SECOND MIDTERM PRACTICE PROBLEMS
MATH 185, SECTION 3

(1) Compute the following integrals.

(a)
$$\int_{|z|=2} \frac{e^z}{z(z-1)} dz.$$

(b)
$$\int_{-\infty}^{\infty} \frac{1}{(x+i)(x+2i)} dx.$$

(c)
$$\int_{|z|=1} \frac{1}{\sin(1/z)} dz.$$

(2) For each of the following functions, classify the singularity at the indicated point z_0 as removable, pole, or essential. For poles, give the order of the pole.

(a)
$$f(z) = \frac{1-\cos(z)}{z^3(z-\pi)}, \ z_0 = 0.$$

(b)
$$f(z) = \frac{(z-3)(\sin(\pi z))^2}{z^2 \sin(\pi z)}, \ z_0 = 1.$$

(c)
$$f(z) = \frac{e^{2z} - 1 - 2z}{\sin(z) - z}, \ z_0 = 0.$$

(3) Determine the number of zeros (counting multiplicities) of
$$f(z) = 2(z-1)^3 - e^{-z}$$
inside the open disk $\mathbb{D}_1(1) = \{z \colon |z-1| < 1\}$.

(4) Let f be a holomorphic function on a neighborhood of $\overline{\mathbb{D}}$, such that $|f(z)| = 1$ for $|z| = 1$ and $f(z) \neq 0$ for $|z| < 1$. Prove that f is a constant function.

(5) Let f be an entire function satisfying $|f(2^{-n})| \leq 2^{-n^2}$ for all positive integer n. Prove that $f(z) = 0$ for all $z \in \mathbb{C}$.

(6) Let $f_1, \ldots, f_n$ be holomorphic functions on $\mathbb{D}$. Suppose that $|f_1(z)| + \cdots + |f_n(z)| = 1$ for all $z \in \mathbb{D}$. Prove that $f_1, \ldots, f_n$ are all constant functions.

(7) Let $\Omega \subseteq \mathbb{C}$ be an open subset (not necessarily simply connected), and let $f \colon \Omega \to \mathbb{C}\backslash\{0\}$ be a non-vanishing holomorphic function. Prove that if there exists a non-vanishing holomorphic function $g \colon \Omega \to \mathbb{C}\backslash\{0\}$ such that $f(z) = e^{g(z)}$ for all

$z \in \Omega$, then we have

$$\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}dz = 0$$

for any closed curve γ in Ω. (Note that Ω may not contain the interior of γ.)