HOMEWORK 2
MATH 104, SECTION 2

Some ground rules:

- You have to submit your homework via **Gradescope** to the corresponding assignment. The submission should be a **single PDF file**.
- Make sure the writing in your submission is clear enough! Answers which are illegible for the reader won't be given credit.
- Write your argument as clear as possible. Mastering mathematical writing is one of the goals of this course.
- Late homework will not be accepted under any circumstances.
- You're allowed to use any result that is proved in the lecture; but if you'd like to use other results, you have to prove them before using them.

Problem set (6 problems; due February 2 at 11am PT)

(1) Determine each of the following sequences is convergent or divergent. For convergent sequences, find the limit and prove it. For divergent sequences, prove that they are divergent.
   (a) $a_n = (\frac{2}{3})^n$.
   (b) $b_n = 2^n$.
   (c) $c_n = \frac{\sin(2n)}{\sqrt{n}}$.
   (d) $d_n = \sin(\frac{n\pi}{2})$.
   (e) $e_n = \sqrt{n^2+4n} - n$.
   (f) $f_n = \frac{2^n}{n!}$.

(2) (Squeeze lemma, **very useful**) Let (a_n) , (b_n) , (c_n) be three sequences satisfying $a_n \leq b_n \leq c_n$ for all n . Suppose that (a_n) and (c_n) both converge with $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = a$. Prove that $\lim_{n\to\infty} b_n = a$.

(3) Let $a_n = \frac{n-\sin(n)}{n}$. Use the squeeze lemma to show that a_n converges and find the limit.

(4) Let $a_1 = 3$ and $a_{n+1} = \sqrt{3a_n + 10}$ for $n \geq 1$. Prove that (a_n) converges, and find the limit.

(5) Show that if (a_n) converges to a , then the sequence of absolute values $(|a_n|)$ converges to $|a|$. Is the converse statement true?

(6) Let (a_n) be a sequence of nonzero real numbers. Suppose that $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = b$ exists and is less than 1 . Prove that $\lim_{n\to\infty} a_n = 0$. (Hint: Choose any c so that $b < c < 1$ and show that there exists $N > 0$ such that $|a_{n+1}| < c|a_n|$ for all $n > N$.)