6.4 微波网络的散射矩阵

必要性：由于在微波频段：

（1）电压和电流已失去明确的物理意义，难以直接测量；

（2）由于开路条件和短路条件在高频的情况下难以实现，故Z参数和Y参数也难以测量。

引入散射参数，简称S参数。

类型：行波散射参量（普通）、功率散射参量（广义）。

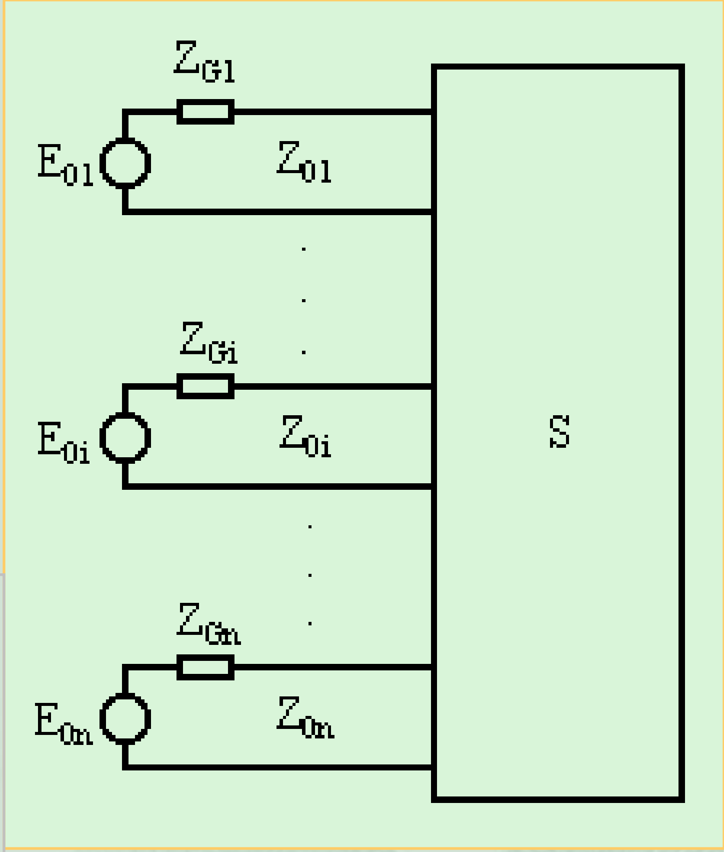
测量技术：电压驻波比VSWR、共轭匹配/失配因子M。

普通散射参数 ⟹ 行波散射参数：物理内涵是以**特性阻抗**Z_0匹配为核心，它在测量技术上的外在表现形态是电压驻波比VSWR.

广义散射参数 ⟹ 功率散射参数：是以**共轭匹配** (最大功率匹配) 为核心，它在测量技术上的外在表现形态是失配因子M。

一、普通散射参数

1.普通散射参数的定义



普通散射参数是用网络各端口的入射电压波*a*和出射电压波*b*来描述网络特性的波矩阵。由传输线理论知在第*i*端口有

$$V_i(z) = V_{0i}^{+}e^{-\gamma z} + V_{0i}^{-}e^{\gamma z} = V_i^{+}(z) + V_i^{-}(z)$$

$$I_i(z) = \frac{V_{0i}^{+}e^{-\gamma z} - V_{0i}^{-}e^{\gamma z}}{Z_{0i}} = I_i^{+}(z) - I_i^{-}(z)$$

则可得

$$V_i^{+} = V_{0i}^{+}e^{-\gamma z} = \frac{1}{2}[V_i(z) + Z_{0i}I_i(z)]$$

$$V_i^{-} = V_{0i}^{-}e^{\gamma z} = \frac{1}{2}[V_i(z) - Z_{0i}I_i(z)]$$

两边除以$\sqrt{Z_{0i}}$，定义如下归一化入射波*a*和归一化出射波*b*。

归一化入射波 $$a_i(z)=\frac{V_i^+}{\sqrt{Z_{0i}}}=\frac{1}{2}\left[\frac{V_i(z)}{\sqrt{Z_{0i}}}+\sqrt{Z_{0i}}I_i(z)\right]$$

归一化出射波 $$b_i(z)=\frac{V_i^-}{\sqrt{Z_{0i}}}=\frac{1}{2}\left[\frac{V_i(z)}{\sqrt{Z_{0i}}}-\sqrt{Z_{0i}}I_i(z)\right]$$

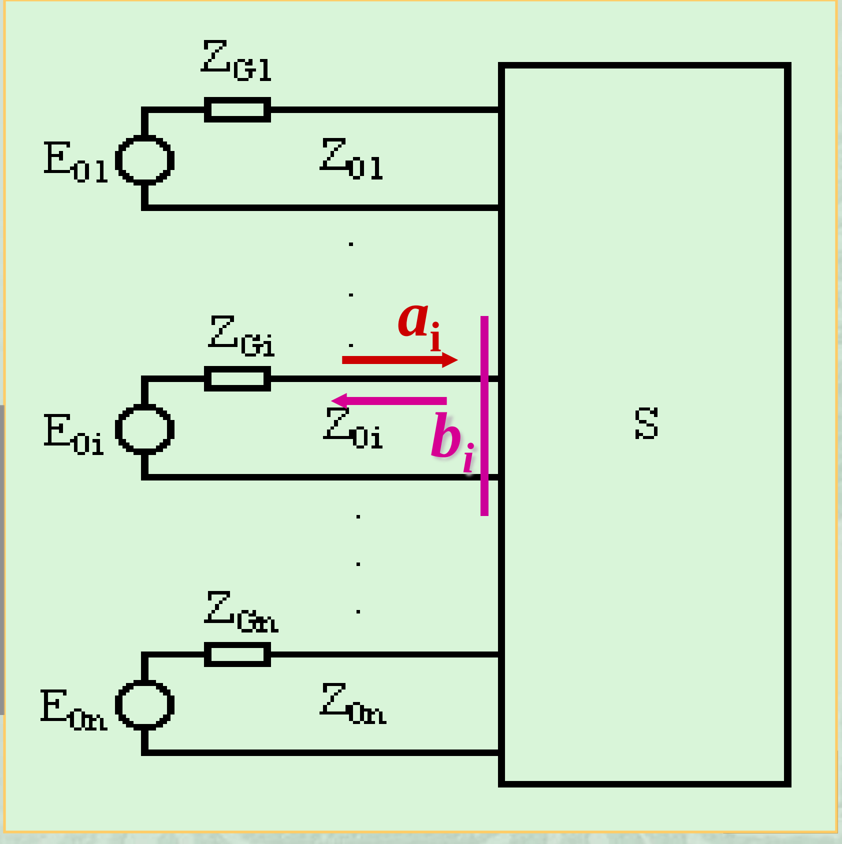
$$V_i(z)=Z_i(z)I_i$$

则第i端口的反射系数为:

$$\frac{b_i(z)}{a_i(z)}=\frac{V_i^-}{V_i^+}=\Gamma_i(z)=\frac{Z_i(z)-Z_{0i}}{Z_i(z)+Z_{0i}} \quad (6.4-3)$$

$$V_i^+=\frac{1}{2}[V_i(z)+Z_{0i}I_i(z)]=\frac{I_i(z)}{2}[Z_i(z)+Z_{0i}]$$

$$V_i^-=\frac{1}{2}[V_i(z)-Z_{0i}I_i(z)]=\frac{I_i(z)}{2}[Z_i(z)-Z_{0i}]$$



由(6.4-3) 式得

$$V_i(z)=\sqrt{Z_{0i}}[a_i(z)+b_i(z)]$$
$$I_i(z)=\frac{1}{\sqrt{Z_{0i}}}[a_i(z)-b_i(z)]$$

$$a_i(z)=\frac{V_i^+}{\sqrt{Z_{0i}}}$$
$$b_i(z)=\frac{V_i^-}{\sqrt{Z_{0i}}}$$

或归一化电压和归一化电流：

$$\overline{V_i(z)}=\frac{V_i(z)}{\sqrt{Z_{0i}}}=a_i(z)+b_i(z)$$
$$\overline{I_i(z)}=I_i(z)\sqrt{Z_{0i}}=a_i(z)-b_i(z)$$

则第*i* 个端口的入射功率和反射功率为:

$$P_i^+=\frac{1}{2}|a_i|^2=\frac{1}{2}\frac{|V^+|^2}{Z_{0i}}$$
$$P_i^-=\frac{1}{2}|b_i|^2=\frac{1}{2}\frac{|V^-|^2}{Z_{0i}}$$

以归一化入射波振幅a_i为自变量，归一化出射波振幅b_i为因变量，则可得线性N端口微波网络的散射矩阵方程为：

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

式中$[a]$、$[b]$为N端口的归一化入射波和归一化出射波的矩阵表示形式：

$$[a] = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \qquad [b] = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}$$

N端口网络的[S]散射矩阵为

$$[S]=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix}$$

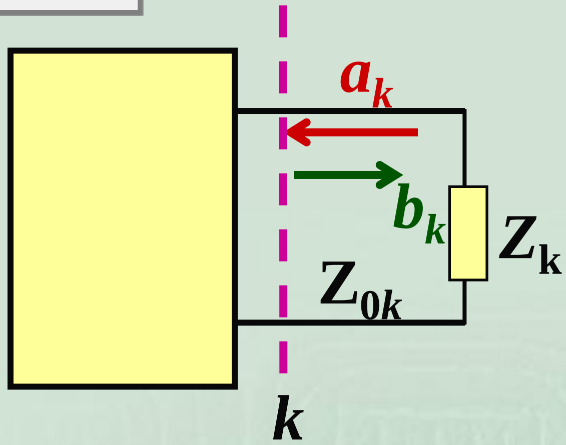
或 $$[b]=[S][a]$$

式中 $$b_i=\sum_{j=1}^{N} S_{ij}a_j = S_{i1}a_1+S_{i2}a_2+\cdots+S_{ij}a_j+\cdots+S_{iN}a_N$$

$$b_i = \sum_{j=1}^{N} S_{ij} a_j = S_{i1}a_1 + S_{i2}a_2 + \cdots + S_{ij}a_j + \cdots + S_{iN}a_N$$

散射矩阵元素的定义为：***i≠j***

$$S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0,k\neq j}$$



当 $a_k=0$时，则k 端口的入射波为零，故要求k 端口：

1) 无源。

2) 无入射。

$Z_k=Z_{0k}$
阻抗匹配

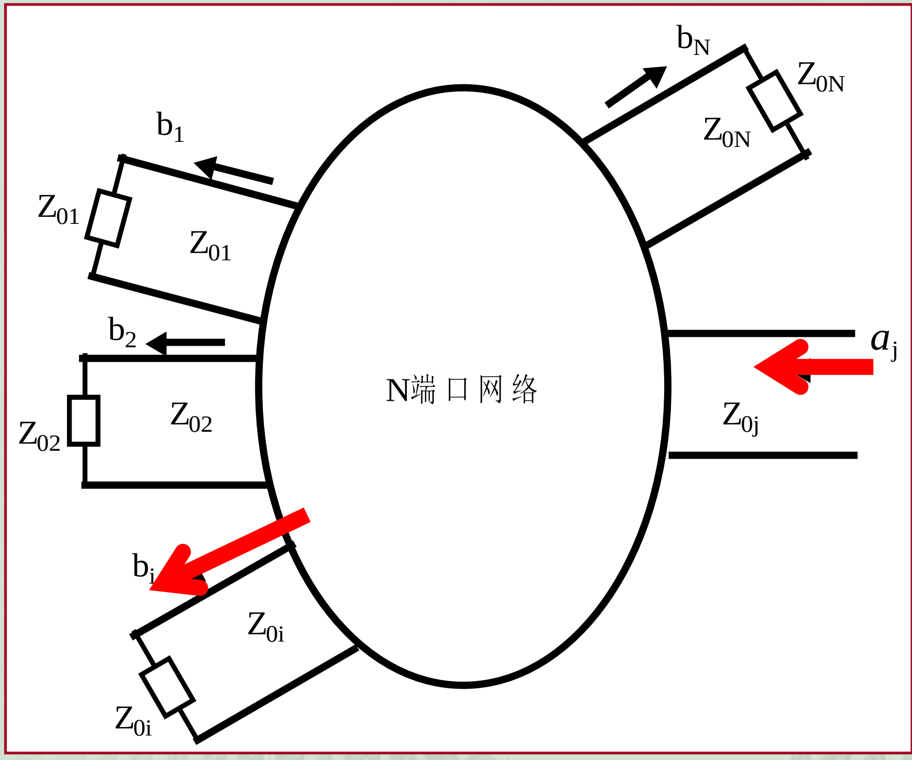
如负载阻抗Z_k无反射，则端口k为无入射。

$$S_{ij}=\left.\frac{b_i}{a_j}\right|_{a_k=0,k\neq j}=\left.\frac{V_i^-/\sqrt{Z_{0i}}}{V_j^+/\sqrt{Z_{0j}}}\right|_{V_k^+=0,k\neq j}=\left.\sqrt{\frac{Z_{0j}}{Z_{0i}}}\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,k\neq j}$$

2. 散射参数的物理意义

只有*j* 端口才有入射波，其他端口为出射电压波

当除*j* 以外的其它端口的入射波为零时(全部接匹配负载时)，S_{ij}为在端口*j* 用入射电压波a_j激励，测量端口*i*的出射电压波振幅b_i来求得.



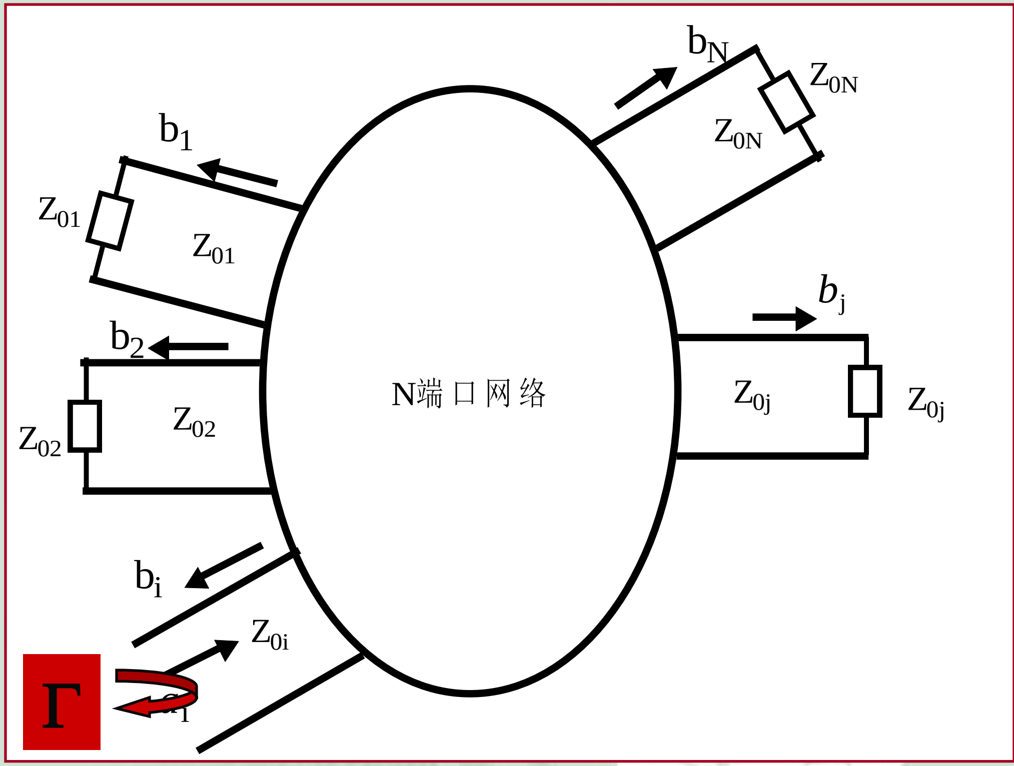
S_{ij}是当所有其它端口接匹配负载时从端口*j*至端口*i*的<u>传输系数.</u>

$$b_i = \sum_{j=1}^{N} S_{ij} a_j = S_{i1} a_1 + S_{i2} a_2 + \cdots + S_{ii} a_i + \cdots + S_{iN} a_N$$

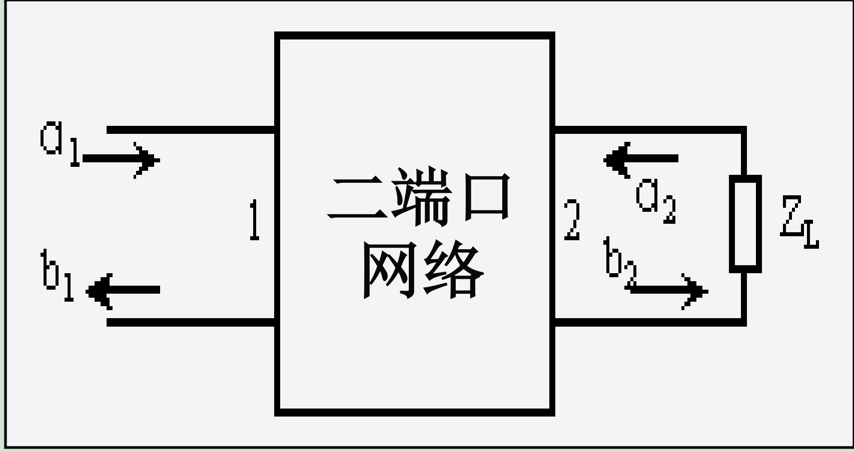
散射矩阵元素(*i=j*)的物理意义：

$$S_{ii} = \frac{b_i}{a_i}\bigg|_{a_k=0, k\neq i} = \frac{V_i^-}{V_i^+}\bigg|_{V_k^+=0, k\neq i}$$

S_{ii}是当所有其它端口接匹配负载时端口*i*的反射系数。



特例：二端口网络



$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

其中 $S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$，$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$，$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$，$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$

S_{11}为端口1的反射系数；
S_{22}为端口2的反射系数；
S_{21}为端口1到端口2的传输系数；
S_{12}为端口2到端口1的传输系数。

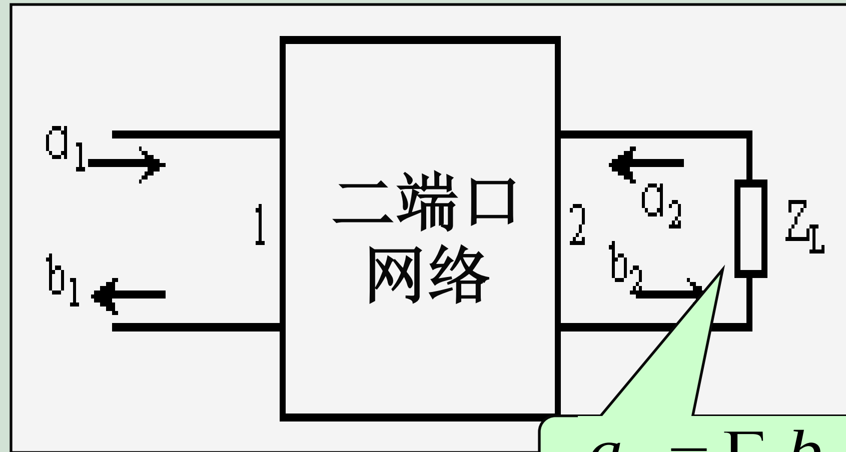
条件是另一端口接匹配负载

其散射矩阵：

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

特色：测试简便

若输出端口接不匹配负载Z_L，设负载的反射系数为Γ_L，有



$$a_2 = \Gamma_L b_2$$

$$a_2 = \Gamma_L b_2 \qquad \begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

则散射矩阵变为

$$\begin{aligned} b_1 &= S_{11}a_1 + S_{12}\Gamma_L b_2 \\ b_2 &= S_{21}a_1 + S_{22}\Gamma_L b_2 \end{aligned}$$

不仅与S参数有关，还与所接负载有关

故输入端口的反射系数为：

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

$$\Gamma_{in}=\frac{b_1}{a_1}=S_{11}+\frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

***对二端口互易网络有 $S_{12}=S_{21}$，则**

$$\Gamma_{in}=\frac{b_1}{a_1}=S_{11}+\frac{S_{12}^2\Gamma_L}{1-S_{22}\Gamma_L}$$

线性互易二端口网络的散射参数可以用三点法测定：当输出端口

- 短路($\Gamma_L=-1$)，
- 开路($\Gamma_L=1$) ，
- 接匹配负载($\Gamma_L=0$) 时.

则有

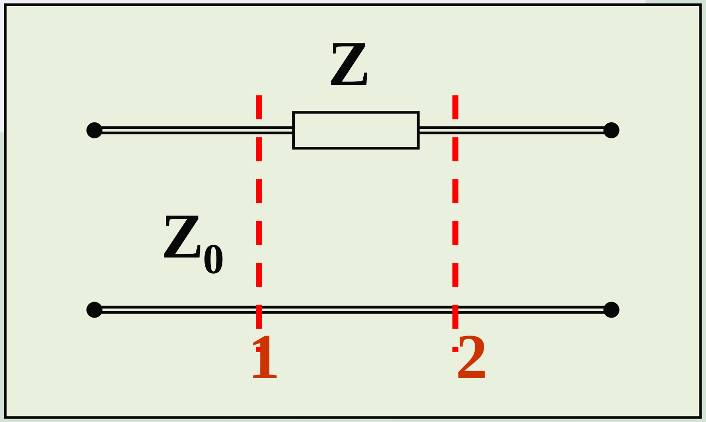
$$\Gamma_{in,sc}=S_{11}-\frac{S_{12}^2}{1+S_{22}}$$

$$\Gamma_{in,oc}=S_{11}+\frac{S_{12}^2}{1-S_{22}} \qquad (6.4-13)$$

$$\Gamma_{in,mat}=S_{11}$$

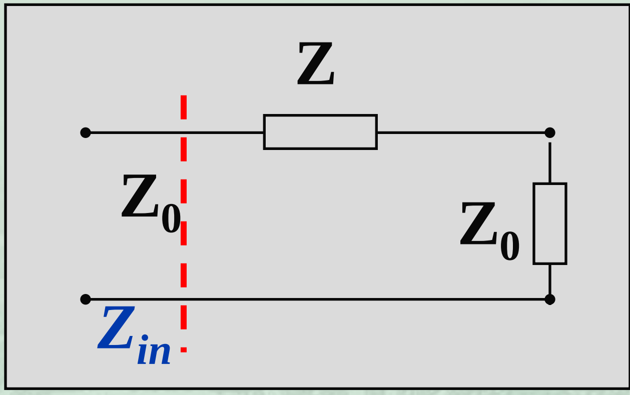
在测量时分别将输出端口短路、开路和接匹配负载，测出 $\Gamma_{in,sc}, \Gamma_{in,oc}, \Gamma_{in,mat}$ 即可由上式计算出S_{11}、S_{12}和S_{22}。

【例6.4-1】 求如图的S参量矩阵。



$$[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

解：选择参考面如图。



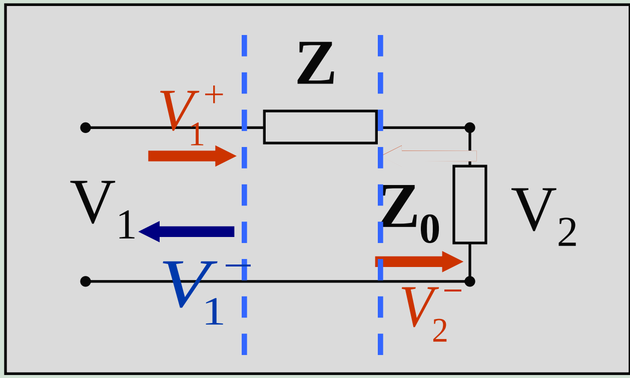
端口2接匹配负载时 $Z_L = Z_0$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \Gamma_{in1}\Big|_{Z_L=Z_0} = \frac{Z_{in}-Z_0}{Z_{in}+Z_0}$$

此时输入阻抗为：$Z_{in} = Z_0 + Z$

故有 $S_{11} = \dfrac{Z_0+Z-Z_0}{Z_0+Z+Z_0} = \dfrac{Z}{2Z_0+Z}$

$$S_{21}=\left.\frac{b_2}{a_1}\right|_{a_2=0}=\left.\frac{V_2^-/\sqrt{Z_0}}{V_1^+/\sqrt{Z_0}}\right|_{V_2^+=0}=\left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$



对于1端口 $V_1=V_1^++V_1^-=V_1^+(1+S_{11})$ ⇒ $V_1^+=\dfrac{V_1}{(1+S_{11})}$

对于2端口 $V_2=V_2^++V_2^-=V_2^-$

$$S_{11}=\frac{Z}{2Z_0+Z}$$

⇒ $S_{21}=\left.\dfrac{V_2^-}{V_1^+}\right|_{V_2^+=0}=\dfrac{V_2}{V_1/(1+S_{11})}=(1+S_{11})\dfrac{V_2}{V_1}$

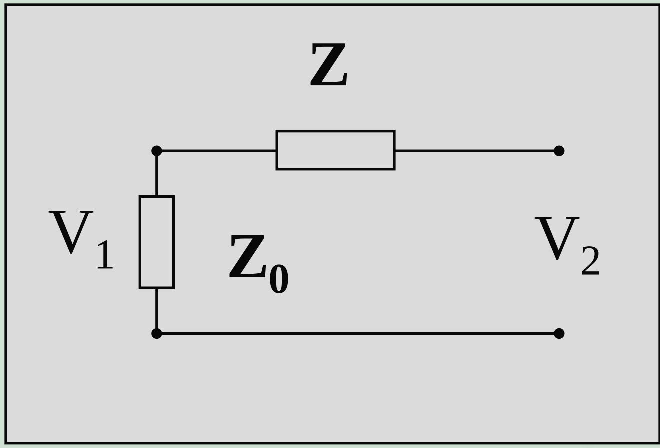
而 $\dfrac{V_2}{V_1}=\dfrac{Z_0}{Z_0+Z}$

$$\therefore\quad S_{21}=\left(1+\frac{Z}{2Z_0+Z}\right)\frac{Z_0}{Z+Z_0}=\frac{2(Z_0+Z)}{2Z_0+Z}\frac{Z_0}{Z+Z_0}=\frac{2Z_0}{Z+2Z_0}$$

由于网络完全对称，则

$$S_{22}=S_{11}=\frac{Z}{2Z_0+Z}$$

$$S_{12}=S_{21}=\frac{b_1}{a_2}\bigg|_{a_1=0}=\frac{2Z_0}{Z+2Z_0}$$



故网络的S参量矩阵为：

$$[S]=\begin{bmatrix}\dfrac{Z}{2Z_0+Z} & \dfrac{2Z_0}{2Z_0+Z}\\ \dfrac{2Z_0}{2Z_0+Z} & \dfrac{Z}{2Z_0+Z}\end{bmatrix}=\frac{1}{2Z_0+Z}\begin{bmatrix}Z & 2Z_0\\ 2Z_0 & Z\end{bmatrix}$$

3. 散射矩阵的特性

1) 互易网络散射矩阵是对称矩阵

对于互易网络，由于其阻抗矩阵[Z]和导纳矩阵[Y]都是对称的，故其散射矩阵[S]也是对称的。即有：

$$\boxed{[S]=[S]^t}$$

对于各参量： $\boxed{S_{ij}=S_{ji}}$

$$\bar{V}=V/\sqrt{Z_0}=a+b$$

$$\bar{I}=I\sqrt{Z_0}=a-b$$

证明： $\because [Z][I]=[V]$

$$\Rightarrow [Z]\boxed{\left[\sqrt{Y_0}\right]([a]-[b])}=\boxed{\left[\sqrt{Z_0}\right]([a]+[b])}$$

$$[Z]\left[\sqrt{Y_0}\right]([a]-[b])=\left[\sqrt{Z_0}\right]([a]+[b]) \qquad \left[\sqrt{Z_0}\right]\left[\sqrt{Y_0}\right]=[U]$$

$$\Rightarrow\left([Z]\left[\sqrt{Y_0}\right]-\left[\sqrt{Z_0}\right]\right)[a]=\left(\left[\sqrt{Z}\right]\left[\sqrt{Y_0}\right]+\left[\sqrt{Z_0}\right]\right)[b]$$

$$\Rightarrow([Z]-[Z_0])\left[\sqrt{Y_0}\right][a]=([Z]+[Z_0])\left[\sqrt{Y_0}\right][b]$$

$$\therefore\ [b]=\left[\sqrt{Z_0}\right]([Z]+[Z_0])^{-1}([Z]-[Z_0])\left[\sqrt{Y_0}\right][a]$$

$$\therefore\ [S]=\left[\sqrt{Z_0}\right]([Z]+[Z_0])^{-1}([Z]-[Z_0])\left[\sqrt{Y_0}\right]$$

$$又[S]^t=\left\{\left[\sqrt{Z_0}\right]([Z]+[Z_0])^{-1}([Z]-[Z_0])\left[\sqrt{Y_0}\right]\right\}^t$$

$$=\left[\sqrt{Y_0}\right]^t([Z]-[Z_0])^t\left\{([Z]+[Z_0])^{-1}\right\}^t\left[\sqrt{Z_0}\right]^t$$

$$\xrightarrow[\text{矩阵求逆、求转置顺序可换}]{\text{对称矩阵的差为对称矩阵}}\underline{\left[\sqrt{Y_0}\right]([Z]-[Z_0])([Z]+[Z_0])^{-1}\left[\sqrt{Z_0}\right]}$$

[S]

$$\therefore\ [S]=[S]^t$$

2) 无耗网络散射矩阵的幺正性

对于一个N端口无耗无源网络，传入系统的功率为

$$P_{in}=\frac{1}{2}|a_1|^2+\frac{1}{2}|a_2|^2+\ldots+\frac{1}{2}|a_N|^2=\sum_{i=1}^{N}\frac{1}{2}|a_i|^2=\sum_{i=1}^{N}\frac{1}{2}\frac{\left|V_i^+\right|^2}{Z_0}$$

系统的出射功率为：

$$P_{out}=\frac{1}{2}|b_1|^2+\frac{1}{2}|b_2|^2+\ldots+\frac{1}{2}|b_N|^2=\sum_{i=1}^{N}\frac{1}{2}|b_i|^2=\sum_{i=1}^{N}\frac{1}{2}\frac{\left|V_i^-\right|^2}{Z_0}$$

因为系统无耗、无源，即损耗功率等于零，因此有：

$$P_{in} - P_{out} = \sum_{i=1}^{N} \frac{1}{2}\left(\left|a_i\right|^2 - \left|b_i\right|^2\right) = 0$$

N端口网络入射功率和出射功率相等

用矩阵形式表示 $[a]^t[a]^* - [b]^t[b]^* = 0$

将 $[b] = [S]\cdot[a]$ 代入上式：

$$[a]^t[a]^* - [a]^t[S]^t[S]^*[a]^* = 0$$

$$[a]^t[a]^* - [a]^t[S]^t[S]^*[a]^* = 0$$

整理得 $[a]^t\{[U]-[S]^t[S]^*\}[a]^* = 0$

只有此项为0

式中 $[U] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & \cdots & 1 \end{bmatrix}$ 为单位矩阵。

故有 $[S]^t[S]^* = [U]$ 此为散射矩阵的幺正性

对于互易网络，由互易性可得 $[S][S]^* = [U]$

即有 $$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$[S][S]^{*} = [U]$$

若 $i = j$，则有 $$\sum_{k=1}^{N} S_{ki} S_{ki}^{*} = 1$$

[S]矩阵的任一列与该列的共轭值的点乘积等于1.

若$i \neq j$，则有 $$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0$$

[S]矩阵任一列与不同列的共轭值的点乘积等于零（正交）.

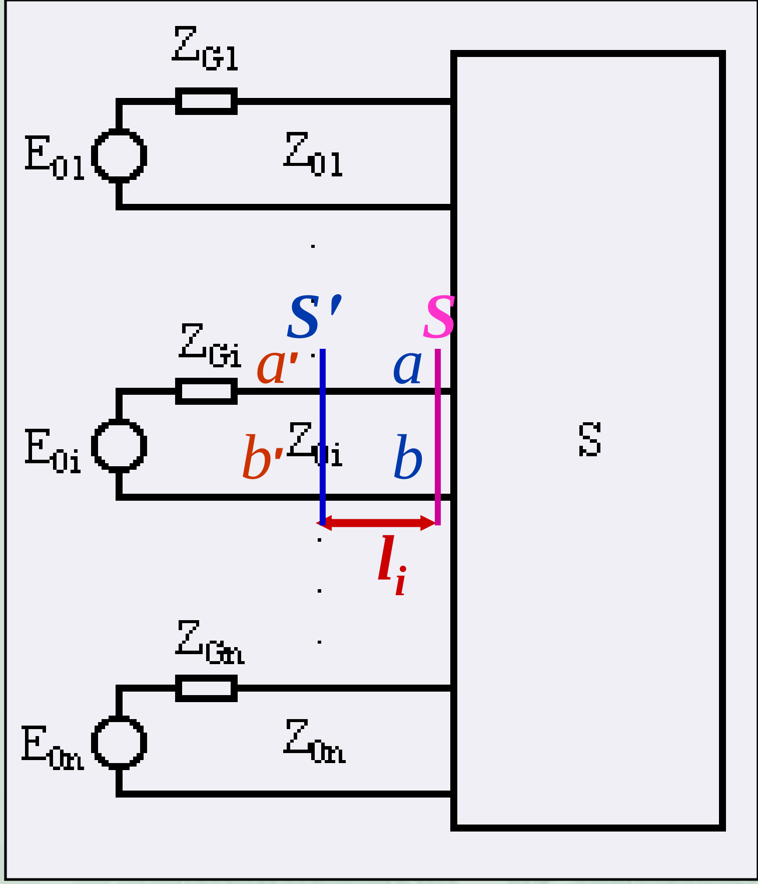
3) 传输线无耗条件下，参考面移动S参数幅值的不变性

S参数表示的是微波网络的出射波振幅与入射波振幅的关系，因此必须规定网络各端口的相位参考面。

参考面移动 $\xrightarrow{\text{传输线无耗}}$

- 散射参数的*幅值不变*
- 散射参数的**相位改变**

设参考面z_i=0 处网络的散射矩阵为[S]，参考面向外移至$z_i=l_i$处网络的散射矩阵为[S′]。



移动距离为l_i, 其相应的相位变化为

$$\theta_i = k_i l_i = 2\pi l_i / \lambda_{gi}$$

由于参考面的移动，各端口出射波b的相位要滞后 (-):

$$b_i' = b_i e^{-j\theta_i}$$

入射波a相位要超前(+):

$$a_i' = a_i e^{+j\theta_i}$$

对于i端口相位:

$$\theta_i = 2\pi l_i / \lambda_{gi}$$

j端口相位:

$$\theta_j = 2\pi l_j / \lambda_{gj}$$

新的散射参量为：

$$S'_{ij}=\frac{b'_i}{a'_j}=\frac{b_i\exp(-j\dfrac{2\pi l_i}{\lambda_{gi}})}{a_j\exp(j\dfrac{2\pi l_i}{\lambda_{gi}})}=S_{ij}e^{-j2\pi[(l_j/\lambda_{gj})+(l_i/\lambda_{gi})]}$$

新的散射矩阵$[S']$与原散射矩阵$[S]$的关系：

$$\boxed{[S']=[P][S][P]}$$

式中：
$$[P]=\begin{bmatrix} e^{-j\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\theta_2} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j\theta_N} \end{bmatrix}$$

4. [S]矩阵与[Z]、[Y]矩阵的关系

$$a_i(z)=\frac{1}{2}\left[\frac{V_i(z)}{\sqrt{Z_{0i}}}+\sqrt{Z_{0i}}I_i(z)\right]$$

$$b_i(z)=\frac{1}{2}\left[\frac{V_i(z)}{\sqrt{Z_{0i}}}-\sqrt{Z_{0i}}I_i(z)\right]$$

由于

$$V_i=\sum_{j=1}^{N}Z_{ij}I_j \qquad i=1,2,\dots,N$$

$$a_i(z)=\frac{1}{2}\left(\sum_{j=1}^{N}\sqrt{Y_{0i}}Z_{ij}I_j+\sqrt{Z_{0i}}I_i\right)=\frac{1}{2}\sum_{j=1}^{N}(\sqrt{Y_{0i}}Z_{ij}+\sqrt{Z_{0i}}\delta_{ij})I_j$$

$$b_i(z)=\frac{1}{2}\left(\sum_{j=1}^{N}\sqrt{Y_{0i}}Z_{ij}I_j-\sqrt{Z_{0i}}I_i\right)=\frac{1}{2}\sum_{j=1}^{N}(\sqrt{Y_{0i}}Z_{ij}-\sqrt{Z_{0i}}\delta_{ij})I_j$$

（6.4-15）

式中当 $i=j$ 时；$\delta_{ij}=1$；当 $i\neq j$ 时，$\delta_{ij}=0$

引入对角矩阵：

$$[Z_0]=\begin{bmatrix} Z_{01} & 0 & \cdots & 0 \\ 0 & Z_{02} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & Z_{0N} \end{bmatrix}$$

$$\left[\sqrt{Z_0}\right]=\begin{bmatrix} \sqrt{Z_{01}} & 0 & \cdots & 0 \\ 0 & \sqrt{Z_{02}} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \sqrt{Z_{0N}} \end{bmatrix}$$

$$[Y_0]=\begin{bmatrix} Y_{01} & 0 & \cdots & 0 \\ 0 & Y_{02} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & Y_{0N} \end{bmatrix}$$

$$\left[\sqrt{Y_0}\right]=\begin{bmatrix} \sqrt{Y_{01}} & 0 & \cdots & 0 \\ 0 & \sqrt{Y_{02}} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \sqrt{Y_{0N}} \end{bmatrix}$$

$$a_i(z)=\frac{1}{2}\sum_{j=1}^{N}(\sqrt{Y_{0i}}Z_{ij}+\sqrt{Z_{0i}}\delta_{ij})I_j,\quad b_i(z)=\frac{1}{2}\sum_{j=1}^{N}(\sqrt{Y_{0i}}Z_{ij}-\sqrt{Z_{0i}}\delta_{ij})I_j$$

（6.4-15）

则（6.4-15）式可以表示成矩阵形式

$$[a]=\frac{1}{2}\left[\sqrt{Y_0}\right]\left([Z]+\left[\sqrt{Z_0}\right][I]\right)$$

$$[b]=\frac{1}{2}\left[\sqrt{Y_0}\right]\left([Z]-\left[\sqrt{Z_0}\right][I]\right)$$

（6.4-17）

由（6.4-17）式中的第一式得到

$$[I]=2\left([Z]+[Z_0]\right)^{-1}\left[\sqrt{Z_0}\right][a]$$

代入（6.4-17）式中的第二式得到

$$[b]=\left[\sqrt{Y_0}\right]\left([Z]-[Z_0]\right)\left([Z]+[Z_0]\right)^{-1}\left[\sqrt{Z_0}\right][a]$$

$$[b]=\left[\sqrt{Y_0}\right]\left([Z]-[Z_0]\right)\left([Z]+[Z_0]\right)^{-1}\left[\sqrt{Z_0}\right][a]$$

而 $[b]=[S][a]$

则[S]与[Z]的关系为：

$$[S]=[\sqrt{Y_0}]([Z]-[Z_0])([Z]+[Z_0])^{-1}[\sqrt{Z_0}]=[\sqrt{Y_0}]\frac{([Z]-[Z_0])}{([Z]+[Z_0])}[\sqrt{Z_0}]$$

同理可求得[S]和[Y]的关系：

$$[S]=[\sqrt{Z_0}]([Y_0]-[Y])([Y_0]+[Y])^{-1}[\sqrt{Y_0}]=[\sqrt{Z_0}]\frac{([Y_0]-[Y])}{([Y_0]+[Y])}[\sqrt{Y_0}]$$

反之 $[Z]=\left[\sqrt{Z_0}\right]([U]+[S])([U]-[S])^{-1}\left[\sqrt{Z_0}\right]$

$$[Y]=\left[\sqrt{Y_0}\right]([U]-[S])([U]+[S])^{-1}\left[\sqrt{Y_0}\right]$$

$$[S]=[\sqrt{Y_0}]\frac{([Z]-[Z_0])}{([Z]+[Z_0])}[\sqrt{Z_0}]$$

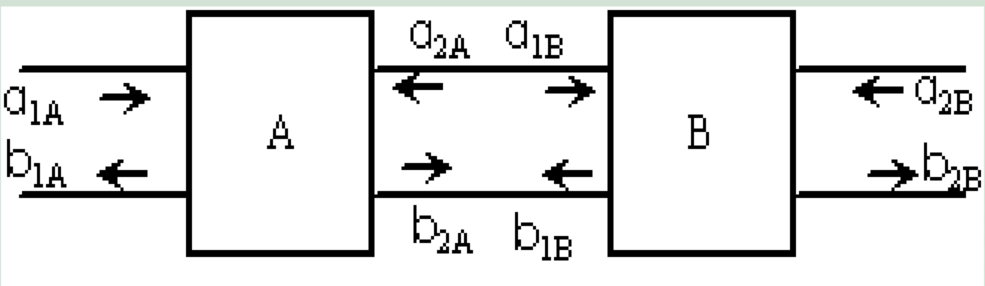
式中 $[U]=\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & \cdots & 1 \end{bmatrix}$ 为单位矩阵

对于一端口网络： $S_{11}=\Gamma_{in}=\dfrac{Z-Z_0}{Z+Z_0}$

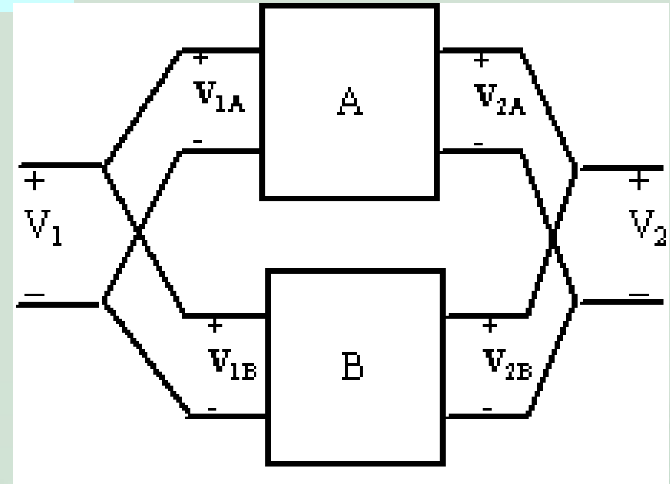
与传输线理论的结果一致。

5. 级联二端口网络的散射矩阵

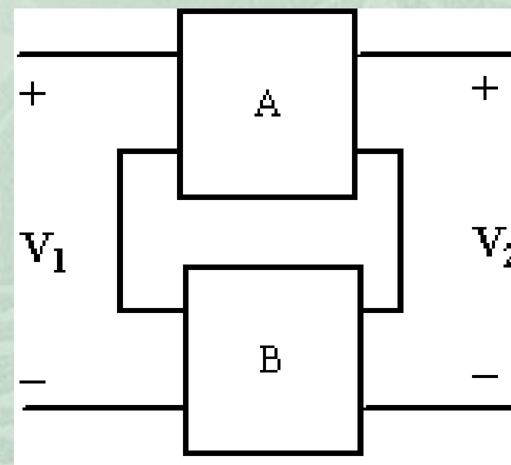
微波网络由基本电路组合而成。
常见的组合形式有三种：



级　联



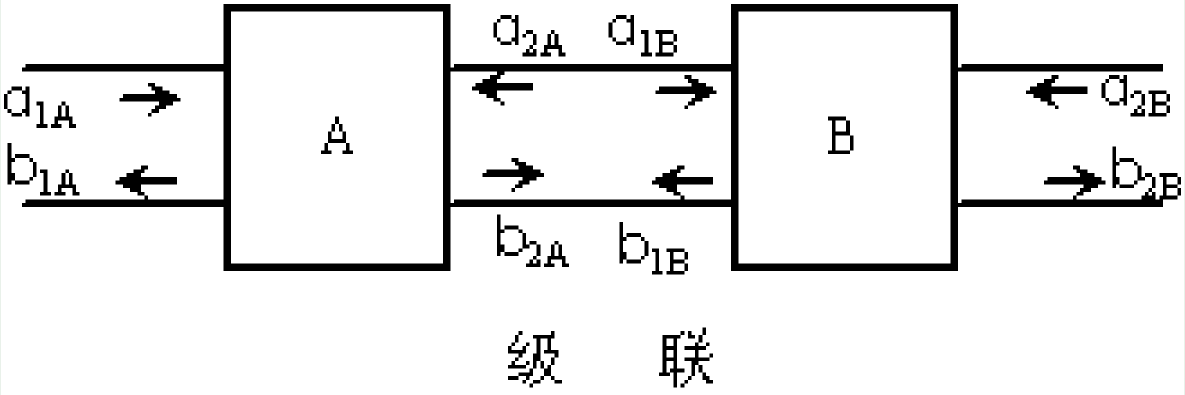
并联—并联



串联—串联

用途：微波CAD——减少矩阵换算。

现有二端口网络A和网络B级联，如图所示。



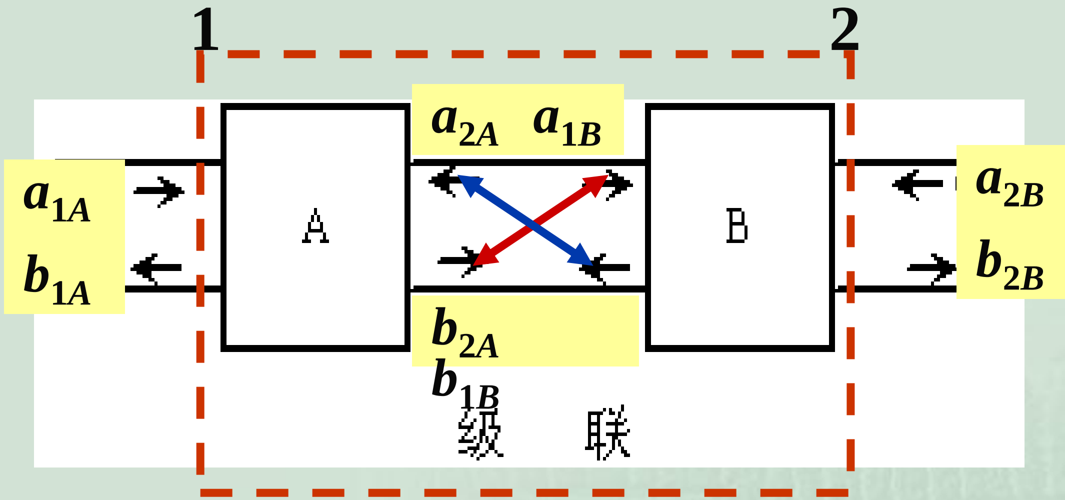
网络A的散射矩阵为$[S]_A$

$$b_{1A} = S_{11}^{A} a_{1A} + S_{12}^{A} a_{2A}$$
$$b_{2A} = S_{21}^{A} a_{1A} + S_{22}^{A} a_{2A}$$

网络B的散射矩阵为$[S]_B$

$$b_{1B} = S_{11}^{B} a_{1B} + S_{12}^{B} a_{2B}$$
$$b_{2B} = S_{21}^{B} a_{1B} + S_{22}^{B} a_{2B}$$

级联之后的两个端口分别为**A**网络的**1**端口，和**B**网络的**2**端口，则其归一化入射波和归一化出射波可表示为：



$$b_{1A} = S_{11}a_{1A} + S_{12}a_{2B}$$
$$b_{2B} = S_{21}a_{1A} + S_{22}a_{2B}$$

∵连接处：　$b_{2A} = a_{1B}$，$b_{1B} = a_{2A}$

代入上式并消去这些中间变量，则可得两级联二端口网络的散射矩阵：

$$[S]_{AB}=\begin{bmatrix} S_{11}^{A}+\dfrac{S_{12}^{A}S_{11}^{B}S_{21}^{A}}{1-S_{22}^{A}S_{11}^{B}} & \dfrac{S_{12}^{A}S_{12}^{B}}{1-S_{22}^{A}S_{11}^{B}} \\ \dfrac{S_{21}^{A}S_{21}^{B}}{1-S_{22}^{A}S_{11}^{B}} & S_{22}^{B}+\dfrac{S_{21}^{B}S_{22}^{A}S_{12}^{B}}{1-S_{22}^{A}S_{11}^{B}} \end{bmatrix}$$

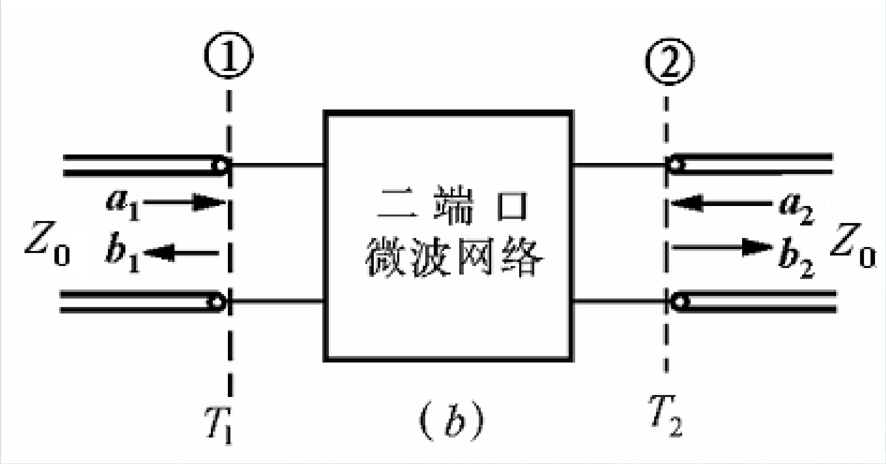
*并联—并联组合：　$[Y]=[Y_1]+[Y_2]$

*串联—串联组合：　$[Z]=[Z_1]+[Z_2]$

在各接口均满足匹配条件时，可连续应用级联组成级联两端口网络总散射阵。

【例6.4-2】 测得某二端口网络的S矩阵为 $[S]=\begin{bmatrix} 0.1 & j0.4 \\ j0.4 & 0.2 \end{bmatrix}$

请问此二端口网络是否互易和无耗？若在端口2短路，求端口1处的驻波比。



解：由于 $\boxed{S_{12}=S_{21}=j0.4}$

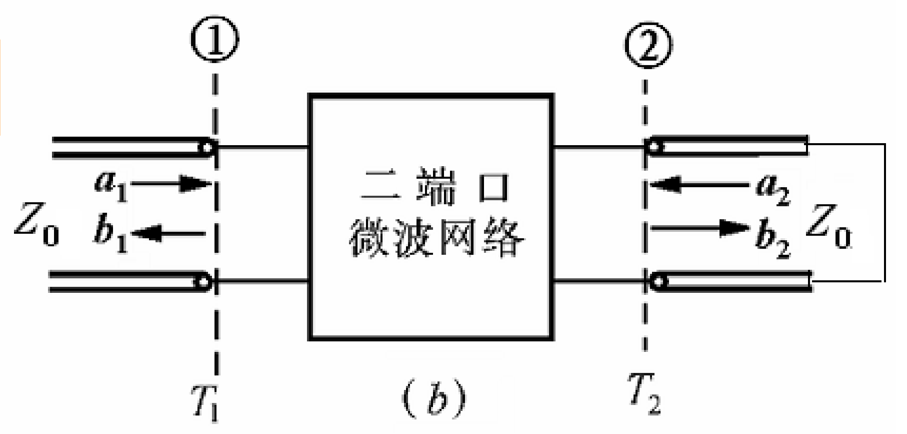
故网络互易。

又由：$\boxed{[S][S]^*=\begin{bmatrix} 0.1 & j0.4 \\ j0.4 & 0.2 \end{bmatrix}\begin{bmatrix} 0.1 & -j0.4 \\ -j0.4 & 0.2 \end{bmatrix}=\begin{bmatrix} 0.17 & -j0.4 \\ -j0.4 & 0.2 \end{bmatrix}\neq[U]}$

不满足幺正性，因此网络为有耗网络。

或 $S_{11}S_{11}^*+S_{21}S_{21}^*=0.01+0.16=0.17\neq 1$

若端口2短路求端口1处的驻波比.



当端口2 短路时：$\Gamma_L=-1$

$$\boxed{a_2=\Gamma_L b_2=-b_2}$$

由二端口网络的S矩阵：

$$b_1=S_{11}a_1+S_{12}a_2=S_{11}a_1-S_{12}b_2 \quad (1)$$

$$b_2=S_{21}a_1+S_{22}a_2=S_{21}a_1-S_{22}b_2 \quad (2)$$

由(2)式得 $b_2=\dfrac{S_{21}}{1+S_{22}}a_1$

代入(1)式消去b_2有

$$\Gamma_{in}=\frac{b_1}{a_1}=S_{11}-\frac{S_{12}^2}{1+S_{22}}=0.1-\frac{-0.16}{1+0.2}=0.233$$

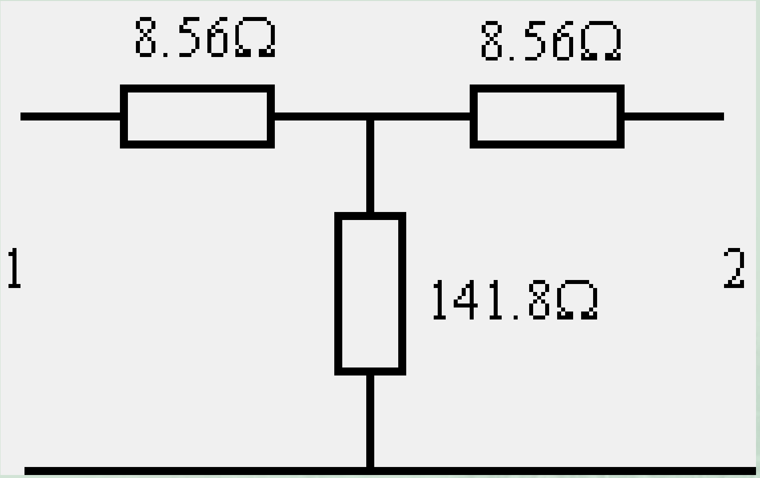
则1端口的驻波比

$$VSWR=\frac{1+|\Gamma_{in}|}{1-|\Gamma_{in}|}=\frac{1.23}{0.77}=1.6$$

则1端口的回波损耗：

$$L_r=-20\lg\Gamma_{in}=-12.6dB$$

【例6.4-3】 求如图所示网络的S参量. $Z_0 = 50\Omega$



解：端口2接匹配负载时

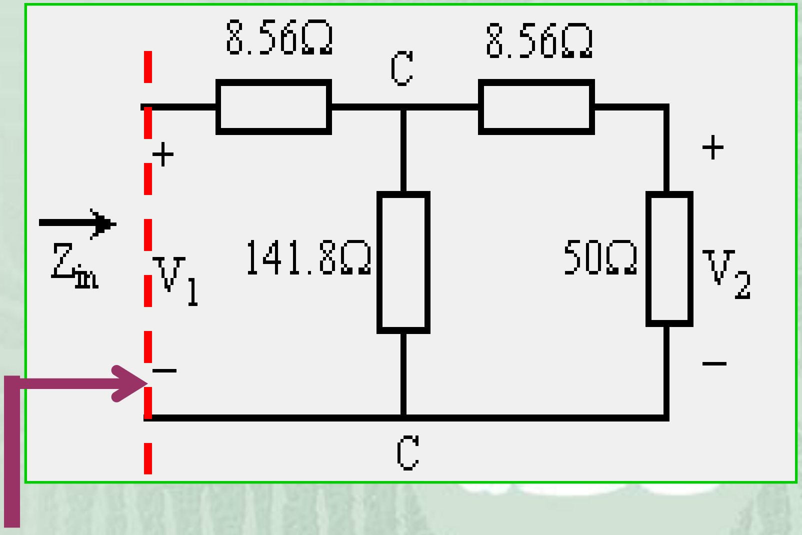
$$Z_L = Z_0$$

故有 $$S_{11} = \Gamma_{in1}\Big|_{Z_L = Z_0} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\because \quad Z_{in} = 8.56 + \frac{141.8(8.56 + 50)}{141.8 + (8.56 + 50)}$$

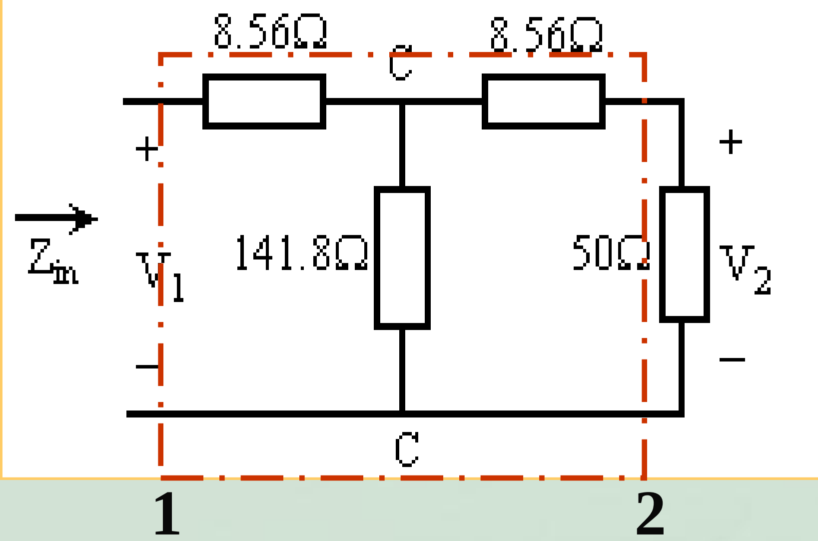
$$= 50\Omega$$

$$\therefore \quad \boxed{S_{11} = 0}$$



又$\because$ 网络完全对称

$\therefore$ $\boxed{S_{22} = S_{11} = 0}$



下面求S_{12}和S_{21}：

$$\text{而}\quad S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

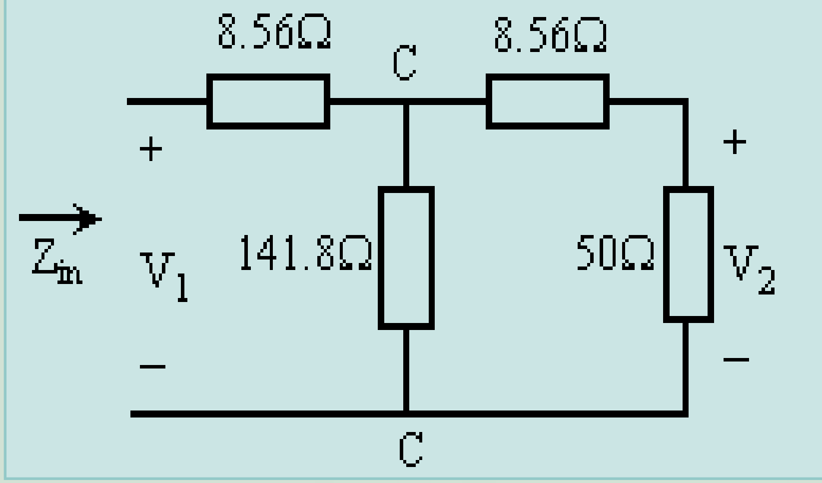
对于1端口 $V_1 = V_1^+ + V_1^- = V_1^+(1+S_{11}) = V_1^+$

对于2端口 $V_2 = V_2^+ + V_2^- = V_2^-$

$$\therefore \quad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \frac{V_2}{V_1}$$

而CC点的等效阻抗为 $Z_{cc} = \dfrac{(8.56+50)\times 141.8}{(8.56+50)+141.8} = 41.44(\Omega)$

则 $V_{cc}=\dfrac{V_1}{Z_{cc}+8.56}Z_{cc}=0.82V_1$



$$Z_{cc}=41.44(\Omega)$$

$$\boxed{\begin{aligned}V_2&=\frac{V_{cc}}{50+8.56}\times 50\\&=0.82\times 0.85V_1=0.707V_1\end{aligned}}$$

$$\therefore\quad \boxed{S_{21}=\frac{V_2}{V_1}=0.707}$$

因为是互易网络， $\boxed{\therefore S_{12}=0.707}$

$$S_{21}=\frac{V_2}{V_1}$$

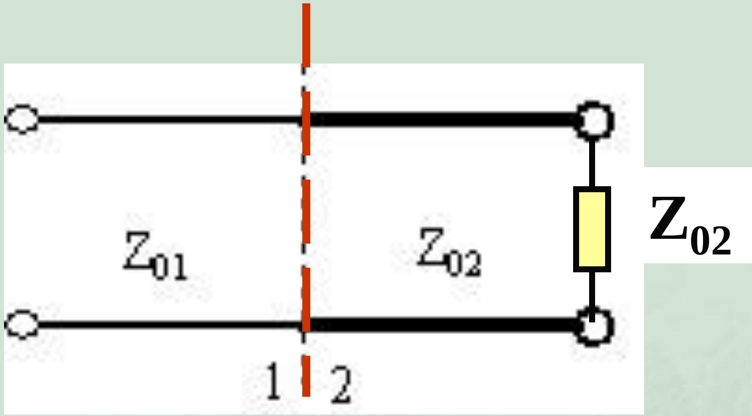
故S参数为 $$\boxed{[S]=\begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}}$$

此网络的输入功率为 $$P_0=\frac{1}{2}\frac{\left|V_1^+\right|^2}{Z_0}$$

输出功率为 $$\frac{1}{2}\frac{\left|V_2^-\right|^2}{Z_0}=\frac{1}{2Z_0}\left|S_{21}V^+\right|^2=\frac{\left|V_1^+\right|^2}{4Z_0}$$

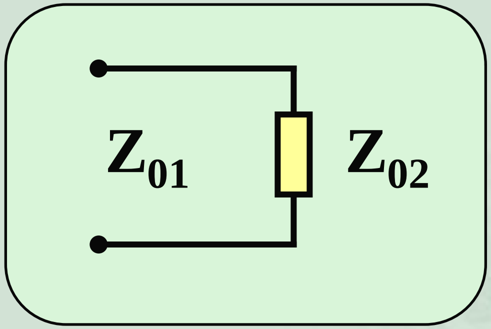
【例6.4-4】求两个不同特性阻抗的传输线接口处的S矩阵.

解：二端口网络只包含接头，参考面的选择：1端口和2端口均在虚线处.



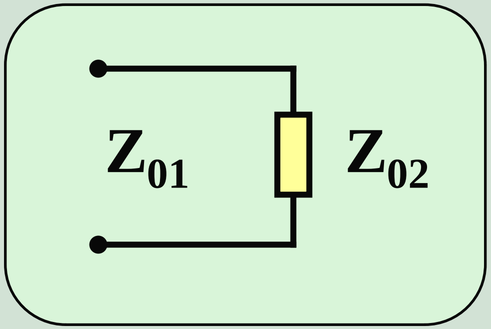
$$S_{11}=\left.\frac{b_1}{a_1}\right|_{a_2=0}=\Gamma_{in}$$

其等效电路为：



$$S_{11}=\left.\frac{b_1}{a_1}\right|_{a_2=0}=\Gamma_{in}=\frac{Z_{in}-Z_{01}}{Z_{in}+Z_{01}}=\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}$$

$$S_{21}=\frac{b_2}{a_1}\bigg|_{a_2=0}=\frac{V_2^-/\sqrt{Z_{02}}}{V_1^+/\sqrt{Z_{01}}}\bigg|_{V_2^+=0}=\sqrt{\frac{Z_{01}}{Z_{02}}}\frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0}$$



对于1端口 $V_1=V_1^+ + V_1^- = V_1^+(1+S_{11})$

对于2端口 $V_2=V_2^+ + V_2^- = V_2^-$

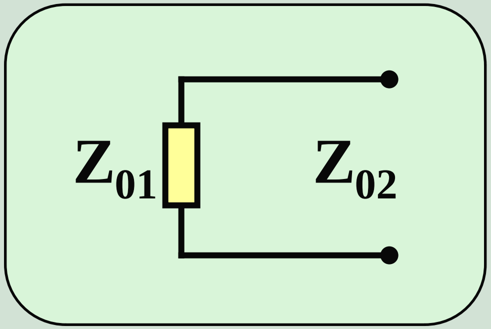
$$S_{11}=\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}$$

由于接头处 $V_1=V_2$

$$\therefore\quad S_{21}=\sqrt{\frac{Z_{01}}{Z_{02}}}\frac{V_2^-}{V_1^+}\bigg|_{V_2^+=0}=\sqrt{\frac{Z_{01}}{Z_{02}}}\frac{V_2}{V_1/(1+S_{11})}=\sqrt{\frac{Z_{01}}{Z_{02}}}(1+S_{11})$$

$$\Rightarrow S_{21}=\sqrt{\frac{Z_{01}}{Z_{02}}}\left(1+\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}\right)=\sqrt{\frac{Z_{01}}{Z_{02}}}\cdot\frac{2Z_{02}}{Z_{02}+Z_{01}}=\frac{2\sqrt{Z_{01}Z_{02}}}{Z_{02}+Z_{01}}$$

对于S_{22}和S_{12}，其等效电路图为



$$S_{22}=\left.\frac{b_2}{a_2}\right|_{a_1=0}=\Gamma_{in}=\frac{Z_{in}-Z_{02}}{Z_{in}+Z_{02}}=\frac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}}$$

$$S_{12}=\left.\frac{b_1}{a_2}\right|_{a_1=0}=\left.\sqrt{\frac{Z_{02}}{Z_{01}}}\frac{V_1^-}{V_2^+}\right|_{V_1^+=0}$$

对于1端口 $V_1=V_1^+ +V_1^- =V_1^-$

对于2端口 $V_2=V_2^+ +V_2^- =V_2^+(1+S_{22})$

由于接头处 $\boxed{V_1 = V_2}$

$$\therefore S_{12} = \sqrt{\frac{Z_{02}}{Z_{01}}}\frac{V_1^-}{V_2^+}\bigg|_{V_1^+=0} = \sqrt{\frac{Z_{02}}{Z_{01}}}\frac{V_1}{V_2/(1+S_{22})} = \sqrt{\frac{Z_{02}}{Z_{01}}}(1+S_{22})$$

$$\because S_{22} = \frac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}}$$

$$\Rightarrow S_{12} = \sqrt{\frac{Z_{02}}{Z_{01}}}\left(1+\frac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}}\right) = \sqrt{\frac{Z_{02}}{Z_{01}}}\frac{2Z_{01}}{Z_{02}+Z_{01}} = \frac{2\sqrt{Z_{01}Z_{02}}}{Z_{02}+Z_{01}}$$

$$\therefore \quad [S] = \begin{bmatrix} \dfrac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}} & \dfrac{2\sqrt{Z_{01}Z_{02}}}{Z_{02}+Z_{01}} \\ \dfrac{2\sqrt{Z_{01}Z_{02}}}{Z_{02}+Z_{01}} & \dfrac{Z_{01}-Z_{02}}{Z_{01}+Z_{02}} \end{bmatrix}$$

互易

6. S参数的特性

$$[S]=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

$\boxed{S_{ii}=0}$ 该端口为匹配，无反射.

$\boxed{|S_{ii}|=1}$ 该端口全反射.

$\boxed{S_{ij}=0}$ 由*j*端口输入，端口*i*无输出；即*j*端口到*i*端口无传输，即两端口隔离.

$\boxed{S_{ij}=S_{ji}}$ 互易.

无耗——幺正性

对于三端口网络：

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$|S_{11}|^2+|S_{21}|^2+|S_{31}|^2=1$$

$$|S_{12}|^2+|S_{22}|^2+|S_{32}|^2=1$$

$$|S_{13}|^2+|S_{23}|^2+|S_{33}|^2=1$$

振幅关系式

$$S_{11}S_{12}^*+S_{21}S_{22}^*+S_{31}S_{32}^*=0$$

$$S_{11}S_{13}^*+S_{21}S_{23}^*+S_{31}S_{33}^*=0$$

$$S_{12}S_{13}^*+S_{22}S_{23}^*+S_{32}S_{33}^*=0$$

相位关系式

二、广义散射矩阵

上述普通的散射矩阵都要求网络所有的端口具有相同的阻抗特性。如不同时，引入功率波（广义参量）。

如图*N*端口网络，定义网络各端口的电压、电流为：

$$V_i = \frac{a_i Z_i^* + b_i Z_i}{\sqrt{\mathrm{Re}\, Z_i}}$$

$$I_i = \frac{a_i - b_i}{\sqrt{\mathrm{Re}\, Z_i}}$$

第*i*端口外接阻抗（一般为复数）

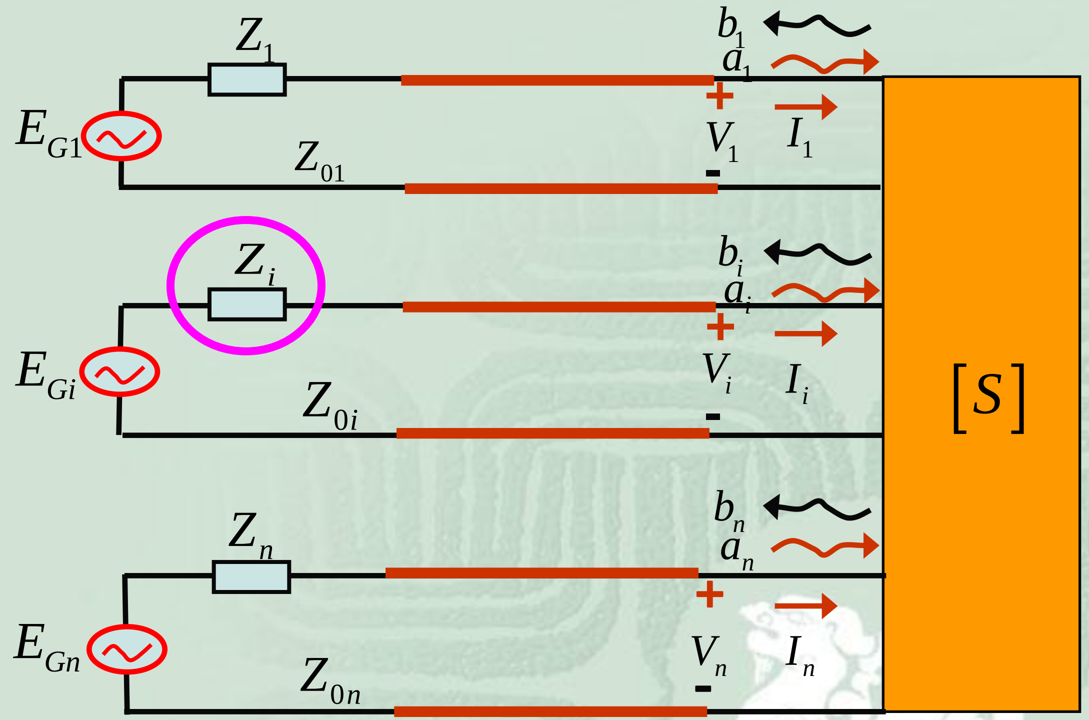


图6.4—3 与N端口网络联系的功率波

由此可解得入射波功率和出射波功率分别为

$$a_i = \frac{V_i^* + Z_i I_i}{2\sqrt{\operatorname{Re} Z_i}} \qquad b_i = \frac{V_i^* - Z_i I_i}{2\sqrt{\operatorname{Re} Z_i}}$$

从而有

$$\Gamma_i = \frac{b_i}{a_i} = \frac{V_i^* - Z_i I_i}{V_i^* + Z_i I_i} = \frac{Z_L - Z_i^*}{Z_L + Z_i^*}$$

功率波的反射系数

Z_L为 i 端口的视入阻抗。

a_i=0：表示该端口无外接源——由别处流来的电流产生电压。

当a_i=0，$b_i \neq 0$时：表示该处实现了共轭匹配。

由分压定理有（第i个支路）：

$$V_i = E_{G,i} - Z_i I_i$$

由此可以得到：$$|a_i|^2 = \frac{|E_{G,i}|^2}{4\,\mathrm{Re}\,Z_i} = P_A$$

信源资用功率

$|b_i|^2$为反射功率，于是净吸收功率为：$|a_i|^2 - |b_i|^2$

可由此定义*N*端口（入端阻抗不同）的广义散射阵：

$$[b]=[S][a]$$

$$S_{ii} = \frac{b_i}{a_i}\bigg|_{a_k=0,k\neq i}$$

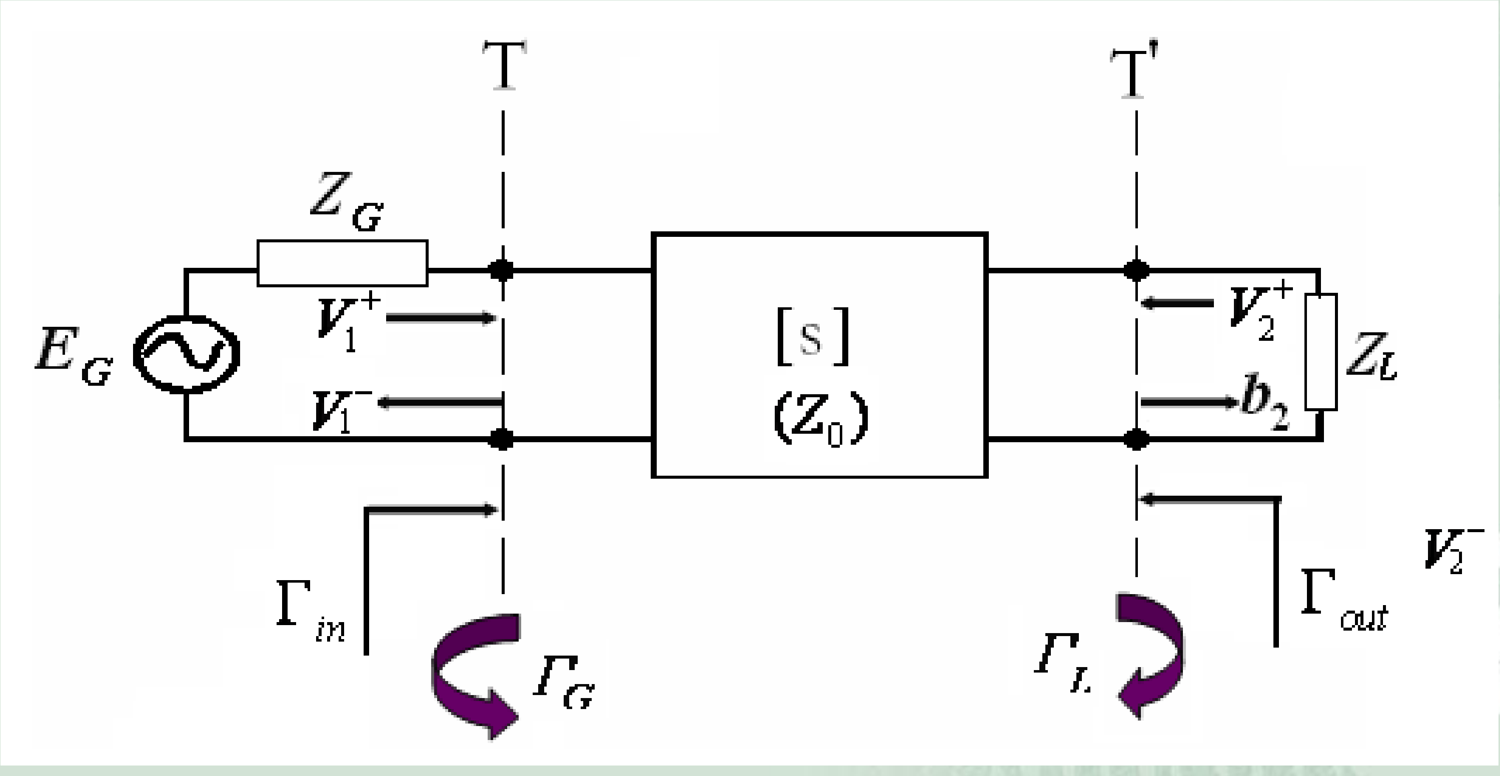
S_{ii}： 自端口的反射系数.

$$S_{ki} = \frac{b_k}{a_i}\bigg|_{a_k=0,k\neq j}$$

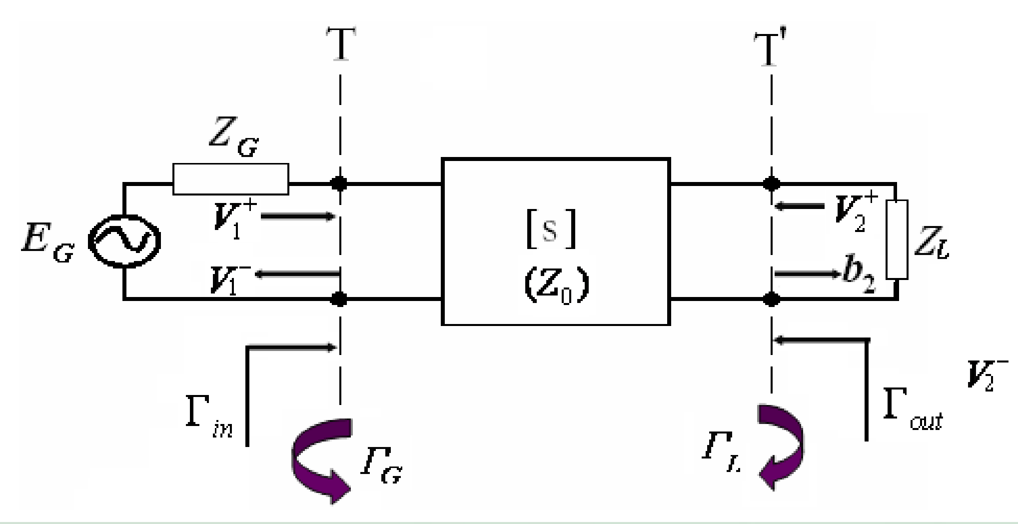
$S_{k,j}$： 互端口的传输系数(功率参量).

三、二端口网络的功率增益

现在考虑任意二端口网络带有任意源和负载阻抗时的功率转移特性。如图所示，实际上是一种滤波器或放大器。下面将利用S参量导出对这种电路有用的三种类型功率增益，以及在信号源和负载上产生的反射系数。



1. 实际功率增益G:



是负载吸收功率与送到二端口网络输入端的功率的比值，此增益一般与Z_G无关，但某些有源电路与Z_G有密切关系。

如图所示, 从网络看向负载的反射系数为 $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

而从网络看向信号源的反射系数为 $\Gamma_G = \dfrac{Z_G - Z_0}{Z_G + Z_0}$

一般情况下，二端口网络的输入端是不匹配的，具有的反射系数为Γ_{in}，可由以下确定。由S参量定义与$a_2=\Gamma_L b_2$，有

$$b_1 = S_{11}a_1 + S_{12}a_2 = S_{11}a_1 + S_{12}\Gamma_L b_2 \qquad (1)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 = S_{21}a_1 + S_{22}\Gamma_L b_2 \qquad (2)$$

$V_2^- \quad V_1^- / V_1^+$

消去上两式中的b_2，并求解b_1/a_1 得

$$b_1 = S_{11}a_1 + S_{12}a_2 = S_{11}a_1 + S_{12}\Gamma_L b_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2 = S_{21}a_1 + S_{22}\Gamma_L b_2$$

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \quad (3)$$

它是任意负载情况下，二端口网络输入端反射系数的一般结果，Z_{in}是看向负载网络端口1的阻抗.

类似地，当端口2用Z_G端接时，看向网络端口2的反射系数为

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_G}{1 - S_{11}\Gamma_G}$$

由分压关系，有 $V_1 = E_G \dfrac{Z_{in}}{Z_G + Z_{in}} = V_1^+ + V_1^- = V_1^+(1 + \Gamma_{in})$

由（3）式和 $Z_{in} = Z_0 \dfrac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$ ，并利用E_G求解V_1^+ 给出

$$V_1^+ = \frac{E_G}{2} \frac{1 + \Gamma_G}{1 - \Gamma_G \Gamma_{in}} \quad (4)$$

$$V_1^+ = \frac{E_G}{2}\frac{1+\Gamma_G}{1-\Gamma_G\Gamma_{in}}$$

如假定所有电压都是峰值，则送到网络的平均功率为

$$P_{in} = \frac{1}{2Z_0}\left|V_1^+\right|^2(1-\left|\Gamma_{in}\right|^2) = \frac{\left|E_G\right|^2\left|1-\Gamma_G\right|^2}{8Z_0\left|1-\Gamma_G\Gamma_{in}\right|^2}(1-\left|\Gamma_{in}\right|^2) \quad (5)$$

送到负载的功率为 $P_L = \frac{1}{2Z_0}\left|V_2^-\right|^2(1-\left|\Gamma_L\right|^2) \quad (6)$

由（2）式求解 V_2^- ，代入到（6）式，并使用（4）式，得

$$P_L = \frac{\left|V_1^+\right|^2}{2Z_0}\frac{\left|S_{21}\right|^2(1-\left|\Gamma_L\right|^2)}{\left|1-S_{22}\Gamma_L\right|^2} = \frac{\left|E_G\right|^2}{8Z_0}\frac{\left|S_{21}\right|^2(1-\left|\Gamma_L\right|^2)\left|1-\Gamma_G\right|^2}{\left|1-S_{22}\Gamma_L\right|^2\left|1-\Gamma_G\Gamma_{in}\right|^2} \quad (7)$$

这时的功率增益为

$$\boxed{G = \frac{P_L}{P_{\text{in}}} = \frac{\left|S_{21}\right|^2(1-\left|\Gamma_L\right|^2)}{\left|1-S_{22}\Gamma_L\right|^2(1-\left|\Gamma_{\text{in}}\right|^2)} \quad (6.4-50)}$$

2.资用功率增益G_A

是负载得到有功功率与由信号源来的资用功率的比值。

由信号源来的资用功率P_{avs}是送到网络去的最大功率，当端接网络的输入阻抗与信号源内阻共轭匹配时才出现P_{avs}，因此由（5）式得

$$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_G^*} = \frac{|E_G|^2}{8Z_0}\frac{|1-\Gamma_G|^2}{1-|\Gamma_G|^2} \qquad (8)$$

类似地，由网络来的资用功率P_{avn}为可能送到负载的最大功率，因此由（7）式得

在(9)式必须利用 $\Gamma_L = \Gamma_{out}^*$时计算$\Gamma_{in}$

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|E_G|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{out}|^2)|1-\Gamma_G|^2}{|1-S_{22}\Gamma_{out}^*|^2|1-\Gamma_G\Gamma_{in}|^2}\Bigg|_{\Gamma_L=\Gamma_{out}^*} \qquad (9)$$

$$P_{in} = \frac{|E_G|^2|1-\Gamma_G|^2}{8Z_0|1-\Gamma_G\Gamma_{in}|^2}(1-|\Gamma_{in}|^2) \qquad (5)$$

$$P_L = \frac{|E_G|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_L|^2)|1-\Gamma_G|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_G\Gamma_{in}|^2} \qquad (7)$$

由（3）式可以证明

$$\Gamma_{in}=\frac{V_1^-}{V_1^+}=S_{11}+\frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}=\frac{Z_{in}-Z_0}{Z_{in}+Z_0} \quad (3)$$

$$\left|1-\Gamma_G\Gamma_{in}\right|^2\Big|_{\Gamma_L=\Gamma_{out}^*}=\frac{\left|1-S_{11}\Gamma_G\right|^2(1-\left|\Gamma_{out}\right|^2)}{\left|1-S_{22}\Gamma_{out}^*\right|^2} \quad (10)$$

它将（9）式归结为 $P_{avn}=\frac{\left|E_G\right|^2}{8Z_0}\frac{\left|S_{21}\right|^2(1-\left|\Gamma_G\right|^2)}{\left|1-S_{11}\Gamma_G\right|^2(1-\left|\Gamma_{out}\right|^2)}$ (11)

由用E_G表示的P_{avs}、P_{avn}结果可见，它们与输入阻抗或负载阻抗无关。如果这些量值用V_1^+表示，由于V_1^+在计算P_L、P_{avs}、P_{avn}是不同的，那么将会出现矛盾。

利用（11）式和（8）式，资用功率增益为

$$G_A=\frac{P_{avn}}{P_{avs}}=\frac{\left|S_{21}\right|^2(1-\left|\Gamma_G\right|^2)}{(1-S_{11}\Gamma_G)^2(1-\left|\Gamma_{out}\right|^2)} \quad (6.4-51)$$

3. (转移)换能器功率增益G_T:

是送到负载的功率P_L与信号源来的资用功率P_A的比值。它与Z_G、Z_L都有关系，所以比前两种增益更有具有优越性。

由（7）式和（8）式，转移功率增益为

P_L负载吸收功率,P_A为信源资用功率

$$G_T = \frac{P_L}{P_A} = \frac{|S_{21}|^2 (1-|\Gamma_G|^2)(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2 |1-\Gamma_G\Gamma_{in}|^2} \quad (6.4-47)$$

$$P_L = \frac{|E_G|^2}{8Z_0} \frac{|S_{21}|^2 (1-|\Gamma_L|^2)|1-\Gamma_G|^2}{|1-S_{22}\Gamma_L|^2 |1-\Gamma_G\Gamma_{in}|^2} (7)$$

$$P_{avs} = \frac{|E_G|^2}{8Z_0} \frac{|1-\Gamma_G|^2}{1-|\Gamma_G|^2} (8)$$

特殊的转移功率增益：

匹配转移功率增益： **($\Gamma_L = \Gamma_G = 0$)**

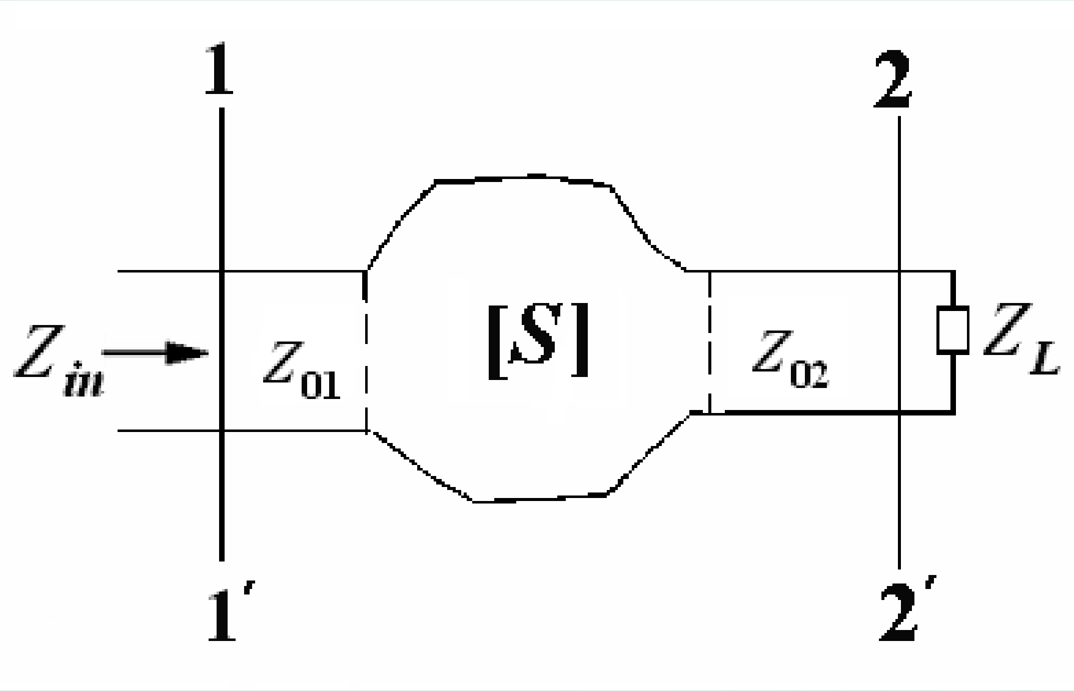
$$G_{Tm} = |S_{21}|^2 \quad (6.4-48)$$

单向转移功率增益 ($S_{12}=0$)

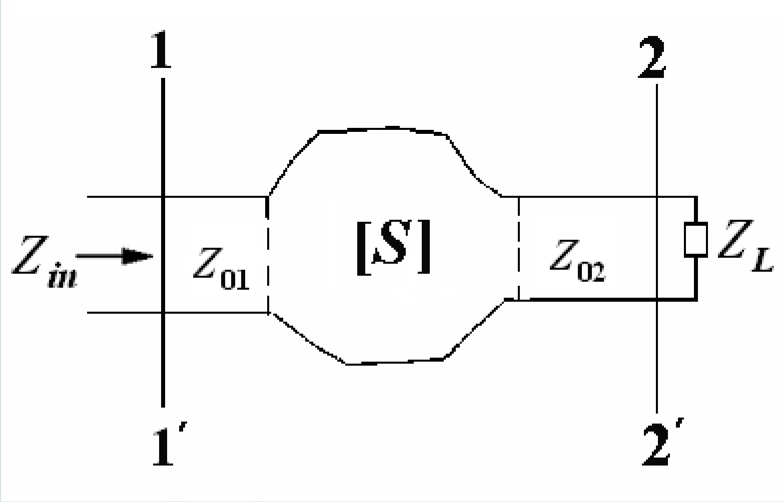
$$G_{Tu} = \frac{|S_{21}|^2 (1-|\Gamma_G|^2)(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2 |1-S_{11}\Gamma_G|^2} \quad (6.4-49)$$

【例6.4-5】一任意二端口网络如图所示，其[S]参数已知。

当2-2端面接负载Z_L时，求1-1端面的输入阻抗Z_{in}。



解：输出端接负载时的反射系数：



$$\Gamma_L = \frac{Z_L - Z_{02}}{Z_L + Z_{02}}$$

输入端反射系数：

$$\Gamma_{in} = \frac{Z_{in} - Z_{01}}{Z_{in} + Z_{01}} \Rightarrow Z_{in} = Z_{01}\frac{\Gamma_{in}+1}{-\Gamma_{in}+1} \quad (1)$$

而

$$\Gamma_{in} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1-s_{22}\Gamma_L} = s_{11} + \frac{s_{12}s_{21}(Z_L - Z_{02})}{(Z_L + Z_{02}) - s_{22}(Z_L - Z_{02})}$$

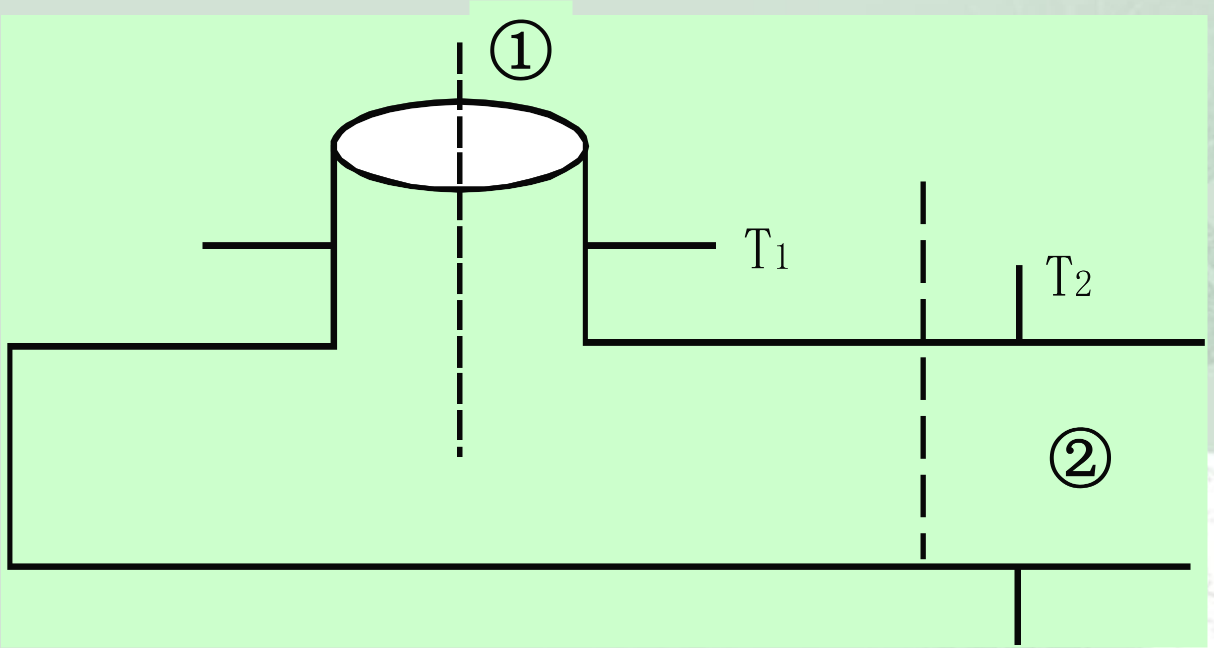
代入(1)式得：

$$Z_{in} = Z_{01}\frac{(1+s_{11})\left[(Z_L+Z_{02})-s_{22}(Z_L-Z_{02})\right]+s_{12}s_{21}(Z_L-Z_{02})}{(1-s_{11})\left[(Z_L+Z_{02})-s_{22}(Z_L-Z_{02})\right]-s_{12}s_{21}(Z_L-Z_{02})}$$

【例6.4-6】已知图示同轴—波导转换移头的S矩阵

$$[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

求:(1)2口接匹配负载时，1口的驻波系数；(2)当2口的反射系数为Γ_2时，1口的反射系数为多少？



解：(1)由散射矩阵定义 $$\boxed{\begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}} \qquad (1)$$

端口2匹配，意味负载处的反射波a_2=0，此时端口1的反射系数为

$$\Gamma_1 = \frac{b_1}{a_1} = S_{11}$$

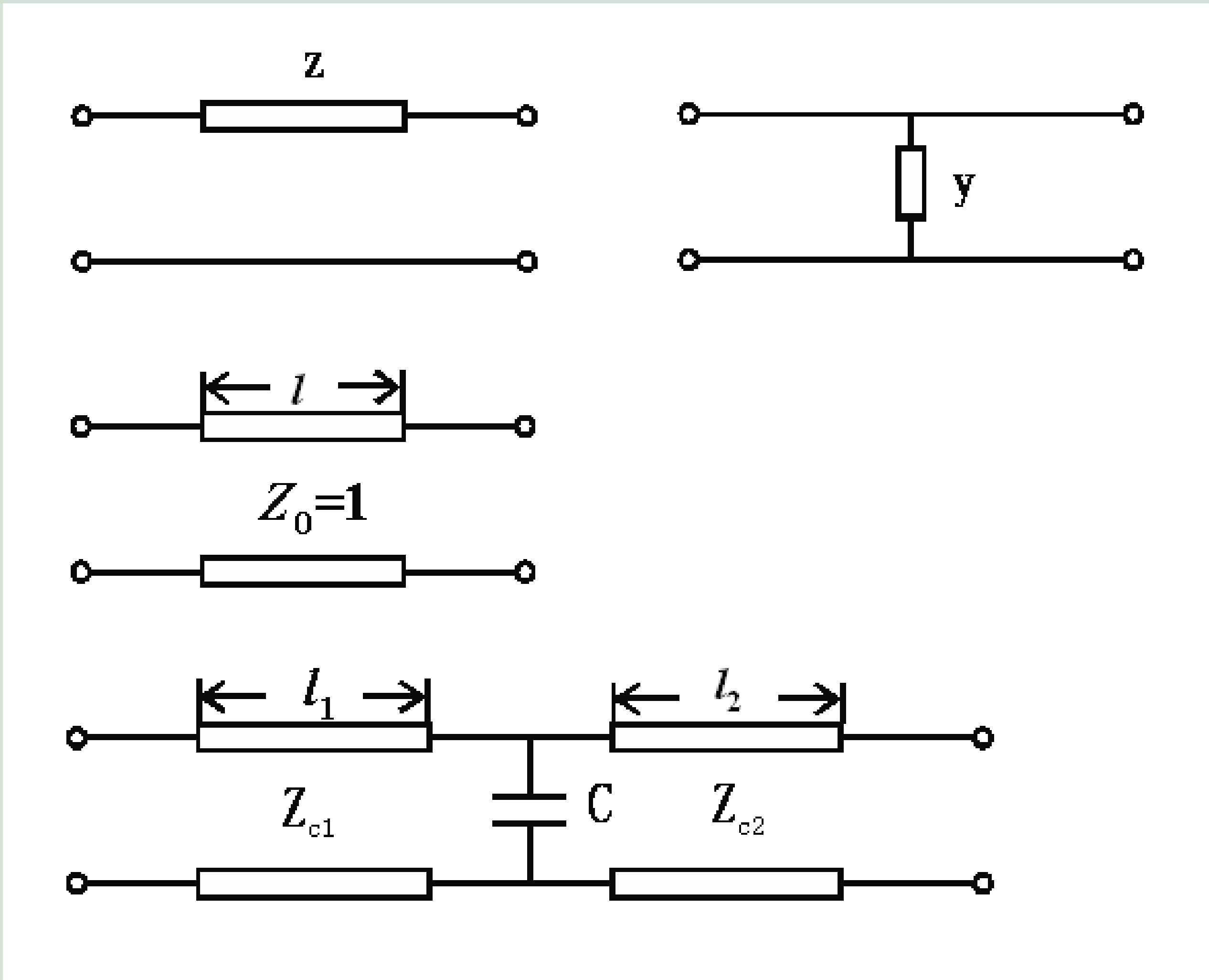
因此端口1的驻波系数为 $\rho_1 = \dfrac{1+|S_{11}|}{1-|S_{11}|}$

(2)当端口2接反射系数为Γ_2的负载时，负载处的入射波b_2与反射波a_2之间满足 $\Gamma_2 = \dfrac{a_2}{b_2} \quad \Rightarrow \quad a_2 = \Gamma_2 b_2$

代入(1)式得：$$\boxed{\begin{aligned} b_1 &= S_{11}a_1 + S_{12}\Gamma_2 b_2 \\ b_2 &= S_{21}a_1 + S_{22}\Gamma_2 b_2 \end{aligned}}$$

因此端口1的反射系数为 $\Gamma_1 = \dfrac{b_1}{a_1} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_2}{1-S_{22}\Gamma_2}$

一、求图示网络的S矩阵



二、已知S矩阵 $[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

试求 Γ_{in} 表达式.