Approaches to Estimating Aggregate Demand for Reserve Balances

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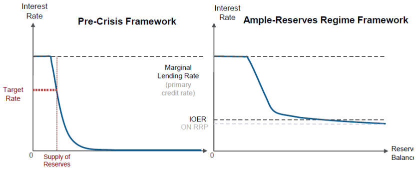
Summary

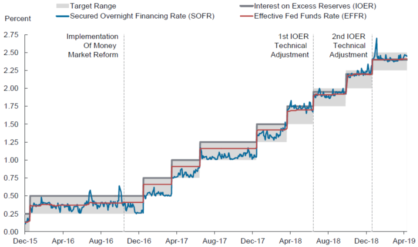
The August 2019 Senior Financial Officer Surveys(SFOS) asked 77 banks to report the approximate lowest level of reserve balances(LCLoR), given the constellation of short-term interest rates.

Based on statistical methods applied in prior FEDS Notes, a point estimate for aggregate demanad, accouting for sampling and non-sampling error, is $800 billion, ranging between $712 billion and $919 billion.

These banks represented a range of asset sizes and business models, and the short-term interest rates include IOER, EFFR, SOFR, OBFR and Treasury bill rate.

Why Does the Demand for Reserve Balances Matter?





An ample supply of reserves made the interest rate insensitive to OMO. Instead, overnight interest rates moved in line with administered rates.

Reserve demand is driven by many factors and it changes continually, so reserve demand and monetary policies codetermine what the interest rate is. ("Observation on Implementing Monetary Policy in an Ample-Reserves Regime", Lorie K.Logan)

Building the Point Estimation

Comparision (just like ”Descriptive Statistics”):

	SFOS Respondent Banks	Non-SFOS Banks	total
LCLoR	652	to be estimated	
Total Reserve Bal.	1152	334	1486
Total Asset	12943	6840	19783
Categories	(1)U.S. Global Systemically Important Banks(G-SIBs)		
	(2)Large Domestic Banks		
	(3)Small Domestic Banks		
	(4)Foreign Banking Organizations(FBOs)		

77 SFOS banks have $\frac{3}{4}$ of reserve holdings, which is \$652 billion. The number of Non-SFOS banks is around 5200 but their size is only half of SFOS banks.

Building the Point Estimation

Formulas for Estimation:

(1) SFOS category ratio = aggregate SFOS LCLoR / aggregate SFOS total assets

(2) NonSFOS LCLoR $\text{estimate}_i = \text{SFOS category ratio} \times \sum_i$ total assets

(3) aggregate point estimate = $\sum LCLoR_i$ for SFOS + estimate of $LCLoR$ for NonSFOS

* The weights are relative to their size! Think about the Fed Reserve Ratio!

Questions needed to be solved:

(1) **Sampling error** : comes from random selection of SFOS banks.

(2) **Non-Sampling error** : this sample(SFOS banks) is not so representative.

$\rightarrow$ **Stratified Sampling , Multiple Imputation(Bootstrap)**

Approach I : Stratified Sampling for Sampling Error

Setup: ("Sampling Techniques, Cochran, Chapter5 Section 5.1-5.4")

(1) There are 4 categories, sorting into L = 4 layers.

(2) The size of population N is 77. Suppose N_j is number of units in *jth* layer, thus $N_1 + N_2 + N_3 + N_4 = N = 77$.

(3) For each layer, random selection is applied for obtaining one series of observations as samples (a subset of all LCLoRs in each category). Suppose n_j is number of units in sample for *jth* layer, y_{ij} is the *ith* unit's value of LCLoR drawn from *jth* layer.

(4) Stratum weight for *jth* layer:

$$W_j = \frac{\text{total assets in j th category in population}}{\text{total population assets}} \tag{3.1}$$

true mean for *jth* layer:

$$\bar{Y}_j = \frac{\sum_{i=1}^{N_j} y_{ij}}{N_j} \tag{3.2}$$

Approach I : Stratified Sampling for Sampling Error

Setup: (Continued)

(4) sample mean for *jth* layer:

$$\bar{y}_j = \frac{\sum_{i=1}^{n_j} y_{ij}}{n_j} \tag{3.3}$$

true variance for *jth* layer:

$$S_j^2 = \frac{\sum_{i=1}^{N_j} \left(y_{ij} - \bar{Y}_j\right)^2}{N_j - 1} \tag{3.4}$$

Properties

- If in every layer the sample estimate $\bar{y}_j$ is unbiased, then population mean per unit $\bar{y}_{st}$ is an unbiased estimate of population mean $\bar{Y}$ **(mean of aggregate LCLoR for SFOC!)**:

$$\bar{y}_{st} = \frac{\sum_{j=1}^{L} N_j \bar{y}_j}{N} = \sum_{j=1}^{L} W_j \bar{y}_j \approx \bar{Y} \tag{3.5}$$

Approach I : Stratified Sampling for Sampling Error

Properties: (Continued)

- For stratified sampling, variance of $\bar{y}_{st}$ and its approximation $s^2\left(\bar{y}_{st}\right)$ **(now we know the standard error!)**:

$$V\left(\bar{y}_{st}\right) = \frac{1}{N^2}\sum_{j=1}^{L} N_j\left(N_j - n_j\right)\frac{S_j^2}{n_j} = \sum_{j=1}^{L} W_j^2\frac{S_j^2}{n_j}\left(1 - \frac{n_j}{N_j}\right) \tag{3.6}$$

$$s^2\left(\bar{y}_{st}\right) = \sum_{j=1}^{L}\frac{W_j^2 s_j^2}{n_j} - \sum_{j=1}^{L}\frac{W_j s_j^2}{N} \tag{3.7}$$

Approach I : Stratified Sampling for Sampling Error

Estimation: **(df part comes from F.E.Satterthwaite)**

* Assumption (1) : $\bar{y}_{st}$ ~Normal Distribution, so MSE$^2 \sim \chi^2$ (and variance of χ^2 would be a good approximation for variance of Complex Estimate)
* Assumption (2) : Compute standard error $s(\bar{y}_{st})$ by Student t-statistics because of only a few degrees of freedom

- $$s^2\left(\bar{y}_{st}\right) = \frac{1}{N^2}\sum_{j=1}^{L} g_j s_j^2, \quad \text{where } g_j = \frac{N_j\left(N_j - n_j\right)}{n_j} \tag{3.8}$$

  the degree of freedom is :

  $$df = \frac{\left(\sum g_j s_j^2\right)^2}{\sum \frac{g_j^2 s_j^4}{n_j - 1}} \tag{3.9}$$

  and the result for range of aggregate LCLoR is:

  $$\bar{y}_{st} \pm ts\left(\bar{y}_{st}\right) \tag{3.10}$$

Approach II : Bootstrap for Non-Sampling Error

Introduction of Bootstrap:

* **resampling to rebuild a new sample, which can represent the distribution of original sample**

(1) **Characteristics :**

- Distribution assumption is not necessary
- Returning drawn observations to the data sample after they have been chosen (some observations may repeat themselves, but it can ensure identical sample size!)

(2) **Service Condition :** small data set, or difficult to classify

(3) **Limitation :** different drawing leads different distribution

Relative Formula:

NonSFOS LCLoR estimate$_i$ = SFOS category ratio * $\times \sum_i$ total assets

Bootstrap can derive a normal distribution for estimating SFOS category ratio *

Approach II : Bootstrap for Non-Sampling Error

"by multiplying individual non-SFOS bank's total assets..., but rather is a randomly selected survey response to total asset ratio for an individual SFOS bank in the appropriate category..."

Procedures:

Step 1 Suppose the individual SFOS ratio ($\frac{\text{individual SFOS LCLoR}}{\text{individual SFOS asset}}$) is [y1, y2, ..., y77], and they all follow an unknown distribution (iid).

Step 2 For the i th sampling, we resample 10 times, noted as $X*_i = [X*_{i1}, X*_{i2}, ..., X*_{i10}]$, so the size of each sample is 10. We use 10-units sample to calculate individual SFOS ratio, noted as $\hat{Y}_i$

Step 3 Repeat Step 2 for 1000 times, we can obtain 1000 estimates for individual SFOS ratio, and the number is large enough to build a normal distribution (central limit theorem):

$$\bar{Y} \approx \frac{1}{1000}\sum_{i=1}^{1000}\hat{Y}_i \quad var(Y) \approx \frac{1}{1000-1}\sum_{i=1}^{1000}(\hat{Y}_i - \bar{Y}_i)^2 \tag{3.11}$$

Conclusions

Accounting for sampling error and non-sampling error, the point estimate for aggregate reserve demand is $800 billion, ranging from $712 billion to $919 billion.

Approach	Low	Point Estimate	High	Notes
	--	800	--	
Stratified sampling	712	--	889	95% confidence interval
Multiple imputation –(estimating uncertainty for large and small domestic bank LCLoRs)	769	--	919	Representative 95% confidence interval
Estimated Range of Aggregate Reserve Demand	712	800	919	Represents the minimum and maximum estimates

Note: If we were to apply the multiple imputation approach to SFOS U.S. G-SIB and FBO banks, the representative 95 percent confidence interval would be $778 billion to $1,148 billion. We excluded SFOS U.S. G-SIB and FBO banks from the multiple imputation approach because of our assumption that the heterogeneity in business models among these types of banks makes it less plausible that a bank's behavior would be more accurately described by the behavior of another bank in the same category.

Short Comings

Short Comings: may be underestimated!

1. Estimating non-sampling error for G-SIB and FBO.
2. Relying on data reported at a point in time. (the asset data are as of March, 2019, while SFOS was conduted in Aug, 2019)
3. Assuming perfect money market efficiency and there are no additional factors affecting supply side.

Thanks for Your Attention

Q & A ?