#1: Amy 
$$\vec{w} \in W$$
 can be written as  $\vec{w} = a_1 \vec{v}_1 + \cdots + a_n \vec{v}_n$ .  
 $\Rightarrow (\vec{x}, \vec{w}) = (\vec{x}, a_1 \vec{v}_1 + \cdots + a_n \vec{v}_n)$ 

$$= a_1 (\vec{x}, \vec{v}_1) + \cdots + a_n (\vec{x}, \vec{v}_n) = 0. \square$$

セス;

(A) If 
$$\vec{x}_1, \vec{x}_2 \in W^{\perp}$$

then . \$1+ \$2 & W SINCE (\$\vec{1}{2}1+\vec{1}{2}1,\vec{1}{2})=(\vec{1}{2}1,\vec{1}{2})+(\vec{1}{2}1,\vec{1}{2})=0.

· CŽIEW suce (CŽI, Z)= C(ŽI, Ž)=0 YEW.

then  $(\vec{x}, \vec{x}) = 0$ .  $\Rightarrow \vec{x} = \vec{o}$ .  $\Box$ 

<u>#3</u>.

Unthogonal = uTu=In

⇒ U.is invertible and u = uT

$$\Rightarrow (\mathcal{U})^T \mathcal{U} = \mathcal{I}_n$$

> UT is orthogonal

> Ut has orthonormal columns.

> U has orthonormal rows. D

 $\frac{\# 4}{1}$ :  $u_1^T = u_1^T$ ,  $u_2^T = u_2^T$ 

 $(u_1 u_2)(u_2^T u_1^T) = I_n.$ 

. (u,u,)<sup>T</sup>.

#5: Notice that  $\langle \vec{z}_1, \vec{v}_2 \rangle_A = (A\vec{z}_1)^T (A\vec{z}_2) = \langle A\vec{z}_1, A\vec{v}_2 \rangle_A$ Standard inner product on  $\mathbb{R}^n$ .

•  $\langle \vec{7}, \vec{7} \rangle_{A} = \langle A\vec{7}, A\vec{7} \rangle = 0$ If and only if  $A\vec{7} = \vec{7}$ .

⇒ V=0 since A is invertible.

· rest of the axioms are easy to check. [

 $\frac{\# b}{\|\vec{c}_{(1)}\vec{c}_{(1)}\|^{2}+\|\vec{c}_{(1)}-\vec{c}_{(1)}\|^{2}} = \langle \vec{c}_{(1)}\vec{c}_{(2)}, \vec{c}_{(1)}\vec{c}_{(2)} \rangle + \langle \vec{c}_{(1)}\vec{c}_{(2)}, \vec{c}_{(2)}\rangle + \langle \vec{c}_{(2)}\vec{c}_{(2)}, \vec{c}_{$ 

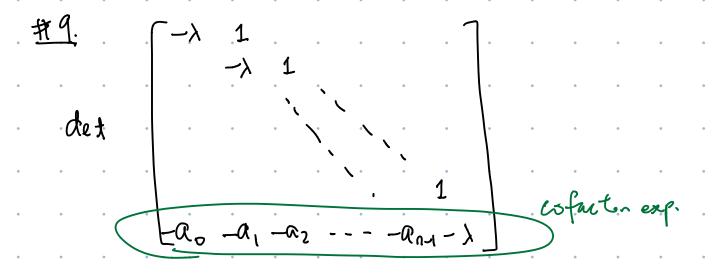
 $\frac{\#7}{||\vec{v}||} > ||pwj = pan \{\vec{v}_i\} (|\vec{v}_i)||, (equality holds <math>\Leftrightarrow$   $\{\vec{v}_i, \vec{v}_i\} \ l.d.$ 

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 $\frac{|\langle \vec{x}, \vec{z} \rangle|}{||\vec{x}||^2} ||\vec{x}|| = \frac{|\langle \vec{x}, \vec{x} \rangle|}{||\vec{x}||}$ 

 $\pm 8$ :  $\|\vec{y}_1 + \vec{y}_2\|^2 = \|\vec{y}_1\|^2 + 2(\vec{y}_1, \vec{y}_2) + \|\vec{y}_2\|^2$ 

 $(B_1 # \frac{1}{2}) \rightarrow \leq (|C_1|^2 + 2|C_1||C_1| + ||C_1|^2 + (|C_1| + ||C_1|^2 + ||C_1||^2 +$ 



$$= (1)^{n+1}(-a_0) \cdot 1 + (-1)^{n+2}(-a_1)(-\lambda) + \cdots + (1)^{n+n-1}(-a_{n-2})(-\lambda)$$

$$+ (-a_{n-1} - \lambda)(-\lambda)^{n-1}$$

= 
$$(1)^{n} a_{0} + (1)^{n} a_{1} \lambda + \cdots + (1)^{n} a_{n-2} \lambda^{n-2} + (1)^{n} a_{n-1} \lambda^{n} + (1)^{n} \lambda^{n}$$

- # 10. (a) follows from the fact that the only continuous fun  $\int_{-1}^{1} f(x)^2 dx = 0$  is the zero fun.
  - (b) follous from certain trigonometre identities.