Lecture 4
Forward Contracts on Foreign Currencies

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Outline

Exchange Rate: Supporting Hong Kong as a World Financial Center?

- **Exchange Rate**: the value of one nation's currency versus the currency of another nation or economic zone.
- US dollar has been the dominating currency for international transactions since World War II.
- Since 1983, the Hong Kong dollar was officially pegged to the US dollar, with a linked exchange rate of HK $ 7.8 = US $ 1.
- Transactions in HKD is almost equivalent to transactions in USD, allowing (US) investors to avoid risks associated to fluctuation in exchange rate.
- This turns out to be key in supporting Hong Kong as a world-class financial center.

Examples: Fluctuation in Exchange Rate

GBP/USD

EUR/USD

AUD/USD

HKD/USD

Currency Forward: Forward Contract Fixing Future Exchange Rate

Definition: A currency forward contract is an agreement to buy or sell a predetermined amount of certain foreign currency at a pre-specified price on a pre-specified date in the future.

- Similar to forward contract on financial assets, it is a contract for purchasing an item (foreign currency) with a fixed price.
- Parties in international trade (manufacturers...) can use such contract to lock down the value of future payment in local currency.

Outline

Finding Forward Exchange Rate 1

Suppose at time t, we enter a forward contract, and agree to buy 1 unit of GBP in the future at time T with K Hong Kong dollars. The spot exchange rate is M_t Hong Kong dollar per GBP.

- Goal: to find the K **such that** the contract has value **zero** at time t. This is the **Forward Exchange Rate** of GBP.

Finding Forward Exchange Rate 2

- At maturity T, the forward contract delivers:
  - 1 GBP, which could be sold at the market immediately with value M_T HKD.
  - $-K$ HKD, the price agreed at time t.
- The replication portfolio targets at yielding the same payoff.
  - (1) We want 1 GBP at T. We can buy $e^{-r_{c,£}(T-t)}$ units of GBP, and save them with risk free rate in UK. The cost is $M_t e^{-r_{c,£}(T-t)}$
  - (2) Borrow $Ke^{-r_{c,\$}(T-t)}$ HKD. We will need to return K HKD at time T.
- The replication portfolio costs $M_t e^{-r_{c,£}(T-t)} - Ke^{-r_{c,\$}(T-t)}$. Making it zero, we get the formula for Forward Exchange Rate:

$$F(t,T) = M_t e^{(r_{c,\$} - r_{c,£})(T-t)}.$$

- Should forward price predict future exchange rate?
  - of a stock: stock price in the future
  - No. As we using r_c, there are risk in market

A Numerical Example

- On 30 Sept 2020, the GBP/HKD spot exchange rate was 9.97 HKD per GBP.
- The riskfree interest rate in UK was 0.1% per annum, continuously-compounded.
- The riskfree interest rate in Hong Kong was 0.5% per annum, continuously-compounded.
- What are the Forward Price on 30 Sept 2020 for 31 Decemeber 2020 delivery?

- $r_{c,\$} - r_{c,£} = 0.4\%;\ T - t = 1/4;\ M_t = 9.97.$
- $F_t(T) = 9.97 \times e^{0.4\% * (1/4)} = 9.98.$

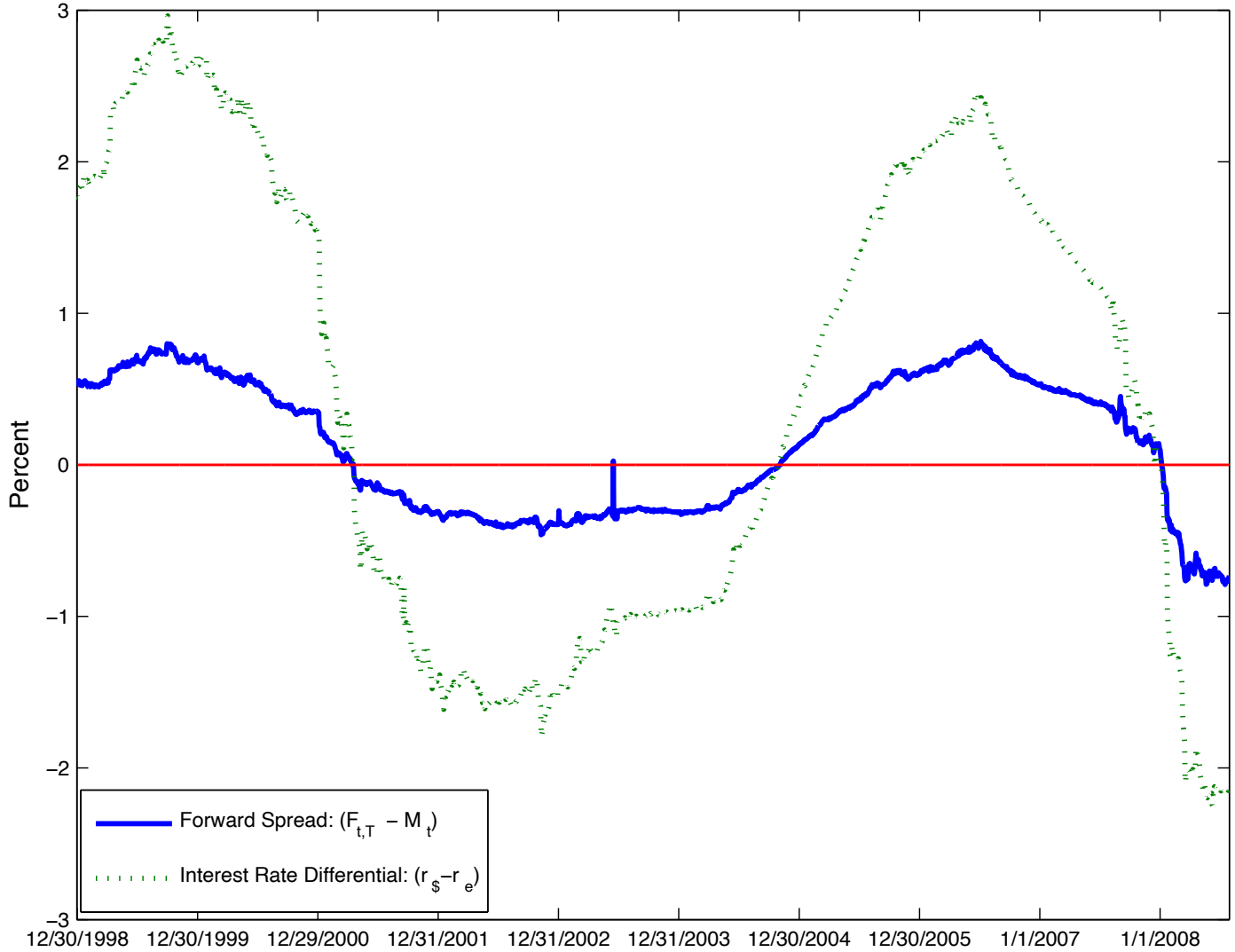
Understanding Forward Exchange Rate

- **Brainstorm**: Suppose as a researcher you find an indicator which predicts that the exchange rate will increase, what's your prediction for the forward rate now?

- The forward exchange rate should **remain the same**, if the corresponding interest rate does not change.

Understanding Forward Exchange Rate (Cont'd)

- The forward exchange rate only depends on the interest rate differential of the two countries.



Outline

Covered Interest Rate Parity

- We have derived the forward exchange rate:

$$F(t, T) = M_t e^{(r_c - r_c^*)(T-t)}.$$

  - M_t: spot exchange rate; $F_t(T)$: forward exchange rate;
  - r_c interest rate of home country (Hong Kong); r_c^* interest rate of foreign country (UK).

- This is called **Covered Interest Rate Parity (CIP)**.

- In reality,
  - r_c and r_c^* are determined by corresponding local authorities.
  - M_t and $F_t(T)$ are determined by local auhtorities and market tradings.

- As a result, they are determined in different spaces, and there might be some violation to the CIP.

Covered Interest Rate Parity (Cont'd)

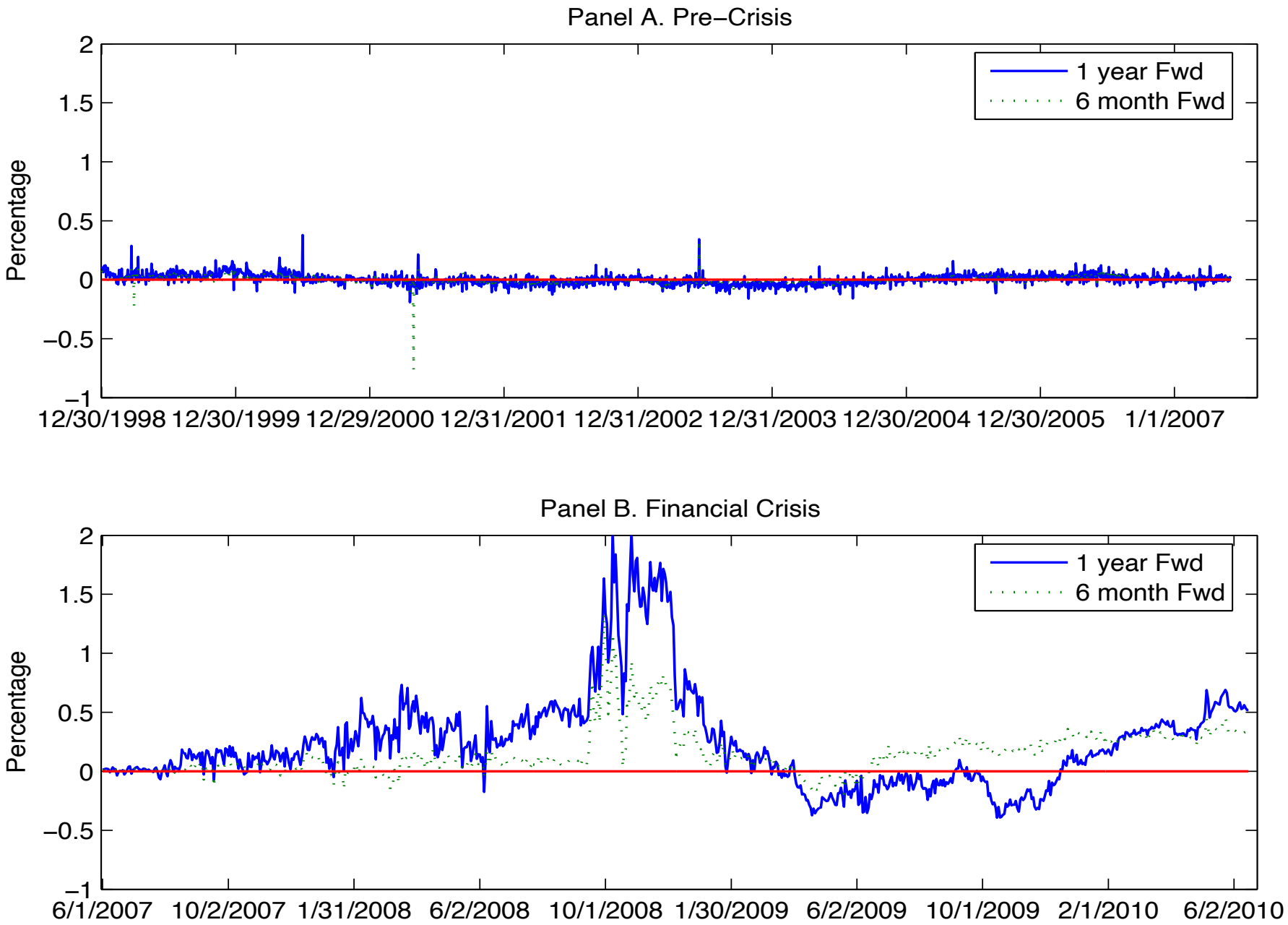
- Consider the difference between the market forward exchange rate and the forward rate implied by the model

$$F(t,T) - M_t e^{(r_c - r_c^*)(T-t)}$$

- If the term above is not close to zero, there is a violation of the covered interest rate parity.

- This was true during the 2008 financial crisis.

CIP Violation: Dollar/Euro, 2007 - 2009



The Failure of CIP during Financial Crisis 1

$\frac{1.02}{1} > e^{(1\% - 1\%)}$

- During the crisis, from the graph
  Market Suppose
  $$F(t,T) > M_t \, e^{(r_{c,\$} - r_{c,e})(T-t)}.$$
- This means that the Forward Exchange Rate was too high, and we want to **sell** Euros in the future.
- The corresponding trading strategy is
  (a) short forward;
  (b) buy Euros at spot market, and deposit them into risk-free accounts;
  (c) borrow U.S dollars to fund the purchase.

The Failure of CIP during Financial Crisis 2

To test which component went wrong

$$F(t,T) > M_t\, e^{(r_{c,\$} - r_{c,e})(T-t)}.$$

- The trade in previous slide requires borrowing US dollar at risk free rate.
  Increase interest rate for borrowing
- What if this is impossible?
  Increase $\Delta r_{c,\$} - r_{c,e}$
  - During the financial crisis, everyone wanted to hold the safest assets, in this case US dollars.
  - Investors had to borrow at a higher rate. So must affect $r_{c,\$}$ but not $r_{c,e}$
- The gap between the market risk-free rate and the 'shadow rate' is the **convenience yield** for US dollars.
- Lesson to learn: when there appear to be an arbitrage opportunity in the market, you should look at the steps of arbitrages carefully, and see if the corresponding trade was possible.

A Deeper Understanding of CIP 1

Let's move back to the case of HKD/USD exchange rate.

- The Hong Kong Monetary Authority promised that the exchange rate between HKD and USD is fixed at 7.8 HKD/USD.

- What is the forward exchange rate between HKD/USD with one month maturity?

A Deeper Understanding of CIP 2

$$F(t,T) = M_t$$

$$F(t,T) = M_t e^{(r_{c,HK\$} - r_{c,HK\$})(T-t)}$$

If CIP must hold (thanks to market power), what is the implication of the interest rates of Hong Kong and US?

$$r_{c,HK\$} = r_{c,US\$}$$

- This means that it would be very hard to take bets on the direction of movement for interest rates between US and HK market.
- What if the equation does not hold?
- Free exchange of foreign currency is not allowed. Can you find one example?
- Why do governments want to control 1) interest rates 2) exchange rate or 3) freedom of financial market?

Summary

- How to price forward contracts on foreign currency.
- Covered interest rate parity (CIP).