HINTS OF PRACTICE PROBLEMS FOR FINAL
MATH H54

(1) Write $P = A + B$, and replace B by $P - A$.

(2) Consider the restrictions of T_A to the images $\mathrm{Im}(T_B)$ and $\mathrm{Im}(T_{BC})$, and apply the rank-nullity theorem.

(5) $A^T A$ is a symmetric matrix with 1's on its diagonal, therefore has trace n (the size of A). Then recall that trace (resp. determinant) is the sum (resp. product) of eigenvalues.

(6) Suppose $\vec{w}$ is a vector in W with $\vec{w} = \vec{w}_1 + \cdots + \vec{w}_n$ its unique decomposition into sum of eigenvectors (with respect to distinct eigenvalues). One can use the fact that $T(W) \subseteq W$ to show that each w_i lies in W.

(7) Apply the rank-nullity theorem.

(8) The eigenvalues of A must be real (why?) and satisfy $\lambda^3 - 2\lambda = 4$.

(10) Make use of HW5, Problem 9.

(13) Show that $(\mathbb{I} - BA)\vec{x} = \vec{0}$ implies $\vec{x} = \vec{0}$.

(14) Prove by contradiction: Suppose there exists $\vec{v}_1 \in W_1 \backslash W_2$ and $\vec{v}_2 \in W_2 \backslash W_1$, consider whether their sum $\vec{v}_1 + \vec{v}_2$ lies in $W_1 \cup W_2$ or not.

(15) First show that A is invertible.