#1. Find $y(t)$ satisfying:

$$y'' - 2y' + y = -2e^t, \quad y(0) = 1, \quad y'(0) = 0.$$

Sol^n: By the computations we did in Lecture 23,

$$y(t) = -t^2 e^t$$

is a sol^n to the diff'l eq'n. Hence the general sol^n are:

$$y(t) = c_1 e^t + c_2 t e^t - t^2 e^t$$

$$= e^t (c_1 + c_2 t - t^2).$$

$$\Rightarrow y'(t) = e^t (c_1 + c_2 t - t^2 + c_2 - 2t)$$

$$\begin{cases} 1 = y(0) = c_1 \\ 0 = y'(0) = c_1 + c_2 \end{cases} \Rightarrow \begin{cases} c_1 = 1 \\ c_2 = -1 \end{cases}$$

$\Rightarrow$ the sol^n to the initial value problem is:

$$y(t) = e^t (1 - t - t^2). \quad \square$$

#2: Find $y(t)$ satisfying: (on $(0, \infty)$)

$$y'' - 4y' + 4y = -t^{-2} e^{2t}, \quad y(1) = 0, \quad y'(1) = e^2.$$

Sol^n: Variation of parameters: Let $y_1 = e^{2t}$, $y_2 = te^{2t}$. $f = -t^{-2} e^{2t}$.

$$c_1' = \frac{-(-t^{-2} e^{2t}) t e^{2t}}{e^{2t}(e^{2t}(1+2t)) - te^{2t} \cdot 2e^{2t}} = \frac{1}{t}$$

$$c_2' = \frac{-t^{-2} e^{2t} \cdot e^{2t}}{\text{same denominator}} = -\frac{1}{t^2}.$$

$c_1 = \log t, \quad c_2 = \frac{1}{t}.$

$\Rightarrow \quad y(t) = (\log t) \cdot e^{2t} + \frac{1}{t} \cdot (t e^{2t})$

$= e^{2t} (\log t + 1)$ is a sol^{n} to the diff^{l} eq^{n}

$\Rightarrow$ general sol^{n}:
$y = e^{2t} (c_1 + c_2 t + \log t)$

$y' = e^{2t} (2c_1 + 2c_2 t + 2 \log t$
$+ c_2 + \frac{1}{t})$

$$\Rightarrow \begin{cases} 0 = y(1) = e^2 (c_1 + c_2) \\ e^2 = y'(1) = e^2 (2c_1 + 3c_2 + 1) \end{cases}$$

$\Rightarrow \quad c_1 = c_2 = 0.$

$\Rightarrow \quad y(t) = \underline{\underline{e^{2t} \log t}}. \quad \square$