Pairs Trading Mean Reversion Approach Using the Ornstein-Uhlenbeck Process

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I. Introduction

In the financial market, the concept of mean reversion is to profit from market movements that deviate from the long-term averages. Also, the asset prices tend to revert to their historical average over time. There are a few mean reversion trading strategies that are used to inform trading decisions. In this paper, we will focus on mean-reversion trading strategies based on the Ornstein-Uhlenbeck(OU) process, a mathematical model designed to capture mean-reverting behavior.

The OU process provides a structured way to model price dynamics that oscillate around a long-term mean, allowing traders to identify opportunities when prices deviate significantly from the historical mean. By systematically buying undervalued assets and selling overvalued assets, traders can profit from the anticipated reversion. Through this approach, the study explores how mean reversion strategies using the OU processes can generate revenue and contribute to the broader field of statistical arbitrage.

Pairs trading, also known as statistical arbitrage, is a market-neutral strategy that seeks to capitalize on the mean-reverting behavior of two historically correlated assets. This strategy involves simultaneously buying and selling two assets whose price movements are believed to be closely related. When the price relationship between the two assets diverges, traders open a position by selling the overpriced asset and buying the underpriced asset, betting that the prices will converge back to their historical relationship. By doing so, pairs trading allows market participants to profit from the relative price movements rather than from the overall market direction, making it a popular choice for hedging and minimizing market risk..

In summary, this paper will explore the combined use of the Ornstein-Uhlenbeck process and pairs trading to develop a robust mean-reversion trading strategy. The study will evaluate the effectiveness of this enhanced model, contributing to the broader understanding of mean-reversion strategies in quantitative finance.

II. Literature Review

Mean reversion has been widely studied in financial markets as an effective strategy to exploit price movements that deviate from their historical averages. Poterba and Summers (1988) provided early evidence of mean-reverting behavior in stock prices, challenging the random walk hypothesis. They demonstrated that stock returns exhibit negative autocorrelation over long horizons, implying a tendency for prices to revert to their historical means. This foundational work laid the groundwork for developing statistical arbitrage strategies that capitalize on such behaviors.

The Ornstein-Uhlenbeck (OU) process, introduced by Uhlenbeck and Ornstein (1930), has emerged as a popular mathematical framework for modeling mean-reverting dynamics in asset prices. The OU process is characterized by a tendency to "pull" prices toward a long-term mean, with stochastic noise causing deviations. Many studies have applied the OU process to various asset classes to develop profitable trading strategies. For example, Zeng and Lee (2014) applied the OU process to commodities markets, demonstrating that mean-reversion-based strategies outperformed simple trend-following methods.

Pairs trading as a strategy was first formalized by Nunzio Tartaglia's quantitative team at Morgan Stanley in the late 1980s, becoming one of the earliest systematic quantitative trading strategies. According to Gatev, Goetzmann, and Rouwenhorst (2006), pairs trading has been widely used in equity markets due to its ability to generate stable, market-neutral profits by exploiting the temporary divergences in the prices of two correlated assets. They demonstrated that pairs trading based on historical price relationships between stocks in the U.S. equity market could generate excess returns, even after accounting for transaction costs. Their research found that price deviations between the two assets typically revert to their historical mean over time, allowing traders to profit from these temporary mispricings.

Despite the widespread use and success of pairs trading, several researchers have pointed out its limitations. Krauss (2017) highlighted that the strategy's performance tends to degrade during periods of heightened market volatility. This occurs because the assumption of mean reversion may break down during market turmoil, as asset prices may deviate more drastically and for longer periods than expected. Jurek and Yang (2007) emphasized the importance of adjusting the traditional pairs trading framework by incorporating risk management techniques to mitigate the impact of market volatility on the strategy’s performance.

Several studies have proposed modifications to the traditional pairs trading model to account for market risk. Avellaneda and Lee (2010) suggested the use of stochastic processes such as the Ornstein-Uhlenbeck (OU) process to model the spread between two assets in a pair, capturing both the mean-reverting behavior and random shocks that can occur due to market volatility. In their framework, the speed of mean reversion and volatility of the spread are both factored into the decision-making process. The incorporation of volatility control, particularly through variance thresholds, has been explored in other quantitative strategies, such as volatility-managed portfolios (Moreira & Muir, 2017), which have shown that constraining trades based on volatility metrics improves risk-adjusted returns.

Building on these findings, this paper introduces a variance constraint into the traditional pairs trading framework. We hypothesize that applying a volatility-based threshold will reduce the likelihood of entering trades during highly volatile periods, where mean reversion may be delayed or invalidated, and improve the overall performance of the strategy by mitigating downside risk.

III. Methodology

This study employs a **Pairs Trading Mean Reversion strategy** using the **Ornstein-Uhlenbeck (OU) process** to model the spread between two correlated assets. By incorporating a **variance constraint** to manage volatility, we aim to enhance the strategy's effectiveness in capturing mean-reverting opportunities while mitigating risk.

3.1. Overview of the Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck (OU) process is a continuous-time stochastic process that models mean-reverting behavior in asset prices. This process is defined by the stochastic differential equation:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t$$

Where:

- **X_t** is the spread between two assets at time t.
- **μ** is the long-term mean of the spread.
- **θ** is the speed of mean reversion, determining how quickly the spread tends to return to μ\muμ.
- **σ** is the volatility of the spread, which represents the magnitude of random fluctuations.
- **dW_t** is a Wiener process representing random noise.

Based on the formula, this process is used to capture how the spread tends to revert to the historical mean over time, making it useful for identifying mean reversion opportunities in pairs trading. After data collection, check a time series of the modeled spread that fits the historical data based on the parameter. Whether the model accurately captures mean-reverting behavior can be determined by how well the spread behaves relative to the estimated mean and reversion speed.

3.2. Pairs Trading Strategy Using Ornstein-Uhlenbeck Process

Pairs trading is a market-neutral strategy where two correlated assets are traded to exploit mean-reverting behavior in their price relationship. The core steps of our pairs trading methodology are pairs selection, spread calculation, and trade execution. We will select pairs of assets based on their historical price correlation or cointegration. Cointegration tests are often used to ensure the pair has a stable, long-term equilibrium relationship, implying that the two prices tend to move together in a predictable way.

Next, the price spread between the two assets is calculated. When the spread deviates from its historical mean by a significant amount (predetermined thresholds), a position is opened: the trader buys the underpriced asset and sells the overpriced one. The trade is closed when the spread reverts to its mean, allowing the trader to capture the profit from mean reversion.

To model the spread and ensure optimal execution, we employ the Ornstein-Uhlenbeck (OU) process, which effectively captures mean-reverting dynamics. Let $S_t^{(1)}$ and $S_t^{(2)}$ represent the prices of two correlated assets. The stock price follows log normal distribution, which make it more reasonable to take logarithm do the calculation. The spread $X_t^{\alpha,\beta}$ between these assets is defined as:

$$X_t^{\alpha,\beta} = \alpha\, log\, S_t^{(1)} - \beta\, log\, S_t^{(2)}$$

where α is the fixed ratio for the first asset (typically set to 1), β is the optimal ratio for the second asset, which is estimated using Maximum Likelihood Estimation (MLE).

3.3. Maximum Likelihood Estimation (MLE)

The spread is modeled using the Ornstein-Uhlenbeck process $dX_t = \theta(\mu - X_t)dt + \sigma dW_t$ introduced above. In order to optimize the α and β in the spread model, we apply Maximum Likelihood Estimation (MLE). This method allows us to estimate these parameters by maximizing the likelihood that the observed historical price data for $S_t^{(1)}$ and $S_t^{(2)}$ conform to the model. Specifically, we calibrate α and β to reflect the most probable configuration of asset quantities, given the observed spread dynamics.

We assume the observed spread $dX_t = \theta(\mu - X_t)dt + \sigma dW_t$ follows the normal distribution with mean μ and variance σ. Then the likelihood function L(α,β) for a series of T observations is given by:

$$L(\alpha, \beta) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} exp(- \frac{(X_t^{\alpha,\beta} - \mu)^2}{2\sigma^2})$$

Taking the natural logarithm of the likelihood function for easier maximization, and substitute $X_t^{\alpha,\beta} = \alpha\, S_t^{(1)} - \beta S_t^{(2)}$

$$log\, L(\alpha, \beta) = -\frac{T}{2} log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T} (\alpha\, log\, S_t^{(1)} - \beta\, log\, S_t^{(2)} - \mu)^2$$

The parameters of the OU process, particularly the optimal ratio β, are estimated through MLE by maximizing the likelihood that the observed spread dynamics conform to the OU model. The likelihood function for the spread is derived from the assumption that the spread follows a normal distribution, with the mean μ and variance σ2 estimated from the spread's first differences. The optimal β is determined by minimizing the negative log-likelihood function.

Once the parameters are calibrated, we monitor the spread over time. When the spread deviates from μ more than a predetermined threshold δ , trades are executed. Specifically:

- Entry Signals
    - When the spread X_t deviates from the mean μ by a certain threshold$(\mu + \delta)$, it generates a signal to short the spread (which could mean selling the overpriced asset and buying the underpriced one).
    - In the other hand, the spread fall below $\mu - \delta$, it generates a long signal (buying the spread, typically buying the underpriced asset and selling the overpriced one).
- Exit Signals
    - When the spread reverts to mean, the trade is closed.

IV. Empirical Research

4.1. Data Selection

In mean reversion strategies, pair selection is a critical process because the success of the strategy hinges on choosing stocks that exhibit a long-term relationship but deviate from short term, providing profitable trading opportunities.

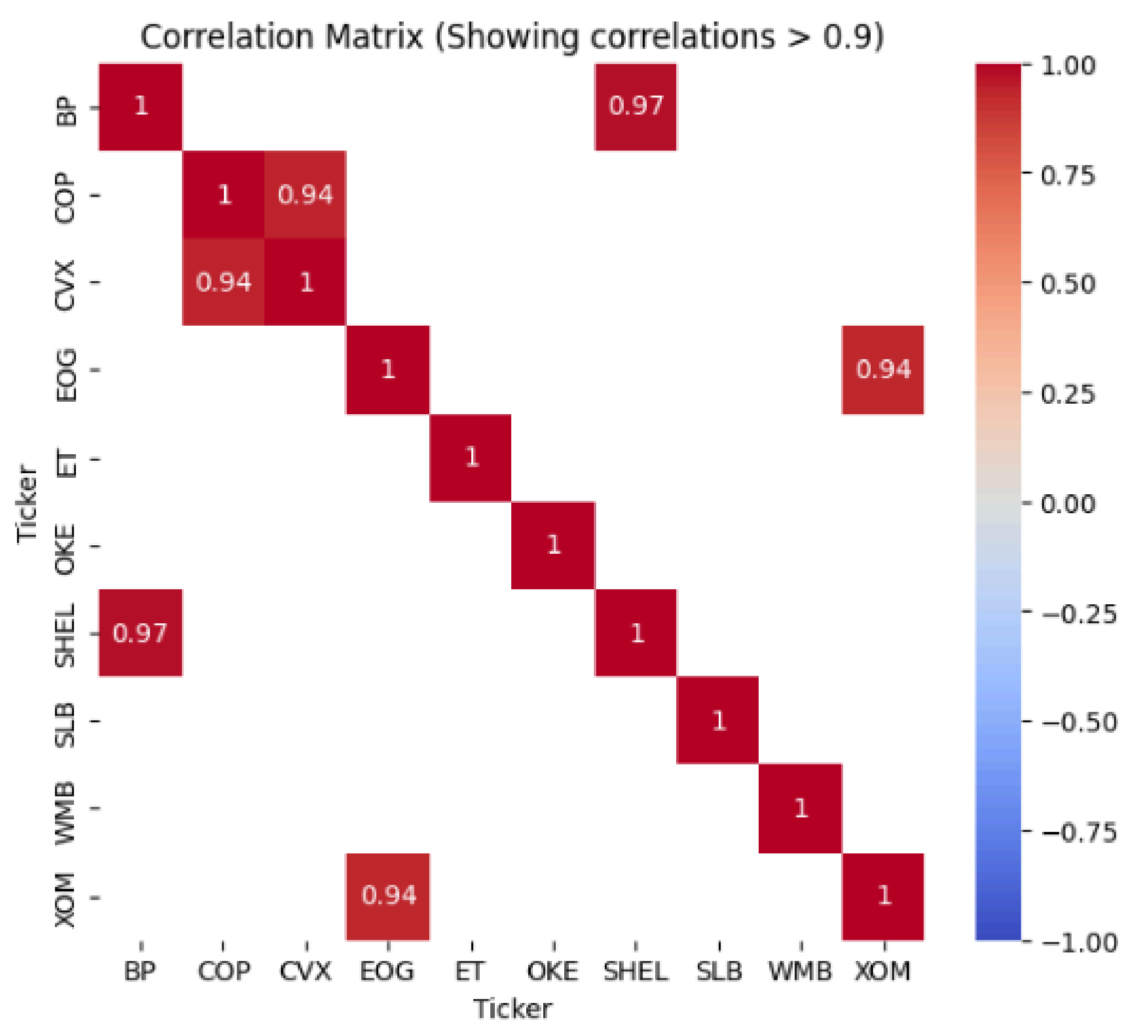
- When the spread between two stocks widens, a trader can short the outperforming stock and buy the underperforming stock.
- When the stocks revert to their historical mean, the trader closes the position, realizing a profit from the reversion

The data, covering a 9-year period from January 2015 to December 2023, is obtained from Yahoo Finance and Bloomberg. Daily adjusted closing prices are used to compute log returns and the spread.To determine the appropriate stock pairs, we used a few methods to evaluate their correlation and validity.

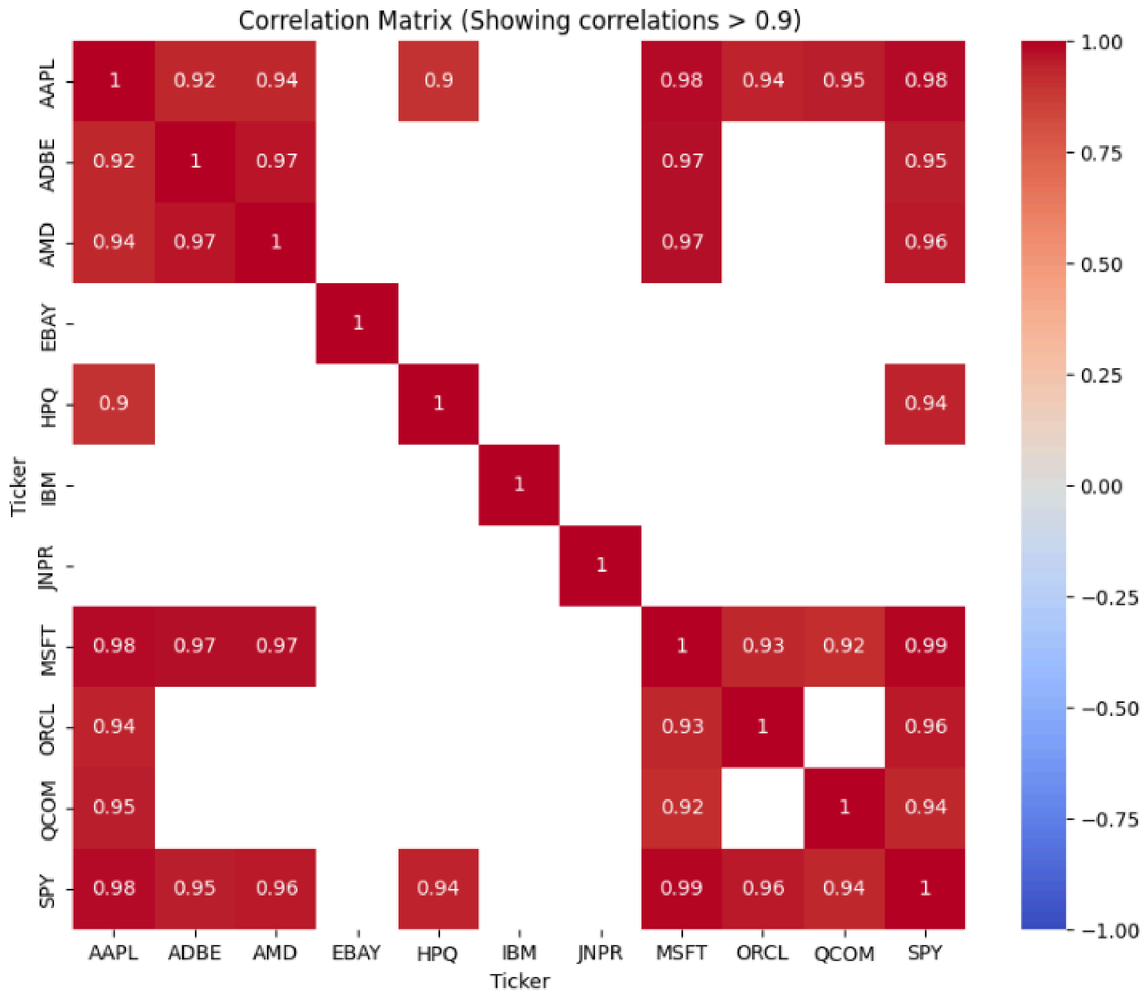
First, Stocks in the same sector or industry tend to be affected by similar macroeconomic factors, reduce the idiosyncratic risk, and increase the likelihood tat the selected stock will move in similar patterns, marking them more suitable for mean reversion strategies. In our project, we select three pair of stocks from the energy sector, technology sector, and real estate sector.

Second, calculating the correlation between stocks is the preliminary step to identify potential pairs that move similarly over time. Highly correlated stocks(correlation close to 1) are more likely to exhibit stable relative price movements. When these highly correlated stocks temporarily deviate from their expected relative price movement, it signals potential trading opportunities. Also, by focusing on highly correlated pairs, we reduce the risk of selecting stocks that move independently of each other, which can lead to unpredictable and unprofitable trades.

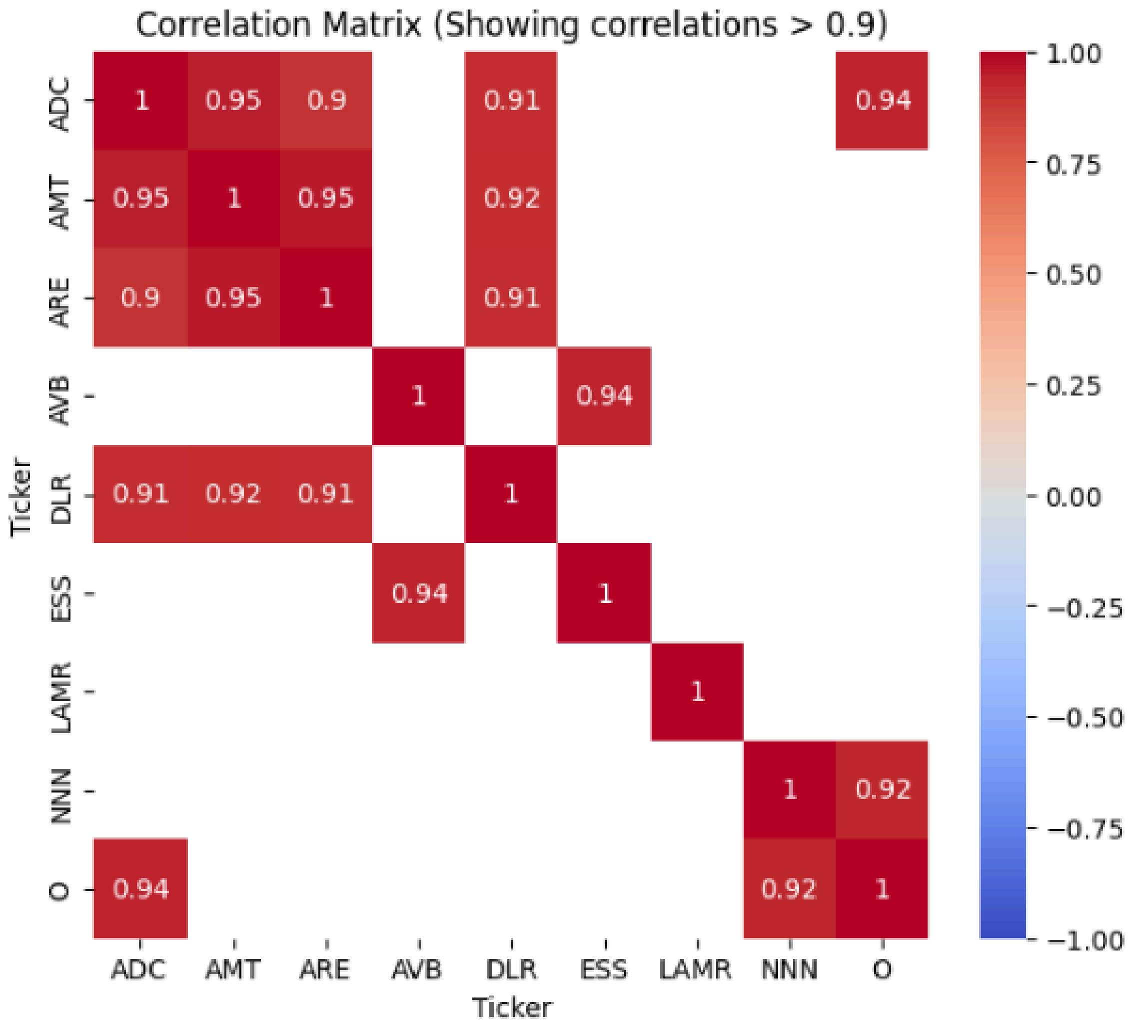
- Energy Sector



- Technology Sector

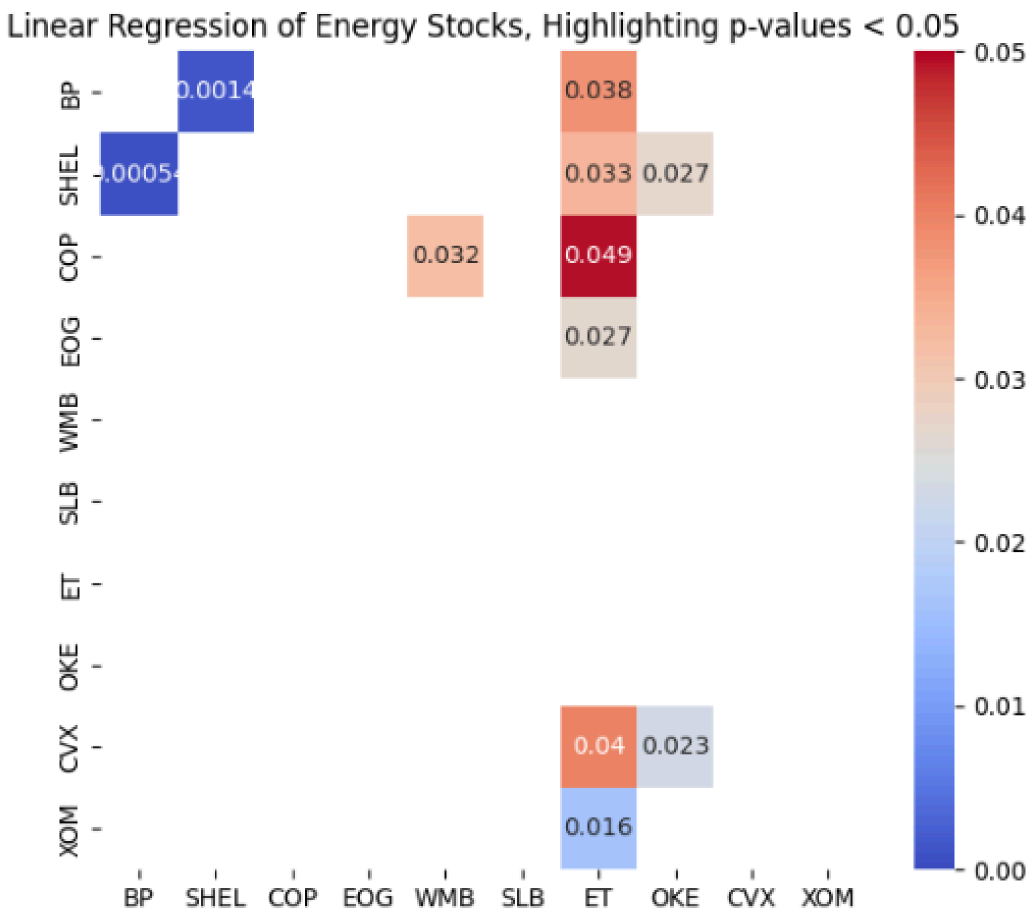


- Real Estate Sector

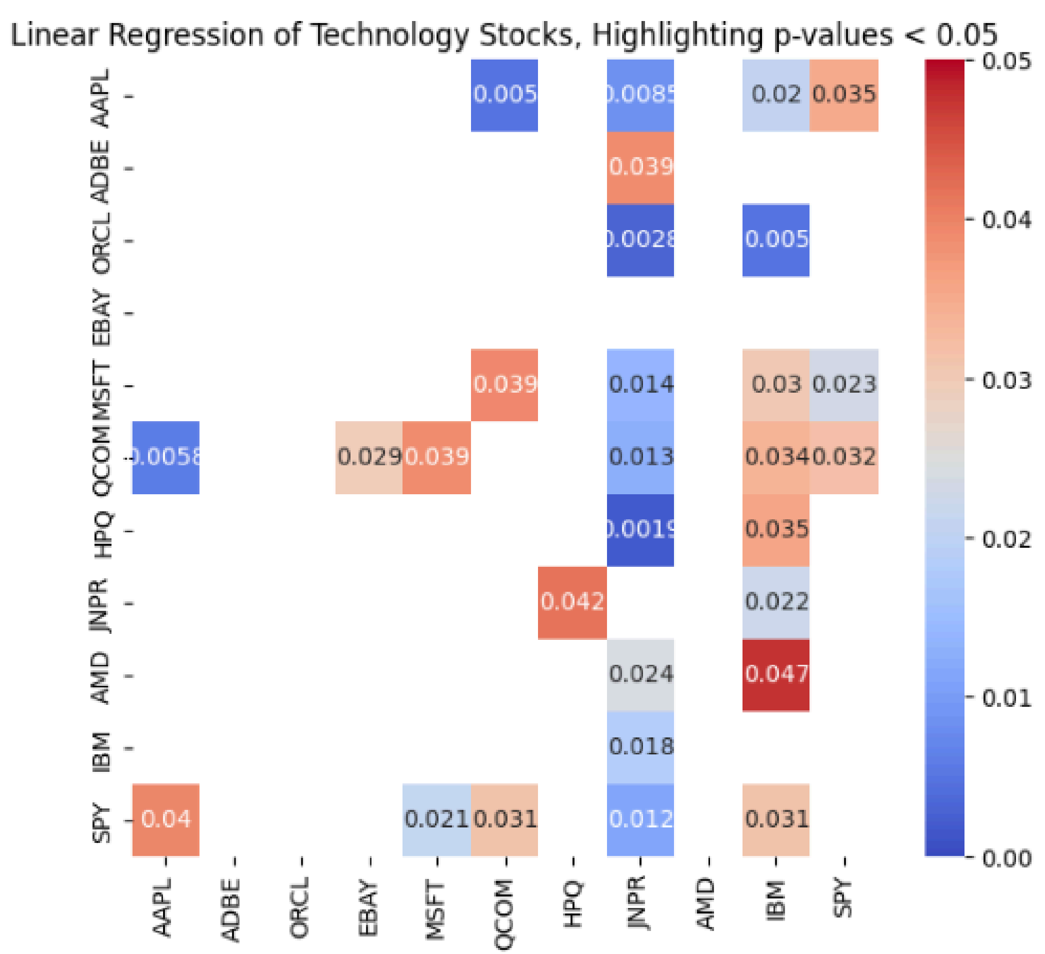


Third, ACF test is part of Cointegration testing, which is crucial in pair selection for mean reversion trading because it helps confirm whether the spread is stationary. Stationary is essential for pair trading as it indicates that the price spread between two assets tends to revert to a mean over time. In our cointegration testing part, we apply the Johansen cointegration testing, which provides a statistical framework for determining whether two time series are cointegrated. In our mode, if the test p-value is less than a set threshold(0.05), we consider the pair of stocks to be cointegrated. After identifying the potential coingrated pairs we use the ADF test on the residuals from linear regression between two stock log prices. From the identified, we consider the value of correlation and results from the cointegration test which filter out those with low p-values from ADF test. And based on the correlation graph and p-value graph for each sector, we chose pair stocks with high correlation including ‘BP’ and ‘SHEL’ as our pair of stock in energy sector with a correlation of 0.97 and p-value of 0.0014, ‘AAPL’ and ‘ GCOM’ in technology sector with a correlation of 0.95 and p-value of 0.005, ‘AMT’ and ‘ ARE’ in real estate sector with a correlation of 0.95 and p-value of 0.036.

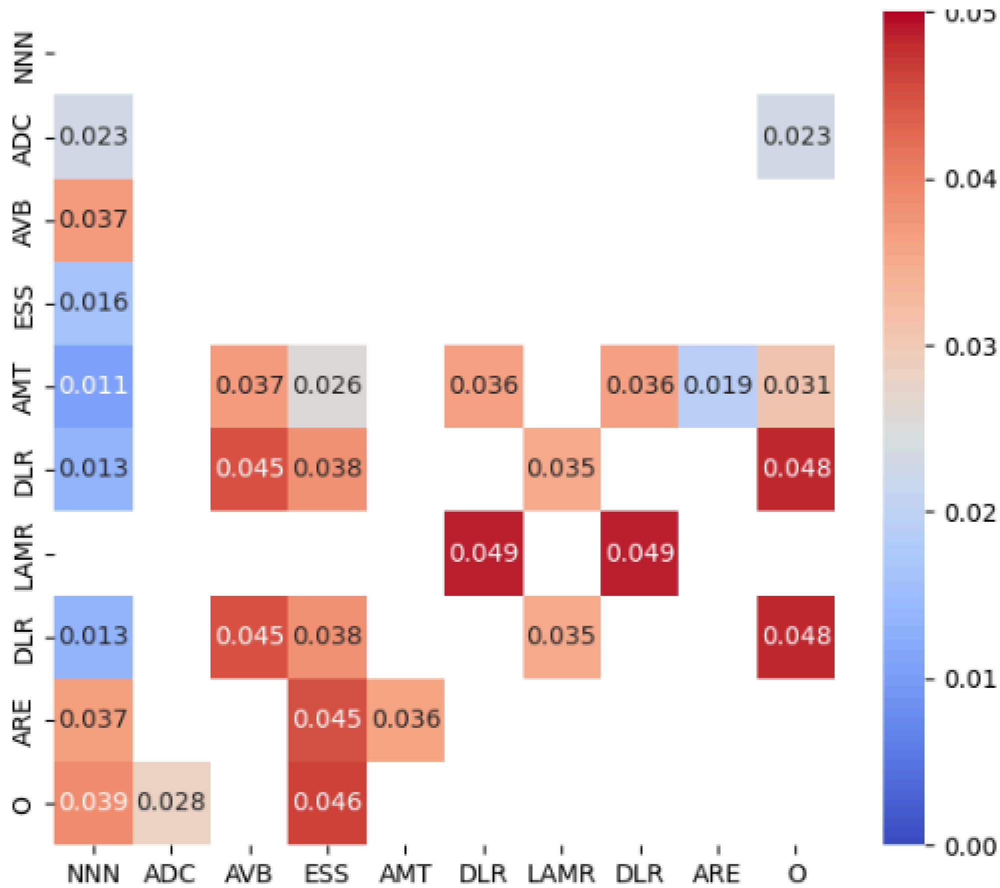
- Energy Sector



- Technology Sector

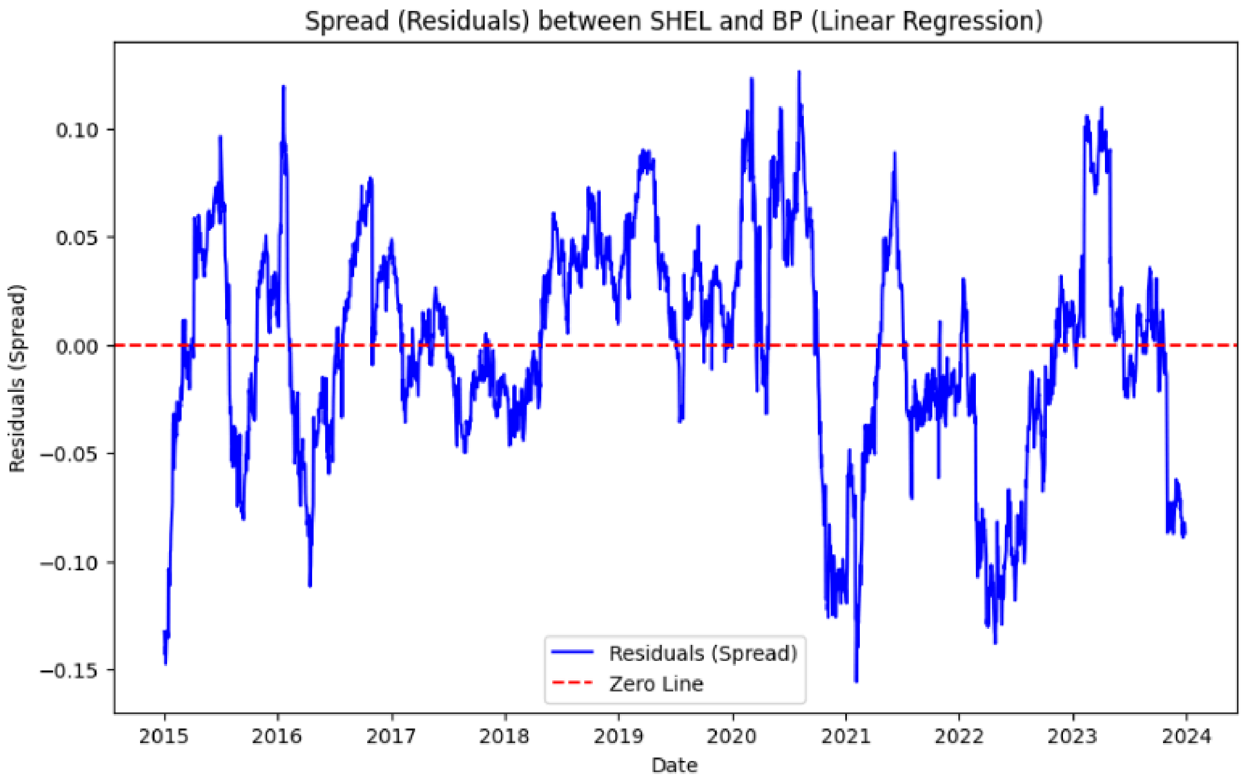


- Real Estate Sector

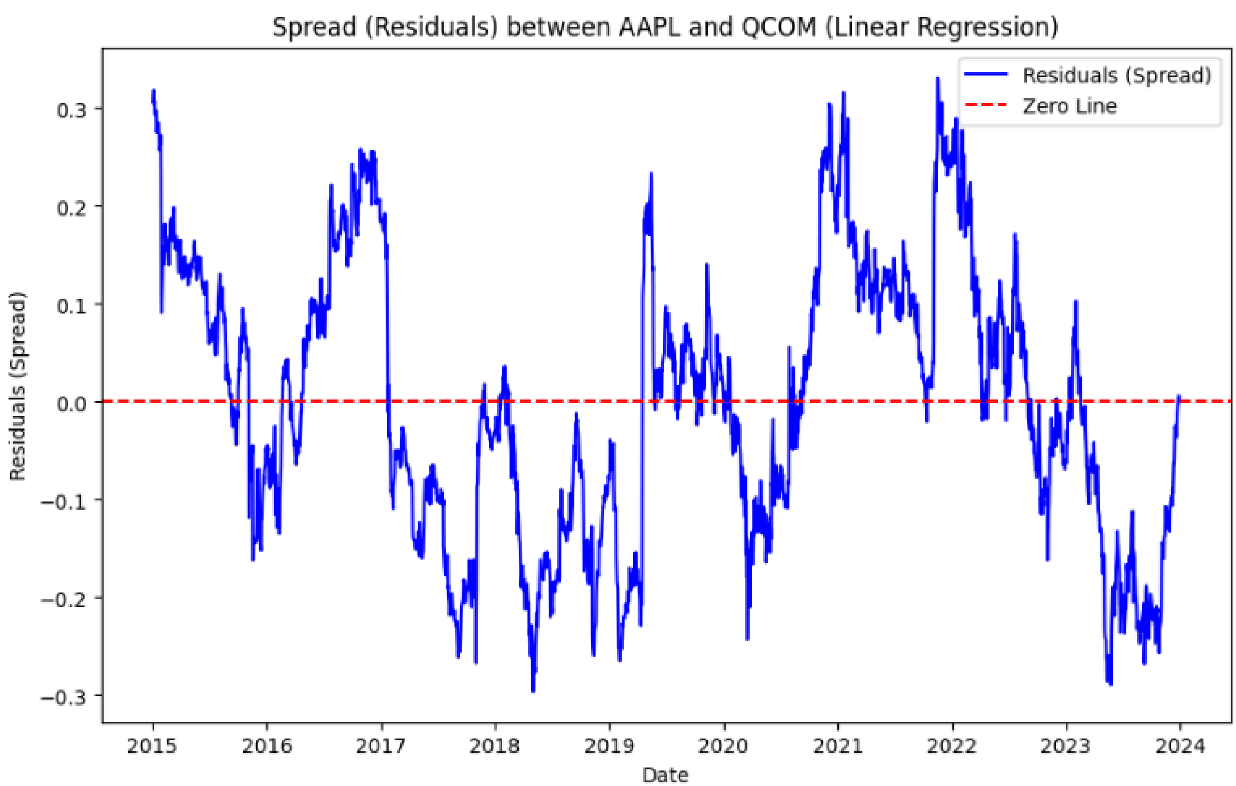


Fourth, the spread is central to pairs trading because it represents the price difference between two stocks. In pair trading, the objective is to take advantage of short-term deviation in price difference, assuming that the two stocks have a stable long-term relationship. In addition, the spread is the main factor used to generate buy and sell signals and affect the entire profitability. When the spread diverges from its historical mean, a trading opportunity arises. When the spread reverts to the mean, the positions are closed and profits are realized. In our research, we extract the log-adjusted price for the three pairs of stocks, then we build a linear regression model to fit the log pair stocks prices. After fitting the model, we use it to predict the X log price(‘BP’ as X in energy sector,’AAPL’ as X in technology sector, ‘AMT’ as X in real estate sector) The spread in this context is residuals which represent the difference between actual log price for pair stock.

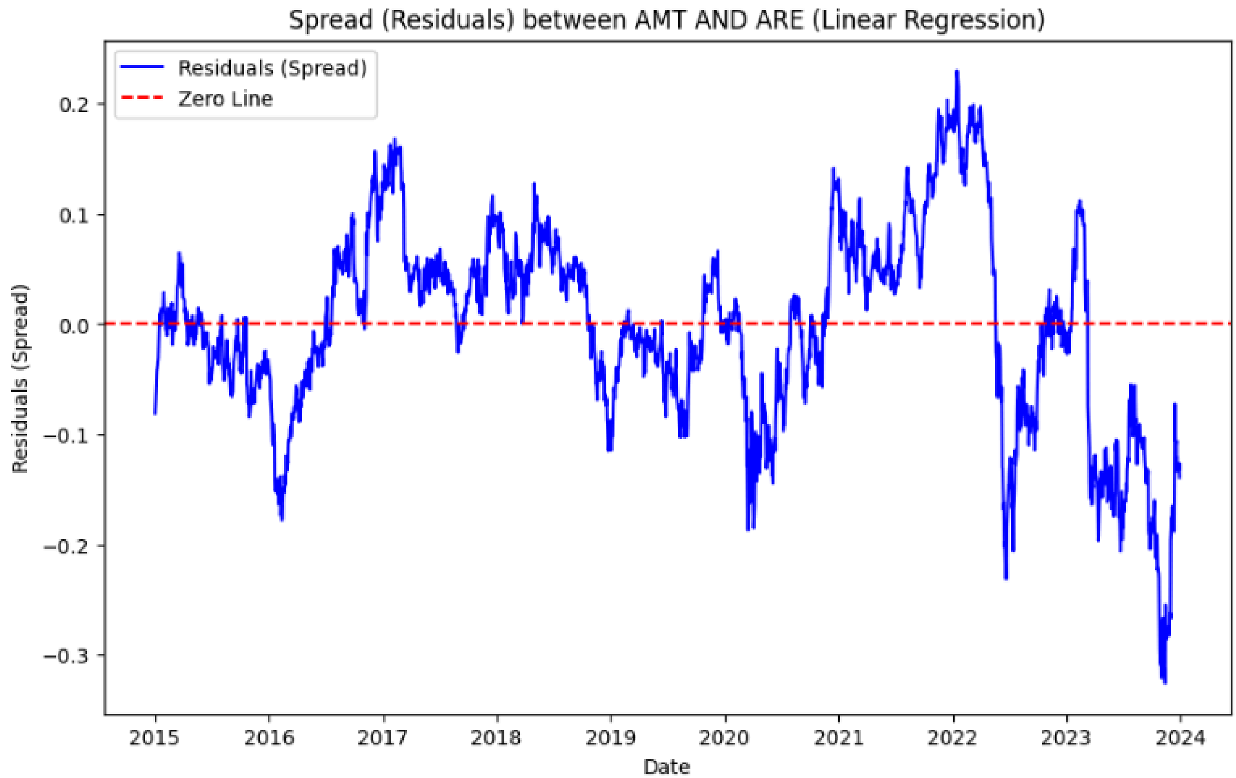
- Energy Sector



- Technology Sector



- Real Estate Sector

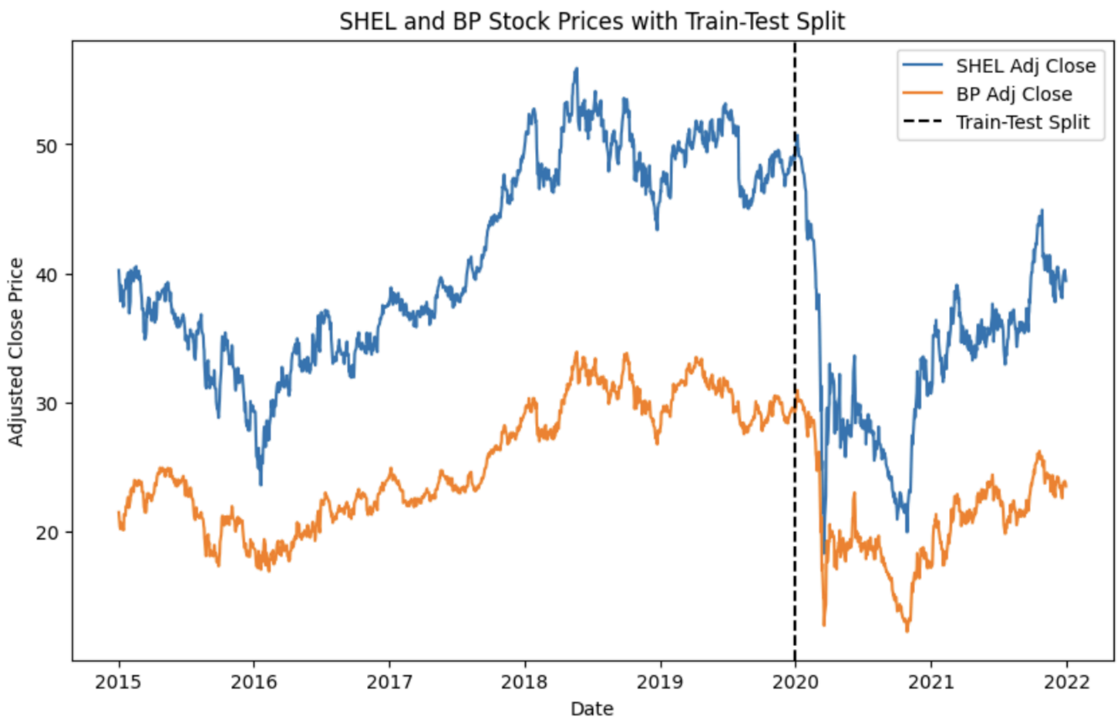


Based on these three spread graphs, we can conclude that ‘SHEL and ‘BP; show potential as a pair for mean reversion trading with spread widening and narrowing visible, although some volatility is observed post 2020. ‘AApl’ and ‘QCOM’is more volatile with large swings above and below the zero line compared to SHEL-BP pair, but it also shows mean reversion behavior over time. For AMT and ARE, it is more stable before 2020, and post-2020 volatility could be a complicated trading decision, but historically, this pair still be good candidates for pairs trading.

4.2. Split Train and Test Dataset

In order to properly evaluate the trading strategy, the dataset was split into two parts: a training set and a testing set. The training period spans from January 1, 2015, to December 31, 2019, while the test period covers January 1, 2020, to December 31, 2023. This split was done to ensure that the strategy is trained on historical data, allowing for the testing phase to evaluate its performance in out-of-sample data.

The training set is used to estimate key parameters such as alpha and beta, and to model the spread between SHEL and BP, which are the two stocks selected for this pair's trading strategy. A vertical line representing the split between training and testing data is added to the stock price plot to clearly visualize the distinction between the two periods. This ensures that no data leakage occurs from the future into the training phase, maintaining the robustness of the backtesting process.



Training set and Test set

4.3. Spread Calculation and OU Process

The spread between SHEL and BP is calculated using a linear combination of their log-adjusted closing prices. The spread serves as the key metric in identifying mean-reverting opportunities in this pair's trading strategy. The initial formula for calculating the spread is as follows:

$$z_t = \alpha * log(SHEL)_t - \beta * log(BP)_t$$

where

- Zt represents the spread at time t
- α and β are the hedge ratio between the two stocks.

In this study, α is fixed to 1, and we aim to optimize β to best capture the mean-reverting behavior of the spread. By fixing α=1, the focus shifts to optimizing β, which simplifies the model and keeps the interpretation straightforward, which means the comparison across different asset pairs remains consistent. The resulting β will represent the number of units of the second asset (BP) required to hedge against one unit of the first asset (SHEL). The variation in β captures the differences in the relationship between the two assets rather than introducing complexity by adjusting both coefficients. This approach ensures that the optimization captures the dynamic relationship between the two assets efficiently.

4.3.1 OU process & MLE

To better model the mean-reversion dynamics of the spread, the parameters of the Ornstein-Uhlenbeck (OU) process were estimated using the spread data from the training period. The OU process is particularly useful for modeling the behavior of the spread as it reverts to its long-term mean, and is described by the following stochastic differential equation:

$$dZ_t = \theta(\mu - Zt)\, d_t + \sigma * dW_t$$

Where:

- μ is the long-term mean of the spread.
- θ is the speed of mean reversion.
- σ is the volatility of the spread.
- dW_t is a Wiener process representing random shocks.

The goal is to **optimize β** using Maximum Likelihood Estimation (MLE). The Ornstein-Uhlenbeck (OU) process assumes that the spread follows a mean-reverting stochastic process, and the MLE method is used to estimate the parameters that maximize the likelihood of observing the given spread data.

$$log\, L(\alpha, \beta) = -\frac{T}{2} log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{t=1}^{T} (\alpha\, log\, S_t^{(1)} - \beta\, log\, S_t^{(2)} - \mu)^2$$

Where:

- Zt is the spread at time t.
- μ is the mean of the spread.
- σ is the volatility (standard deviation) of the spread.

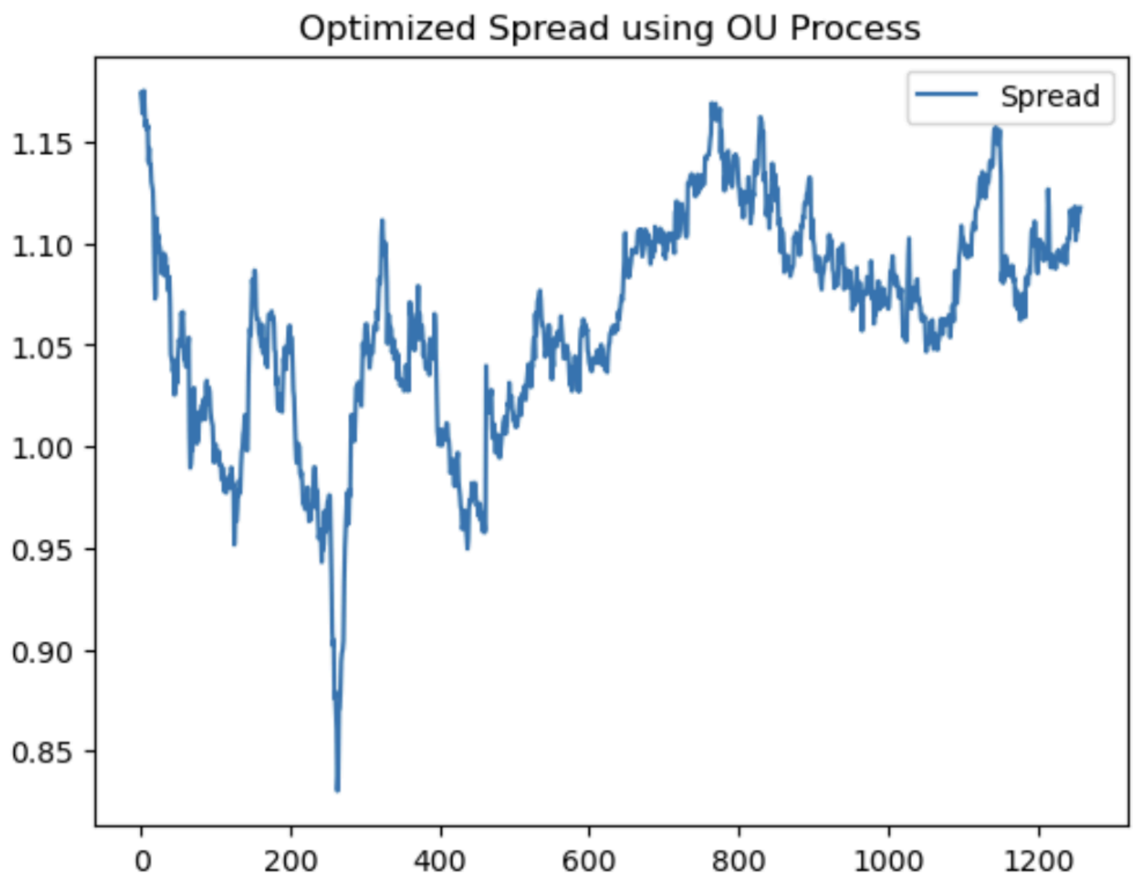
To optimize β, we minimize the negative log-likelihood function using the L-BFGS-B method, with a constraint that β must be positive. The optimal values of α and β for this strategy were found to be:

- β_opt=0.823
- α_opt=1 is fixed

4.3.2 Spread Calculation and Visualization

After finding the optimal β, we recalculate the spread using the optimized hedge ratio and we plot the spread:

$$z_t = \alpha_{opt} * log(SHEL)_t - \beta_{opt} * log(BP)_t$$

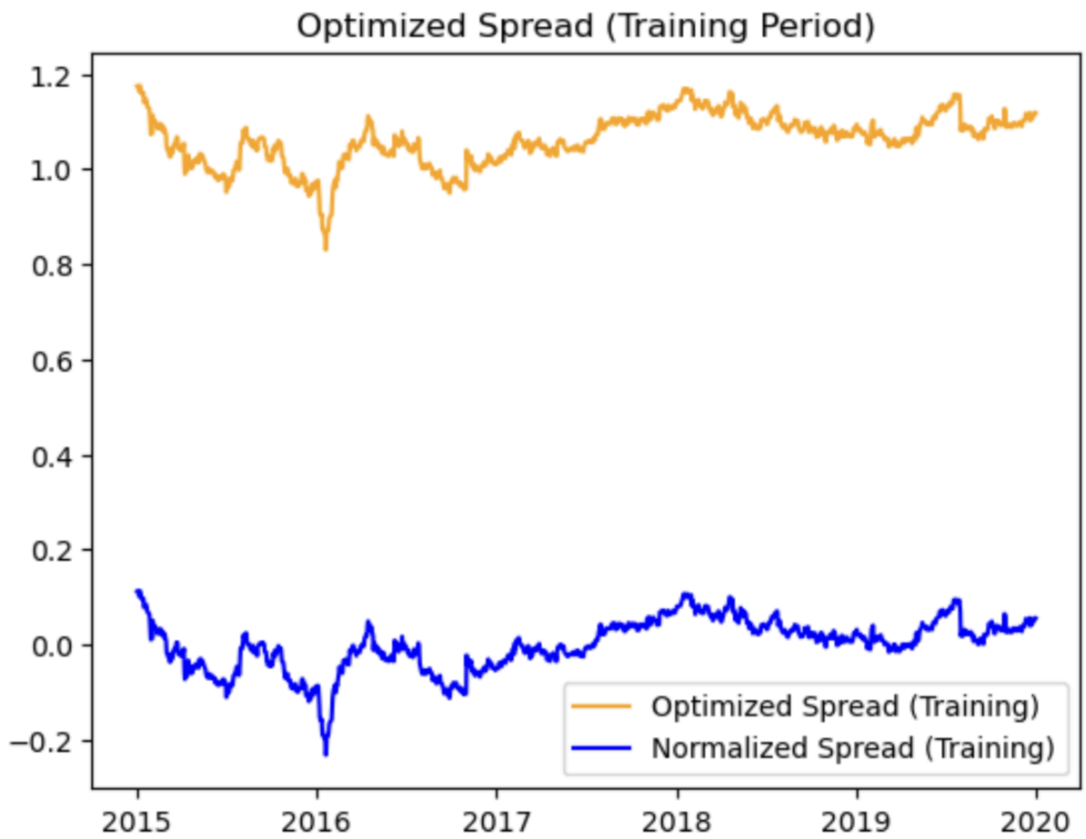


The optimized spread serves as the primary signal for our pairs trading strategy. The spread shows clear mean-reverting behavior over the training period, fluctuating within a stable range, which indicates potential trading opportunities whenever it deviates significantly from its mean.

Normalized Spread

To ensure that the methodology and visualization are well-connected, we also apply a normalized version to assess the mean-reverting characteristics over the training period. Normalized spread is obtained by subtracting the mean from the optimized spread. Normalization allows us to better observe how far the spread deviates from its long-term average:

$$z_{normal} = z_{opt} - mean(z_{opt})$$



Spread Trend Plot

The plot effectively demonstrates the mean-reverting nature of the spread between SHEL and BP. The optimized spread (orange) fluctuates within a bounded range, and the normalized spread (blue) emphasizes the deviations from the mean. These deviations provide clear signals for executing pairs trades when the spread exceeds certain thresholds, as we will define in the next steps of the strategy.

V. Result Analysis

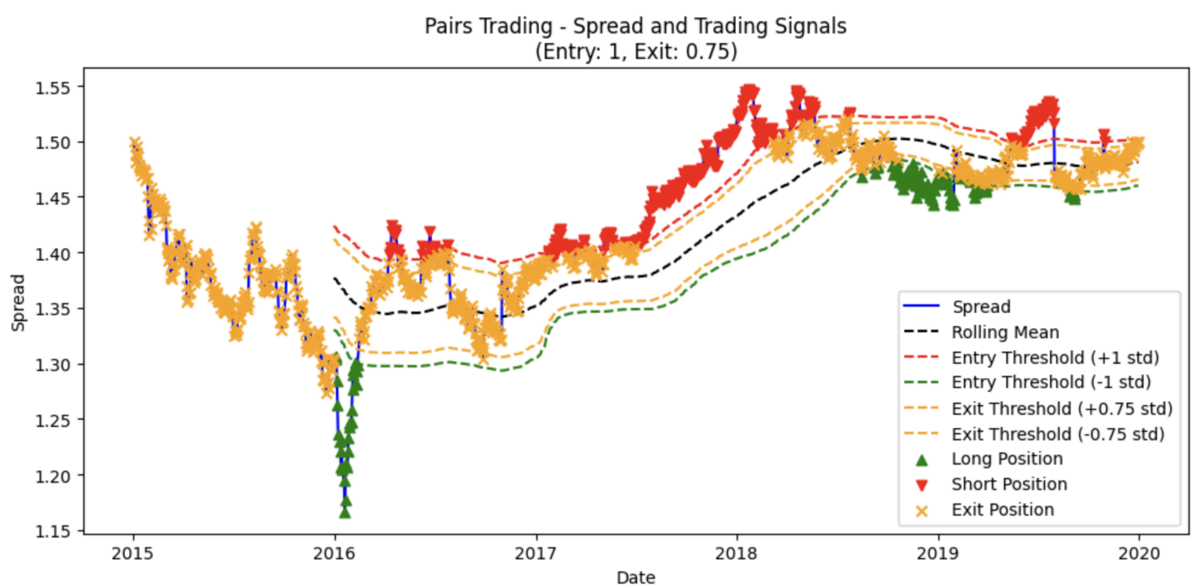
5.1 Train Data analysis

After implementing the pairs trading strategy based on mean reversion, we conducted multiple tests by adjusting both entry and exit thresholds on the training data set. The goal was to determine the optimal combination of parameters that maximizes returns while minimizing risk. Specifically, we tested three different entry thresholds (1.0, 1.5, and 2.0 standard deviations) and three exit thresholds (0.75, 1.0, and 1.25 standard deviations). These thresholds dictate when positions are initiated (entry) and closed (exit) based on the spread deviation from its rolling mean. Three trading signal graphs are attached as examples.

These specific values (1.0, 1.5, and 2.0 for entry and 0.75, 1.0, and 1.25 for exit) are chosen because they balance frequency of trades, risk management, and profit potential:

- 1.0 Standard Deviation entry allows for more frequent trades, though they might offer smaller returns. 2.0 Standard Deviation entry provides fewer signals but with potentially larger profits due to the larger spread deviations.
- The exit thresholds of 0.75, 1.0, and 1.25 allow for varying degrees of risk management. A 0.75 Standard Deviation exit is more conservative, ensuring quicker profit realization, while 1.25 Standard Deviation exit is more aggressive, allowing the trade to run longer for potentially larger gains.

It's important to note that both the mean and standard deviation in this strategy are calculated using rolling windows to adjust dynamically to the market's trend and maintain real-time responsiveness.



5.1.1. Key Performance Metrics

To evaluate the effectiveness of each threshold combination, several performance metrics were computed, including expected gain, expected loss, trading frequency, hit ratio, annualized return, annualized volatility, Sharpe ratio, and maximum drawdown. These metrics provide a comprehensive view of the strategy's profitability, risk, and trading behavior.

	Entry Threshold	Exit Threshold	Annualized Return	Annualized Volatility	Sharpe Ratio	Expected Gain	Expected Loss	Trading Frequency	Hit Ratio	Max Drawdown
0	1.0	0.75	0.042764	0.074851	0.571329	0.005678	-0.004871	11.614626	0.975000	0.142604
1	1.0	1.00	0.042764	0.074851	0.571329	0.005678	-0.004871	11.614626	0.975000	0.142604
2	1.0	1.25	0.042764	0.074851	0.571329	0.005678	-0.004871	11.614626	0.975000	0.142604
3	1.5	0.75	0.038284	0.056351	0.679392	0.005836	-0.004438	10.162798	0.942857	0.103547
4	1.5	1.00	0.038284	0.056351	0.679392	0.005836	-0.004438	10.162798	0.942857	0.103547
5	1.5	1.25	0.038284	0.056351	0.679392	0.005836	-0.004438	10.162798	0.942857	0.103547
6	2.0	0.75	0.012710	0.038165	0.333022	0.006350	-0.005578	4.355485	0.933333	0.076755
7	2.0	1.00	0.012710	0.038165	0.333022	0.006350	-0.005578	4.355485	0.933333	0.076755
8	2.0	1.25	0.012710	0.038165	0.333022	0.006350	-0.005578	4.355485	0.933333	0.076755

Expected Gain and Loss: The expected gain is quite consistent across all combinations, around 0.0056, but the expected loss decreases slightly with higher entry thresholds. This aligns with the overall reduced risk profile at higher entry thresholds.

Trading Frequency and Hit Ratio: A higher trading frequency (like 11.6 trades per year for a 1.0 entry threshold) is associated with a higher hit ratio (0.975), meaning the strategy captures more opportunities and closes a higher percentage of profitable trades. As the trading frequency decreases with more conservative thresholds (4.35 trades per year for a 2.0 entry threshold), the hit ratio slightly drops (0.933), reflecting fewer but more selective trades that are still largely successful.

Annualized Return and Volatility: Annualized returns are highest (~4.27%) with a 1.0 entry threshold, but they come with higher annualized volatility (~0.0749), reflecting increased risk from more frequent trades.
As we move to a more conservative threshold, the annualized return decreases (~1.27% at 2.0 entry threshold), but volatility also drops significantly (~0.038), indicating lower risk but at the expense of profitability.

Sharpe Ratio: The 1.5 entry threshold with any exit threshold (0.75, 1.0, 1.25) has the highest Sharpe ratio (~0.679), suggesting this is the most favorable combination for risk-adjusted returns.
While the 1.0 entry threshold also delivers decent Sharpe ratios (~0.571), it doesn't balance risk and return as effectively.

Maximum Drawdown: Drawdowns are lower with higher entry thresholds, with the 2.0 SD entry threshold having the smallest maximum drawdown (~0.0767), indicating a more conservative risk profile.
In contrast, the 1.0 entry threshold has the highest drawdown (~0.1426), reflecting the increased risk from more frequent trades.

5.1.2. Summary

1.0 Standard Deviation entry threshold results in the highest annualized returns (~4.27%) and most frequent trades, but also comes with the highest volatility and maximum drawdown. This strategy is aggressive, capitalizing on smaller deviations but exposes the portfolio to higher risk.

1.5 Standard Deviation entry threshold strikes the best balance between risk and reward. It delivers moderate returns (~3.8%) with a lower volatility (~0.056) and best Sharpe ratio (~0.679). This makes it an optimal choice for a more balanced risk-adjusted return profile.

2.0 Standard Deviation entry threshold is the most conservative, with fewer trades and lower risk (volatility ~0.038, max drawdown ~0.0767). However, it sacrifices profitability, achieving the lowest annualized return (~1.27%).

Exit thresholds are less significant but affect how quickly profits are realized. A 0.75 exit threshold ensures faster profit-taking, while a 1.25 exit allows trades to run longer, with marginal effects on returns and risk.

In conclusion, the 1.5 entry threshold seems to offer the best overall performance, balancing frequent trades, acceptable risk, and solid returns, while more aggressive or conservative strategies may appeal depending on risk tolerance and return expectations.

5.2. Backtesting and Performance on Test Data

For the backtesting phase, the model was tested on a dataset spanning January 2020 to December 2023, which was not used during the training process. This allows for an unbiased evaluation of how the pairs trading strategy performs on unseen data, simulating real-world trading conditions.

The testing process involved recalculating the logarithmic prices for SHEL and BP based on their adjusted closing prices, consistent with the method used in the training phase. The spread between the two assets was then computed using the optimal alpha and beta values derived from the training data, ensuring the spread reflects the relationship between the two assets as modeled during the training phase.

Multiple combinations of entry and exit thresholds were once again tested to identify the optimal parameters. The same performance metrics—annualized return, annualized volatility, Sharpe ratio, expected gain/loss, trading frequency, hit ratio, and maximum drawdown—were computed across various time periods: 3 months, 6 months, 9 months, 1 year, 2 year and 3 year. This allows us to assess the strategy's robustness over different market conditions and time horizons. The results for different time periods are shown below.

Results for 3_months:

	Entry Threshold	Exit Threshold	Annualized Return	Annualized Volatility	Sharpe Ratio	Expected Gain	Expected Loss	Trading Frequency	Hit Ratio	Max Drawdown
0	1.0	0.75	-0.005529	0.002257	-2.44949	0	-0.000921	8.494884	0.0	0.000921
1	1.0	1.00	-0.005529	0.002257	-2.44949	0	-0.000921	8.494884	0.0	0.000921
2	1.0	1.25	-0.005529	0.002257	-2.44949	0	-0.000921	8.494884	0.0	0.000921
3	1.5	0.75	-0.005529	0.002257	-2.44949	0	-0.000921	8.494884	0.0	0.000921
4	1.5	1.00	-0.005529	0.002257	-2.44949	0	-0.000921	8.494884	0.0	0.000921
5	1.5	1.25	-0.005529	0.002257	-2.44949	0	-0.000921	8.494884	0.0	0.000921
6	2.0	0.75	0.000000	0.000000	NaN	0	0.000000	8.494884	0.0	0.000000
7	2.0	1.00	0.000000	0.000000	NaN	0	0.000000	8.494884	0.0	0.000000
8	2.0	1.25	0.000000	0.000000	NaN	0	0.000000	8.494884	0.0	0.000000

Results for 6_months:

	Entry Threshold	Exit Threshold	Annualized Return	Annualized Volatility	Sharpe Ratio	Expected Gain	Expected Loss	Trading Frequency	Hit Ratio	Max Drawdown
0	1.0	0.75	0.234696	0.210161	1.116745	0.024239	-0.016958	25.191724	0.833333	0.134013
1	1.0	1.00	0.234696	0.210161	1.116745	0.024239	-0.016958	25.191724	0.833333	0.134013
2	1.0	1.25	0.234696	0.210161	1.116745	0.024239	-0.016958	25.191724	0.833333	0.134013
3	1.5	0.75	-0.126175	0.181211	-0.696286	0.024753	-0.020853	8.397241	0.500000	0.134013
4	1.5	1.00	-0.126175	0.181211	-0.696286	0.024753	-0.020853	8.397241	0.500000	0.134013
5	1.5	1.25	-0.126175	0.181211	-0.696286	0.024753	-0.020853	8.397241	0.500000	0.134013
6	2.0	0.75	0.076590	0.149826	0.511192	0.024838	-0.036607	12.595862	0.666667	0.059786
7	2.0	1.00	0.076590	0.149826	0.511192	0.024838	-0.036607	12.595862	0.666667	0.059786
8	2.0	1.25	0.076590	0.149826	0.511192	0.024838	-0.036607	12.595862	0.666667	0.059786

Results for 9_months:

	Entry Threshold	Exit Threshold	Annualized Return	Annualized Volatility	Sharpe Ratio	Expected Gain	Expected Loss	Trading Frequency	Hit Ratio	Max Drawdown
0	1.0	0.75	0.335198	0.182252	1.839196	0.020821	-0.015629	33.718154	0.916667	0.134013
1	1.0	1.00	0.335198	0.182252	1.839196	0.020821	-0.015629	33.718154	0.916667	0.134013
2	1.0	1.25	0.335198	0.182252	1.839196	0.020821	-0.015629	33.718154	0.916667	0.134013
3	1.5	0.75	-0.084116	0.147717	-0.569444	0.024753	-0.020853	5.619692	0.500000	0.134013
4	1.5	1.00	-0.084116	0.147717	-0.569444	0.024753	-0.020853	5.619692	0.500000	0.134013
5	1.5	1.25	-0.084116	0.147717	-0.569444	0.024753	-0.020853	5.619692	0.500000	0.134013
6	2.0	0.75	0.051060	0.122115	0.418131	0.024838	-0.036607	8.429538	0.666667	0.059786
7	2.0	1.00	0.051060	0.122115	0.418131	0.024838	-0.036607	8.429538	0.666667	0.059786
8	2.0	1.25	0.051060	0.122115	0.418131	0.024838	-0.036607	8.429538	0.666667	0.059786

Results for 1_year:

	Entry Threshold	Exit Threshold	Annualized Return	Annualized Volatility	Sharpe Ratio	Expected Gain	Expected Loss	Trading Frequency	Hit Ratio	Max Drawdown
0	1.0	0.75	0.224957	0.168511	1.334969	0.015736	-0.014309	31.489655	0.933333	0.134013
1	1.0	1.00	0.224957	0.168511	1.334969	0.015736	-0.014309	31.489655	0.933333	0.134013
2	1.0	1.25	0.224957	0.168511	1.334969	0.015736	-0.014309	31.489655	0.933333	0.134013
3	1.5	0.75	-0.064621	0.137388	-0.470356	0.017301	-0.018114	6.297931	0.666667	0.134013
4	1.5	1.00	-0.064621	0.137388	-0.470356	0.017301	-0.018114	6.297931	0.666667	0.134013
5	1.5	1.25	-0.064621	0.137388	-0.470356	0.017301	-0.018114	6.297931	0.666667	0.134013
6	2.0	0.75	0.012382	0.115395	0.107301	0.021505	-0.024106	8.397241	0.750000	0.059786
7	2.0	1.00	0.012382	0.115395	0.107301	0.021505	-0.024106	8.397241	0.750000	0.059786
8	2.0	1.25	0.012382	0.115395	0.107301	0.021505	-0.024106	8.397241	0.750000	0.059786

Results for 2_year:

	Entry Threshold	Exit Threshold	Annualized Return	Annualized Volatility	Sharpe Ratio	Expected Gain	Expected Loss	Trading Frequency	Hit Ratio	Max Drawdown
0	1.0	0.75	0.021287	0.068168	0.312277	0.007361	-0.007955	2.099310	0.500000	0.075084
1	1.0	1.00	0.021287	0.068168	0.312277	0.007361	-0.007955	2.099310	0.500000	0.075084
2	1.0	1.25	0.021287	0.068168	0.312277	0.007361	-0.007955	2.099310	0.500000	0.075084
3	1.5	0.75	0.032615	0.048668	0.670152	0.007505	-0.008197	7.347586	0.857143	0.034045
4	1.5	1.00	0.032615	0.048668	0.670152	0.007505	-0.008197	7.347586	0.857143	0.034045
5	1.5	1.25	0.032615	0.048668	0.670152	0.007505	-0.008197	7.347586	0.857143	0.034045
6	2.0	0.75	0.028151	0.023343	1.205957	0.019382	0.000000	3.148966	0.666667	0.000000
7	2.0	1.00	0.028151	0.023343	1.205957	0.019382	0.000000	3.148966	0.666667	0.000000
8	2.0	1.25	0.028151	0.023343	1.205957	0.019382	0.000000	3.148966	0.666667	0.000000

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Results for 3_year:
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	Entry Threshold	Exit Threshold	Annualized Return	Annualized Volatility	Sharpe Ratio	Expected Gain	Expected Loss	Trading Frequency	Hit Ratio	Max Drawdown
0	1.0	0.75	0.039575	0.073905	0.535483	0.008696	-0.008301	4.198621	0.833333	0.075084
1	1.0	1.00	0.039575	0.073905	0.535483	0.008696	-0.008301	4.198621	0.833333	0.075084
2	1.0	1.25	0.039575	0.073905	0.535483	0.008696	-0.008301	4.198621	0.833333	0.075084
3	1.5	0.75	0.045804	0.052648	0.870013	0.009730	-0.008197	5.598161	0.875000	0.034045
4	1.5	1.00	0.045804	0.052648	0.870013	0.009730	-0.008197	5.598161	0.875000	0.034045
5	1.5	1.25	0.045804	0.052648	0.870013	0.009730	-0.008197	5.598161	0.875000	0.034045
6	2.0	0.75	0.042831	0.039480	1.084881	0.029517	0.000000	2.799080	0.750000	0.000000
7	2.0	1.00	0.042831	0.039480	1.084881	0.029517	0.000000	2.799080	0.750000	0.000000
8	2.0	1.25	0.042831	0.039480	1.084881	0.029517	0.000000	2.799080	0.750000	0.000000

5.2.1. Shorter Time Horizons (3 and 6 months):

- In the 3-month period, the strategy shows negative returns for most entry and exit threshold combinations, with low trading frequency and a hit ratio of zero. This suggests that the pairs trading strategy struggles in very short time frames, potentially due to market noise or insufficient spread deviation.
- By contrast, in the 6-month period, the strategy becomes more profitable with a 1.0 entry threshold, yielding the highest annualized return (23.47%) and Sharpe ratio (1.12). However, more conservative entry points (like 1.5 SD) lead to negative returns. The trading frequency is much higher in this period (~25 trades), suggesting that the strategy finds more opportunities over a slightly longer horizon.

5.2.2. Medium Time Horizons (9 months to 1 year):

- In the 9-month backtest, the 1.0 entry threshold continues to deliver the best performance, with annualized returns of 33.5% and an impressive Sharpe ratio of 1.83. This period also shows high trading frequency (~33.7 trades/year) and a strong hit ratio of 91.67%, indicating that the strategy is highly effective in identifying profitable trades over a medium-term horizon.
- Over the 1-year period, the results are similar, with the 1.0 entry threshold maintaining a 22.5% annualized return and Sharpe ratio of 1.33. The hit ratio (93.33%) and trading frequency (31.5 trades/year) continue to support the strategy's ability to capture deviations effectively.

5.2.3. Longer Time Horizons (2 years to 3 years):

- Over longer horizons, the performance stabilizes. In the 2-year period, a 1.5 entry threshold provides the highest annualized return (3.26%) and a balanced Sharpe ratio (0.67), with a moderate trading frequency (~7.3 trades/year). The 1.0 entry threshold shows much lower returns (~2.1%), likely due to the increased noise in long-term spread deviations.
- In the 3-year period, the 1.5 entry threshold remains the optimal strategy, offering 4.58% returns with a Sharpe ratio of 0.87 and solid risk control (low volatility and drawdown). However, the 2.0 entry threshold yields slightly lower returns (4.28%) but with better risk

management (higher Sharpe ratio ~1.08 and zero drawdowns), suggesting that a more conservative approach works better over extended periods.

5.2.4 Summary

In summary, the 1.0 entry threshold performs best in shorter to medium time frames (up to 1 year), delivering the highest returns and a strong hit ratio. However, for longer horizons (2 to 3 years), a more conservative 1.5 entry threshold strikes a better balance between risk and reward, with lower volatility and consistent profitability. The strategy's performance improves as the trading period increases, but it is important to select entry and exit thresholds that match the desired time horizon and risk tolerance.

VI. Conclusion

This paper explored the application of pairs trading based on the mean-reverting Ornstein-Uhlenbeck (OU) process, focusing on two highly correlated assets—SHEL and BP—within the energy sector. The strategy aimed to capitalize on temporary deviations in the price spread between the assets, leveraging a statistical arbitrage framework. Using a robust modeling process, the strategy employed maximum likelihood estimation (MLE) to optimize hedge ratios and identify profitable trading opportunities.

Through multiple tests on training and out-of-sample test datasets, the results demonstrated that **entry thresholds** played a critical role in determining the profitability and risk of the strategy. Higher entry thresholds led to lower volatility and drawdown, but also resulted in fewer trading opportunities and lower returns. The **exit thresholds**, on the other hand, had minimal impact on performance, indicating that entry points are more significant for maximizing the effectiveness of pairs trading strategies.

The 1.0 standard deviation entry threshold delivers the highest returns and frequency of trades in shorter time frames (up to 1 year), making it an optimal choice for traders seeking higher profitability in shorter windows, despite the higher associated risks. For longer-term strategies, the 1.5 entry threshold provides a more balanced approach, offering stable returns, lower volatility, and better risk-adjusted performance.

The study demonstrates that the OU process, combined with a well-tuned pairs trading strategy, can generate consistent returns, particularly when selecting appropriate entry and exit thresholds for the given time horizon. This research contributes valuable insights into optimizing statistical arbitrage strategies using mean reversion in quantitative finance.

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