REVIEW FOR FIRST MIDTERM
MATH 104

You are expected to be able to...

- State the completeness axiom of $\mathbb{R}$ (§4.4). Use the completeness axiom to prove the denseness of $\mathbb{Q}$ (§4.7), any bounded monotone sequence converges (§10.2).
- State the definition of a convergent sequence (§7.1), a Cauchy sequence (§10.8), the $\liminf$ and $\limsup$ of a bounded sequence (§10.6).
- Prove the convergent of a sequence based on the definition (§8, 9.7), see for instance practice exam Problem 2.
- Prove the limit theorems (§9.1, 9.2, 9.3, 9.4, 9.5, 9.6) based on the definition, see for instance practice exam Problem 1(b).
- Prove the convergent of a sequence by showing that it is bounded and monotone, see for instance §10.2 Example 2, practice exam Problem 3(a), and the example of continued fraction
$$1+\frac{1}{1+\frac{1}{1+\frac{1}{\cdots}}}$$
we discussed in class.
- Prove that all of the following statements are equivalent:
  - (a_n) converges.
  - (a_n) is a Cauchy sequence.
  - (a_n) is bounded and $\liminf a_n = \limsup a_n$.
- Prove basic (but important!) statements like Exercises 8.5(a), 8.9(a), 9.12(a).
- Understand possible subsequential limits of a bounded sequence.