A Fast Power Grid Frequency Estimation Approach Using Frequency-Shift Filtering

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***Abstract*—In this letter, a fast frequency estimation approach is proposed using frequency-shift filtering. The original sampled signal is first frequency-shifted to 0 Hz nearby via multiplying a reference signal. Then a convolution average filter is applied on the shifted signal to eliminate the spectral interference caused by asynchronous sampling. Finally, frequency of power system can be estimated using the phase difference between arbitrary two points of the filtered signal. The approach has major advantages of low computational burden and strong anti-noise performance, which makes it appropriate for high precision and reporting rate frequency measurement in embedded systems. Comprehensive simulations are conducted to validate the accuracy and efficiency of the proposed approach.**

***Index Terms*—Convolution average filter, frequency estimation, frequency-shift filtering, high reporting rate**

I. Introduction

FREQUENCY is one the most critical parameters to indicate the operation status of power grid. Therefore, accurate and prompt estimation of frequency is a critical task of wide-area measurement system.

Several digital signal processing proposed in the literature to estimate the power grid frequency such as: Discrete Fourier transform (DFT)[1], wavelet transform[2], Prony[3], and Kalman filter[4].

Among them, DFT based methods is the most commonly used techniques in commercial Phasor Measurement Units (PMUs) and digital relays for frequency estimation due to its advantage of easy hardware implementation and harmonic immunity[5], [6]. To reduce the influence of spectral leakage, Windowed Interpolation DFT (WIDFT) algorithm was used to improve the performance under the conditions of asynchronous sampling[7]. However, the WIDFT involve computationally expensive procedures of solving high-order equations. A Recursive DFT (RDFT) calculation method was developed by A. G. Phadke and J. S. Thorp with significant reduction on computational complexity[8]. To achieve the high accuracy under off-nominal condition, calculation of least square estimation and resample is applied on RDFT[9]. This approach has been successfully implemented in Frequency Disturbance Recorders (FDRs) of FNET/Grideye with reporting rate 10 Hz for providing synchronized frequency measurement at worldwide distribution level power grid[10].

However, for applications such as instantaneous relay protection[11], fast system dynamic response prediction[12], [13], [14], sub-synchronous resonance measurement [15] and instant frequency-based load control [16], a much higher reporting rate of measurements, e.g. 120Hz or 240 Hz, is needed to capture the transient behavior. For example, according the result in Ref.[13], [14], a higher reporting rate leads to a faster prediction of system dynamic response under disturbance. Unfortunately, the execution time of RDFT in real hardware implementation can not meet the real-time requirement. For example, the digital signal processor used in FDR needs more than 50 ms for executing a RDFT. As a result, there is no room for FDR to increase reporting rate even from 10Hz to 30Hz. . Therefore, the necessity to explore a more efficient frequency estimation approach with a satisfactory performance arises.

In this paper, a fast frequency estimation approach is proposed for practical application using frequency-shift filtering. The spectrum shifting in frequency domain is a digital signal processing technique used for removing interference and noise from a wanted signal[17]. The main idea of our approach is to shift fundamental components of power grid signal to 0 Hz nearby, and then use a Convolution Averaging Filter (CAF) to eliminate spectral interferences and white noise. The extensive computation of least square estimation and resample process in RDFT can be avoided. It has major advantages of high accuracy and low computational complexity.

II. Proposed algorithm

Assuming actual power grid signal distorted by harmonic is sampled at an interval $T_s = 1/f_s$ in a measurement device, the sampled discrete-time signal $x(n)$ can be represented by

$$x(n) \equiv x_a(nT_s) = \sum_{h=1}^{H} A_h \cos(h 2\pi f_r/f_s + \varphi_h) \tag{1}$$

where f_r is the real fundamental frequency and f_s is the sampling rate. h stands for the order of harmonic, A_h and φ_h denote the amplitude and phase of harmonics, respectively. We assume the sampling frequency f_s is an integral multiple of the nominal frequency f_{nom} as $f_s = M f_{\text{nom}}$ and define normalized angular frequency as $\omega_r = 2\pi f_r/(M f_{\text{nom}})$.

According to the Euler identity, $x(n)$ can be rewritten as

$$x(n) = \sum_{h=1}^{H} \frac{A_h}{2}\left(e^{j(h\omega_r n+\varphi_h)} + e^{-j(h\omega_r n+\varphi_h)}\right) \tag{2}$$

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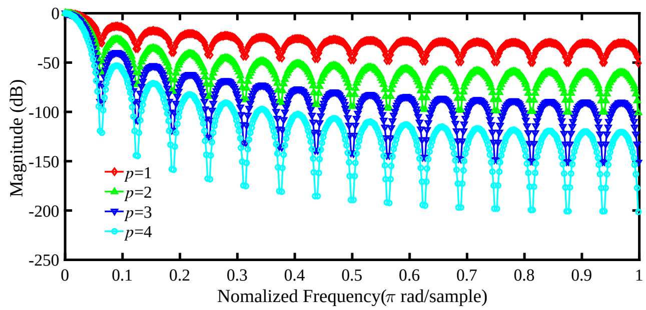


Fig. 1. Magnitude spectrum of the pth order CAF with $p = 1, 2, 3, 4$.

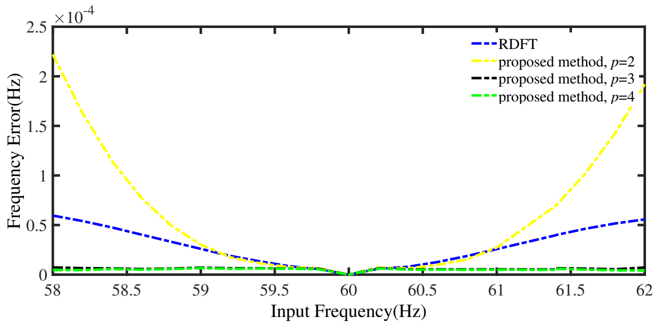


Fig. 2. Frequency range test result

Defining a reference signal to shift original sampled signal, it can be expressed as

$$r(n) = e^{j\omega_{\text{nom}} n} \tag{3}$$

where $\omega_{\text{nom}} = 2\pi/M$. Multiplying both sides of (2) by the exponential signal $r(n)$ in time domain, we can get the shifted signal $x_s(n)$ as

$$\begin{aligned} x_s(n) = \sum_{h=1}^{H} \frac{A_h}{2} \Big(& e^{j((h\omega_r + \omega_{\text{nom}})n + \varphi_h)} \\ & + e^{-j((h\omega_r - \omega_{\text{nom}})n + \varphi_h)} \Big) \end{aligned} \tag{4}$$

It can be seen from (4) that the frequency components of $x(n)$ are all shifted from $h\omega_r$ and $-h\omega_r$ to $h\omega_r + \omega_{\text{nom}}$ and $-h\omega_r + \omega_{\text{nom}}$, respectively. Consequently, the spectral line of fundamental components are shifted around 0 Hz in frequency domain since the f_r slight fluctuates around f_{nom}.

To eliminate the spectral interferences of harmonic, a moving average filter is applied on the shifted signal $x_s(n)$. The moving average filter h_{av} of length M can be expressed as

$$h_{\text{av}}(m) = \begin{cases} 1/M, & m = 0, 1, \ldots, M-1 \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The frequency response of $h_{\text{av}}(m)$ is

$$H(e^{j\omega}) = \frac{1}{M} \sum_{n=0}^{M-1} e^{j\omega n} = \frac{\sin(\omega M/2)}{M \sin(\omega/2)} e^{-j\omega(M-1)/2} \tag{6}$$

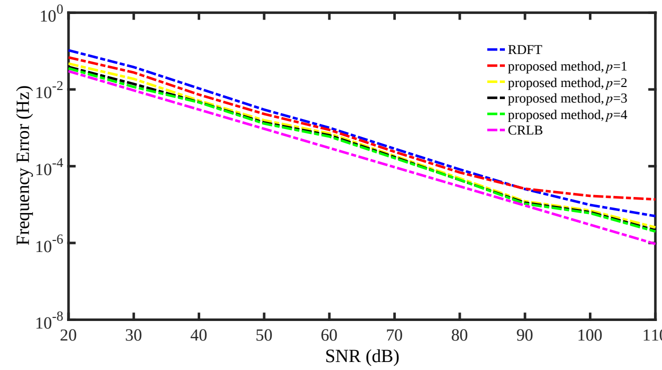


Fig. 3. Noise tolerance test result

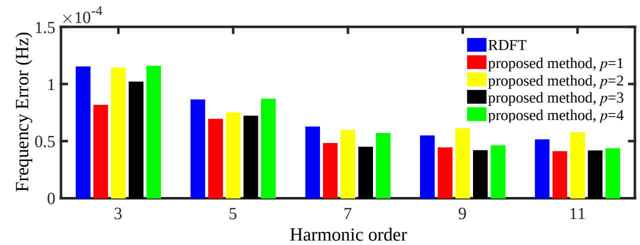


Fig. 4. Harmonic distortion test result

and its magnitude frequency responses is

$$|H(e^{j\omega})| = \left| \frac{\sin(\omega M/2)}{M \sin(\omega/2)} \right|. \tag{7}$$

Thus, $|H(e^{j\omega})|$ yields 0 for the cases $\omega = 2\pi k/M, k = 0, 1, \ldots, M-1$.

Let us divide the normalized angular frequencies into two parts after frequency shifting, that is, the negative angular frequencies $\omega_n(h) = -h\omega_r + \omega_{\text{nom}}$ and the positive angular frequencies $\omega_p(h) = h\omega_r + \omega_{\text{nom}}$. Since $\omega_n(1) = \omega_{\text{nom}} - \omega_r \approx 0$ for $h = 1$, from (7) we have $|H(e^{j\omega_n(1)}| \approx 1$. Similarity, it can be obtained that $|H(e^{j\omega_n(h)})| \approx 0$ for $h \neq 1$ and $|H(e^{j\omega_p(h)})| \approx 0$, respectively. As as result, only the frequency component $\omega_{\text{nom}} - \omega_r$ is reserved and the other frequency components are suppressed after applying the filter.

To achieve better suppression effect on the interference, a pth order CAF is constructed as

$$h_p(n) = \underbrace{h_{av}(m) * \cdots * h_{av}(m)}_{p} \tag{8}$$

where $*$ denotes convolution operation, p represents the number of the CAF. The length of the pth order CAF is $p(M-1)+1$. Since coefficients of CAF are fixed with a certain p, they can be pre-computed to decrease computational burden and only one time convolution calculation between $x_s(n)$ and $h_p(n)$ is needed.

The magnitude frequency response of the first order CAF to the fourth order CAF are illustrated in Fig. 1. It can bee observed in Fig. 1 that the capacity of interference inhibition of CAF is proportional to the convolution order p. Defining the

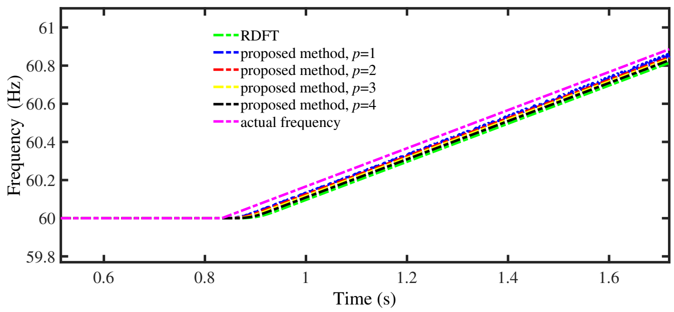
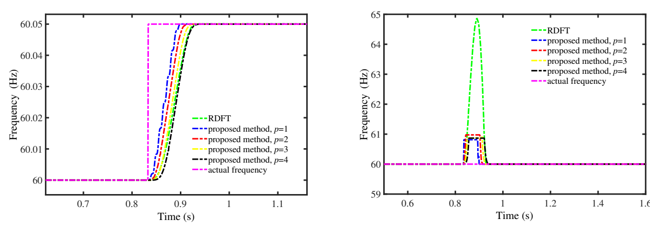


Fig. 5. Frequency ramping test result



(a) (b)

Fig. 6. Step change test results. (a)Frequency step change; (b)Angle step change;

length of truncation window for original filter is L, the computational complexity of the proposed method is $O(pML)$, where $O()$ means time complexity notation. The signal after applying frequency shifting and CAF can be approximated as

$$x_f(n) = h_p(n) * x_s(n) \approx \sum_{n=-\infty}^{\infty} \frac{A_h}{2} e^{j((\omega_{\text{nom}}-\omega_r)n+\varphi_1)} \quad (9)$$

Then the frequency deviation can obtained using phase difference of two points in $x_f(n)$ as

$$\Delta f = \frac{(\arg(x_f(n_1)) - \arg(x_f(n_2))M f_{\text{nom}}}{2\pi(n1 - n2)}. \quad (10)$$

Consequently, the estimated fundamental frequency is

$$f_{\text{est}} = f_{\text{nom}} + \Delta f \quad (11)$$

III. Simulation Results and Analysis

Simulations are conducted to validate the performance of the proposed approach. The performance is also compared with FDR algorithm[9], [10], which is denoted as RDFT in this section. The nominal frequency f_{nom} is 60 Hz and the sampling rate f_s is 1440 Hz. Note that both algorithms were implemented in a "sliding window" approach, i.e., continuously shifting the estimation window by a single sample, which means that the frequency estimation rate is equal to sampling rate f_s. Since the order p influences of accuracy and computational burden, the proposed approach with p from 1 to 4 are evaluated. For each test scenario, 10000 simulation runs have been performed to evaluate the statistical properties.

For the frequency range test, the frequency of input sinusoidal signal varies from 58 Hz to 62 Hz with according to IEEE Standard C37.118.1. The results is shown in Fig. II. It can be seen that accuracy of the proposed method is proportional to the p. Moreover, the proposed method with $p > 2$ can offer a better accuracy compared to the RDFT.

To evaluate the performance with white noise, Gaussian white noise with zero mean is superposed on the 59.95 Hz test signal. The accuracy is evaluated under different Signal-to-Noise Ratios (SNR) ranging from 20 to 100 dB at an increment of 10 dB. The Cramer–Rao lower bound (CRLB) is also used to provide a quantitative benchmark of measurement error as shown in Fig. 3. In this semilog plot, it can be seen that the proposed method with larger p is closer to the CRLB. The results also demonstrates that the proposed method with $p \geq 2$ has high reliability and accuracy in a noisy signal environment than conventional RDFT, which proves robustness of the proposed method.

For harmonic distortion test, the test signal consists of a fundamental frequency component and an odd harmonic component ranging from 3rd order to 11th order. The magnitude is 0.1 p.u. (20 dB with respect to the fundamental component), which is the worst condition according to the IEEE C37.118 standard[18]. Besides, 80 dB white noise is added, which is selected based on the study of noise at the distribution level grid in Ref. [19]. From Fig. 4, the error of the proposed method is within 0.2 mHz and much less than the 5 mHz requirement for P-class PMU in the C37 standard. We can safely conclude that the proposed method is immune to the harmonic distortion.

To assess dynamic performance of the proposed method, frequency ramp and step-change tests are conducted. For the ramping test, it can be seen in Fig. II that the frequency change trend can be well-captured using proposed method. For the step change test results in Fig. II, the response time of the proposed is as short as 0.3 seconds when $p \leq 3$, which is faster than the conventional RDFT method. From the Fig. II(b), the frequency spike of the proposed method are much smaller than RDFT.

TABLE I
Comparison of Execution Time for 10000 Simulation Run

Method	RDFT	Proposed method			
		p=1	p=2	p=3	p=4
Time(s)	2.57	0.423	0.444	0.523	0.796

The comparative results of computation time for simulation on a 3.2 GHz Dual-core computer are listed in Table I.It is noted that in the hardware implementation, the calculation time will be longer than the simulation scenario since the speed of computation unit in an embedded system is lower than the simulation platform. The simulation results confirm the superiority of the proposed algorithm in lower computational complexity. Taking account all the above results, to balance the trade-off between accuracy and estimation time, $p = 2$ can be considered as reasonable selection in practical implementation since keeping increasing p does not improve the performance significantly but introducing much higher computational effort.

IV. Conclusion

In this letter, a novel frequency estimation approach combining frequency shifting and convolution average filtering is proposed. The simulation results show that the proposed method has merit of low computational complexity, robust performance in the presence of noise, harmonic and off-nominal conditions. There advantages make it appropriate and promising for the real-time application in digital relays and high reporting rate synchronized measurement devices.

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