linear algebra

3. solutions

$$A=\begin{vmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{vmatrix} \qquad B=\begin{vmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{vmatrix} \qquad C=\begin{vmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{vmatrix}$$

(a)

$$AB=\begin{vmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{vmatrix}\begin{vmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{vmatrix}=\begin{vmatrix} -10+6-16 & 3-16 & 14-9 \\ -5-6-14 & -3-14 & 7+9 \\ 15-8 & -8 & -21 \end{vmatrix}$$

$$=\begin{vmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{vmatrix}$$

(b)

$$BA=\begin{vmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{vmatrix}\begin{vmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{vmatrix}=\begin{vmatrix} -10-21 & 5 & 40-28 \\ -12-3-27 & 6-3 & 48+21-36 \\ 4+2 & -2+2 & -16-14 \end{vmatrix}$$

$$=\begin{vmatrix} -31 & 5 & 12 \\ -42 & 3 & 33 \\ 6 & 0 & -30 \end{vmatrix}$$

(c)

$$AB-BA=\begin{vmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{vmatrix} - \begin{vmatrix} -31 & 5 & 12 \\ -42 & 3 & 33 \\ 6 & 0 & -30 \end{vmatrix} = \begin{vmatrix} 11 & -18 & -7 \\ 17 & -20 & -17 \\ 1 & -8 & 9 \end{vmatrix}$$

(d) $$ABC=(AB)C=\begin{vmatrix} -20 & -13 & 5 \\ -25 & -17 & 16 \\ 7 & -8 & -21 \end{vmatrix}\begin{vmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{vmatrix}$$

$$=\begin{vmatrix} -120-26-5 & -60-52-5 & 20-65+40 \\ -150-34-16 & -75-68-16 & 25-75+128 \\ 42-16+21 & 21-32+21 & -7-40-168 \end{vmatrix}$$

$$=\begin{vmatrix} -151 & -117 & -5 \\ -200 & -159 & 68 \\ 47 & 10 & -215 \end{vmatrix}$$

4. Solutions

$$A=\begin{vmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{vmatrix}$$

To get the eigenvales of A, $|\lambda I-A|=0$

$$|\lambda I-A|=\begin{vmatrix} \lambda+2 & -1 & -8 \\ 1 & \lambda+1 & -7 \\ -3 & 0 & \lambda-4 \end{vmatrix}=(\lambda+2)\begin{vmatrix} \lambda+1 & -7 \\ 0 & \lambda-4 \end{vmatrix}+\begin{vmatrix} 1 & -7 \\ -3 & \lambda-4 \end{vmatrix}-8\begin{vmatrix} 1 & \lambda+1 \\ -3 & 0 \end{vmatrix}$$

$$=(\lambda+2)(\lambda+1)(\lambda-4)+(\lambda-4)-21-24(\lambda+1)$$

$$=\lambda^3-\lambda^2-33\lambda-57$$

To solve $|\lambda I-A|=0$

$$\lambda^3-\lambda^2-33\lambda-57=0$$

therefore $\lambda=-2.192, -3.747, 6.939$

So the eigenvales of A is $-2.192, -3.747, 6.939$

the trace of matrix A is:

$$tr A=\sum_{i=1}^{3}\lambda_i=1$$

4.

$$A = \begin{vmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{vmatrix}$$

To get the eigenvectors

$$Ax = \lambda x$$

λ can be $-2.192, -3.747, 6.939$ State here $\lambda = 6.939$

$$\begin{vmatrix} -2 & 1 & 8 \\ -3 & -1 & 7 \\ 3 & 0 & 4 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = 6.939 \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$-2x_1 + x_2 + 8x_3 = 6.939x_1$$
$$-3x_1 - x_2 + 7x_3 = 6.939x_2 \Rightarrow$$
$$3x_1 + 4x_3 = 6.939x_3$$

$$x_1 \approx x_3$$
$$x_3 \approx 2x_2$$

So one of the eigenvectors of matrix A is:

$$\begin{vmatrix} 2 \\ 1 \\ 2 \end{vmatrix}$$

Statitics

1. Solutions

(1) $P(h) = (\frac{1}{2})^{10} = \frac{1}{1024}$

(2) $P = \frac{1}{1000} \times \frac{1}{1024} = \frac{1}{1024000}$

(3) $P = 1 - (\frac{1023}{1024})^{1000} \approx 0.62487$

2. Solutions

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-u}{\sigma}\right)^2}$$

$$\int_{32}^{38} f(x) = \frac{1}{5\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-36}{5}\right)^2} \Big|_{32}^{38}$$ (get the result from cdf)

$= 0.443566$

3. Solutions

$$\bar{x} - \frac{t_a}{2}\left(\frac{s}{\sqrt{n}}\right) < u < \bar{x} + \frac{t_a}{2}\left(\frac{s}{\sqrt{n}}\right)$$

$\bar{x} = 842.6$ $\qquad$ $n = 10.$

$S = 534.3$ $\qquad$ $\frac{t_a}{2} = 2.262$

we can get:

$460.385 < u < 1224.815$

4. Solutions

By applying t-test

$$t = \frac{\bar{x} - \bar{u}}{\frac{s}{\sqrt{n}}}$$

$n = 40$ $\qquad$ $\bar{u} = 8.5$

$\bar{x} = 9.6$ $\qquad$ $s = 3.2$

$$t = \frac{1.1}{0.51} = 2.15$$

When checking t distribution table

$df = 39$, $t = 2.15$, α is smaller than 5%.

Therefore, we should accept it.

It represents a difference!

5. Solutions

We use the chi-square test for the variance.

$$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2} = \frac{14 \times 183.8}{100} = 25.7$$

By checking chi-square table,

when $df=14$, $\alpha=0.05$, $\chi^2=23.685$.

In other words, when $df=14$, $\chi^2=25.7$,

α is smaller than 0.05

Therefore, we should accept it.

The variance in grades exceeds 100.