Linear Algebra [50pt]

1. [10pt] Create a matrix array A and vector array in Numpy with **random** integers. For example

$A = \begin{bmatrix} 5 & -2 \\ -3 & -1 \\ 7 & 9 \end{bmatrix}, x = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$. Your A matrix is **5x2** and your x vector is **2x1**. Show your code and result.

Create a diagonal matrix B, with 1,2,3,4,5 in the diagonal. The order doesn't matter. You can have

$B = \begin{bmatrix} 3 & & & & \\ & 4 & & & \\ & & & & \\ & & & 1 & \end{bmatrix}$. Note your diagonal matrix is 5x5.

Calculate Ax, and BA. Show your code and result.

2. [10pt] Calculate the rank of your matrix A and the rank of your matrix B. What about the rank of BA? Are they different? Why?

Consider the following 3 x 3 real matrices:

$$A = \begin{bmatrix} -2 & 1 & 8 \\ -1 & -1 & 7 \\ 3 & 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 0 & -7 \\ 6 & 3 & -9 \\ -2 & -2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 3 & -1 \\ 2 & 4 & 5 \\ -1 & -1 & 8 \end{bmatrix}.$$

3. [10pt] Find the following expressions by hand. Show your steps.
   (a) AB
   (b) BA
   (c) AB – BA
   (d) ABC
4. [10pt] Calculate the eigenvalues and eigenvectors of matrix A above by hand.
   a. Show your steps below. You can use external tools to solve for a polynomial equation only.
   b. Show the trace of matrix A.
5. [10pt] Use Numpy to concatenate A, B, C to a 9 x 3 matrix. The order doesn't matter. Let's call the new matrix D. Create b=[3, -10, 2]^T. Find the least squares solution x that minimize $\|Dx - b\|^2$. Show your code and results.

Statistics [50pt]

1. [10pt] Flip an unbiased coin 10 times. What is the probability of getting 10 heads? If we call flipping a coin 10 times one trial, let's do 1000 trials. What is the probability of

getting 10 heads in exactly one of the 1000 trials? What about getting 10 heads in at least one of the 1000 trials?

2. [10pt] A report stated that the average number of times a cat returns to its food bowl during the day is 36. Assuming the variable is normally distributed with a standard deviation of 5, what is the probability that a cat would return to its dish between 32 and 38 times a day?

3. [10pt] 6 Digital Camera Prices The prices (in dollars) for a particular model of digital camera with 18.0 megapixels and a f/3.5−5.6 zoom lens are shown here for 10 randomly selected online retailers. Estimate the true mean price for this particular model with 95% confidence.

Prices: [999, 1499, 1997, 398, 591, 498, 798, 849, 449, 348]

4. [10pt] The average “moviegoer” sees 8.5 movies a year. A moviegoer is defined as a person who sees at least one movie in a theater in a 12-month period. A random sample of 40 moviegoers from a large university revealed that the average number of movies seen per person was 9.6. The population standard deviation is 3.2 movies. At the 0.05 level of significance, can it be concluded that this represents a difference from the national average?

5. [10pt] A statistics professor is used to having a variance in his class grades of no more than 100. He feels that his current group of students is different, and so he examines a random sample of midterm grades as shown. At $\alpha = 0.05$, can it be concluded that the variance in grades exceeds 100?

The grades: [92.3, 89.4, 76.9, 65.2, 49.1, 96.7, 69.5, 72.8, 67.5, 52.8, 88.5, 79.2, 72.9, 68.7, 75.8]