HOMEWORK 6
MATH H54

Office Hours: Tuesday 2:30-4pm and Wednesday 5:15-6:45pm at 735 Evans.
Kubrat's Office Hours: Friday 9-11am at 741 Evans.

Submit your homework at the beginning of the discussion section on Wednesday. *Late homework will not be accepted under any circumstances.*

You are encouraged to discuss the problems with your classmates, but you must write your solutions on your own and acknowledge collaborators/cite references if any.

The following exercises are from the corresponding sections of the UC Berkeley custom edition of Lay, Nagle, Saff, Snider, *Linear Algebra and Differential Equations*. Note that the section numbers and problem numbers may not be the same as in Lay, *Linear Algebra*.

Due October 16:

- **Exercise 5.1:** 26, 27, 29, 31
- **Exercise 5.2:** 14, 18, 19
- **Exercise 5.3:** 23, 26, 27, 28, 31, 32
- **Exercise 5.4:** 22, 25
- **Additional Problem 1:** Show that if A and B are similar, then they have the same rank. (Hint: Use $A = PBP^{-1}$ and $B = P^{-1}AP$.)
- **Additional Problem 2:** The *trace* of a square matrix A is the sum of the diagonal entries in A, and is denoted by $\mathrm{tr}A$. Show that $\mathrm{tr}A$ equals the sum of the eigenvalues of A. (Hint: Look at the characteristic polynomial of A.)
- **Challenge Problem (no need to turn in):** Show that $\mathrm{tr}(AB) = \mathrm{tr}(BA)$ holds for any square matrices A and B. Moreover, show that if a function $f : M_{n\times n}(\mathbb{R}) \to \mathbb{R}$ satisfies:
  (1) $f(A+B) = f(A) + f(B)$;
  (2) $f(cA) = cf(A)$ for any $c \in \mathbb{R}$;
  (3) $f(AB) = f(BA)$;
  (4) $f(\mathbb{I}_n) = n$,
  then $f = \mathrm{tr}$. In other words, (1)–(4) gives a characterization of the trace function on square matrices.