

## HOMEWORK 6

### MATH H54

**Office Hours:** Tuesday 2:30-4pm and Wednesday 5:15-6:45pm at 735 Evans.

**Kubrat's Office Hours:** Friday 9-11am at 741 Evans.

Submit your homework at the beginning of the discussion section on Wednesday. *Late homework will not be accepted under any circumstances.*

You are encouraged to discuss the problems with your classmates, but you must write your solutions on your own and acknowledge collaborators/cite references if any.

Write clearly! Mastering mathematical writing is one of the goals of this course.

You have to staple your work if it is more than one page.

The following exercises are from the corresponding sections of the UC Berkeley custom edition of Lay, Nagle, Saff, Snider, *Linear Algebra and Differential Equations*. Note that the section numbers and problem numbers may not be the same as in Lay, *Linear Algebra*.

#### Due October 16:

- **Exercise 5.1:** 26, 27, 29, 31
- **Exercise 5.2:** 14, 18, 19
- **Exercise 5.3:** 23, 26, 27, 28, 31, 32
- **Exercise 5.4:** 10, 16, 22, 25
- **Exercise 5.5:** 6
- **Additional Problem 1:** Show that if  $A$  and  $B$  are similar, then they have the same rank. (Hint: Use  $A = PBP^{-1}$  and  $B = P^{-1}AP$ .)
- **Additional Problem 2:** The *trace* of a square matrix  $A$  is the sum of the diagonal entries in  $A$ , and is denoted by  $\text{tr}A$ . Show that  $\text{tr}A$  equals the sum of the eigenvalues of  $A$ . (Hint: Look at the characteristic polynomial of  $A$ .)
- **Challenge Problem (no need to turn in):** Show that  $\text{tr}(AB) = \text{tr}(BA)$  holds for any square matrices  $A$  and  $B$ . Moreover, show that if a function  $f : M_{n \times n}(\mathbb{R}) \rightarrow \mathbb{R}$  satisfies:
  - (1)  $f(A + B) = f(A) + f(B)$ ;
  - (2)  $f(cA) = cf(A)$  for any  $c \in \mathbb{R}$ ;
  - (3)  $f(AB) = f(BA)$ ;

$$(4) \quad f(\mathbb{I}_n) = n,$$

then  $f = \text{tr}$ . In other words, (1)–(4) gives a characterization of the trace function on square matrices.