

**HOMEWORK 3: COMMON MISTAKES**  
**MATH 104, SECTION 6**

Problem (3): (Squeeze lemma) Let  $(a_n)$ ,  $(b_n)$ ,  $(c_n)$  be three sequences satisfying  $a_n \leq b_n \leq c_n$  for all  $n$ . Suppose that  $(a_n)$  and  $(c_n)$  both converge with  $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} c_n = a$ . Prove that  $\lim_{n \rightarrow \infty} b_n = a$ .

- Simply saying that there exists some  $N > 0$  and then apply it to both  $(a_n)$  and  $(c_n)$  is not good enough. For any  $\epsilon > 0$ , you'll get some  $N_1 > 0$  for  $(a_n)$  and some  $N_2 > 0$  for  $(c_n)$ , and then you can take  $N = \max\{N_1, N_2\}$ .

Problem (5): Let  $(a_n)$  and  $(b_n)$  be two convergent sequences with limits  $a$  and  $b$  respectively. Suppose that  $a_n \leq b_n$  for all but finitely many  $n$ . Prove that  $a \leq b$ .

- the most common error was excluding the part about there exists  $M > 0$  such that  $a_n \leq b_n$  for all  $n > M$ , or forgetting to include it in the maximum  $N = \max\{M, N_1, N_2\}$ .