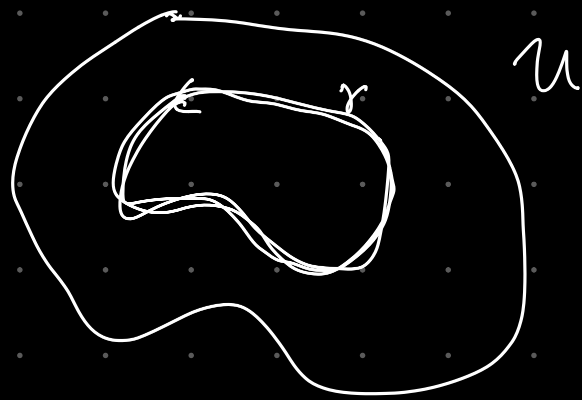
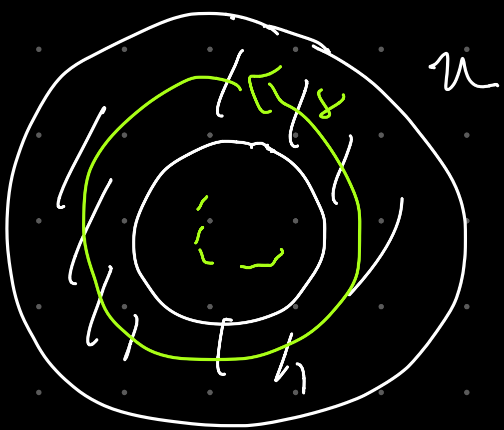



$f : \mathcal{U} \to \mathbb{C}$ holo has a primitive.

$$\boxed{\int_\gamma f(z)\,dz = 0}$$




$\boxed{X \text{ disconnected}}$

if $\exists\ \mathcal{U}_1, \mathcal{U}_2 \subseteq X$,

$\mathcal{U}_1 \neq \phi$, $\mathcal{U}_2 \neq \phi$,

$\mathcal{U}_1 \neq X$ $\mathcal{U}_2 \neq X$

$\mathcal{U}_1 \cap \mathcal{U}_2 = \phi$

$\mathcal{U}_1 \cup \mathcal{U}_2 = X$

---

#2 $f : \mathbb{D} \to \mathbb{C}$ holo, $|f|$ const.

$$\Downarrow$$

$$\boxed{|f|^2} \text{ const.}$$

$$\| \quad f \cdot \bar{f}$$

$$0 = \boxed{\frac{\partial}{\partial z}\left(\boxed{\frac{\partial}{\partial \bar{z}} |f|^2}\right)}$$

$$= \frac{\partial}{\partial z}\left(\frac{\partial}{\partial \bar{z}}(f\bar{f})\right)$$

$$= \frac{\partial}{\partial z}\left(\cancel{\frac{\partial f}{\partial \bar{z}}} \cdot \bar{f} + f \cdot \boxed{\frac{\partial \bar{f}}{\partial \bar{z}}}\right) \quad \overline{\frac{\partial f}{\partial z}}$$

$$= \frac{\partial}{\partial z}\left( f \cdot \frac{\partial \bar{f}}{\partial \bar{z}} \right)$$

$$= \frac{\partial f}{\partial z} \cdot \frac{\partial \bar{f}}{\partial \bar{z}} + f \cdot \boxed{\frac{\partial^2 \bar{f}}{\partial z \partial \bar{z}}}$$

$$\boxed{\frac{\partial^2 \bar{f}}{\partial z \partial \bar{z}}} = 0$$

$$\frac{\partial \bar{f}}{\partial z} = \overline{\frac{\partial f}{\partial \bar{z}}} = 0$$

$$= \left| \frac{\partial f}{\partial z} \right|^2$$

$$\Rightarrow \frac{\partial f}{\partial z} \equiv 0 \quad \Rightarrow f \text{ is a const. fn.}$$

$\square$

---

$f = u + iv$

holo $\Rightarrow \begin{cases} u_x = v_y \\ v_x = -u_y \end{cases}$

$|f| = \text{const.}$

$\quad \parallel$

$\sqrt{u^2+v^2}$ $\Downarrow$ $\boxed{u^2+v^2 \equiv \text{const.}}$

$\frac{\partial}{\partial x} \Rightarrow \cancel{2} u u_x + \cancel{2} v v_x = 0$

$\frac{\partial}{\partial y} \Rightarrow u u_y + v v_y = 0$

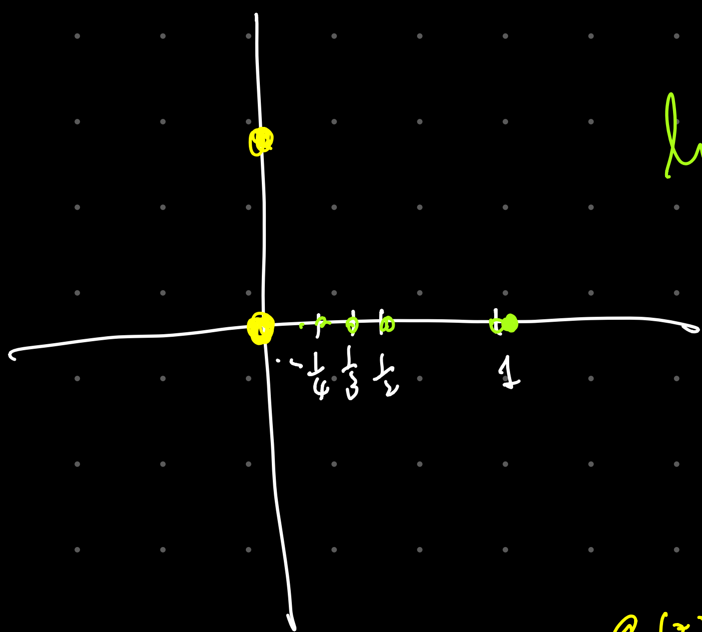
$$\left.\begin{aligned} u u_x &= v u_y \\ u u_y &= v u_x \end{aligned}\right\} \Rightarrow u_x^2 + u u_{xx} = v_x u_y + v u_{xy}$$

$$\left(\frac{\partial}{\partial x}\right)^2 \rightsquigarrow u_x^2 + uu_{xx} + v_x^2 + vv_{xx} = 0$$

$$u_y^2 + uu_{yy} + v_y^2 + vv_{yy} = 0$$

---

#8 Assume $f: \mathbb{C} \to \mathbb{C}$ hol.




local - determine - global

$\Downarrow$

$f \equiv 1.$

---

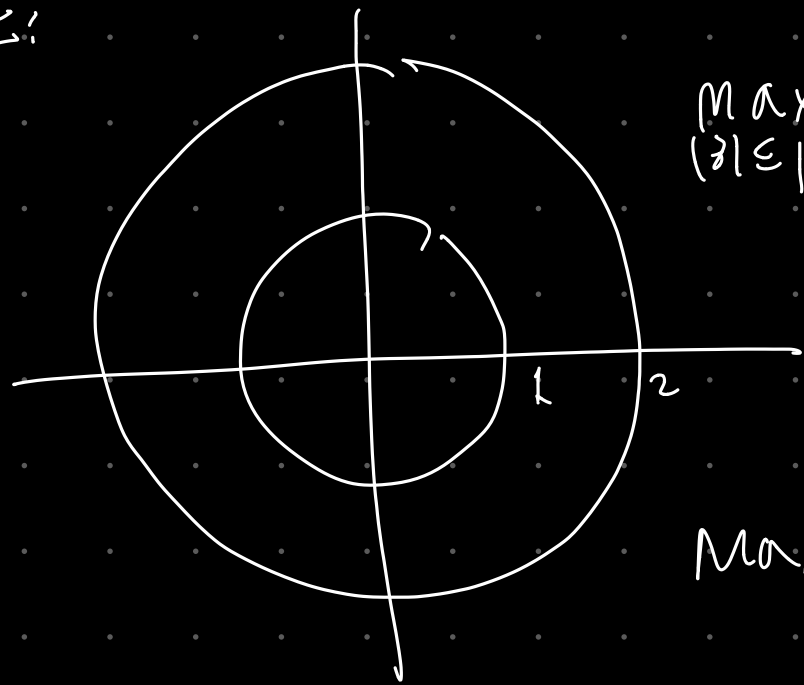
$g(z)$ $\quad g: \mathbb{C} \to \mathbb{R}$

#10 $f$: entire. $\quad \boxed{\underbrace{\operatorname{Re}(f(z))}_{\text{harmonic}}} = |z|^2 = x^2+y^2. \quad z = x+iy$

$$0 = \Delta g = g_{xx} + g_{yy}$$

$$0 \neq \underbrace{(x^2+y^2)_{xx}}_{2} + \underbrace{(x^2+y^2)_{yy}}_{2}$$

#12:




$$\max_{|z|\le 1}|f(z)| \le \max_{|z|\le 2}|f(z)|$$

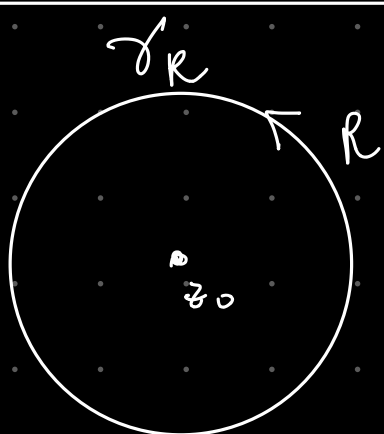
Max. modulus principle

$$\Downarrow$$

equality can only achieved when $f$ is a const. fn.

---

#6.




$$f(z_0) = \frac{1}{2\pi i}\int_{\gamma_R} \frac{f(z)}{z-z_0}\,dz.$$

$\gamma_R$ $\boxed{\begin{array}{c} z_0 + Re^{i\theta} \\ 0 \le \theta \le 2\pi \end{array}}$

$$= \frac{1}{2\pi \cancel{i}}\int_0^{2\pi} \frac{f(z_0+Re^{i\theta})}{\cancel{Re^{i\theta}}} \cdot \cancel{i}\cancel{Re^{i\theta}}\,d\theta$$

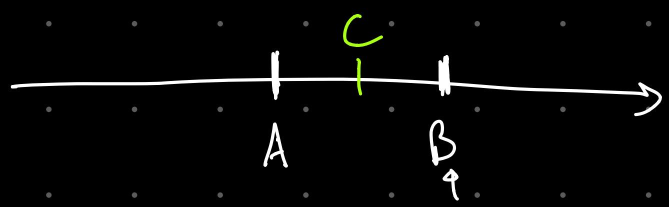
$$= \frac{1}{2\pi}\int_0^{2\pi} f(z_0+Re^{i\theta})\,d\theta.$$

---

$e^{icz}$ $P$ - poly, $c>0$.

$P(z)$

---




f

$$\boxed{\sum a_n (X-A)^n}$$

$$\downarrow$$

$$\frac{f^{(n)}(A)}{n!}$$

$\exists C$

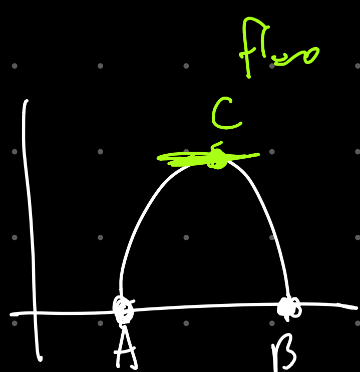
$$R_K(B) = f^{(K)}(C)$$

$$\|$$

$$f(B) - \sum_{n=0}^{K-1} a_n (B-A)^n$$

---

Real Analysis




$f(A) = f(B)$

$\Downarrow$

$\exists\, C \in (A,B)$

st. $f'(C) = 0$

---

$e^x$