Problem Set 1

1. We will learn AR models soon. The AR(1) is defined as

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t,$$

   where the explanatory variable is the lag of the dependent variable. Suppose you observe $\{y_t\}_{t=1}^T$ as your data.

   (a) Construct an OLS estimator of α and β. You need to list the objective function and provide the details of how you obtain the estimator.

   (b) Suppose $\{\varepsilon_t\}_{t=1}^T$ are i.i.d. with finite fourth moment. Does the regression you just run satisfy all the required assumptions (check Page 16 of the slides for the assumptions) for the OLS?

2. We continue to consider the above model with the same assumptions. On top of that, suppose $\{\varepsilon_t\}_{t=1}^T$ are i.i.d. normally distributed with mean 0 and variance σ^2. Construct a MLE of α, β, and σ^2. You need to provide the log likelihood function and the details of finding the solutions. Hint: get the likelihood function for $y_2, y_3, ..., y_T$ and ignore y_1.