ALGEBRAIC COMBINATORICS II, HOMEWORK 3
DUE AUGUST 8 AT 5:30PM

Some ground rules:

- Feel free to use English, Chinese, or both, in your solutions.
- Write your argument as clear as possible, and make sure the writing in your submission is clear.
- Feel free to use results that are proved in class. If you'd like to use other results, you have to prove them before using them.
- You're encouraged to work together on the assignments. In your solutions, you should acknowledge the students with whom you worked, and should **write solutions on your own**.

Problems:

(1) Prove that the semidirect product we defined in class is a group (find the identity, inverses; verify associativity, etc.).
(Hint: The inverse of an element (h, k) is not *necessarily (h^{-1}, k^{-1})!)*

(2) Let H and K be two groups, and let $H \rtimes_\varphi K$ be the semidirect product associated to an action $\varphi\colon K \to \mathrm{Aut}(H)$.

Prove that both $\{(h,1) \mid h \in H\}$ and $\{(1,k) \mid k \in K\}$ are subgroups of $H \rtimes_\varphi K$, which isomorphic to H and K, respectively. Also, show that the map $H \rtimes_\varphi K \to K$ defined by $(h,k) \mapsto (1,k)$ is a group homomorphism.

(3) Let $G = G_1 \times G_2$ be the direct product of two groups G_1 and G_2. Prove that the subgroups $G_1 \times \{e_2\}$ and $\{e_1\} \times G_2$ of G are both normal.

(4) For each of the frieze patterns in the next page, find the corresponding IUC notation (cf. the lecture notes for the IUC notations).
(Hint: Each of (p1), (p2), (p11m), (p11g), (p1m1), (p2mm), (p2mg) appears exactly once.)