HINTS OF HW3
MATH 185

1. While doing the estimates, you might need to use the fact that the cosine function is concave on $[-\frac{\pi}{2}, \frac{\pi}{2}]$, therefore $\cos(x) \geq 1 - \frac{2x}{\pi}$ for any $x \in [0, \frac{\pi}{2}]$.

2. To show that

$$\int_0^\infty \frac{\sin x}{x} dx = \frac{1}{2i} \int_{-\infty}^\infty \frac{e^{ix} - 1}{x},$$

first show that

$$\begin{aligned}
\int_0^\infty \frac{\sin x}{x} dx &= \lim_{\substack{\epsilon \to 0 \\ R \to \infty}} \int_\epsilon^R \frac{\sin x}{x} dx \\
&= \lim_{\substack{\epsilon \to 0 \\ R \to \infty}} \int_\epsilon^R \frac{e^{ix} - e^{-ix}}{2ix} dx \\
&= \frac{1}{2i} \lim_{\substack{\epsilon \to 0 \\ R \to \infty}} \Big(\int_\epsilon^R \frac{e^{ix}}{x} dx + \int_{-R}^{-\epsilon} \frac{e^{ix}}{x} dx \Big).
\end{aligned}$$

Note that the reason that the hint suggests to consider the function $\frac{e^{ix}-1}{x}$ rather than $\frac{e^{ix}}{x}$ is because $\frac{e^{ix}}{x}$ blows up at $x = 0$, while $\lim_{x \to 0} \frac{e^{ix}-1}{x}$ exists.

4. Note that the problem is slightly different from what we did in class: in class, we only considered $\xi \in \mathbb{R}$, and the function we considered is $e^{-\pi x^2} e^{-2\pi i x \xi}$ rather than $e^{-\pi x^2} e^{2\pi i x \xi}$ in the problem. (They're called the *Fourier transform* and the *inverse Fourier transform*.) In any case, the proof is very similar to what we discussed in class.

6. You might need to apply the keyhole argument we discussed in class, and the fact (which we also proved in class) that if $\gamma \colon [a, b] \to \Omega$ is a parametrized curve, then

$$\Big| \int_\gamma f(z) dz \Big| \leq \max_{t \in [a,b]} \Big| f(\gamma(t)) \Big| \cdot \text{length}(\gamma).$$