RESEARCH STATEMENT

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My research interest lies broadly in *algebraic geometry* and *dynamical systems*. The present text starts with a very brief overview of some of my work in the first two pages, then follows with more details of these works, and ends with summary of my ongoing projects.

(§1) One of my main research interests is the study of relationships among *derived categories*, *birational geometry*, and *symplectic geometry*. In particular:

- In [FL20] (see §1.1), we prove the rationality of certain cubic fourfolds (that is not known before) via the technique of Cremona transformations. More precisely, we show that there is a birational involution on the space of cubic fourfolds containing a Veronese surface, and use the involution to produce new rational cubic fourfolds. Moreover, we prove the conjectural relationship between birational geometry of cubic fourfolds and their derived categories for these cubics.
- In [FHLY18] (see §1.2), we construct an operation on symplectic 6-manifolds, which is compatible with the well-known operation, Atiyah flop, in algebraic geometry, under mirror symmetry relating algebraic and symplectic geometry. The operation we constructed can also be interpreted as wall-crossing of Bridgeland stability conditions on certain Fukaya categories.

(§2) Another of my main research areas is *categorical dynamical systems*, which studies the properties of endofunctors of triangulated categories under large iterations. My work on this subject demonstrates its close relationships with holomorphic dynamics, Teichmüller theory, symplectic mapping class groups, and rotation theory among others. In particular:

- In [Fan18a] (see §2.1), I examine categorical dynamical systems which encompass both holomorphic and symplectic dynamical systems, and prove that the systems are strictly more complicated than holomorphic dynamical systems alone in the sense of *entropy*. A class of examples of such triangulated categories is provided by the derived categories of Calabi–Yau manifolds and mirror symmetry between algebraic and symplectic geometry.
- In [FF20] (see §2.2), we introduce new canonical invariants, called the *shifting numbers*, that measure the asymptotic amount by which an endofunctor is translates inside the triangulated category. These invariants are exactly the complement of the *entropy*, and can be considered as the categorical generalization of the Poincaré rotation numbers in classical dynamics. Moreover, we show that the shifting numbers give rise to quasimorphisms on the group of autoequivalences in certain examples.
- In [FFH+19] (see §2.3), we propose a categorical generalization of the notion of *pseudo-Anosov maps* in the theory of mapping class groups of Riemann surfaces. These mapping classes are important since they are generic in the mapping class groups. Therefore, the property we describe is conjecturally satisfied by a *generic* autoequivalence of certain triangulated categories. We provide examples of pseudo-Anosov autoequivalences of certain Calabi–Yau categories of dimension at least three to justify the genericity in these cases.

(§3) Recently, I work on the *arithmetic dynamics* of *local systems on Riemann surfaces*, and their connection with *Stokes matrices* and *exceptional collections* among others.

- Moduli spaces of points on n-spheres carry natural actions of braid groups. In [FW20] (see §3), we prove that for $n = 0, 1,$ or 3, these symmetries extend to actions of mapping class groups of positive genus surfaces, through braid group equivariant isomorphisms with certain moduli of $\mathrm{SL}_2(\mathbb{C})$-local systems on the surfaces. Moreover, the Coxeter invariants on moduli of points on spheres are identified with the boundary traces of the surfaces. As a corollary of arithmetic dynamical results of character varieties, we prove the finiteness of possible Gram matrices of certain exceptional collections of length 4 up to mutations.

(§4) I also work on *Bridgeland stability conditions* on triangulated categories. My work introduces new geometric structures/invariants on stability conditions motivated from their connections with Teichmüller theory and Calabi–Yau geometry. In particular:

- In [Fan18b] (see §4.1), I introduce the notions of systole and volume of Bridgeland stability conditions, and prove the systolic inequality relating systole and volume for any K3 surface. This is a categorical generalization of the classical systolic inequality on 2-torus by Loewner: $\mathrm{sys}(\mathbb{T}^2, g)^2 \leq \frac{2}{\sqrt{3}}\mathrm{vol}(\mathbb{T}^2, g)$.
- In [FKY17] (see §4.2), we introduce the notion of Weil–Petersson geometry on the stability manifolds, and prove that they coincide with the classical Weil–Petersson metric on the moduli of Calabi–Yau manifolds under mirror symmetry for certain examples.

1. Derived categories, birational geometry, and symplectic geometry

1.1. **Cremona transformations and cubic fourfolds.** The relationship between derived categories and birational geometry has been an active and fruitful research area in the last two decades or so. In 2008, Kuznetsov proposed a conjecture that relates the rationality problem of cubic fourfolds, one of the most important problems in birational geometry, with a property of their derived categories. For any cubic fourfold $X \subseteq \mathbb{P}^5$, its bounded derived category of coherent sheaves admits a semiorthogonal decomposition $\mathcal{D}^b\mathrm{Coh}(X) = \langle \mathcal{A}_X, \mathcal{O}, \mathcal{O}(1), \mathcal{O}(2)\rangle$, where $\mathcal{A}_X$ is a K3 category. Kuznetsov conjectured that X is rational if and only if $\mathcal{A}_X \cong \mathcal{D}^b\mathrm{Coh}(S)$ for some K3 surface S. Another conjecture that relates birational geometry and derived categories of cubic fourfolds can be formulated as follows.

Question 1.1 (Huybrechts). *Let X and X' be cubic fourfolds with equivalent associated K3 categories $\mathcal{A}_X \cong \mathcal{A}_{X'}$. Does this imply that X and X' are birational?*

We give an affirmative answer to the above question when one of the cubic fourfolds lies in $\mathcal{C}_V$, the divisor in the moduli space of cubic fourfolds that parametrizes cubics containing a Veronese surface $\mathbb{P}^2 \cong V \subseteq X \subseteq \mathbb{P}^5$. The key tool in our argument is certain Cremona transformations on $\mathbb{P}^5$ defined as follows. Recall that a Veronese surface $V \subseteq \mathbb{P}^5$ is given by six quadratic defining polynomials $Q_0 = \cdots = Q_5 = 0$. One can use these polynomials to define a birational map

$$F := (Q_0, ..., Q_5) : \mathbb{P}^5 \dashrightarrow \mathbb{P}^5.$$

Theorem 1.2 (F.–Lai [FL20]). *Let $X \in \mathcal{C}_V$ be a generic cubic fourfold containing a Veronese surface, then*

- *its strict transform $X' = F(X)$ is again a cubic fourfold in $\mathcal{C}_V$,*
- *the induced birational involution on $\mathcal{C}_V$ is non-trivial (i.e. $X \ncong X'$),*
- *their K3 categories are equivalent $\mathcal{A}_X \cong \mathcal{A}_{X'}$, and*
- *there is no other cubic fourfolds Y such that $\mathcal{A}_X \cong \mathcal{A}_Y$.*

This gives an affirmative answer to Huybrechts' question for generic cubic fourfolds in $\mathcal{C}_V$.

As a byproduct of the birational involution on $\mathcal{C}_V$ we constructed, we prove the rationality of certain cubic fourfolds that are not known before.

Theorem 1.3 (F.–Lai [FL20]). *For each $d = 26, 38, 42$, the birational involution on $\mathcal{C}_V$ maps a component of $\mathcal{C}_V \cap \mathcal{C}_d$ birationally onto a component $\overline{\mathcal{C}_V} \cap \mathcal{C}_{d'}$, where d' cannot be in the list $\{2, 6, 8, 14, 18, 26, 38, 42\}$. Therefore, there exists at least three irreducible divisors in $\mathcal{C}_V$ parametrizing rational cubic fourfolds which are not known before.*

1.2. **The mirror operation of Atiyah flop in symplectic geometry.** The idea of *mirror symmetry* was originated from physics. Its mathematical formulations were proposed by Kontsevich and Strominger–Yau–Zaslow around 1995. They are best understood as conjectural dualities between *algebraic* and *symplectic geometry*. A natural question arising from the mirror symmetry conjectures is that, given an "object" (e.g. operation, invariant, etc.) in algebraic (resp. symplectic) geometry, can one construct its *mirror* counterpart in symplectic (resp. algebraic) geometry?

We construct the mirror operation of one of the most fundamental operations in birational geometry, the *Atiyah flop*. Recall that an Atiyah flop $\hat{X} \to X \leftarrow \hat{X}^\dagger$ contracts a $(-1,-1)$-rational curve C in a complex threefold $\hat{X}$ and resolves the resulting conifold singularity with another $(-1,-1)$-curve $C^\dagger$.

Theorem 1.4 (F.–Hong–Lau–Yau [FHLY18])**.** *Given a symplectic sixfold (Y,ω) and a Lagrangian three-sphere $S \subseteq Y$, we construct another symplectic sixfold $(Y^\dagger,\omega^\dagger)$ with a corresponding Lagrangian three-sphere $S^\dagger \subset Y^\dagger$, together with a symplectomorphism $f^{(Y,S)} : (Y,\omega) \to (Y^\dagger,\omega^\dagger)$. It has the property that $f^{(Y^\dagger,S^\dagger)} \circ f^{(Y,S)} = \tau_S^{-1}$, where τ_S is the Dehn twist along S.*

The symplectomorphism $f^{(Y,S)}$ in Theorem 1.4 is the *mirror of Atiyah flop*. The contracted $(-1,-1)$-curve in algebraic geometry corresponds to the Lagrangian three-sphere in symplectic geometry. Recall that threefolds related by a flop are derived equivalent by a result of Bridgeland. The property $f^{(Y^\dagger,S^\dagger)} \circ f^{(Y,S)} = \tau_S^{-1}$ is *mirror* to the fact that the composition of two flop functors $\mathcal{D}^b(\hat{X}) \xrightarrow{\simeq} \mathcal{D}^b(\hat{X}^\dagger) \xrightarrow{\simeq} \mathcal{D}^b(\hat{X})$ is the inverse of the Seidel–Thomas spherical twist by $\mathcal{O}_C(-1)$.

Let $\mathcal{D}_{\hat{X}/X} \subset \mathcal{D}^b(\hat{X})$ be the subcategory which consists of objects supported on C. Then Bridgeland's equivalence restricts to $\mathcal{D}_{\hat{X}/X} \cong \mathcal{D}_{\hat{X}^\dagger/X}$. It is proved by Chan–Pomerleano–Ueda that $\mathcal{D}_{\hat{X}/X}$ is equivalent to certain derived Fukaya category $\mathcal{D}^b\mathcal{F}_Y$. We prove the following compatibility result between the Atiyah flop and our mirror Atiyah flop.

Theorem 1.5 (F.–Hong–Lau–Yau [FHLY18])**.** *The symplectomorphism $f^{(Y,S)}$ induces an equivalence between the derived Fukaya categories $\mathcal{D}^b\mathcal{F}_Y \cong \mathcal{D}^b\mathcal{F}_{Y^\dagger}$. Moreover, the equivalence is the same as the composition $\mathcal{D}^b\mathcal{F}_Y \cong \mathcal{D}_{\hat{X}/X} \cong \mathcal{D}_{\hat{X}^\dagger/X} \cong \mathcal{D}^b\mathcal{F}_{Y^\dagger}$, where the first and third equivalences are given by Chan–Pomerleano–Ueda, and the second equivalence is Bridgeland's flopping equivalence.*

Note that unlike the Atiyah flop which produces different complex manifolds, its mirror $f^{(Y,S)}$ is a symplectomorphism. It is not very surprising since symplectic geometry is much softer than complex geometry. However, the transitions between different varieties on the algebraic side appear as *wall-crossings of Bridgeland stability conditions* on the symplectic side.

Theorem 1.6 (F.–Hong–Lau–Yau [FHLY18])**.** *Let $Y = \{u_1v_1 = z+q, u_2v_2 = z+1, z \neq 0\}$ be the deformed conifold and $\Omega_Y = dz \wedge du_1 \wedge du_2$ be a holomorphic volume form on Y. Then there exists a collection $\mathcal{P}$ of graded special Lagrangian submanifolds which defines a geometric stability condition $(\mathcal{Z},\mathcal{P})$ on $\mathcal{D}^b\mathcal{F}_Y$. Moreover, the mirror Atiyah flop $f^{(Y,S)}$ defines another geometric stability condition $(\mathcal{Z}^\dagger,\mathcal{P}^\dagger)$ with respect to $(f^{(Y,S)})^*\Omega_{Y^\dagger}$. Finally, $(\mathcal{Z},\mathcal{P})$ and $(\mathcal{Z}^\dagger,\mathcal{P}^\dagger)$ are related by a wall-crossing in the space of Bridgeland stability conditions* $\mathrm{Stab}(\mathcal{D}^b\mathcal{F}_Y)$*, which matches with wall-crossing in* $\mathrm{Stab}(\mathcal{D}_{\hat{X}/X})$ *on the mirror in a work of Toda.*

Another way to see the effect of mirror Atiyah flop is by equipping the symplectic sixfold Y with a Lagrangian fibration. See [FHLY18, Theorem 1.2] for more details.

2. Categorical dynamical systems

A pair $(\mathcal{D},F)$ is called a *categorical dynamical system* if $\mathcal{D}$ is a triangulated category and $F\colon \mathcal{D} \to \mathcal{D}$ is an endofunctor of $\mathcal{D}$. One of the goals in this subject is to study long-term behaviors of large iterations $F \circ \cdots \circ F\colon \mathcal{D} \to \mathcal{D}$. This is a fascinating research field because it encompasses holomorphic dynamics, Teichmüller theory, and symplectic mapping class groups among others. For instance, given a holomorphic self-map of a smooth complex projective variety $f\colon X \to X$, one can associate the categorical dynamical system $\mathbb{L}f^*\colon \mathcal{D}^b\mathrm{Coh}(X) \to \mathcal{D}^b\mathrm{Coh}(X)$ on the bounded derived category of coherent sheaves on X via the pullback functor.

2.1. **Categorical dynamical systems of Calabi–Yau manifolds.** In this work, we study categorical dynamical systems via the notion of *categorical entropy*. We start with recalling a fundamental result of Gromov and Yomdin on the *topological entropy* of holomorphic dynamical systems. Let X be a compact

Kähler manifold and $f\colon X \to X$ be a holomorphic self-map. Then the topological entropy $h_{\mathrm{top}}(f)$, which roughly speaking measures the complexity of the dynamical system formed by the iterations of f, satisfies

$$h_{\mathrm{top}}(f) = \log \rho(f^*).$$

Here f^* denotes the induced linear map on $H^*(X;\mathbb{C})$, and $\rho(f^*)$ is its spectral radius.

Now we consider a categorical dynamical system $(\mathcal{D}, F)$, i.e an endofunctor $F\colon \mathcal{D} \to \mathcal{D}$ of a triangulated category $\mathcal{D}$. There is a notion of *categorical entropy* of F, denoted by $h_{\mathrm{cat}}(F)$, introduced by Dimitrov, Haiden, Katzarkov, and Kontsevich in 2014. It is the categorical generalization of the notion of topological entropy. For instance, it is proved by Kikuta and Takahashi that if $\mathcal{D} = \mathcal{D}^b\mathrm{Coh}(X)$ and $F = \mathbb{L}f^*$, then $h_{\mathrm{cat}}(\mathbb{L}f^*) = h_{\mathrm{top}}(f)$. It is then natural to make the following conjecture which generalizes the result of Gromov and Yomdin.

Conjecture 2.1 (Kikuta–Takahashi). *Let X be a smooth complex projective variety and $F\colon \mathcal{D}^b(X) \to \mathcal{D}^b(X)$ be an autoequivalence. Then*

$$h_{\mathrm{cat}}(F) = \log \rho([F]).$$

Here $[F]$ denotes the induced linear map on the numerical Grothendieck group of $\mathcal{D}^b(X)$.

I find the first counterexamples of Conjecture 2.1. Heuristically, this shows that categorical dynamical systems on $\mathcal{D}^b(X)$ are in general more complicated than holomorphic dynamical systems on X. The counterexamples arise from a combination of *holomorphic* and *symplectic* dynamical systems. The source of such examples are provided by *Calabi–Yau manifolds* and the *mirror symmetry conjecture*. Let X be a Calabi–Yau manifold. It is conjectured by mirror symmetry that there exists another Calabi–Yau Y, such that $\mathcal{D}^b\mathrm{Coh}(X) \cong \mathcal{D}^\pi\mathrm{Fuk}(Y)$, where $\mathcal{D}^b\mathrm{Fuk}$ denotes the derived Fukaya category. Note that any symplectomorphism of Y induces an autoequivalence of $\mathcal{D}^\pi\mathrm{Fuk}(Y)$. Therefore, the triangulated category $\mathcal{D} := \mathcal{D}^b\mathrm{Coh}(X) \cong \mathcal{D}^\pi\mathrm{Fuk}(Y)$ admits autoequivalences arise from both holomorphic dynamics of X and symplectomorphisms of Y.

Theorem 2.2 (F. [Fan18a]). *Let X be a Calabi–Yau hypersurface in $\mathbb{P}^{d+1}$ and $d \geq 4$ be an even integer. Consider the autoequivalence $F := T_{\mathcal{O}_X} \circ (- \otimes \mathcal{O}(-1))$ on $\mathcal{D}^b(X)$, where $T_{\mathcal{O}_X}$ is the spherical twist with respect to the structure sheaf $\mathcal{O}_X$. Then*

$$h_{\mathrm{cat}}(F) > 0 = \log \rho([F]).$$

This gives a counterexample to Conjecture 2.1. In fact, $h_{\mathrm{cat}}(F)$ is given by the unique positive real number $\lambda > 0$ satisfying $\sum_{k\geq 1} \frac{\chi(\mathcal{O}(k))}{e^{k\lambda}} = 1$.

Note that in the construction of F, the autoequivalence $(- \otimes \mathcal{O}(-1))$ is one of the *standard* autoequivalences of $\mathcal{D}^b(X)$ arises from complex geometry of X; while the spherical twist $T_{\mathcal{O}}$, first studied by Seidel and Thomas, corresponds to Dehn twist along a Lagrangian sphere in a mirror Calabi–Yau of X under mirror symmetry, which should be considered as an input from the symplectic mapping class group of the mirror. Theorem 2.2 shows that categorical dynamical systems formed by combinations of holomorphic and symplectic ones in general can have larger entropy than expected from the theory of holomorphic dynamics.

2.2. **Shifting numbers of autoequivalences.** In this work, we introduce new canonical invariants, the *shifting numbers*, that measure the asymptotic amount by which an endofunctor $F\colon \mathcal{D} \to \mathcal{D}$ is *translates* inside the triangulated category. The concept is analogue to the notion of *Poincaré translation number* which we now recall. There is a central extension of the group of orientation-preserving homeomorphisms of the circle $S^1 \cong \mathbb{R}/\mathbb{Z}$:

$$0 \to \mathbb{Z} \to \mathrm{Homeo}^+_Z(\mathbb{R}) \to \mathrm{Homeo}^+(\mathbb{R}/\mathbb{Z}) \to 1.$$

The Poincaré translation number of $f \in \mathrm{Homeo}^+_{\mathbb{Z}}(\mathbb{R})$ is defined by $\rho(f) := \lim_{n\to\infty}(f^{(n)}(x_0) - x_0)/n$ for some $x_0 \in \mathbb{R}$. It is a classical result that the limit exists and is independent of the choice of x_0.

Let $\mathcal{D}$ be a ($\mathbb{Z}$-graded) triangulated category. We have a similar central extension

$$0 \to \mathbb{Z} \to \mathrm{Aut}(\mathcal{D}) \to \mathrm{Aut}(\mathcal{D})/[1] \to 1,$$

in which the shift functor [1] plays the role of the integral shift by 1 in the classical setting. Instead of the basepoint $x_0 \in \mathbb{R}$, we consider a split generator $G \in \mathcal{D}$, and define

$$\epsilon^+(G, F^n G) \coloneqq \max\{k \colon \mathrm{Hom}(G, F^n G[-k]) \neq 0\} \text{ and } \epsilon^-(G, F^n G) \coloneqq \min\{k \colon \mathrm{Hom}(G, F^n G[-k]) \neq 0\},$$

which serve as categorical analogues of the difference "$(f^{(n)}(x_0) - x_0)$". We then establish the definition of shifting numbers of endofunctors of triangulated categories.

Theorem 2.3 (F.–Filip [FF20])**.** *Let $F \colon \mathcal{D} \to \mathcal{D}$ be an endofunctor of a triangulated category $\mathcal{D}$ and let G be a split generator of $\mathcal{D}$.*

(1) The following limits exist and are finite real numbers:

$$\tau^+(F) \coloneqq \lim_{n\to\infty} \frac{\epsilon^+(G, F^n G)}{n} \text{ and } \tau^-(F) \coloneqq \lim_{n\to-\infty} \frac{\epsilon^-(G, F^n G)}{n}.$$

They are defined be to the upper/lower shifting numbers *of F.*

(2) The shifting numbers can be computed via the categorical entropy function *$h_t(F)$: We prove that $h_t(F)$ is a real-valued convex function that satisfies*

$$t \cdot \tau^+(F) \leq h_t(F) \leq h_0(F) + t \cdot \tau^+(F) \quad \text{for} \quad t \geq 0,$$

$$t \cdot \tau^-(F) \leq h_t(F) \leq h_0(F) + t \cdot \tau^-(F) \quad \text{for} \quad t \leq 0.$$

In particular, we have $\lim_{t\to\pm\infty} \frac{h_t(F)}{t} = \tau^\pm(F)$.

(3) The shifting numbers also can be computed via Bridgeland stability conditions *on $\mathcal{D}$. Let $\sigma \in \mathrm{Stab}(\mathcal{D})$, and let $\phi^\pm_\sigma \colon \mathrm{Ob}(\mathcal{D}) \to \mathbb{R}$ be the phase functions with respect to σ. Then we have*

$$\lim_{n\to\infty} \frac{\phi^\pm_\sigma(F^n G) - \phi^\pm_\sigma(G)}{n} = \tau^\pm(F).$$

This is closer in spirit to the Poincaré translation number, but requires the existence of Bridgeland stability conditions.

We show that shifting numbers satisfy the standard properties of Poincaré translation numbers.

Theorem 2.4 (F.–Filip [FF20])**.** *We have the following properties of shifting numbers.*

(1) For any $k \in \mathbb{Z}$, we have $\tau^\pm(F \circ [k]) = \tau^\pm(F) + k$. In particular, $\tau^\pm([k]) = k$.

(2) $\tau^\pm(F_1 F_2) = \tau^\pm(F_2 F_1)$, i.e. they are conjugacy invariant.

(3) For any $n \in \mathbb{Z}_{>0}$, we have $\tau^\pm(F^n) = n \cdot \tau^\pm(F)$. If F is an autoequivalence and $\mathcal{D}$ admits a Serre functor, then $\tau^\pm(F^{-1}) = -\tau^\mp(F)$. In particular, consider the shifting number *$\tau(F) \coloneqq (\tau^+(F) + \tau^-(F))/2$. Then $\tau(F^n) = n \cdot \tau(F)$ for any $n \in \mathbb{Z}$.*

An important property of the Poincaré translation number is that it gives a nontrivial *quasimorphism* on $\mathrm{Homeo}^+_{\mathbb{Z}}(\mathbb{R})$, i.e. there exists a constant $C > 0$ such that $|\rho(fg) - \rho(f) - \rho(g)| \leq C$ for any $f, g \in \mathrm{Homeo}^+_{\mathbb{Z}}(\mathbb{R})$. We prove that the shifting number also gives a quasimorphism on $\mathrm{Aut}(\mathcal{D})$ in the following cases. Note that the existence of quasimorphisms on a group has purely algebraic consequences for the structure of the group, for instance the stable commutator length and the group cohomology.

Theorem 2.5 (F.–Filip [FF20])**.** *The shifting number $\tau \colon \mathrm{Aut}(\mathcal{D}) \to \mathbb{R}$ is a quasimorphism if*

- *$\mathcal{D} = \mathcal{D}^b(X)$ for an elliptic curve X, in which case τ can be decomposed as $\tau = \rho \circ f$, where $f \colon \mathrm{Aut}(\mathcal{D}) \to \mathrm{Homeo}^+_{\mathbb{Z}}(\mathbb{R})$ is a group homomorphism and ρ is the Poincaré translation number;*
- *$\mathcal{D} = \mathcal{D}^b(X)$ for an abelian surface X, in which case τ is compatible with certain quasimorphism on the Lie group $\mathrm{SO}(2, \rho)$ of Hermitian type;*
- *$\mathcal{D} = \mathcal{D}_{A_2}$ is the 3-Calabi–Yau category associated to the A_2 quiver, then τ gives a quasimorphism on $\mathrm{Aut}_*(\mathcal{D}) = \langle T_1, T_2, [1] \rangle$, which is closely related to the classical Rademacher quasimorphism on $\mathrm{PSL}(2, \mathbb{Z})$.*

2.3. **Pseudo-Anosov autoequivalences.** In this work, we propose a categorical generalization of the notion of *pseudo-Anosov maps* in the theory of mapping class groups of Riemann surfaces. These mapping classes are important in Teichmüller theory because a *generic* mapping class is of pseudo-Anosov type. Therefore, the property we describe is conjecturally satisfied by a *generic* autoequivalence of certain triangulated categories.

Our work is motivated by the connections between Teichmüller theory and the theory of stability conditions established by Bridgeland–Smith and Haiden–Katzarkov–Kontsevich in recent years. Recall a classical result of Thurston which states that for any diffeomorphism f on a Riemann surface, the growth rate $\lim_{n\to\infty}|\ell_g(f^{(n)}\alpha)|^{1/n}$ of length of any curve α is independent of the choice of metric g. Moreover, for a pseudo-Anosov map, the growth rate of length of any curve is the same positive number.

We prove the following categorical generalization of Thurston's result.

Theorem 2.6 (F.–Filip–Haiden–Katzarkov–Liu [FFH+19])**.** *Let $\mathcal{D}$ be a triangulated category and assume the existence of Bridgeland stability conditions* $\mathrm{Stab}(\mathcal{D}) \neq \emptyset$. *Let* $\mathrm{Stab}^\dagger(\mathcal{D})$ *be a distinguished connected component of* $\mathrm{Stab}(\mathcal{D})$. *Then*

- *Any autoequivalence $F \in \mathrm{Aut}(\mathcal{D})$ induces a filtration $\{\mathcal{D}_\lambda\}_{\lambda\in\mathbb{R}}$ of F-invariant thick triangulated subcategories of $\mathcal{D}$. The filtration is given by the mass growth rate of objects with respect to a Bridgeland stability condition $\sigma \in \mathrm{Stab}^\dagger(\mathcal{D})$.*
- *The filtration is* canonical *in the sense that it is independent of the choice of $\sigma \in \mathrm{Stab}^\dagger(\mathcal{D})$.*

Motivated by Thurston's result, we then define an autoequivalence to be pseudo-Anosov *if its associated filtration has only one step $0 \subset \mathcal{D}_{\lambda>1} = \mathcal{D}$.*

Note that there is another proposed definition of pseudo-Anosov autoequivalences by Dimitrov, Haiden, Katzarkov, and Kontsevich, which we will call 'DHKK pseudo-Anosov'. However, DHKK pseudo-Anosov is too restrictive in some cases. In fact, there is no known example of DHKK pseudo-Anosov autoequivalences on Calabi–Yau categories of dimension greater than one. This contradicts with the idea that pseudo-Anosovness should be satisfied by a generic autoequivalence.

Theorem 2.7 (F.–Filip–Haiden–Katzarkov–Liu [FFH+19])**.** *If $F \in \mathrm{Aut}(\mathcal{D})$ is DHKK pseudo-Anosov, then it is pseudo-Anosov (under our definition).*

We provide several examples of pseudo-Anosov autoequivalences on 3-Calabi–Yau categories, and show that our definition is strictly more general than DHKK pseudo-Anosov.

Theorem 2.8 (F.–Filip–Haiden–Katzarkov–Liu [FFH+19])**.** *We find pseudo-Anosov autoequivalences in the following examples.*

(1) Let $\mathcal{D}_{A_2}$ be the 3-Calabi–Yau category associated to the A_2 quiver. Then
  - *Any composition of spherical twists T_1 and T_2^{-1} that is neither T_1^a nor T_2^{-b} is a pseudo-Anosov autoequivalence on $\mathcal{D}_{A_2}$.*
  - *There is no DHKK pseudo-Anosov autoequivalences on $\mathcal{D}_{A_2}$.*

(2) Let X be a quintic Calabi–Yau hypersurface in $\mathbb{P}^4$. Then $F := T_{\mathcal{O}} \circ (- \otimes \mathcal{O}(-1))$ considered in §2.1 is a pseudo-Anosov autoequivalence.

3. Character varieties and moduli of points on spheres

We start with introducing two seemly unrelated spaces, each with natural braid group actions and braid group invariants. Our main theorem gives a rather surprising connections between them.

Let $\Sigma_{g,n}$ be a surface of genus g with n boundary curves, and G be a reductive algebraic group. One can define the moduli space of G-local systems on $\Sigma_{g,n}$ as $X(\Sigma, G) := \mathrm{Hom}(\pi_1(\Sigma_{g,n}), G) /\!\!/ G$. Now let $r = 2g + n \geq 3$ with $n \in \{1, 2\}$. The braid group B_r embeds into the pure mapping class group of $\Sigma_{g,n}$ as a subgroup generated by Dehn twists along a series of interlocking simple loops in $\Sigma_{g,n}$. Therefore there is a natural B_r-action on $X(\Sigma_{g,n}, G)$, and the monodromy along the boundary curves (as elements of conjugacy classes $G /\!\!/ G$) gives B_r-invariants. The subvariety of $X(\Sigma_{g,n}, G)$ consisting of local systems with fixed boundary monodromy data k is denoted by $X_k(\Sigma_{g,n}, G)$.

On the other hand, let $S(m)$ denotes the complex affine hypersurface in $\mathbb{C}^m$ defined by the quadratic equation $x_1^2+\cdots+x_m^2=1$. We define the *moduli space of r points on $S(m)$* to be the geometric invariant theoretic quotient $A(r,m) \coloneqq S(m)^r /\!\!/ \mathrm{SO}(m)$. There is a natural braid group B_r-action on $A(r,m)$, arising from the *quandle* structure on $S(m)$: $u \triangleleft v = s_u(v) = 2\langle u,v\rangle u - v$. We introduce the *Coxeter invariants* on $A(r,m)$, which is a natural B_r-invariant morphism defined by

$$c\colon A(r,m) = S(m)^r /\!\!/ \mathrm{SO}(m) \to \mathrm{Pin}(m) /\!\!/ \mathrm{SO}(m), \quad [u_1,\dots,u_r] \mapsto [u_1 \otimes \cdots \otimes u_r].$$

The moduli space $A(r,m)$ is foliated by the fibers $A_P(r,m) \coloneqq c^{-1}(P)$, which each carries an action of B_r.

Theorem 3.1 (F.–Whang [FW20]). *Let $r = 2g+n \geq 3$ with $n \in \{1,2\}$. We have B_r-equivariant isomorphisms of complex affine varieties*

- $X_k(\Sigma_{g,n}, \{\pm 1\}) \cong A_P(r,1)/\{\pm 1\}$,
- $X_k(\Sigma_{g,n}, \mathbb{C}^*) \cong A_P(r,2)$, *and*
- $X_k(\Sigma_{g,n}, \mathrm{SL}_2) \cong A_P(r,4)$,

where the boundary monodromy k is determined by the Coxeter invariant P, and vice versa. Thus, for $m \in \{1,2,4\}$, the action of B_r on each $A_P(r,m)$ extends to an action of the mapping class group of $\Sigma_{g,n}$.

Note that the definitions of moduli of points on spheres $A(r,m)$ and the Coxeter invariants P do not involve any Riemann surface $\Sigma_{g,n}$ a priori. Therefore it is very interesting that the braid group actions on $A_P(r,m)$ can be extended to a mapping class group actions for certain m.

We also establish a connection between the moduli of points on spheres and the space of Stokes matrices $V(r)$, i.e. unipotent upper triangular $r \times r$ matrices. Denote $V(r,m) \subseteq V(r)$ the subvariety of Stokes matrices s such that $\mathrm{rank}(s+s^T) \leq m$.

Theorem 3.2 (F.–Whang [FW20]). *There is an isomorphism $A(r,m)/\{\pm 1\} \cong V(r,m)$ defined by sending $[(u_1,\dots,u_r)]$ to the unique Stokes matrix s such that $s+s^T = [2\langle u_i,u_j\rangle]$. Moreover, under the B_r-invariant morphisms $V(r) \xrightarrow{\sim} A(r,r)/\{\pm 1\} \xrightarrow{c} \mathrm{Pin}(r) /\!\!/ \mathrm{O}(r) \to \mathrm{O}(r) /\!\!/ \mathrm{O}(r)$, the image of $s \in V(r)$ has the same characteristic polynomial as $-s^{-1}s^T$. In other words, we can use the Coxeter invariants of points on spheres to give a finer invariant than the characteristic polynomial of $-s^{-1}s^T$, which has been widely used in the study of Stokes matrices.*

Combining with our main theorem, we have a sequence of B_r-equivariant morphisms

$$X(\Sigma_{g,n}, \mathrm{SL}_2(\mathbb{C})) \cong S(4)^r /\!\!/ \mathrm{SO}(4) \xrightarrow{2:1} S(4)^4 /\!\!/ \mathrm{O}(4) \cong V(r,4) \hookrightarrow V(r).$$

This gives a conceptual clarification of a result of Chekhov–Mazzocco on embedding of Teichmüller space into the space of Stokes matrices.

One of the main applications of our work is to establish arithmetic dynamical results for the varieties $A_P(r,4)$. In particular, when $r=4$, using the arithmetic dynamical results for character varieties by Whang, together with our main theorem $X_k(\Sigma_{g,n}, \mathrm{SL}_2) \cong A_P(r,4)$, we show the following.

Corollary 3.3 (F.–Whang [FW20]). *Let p be a monic palindromic polynomial of degree 4 with $\mathrm{disc}(p) \neq 0$. Then $V_p(4) \coloneqq \{s \in V(4)\colon \det(\lambda + s^{-1}s^T) = p(\lambda)\}$ consists of only finitely many integral B_4-orbits.*

When a triangulated category $\mathcal{D}$ admits a full exceptional collection and a Serre functor $\mathbf{S}_{\mathcal{D}}$, the action of $\mathbf{S}_{\mathcal{D}}$ on the numerical Grothendieck group of $\mathcal{D}$ is conjugate to $s^{-1}s^T$, where s is the Gram matrix s of a full exceptional collection of $\mathcal{D}$. We also have the following corollary.

Corollary 3.4 (F.–Whang [FW20]). *Let $\mathcal{D}$ be a triangulated category admitting a full exceptional collection of length 4. Suppose that the Serre functor $\mathbf{S}_{\mathcal{D}}$ of $\mathcal{D}$ satisfies $\mathrm{disc}(\det(\lambda + \mathbf{S}_D^{\mathrm{num}})) \neq 0$. Then there is a finite list of Stokes matrices of rank 4 such that, up to mutations, the Gram matrix of any full exceptional collection of $\mathcal{D}$ belongs to this list.*

4. Bridgeland stability conditions

The notion of stability conditions on triangulated categories was introduced by Bridgeland in 2002. It has been one of the most active research fields in algebraic geometry in recent years, due to its rich connections with birational geometry, Calabi–Yau geometry, Teichmüller theory, hyperkähler geometry, counting invariants, cluster varieties, and so forth. I introduce and study various new invariants/structures on stability conditions motivated from Teichmüller theory and Calabi–Yau geometry.

4.1. **Systolic inequalities for K3 surfaces via stability conditions.** For a Riemannian manifold M with a metric g, its *systole* $\mathrm{sys}(M,g)$ is defined to be the least length of a non-contractible loop in M. There is a classical systolic inequality on the two-torus $\mathbb{T}^2$ proved by Loewner in 1949: For *any* metric g on $\mathbb{T}^2$, we have $\mathrm{sys}(\mathbb{T}^2,g)^2 \leq \frac{2}{\sqrt{3}}\mathrm{vol}(\mathbb{T}^2,g)$.

We propose the following question that naturally generalizes Loewner's torus systolic inequality from the viewpoint of *Calabi–Yau geometry*, in which straight lines in the two torus correspond to special Lagrangian submanifolds in Calabi–Yau manifolds.

Question 4.1 (F. [Fan18b])**.** *Let Y be a Calabi–Yau manifold, and let ω be a symplectic form on Y. Does there exist a constant $C>0$ such that*

$$\inf_{L:\mathrm{sLag}}\left|\int_L \Omega\right|^2 \leq C\cdot\left|\int_Y \Omega\wedge\overline{\Omega}\right|$$

holds for any holomorphic top form Ω on Y? Here "sLag" denotes the special Lagrangian submanifolds in Y with respect to ω and Ω.

Motivated by the conjectural description of Bridgeland stability conditions on Fukaya category by Bridgeland and Joyce, we define the categorical analogues of the quantities $\inf_{L:\mathrm{sLag}}\left|\int_L\Omega\right|$ and $\int_Y\Omega\wedge\overline{\Omega}$ in terms of Bridgeland stability conditions, which are denoted by $\mathrm{sys}(\sigma)$ and $\mathrm{vol}(\sigma)$ for a Bridgeland stability condition σ.

We prove the following systolic inequality for stability conditions on the derived category of any complex projective K3 surface. Assuming the mirror symmetry conjecture, this also gives an affirmative answer to Question 4.1.

Theorem 4.2 (F. [Fan18b])**.** *Let X be a complex projective K3 surface. Then*

$$\mathrm{sys}(\sigma)^2 \leq C\cdot\mathrm{vol}(\sigma)$$

holds for any $\sigma\in\mathrm{Stab}^\dagger(\mathcal{D})$ in the distinguished connected component that contains geometric stability conditions. Here $C>0$ is an explicit constant depends only on the rank ρ and the discriminant of the Néron–Severi group of X:

$$C=\frac{((\rho+2)!)^2|\mathrm{discNS}(X)|}{2^\rho}+4.$$

4.2. **Weil–Petersson geometry on the space of stability conditions.** The moduli space of complex structures $\mathcal{M}_{\mathrm{cpx}}(Y)$ on a Calabi–Yau manifold Y has a canonical Kähler metric, the *Weil–Petersson metric*. Our goal is to construct the *mirror* object of the Weil–Petersson metric. Under mirror symmetry, $\mathcal{M}_{\mathrm{cpx}}(Y)$ should be identified with the so-called "*stringy Kähler moduli space*" $\mathcal{M}_{\mathrm{Kah}}(X)$ of a mirror Calabi–Yau manifold X. When $\dim(X)\leq 2$, $\mathcal{M}_{\mathrm{Kah}}(X)$ can be defined in terms of Bridgeland stability conditions. When $\dim(X)\geq 3$, it is conjectured by Bridgeland that there is an embedding $\mathcal{M}_{\mathrm{Kah}}(X)\hookrightarrow \mathrm{Aut}(\mathcal{D}^b(X))\backslash\mathrm{Stab}(\mathcal{D}^b(X))/\mathbb{C}$. In other words, the stringy Kähler moduli space is encoded in the space of Bridgeland stability conditions on the derived category $\mathcal{D}^b(X)$.

Our strategy therefore is to first define the Weil–Petersson geometry on the space of Bridgeland stability conditions, then restrict to the stringy Kähler moduli space.

Theorem 4.3 (F.–Kanazawa–Yau [FKY17])**.** *For any Calabi–Yau category $\mathcal{D}$, we define the Weil–Petersson metric on $\mathrm{Stab}^+(\mathcal{D})/\mathbb{C}$ for an appropriate subset $\mathrm{Stab}^+(\mathcal{D})\subseteq\mathrm{Stab}(\mathcal{D})$. The metric descends to the double quotient space $\mathrm{Aut}(\mathcal{D})\backslash\mathrm{Stab}^+(\mathcal{D})/\mathbb{C}$. We compute several low-dimensional examples to justify our definition of Weil–Petersson metric:*

- *Let E be an elliptic curve. Then*
$$\mathcal{M}_{\mathrm{Kah}}(E) \cong \mathrm{Aut}(\mathcal{D}^b(E))\backslash \mathrm{Stab}^+(\mathcal{D}^b(E))/\mathbb{C} \cong \mathrm{PSL}(2,\mathbb{Z})\backslash\mathbb{H};$$
our Weil–Petersson metric coincides with the Poincaré metric *on* $\mathbb{H}$.
- *Let $A = E_\tau \times E_\tau$ be the self-product of a generic elliptic curve. Then*
$$\mathcal{M}_{\mathrm{Kah}}(A) \cong \overline{\mathrm{Aut}}_{\mathrm{CY}}(\mathcal{D}^b(A))\backslash \mathrm{Stab}^+(\mathcal{D}^b(A))/\mathbb{C} \cong \mathrm{Sp}(4,\mathbb{Z})\backslash\mathfrak{H}_2$$
is the Siegel modular variety; our Weil–Petersson metric coincides with the Bergman metric *on* $\mathrm{Sp}(4,\mathbb{Z})\backslash\mathfrak{H}_2$.

The Weil–Petersson metric on $\mathrm{Aut}(\mathcal{D})\backslash\mathrm{Stab}^+(\mathcal{D})/\mathbb{C}$ is a degenerate metric in general. However, the Weil–Petersson metric on the complex moduli $\mathcal{M}_{\mathrm{cpx}}(Y)$ is non-degenerate. Hence we expect the non-degeneracy condition can be used to characterize the stringy Kähler moduli space $\mathcal{M}_{\mathrm{Kah}}(X)$.

Conjecture 4.4 (F.–Kanazawa–Yau [FKY17], refining Bridgeland's conjecture)**.** *For* $\dim(X) \geq 3$*, there exists an embedding of the stringy Kähler moduli space*
$$i : \mathcal{M}_{\mathrm{Kah}}(X) \hookrightarrow \mathrm{Aut}(\mathcal{D}^b(X))\backslash \mathrm{Stab}^+(\mathcal{D}^b(X))/\mathbb{C}.$$
Moreover, the pullback of our Weil–Petersson metric is a non-degenerate Kähler metric on $\mathcal{M}_{\mathrm{Kah}}(X)$.

Ongoing work

Ongoing work on categorical dynamics. I currently have four ongoing projects on categorical dynamical systems. Firstly, I am investigating applications of categorical dynamics to algebro-geometric problems. It is a general suspicion that maps of polynomial degree growth usually preserve a fibration. From the point of view of categorical dynamics, one would like to show that if a map has zero categorical entropy (but positive categorical polynomial entropy [FFO20]), then it preserves certain objects in the derived category. If one can further improve the objects into nef line bundles, then the map should preserves a fibration.

Secondly, I am working on the categorical analogue of Nielsen–Thurston classification of mapping class groups. That is, define a reasonable notion of 'reducible autoequivalences', such that any autoequivalence is either pseudo-Anosov (in the sense of our definition [FFH+19]), reducible, or of finite order up to shifts. One of my recent works studies this kind of categorical trichotomy from the point of view of categorical polynomial entropy [FFO20].

Thirdly, I am studying further properties of the shifting numbers we introduce in [FF20]. It would be very surprising if the shifting number indeed gives a quasimorphism on $\mathrm{Aut}(\mathcal{D})$ in complete generality. On the other hand, it is possible to prove a weaker statement like the subadditivity $\tau(F_1F_2) \leq \tau(F_1) + \tau(F_2)$. Such a statement is true in the context of contact diffeomorphisms as well as certain constructions from Floer theory. I am working on categorical generalizations of these ideas from symplectic geometry.

Finally, I have a long term project on investigating categorical Lyapunov exponents and their relationship with certain Harder–Narasimhan filtrations. Roughly speaking, Lyapunov exponents are dynamical invariants associated to local systems over hyperbolic Riemann surfaces, which turns out to be closely related to the Harder–Narasimhan filtration of certain variation of Hodge structures. I am studying the categorical generalization of such connections. There are plenty of examples of 'categorical local systems' provided by moduli space of Calabi–Yau manifolds. For instance, the family of quintic Calabi–Yau threefolds gives rise to a representation $\pi_1(\Sigma_{0,3}) \to \mathrm{Aut}(\mathcal{D}^b(X))$.

Ongoing program on birational/symplectic/cluster interpretation of '$(N-1)$-ality of colored quivers'. It is well-known that two threefolds have equivalent derived categoires if they are related by flops. Moreover, the composition of two flop functors $\mathcal{D}^b(\hat{X}) \to \mathcal{D}^b(\hat{X}^\dagger) \to \mathcal{D}^b(\hat{X})$ give an autoequivalence, the spherical twist $T^{-1}_{\mathcal{O}_C(-1)}$. (The symplectic analogue of the flop-flop functor is considered in our work [FHLY18].) This can be regarded as the categorification of the duality of quiver mutations: if one mutates at the same vertex twice then one gets back to the original quiver.

There is a generalization of the duality of quiver mutations that naturally arises when one considers tilting in a N-Calabi–Yau category for $N > 3$. One needs to tilt the same spherical object $N-1$ times to get an autoequivalence (spherical twist). I am establishing a program on realizing such '$(N-1)$-ality' from the point of view of birational geometry, symplectic geometry, and cluster algebras. For instance, when $N=4$, one would like to find certain birational operation on complex fourfolds such that the composition of *three* of them gives a spherical twist. A natural candidate to consider is complex fourfolds containing $\mathcal{O}(-1)\boxtimes\mathcal{O}(-1)\boxtimes\mathcal{O}(-1)\to\mathbb{P}^1\times\mathbb{P}^1\times\mathbb{P}^1$ and blowdown in three different ways. One would also like to find the symplectic analogue of this operation by generalizing our work in [FHLY18].

Ongoing work on stability conditions. I currently have two ongoing projects on stability conditions. Firstly, with Cheung and Lin, I study a new geometric interpretation of Bridgeland stability conditions on the 3-Calabi–Yau category associated to A_2 quiver. Consider the Lefschetz fibration given by $\pi\colon\{y^2=x^3+ax+b\}\to\mathbb{C}_b$, where $a\in\mathbb{C}$ is generic. The generic fiber of π is a smooth elliptic curve and there are two nodal fibers. There is a 'wall' $\mathcal{W}_b$ on $\mathbb{C}_b$ which is a simple closed curve passing through the two singular points. It has the property that there are two special Lagrangian Lefschetz thimbles whose boundary circles lie in $\pi^{-1}(b)$ for any b inside the wall, and there are three special Lagrangian Lefschetz thimbles whose boundary circles lie in $\pi^{-1}(b)$ for any b outside the wall. This resembles the wall-crossing phenomenon on the space of stability conditions. More precisely, we show that there is a holomorphic embedding $\rho_b:\widetilde{\mathbb{C}_b}\hookrightarrow\mathrm{Stab}(\mathcal{D}_{A_2})$ such that

- for any $z\in\mathbb{C}_b$ with regular fiber, the special Lagrangian Lefschetz thimbles whose boundary circles lie in $\pi^{-1}(z)$ have a one-to-one correspondence with the stable objects (up to shifts) of $\rho_b(z)$. Moreover, the central charges of the stable objects coincide with the integrations of the natural holomorphic top form on the total space along the special Lagrangian thimbles;
- the deck transformation of $\widetilde{\mathbb{C}_b}\to\mathbb{C}_b$ gives the Picard–Lefschetz transformations of the vanishing cycles, and corresponds to the spherical twists on $\mathcal{D}_{A_2}$ under the embedding.

As a varies, this construction gives a foliation of (an open submanifold of) $\mathrm{Stab}(\mathcal{D}_{A_2})$ where each leaf is the base of certain Lefschetz fibration. We also are working on defining the 'split attractor flow' on the space of Bridgeland stability conditions, and identify the flow trees with the tropical curves obtained from the special Lagrangian Lefschetz thimbles of our Lefschetz fibrations. This should enable us to establish an equivalence between BPS counting and tropical curve counting.

Secondly, with Athreya and Lee, I study the growth rate of the number of semistable (or special Lagrangian) classes of K3 surfaces with central charge less than L, as L goes to infinity. The classical analogue of this problem is the growth rate of the number of closed geodesics with length less than L in a Riemann surface, which is one of the most establishing subject of study in Teichmüller theory. We are able to express the polynomial order of the growth rate, as well as the leading coefficient, in terms of the geometric data (complex structure and an ample class) that gives rise to a stability condition and the systole of the stability condition introduced in my work [Fan18b].

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