Lecture 5
Forward Contracts on Commodities

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Outline

Commodities as Underlying Assets

- Energy
  - crude oil, gasoline, natural gas, propane, coal, etc.
- Metals
  - gold, silver, copper, aluminum, etc.
- Agriculture
  - corn, wheat, etc.

Revisiting the Replication Argument 1

- Consider the replication portfolio of a forward contract on a stock with strike price K:
  - (1) At time t: buy one unit of stock with price S_t;
  - (2) Borrow $Ke^{-r_c(t-T)}$ with riskfree rate.

$S_T - K$

Revisiting the Replication Argument 2

- The asset / liability in the portfolio is given by:
  - (1) Time t:
    - Cash Flow: $-S_t$ for buying the stock; $Ke^{-r_c(t-T)}$ from borrowing;
    - Financial Asset: 1 unit of the stock; liability of paying K at time T.
  - (2) Time T:
    - Financial Asset: 1 unit of the stock.
    - Cash Flow: $-K$ for debt payment.

$S_T - K$.

Revisiting the Replication Argument 3

- There are several implicit assumptions in the replication
  - The asset can be stored for free.
  - The asset in the future could be purchased today.
- These assumptions are sometimes wrong for commodities.
- Such issues will cause the failure of the formula for forward prices of financial assets in the space of commodity forward contracts.

Outline

Pricing of Commodity Forwards

There are various reasons why we can not use the formula for financial assets to get the forward prices of commodities, including

- Storage cost (carry cost)
- Convenience yield
- Spot price is not available
- etc....

We will discuss them and see what we can learn.

Storage Costs

- For financial assets, we can simply hold them and create replicate portfolio without a direct cost.
- For commodities, however, 'holding' them are generally costly.
  - space, equipment needed
- The spot price S_t does not incorporate the costs.

Replicating the Payoff of a Forward Contract with Storage Cost

- Suppose that there is a storage cost Λ paid at time t to store 1 unit of the underlying commodity.
- What is the cash flow of the replicating portfolio for the forward contract?
  - Portfolio 1: Long the forward contract at forward price.
    - Time-T cash flow: $S_T - K$ $S_T - \Lambda$
  - Portfolio 2: Long 1 unit of the underlying asset, borrow K units of zero-coupon bonds.
    - Total cost of puchasing: $S_t + \Lambda$.
    - Cost of borrowing: $-KZ(t, T)$.
    - Total cost of the portfolio: $S_t + \Lambda - KZ(t, T)$.

If buy the commodity and sell it directly, the new lender will take care of Λ

borrow less if Λ at T

Forward Price with Storage Cost

$$S_t + \Lambda - KZ(t, T).$$

- This must also be the price of the forward contract.
- Making it 0, and we can solve for the forward price of asset with storage cost:

$$F_t = (S_t + \Lambda)e^{r_c(T-t)}.$$

- $S_t + \Lambda$: By entering the forward contract, you save the storage cost. However, the counter party effectively paid the cost and you need to cover it.

Continuously Compounded Carry Costs

Not necessary cash flow

- There could be other costs associated with storing a commodity asset, which are called **carry costs**.

  This continuously

  - One unit of commodity at t becomes $e^{-\lambda(T-t)}$ unit at T due to carry costs, where λ is continuously-compounded carry cost.
  - **Example:** spoilage, depreciation.

- In the replication portfolio, we need

$$e^{\lambda(T-t)}$$

  units of underlying asset at the time of contract initiation.

- **Forward price** when there is continuously-compounded carry cost λ:

$$F_t(S_t, T) = S_t e^{\lambda(T-t)} e^{r_c(T-t)} = S_t e^{(r_c+\lambda)(T-t)}.$$

- $S_t e^{\lambda(T-t)}$: actual cost of getting 1 unit of commodity at time T.

Lease Gain

Holding the commodity could also yield some returns, which is characterized by lease rate q (continuously compounded).

- You can lend the commodity to a third part in a centralized market, and generate some profit.
- Suppose there is an active lease market for the commodity and with lease rate q (continuously compounded).
- From t to T, 1 unit of commodity grows to $e^{q(T-t)}$ units.
- In the replication portfolio, you only need $e^{-q(T-t)}$ at the beginning.

$$F_t(S_t, T) = S_t e^{-q(T-t)} e^{r_c(T-t)} = S_t e^{(r_c-q)(T-t)}.$$

- **Key insight:** lease rate is the same as dividend yield for financial assets.

The Convenience Yield 1

Definition: Sometimes holding a commodity asset can provide extra benefit, and this is called **Convenience Yield**.

Example:

- Corn is an essential input for a food manufacturer
- Stop producing if running out of corn; idling workers and machines
- Manufacturer is willing to pay in order to ensure a stable flow of corn – the convenience yield.

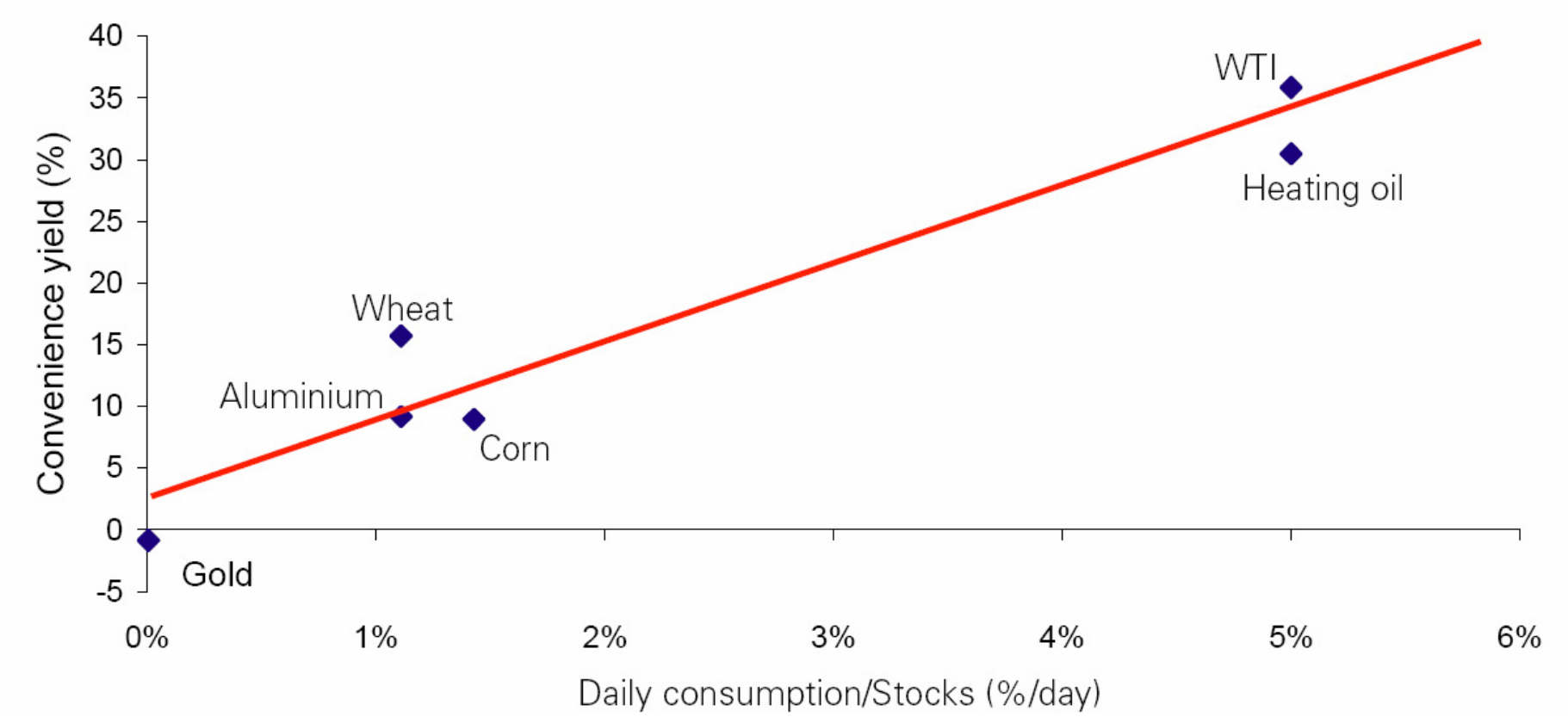
Key property:

- The convenience yield is 'shadow': it could be hardly directly observed.
- Demand/Inventory is a key determinant of the convenience yield

$$\text{Demand/Inventory} \uparrow \quad \Longrightarrow \text{the convenience yield} \uparrow$$

- Both at the individual level and the aggregate level

The Convenience Yield 2



Source: DB Global Markets Research

- Convenience yields are strongly positively correlated with demand-to-inventory ratios

The Convenience Yield 3

- If everyone has the same convenience yield c (again, continuously compounded...)
- We get a simple formula for forward price:

$$F_t = S_t e^{-c(T-t)} e^{r_c(T-t)} = e^{(r_c-c)(T-t)}.$$

- In practice, different investors might have different convenient yields.
- Then, there could be many forward prices available in the market, depending on the convenience yields of two counter parties.
- This implies a range of forward prices satisfying non-arbitrage conditions:

willing to sell at $S_t e^{(r_c-c)(T-t)}$ $\leq F_t \leq$ $S_t e^{r_c(T-t)}$ willing to buy at

Long buy from sell Short

- This is **not** a violation to absence of arbitarge: forward contracts are OTC, and investors can not form a trading strategy to take advantage of the difference in convenience yield.
- We use forward prices observed to **back out** the convenience yields.
- Economically, convenience yields are used to explain deviation from standard pricing formula when physical costs / benefits are all considered.

Spot Price Unavailable 1

Consider the case of electricity.

- Compared to generation, storage of electricity is extremely costly.
- No one stores electricity. As a result, we can not buy '1 unit' of electricity and store it to maturity of our forward contrtact.
- As a result, we could not use spot price as the cost of replicating the electricity delivery in the future.

Spot Price Unavailable 2

- In practice, the forward prices are available via market trading.
- Consider the formula for forward price of simple financial assets:

$$F_t(S_t, T) = S_t e^{r_c(T-t)}.$$

- What does S_t mean?
- **The present value of S_T.**
- Let $\text{PV}_t(S_T)$ denote the present value of S_T, or the underlying asset.
- We have

$$F_t = \text{PV}_t(S_T) e^{r_c(T-t)}.$$

- Then we can use the forward prices to **back out** the present values of commodities in the future

$$\text{PV}_t(S_T) = F_t e^{-r_c(T-t)}.$$

- This is called **price discovery**, a very important function of the financial market.
- What does this remind you of what we have covered?

What Does Data Say?

- The market **should** incorporate the costs of replication strategies.
- It is then helpful to look at data first to see what the data says.
- Empirically, we look at **Forward Curves**.
- **Definition:** "Forward curve" is a curve that plots the forward prices of assets against the maturity.

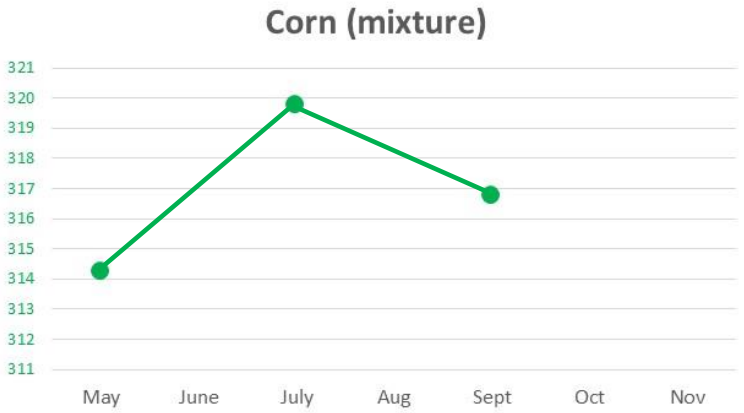
What Does Data Say? (Cont'd)

May 5, 2004. Corn and soybeans are from CBOT and unleaded gasoline, oil, and gold from NYMEX.

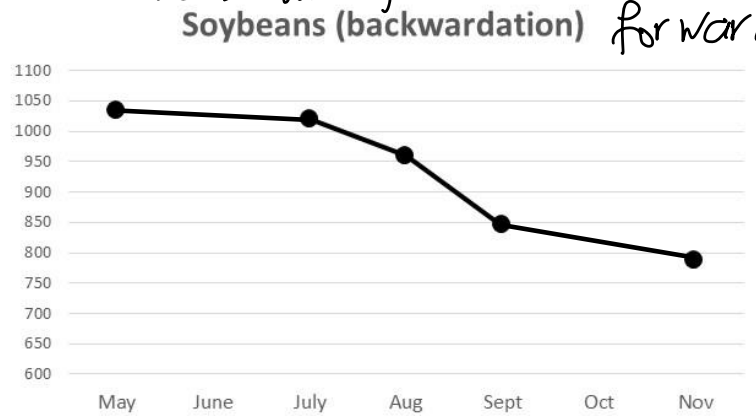
Expiration Month	Corn (cents/bushel)	Soybeans (cents/gallon)	Gasoline (cents/gallon)	Crude Oil (dollars/barrel)	Gold (dollars/ounce)
May	314.25	1034.5	-	-	393.40
June	-	-	131.25	39.57	393.80
July	319.75	1020.00	127.15	39.36	394.30
August	-	959.00	122.32	38.79	394.80
September	316.75	845.00	116.57	38.13	-
October	-	-	109.64	37.56	395.90
November	-	786.5	105.49	37.04	-

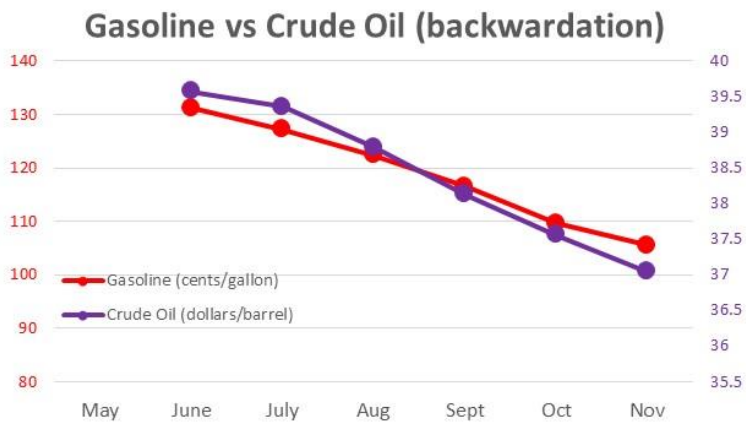
Source: Futures data from Datastream

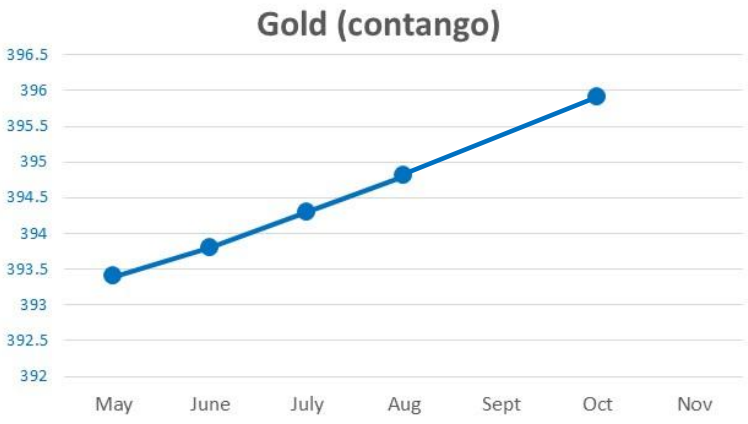
What Does Data Say? (Cont'd)



we know there will be lots of beans during harvest. No need forward







Empirical Patterns of Commodity Forwards

- The curves could be both upward or downward sloping.
  - For simple finance assets, the curve should be upward-sloping if interest rate is positive.
  - The deviation from simple model implies the costs in the replicating strategy.
    - Convenience yield
    - Storage cost
    - Etc...

Summary

- The spot price S_t is not always available due to special properties of the commodities.
- The spot price S_t does not necessarily contain the (physical) cost or benefit of holding the asset.
- When determining the forward price, you need to back out how much you need to pay to get **1 unit** of the underlying at maturity.

Outline

Non-Storable Commodities: electricity

- **Instantaneous process (almost):**

$$\text{production} \Rightarrow \text{transmission to end-users} \Rightarrow \text{consumption}$$

- **Three features of electricity as a commodity:**

  (1) Non-storability
  - extremely costly to store

  (2) Limited capacity of producing new electricity power at any time
  - existing power station for electricity

  (3) Substantial variability of demand during every day
  - inactive nighttime vs. active daytime

- **Implications:**
  - Pricing with replication can hardly work, due to (1).
  - Fluctuating forward prices within a day, due to (1), (2), and (3).

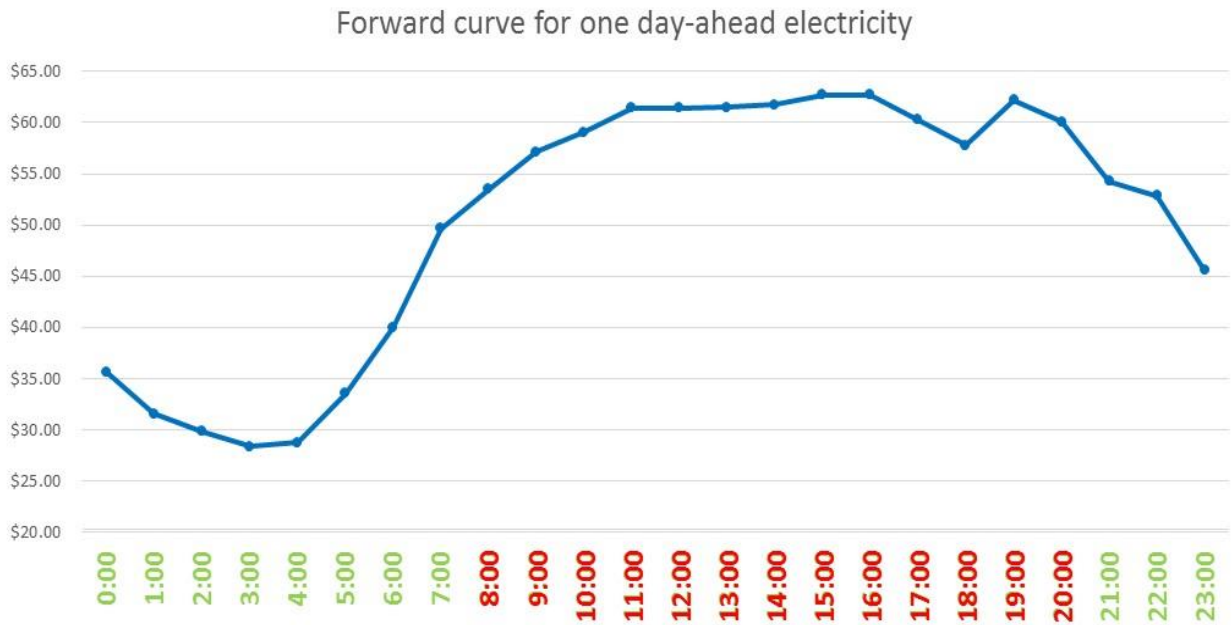
Non-storable Commodities: Electricity (cont'd)

One day-ahead forward price, by hour, for 1 megawatt-hour of electricity in New York City, September 7, 2004

Time	Price	Time	Price	Time	Price	Time	Price
00:00	$35.68	06:00	$40.03	12:00	$61.46	18:00	$57.81
01:00	$31.59	07:00	$49.64	13:00	$61.47	19:00	$62.18
02:00	$29.85	08:00	$53.48	14:00	$61.74	20:00	$60.12
03:00	$28.37	09:00	$57.15	15:00	$62.71	21:00	$54.25
04:00	$28.75	10:00	$59.04	16:00	$62.68	22:00	$52.89
05:00	$33.57	11:00	$61.45	17:00	$60.28	23:00	$45.56

Source: Bloomberg

Non-storable Commodities: Electricity (cont'd)



Electricity Markets in the U.S.

- Two types of markets:
  - **Spot market**: balances demand and supply 30 mins before each operating hour.
  - **Forward market**: schedules production for 24 hours in advance. Buyers and sellers submit bids before 11AM and market clears separately for each hour.

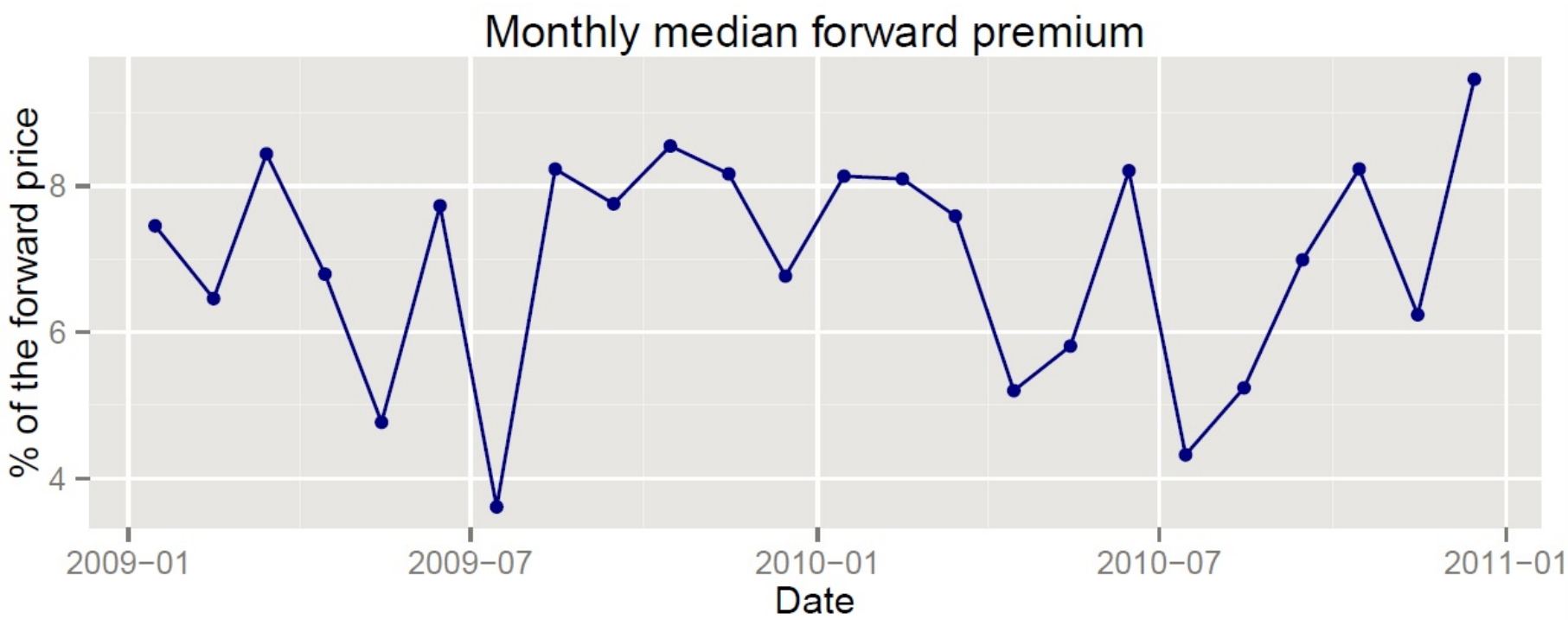
Forward market allows the market to anticipate the demand, allowing generators to schedule the production in advance.

The Forward Premium

- Recall that in theory: $F_t = \text{PV}_t(S_T)e^{r_c(T-t)}$.
- With $T - t < 1/365$, we have $F_t \approx \mathbb{E}_t(S_T)$.
- S_T is fairly predictable due to the special property of the industry.
- $F_t - \mathbb{E}_t(S_T)$ should be zero.
- The reality was not the case.

The Forward Premium (Cont'd)

Define $F_t - \mathbb{E}_t(S_T)$ as the forward premium.



- Why there is a forward premium in reality?
  - Generators engage in inter-temporal price discrimination by withholding sales in the forward market.

Forward price > spot price seller makes profit

(Limited) Arbitrage Opportunities

- In the presence of forward premium, profits can be made (in expectation) by short-selling in the forward market.
  - Selling 1MWh in the forward markets yields $F_t - S_T$.
  - In expectation, profits are $F_t - \mathbb{E}_0(S_T) > 0$
- To avoid inefficiencies caused by market power, in 2005, financial traders are legalized.
  - Virtual participants who buy/sell electricity purely for financial purposes.
  - However, they did not work well as the generators have market power.

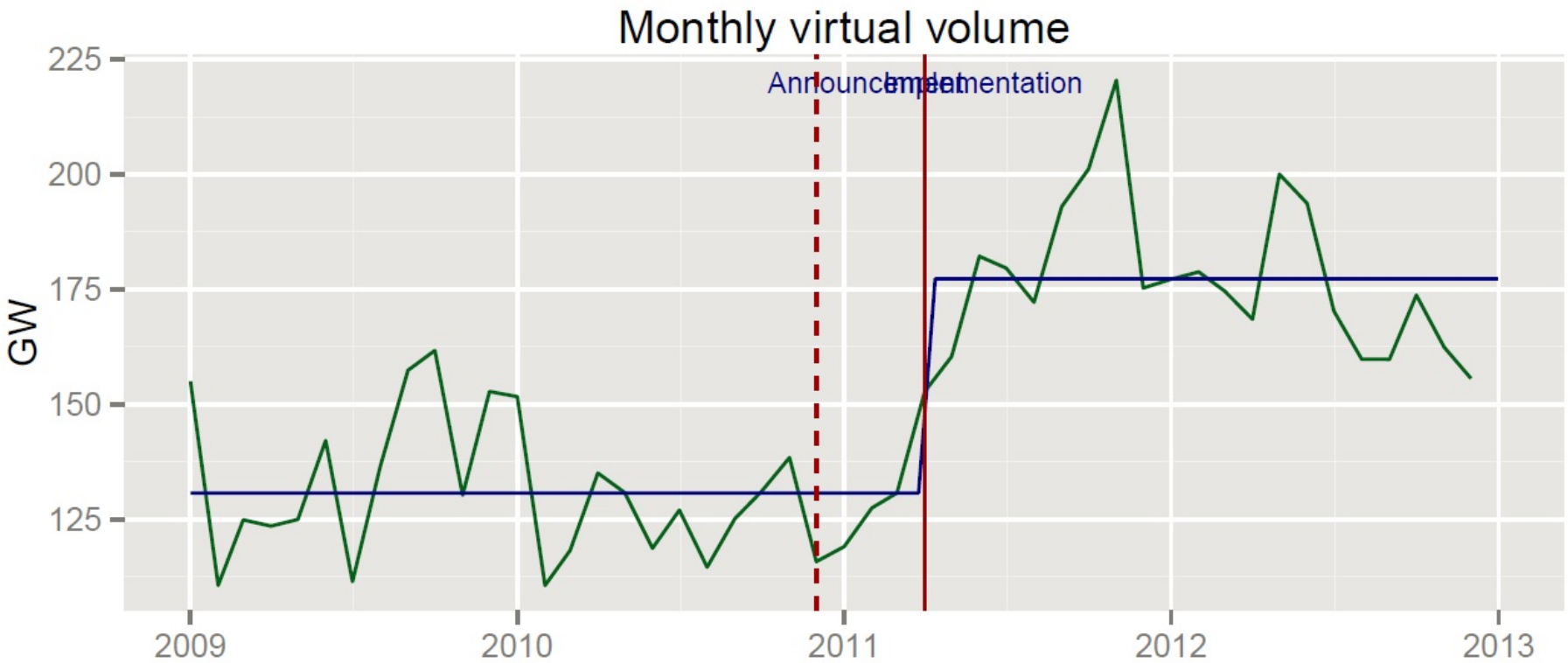
(Limited) Arbitrage Opportunities (Cont'd)

- Financial traders could not trade as the prices of electricity contains 'guaranteed' profit for the generators.
  - The average forward premium was \$0.9 per MWh.
  - The average RSG (Revenue Sufficiency Guarantee) charge is \$1.8 per MWh (transaction costs) to cover ramping/start up costs.
  - The prices of electricity deviate a lot from the marginal cost of generation, leaving the generators rooms to play and as a result market power.

Regulatory change on RSG charges

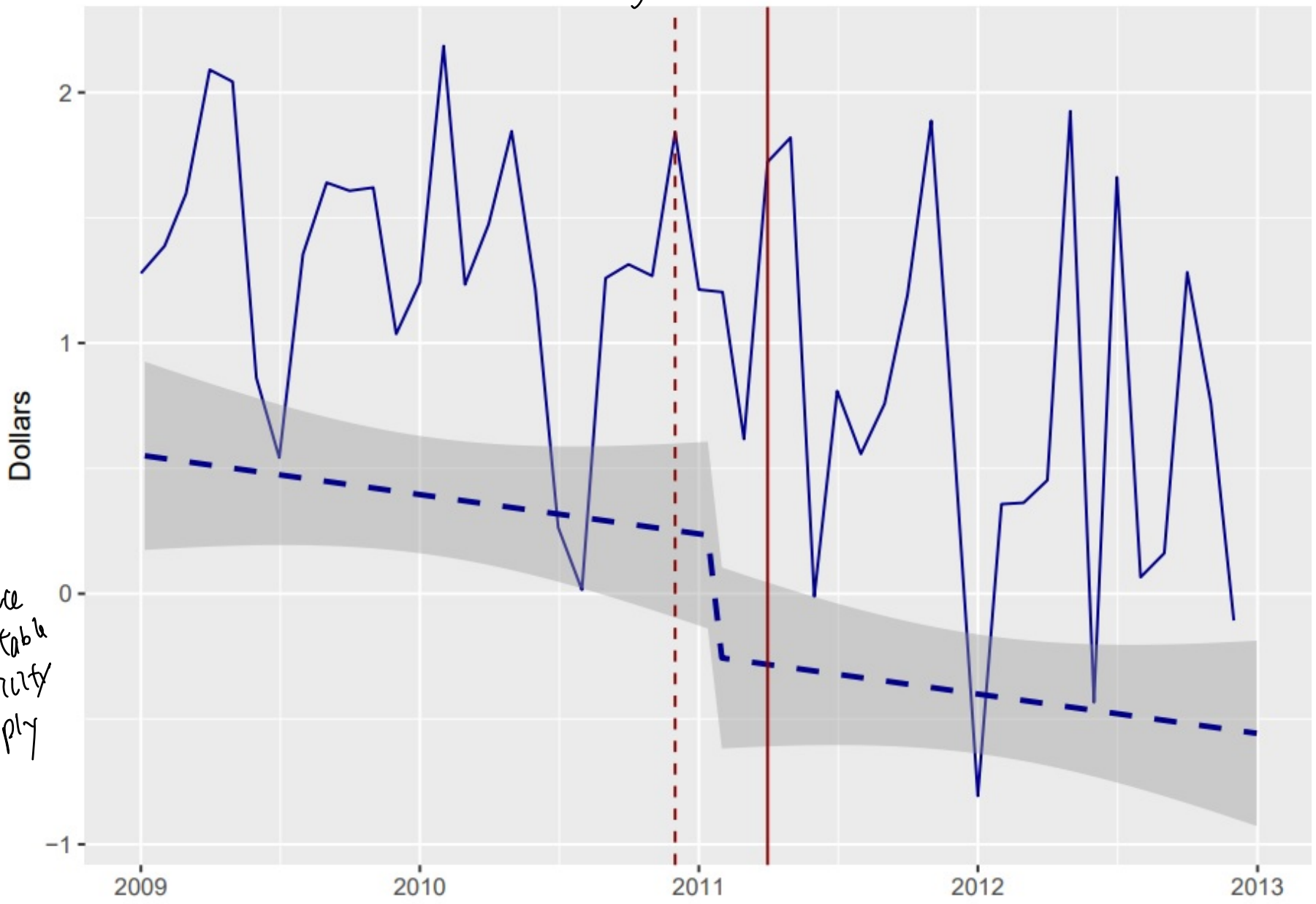
- In April, 2011, the regulator modified the way RSG were calculated. As a result, RSG charge dropped from \$1.8 to \$0.3 per MWh.

- The room to play for generators is much smaller; as a result the market becomes more competitive and financial traders are more willing to participate.



Forward premium decreases

Buyers in forward market is paying the forward premium

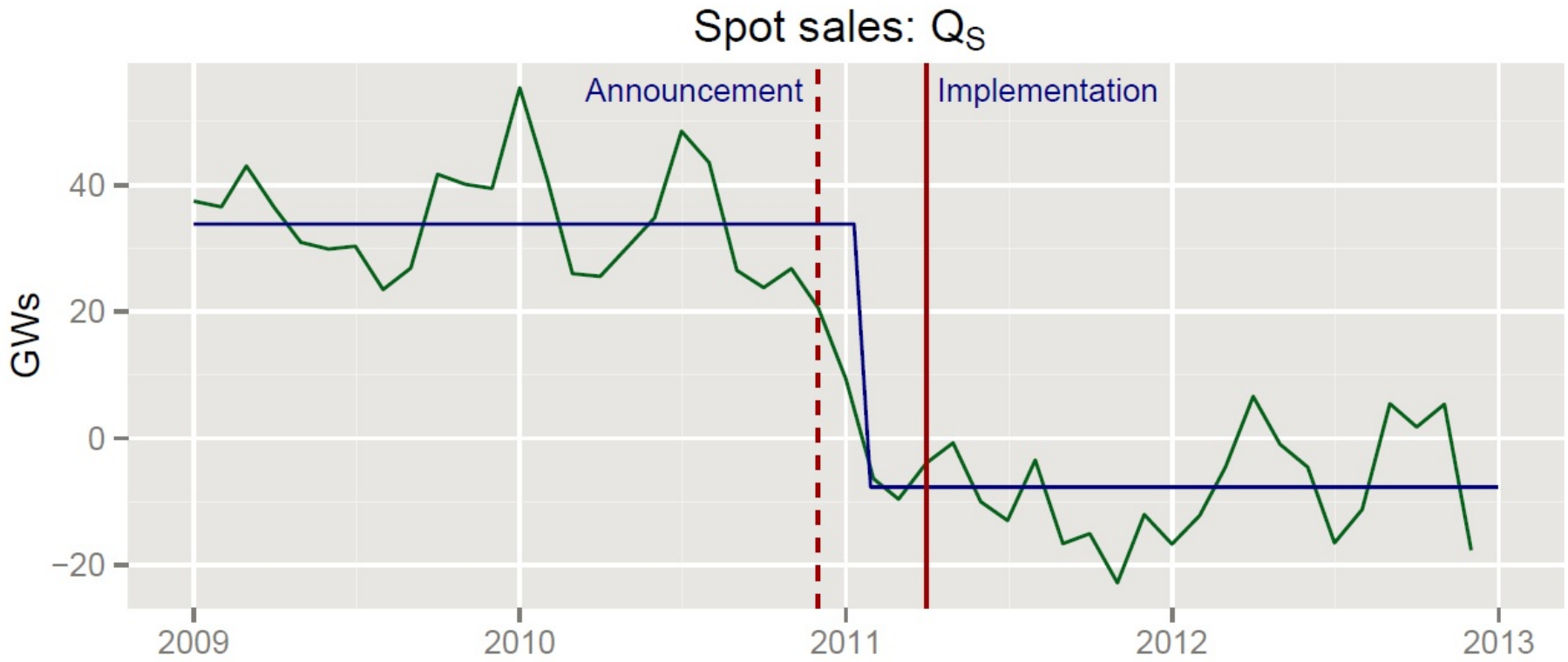


insurance of stable electricity supply

Who will engage in the forward market? Industrial users, manufacturer secure a stable electricity supply

Market power diminishes after regulatory change

- Before 2011, spot sales from generators are on average positive, implying the existence of market power.
- After 2011, market power decreases due to increased trading from financial traders.



What do we learn from this case?

- An example to illustrate the idea of using expected commodity price (on the next day) to back out forward price.
- The limits to arbitrage opportunities.
- Opening up forward markets to financial traders reduces market power.

Summary

- What is a commodity forward?
- What are the key features of commodity forwards?
- How do these features affect commodity forward pricing?