TR-GY 7133: Urban Public Transportation
Spring 2018

Prof. Joseph Chow

Assignment 3:

Modeling Transit Demands

Yuwen Chang

M.S. in Applied Urban Science and Informatics

Center for Urban Science and Progress

New York University

04/04/2018

1. **Leveraging the Direct Demand Econometric Model with Modern Data and Commuting Patterns**

   a) Discuss how the models used in that study could be applied to transit travel in NYC using today's data (i.e. what zones, data sources, how would you validate the demand model).

The paper (Talvitie, 1973) adopts econometric models to map the southbound travel demands from suburbs to CBD, a corresponding comparison to be made in New York may be for commuters in Bronx traveling to downtown Manhattan. It is also possible to model similar demands from Queens, Brooklyn, New Jersey into Manhattan.

Given the availability and accessibility to high quality data we have today, it is also possible to validate the model with more and granular data. For static travel survey, we have U.S. Census Data including American Community Survey (ACS) and Longitudinal Employer-Household Dynamics (LEHD) which estimate the commuting behaviors, vehicle ownership, job-home location pairs, and so forth. For dynamic data of each transportation mode, we have loop detectors on highways, real-time GPS trajectory data of buses, and real-time train schedules that would allow us to monitor the demands and change in a timely manner and for as many directions and time period as possible. These advantages allow us to overcome many of the problems encountered by the authors, such as lack of automobile data, limited to one direction, and so on. The demand generation, demand distribution, traffic situation, travel time and other variables can also be estimated more accurately. This is made even more possible with on-demand services data, such as Taxi trips.

As to the consumer profile, other than traditional ACS data, we now have better means to approximate the constantly changing characteristics of neighborhoods and residents. These include rents, commercial activities, geo-tagged social media data, GPS popularity, and even mobile communication data (e.g., Wi-Fi probes). We can then attempt to derive the population, their profile, and potential behaviors/preferences, similar to how one would try to trace the happening of gentrification.

We may thus validate the econometrics models proposed in the paper by constructing the model with newly available data between outer boroughs/areas and downtown Manhattan. Then, based on the change in services (bus schedule change, road maintenance, fare adjustment, etc.), examine how the demands among modes alter. This will return us with a set of parameters for the (cross-)elasticities for comparison and examine how the model is different from the past and between Chicago and NYC.

b) Do you think there would be a significant change in elasticities (given the increased number of mobility options we have now)? Explain.

It depends. I think the general relationships may remain true in the U.S. context. For example, the pricing cost, transferring time, and the linehaul time may have similar effects for even for passengers today. Or, with linehaul time, bus competes with rail and rail competes with car. This is because the modal selections may remain similar, those who have car may only consider driving alone or taking a train, and those considering public transportation, may only be choosing between bus and rail. Unless there is a significant improvement (or deterioration), such (cross-)elasticities may more or less be similar. This is especially the case for longer commutes from outer suburbs to a CBD, such as Long Island New Jersey, and Upstate NY.

What may have changed drastically, however, is the urban transportation landscape. As more options pop up in recent years, users can easily switch between cabs, carpools, sharing bikes, buses, and subways. There are many competing services in big cities like NYC. This means passengers are more elastic towards the mode of transportation and are more likely to jump into a different mode or even switching among a set of attractive services instead of sticking to a single one. This may be true for neighboring boroughs, such as Bronx, Brooklyn, and Queens as well. Depending on the characteristics of communities, services like Chariot may be attractive over buses and subways for example.

However, all these modes are still competing on a similar set of the key variables that affect the elasticities, such as traveling times and fare costs. Comfort may be an extra consideration for modern population. For example, a more comfortable riding experience means lower cost for in-vehicle time.

2. Fare Elasticity

A single, one-directional bus loop serves travelers between three zones as shown in Figure 1. The estimated direct demand model is $D = D_0 t^{0.0638} p^{-0.41}$, where t = travel time (min), and p = fare price ($), and demand is in passengers/hr, as specified in Table 1.

Figure 1. Bus Loop

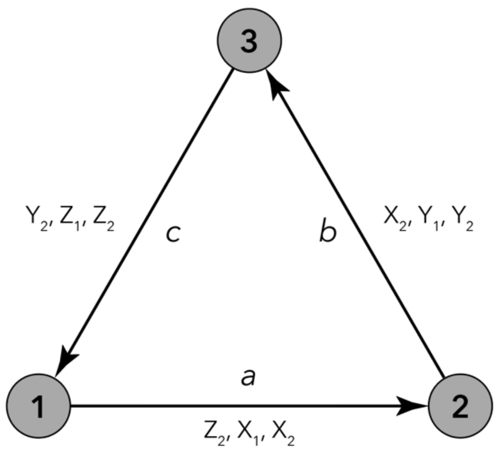


Table 1. Demands, Prices, and Travel Times

Demand	1	2	3
1	-	150	50
2	300	-	100
3	450	350	-

New Demand	1	2	3
1	-	X1'	X2'
2	Y2'	-	Y1'
3	Z1'	Z2'	-

Price	1	2	3
1	-	3	3
2	3	-	3
3	3	3	-

New Price	1	2	3
1	-	a	a+b
2	b+c	-	b
3	c	c+a	-

Travel Time	1	2	3
1	-	31	42
2	32	-	11
3	21	52	-

Demand Base	1	2	3
1	-	3x	x
2	3y	-	y
3	z	7/9z	-

a) Determine the elasticity of demand w.r.t. fare price. If a flat fare of $3 is currently in operation and travelers for each OD pair are assigned onto this loop on each link needed to get them from O to D, which link is most sensitive in terms of absolute ridership?

Currently, there are a total of 550, 450, and 1100 passengers on link a, b, and c respectively. The elasticity for price derived from the demand model is $e_{(D|p)} = -0.41$. In other words, for each percent increase in price, the ridership for each route will drop by 0.41%. Given the same percent change in ridership, link c (3→1) with the most demand will be most sensitive in terms of absolute ridership.

b) Suppose this system has a line capacity of 500 passengers/hr for each segment. Assuming FCFS rule along the loop (i.e. someone who gets on at station 1 has priority in being at segment (2,3) versus someone boarding at station 2) but random priority for people boarding at the same station, then excess demand would not be served if capacity is exceeded. Under this setting, design a station-to-station fare pricing to <u>eliminate unserved demand and reduce unused capacity</u>. Compare the revenue and ridership between this design with the flat $3 design.

As I shown in Table 1, let X, Y, Z be the demands for origins at 1, 2, 3, and the subscript number be the zones traveled (i.e., X_2 denotes OD pair from station 1 traveling 2 zones through link a and b to station 3). Since the longer commuters (2 zones) always stay onboard, and the rest of boarding passengers board randomly, we may develop the conditions according to the capacity limit as follow:

$$link\ a: Z_2 + X_1 + X_2 \leq 500$$

$$link\ b: X_2 + Y_1 + Y_2 \leq 500$$

$$link\ a: Y_2 + Z_1 + Z_2 \leq 500$$

Let P be the price and D be the demand of each link segment and the subscript a1, b1, c1, a2, b2, c2 be the specific price for each OD pair and the zones traveled (i.e., a1 means the fare for traveling on link a alone and a2 means traveling on link a and its consecutive link, b). The flat fare design ensures $3 for any OD pair, but the new design (P', D') sets different prices for each link respectively.

$$\Delta P_{a1} = \frac{(P'_{a1} - P_{a1})}{P_{a1}} \%$$

$$\Delta D_{a1} = \Delta P_{a1} \times (-0.41)$$

$$D'_{a1} = D_{a1} \times (1 + \Delta D_{a1})$$

We can then build up these relationships in Excel and then run a Solver to maximize our objectives by varying the fares respectively. The result, as shown as Table 2, for maximizing ridership, which aims to eliminate unserved demand and reduce unused capacity, is rather unexpected. While the fare for link c increased more than doubled to $7, the other two links are free. This may be due to the nature that link c is the most sensitive link that has significantly higher demands than the sum of a and b. Thus, even when other links are free, there are just not as much demand that meets the capacity as link c would.

However, if we switch our objective to maximizing revenue, the result shown as Table 3 makes more sense in implementation reality. We can see the fares for link a and b drop slightly and the fare for link c, as expected, increased in order to ease the demand to meet the capacity, but is not as high as the previous case.

Table 2. Fare Optimization for Maximum Ridership

fare		original fare		new fare		fare diff		demand diff		orig. demand		new demand		constraints		revenues	
a	0	Pa1	3	Pa1'	0	dPa1	-100%	dDa1	41%	Da1	150	Da1'	211.5	cons a	441.09	rev a	0
b	0	Pb1	3	Pb1'	0	dPb1	-100%	dDb1	41%	Db1	100	Db1'	141	cons b	347.86	rev b	0
c	6.991	Pc1	3	Pc1'	6.991	dPc1	133%	dDc1	-55%	Dc1	450	Dc1'	204.5	cons c	500	rev c	3495.6
flat	3	Pa2	3	Pa2'	0	dPa2	-100%	dDa2	41%	Da2	50	Da2'	70.5	Total	1289	Total	3495.6
elasticity	-0.41	Pb2	3	Pb2'	6.991	dPb2	133%	dDb2	-55%	Db2	300	Db2'	136.4				
		Pc2	3	Pc2'	6.991	dPc2	133%	dDc2	-55%	Dc2	350	Dc2'	159.1				

inputs variables constraints objectives

Table 3. Fare Optimization for Maximum Revenue

fare		original fare		new fare		fare diff		demand diff		orig. demand		new demand		constraints		revenues	
a	2.603	Pa1	3	Pa1'	2.603	dPa1	-13%	dDa1	5%	Da1	150	Da1'	158.1	cons a	300.09	rev a	781.19
b	2.423	Pb1	3	Pb1'	2.423	dPb1	-19%	dDb1	8%	Db1	100	Db1'	107.9	cons b	242.11	rev b	586.64
c	5.502	Pc1	3	Pc1'	5.502	dPc1	83%	dDc1	-34%	Dc1	450	Dc1'	296.1	cons c	500	rev c	2751
flat	3	Pa2	3	Pa2'	5.026	dPa2	68%	dDa2	-28%	Da2	50	Da2'	36.15	Total	1042.2	Total	4118.8
elasticity	-0.41	Pb2	3	Pb2'	7.925	dPb2	164%	dDb2	-67%	Db2	300	Db2'	98.07				
		Pc2	3	Pc2'	8.105	dPc2	170%	dDc2	-70%	Dc2	350	Dc2'	105.8				

inputs variables constraints objectives

Table 4. Ridership and Revenue under Flat Fare

orig. base		new base		ridership	
x	50	x'	102.9	link a	500
y	100	y'	99.3	link b	500
z	450	z'	113.7	link c	500
		flat-fare	3	Total	1500

inputs
variables
constraints
objectives

revenue w capacity 3032
revenue w/o capacity 4200

In order to compare with the flat fare scenario, we also need to build the constraints. However, since it is the same price to travel at any OD pair, we are not calculating the demand change for each OD pair. Instead, based on the information we have (FCFS and random boarding), we assume that at any given station, the passengers traveling at any length (1 or 2 zones) have the same chance of boarding, leading to the same proportion of the demands onboard. This leaves us with 3 base demand variables x, y, and z to solve, as shown in Table 1. For example, for link a, Z_2, X_1, and X_2 will thus be derived as $7/9z$, $3x$, and x. We may then optimize this problem with Solver in Excel

The result is displayed in Table 4. Given the maximum ridership of 1,500, the base will be approximately 100 for each base. Following the previous example, this means there are a total of $4x'$ passengers that board on station 1 with $7/9z'$ passengers already onboard. The final comparison is shown as Table 5. As a result, different fares solutions tend to have better revenues than flat fare. However, there still exists a gap of unused capacity and unserved demand due to the interactions of passengers with the fare change and the capacity limits at each link.

Table 5. Comparison of Different Optimization Results

	Ridership	Unused	Unserved	Revenue
flat fare w/o capacity	2,100	-	-	4,200
flat fare w/ capacity	1,500	-	600	3,032
different fare w/ max ridership	1,289	211	811	3,496
different fare w/ max revenue	1,042	458	1,058	4,119

3. Path Selection

Consider the transit network shown as Figure 2 and Table 6. Travelers perceive the line schedules to be practically independent of each other. The headways are deterministic, but a traveler's arrival incident on the headway is assumed to be uniformly distributed between 0 and the headway. The main analysis is carried out with wait time assumed to be valued the same as in-vehicle travel time, and then discussed with at 1.76 times in-vehicle travel time.

Figure 2. Line Route Map

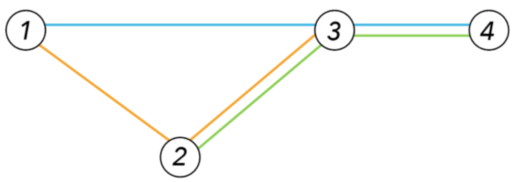


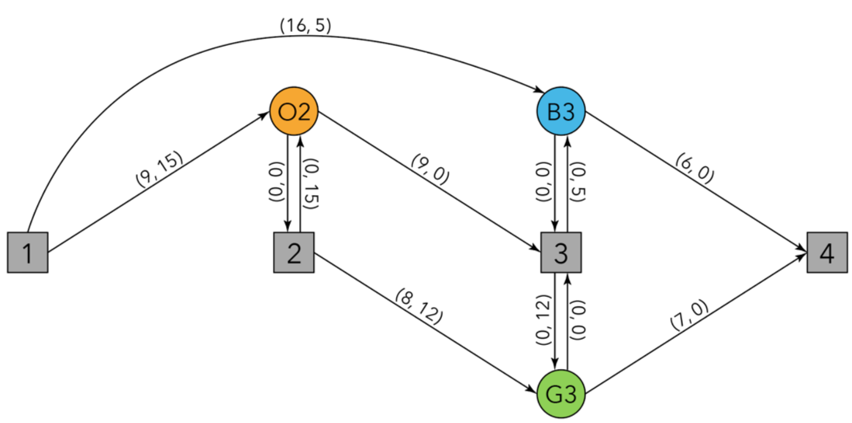
Table 6. Line Information

	Travel Time for (O,D) (min)				(/hr)
Lines	**(1,2)**	**(1,3)**	**(2,3)**	**(3,4)**	**Freq.**
Orange	9	-	9	-	15
Blue	-	16	-	6	5
Green	-	-	8	7	12

a) Convert the network representation below into a link representation.

The route map is first converted into a network representation with nodes and links (see appendix for the draft). Then the links are simplified by removing unnecessary links and nodes. The finalized link representation result is shown as Figure 3.

Figure 3. Link Representation of the System Illustrating OD Pair of 1→4 (cost, frequency)



b) For OD (1,4), determine the shortest *a priori* expected path travel time (and which line).

The shortest available *a priori* single route is selected by the evaluating the total travel time for each path independently. Given deterministic headways and uniform arrival distribution of passengers, all expected waiting time ($E[W]$) here and for the following problem are calculated with $\alpha = 0.5$.

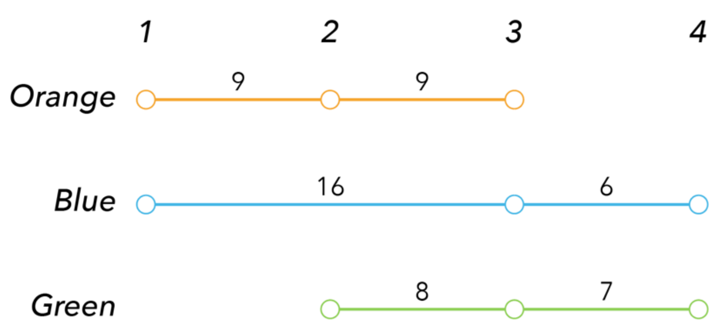
$$E[W(A)] = \alpha \times \frac{1}{f_A} = \frac{1}{2f_A}(hour)$$

$$E[W(A,B)] = \alpha \times \frac{1}{f_A + f_B}(hour)$$

As a result, based on the calculation of $E[W]$, which can be found in Table 7, and the travel times, as in Figure 4, the shortest a priori route to be considered is 1→B→4, which will take 28 minutes (6 + 16 + 6) to get from station 1 to 4.

If the wait time is assumed to be valued at 1.76 times in-vehicle travel time, then all wait time would simply be multiplied by 1.76. The new $E[W]$ can be regarded as 3.52, 10.56, and 4.4 minutes respectively for Orange, Blue, and Green Lines. The shortest *a priori* route remains the same, but the cost is valued as 32.56 minutes instead. The second option will be 1→O→3→G→4, which has a travel cost equivalent to 32.92 minutes.

Figure 4. Path Travel Times (mins)



c) For the same OD (1,4), identify the set of attractive routes and determine the expected shortest path and travel time under an "optimal strategy" route choice policy. Discuss the difference with an *a priori* shortest path assumption.

In contrast to the *a priori* shortest path, which passengers also follow this route no matter what, the optimal strategy is more flexible and goes with whatever line comes first in a set of attractive routes. The advantage of this approach is to minimize the expected total travel time even more by reducing the average expected waiting time.

To utilize this strategy, a set of $E[W]$ is calculated in Table 7, outlining the average waiting times when passengers get on whatever line comes first. The benefit is obvious, as the waiting time is shorter in average than any specific line when we treated them independently.

Table 7. Line Probabilities and Waiting Time at Each Origin Node under Optimal Strategy

				Line Probabilities		
Origin	**Attractive Lines**		**E[W]**	**Orange**	**Blue**	**Green**
1	O→2		2	1		
1	O→3		2	1		
1	B→3		6		1	
1	B→4		6		1	
1	O→2	B→3	1.5	0.75	0.25	
1	O→2	B→4	1.5	0.75	0.25	
1	O→3	B→3	1.5	0.75	0.25	
1	O→3	B→4	1.5	0.75	0.25	
2	O→3		2	1		
2	G→3		2.5			1
2	G→4		2.5			1
2	O→3	G→3	1.1	0.56		0.44
2	O→3	G→4	1.1	0.56		0.44
3	B→4		6		1	
3	G→4		2.5			1
3	B→4	G→4	1.8		0.29	0.71

Later on, we may use the line probabilities to construct the hyperpath, as shown in Figure 5. There is a 25% probability of getting on the Blue Line first, and since there is no incentive for the passenger to get off at station 3, this will directly lead the passenger to the final destination (station 4). For the 75% probability that gets on the Orange Line first, there is also no reason to get off at station 2, leaving G(2,3) unattractive. However, Orange Line ends at station 3 and a transfer is required. We then recalculate the probabilities (75%) using the line probabilities at station 3, which is 29% and 71% respectively for transferring to the Blue and Green Lines. This yields us with the following results:

$Path\ A$ (25%) $B{:}\,4$ $\quad E[TT_A] = 0.25\times(1.5 + 22) = 5.88$

$Path\ B$ (22%) $O{:}\,3; B{:}\,4$ $\quad E[TT_B] = 0.22\times(1.5 + 18 + 1.8 + 6) = 6.01$

$Path\ C$ (53%) $O{:}\,3; G{:}\,4$ $\quad E[TT_C] = 0.53\times(1.5 + 18 + 1.8 + 7) = 15.00$

Figure 5. The Probabilities of a Hyperpath Set

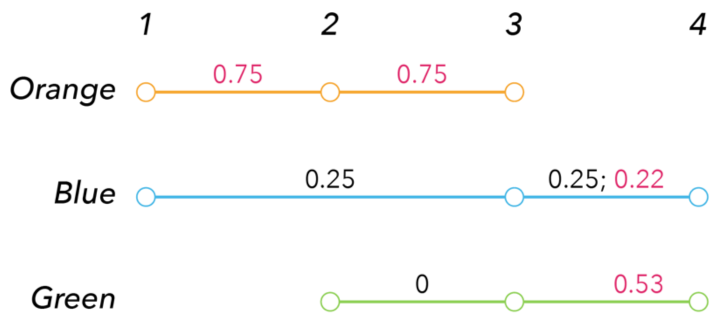


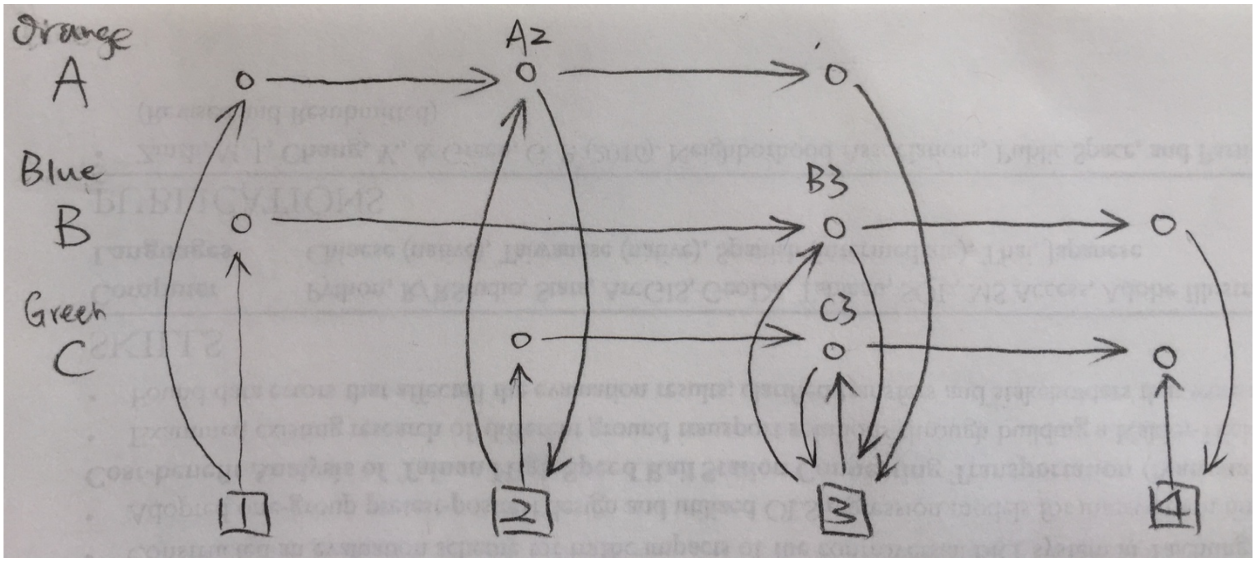
Table 8. Total Valued Travel Time Comparison

	Wait Time Value	
Travel Time	**1**	**1.76**
Shortest	28.00	32.56
Optimal	26.89	29.05
Difference	1.11	3.51

Thus, the total travel time in average for an optimal strategy approach is estimated to be 26.89 minutes, which is 1.11 minutes fewer than the *a priori* approach. Although this may not seem much, it reveals the concept of how having a set of attractive routes and taking the first that comes can lead to even shorter traveling time than just sticking to one of them.

In order to compare with the assumption of a higher cost for waiting time (1.76 times), we need to penalize waiting times by multiplying it with 1.76 (i.e., every minute waiting for the vehicle is consider as costly as 1.76 times of that traveling in-vehicle). As a result, the updated calculation yields us with 29.05 minutes equivalent of travel time. Now the gap between optimal strategy and the *a priori* shortest route widens to 3.51 minutes. This echoes the previous discussion about how the cost of waiting time is preferably reduced by the optimal strategy, especially when you hate waiting. The final comparison is demonstrated in Table 8.

Appendix A. Draft of Complete Link Representation Network



References

Talvitie, A. (1973). A direct demand model for downtown work trips. *Transportation*, *2*(2), 121-152.