HOMEWORK 4
MATH H54

Office Hours: Tuesday 2:30-4pm and Wednesday 5:15-6:45pm at 735 Evans.
Kubrat's Office Hours: Friday 9-11am at 741 Evans.

Submit your homework at the beginning of the discussion section on Wednesday. *Late homework will not be accepted under any circumstances.*

You are encouraged to discuss the problems with your classmates, but you must write your solutions on your own and acknowledge collaborators/cite references if any.

Write clearly! Mastering mathematical writing is one of the goals of this course.

You have to staple your work if it is more than one page.

The following exercises are from the corresponding sections of the UC Berkeley custom edition of Lay, Nagle, Saff, Snider, *Linear Algebra and Differential Equations*. Note that the section numbers and problem numbers may not be the same as in Lay, *Linear Algebra*.

Due September 25:

- **Exercise 3.2:** 24, 34, 43
- **Exercise 3.3:** 24, 30
- **Exercise 4.1:** 16, 22, 32, 33, 34
- **Exercise 4.2:** 4, 16, 28, 35, 36
- **Additional Problem 1:** Show that if A is invertible and both A and A^{-1} have only integer entries, then $\det A = \pm 1$.
- **Additional Problem 2:** Prove that the set

$$\mathrm{SL}_n(\mathbb{Z}) := \{A \in M_{n\times n}(\mathbb{Z}) : \det A = 1\}$$

of $n \times n$ matrices whose entries are all integers and has determinant 1 forms a *group*. In particular, prove that:

  - If $A, B \in \mathrm{SL}_n(\mathbb{Z})$, then $AB \in \mathrm{SL}_n(\mathbb{Z})$;
  - If $A \in \mathrm{SL}_n(\mathbb{Z})$, then $A^{-1} \in \mathrm{SL}_n(\mathbb{Z})$.

The second part is actually Exercise 3.3.18 in the textbook.