HOMEWORK 1
MATH H54

Office Hours: Tuesday 2:30-4pm and Wednesday 5:15-6:45pm at 735 Evans.

Submit your homework at the beginning of the discussion section on Wednesday. *Late homework will not be accepted under any circumstances.*

You are encouraged to discuss the problems with your classmates, but you must write your solutions on your own and acknowledge collaborators/cite references if any.

Write clearly! Mastering mathematical writing is one of the goals of this course.

The following exercises are from the corresponding sections of the UC Berkeley custom edition of Lay, Nagle, Saff, Snider, *Linear Algebra and Differential Equations*. Note that the section numbers and problem numbers may not be the same as in Lay, *Linear Algebra*.

Due September 4:

- **Exercise 1.1:** 12, 20, 32
- **Exercise 1.2:** 12, 20, 29, 30, 31
- **Exercise 1.3:** 14, 18, 20, 26
- **Additional Problem 1:** Prove that any system of linear equations
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$
has either no solution, a unique solution, or infinitely many solutions. (Hint: Prove that if $\vec{u}, \vec{v}$ are solutions of the linear system, then so is $t\vec{u} + (1-t)\vec{v}$ for any $t \in \mathbb{R}$. What's the geometric interpretation of $t\vec{u} + (1-t)\vec{v}$?)
- **Additional Problem 2:** We say two matrices are *row equivalent* if there is a sequence of elementary row operations that transforms one matrix to the other. Prove that if the augmented matrices of two linear systems are row equivalent, then the linear systems have the same solution set.