```

- ❑ The output is

```
Current date and time: 2017-10-19 03:22:31.417640
17/10/19 03:22
Current year: 2017
Month of year: October
Day of the month : 19
```

# Differencing Dates

- ❑ First, we create two dates

```
import datetime as dt
tradeDate = dt.date(2017,3,13) # 13 March 2017
expiryDate = dt.date(2020,3,20) # 20 March 2020
```

- ❑ We can use simple subtraction to get a **timedelta** object

```
dateDiff = expiryDate - tradeDate
print(dateDiff)
1103 days, 0:00:00
```

- ❑ We want the number of days as an integer

```
# That is a time delta object - to get the number of days
dateDiff.days
1103
```

# **Lists, Tuples and Dictionaries**

# Python Lists

- ❑ If we have a list of data, we can store it in a list! A list can be empty initially

```
list1 = []
```

- ❑ Or we can populate it – data types can be mixed

```
list1 = ['Edhec', 'Business School', 493, 'Nice']
```

- ❑ We access members using square brackets

```
print (list1[0])  
>>>Edhec
```

- ❑ We can add new members using **append**

```
list1.append('France')
```

- ❑ The command delete can be used to delete an entry

- ❑ Using colons [start:end] we can access sublists list1[1:3]

# Python Matrices

- ❑ We can also define a matrix using Python lists
- ❑ A matrix is simply a list of lists

```
m = [[1,4,5],[3,2,7]]
```

- ❑ We can access the elements as

```
m[0][0]  
1
```

```
m[1][2]  
7
```

前面是行，后面是列

- ❑ Note that the first element has indices `m[0][0]`
- ❑ **However, we do not use Python lists for any computationally heavy matrix calculations**
- ❑ We use **Numpy arrays and matrices** - these are 10-30 times faster to use – we will introduce Numpy later

# Python Tuples

- ❑ A Tuple is like a list, but it is an immutable (unchanging) sequence
- ❑ It is defined using round brackets

```
tup1 = ('Edhec', 'Business School', 493, 'Nice')
```

- ❑ As with lists we access members using square brackets
- ❑ It is faster to iterate a tuple than a list
- ❑ The data in a tuple is write-protected

# Python Dictionary

- ❑ A Dictionary is a list of key and value pairs
- ❑ A key is what you use to look up the value
- ❑ Keys and values are separated by a colon

```
dict = {'Name' : 'Pierre', 'Age': 23, 'Nationality' : 'French'}
```

- ❑ We access the members as follows

*Type 'Tab' to get the list of function key words*

```
dict['Name']
```

**Pierre**

```
dict['Age']
```

**23**

- ❑ Elements can be amended directly using their key

```
dict['Age'] = 24
```



# List Comprehensions

# List Comprehensions using Text

- ❑ Powerful tool in Python to do complex operations in a single line

```
list1 = []  
for char in 'EDHEC':  
    list1.append(char)  
  
print(list1)  
['E', 'D', 'H', 'E', 'C']
```

- ❑ The list comprehension looks like this

```
list2 = [ char for char in 'EDHEC' ]  
print(list2)  
['E', 'D', 'H', 'E', 'C']
```

# List Comprehensions in Maths

- ❑ Powerful tool in Python to do complex operations in a single line

```
list1 = []  
for x in range(0,11):  
    list1.append(x**2)  
  
print(list1)  
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

- ❑ The list comprehension looks like this

```
list2 = [ x**2 for x in range(0,11)]  
print(list2)  
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

- ❑ It seems sort of backwards

# **Program Control Flow in Python**

# Loops using for

- We can loop over any list using the **for command**

```
words = ['cat', 'monkey', 'hippopotamus']  
for w in words:  
    print(w, len(w))
```

```
cat 3  
monkey 6  
hippopotamus 12
```

- If we need to loop over a series then we use range which returns series from the first input to last number exclusive

```
for i in range(0,4):  
    print(i,i**2)
```

```
0 0  
1 1  
2 4  
3 9
```

- If you only give range one number, range will always start at zero

# Control Flow using if, elif and else

- ❑ Python requires indentation instead of brackets to control program flow and a colon after each condition

```
x=5

if x < 0:
    print("X is negative")
elif x < 10:
    print("X is positive but less than 10")
else:
    print("X is positive and greater than 10")
```

- ❑ Indentation must be 4 spaces or a tab or the program will not run
- ❑ You will quickly get use to this style format as you use Python

# Control Flow using break

- ❑ Sometimes you want to jump out of an inner loop
- ❑ You can use the break statement to do this
- ❑ The code is on the left and the output on the right

```
for n in range(2, 10):  
    prime = True  
    for x in range(2, n):  
        if n % x == 0:  
            print(n, 'equals', x, '*', n/x)  
            prime = False  
            break  
  
    if prime == True:  
        print(n, 'is a prime number')
```

```
2 is a prime number  
3 is a prime number  
4 equals 2 * 2.0  
5 is a prime number  
6 equals 2 * 3.0  
7 is a prime number  
8 equals 2 * 4.0  
9 equals 3 * 3.0
```

# Functions in Python



# Functional Programming

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- ❑ In Python we can write functions
- ❑ Split your code into logical blocks that can be reused
- ❑ Python functions can
  - ❑ Manage files
  - ❑ Draw graphs
  - ❑ Calculate and Return values
  - ❑ Lots more ..
- ❑ We only need to give a name and list of arguments (inputs)
- ❑ No need to specify datatypes

# Functions: Using def

- ❑ We use the **def** keyword to define the function
- ❑ Don't give the function the same name as a built-in function

```
from math import exp
```

```
def sigmoid(x):
```

```
    """ This function returns the value of the sigmoid function """
```

```
    e = exp(-x)
```

```
    s = 1.0/(1.0+e)
```

```
    return s
```

- ❑ Type help(sigmoid) and you will see the docstring

```
help(sigmoid)
```

```
Help on function sigmoid in module __main__: sigmoid(x)
```

```
    This function returns the value of the sigmoid function
```

# Functions: A function can return multiple arguments

- ❑ Suppose we want to return the value of the exponential

```
def sigmoid2(x):  
    e = exp(-x); s = 1.0/(1.0+e)  
    return s, e
```

- ❑ We then call it as follows to get back a tuple (the return type)

```
ret = sigmoid2(1.0)  
print(ret)  
(0.7310585786300049, 0.36787944117144233)
```

- ❑ Or to get the individual values directly we use the syntax

```
x, y = sigmoid2(1.0)  
print(x,y)  
0.7310585786300049, 0.36787944117144233
```

# Function Default Arguments

- ❑ We use default arguments to set some which almost never change which saves the user from having to input a value

```
import datetime as dt
def yearFraction(d1,d2,daysInYear = 365.252):
    dateDiff = d2 - d1
    yearFraction = dateDiff.days / daysInYear
    return yearFraction
```

- ❑ Here is an example

```
startDt = dt.datetime(2011,1,1)
endDt = dt.datetime(2011,6,1)
yearFraction(startDt, endDt)
0.41341320512960916
```

# Function Keyword Arguments

- ❑ We can call the function using keywords to specify which argument is which

```
startDt = dt.datetime(2011,1,1)
endDt = dt.datetime(2011,6,1)
yearFraction(startDt, endDt)
0.41341320512960916
yearFraction(d1=startDt, d2=endDt)
0.41341320512960916
yearFraction(d2=startDt, d1=endDt)
-0.41341320512960916
```

- ❑ This can be very useful when there are lots of arguments and many of them are default arguments

# Lambda Functions in Python

- ❑ Sometimes you only call a function from one place
- ❑ Maybe it needs to know the value of some local variables
- ❑ A lambda function can do this better than a function

```
def f(x): return x*x
```



```
f = lambda x: x*x
```

- ❑ Think of lambda as the word function and x is the argument
- ❑ After the : we have the function itself
- ❑ Some simple examples (lambda functions can get complex!)

```
f = lambda x: x**2 + 2*x + 5
```

```
f = lambda x, y, z: x+y+z
```

# List Comprehensions

# List Comprehensions using Text

- ❑ Powerful tool in Python to do complex operations in a single line
- ❑ You will see lots of Python people using it
- ❑ Consider a standard Python loop

```
list1 = []  
for char in 'EDHEC':  
    list1.append(char)  
print(list1)  
['E', 'D', 'H', 'E', 'C']
```

- ❑ The list comprehension that does the same looks like this

```
list2 = [ char for char in 'EDHEC' ]  
print(list2)  
['E', 'D', 'H', 'E', 'C']
```



# List Comprehensions in Maths

- ❑ Powerful tool in Python to do complex operations in a single line

```
list1 = []  
for x in range(0,11):  
    list1.append(x**2)  
  
print(list1)  
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

- ❑ The list comprehension looks like this

```
list2 = [ x**2 for x in range(0,11)]  
print(list2)  
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
```

- ❑ It seems sort of backwards as the function is before the loop

# Introduction to Python Notebook 1

Add your notes here

# **Python String Manipulation**

# Python String Manipulation

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- ❑ Here are some useful commands for manipulating string

**s = "EDHEC is a Business School"**

- ❑ len(s) returns the length of the string which is 26
- ❑ s[4] returns 'C'
- ❑ s[10:15] returns 'Busi'
- ❑ s.count('s') returns 4
- ❑ s.find('B') returns 11
- ❑ s.find('X') returns -1
- ❑ s.replace('a','the') returns 'EDHEC is the Business School'
- ❑ s.lower() returns 'edhec is a business school'
- ❑ s.upper() returns 'EDHEC IS A BUSINESS SCHOOL'

# Python File I/O

# Getting a List of Files

- ❑ We often need to find the file before we open it (or many)

```
mypath = "./data/"  
import os  
files = os.listdir(mypath)  
print(files)  
['.ipynb_checkpoints', 'bondPrice.py', 'giltBondPrices.txt',  
'optionPortfolio.csv', 'stocks', 'timeSeriesData.pkl']
```

- ❑ To use a **pattern matching filter**, the **glob** library can be used

```
import glob  
files = glob.glob("./data/*.txt")  
print(files)  
['./data\\giltBondPrices.txt']
```

# Python File I/O: Reading from a File

- ❑ We open a file and prepare to read from it using `open()` with 'r'
- ❑ This creates a **file object** that we use to extract the file contents
- ❑ We read the whole file into a list object using `readlines()`
- ❑ It detects EOL characters in the file and uses these

```
filename = "./data/giltBondPrices.txt"
f = open(filename, 'r')
lines = f.readlines()
f.close()
```

- ❑ Once you have used the file, close it to release any resources
- ❑ The `with` command can do this for you automatically

```
with open(filename, 'r') as f:
    lines = f.readlines()
```

# Contents of File

- ❑ When we read a line from a file, it is usually a string of data fields
- ❑ We can simply print the contents

```
lines
```

```
['epic\tdescription\tcoupon\tmaturity\tbid\task\tchange\tincome yield\tgross redemption yield\n',
 'TR13\tUk Gilt Treasury Stk\t4.5\t07-Mar-13\t101.92\t102.07\t-0.01\t4.41\t0.22\n',
 'T813\tUk Gilt Treasury Stk\t8\t27-Sep-13\t107.86\t107.98\t-0.03\t7.41\t0.23\n',
 'TR14\tUk Gilt Treasury Stk\t2.25\t07-Mar-14\t102.9\t103.05\t0.01\t2.18\t0.22\n',
 'T514\tUk Gilt Treasury Stk\t5\t07-Sep-14\t109.28\t109.43\t0.02\t4.57\t0.23\n',
 'TR15\tUk Gilt Treasury Stk\t2.75\t22-Jan-15\t105.57\t105.68\t0.05\t2.6\t0.33\n',
 'T4T\tUk Gilt Treasury Stk\t4.75\t07-Sep-15\t112.92\t113.04\t0.04\t4.2\t0.35\n',
 'TY8\tUk Gilt Treasury Stk\t8\t07-Dec-15\t124.39\t124.55\t0.04\t6.43\t0.34\n',
 'T516\tUk Gilt Treasury Stk\t2\t22-Jan-16\t104.92\t105.04\t0.07\t1.91\t0.49\n',
 'T16\tUk Gilt Treasury Stk\t4\t07-Sep-16\t113.44\t113.55\t0.08\t3.52\t0.56\n']
```

- ❑ The first column is some bond ID
- ❑ Then we have string and numeric values
- ❑ Looking at the file we see that it is tab separated – “\t”
- ❑ There is an end of line character – “\n”



# Parsing Data In a File

- ❑ We can use replace to remove the end of line character
- ❑ We can split the data into columns using the split command

```
dataTable = []  
for line in lines:  
    line2 = line.replace("\n", "")  
    dataFields = line2.split("\t")  
    dataTable.append(dataFields)
```

- ❑ dataFields becomes a list of strings
- ❑ dataTable becomes a list of lists
- ❑ We may need to do more work e.g. converting date strings
- ❑ In practice we will use Pandas for this sort of work

# Python File I/O: Writing to a File

- ❑ You **open a new file using open() with 'w'**
- ❑ Or you can **open an existing one with 'a' for append**
- ❑ We can write to the file using the write(string) command
- ❑ At the end you shouldn't forget to close the file
- ❑ Or better, use the command **with**

```
with open(filename,'w') as f:  
    f.write("Hello World")  
    .....
```

# Introduction to File IO and Strings Notebook 2

Add your notes here

# Using the Libraries

# The Python Stack

- ❑ The power of Python is partly due to its simple language
- ❑ It is **MOSTLY** due to its set of excellent libraries listed below

Library	Description
<b>Numpy</b>	Fast mathematics library with vectorization
<b>Scipy</b>	Math functions, statistics and optimizers
<b>Pandas</b>	Advanced manipulation of data tables
<b>Matplotlib</b>	Advanced plotting and visualisation
<b>Statsmodels</b>	Time series analysis / econometrics
<b>Numba</b>	High performance calculations

- ❑ These libraries will be our initial focus and then I will show how to combine them to do useful things

# Importing Modules

---

- ❑ Most of the power of Python comes from using its very extensive libraries – or in Python we call them **modules**
- ❑ These have been written by various individuals and groups and made available for free
- ❑ All of the main ones we need came with the Anaconda package
- ❑ To access a library from your code you need to import it
- ❑ You should import it with a name to prevent names clashing

**import numpy as np**

- ❑ The np means that to access any Numpy function called **function** we call it with **np.function(...)**

# Using Pip

- ❑ **Pip** is the name of the program that you use to **install new or update existing libraries in your Python package**
- ❑ Use it from the Mac or Windows **command line** (not in Python)
- ❑ The path to pip should be in your user-defined path
- ❑ You can install a project called SomeProject as follows

```
C:\Users>pip install 'SomeProject'
```

- ❑ If you want a specific version then use

```
C:\Users> pip install 'SomeProject==1.4'
```

- ❑ To upgrade a specific project

```
C:\Users> pip install --upgrade SomeProject
```

# Deprecations

- ❑ As you work with Python from time to time you will receive warnings such as

DeprecationWarning: Implicitly casting between incompatible kinds...

- ❑ The purpose of Deprecation warnings is to tell you that the form of the library you are using has changed and that this function you called is to be removed
- ❑ What happens is that developers fix and improve libraries and may change the way a function is named or called
- ❑ They leave the old code so your library does not break
- ❑ But they warn you to change your code as the old way will no longer be supported and may not work in future



**Libraries: NumPy and Numba**

# Numpy is the Core Python Scientific Library

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- ❑ NumPy = Numerical Python
- ❑ It's a dedicated library for numerical work
- ❑ While we can use Python's built-in lists, they are not efficient
- ❑ NumPy uses less memory as it uses numpy arrays
- ❑ These assume that the data members are all the same
- ❑ For this reason, the calculations are faster
- ❑ They have been optimised
- ❑ We always import NumPy as follows

```
import numpy as np
```

- ❑ Calling it “np” is a widely-used convention

# NumPy Functions

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- ❑ NumPy includes the following features
  - ❑ A fast and efficient **multidimensional array type ndarray**
  - ❑ This includes powerful shape manipulation functions
  - ❑ Indexing and slicing of multidimensional arrays
  - ❑ Linear algebra – transpose, dot product, eigenvalues
  - ❑ A comprehensive library of mathematical functions
  - ❑ Vectorized fast calculations
  - ❑ Deep and shallow copies for memory management
- ❑ I am not going to do every function here, just the basics
- ❑ I will introduce extra functions later as we encounter them

# The Foundation of NumPy is its ndarray data type

- ❑ **ndarray** is a homogeneous multidimensional array data type
- ❑ It has a number of methods which we can call including
  - ❑ `ndarray.ndim` gives the number of dimensions
  - ❑ `ndarray.shape` gives shape tuple of the size in each dimension
- ❑ There are several ways to create an **ndarray** which include:
  - ❑ It can be created by passing in a Python list (or list of lists)
  - ❑ It can be initialised using `np.zeros(shape tuple)`
  - ❑ Or it can be initialised using `np.ones(shape tuple)`

# Creating a NumPy Array

- ❑ Creating a NumPy array by converting a Python list

```
a = np.array([3,5,4,2])
```

- ❑ We can use zeros to create an 10-element array of **zeros**

```
b = np.zeros( 0) Here is 10 not 0
```

- ❑ Use the **shape tuple** (d1,d2,...) to create multi-dimensional arrays
- ❑ To create a 5x9x2 = 3-dimensional array of zeros

```
b = np.zeros( (5,9,2) )
```

- ❑ We can create a 2x5 = 2-dimensional array of **ones**

```
C = np.ones( (2,5) )
```

# Indexing Arrays

```
v = np.array([12,22,13,44,22,43,35,36])
```

Create Array

```
v[0]
```

Starts at zero

```
12
```

```
v[2:5]
```

2:5 has elements 2,3,4

```
array([13, 44, 22])
```

```
v[5:]
```

5: gives all elements after 5

```
array([43, 35, 36])
```

```
v[5:-1]
```

-1 is from the end

```
array([43, 35])
```

Broadcasting

```
v[1:4] = 100
```

Array has changed

```
v
```

```
array([ 12, 100, 100, 100, 22, 43, 35, 36])
```

## Useful Functions: linspace and arange

- ❑ Want to get a list of values evenly spaced between two values
- ❑ Values between 0 and 10 with 5 evenly spaced values returned

```
np.linspace(0,10,5)  
array([ 0. ,  2.5,  5. ,  7.5, 10. ])
```

- ❑ Note you don't pass 4 (even though it may seem more intuitive)
- ❑ If you want a list of integers, then use arange

```
l = np.arange(10)  
print(l)  
[0 1 2 3 4 5 6 7 8 9]
```

# Random Numbers in NumPy

- ❑ Numpy has a lot of random number distributions built-in
- ❑ You need to look under `numpy.random`
- ❑ Everything you need is there
- ❑ For example, a random int between 1 and 100 is found by

```
np.random.randint(1, 101)  
53
```

- ❑ For reproducibility set the seed

```
np.random.seed(1828)
```

- ❑ This will give you the same sequence of random numbers each time you run your code



# Uniform and Gaussian Random Numbers in NumPy

## □ Uniform random numbers

```
x = np.random.rand(10)
print(x)
array([0.26406512, 0.4857891 , 0.8550659 , 0.54195836, 0.48441921,
       0.54879056, 0.40970468, 0.98964574, 0.05902159, 0.83340854])
```

## □ Gaussian random numbers now returned in a 2D array

```
# Gaussian random numbers
r=np.random.normal(size=(5,5))
print(r)
[[-1.33171433 -0.74678445  1.13349986 -1.52567032  0.09068109]
 [ 0.77295278  1.15462762  0.14064668 -0.20941348 -0.18446004]
 [-0.25627145  0.59579659 -1.80004184 -0.96032238  0.29522222]
 [ 0.43145469  0.84674349  0.10065509 -0.69472277  0.03555611]
 [ 0.05831341  0.64811795 -0.05200189  1.12076513 -0.71721171]]
```

# NumPy Vectorisation

- ❑ NumPy's functions allow vectorisation- Python's functions do not
- ❑ You pass in a vector of values; you get back a vector of values
- ❑ Here is an example using the exponential function

```
x = np.random.rand(10)
```

```
y = np.exp(x)
```

```
print(y)
```

```
[1.2179072  2.66579699 2.50019139 1.63080461 2.20330939 1.16205315  
2.24043983 1.47829626 1.30233741 1.15762022]
```

- ❑ This works for all NumPy's mathematical functions
- ❑ Not only does this save us from writing a loop, but it is also much faster too than doing this in Python
- ❑ Let us see ...

# Speed Test: Compare Python vs NumPy

- ❑ First we generate 10,000 random numbers

```
from math import exp
numElements = 10000
rarray=np.random.rand(numElements)
```

- ❑ Function f1 stores 10,000 values of the exponential in array x

```
def f1(rv):
    x=[]
    for r in rarray: x.append(exp(r))
    return x
```

- ❑ Timing

```
x=[]
%timeit x=f1(rarray)
100 loops, best of 3: 1.99ms per loop
```

# NumPy Vectorized Calculations are fast

- ❑ First we generate 10,000 random numbers
- ❑ We then store 10,000 values of the exponential in array x

```
def f2(rv):  
    x=np.exp(rv)  
    return x
```

- ❑ We also store 10,000 of the exponential in ndarray x
- ❑ The calculation is about 50 times faster

```
%timeit x = f2(rarray)  
10000 loops, best of 3: 42 µs per loop
```

- ❑ The looping is done in C code inside Numpy – not in Python

# Numba provides high-performance but does not win

- ❑ It uses a just-in-time (JIT) compiler to convert code to native machine code using – Numba decides what to optimise

```
from numba import njit
@njit
def f3(rv):
    x = []
    for r in rv:
        x.append(exp(r))
    return x
```

**Basic Python 1990.00000 ms**

**Numpy Python 42.00000 ms**

**Numba Python 254.00000 ms**

- ❑ Numba works best when NumPy vectorization is not possible

# Introduction to Numpy and Numba Notebook 3

Add your notes here

**Libraries: Matplotlib**

# Matplotlib

---

- ❑ Matplotlib is a Python 2D and 3D plotting library which produces publication quality figures in a variety of hardcopy formats
- ❑ It is the most widely used 2D plotting package in Python
- ❑ Matplotlib has a subsection called **pyplot**
- ❑ This provides a plotting interface that is similar to Matlab
- ❑ We load it as follows

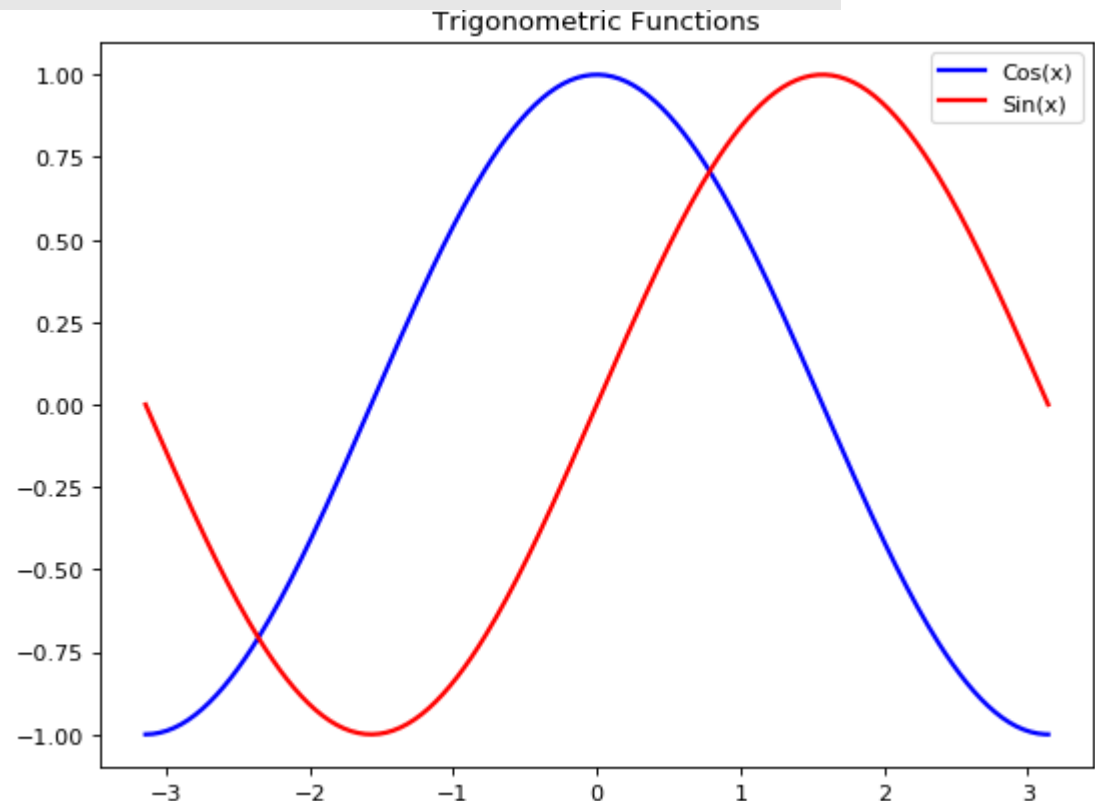
```
import matplotlib.pyplot as plt
```



# Simple Plots

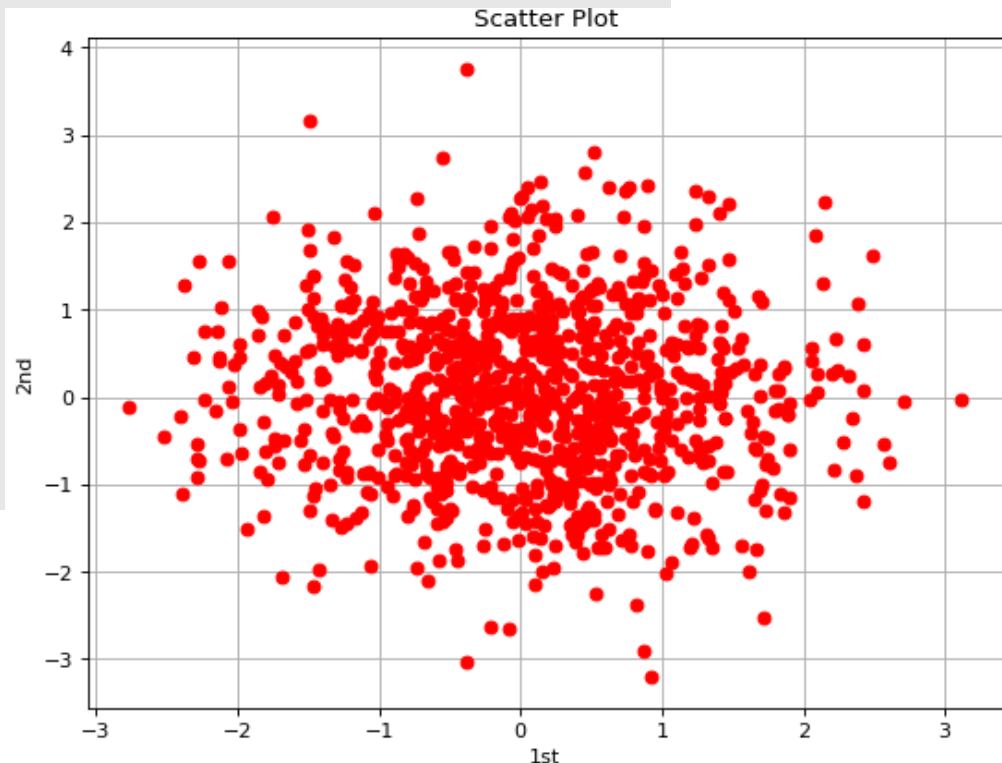
```
x = np.linspace(-np.pi,np.pi,256,endpoint=True)
cx, sx = np.cos(x), np.sin(x)
plt.figure(figsize=(8,6), dpi=80)
plt.plot(x, cx, color='blue', label="Cos(x)", linewidth=2)
plt.plot(x, sx, color='red', label="Sin(x)", linewidth=2)
plt.title("Trigonometric Functions")
plt.legend()
```

- ❑ Make sure you understand each line above
- ❑ You should be able to do this without looking at the documentation



# Scatterplots

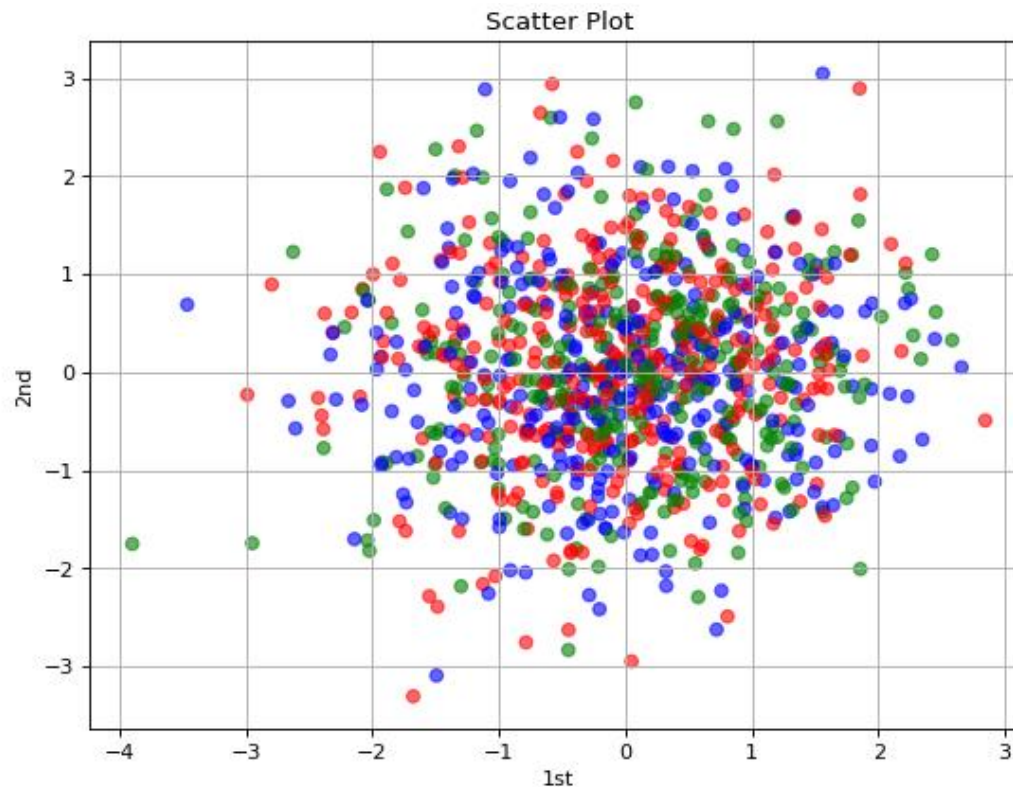
```
y = np.random.standard_normal((1000, 2))  
plt.figure(figsize=(8, 6), dpi=80)  
plt.plot(y[:, 0], y[:, 1], 'ro')  
plt.grid(True)  
plt.xlabel('1st')  
plt.ylabel('2nd')  
plt.title('Scatter Plot')
```



- ❑ 'ro' means red dot
- ❑ **Colours** can include b=blue, g=green, y=yellow, w=white, m=magenta, k=black
- ❑ **Markers** can take many values including .=dot, o=circle, \*=star, -=lines, v=triangle down, s=square, h=hexagon, +=plus, ....

# Scatterplots with coloured points – use a ColorMap

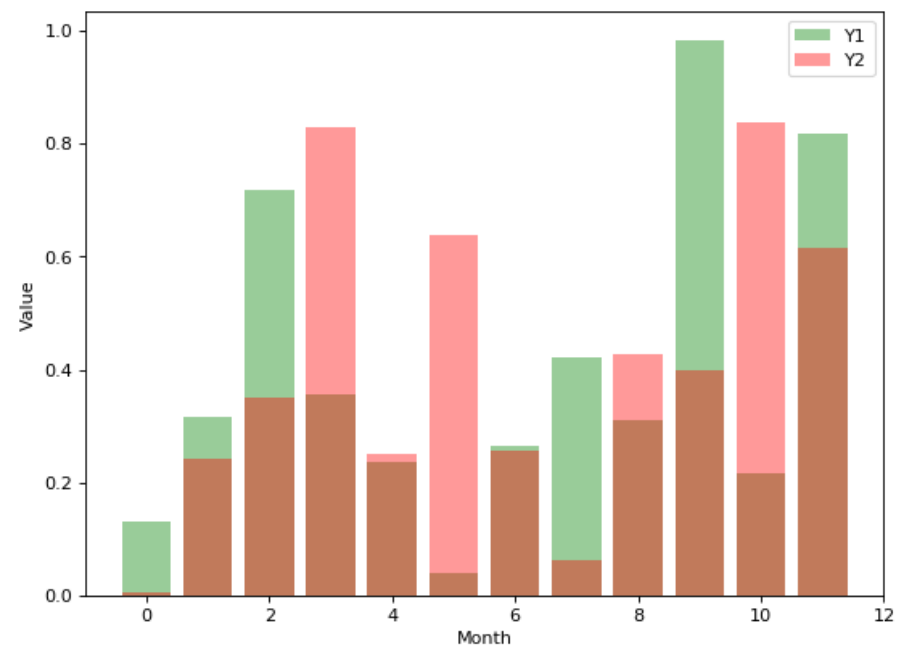
```
from matplotlib import colors
colours = ['red','green','blue']
c = np.random.randint(0, len(colours), len(y))
cmap = colors.ListedColormap(colours)
plt.scatter(y[:, 0], y[:, 1], c=c, marker='o', cmap=cmap, alpha=0.6)
```



# Bar Charts

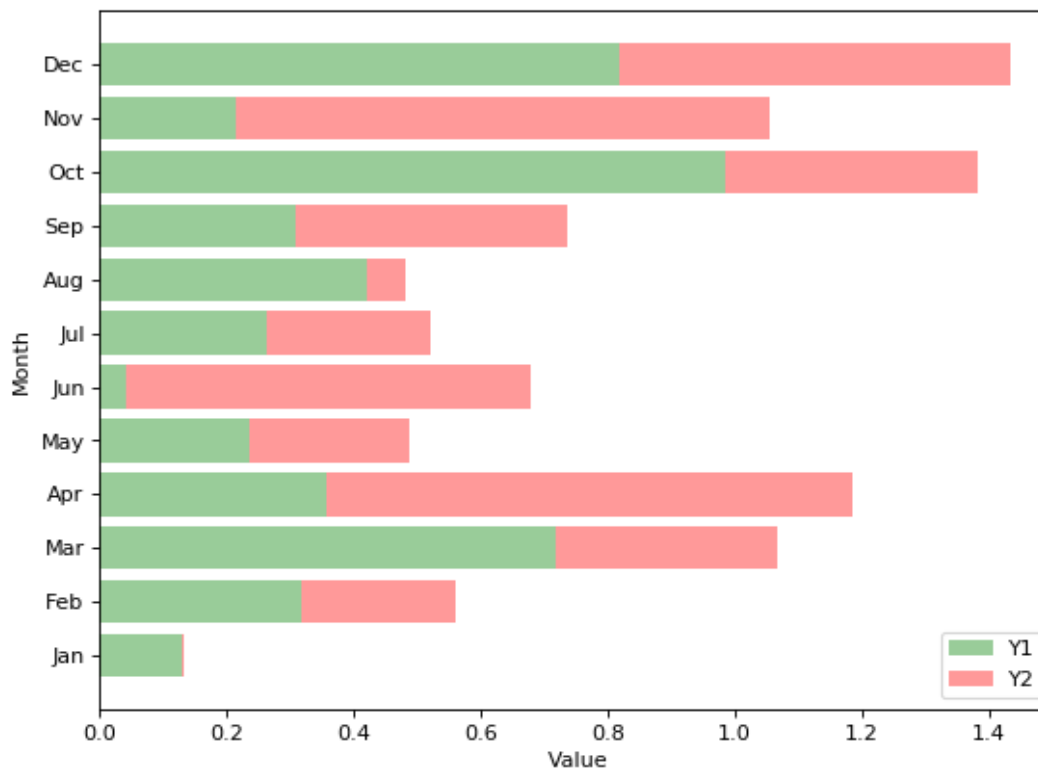
```
n = 12
x = np.arange(n)
y1 = np.random.uniform(0.0,1.0,n)
y2 = np.random.uniform(0.0,1.0,n)
plt.figure(figsize=(8,6), dpi=80)
plt.bar(x, y1, facecolor='green', alpha=0.4, label="Y1")
plt.bar(x, y2, facecolor='red', alpha=0.4, label = "Y2")
plt.xlabel("Month")
plt.ylabel("Value")
plt.legend()
```

- Alpha is the transparency so that we can see both columns



# Horizontal Stacked Bar Charts and Ticks

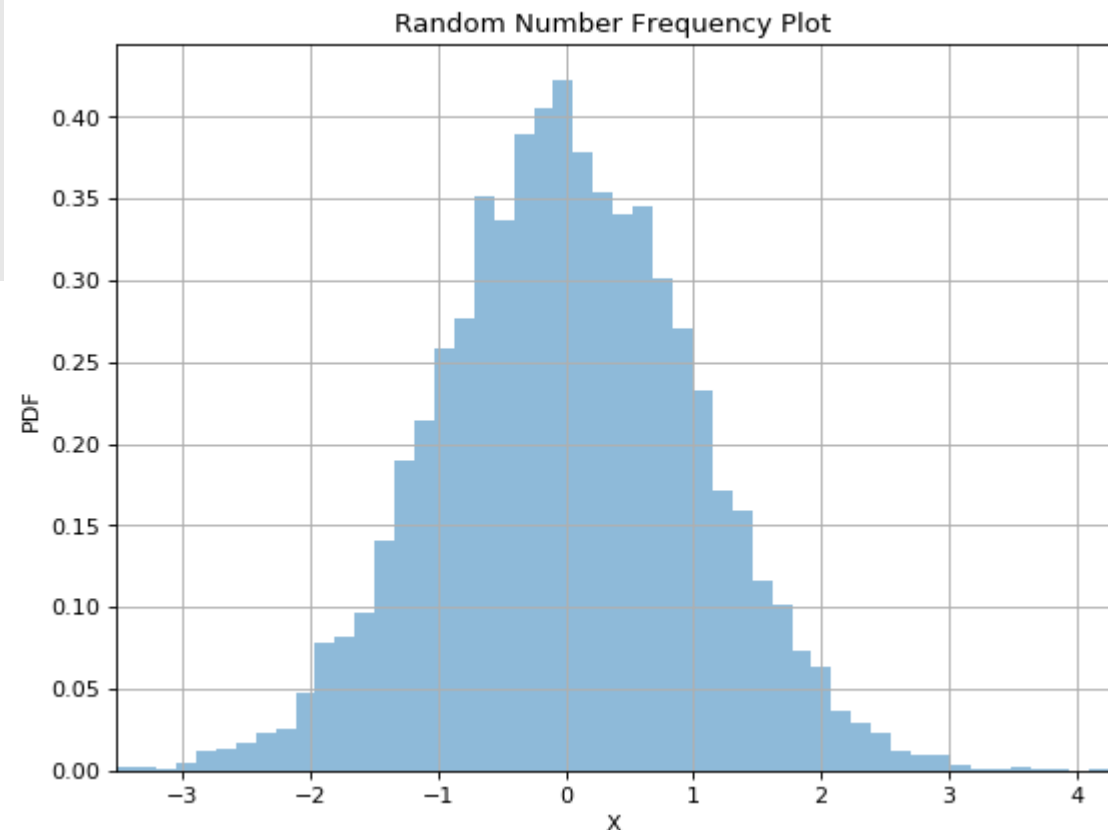
```
plt.barh(x, y1, facecolor='green', alpha=0.4, label="Y1")  
plt.barh(x, y2, left = y1, facecolor='red', alpha=0.4, label = "Y2")  
plt.xlabel("Value")  
plt.ylabel("Month")  
plt.yticks(x, ('Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sep', 'Oct', 'Nov', 'Dec'))  
plt.legend()
```



# Histograms

```
n = np.random.randn(10000)
plt.figure(figsize=(8,6), dpi=80)
plt.hist(n, bins=20, alpha=0.5, normed=1)
plt.title("Random Number Frequency Plot")
plt.xlim((min(n), max(n)))
plt.xlabel("X")
plt.ylabel("PDF")
plt.grid(True)
```

- ❑ **normed=1** converts frequencies to probabilities



# Use Styles to make good-looking figures easily

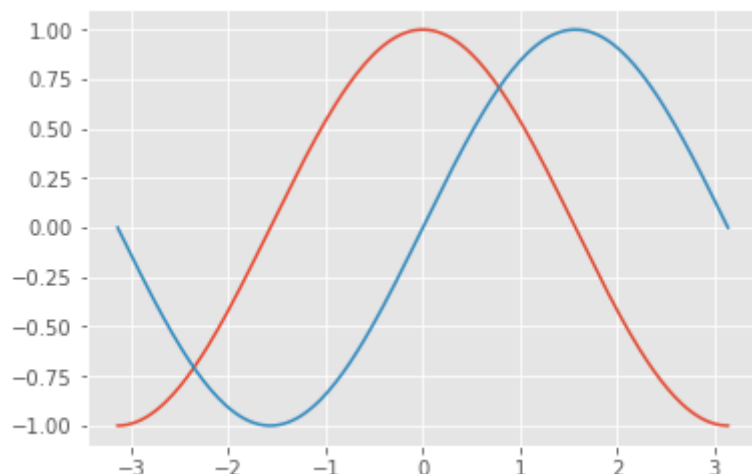
- ❑ The whole look of the graphs can be changed using the style

```
plt.style.use('ggplot')
```

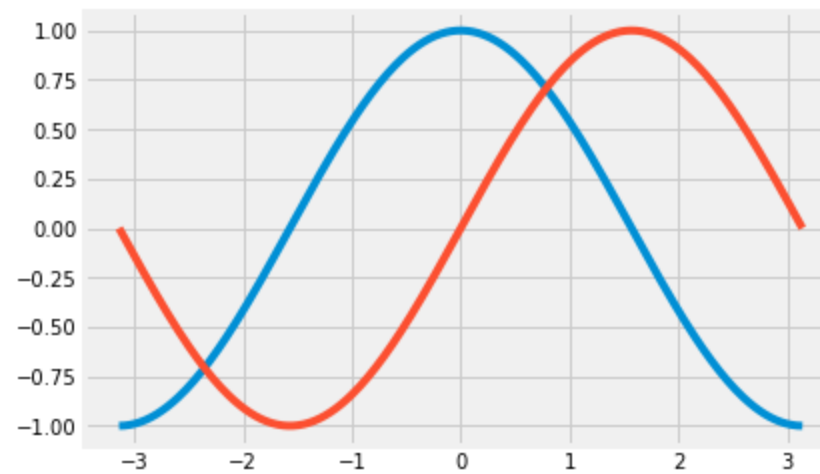
- ❑ The style 'ggplot' is similar to the style used in ggplot in R
- ❑ This is widely liked
- ❑ There are lots of styles – look at documentation of matplotlib
- ❑ To recover the default settings you need to use

```
import matplotlib as mpl  
mpl.rcParams.update(mpl.rcParamsDefault)
```

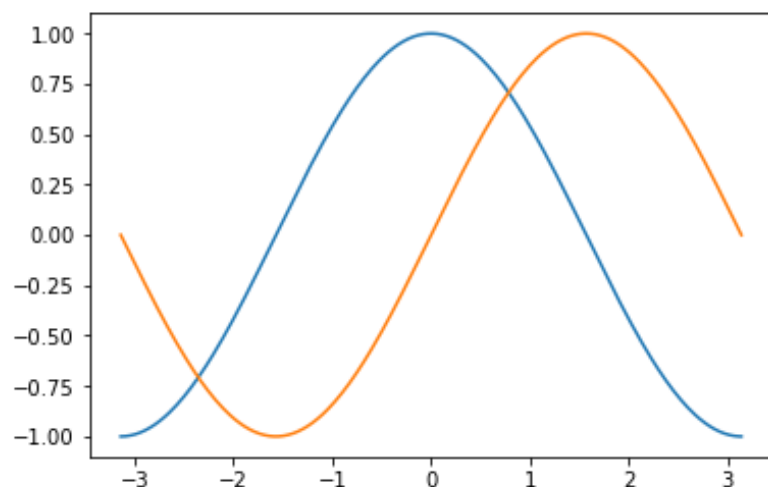
# Different Styles for a Graph with no formatting



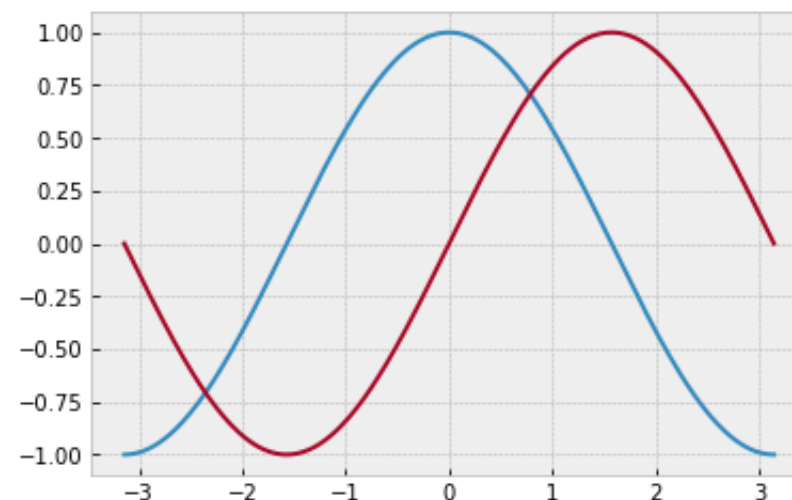
ggplot



fivethirtyeight



default



bmh



# Saving Plots to File

---

- ❑ We often need to store the plots in a file
- ❑ Matplotlib can handle a range of formats

```
plt.savefig('surfaceplot.png')  
plt.savefig('surfaceplot.jpeg')  
plt.savefig('surfaceplot.pdf')
```

- ❑ If you want better looking statistical plots, try Seaborn
- ❑ Seaborn is a separate library that sits on top of matplotlib
- ❑ Its purpose is to produce attractive and informative statistical plots in Python and is closely integrated with Numpy and Pandas

# Introduction to Matplotlib Notebook 4

Add your notes here

**Libraries: Pandas**

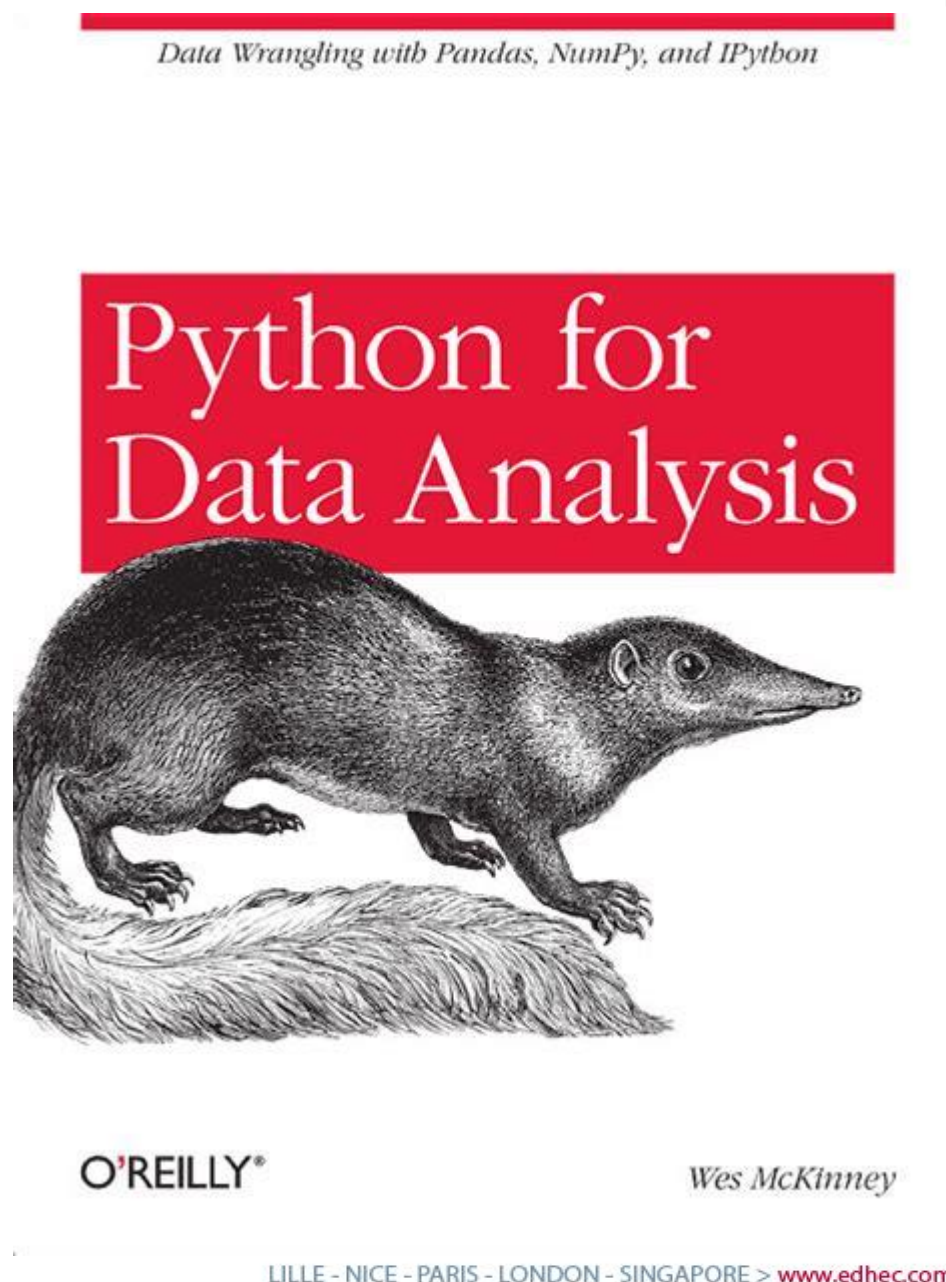
# Pandas

- ❑ **Pandas = Panel Data** enables you to do efficient data analysis
- ❑ Has two key data types:
  - ❑ **Series** – A data array with a named index
  - ❑ **DataFrame** – A data matrix with labelled index and columns
- ❑ It can handle huge data sets with high efficiency
- ❑ It has been highly optimised to deal with time series data
- ❑ Many standard calculations are built in and so are fast
- ❑ **You should use it instead of Excel for any data work**
- ❑ It provides SQL like ways to join data and create useful reports
- ❑ You should import it as follows:

```
import pandas as pd
```

# Pandas Reference

- ❑ The book Python for Data Analysis is a good reference
- ❑ The author Wes McKinney wrote the Pandas Library



# Pandas DataFrame

---

- ❑ The **DataFrame** is the core Pandas object and borrows from R
- ❑ It is similar in format to an Excel spreadsheet
- ❑ Has tools for easy reading in of data from different sources including csv files, excel sheets, SQL databases and others
- ❑ Can do pivot table style operations on the data
- ❑ Time series functionality so understands date ranges, lagging, statistical functions
- ❑ Can easily slice the data into smaller subsets
- ❑ Partly coded in Cython (Python compiled to C) to make it fast

# Pandas Series

- ❑ A Pandas Series is like a numpy array with a named index
- ❑ It has a few inputs – the main ones are **Data** and **Index**
- ❑ Here we define, access and plot a series

```
sales = pd.Series(data=[303,391,374, 401], index=['Q1','Q2','Q3', 'Q4'])
```

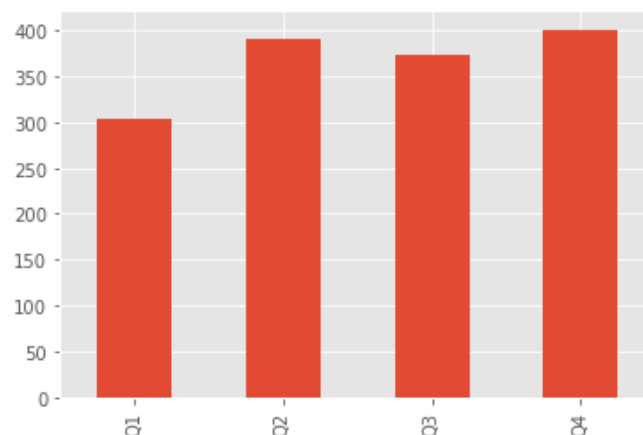
```
sales
```

```
Q1    303
Q2    391
Q3    374
Q4    401
dtype: int64
```

```
sales['Q1']
```

```
303
```

```
plt.style.use('ggplot')
sales.plot.bar();
```



# The Pandas DataFrame

- A DataFrame can be created in many ways including this way

```
df = pd.DataFrame([1,3,5,7], columns = ['odd'], index = ['a','b','c','d'])
df
```

	odd
a	1
b	3
c	5
d	7

**Create DataFrame  
with one column**

```
df['even'] = [2,4,6,8]
```

```
df
```

	odd	even
a	1	2
b	3	4
c	5	6
d	7	8

**Add a new  
column**

```
type(df['odd'])
```

```
pandas.core.series.Series
```

**Each column is a  
Pandas Series**



# Accessing the Pandas DataFrame

- ❑ The first column, the **index**, can be accessed using **df.index**
- ❑ The column names can be accessed using **df.columns**
- ❑ Access rows by index using **df.loc['a']** or row number **df.iloc[0]**

```
df.loc['a']
```

```
odd      1  
even     2  
Name: a, dtype: int64
```

```
df.iloc[2]
```

```
odd      5  
even     6  
Name: c, dtype: int64
```

- ❑ We don't do this much but it's worth knowing

# Removing Columns

---

- ❑ To **remove a column**, we call drop

```
df = df.drop(['Currency'], axis=1)
```

- ❑ This returns a new data frame without the dropped column
- ❑ It does not change df, so we must assign the result to change df

# Examining Datasets

- ❑ Loading from a CSV file is quite straightforward

```
df = pd.read_csv("./data/optionPortfolio.csv")
```

- ❑ Examining the DataFrame

*df.shape* 没有括号

- ❑ **df.shape()** prints the number of rows and columns
  - ❑ **df.head(n)** prints the top n rows of the DataFrame
  - ❑ **df.tail(n)** prints the bottom n rows of the DataFrame
  - ❑ **df.info()** prints all the columns and the non-null values
  - ❑ **df.describe()** gets a statistical description of the data
- ❑ The last of these is very useful – I use it a lot in ML

# Examining the DataFrame using head() and info()

	Trade_ID	TradeDate	Currency	OptionType	Ticker	TradedStockPrice	NumOptions	Strike	ExpiryDate
0	OPT_201313_0	2013-01-03	USD	PUT	CHK	15.865658	490	15.0	2013-12-29
1	OPT_201313_1	2013-01-03	USD	PUT	AAPL	77.442856	280	76.0	2013-12-29
2	OPT_201313_2	2013-01-03	USD	PUT	FB	27.770000	0	26.0	2013-12-29
3	OPT_201313_3	2013-01-03	USD	PUT	AAPL	77.442856	620	74.0	2013-07-02
4	OPT_201318_1	2013-01-08	USD	CALL	AAPL	75.044289	750	75.0	2013-04-08

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 4253 entries, 0 to 4252
Data columns (total 9 columns):
Trade_ID          4253 non-null object
TradeDate         4253 non-null object
Currency          4253 non-null object
OptionType        4253 non-null object
Ticker            4253 non-null object
TradedStockPrice  4253 non-null float64
NumOptions        4253 non-null int64
Strike            4253 non-null float64
ExpiryDate        4253 non-null object
dtypes: float64(2), int64(1), object(6)
memory usage: 299.1+ KB
```

# Examining an Option Portfolio

- ❑ We can get a unique list of tickers

```
df['Ticker'].unique()  
array(['CHK', 'AAPL', 'FB', 'MSFT', 'XRX', 'AMZN', 'BA', 'BLCM'],  
      dtype=object)
```

- ❑ We can see how many option trades there are by ticker

```
df['Ticker'].value_counts()  
CHK      589  
AMZN     574  
XRX      566  
FB       566  
AAPL     562  
MSFT     556  
BA       556  
BLCM     284  
Name: Ticker, dtype: int64
```

# Apply is used to amend a column

- ❑ The trade and expiry dates are non specific objects
- ❑ Want to convert them to datetimes
- ❑ Need to create a function which takes as input an element of the column and then use the **apply method**

```
def dateConverter(dt):  
    dt = dt.replace('-', ' ')  
    return pd.to_datetime(dt, format='%Y %m %d',dayfirst=True)  
  
df['TradeDate'] = df['TradeDate'].apply(dateConverter)  
df['ExpiryDate'] = df['ExpiryDate'].apply(dateConverter)
```

- ❑ This is a very powerful way to do complex operations simply

# Conditionals on the Rows

- ❑ We can select subsets of the data using Boolean filters
- ❑ We select all Put options using “==” comparison not assignment

```
df['OptionType'] == "PUT"
```

```
0      True
1      True
2     False
3      True
4      True
```

```
...
```

```
4193   False
4194     True
4195     True
4196     True
4197     True
```

```
Name: OptionType, Length: 4198, dtype: bool
```

- ❑ This returns an array of Boolean Trues and False values
- ❑ We can then use this to select the True valued rows

# Filters are very powerful

- ❑ We can select subsets of the data using Boolean filters
- ❑ Here we select all of the Put options

```
df[df['OptionType'] == "PUT"].head()
```

- ❑ Here we select PUTS on Apple – use the & operator for AND

```
df[(df['Ticker'] == "AAPL") & (df['OptionType'] == "PUT")].head()
```

- ❑ Here we use a condition requiring Puts where the expiry date has to be before the 2 October 2013

```
import datetime as dt  
df[(df['ExpiryDate'] < dt.datetime(2013,10,2)) & (df['OptionType'] ==  
"PUT")]
```



## Group By

---

- ❑ This enables us to do a breakdown by a specific field
- ❑ We want to count the number of options by Ticker

```
df[['NumOptions','Ticker']].groupby(['Ticker']).count()
```

- ❑ We want to calculate the average strike by Ticker

```
df[['Strike','Ticker']].groupby(['Ticker']).mean()
```

- ❑ See the notebook for examples

# Introduction to Pandas Notebook 5

Add your notes here

**Libraries: StatsModels**

# Time Series Data

---

- ❑ I created a file of time series data
- ❑ I have a set of individual stock files from the NASDAQ
- ❑ I have downloaded them from Kaggle
- ❑ Some are in the course project
- ❑ I then read them in using Python and created a data frame
- ❑ I then save them as a Pickle file - Pickle is a way to convert some Python object to a format where it can be stored in a file
- ❑ It is Python specific

# Reading in Data

- ❑ How to construct the large Pandas timeseries DataFrame
- ❑ I use the pandas **read\_csv** function to load the stock files

```
df_all = pd.DataFrame()

for ticker in ['ba', 'dis', 'ge', 'hpq', 'ibm', 'intc', 'jnj', 'jpm', 'ko',
               'mcd', 'mo', 'mrk', 'pg', 'utx', 'xom']:

    filename = ticker + ".us.txt"
    full_filename = ".\\data\\stocks\\" + filename
    df = pd.read_csv(full_filename)
    df["Ticker"] = ticker
    df['Date'] = df['Date'].apply(dateConverter)

    df_all = pd.concat([ df_all, df], axis=0)
```

- ❑ I then concatenate each file to the large time series data and save it to a pickle file

```
df_all.to_pickle("../data//timeSeriesData.pkl")
```

# Constructing Time Series Data File Notebook 6

Add your notes here

# Loading Financial Time Series Data

- ❑ There are several ways of loading data from public sources
- ❑ Problem is they break every few years – so I use my own data which was created in Notebook 06

```
df_all = pd.read_pickle('.\\data\\timeSeriesData.pkl')
```

```
df_all.info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 184562 entries, 0 to 12073
Data columns (total 8 columns):
#   Column      Non-Null Count  Dtype
---  -
0   Date        184562 non-null  datetime64[ns]
1   Open        184562 non-null  float64
2   High        184562 non-null  float64
3   Low         184562 non-null  float64
4   Close       184562 non-null  float64
5   Volume      184562 non-null  int64
6   OpenInt     184562 non-null  int64
7   Ticker      184562 non-null  object
dtypes: datetime64[ns](1), float64(4), int64(2), object(1)
memory usage: 12.7+ MB
```

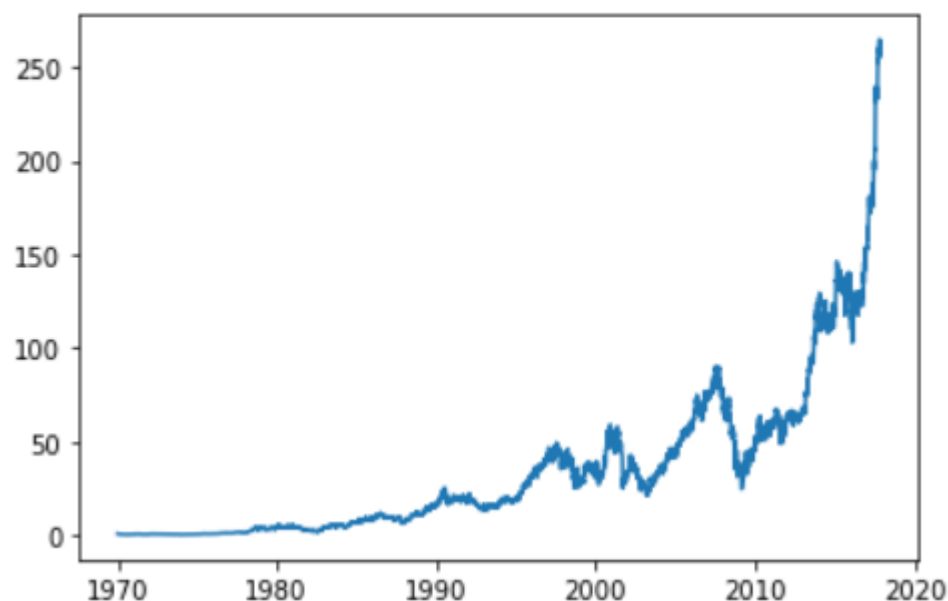
# Plotting the Close

- We extract the dates and the close prices

```
df = df_all[df_all.Ticker=="ba"]  
dates = df['Date']  
timeSeries = df['Close']
```

- Plotting is simple

```
plt.plot(dates, timeSeries);
```





# Statsmodels

---

- ❑ Statsmodels is a library that sits on top of NumPy and SciPy
- ❑ It contains statistical models that have a similar interface to R
- ❑ Includes linear (regression) models of many forms
- ❑ Descriptive statistics
- ❑ Statistical tests
- ❑ Time Series analysis including
  - ❑ VAR and SVAR models
  - ❑ AR/ARMA Kalman Filter, Macro filters
  - ❑ ARCH and GARCH

# Linear Regression in Statsmodels

- ❑ We have done this in Scikit but sometimes we want to use a dedicated statistics library to do all of our time series analysis
- ❑ We generate a noisy linear relationship and fit using OLS

```
numPoints = 20
x = np.linspace(-5, 5, numPoints)
np.random.seed(1)

# normal distributed noise
y = -5 + 3*x + 4 * np.random.normal(size=x.shape)

# Create a data frame containing all the relevant variables
data = pd.DataFrame({'x': x, 'y': y})

from statsmodels.formula.api import ols
model = ols("y ~ x", data).fit()
```

# Linear Regression Results

- We get the results of the model fit using

```
print(model.summary())
```

- These are as follows

```

=====
                        OLS Regression Results
=====
Dep. Variable:          y      R-squared:                0.804
Model:                  OLS    Adj. R-squared:           0.794
Method:                 Least Squares    F-statistic:        74.03
Date:                   Fri, 23 Feb 2018    Prob (F-statistic):   8.56e-08
Time:                   14:48:29    Log-Likelihood:      -57.988
No. Observations:       20    AIC:                  120.0
Df Residuals:           18    BIC:                  122.0
Df Model:                1
Covariance Type:        nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	-5.5335	1.036	-5.342	0.000	-7.710	-3.357
x	2.9369	0.341	8.604	0.000	2.220	3.654

```

=====
Omnibus:                 0.100    Durbin-Watson:           2.956
Prob(Omnibus):           0.951    Jarque-Bera (JB):        0.322
Skew:                    -0.058    Prob(JB):                0.851
Kurtosis:                 2.390    Cond. No.                 3.03
=====

```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

# Time Series

- ❑ Load up the stock prices

```
# Function loads historical stock prices
df_all = pd.read_pickle('.\\data\\timeSeriesData.pkl')
```

- ❑ We have a time series with 185k row – lots of tickers

```
df_all.info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 184562 entries, 0 to 12073
Data columns (total 8 columns):
 #   Column      Non-Null Count  Dtype
---  -
 0   Date        184562 non-null  datetime64[ns]
 1   Open        184562 non-null  float64
 2   High        184562 non-null  float64
 3   Low         184562 non-null  float64
 4   Close       184562 non-null  float64
 5   Volume      184562 non-null  int64
 6   OpenInt     184562 non-null  int64
 7   Ticker      184562 non-null  object
dtypes: datetime64[ns](1), float64(4), int64(2), object(1)
memory usage: 12.7+ MB
```

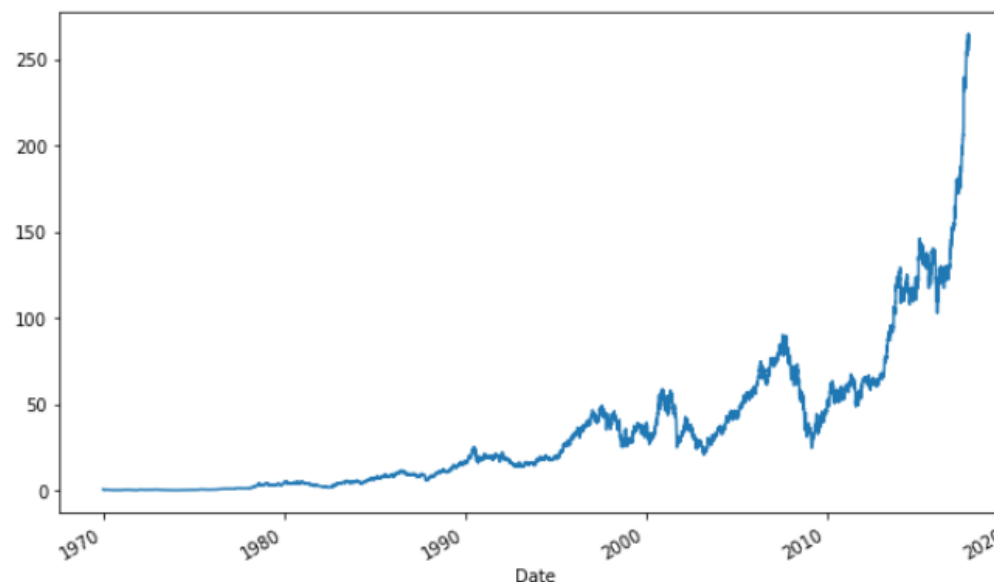
	Date	Open	High	Low	Close	Volume	OpenInt	Ticker
0	1970-01-02	0.7587	0.8092	0.7587	0.8092	753088	0	ba
1	1970-01-05	0.8263	0.8429	0.8263	0.8345	879203	0	ba
2	1970-01-06	0.8429	0.8598	0.8429	0.8429	1607067	0	ba
3	1970-01-07	0.8429	0.8598	0.8429	0.8512	767501	0	ba
4	1970-01-08	0.8512	0.8512	0.8263	0.8429	958476	0	ba

# Focus on the Close Price for Boeing (BA)

- ❑ Grab all the rows for Boeing and drop all but Close

```
df_ba = df_all[df_all.Ticker=="ba"]  
df_ba = df_ba.drop(['Open', 'High', 'Low', 'Volume', 'OpenInt', 'Ticker'], axis=1)  
# Make the date the index  
df_ba.index = df_ba['Date']  
df_ba['Close'].plot(figsize=(10,6));
```

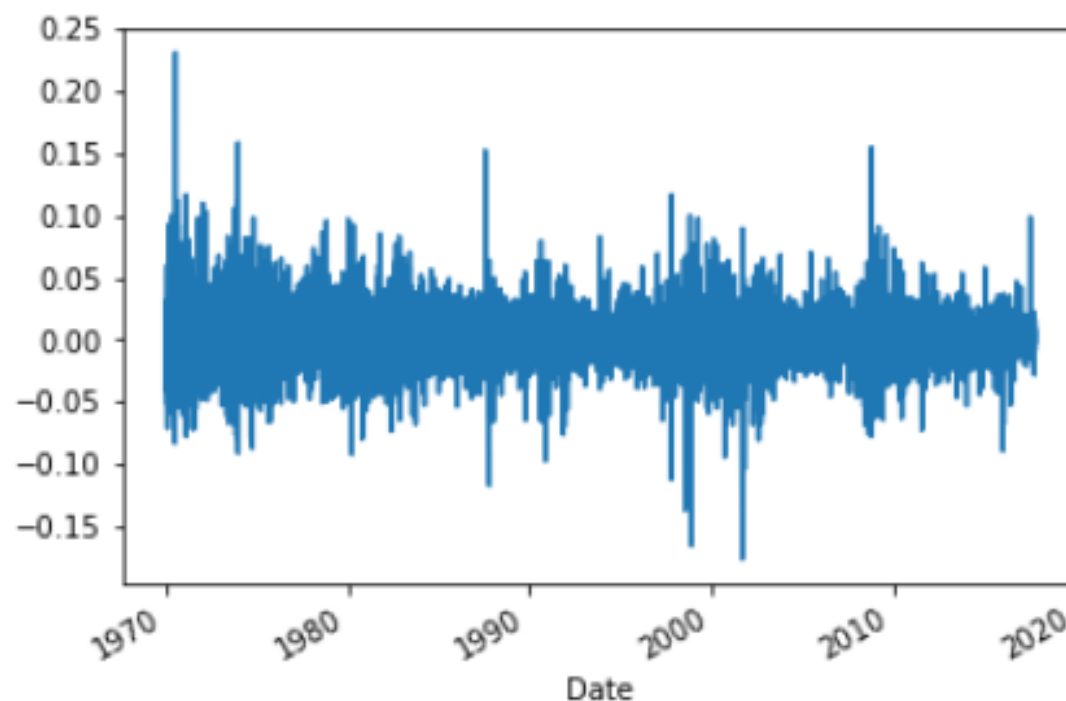
- ❑ These are as follows



# Percentage Changes

- ❑ Want a stationary time series – we take percentage differences using a one-day lag

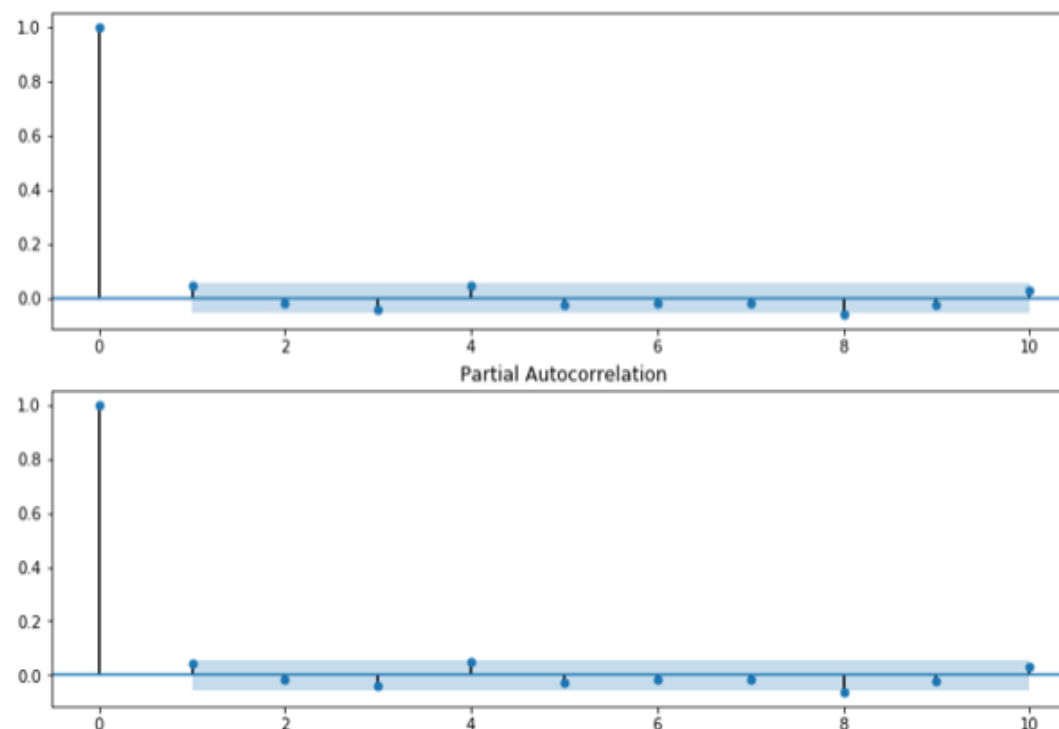
```
df_ba_close = df_ba['Close']  
diffs = df_ba_close.pct_change(1)  
diffs.plot();
```



# Time Series: Autocorrelation Tests

- ❑ We can easily calculate and plot the Autocorrelation and PACF
- ❑ Squeeze converts the dataframe to a Pandas Series

```
import statsmodels as sm
s = diffs.values.squeeze()
sm.graphics.tsaplots.plot_acf(s[1:], lags = 5);
sm.graphics.tsaplots.plot_pacf(s[1:], lags = 10);
```



# Testing Stationarity using ADF

- The standard test is known as Augmented Dickey-Fuller

```
from statsmodels.tsa.stattools import adfuller
dfTest = adfuller(s[1:])
print('Test Statistic %9.5f' % dfTest[0])
print('p-value %9.5f' % dfTest[1])
print('Number of Lags Used %9.5f' % dfTest[2])
print('Number of Observations Used',dfTest[3])
for conf in dfTest[4]:
    print('Critical Value at %s: %9.5f' % (conf, dfTest[4][conf]))
```

**Test Statistic -79.29879**

**p-value 0.00000**

**Number of Lags Used 1.00000**

**Number of Observations Used 12071**

**Critical Value at 1%: -3.43089**

**Critical Value at 5%: -2.86178**

**Critical Value at 10%: -2.56690**



# Introduction to StatsModels for Time Series Notebook 7

Add your notes here

**Libraries: Scipy**

# Scipy

---

- ❑ SciPy is a Python-based ecosystem of open-source software for mathematics, science, and engineering
- ❑ It includes a number of modules that may be of interest to us
  - ❑ Special functions – `scipy.special`
  - ❑ Integration – `scipy.integrate`
  - ❑ Optimization – `scipy.optimize`
  - ❑ Interpolation – `scipy.interpolate`
  - ❑ Linear Algebra – `scipy.linalg`
  - ❑ Statistics – `scipy.stats`
- ❑ We will discuss just a few of these in this course
- ❑ For more information check out <https://docs.scipy.org/doc/scipy/reference/tutorial/index.html>

# Scipy: A Simple One-Dimensional Optimiser

- We define a one-dimensional function with a minimum

```
def f(x):  
    return -np.exp(-(x-0.7)**2)
```

- We call the function **minimize\_scalar** as follows

```
from scipy import optimize  
result = optimize.minimize_scalar(f)  
print(result)  
fun: -1.0  
nfev: 10  
nit: 9 success:  
True x: 0.69999999997839409
```

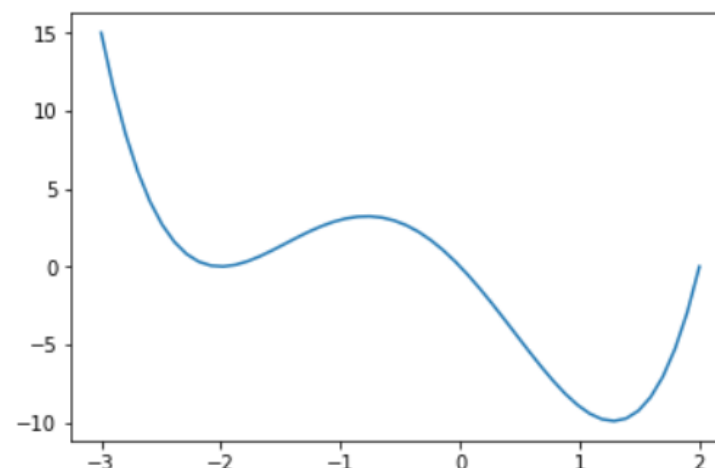
- It allows bounds on the range of solutions, but no other constraints, a choice of algorithms and tolerances

# Scipy: Minimising a Non-Convex Function

- We define a function as follows

```
def f(x):
    return (x - 2.0) * x * (x + 2.0)**2
```

- I want the left solution not the global minimum so need to use the bounded method



```
result = optimize.minimize_scalar(f, bounds=(-3,-1), method='bounded')
```

**fun: 3.2836517984978577e-13**

**message: 'Solution found.'**

**nfev: 12**

**status: 0**

**success: True**

**x: -2.000000202597239**

*If it has two minimum, we need to use the bounded method. 不然到第一个最低点就停止继续寻找了*

# Scipy: Multidimensional Optimisation

- ❑ I define a multidimensional objective function

```
def fn(x):  
    n = len(x); v = 0.0  
    for i in range(0,n):  
        v = v + (x[i]-i)*(x[i]-i)  
    return v
```

- ❑ We set the bounds and starting point and call the optimizer

```
bnds = ((0, 5), (0, 5), (0,5), (0,5)); x0 = (0,0,0,0)  
res = optimize.minimize(fn, x0, bounds=bnds)
```

- ❑ The main return values are the following

```
fun: 1.1868227899471725e-15  
success: True  
x: array([ 0. , 0.99999999, 2.00000003, 3. ])
```

# Scipy: Constrained Multidimensional Optimisation

- ❑ I define constraints as a Python dictionary using lambda functions

```
cons = ({'type': 'ineq', 'fun': lambda x: x[0] - 2 * x[1] + 2},  
        {'type': 'ineq', 'fun': lambda x: -x[1] - 2 * x[3] + 6},  
        {'type': 'ineq', 'fun': lambda x: -x[2] + 2 * x[3] + 2})
```

- ❑ These inequalities are all greater than zero constraints

```
res = optimize.minimize(fun, x0, bounds=bnds, constraints=cons)
```

- ❑ The main return values are the following

```
fun: 0.1999999999999998  
success: True  
x: array([ 0 , 0.8, 2.0, 2.6 ])
```

- ❑ The constraints have stopped the optimiser from finding the previous solution which had a minimum function value of 0.0

# Using Scipy for Optimisation Notebook 8

Add your notes here



**Case Study:  
Option Valuation  
with Monte Carlo**

# Black Scholes Analytical

- ❑ The valuation of a call option can be programmed easily

```
from math import log, exp, sqrt
from scipy.stats import norm
```

```
def priceCallOptionAnalytical(S0,K,T,r,q,sigma):
    d1 = (log(S0/K) + (r - q + 0.5*sigma*sigma)*T) /sigma*sqrt(T)
    d2 = (log(S0/K) + (r - q - 0.5*sigma*sigma)*T) /sigma*sqrt(T)
    value = S0 * exp(-q*T) * norm.cdf(d1,0.0,1.0) - K * exp(-r*T) * norm.cdf(d2,0.0,1.0)
    return value
```

- ❑ This is very fast to execute, as we would expect.

```
%timeit priceCallOptionAnalytical(S0,K,T,r,q,sigma)
```

**133 µs per loop**

# Basic Python

- ❑ Monte Carlo evaluation can be done easily too

```
import random
def priceCallOptionMC(S0,K,T,r,q,sigma,numPaths):
    payOff = 0.0
    for i in range(0,numPaths):
        z = random.gauss(0.0,1.0)
        S = S0 * exp((r-q-sigma*sigma/2.0) * T + sigma * sqrt(T) * z)
        payOff += max(0,S-K)
    value = payOff * exp(-r*T) / numPaths
    return value
```

- ❑ But it is very slow – about 1,000 times slower than analytical

```
%timeit priceCallOptionMC(S0,K,T,r,sigma,numPaths)
98.4 ms per loop
```

# Numpy Vectorization

- ❑ Using Numpy vectorization works here as we have a simply one-dimensional loop over paths

```
import numpy as np
def priceCallOptionMC_Numpy(S0,K,T,r,q,sigma,numPaths):
    z = np.random.normal(size=numPaths,loc=0.0,scale=1.0)
    S = S0 * np.exp((r-q-sigma*sigma/2.0) * T + sigma * sqrt(T) * z)
    payoff = np.maximum(S-K,0)
    value = np.sum(payoff)/numPaths * np.exp(-r*T)
    return value
```

- ❑ This can be memory intensive due to need to hold randoms
- ❑ Execution time is about 30 times faster using Numpy

```
%timeit priceCallOptionMC_Numpy(S0,K,T,r,q,sigma,numPaths)
3.7 ms per loop
```

# Numba JIT Wins!

- ❑ Using Numba we can return to the basic python version

```
from numba import njit
@njit
def priceCallOptionMC_Numba(S0,K,T,r,q,sigma,numPaths):
    payOff = 0.0
    for i in range(0,numPaths):
        z = random.gauss(0.0,1.0)
        S = S0 * exp((r-q-sigma*sigma/2.0) * T + sigma * sqrt(T) * z)
        payOff += max(0,S-K)
    value = payOff * exp(-r*T) / numPaths
    return value
```

- ❑ Execution time is even faster than using Numpy - low memory

```
%timeit priceCallOptionMC_Numba(S0,K,T,r,q,sigma,numPaths)
2.84 ms per loop
```

# Conclusions

---

- ❑ Using Numba, Python code becomes very fast
- ❑ Yet we can retain readability and flexibility over the code which is not possible with Numpy vectorizations
- ❑ Applying Numba to Numpy seems to make it slower !
- ❑ With Numba, Python becomes C-like in its speed
- ❑ And it is 10-100 times faster than VBA

# Option Pricing Using Monte Carlo Notebook 9

Add your notes here

**Case Studies**  
**Bond Yield Curves**  
**Fitting and Interpolation**



# Generate Cashflow Times

- ❑ **# of payments** left is **maturity x frequency** rounded down
- ❑ The first payment time is then the maturity minus all full periods
- ❑ We use **Numpy's linspace** to generate the times

```
def flowTimes(maturity, frequency):
    small = 1e-10
    numPaymentsMinusOne = int(maturity * frequency - small)
    firstPayment = maturity - numPaymentsMinusOne / frequency
    return np.linspace(firstPayment, maturity, numPaymentsMinusOne + 1)
```

- ❑ Some examples make it clear

```
print(flowTimes(2.75, 2))
[ 0.25  0.75  1.25  1.75  2.25  2.75]
print(flowTimes(10.1, 2))
[ 0.1  0.6  1.1  1.6  2.1  2.6  3.1  3.6  4.1  4.6  5.1  5.6  6.1  6.6  7.1  7.6  8.1  8.6  9.1
 9.6 10.1]
```

# Calculate Full Bond Price from Yield

- It is then straightforward to calculate the full price of a bond

```
def bondFullPriceFromYield(y,maturity,coupon,frequency):
    paymentTimes = flowTimes(maturity,frequency)
    price = 0.0; df = 1.0
    for t in paymentTimes:
        df = 1.0/(1.0 + y/frequency)**(t*frequency)
        price += ( coupon / frequency ) * df
    price += df
    return price
```

- We also need to calculate accrued interest

```
def accruedInterest(maturity,coupon,frequency):
    paymentTimes = flowTimes(maturity,frequency)
    accruedPeriod = 1.0/frequency - paymentTimes[0]
    return accruedPeriod * coupon
```

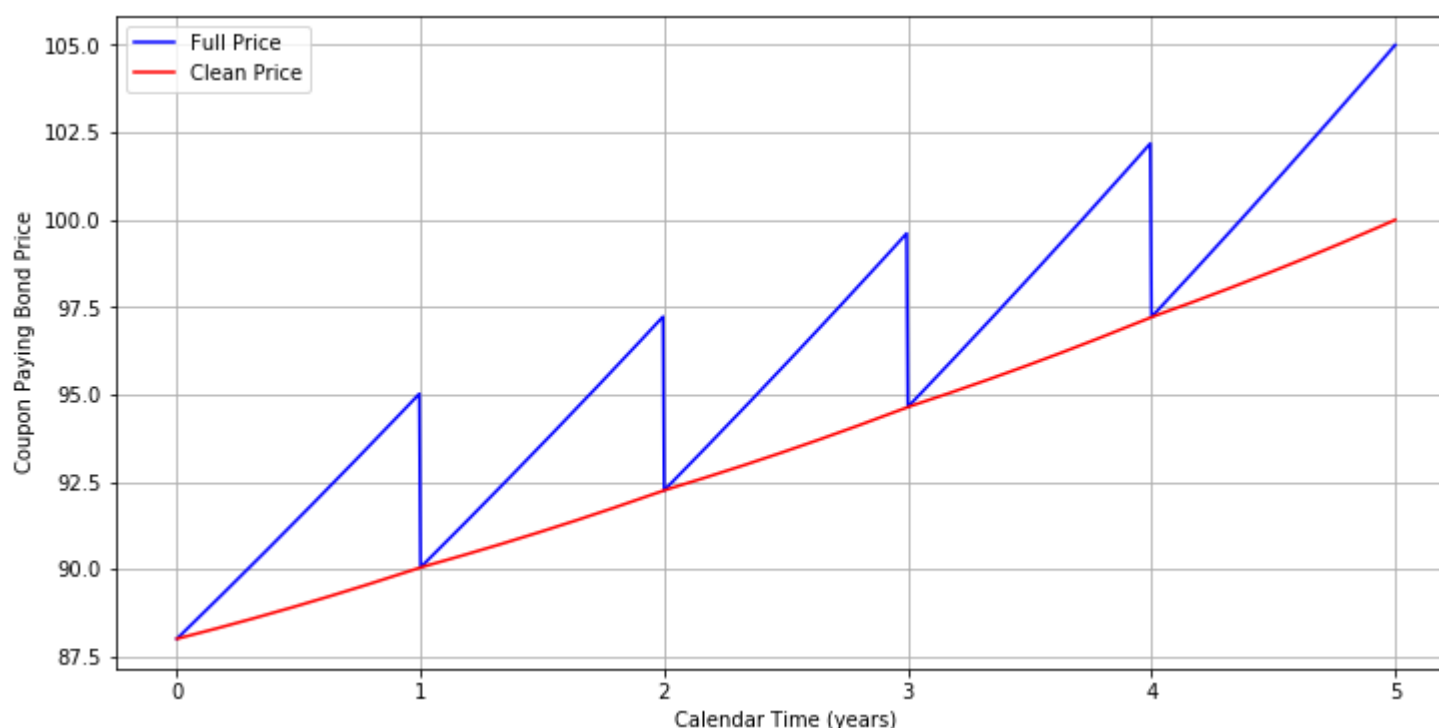
# Bond Price Action

- ❑ The price action of a bond through time assuming a constant yield is then easy to generate

```
def plotFullPriceAction(y, maturity, coupon, frequency):  
    calendarTimes = np.linspace(0.0,maturity,1001)  
    fullPrices = []  
    cleanPrices = []  
    for t in calendarTimes:  
        yearsToMaturity = maturity - t  
        fullPrice = bondFullPriceFromYield(y, yearsToMaturity, coupon, frequency)  
        accrued = accruedInterest(yearsToMaturity, coupon, frequency)  
        cleanPrice = fullPrice - accrued  
  
    ... plotting code ...
```

# Bond Price Action

- ❑ Setting the maturity to 5 years, coupon of 5%, annual frequency and a yield of 8% we have the following price action
- ❑ We see the clean price and the full price



- ❑ Plotting such graphs is easy in Python Jupyter notebooks

# Loading Bond Data

- ❑ I have loaded a dataset of US Gilts with prices from 19 Sep 2012
- ❑ It is tab separated so use `sep = '\t'` to load into a dataframe

```
bondDf = pd.read_csv('./data/giltbondprices.txt', sep='\t')
```

	epic	description	coupon	maturity	bid	ask	change	income yield	gross redemption yield
0	TR13	Uk Gilt Treasury Stk	4.50	07-Mar-13	101.92	102.07	-0.01	4.41	0.22
1	T813	Uk Gilt Treasury Stk	8.00	27-Sep-13	107.86	107.98	-0.03	7.41	0.23
2	TR14	Uk Gilt Treasury Stk	2.25	07-Mar-14	102.90	103.05	0.01	2.18	0.22
3	T514	Uk Gilt Treasury Stk	5.00	07-Sep-14	109.28	109.43	0.02	4.57	0.23
4	TR15	Uk Gilt Treasury Stk	2.75	22-Jan-15	105.57	105.68	0.05	2.60	0.33

- ❑ I then added a new column with the mid price

```
bondDf['mid'] = 0.5*(bondDf['bid'] + bondDf['ask'])
```

- ❑ Note that these are clean prices as this is market convention and so we need to add on accrued to use them in our bond math

# Calculating the Yield Curve

- ❑ Want to calculate the yield to maturity using the full price
- ❑ I use a `lambda function inside the scipy optimize.newton function`

```
def bondFullPriceToYield(fullPrice,maturity,coupon,frequency):

    paymentTimes = flowTimes(maturity,frequency)

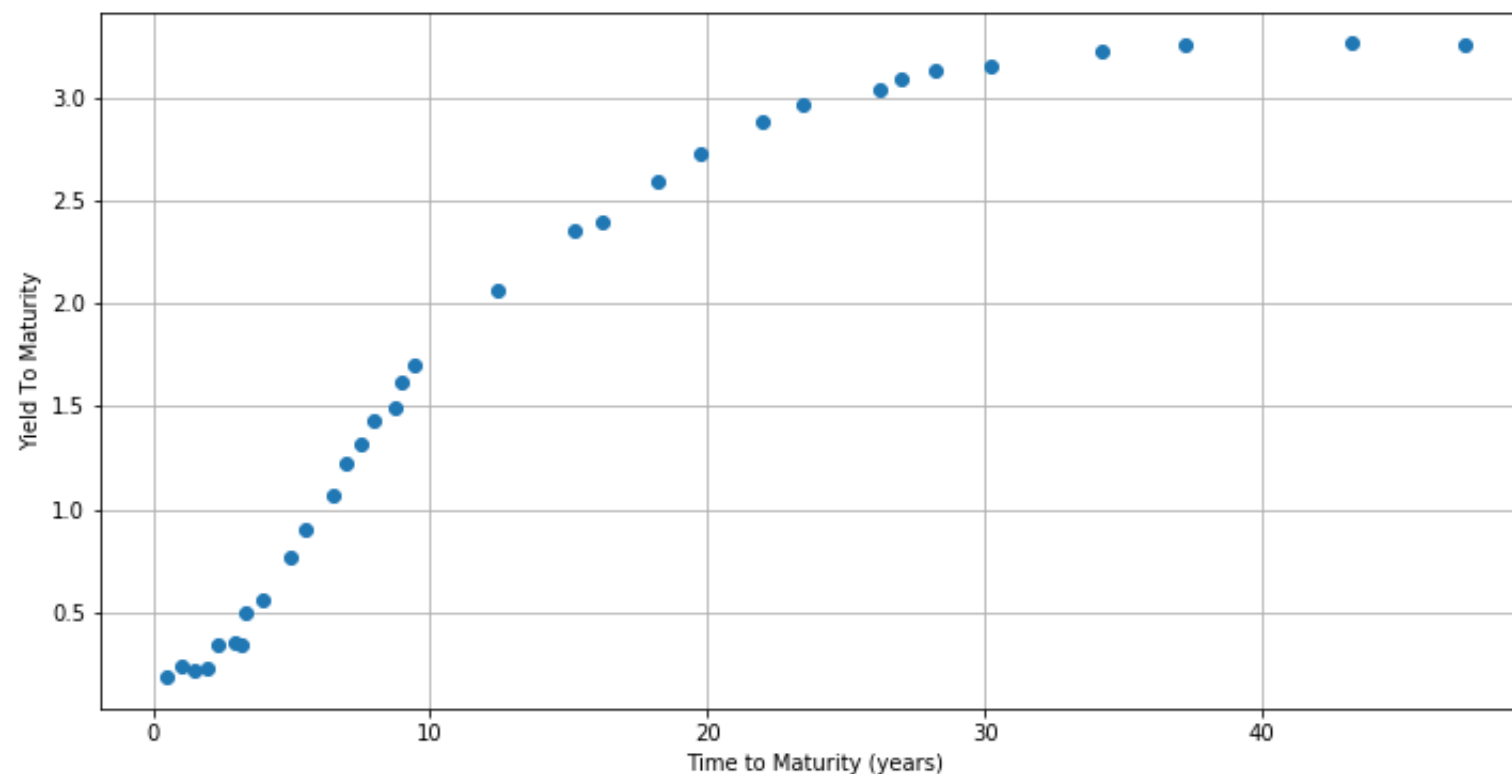
    ytm_func = lambda y: \
        sum([(coupon/frequency)/(1.0+y/frequency)**(frequency*pmtTime) for pmtTime
in paymentTimes ]) + \
        1.0/(1.0+y/frequency)**(frequency*paymentTimes[-1]) – fullPrice

    initial_guess = 0.05
    return optimize.newton(ytm_func, initial_guess)
```

- ❑ This is pushing the lambda function to its limits
- ❑ We will see how to handle more complex functions later

# The Yield Curve Points

```
plt.figure(figsize=(12, 6))  
plt.plot(bondDf['yearsToMaturity'], bondDf['ytm'], 'o')  
plt.grid(True)  
plt.xlabel('Time to Maturity (years)')  
plt.ylabel('Yield To Maturity')
```



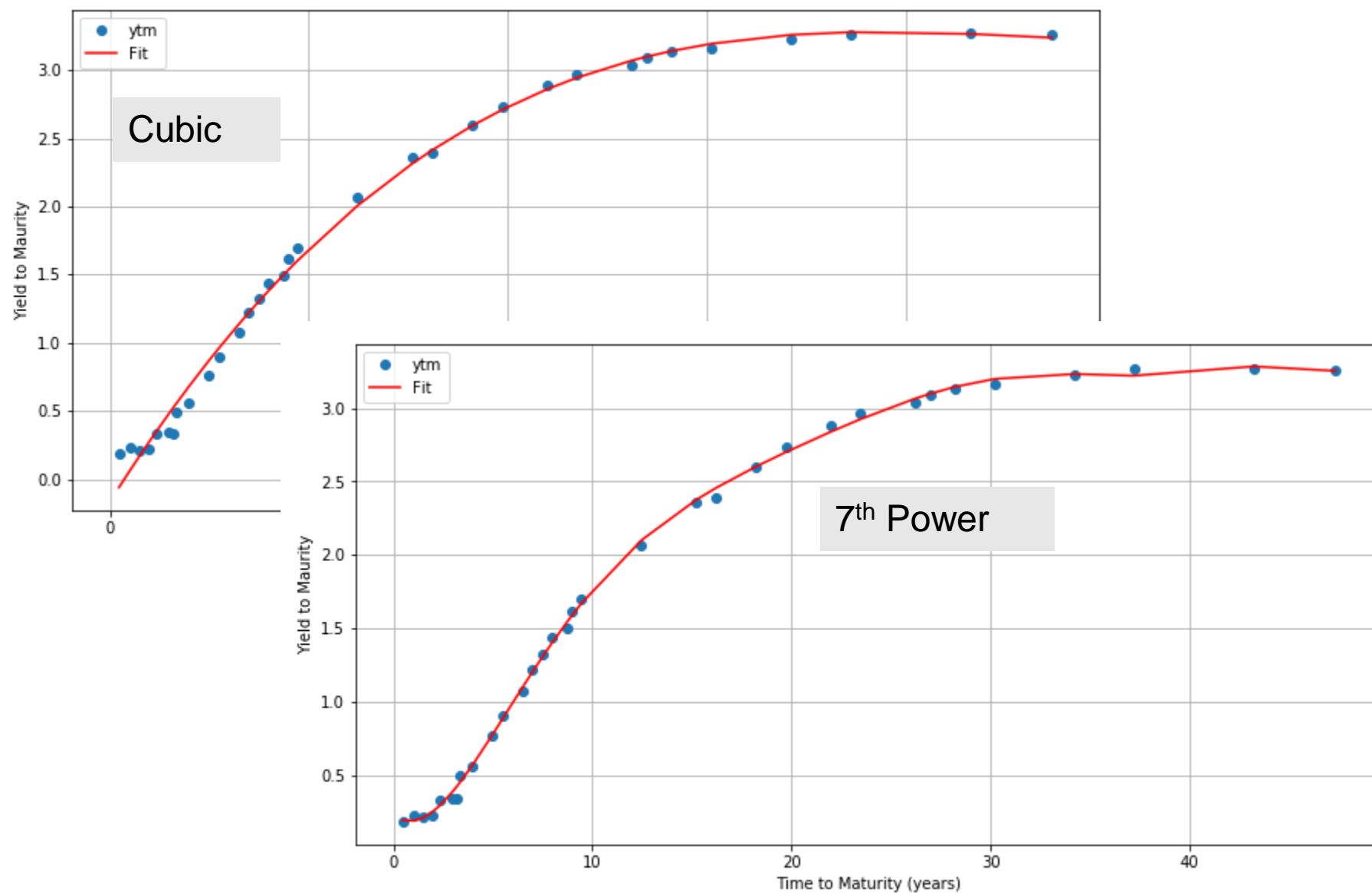
# Fit and Plot

- ❑ It is not hard to fit a polynomial function to the curves
- ❑ I wrote a function to do this which is shown below

```
def fitAndPlot(x,f,d):  
    # x is the vector of years and f is the vector of yields  
    coeffs = np.polyfit(x, f, deg=d)  
    ry = np.polyval(coeffs, x)  
  
    plt.figure(figsize=(12, 8))  
    plt.plot(x, f, 'o')  
    plt.plot(x, ry, 'r', label='Fit')  
    plt.legend(loc=0)  
    plt.grid(True)  
    plt.xlabel('Time to Maturity (years)')  
    plt.ylabel('Yield to Maturity')
```



# Fitting Using Polynomials



# Fitting Bond Yield Curves Notebook 10

Add your notes here

**Case Studies**  
**Mean-Variance**  
**Portfolio Optimisation**

# The Theory

- ❑ We have the time series of returns for a universe of N assets
- ❑ We wish to determine the optimal allocation on the basis of a mean-variance criteria
- ❑ The **portfolio return** is estimated on T historical daily returns

$$\mu_P = \sum_{i=1}^N w_i \mu_i = 252 \times \sum_{i=1}^N w_i \sum_{t=1}^T r_{it}$$

- ❑ The **portfolio variance** is given by

$$\sigma_P^2 = \sum_{i,j=1}^N w_i w_j \sigma_{ij}^2$$

- ❑ The  $\sigma_{ij}^2$  is the covariance of historical returns between i and j

# The Stock Price Data

- ❑ I loaded equity prices **stored in single ticker format**

```
timeSeriesData = pd.read_pickle('.\\data\\timeSeriesData.pkl')
```

- ❑ The format of this dataframe is

	Date	Open	High	Low	Close	Volume	OpenInt	Ticker
0	1970-01-02	0.7587	0.8092	0.7587	0.8092	753088	0	ba
1	1970-01-05	0.8263	0.8429	0.8263	0.8345	879203	0	ba
2	1970-01-06	0.8429	0.8598	0.8429	0.8429	1607067	0	ba
3	1970-01-07	0.8429	0.8598	0.8429	0.8512	767501	0	ba
4	1970-01-08	0.8512	0.8512	0.8263	0.8429	958476	0	ba

- ❑ I want to **have the tickers as columns and just examine the close price and have the date as the index**

# Aligning the Time Series

- ❑ The code is a bit complicated at first sight

```
df_all = pd.DataFrame()
for ticker in tickers:
    df_ticker = closePrices[closePrices.Ticker == ticker]
    df_ticker = df_ticker.set_index('Date')
    df_ticker.columns = [['Ticker', ticker]]
    df_ticker = df_ticker.drop(['Ticker'], axis=1)
    df_all = pd.concat([df_ticker, df_all], axis=1, join="outer")
df_all = df_all.dropna()
```

- ❑ The new dataframe looks like this - which is what we want

	xom	utx	pg	mrk	mo	mcd	ko	jpm	jnj
Date									
1972-01-07	1.8452	0.31680	1.9119	0.7077	0.04379	0.7691	0.9870	2.9672	0.9929
1972-01-14	1.8132	0.32548	1.9523	0.6998	0.04379	0.7448	0.9870	3.0419	0.9685
1972-01-21	1.8452	0.33400	1.9927	0.6840	0.04379	0.7530	0.9747	3.0088	0.9767

# Extracting the Returns

- ❑ We have closing prices but need returns

```
returns = df_all.pct_change(periods = 1)
returns.dropna(inplace=True)
returns.head()
```

- ❑ Using the `pct_change` function I calculated the daily returns
- ❑ I dropped any NA values and filtered out a list of tickers
- ❑ Passing this into the dataframe selected just those asset return
- ❑ I calculated the average returns, covariance and correlations

```
assetReturns = newReturns.mean()
assetCovariance = newReturns.cov()
assetCorrelations = newReturns.corr()
```

# Portfolio Measures of Risk and Return

- ❑ In the Jupyter notebook all variables are in memory scope and do not need to be input - we only explicitly pass the weight vector

```
def portfolioVolatility(weights):  
    return np.sqrt(np.dot(weights.T, np.dot(assetCovariance * 252, weights)))  
  
def portfolioReturn(weights):  
    return np.sum(assetReturns * weights) * 252  
  
def portfolioSharpeRatio(weights):  
    return (portfolioReturn(weights) - rfr) / portfolioVolatility(weights)
```

- ❑ If we were to write this as standalone python code we would need to pass in the **assetReturns** and **assetCovariance** and **rfr**



# Generating the Efficient Frontier

- ❑ We iterate over returns from the lowest to the highest
- ❑ For each we find the portfolio with lowest variance or volatility

```
minRet = min(assetReturns*252)
maxRet = max(assetReturns*252)
trets = np.linspace(minRet, maxRet, 50)
```

- ❑ We have fifty steps in our loop over the different return values
- ❑ We store results of the optimiser in an array so we can plot them

```
for tret in trets:
    ...
    tvols.append(res['fun'])
```

- ❑ Each return value has to become a constraint of the optimiser

# Constrained Optimisation

- ❑ We now have three constraints – each weight has to be in range 0-100% (no short selling), we are fully invested and a fixed return
- ❑ The weights constraint is set by simple bounds on each variable

```
bnds = tuple((0, 1) for x in weights)
```

- ❑ The investment constraint states that sum of allocations is 100%
- ❑ The return constraint sets the average portfolio return
- ❑ These take the form of a tuple of dictionaries

```
cons = ({'type': 'eq', 'fun': lambda x: portfolioReturn(x) - tret},  
        {'type': 'eq', 'fun': lambda x: np.sum(x) - 1})
```

- ❑ We use lambda functions for the constraints - the optimiser looks for the value of  $x$  that sets the function value to zero

# Constrained Optimisation

- ❑ We pass these to the optimizer as follows

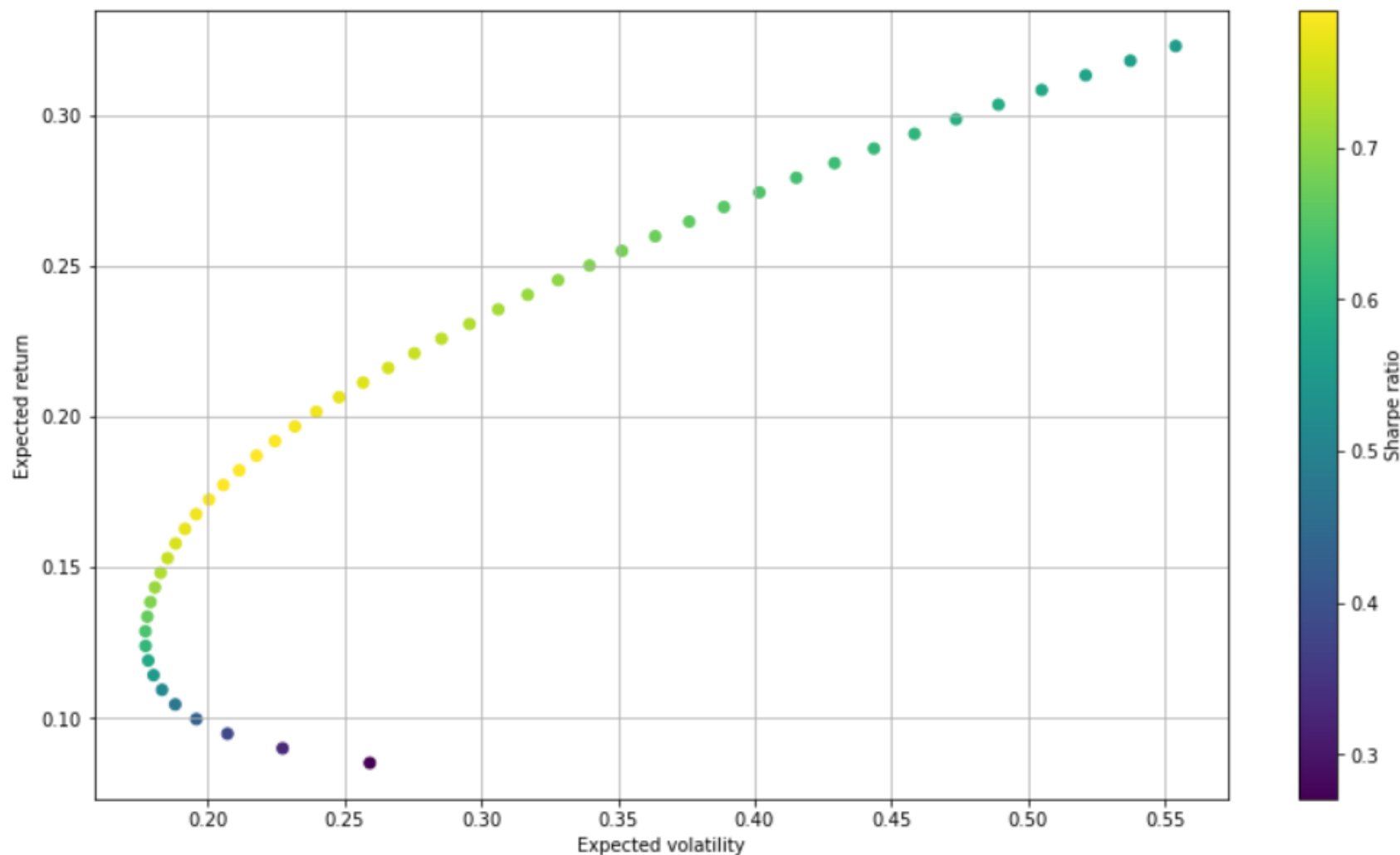
```
res = sco.minimize(portfolioVolatility, initialWeights, method='SLSQP',
bounds=bnds, constraints=cons)
```

- ❑ I use the Sequential Least Squares Programming (**SLSQP**) method
- ❑ I plot the results as a scatterplot with a grid - easier to read
- ❑ I set the color of the dots to be the Sharpe Ratio
- ❑ A colorbar indicates the value of the Sharpe Ratio

```
plt.figure(figsize=(14, 8))
plt.scatter(tvols, trets, c=(trets-rfr) / tvols, marker='o')
plt.grid(True)
plt.xlabel('Expected volatility')
plt.ylabel('Expected return')
plt.colorbar(label='Sharpe ratio')
```

# Efficient Frontier

- And we get the following plot



# Mean Variance Portfolio Optimization Notebook 11

Add your notes here

# Python Modules

# Python Modules

---

- ❑ Jupyter notebooks are great if we are experimenting
- ❑ Or if we want to share ideas and teach people how to code
- ❑ In practice we want to use Python to automate processes
- ❑ We need it to do large complex calculations
- ❑ We want it to interface to other stages in some process
- ❑ We need it to pull in other code we have already written
- ❑ For example it needs to generate automatic daily risk management reports that get emailed to all of the traders
- ❑ At this point we need a body of code that works
- ❑ We develop in Python modules – a **module** is a .py file that contains Python code

# Integrated Development Environment

---

- ❑ We develop modules in an Integrated Development Environment (IDE)
- ❑ This is a graphical user interface that combines an editor with a console for running plus lots of other useful tools
  - ❑ Editing multiple files
  - ❑ Code syntax checking
  - ❑ Debugger
  - ❑ Variable watcher
  - ❑ Console window
- ❑ There are several Python IDEs
- ❑ I prefer Spyder – it comes with your Anaconda installation.



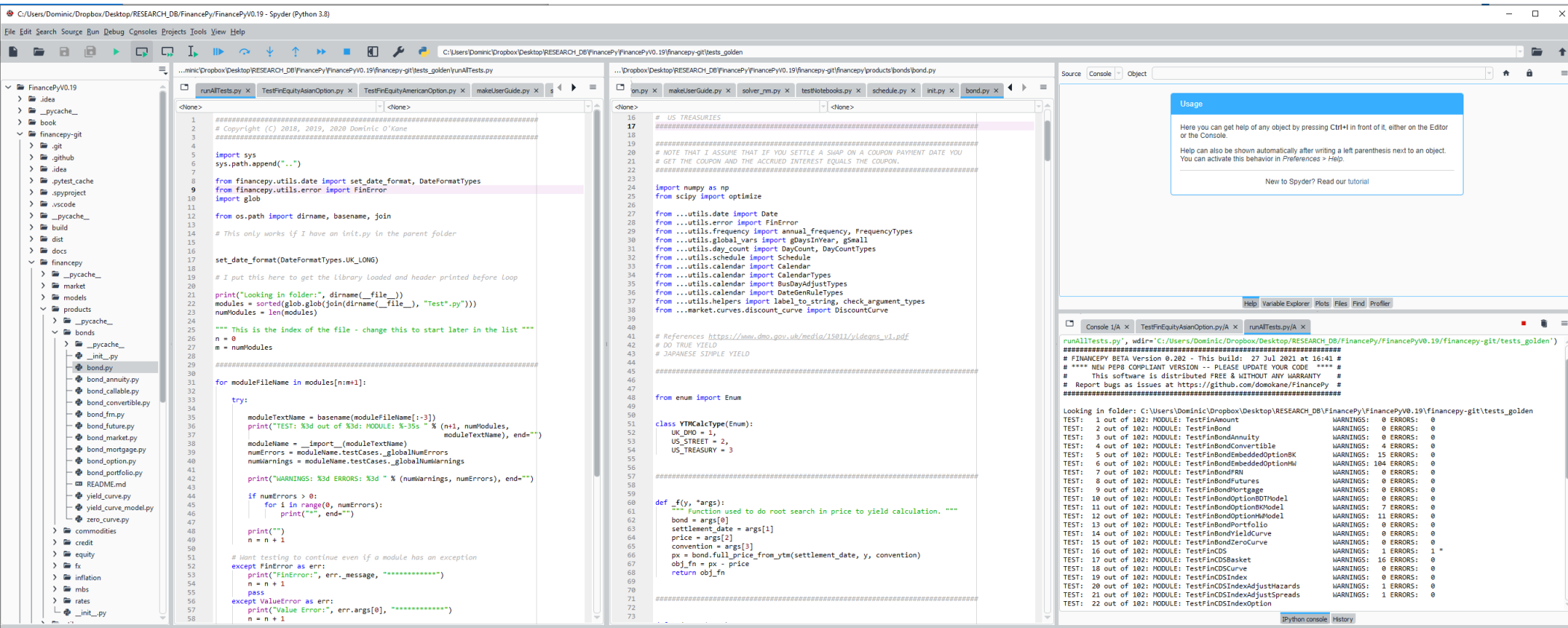
# To Run Spyder

---

- ❑ In Windows, hit the Windows Key
- ❑ Choose the Anaconda menu
- ❑ Then you will see Spyder
- ❑ Click on it

# Here is my FinancePy Project in Spyder

- Multiple editors, file manager, command window, debugger ...



# Executing a Python File in Spyder

---

- ❑ Python code is usually stored in a module of form **filename.py**
- ❑ Load this file so that you can see its contents
- ❑ This file can contain classes, functions and commands
- ❑ Click on the file so that the text is visible
- ❑ Press the play button to execute the file
- ❑ Running the file in Spyder will execute any code that is left-aligned – functions will be created but not executed
- ❑ The output will be displayed in the Command window

# Structure of a Python Module File

---

- ❑ Python code is stored in a module file of form **filename.py**
- ❑ This module can contain classes, functions and commands
- ❑ Running the file will load functions and classes into memory and will execute any code that is not inside a class or function

# Simple Self-Contained MC Option module

- ❑ We import dependencies, define a function and call it.
- ❑ This module is called BlackScholesMC\_1.py

```
import numpy as np
```

**We need Numpy**

```
def blackScholesMonteCarlo(numPaths,s0,k,T,r,sigma):
```

**Define a function**

```
    payoff = 0.0
```

```
    for i in range(1,numPaths):
```

```
        z = np.random.normal(0.0,1.0)
```

```
        sT = s0 * np.exp((r-0.5*sigma**2) * T + sigma * np.sqrt(T) * z)
```

```
        payoff += max(sT-k,0)
```

```
    value = np.exp(-r*T) * payoff/numPaths
```

```
    return value
```

```
numPaths = 50000; s0 = 100.0; k = 100.0; T=1.0; r=0.05; sigma = 0.20
```

```
price = blackScholesMonteCarlo(numPaths,s0,k,T,r,sigma)
```

```
print(price)
```

**Call the function and  
print values**

```
10.427788307249811
```

# You can import your functions from other modules

- ❑ You can access the functions you have written in other modules
- ❑ You just need to import the other module

```
import filename as fn
```

- ❑ You can then access the function using

```
fn.functionname(x)
```

- ❑ Or you can write

```
from filename import functionname
```

- ❑ And simply call it as

```
functionname(x)
```

# We can isolate Functions so they can be reused

- ❑ I want to call this function from another module
- ❑ I create a new module called callBlackScholesMC\_1.py

```
from BlackScholesMC_1 import blackScholesMonteCarlo

numPaths = 50000; s0 = 100.0; k = 100.0
T=1.0; r=0.05; sigma = 0.20
price = blackScholesMonteCarlo(numPaths,s0,k,T,r,sigma)
print(price)
10.520588738511313
10.501508113310235
```

**I import my function  
from the module I just  
created**

**Call function and  
print result**

- ❑ It runs twice – as it's random I get two slightly different results
- ❑ Why did it run twice ?
- ❑ Because when BlackScholesMC\_1 is imported it runs that module

# Importing a Module from another Programme

- ❑ Sometimes we want to include functionality, e.g. unit tests, in the same module as the function
- ❑ If we run the module then run the tests
- ❑ But I do not want these tests to run when I import the module
- ❑ To achieve this I use the following trick

```
if __name__ == '__main__':  
    INDENTED TEST CODE HERE
```

- ❑ If the `__name__` variable which is internal to Python equals `"__main__"` then the file is being run directly, not imported
- ❑ This ensures the INDENTED CODE will not be run if the file is imported, it will only run if we run the module directly



# We can isolate Functions so they can be reused

- ❑ I want to call this function from another module  
BlackScholesMC\_2 with this conditional around the test code

```
from BlackScholesMC_2 import blackScholesMonteCarlo

numPaths = 50000; s0 = 100.0; k = 100.0
T=1.0; r=0.05; sigma = 0.20
price = blackScholesMonteCarlo(numPaths,s0,k,T,r,sigma)
print(price)
10.520588738511313
```

**I import my function  
from the module I just  
created**

**Call function and  
print result**

- ❑ It runs once !
- ❑ This is very powerful – now you can start reusing functions

# **Object Oriented Python**

# Object Oriented Programming

---

- ❑ OO was one of the big revolutions in programming languages
- ❑ The idea is to unify functionality and data into a single entity
- ❑ This entity is called a Class
- ❑ The functions are called Class Methods
- ❑ The data are called Class Members
- ❑ A specific use of a class is called objects
- ❑ **Example:** Class Humans
  - ❑ Class Methods: Walk, Talk, Eat, Drink, Sleep, ...
  - ❑ Class members: Sex, Height, Weight, Hair colour, ...
  - ❑ Object: Me, You

# Object Oriented Programming

---

- ❑ OO is different from functional programming
- ❑ OO languages usually provide the following:
  - ❑ Data Encapsulation
  - ❑ Data Abstraction
  - ❑ Inheritance
  - ❑ Modularity
  - ❑ Polymorphism
- ❑ I will not explain all of these, but they are all important
- ❑ It is a revolutionary new way for designing your code
- ❑ Code becomes easier to write, easier to organise and re-usable

# Creating a Python Classes

- ❑ By convention we ALWAYS start a class name with a capital letter
- ❑ Here we define a class called **Circle**

```
class Circle(object):  
    pass
```

- ❑ We can now create this class

```
b = Circle(object)
```

- ❑ And we can check the type to confirm what happened

```
type(x)  
__main__.Circle
```

# Python Circle Class: Attributes

- ❑ Let's add some methods and attributes to the class

```
class Circle ():  
    def __init__(self, radius=1):  
        print('Creating circle')  
        self._radius = r
```

- ❑ The function `__init__` inside a class is called a **method**
- ❑ This special function `__init__` is **automatically** called if you create a Circle type – in C++ this is called a **constructor**
- ❑ The **self** input is a reference to the object itself
- ❑ The radius is an attribute of the Circle
- ❑ If it is not passed in, then a default value of 1 is used
- ❑ I prefix all class members with `_` to signify that it is protected

# Python Classes : Naming data members

- ❑ Changing the radius can be done explicitly

```
self._radius = 0.25
```

- ❑ This may not be desirable
- ❑ In other OO languages like C++, some class members can be made **private** so that you cannot change them directly
- ❑ The reason is that the coder wants to implement some validation or perhaps there is some sort of order dependency in value setting that needs to be enforced
- ❑ This does not exist in Python – it is only enforced by convention
- ❑ **The convention is that if you prefix a class member with a \_ then you signal to users that it should not be changed like this**

# Python Circle Class : Methods

- ❑ Methods are functions inside the Class – you must pass **self** as the first argument – this tells the function which object its from

```
PI = 3.14159  
class Circle():  
...  
    def area(self):  
        a = PI * self._radius ** 2  
        return a
```

- ❑ We can call the area method as follows

```
c = Circle(4.0)  
c.area()  
50.26544
```



# Python Circle Class : Using a set method

```
class Circle (object):  
    def __init__(self,r):  
        self._radius = r  
    def area(self):  
        a = PI * self._radius ** 2  
        return a  
    def setRadius(self, r):  
        self._radius = r
```

- ❑ You shouldn't really do this outside a module

```
circle._radius = 0.25
```

- ❑ Do this instead or just create a new Circle object

```
circle.setRadius(0.25)
```

# Summary

---

- ❑ The Object-Oriented paradigm is very different to pure functional programming and takes a while to understand
- ❑ It makes your code much easier to organize and to understand
- ❑ Your classes can be re-used in different projects without any additional effort
- ❑ You can control access to data members and ensure that validation is performed
- ❑ There is a lot more to object-oriented programming including inheritance that we do not have time to cover here

# **Case Study: A Vanilla Option Class**

# A Vanilla Option Class

---

- ❑ What are the attributes of a Vanilla Option
- ❑ These are the things you would find on a term sheet
- ❑ They are
  - ❑ Option Expiry Date
  - ❑ Option Strike
  - ❑ Option Type – Call or Put
- ❑ Should the following be in the class
  - ❑ Stock price ?
  - ❑ Trade date ?
  - ❑ Volatility ?
  - ❑ Risk-free rate ?

# Initiating the Option Class

- ❑ We create and instantiate the class in the usual way

```
class Option(object):  
  
    def __init__(self, expiry_date, strike_price, option_type ):  
  
        self._expiry_date = expiry_date  
        self._strike_price = float(strike_price)  
        self._option_type = option_type.upper()  
  
        if self._option_type != "CALL" and self._option_type != "PUT":  
            print("Unknown option type")
```

- ❑ Once we have checked option\_type here we don't need to do it again

# Doing some Type Checking

- ❑ We may want to do some type checking on inputs
- ❑ For example, we may want to check that `expiry_date` is a date
- ❑ We use the command **`isinstance`** to check and return a message

```
class Option(object):  
  
    def __init__(self, expiry_date, strike_price, option_type ):  
  
        if isinstance(expiry_date, date) == False:  
            print("Expiry date is not a date")  
  
        self._expiry_date = expiry_date  
        ...
```

- ❑ There are better ways to handle errors, but this is a good start

# I need to import my dependencies

- ❑ I import the functions, classes and set the constants I will need

```
from math import exp, log, sqrt  
from scipy import optimize  
from scipy.stats import norm  
from datetime import date
```

```
DAYS_IN_YEAR = 365.242
```

- ❑ We need scipy for the optimizer and for the NORMCDF
- ❑ Later when I would like vectorised calculations I switch to Numpy

# Valuing the Option

- We create and instantiate the class in the usual way

```
def value(self, value_date, stock_price, interest_rate, dividend_yield, volatility ):
    t = abs(self._expiry_date - value_date).days / DAYS_IN_YEAR
    r = interest_rate; q = dividend_yield; s = stock_price;
    k = self._strike_price; v = volatility

    d1 = (log(s/k) + (r - q + v*v / 2.0) * t) / (v * sqrt(t))
    d2 = (log(s/k) + (r - q - v*v / 2.0) * t) / (v * sqrt(t))
    if self._option_type == "CALL":
        v = s * exp(-q * t) * norm.cdf(d1)
        v = v - k * exp(-r * t) * norm.cdf(d2)
    elif self._option_type == "PUT":
        v = k * exp(-r * t) * norm.cdf(-d2)
        v = v - s * exp(-q * t) * norm.cdf(-d1)
    return v
```

**I often rename the variables to make the formulae shorter and easier to read**



# Implied Volatility

- ❑ The implied volatility calculation involves a root search
- ❑ We use Scipy's Newton function to do this

```
def impliedVolatility(self, value_date, option_mkt_value, stock_price,  
                      dividend_yield, interest_rate):  
  
    argtuple = (self, value_date, stock_price, dividend_yield,  
                interest_rate, option_mkt_value)  
  
    sigma = optimize.newton(f,x0=0.2, args=argtuple, tol=1e-8, maxiter=50)  
    return sigma
```

- ❑ We use the tuple args to pass in extra information

# Implied Volatility – Objective Function

- ❑ We need a function that gives zero at the implied volatility
- ❑ This is declared outside the class – so we need to pass all the class information

```
def f(volatility, *args):  
  
    self = args[0]  
    valueDate = args[1]  
    stockPrice = args[2]  
    divYield = args[3]  
    interestRate = args[4]  
    value = args[5]  
  
    objFn = self.value(valueDate,stockPrice,divYield,volatility,interestRate) - value  
  
    return objFn
```

# Calling the Option Class

- We can now call the class and value a call option

```
expiry_date = date(2022, 6, 1)
stockPrice = 100
volatility = 0.30
interest_rate = 0.05
dividend_yield = 0.0
stockPrices = 100.0
value_date = date(2022, 1, 1)

callOption = Option(expiry_date, 100.0, "CALL")

value = callOption.value(value_date, stockPrice, interest_rate,
                        dividend_yield, volatility)
```

# Object-Oriented Code

---

- ❑ We have only just introduced the idea of OO code
- ❑ There are a number of books and I encourage you to read them
- ❑ As soon as your project gets large, OO becomes a very powerful way to organise and structure code
- ❑ You will get a chance to extend the Option class in the coursework

## **Case Study: A Bond Class**

# Objective

---

- ❑ We build a class which gives the price & yield of a standard bond
- ❑ It's a very simple class that ignores basis conventions and actual cashflow dates
- ❑ The class function takes in
  - ❑ Remaining Maturity in years
  - ❑ Coupon
  - ❑ Frequency
- ❑ It's a class you could easily extend for your own needs
- ❑ We have already defined a number of bond functions so we re-use those

# Bond Class

- ❑ What should a bond contain as members ?
- ❑ Use “Bond **has** a X” to decide ...

```
from HelperFunctions import flowTimes
import scipy.optimize as optimize
from datetime import date
DAYS_IN_YEAR = 365.242

class Bond():
    def __init__(self, maturity_date, coupon, frequency):
        self._maturity_date = maturity_date
        self._coupon = coupon
        self._frequency = frequency
```

- ❑ We cannot compute the payment dates as we don't know the value date or the bond issue date

# Calculate the Full Price from the Yield

- ❑ The full price is the discounted sum of all payments

```
def fullPriceFromYield(self, value_date, yld):
    years = abs(self._maturity_date - value_date).days / DAYS_IN_YEAR
    paymentTimes = flowTimes(years, self._frequency)
    price = 0.0
    for t in paymentTimes:
        df = 1.0/(1.0 + yld/self._frequency)**(t*self._frequency)
        price += ( self._coupon / self._frequency ) * df
    price += df # par
    return price
```

- ❑ However, bond prices are usually quoted clean
- ❑ We need to subtract the accrued interest



# Calculate the Clean Price from the Yield

- ❑ The full price is the discounted sum of all payments

```
def cleanPriceFromYield(self, value_date, yld):  
    full_price = self.fullPriceFromYield(value_date, yld)  
    clean_price = full_price - self.accruedInterest(value_date, )  
    return clean_price
```

- ❑ We re-use the full price function – avoid duplication of code
- ❑ We then subtract the accrued interest

# Calculate the Accrued Interest

- ❑ The accrued is the year fraction from the last coupon to the value / settlement date times the annualised coupon

```
def accruedInterest(self, value_date):  
    years = abs(self._maturity_date - value_date).days / DAYS_IN_YEAR  
    paymentTimes = flowTimes(years, self._frequency)  
    accruedPeriod = 1.0/self._frequency - paymentTimes[0]  
    return accruedPeriod * self._coupon
```

- ❑ I am not sure if this is optimal – why ?
- ❑ How might you improve the code ?

# OAT Comparison Revisited

- ❑ The full price is the discounted sum of all payments

```
maturity_date = date(2016, 10, 25)
```

```
cpn = 0.05
```

```
freq = 1
```

```
bond = Bond(maturity_date, cpn, freq)
```

```
clean_price = 1.1462
```

```
yld = bond.yieldFromCleanPrice(value_date, clean_price)
```

```
print('Yield:', yld*100)
```

**Yield: 1.9280277547132966**

YA  
Enter all values and hit <GO>.

FRANCE O.A.T.	FRTR 5	10/25/16	114.4700/
PRICE	114.620000		
		Worst	
YIELD			
CALCULATIONS	MATURITY	10/25/2016	
		10/25/2016	@100.000
		1.927	1.927

- ❑ Bloomberg gets 1.927% so we are out by 0.001%
- ❑ Why is it not exactly right ? We are not calculating the payment sizes exactly using Actual 360.

# **An External Library: FinancePy**

# What is FinancePy ?

---

- ❑ FinancePy is a Python-based library for the valuation of financial securities, **with a special focus on financial derivatives**
- ❑ I have developed this as a teaching tool, and it can also be used by practitioners to do valuation and risk
- ❑ It's an example of how you can easily grow the Python ecosystem
- ❑ Handles a broad range of asset classes including:
  - ❑ bonds
  - ❑ equities
  - ❑ currencies
  - ❑ interest rates
  - ❑ inflation
- ❑ And derivatives on all of these

# FinancePy at Github

❏ <https://github.com/domokane/FinancePy>

domokane / FinancePy lines 136.6k

Unwatch 36 Unstar 525

Code Issues 50 Pull requests 2 Actions Projects Wiki Security Insights Settings

master 2 branches 1 tag Go to file Add file Code

File/Folder	Commit Message	Time Ago
domokane	Test case for HW Bond Option	5fef706 2 days ago 536 commits
.github/workflows	Merging pep8 compliant version with master	8 days ago
docs	Merging pep8 compliant version with master	8 days ago
financepy	Test case for HW Bond Option	2 days ago
notebooks	Test case for HW Bond Option	2 days ago
tests	Merge branch 'master' of <a href="https://github.com/domokane/FinancePy">https://github.com/domokane/FinancePy</a>	8 days ago
tests_golden	Test case for HW Bond Option	2 days ago
.gitignore	Merging pep8 compliant version with master	8 days ago
LICENSE	Create LICENSE	14 months ago
README.md	Readme#	7 days ago
THANKS.md	Comments added to files for documentation. Added markdown parser to...	2 years ago
TODO.md	Added newton_secant to speed FX vol surface calibration	7 months ago
__init__.py	Merging pep8 compliant version with master	8 days ago
requirements-dev.txt	Merging pep8 compliant version with master	8 days ago
requirements.txt	Merging pep8 compliant version with master	8 days ago
setup.py	Minor fixea	9 months ago
version.py	Merging pep8 compliant version with master	8 days ago

**About**

A Python Finance Library that focuses on the pricing and risk-management of Financial Derivatives, including fixed-income, equity, FX and credit derivatives.

[financepy.com/](https://financepy.com/)

python students finance risk  
currency pricing valuation  
derivatives investment numba  
bonds asset-allocation credit  
fixed-income risk-management  
derivatives-pricing

Readme  
GPL-3.0 License

**Releases**

1 tags  
[Create a new release](#)

**Packages**

No packages published  
[Publish your first package](#)

# FinancePy Design

---

- ❑ Utils
  - ❑ Basic functionality used across the library
- ❑ Market
  - ❑ Holders, processors of market data as Python Classes
- ❑ Models
  - ❑ Quantitative valuation model library as Python Classes
- ❑ Products
  - ❑ Financial securities including derivatives as Python classes

# Utils

---

- ❑ There are a lot of market conventions used in finance
- ❑ We ensure these are followed as exactly as possible in FinancePy
- ❑ Date
  - ❑ In finance, dates are key to determining valuation
  - ❑ There are certain key dates (CDS, IMM dates)
- ❑ Calendar
  - ❑ Need to know all holiday dates in NY, Europe, London, ...
- ❑ Schedule
  - ❑ Need to calculate series of cashflow payment dates in accordance with market conventions
  - ❑ Getting the date correct is essential as the timing of payments plus the right discount rate determines the present value



# Products

## Bonds

- ❑ Bond
- ❑ BondAnnuity
- ❑ BondConvertible
- ❑ BondEmbeddedOption
- ❑ BondFRN
- ❑ BondFuture
- ❑ BondMortgage
- ❑ BondOption

## Credit

- CDS
- CDSBasket
- CDSCurve
- CDSIndexOption
- CDSIndexPortfolio
- CDSOption
- CDSTranche

## Funding

- FixedLeg
- FloatLeg
- IborBasisSwap
- IborCallableSwap
- IborDeposit
- IborFuture
- IborFRA
- IborSwap
- IborCapFloor
- IborSwaption
- IborSingleCurve
- IborDualCurve
- IborOIS
- OIS
- OISCurve
- IborBermudanSwaption

## Equity

- EquityAmericanOption
- EquityAsianOption
- EquityBarrierOption
- EquityBasketOption
- EquityChooserOption
- EquityCliquetOption
- EquityCompoundOption
- EquityDigitalOption
- EquityFixedLookbackOption
- EquityFloatLookbackOption
- EquityRainbowOption
- EquityOneTouchOption
- EquityVanillaOption
- EquityVarianceSwap

## FX

- FXForward
- FXVanillaOption
- FXBarrierOption
- FXBasketOption
- FXRainbowOption
- FXDigitalOption
- FXFixedLookbackOption
- FXFloatLookbackOption
- FXVarianceSwap

## Inflation

- InflationBond
- InflationSwap

- Each of these is a Python class under **Products**

# Market

---

- ❑ Discounting future cashflows correctly is essential

## Discount Curves

- DiscountCurve
- DiscountCurveFlat
- DiscountCurveNS
- DiscountCurveNSS
- DiscountCurvePoly
- DiscountCurvePWF
- DiscountCurvePWL
- DiscountCurveZeros

- ❑ Managing the volatility assumptions for options is key

## Volatility

- EquityVolCurve
- FXVolSurface
- IborCapVolCurve
- IborCapVolCurveFn

# Models

---

- Models are not product-specific

## Lognormal

- GBMProcess
- ModelBlack
- ModelBlackScholes
- ModelBlackScholesAnalytical
- ModelBlackScholesShifted
- ModelCRRTree

## Credit

- ModelGaussianCopula
- ModelLossDbnBuilder
- ModelLHPlus
- ModelMertonCredit
- ModelMertonCreditMkt

## Rates

- ModelRatesBDT
- ModelRatesBK
- ModelRatesCIR
- ModelRatesHL
- ModelRatesLMM

## Normal

- ModelBachelier
- ModelRatesVasicek

## Stochastic Vol

- ModelHeston
- ModelSABR

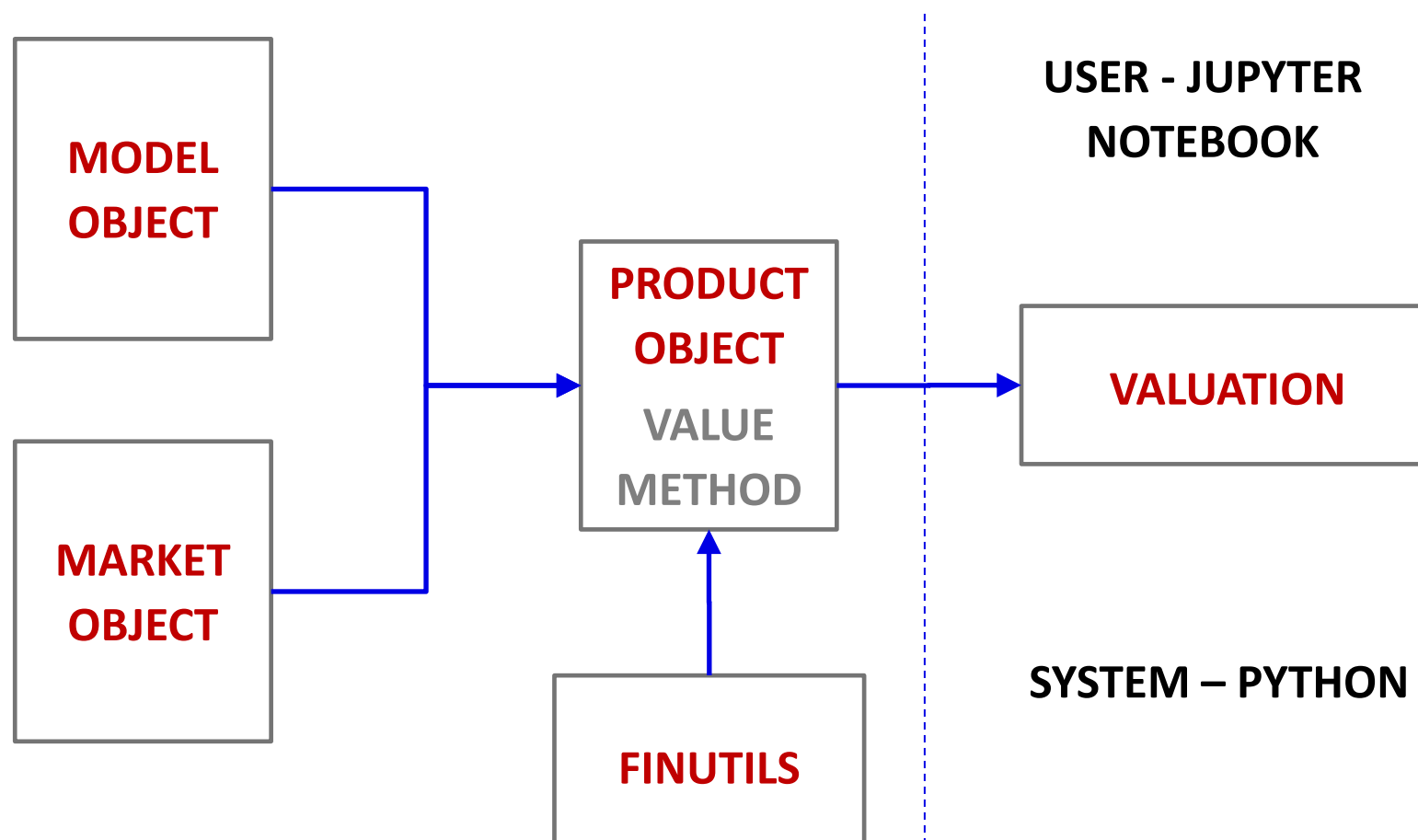
# Modelling Highlights

---

- ❑ Bond yield-curve fitting with multiple parametric forms
- ❑ IBOR discount curves with different interpolation schemes
- ❑ Two curve construction using Overnight Index Swaps
- ❑ Trinomial Trees for interest rate option pricing
- ❑ Multi-factor Libor Market Model
- ❑ Convertible bond pricing model
- ❑ Valuation of Synthetic CDO tranches
- ❑ Full calibration to FX volatility surface
- ❑ Multi-process simulator with stochastic volatility
- ❑ Fast Sobol random number generator
- ❑ Variance reduction methods for path dependent options
- ❑ and lots more ...

# Design

- ❑ Stage I: Create a Product Object e.g. a call option
- ❑ Stage II: Value the product by passing in a model and market



# Installing

- ❑ Use Pip to install financepy

```
pip install financepy
```

- ❑ If you import it then you get a message as follows

```
C:\Users\Dominic>python
Python 3.8.10 (tags/v3.8.10:3d8993a, May  3 2021, 11:48:03) [MSC v.1928 6
Type "help", "copyright", "credits" or "license" for more information.
>>> import financepy as fp
#####
# FINANCEPY BETA Version 0.202 - This build:  16 Jul 2021 at 18:46 #
# **** NEW PEP8 COMPLIANT VERSION -- PLEASE UPDATE YOUR CODE **** #
#       This software is distributed FREE & WITHOUT ANY WARRANTY   #
# Report bugs as issues at https://github.com/domokane/FinancePy  #
#####
>>>
```

- ❑ You are now ready to use it

# Creating and Valuing a Call Option

```
valuation_date = Date(1, 1, 2015)
expiry_date = valuation_date.add_tenor("6M")
strike_price = 50.0
```

```
call_option = EquityVanillaOption(expiry_date, strike_price, FinOptionTypes.EUROPEAN_CALL)
```

```
stock_price = 50
volatility = 0.20
interest_rate = 0.05
dividend_yield = 0.0
```

```
discount_curve = DiscountCurveFlat(valuation_date, interest_rate)
dividend_curve = DiscountCurveFlat(valuation_date, dividend_yield)
model = BlackScholes(volatility)
```

```
call_option.value(valuation_date, stock_price, discount_curve, dividend_curve, model)
```

```
3.4276581469416914
```

```
print(call_option)
```

```
OBJECT TYPE: EquityVanillaOption
EXPIRY DATE: 01-JUL-2015
STRIKE PRICE: 50.0
OPTION TYPE: FinOptionTypes.EUROPEAN_CALL
NUMBER: 1.0
```

# Vectorisation

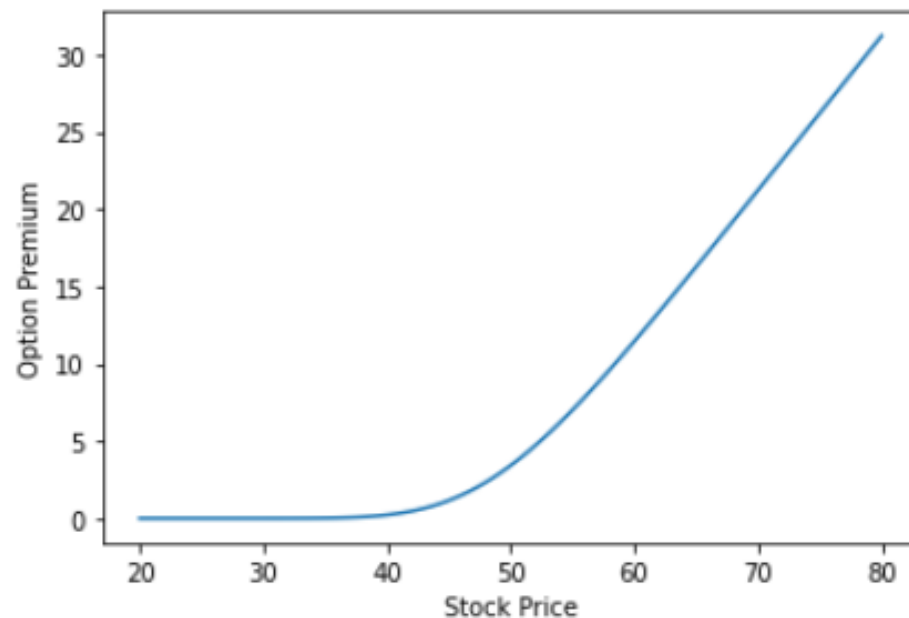
- ❑ Make stock prices a vector - can plot a function with a single call

```
stock_prices = np.linspace(20,80,100)
```

```
value = call_option.value(valuation_date, stock_prices, discount_curve, dividend_curve, model)
```

```
plt.plot(stock_prices, value)  
plt.xlabel("Stock Price")  
plt.ylabel("Option Premium")
```

```
Text(0, 0.5, 'Option Premium')
```





# Interest Rate Swap

- We define a fixed-floating IBOR swap

```
swap_calendar_type = CalendarTypes.NONE
bus_day_adjust_type = BusDayAdjustTypes.FOLLOWING
date_gen_rule_type = DateGenRuleTypes.BACKWARD

fixed_coupon = 0.05
fixed_freq_type = FrequencyTypes.ANNUAL
fixed_day_count_type = DayCountTypes.THIRTY_E_360_ISDA

float_spread = 0.0
float_freq_type = FrequencyTypes.ANNUAL
float_day_count_type = DayCountTypes.THIRTY_E_360_ISDA

swapType = SwapTypes.RECEIVE
notional = ONE_MILLION

start_date = Date(20, 6, 2020)
maturity_date = start_date.add_tenor("5Y")
```

# Creating the IborSwap

- ❑ We create the swap and can examine the payments

```
swap = IborSwap(start_date, maturity_date, swapType,
                fixed_coupon, fixed_freq_type, fixed_day_count_type,
                notional,
                float_spread, float_freq_type, float_day_count_type,
                swap_calendar_type, bus_day_adjust_type, date_gen_rule_type)
```

```
swap._fixed_leg.print_payments()
```

START DATE: 20-JUN-2020

MATURITY DATE: 20-JUN-2025

COUPON (%): 5.0

FREQUENCY: FrequencyTypes.ANNUAL

DAY COUNT: DayCountTypes.THIRTY\_E\_360\_ISDA

PAY_DATE	ACCR_START	ACCR_END	DAYS	YEARFRAC	RATE	PAYMENT
21-JUN-2021	20-JUN-2020	21-JUN-2021	361	1.002778	5.000000	50138.89
20-JUN-2022	21-JUN-2021	20-JUN-2022	359	0.997222	5.000000	49861.11
20-JUN-2023	20-JUN-2022	20-JUN-2023	360	1.000000	5.000000	50000.00
20-JUN-2024	20-JUN-2023	20-JUN-2024	360	1.000000	5.000000	50000.00
20-JUN-2025	20-JUN-2024	20-JUN-2025	360	1.000000	5.000000	50000.00

## We value the Swap using a Flat Curve at 6%

- ❑ We have several ways to construct market discount curves
- ❑ The simplest is the flat discount curve which takes a zero rate

```
from financepy.market.curves.discount_curve_flat import DiscountCurveFlat
```

```
valuation_date = Date(20,6,2018)
settlement_date = valuation_date
```

```
rate = 0.06
discount_curve = DiscountCurveFlat(valuation_date, rate, FrequencyTypes.ANNUAL,
                                   DayCountTypes.THIRTY_E_360_ISDA)
```

```
swap.value(settlement_date, discount_curve, discount_curve)
```

```
-37490.10023311118
```

- ❑ We value the 5% fixed receiver swap with a 6% discount rate
- ❑ The MTM is negative, as expected – a new swap would pay us 6% instead of the 5% we have locked in.

# CDS Contracts can be created

- ❑ We create a CDS contract

```
trade_date = Date(3, 2, 2011)
effective_date = Date(4, 2, 2011)
settlement_date = Date(6, 2, 2011)
```

```
maturity_date = Date(20, 3, 2016)
cdsCoupon = 0.010
notional = ONE_MILLION * 10
long_protection = True
```

```
cds_contract = CDS(effective_date, maturity_date, cdsCoupon, notional, long_protection)
```

- ❑ This is \$10m face amount of long protection 5Y contract with a 100bp running coupon

# We Build a Swap Curve to Discount with

## □ Calibrate to deposits and swaps

```

depos = []
depoDCCType = DayCountTypes.ACT_360

depo = IborDeposit(effective_date, "1M", 0.002630, depoDCCType); depos.append(depo)
depo = IborDeposit(effective_date, "2M", 0.002870, depoDCCType); depos.append(depo)
depo = IborDeposit(effective_date, "3M", 0.003105, depoDCCType); depos.append(depo)
depo = IborDeposit(effective_date, "6M", 0.004608, depoDCCType); depos.append(depo)
depo = IborDeposit(effective_date, "9M", 0.006205, depoDCCType); depos.append(depo)

swaps = []
fixedDCCType = DayCountTypes.THIRTY_E_360_ISDA
fixedFreqType = FrequencyTypes.SEMI_ANNUAL
swapType = SwapTypes.PAY

swap = IborSwap(effective_date, "1Y", swapType, 0.007861, fixedFreqType, fixedDCCType); swaps.append(swap)
swap = IborSwap(effective_date, "2Y", swapType, 0.008799, fixedFreqType, fixedDCCType); swaps.append(swap)
swap = IborSwap(effective_date, "3Y", swapType, 0.013958, fixedFreqType, fixedDCCType); swaps.append(swap)
swap = IborSwap(effective_date, "4Y", swapType, 0.018825, fixedFreqType, fixedDCCType); swaps.append(swap)
swap = IborSwap(effective_date, "5Y", swapType, 0.023251, fixedFreqType, fixedDCCType); swaps.append(swap)

libor_curve = IborSingleCurve(effective_date, depos, [], swaps, interp_type = InterpTypes.LINEAR_FWD_RATES)

```

## □ You can do a two-curve bootstrap too – but this is beyond the scope of this course

## Can then construct a CDS Issuer Curve

- ❑ We can calibrate to a set of CDS to create an issuer curve
- ❑ We need to pass in the IBOR discount curve
- ❑ This is a flat CDS curve at 70bps

```
cds1 = CDS(effective_date, "1Y", 0.0070)
cds2 = CDS(effective_date, "2Y", 0.0070)
cds3 = CDS(effective_date, "3Y", 0.0070)
cds4 = CDS(effective_date, "4Y", 0.0070)
cds5 = CDS(effective_date, "5Y", 0.0070)
```

```
cdss = [cds1, cds2, cds3, cds4, cds5]
```

```
recovery_rate = 0.40
```

```
issuer_curve = CDSCurve(effective_date, cdss, libor_curve, recovery_rate)
```



# CDS Valuation

```
cds_contract.value(settlement_date, issuer_curve, recovery_rate)
```

```
{'full_pv': -157675.65231979737, 'clean_pv': -144897.87454201956}
```

```
cds_contract.print_flows(issuer_curve)
```

PAYMENT_DATE	YEAR_FRAC	FLOW	DF	SURV_PROB	NPV
21-MAR-2011	0.252778	25277.78	0.999660	0.998541	25232.30
20-JUN-2011	0.252778	25277.78	0.998602	0.995596	25131.29
20-SEP-2011	0.255556	25555.56	0.996684	0.992629	25283.05
20-DEC-2011	0.252778	25277.78	0.993950	0.989702	24866.09
20-MAR-2012	0.252778	25277.78	0.990722	0.986783	24712.25
20-JUN-2012	0.255556	25555.56	0.987955	0.983850	24839.98
20-SEP-2012	0.255556	25555.56	0.985537	0.980926	24705.56
20-DEC-2012	0.252778	25277.78	0.983489	0.978042	24314.52
20-MAR-2013	0.250000	25000.00	0.981223	0.975198	23922.17
20-JUN-2013	0.255556	25555.56	0.977037	0.972305	24277.21
20-SEP-2013	0.255556	25555.56	0.971051	0.969420	24056.89
20-DEC-2013	0.252778	25277.78	0.963393	0.966575	23538.44
20-MAR-2014	0.250000	25000.00	0.955926	0.963769	23032.31
20-JUN-2014	0.255556	25555.56	0.948942	0.960912	23302.84
22-SEP-2014	0.261111	26111.11	0.940687	0.958003	23530.84
22-DEC-2014	0.252778	25277.78	0.931648	0.955194	22494.81
20-MAR-2015	0.244444	24444.44	0.923029	0.952486	21490.87
22-JUN-2015	0.261111	26111.11	0.914059	0.949604	22664.28
21-SEP-2015	0.252778	25277.78	0.904536	0.946822	21648.76
21-DEC-2015	0.252778	25277.78	0.894215	0.944049	21339.06
21-MAR-2016	0.252778	25277.78	0.884138	0.941283	21036.78